

### Problem Set 3

**# 1:** Suppose company A stock is trading for \$80/share on the NYSE but is also trading on the OTC markets for \$72/share. What steps could you take to take advantage of this?

**# 2:** Consider a 1 period model with 2 states of the world at time 1. We have 3 stocks, A,B, and C with prices at time 0

$$P_A(0) = \$100$$

$$P_B(0) = \$50$$

$$P_C(0) = \$220$$

and prices in the the two states of the world

	State I	State II
$P_A(1)$	$= \$60$	$\$250$
$P_B(1)$	$= \$80$	$\$40$
$P_C(1)$	$= \$200$	$\$540$

Find an arbitrage portfolio involving these stocks.

**# 3:** Suppose the spot (current market) price for Brent crude oil is \$64/barrel. Suppose the current 1 year forward price for Brent crude is \$67/barrel, i.e. it is possible now to enter a contract (without payment) to sell or buy Brent crude in 1 year at a price of \$67/barrel. Suppose the current risk free, continuously compounded, interest rate for a 1 year term is  $r = 3\%$ . Is there an arbitrage opportunity? Ignore any storage costs or convenience yields.

**Hint:** What happens if you borrow \$64 at the risk free rate, buy a barrel of oil, and enter this forward contract as seller of oil?

# 4: Consider a 1 period model with 3 states of the world at time 1 and 4 assets with prices at time 0

$$\begin{aligned}P_A(0) &= \$15 \\P_B(0) &= \$10 \\P_C(0) &= \$14 \\P_D(0) &= \$10\end{aligned}$$

and prices at time 1 in the the three states of the world

	State I	State II	State III
$P_A(1) =$	\$20	\$10	\$40
$P_B(1) =$	\$20	\$12	\$8
$P_C(1) =$	\$20	\$38	\$12
$P_D(1) =$	\$12	\$12	\$12

Find an arbitrage portfolio involving these assets.

# 5: Justify the 2 extended versions of the Law of One Price stated in the lecture. We recall these extensions here.

**Law of One Price Extension #1:** *If two securities, with certainty, pay the exact same cash flows then they should have the same price at any time during their lives.*

**Hint:** Try applying the Law of One Price to each individual cash flow.

**Law of One Price Extension #2:** *If  $A$  and  $B$  are 2 assets with prices  $P_A(t)$  and  $P_B(t)$  and if at some time  $T > 0$  we can say*

$$P_A(T) \geq P_B(T)$$

*with probability 1, then we must have*

$$P_A(0) \geq P_B(0)$$

**Hint:** Proceed with the same argument used for the original version of the Law of One Price. Assume that instead

$$P_A(0) < P_B(0)$$

You can then construct an arbitrage portfolio by shorting  $B$  and taking a long position in  $A$ .