Problem Set 8

**# 1:** In the example from the first 1 step binomial model lecture, show that the riskless portfolio that was constructed, with value

$$V_1 = \Delta S_1 - D_1$$

has the same value in both states of the world at time 1. Explain the argument from the Law of One Price that implies that at time 0

$$V_0 = \frac{16}{1.08}$$

**# 2:** In the 1 step binomial model, work through arbitrage argument explicitly that rules out (in the absence of arbitrage) either of the inequalities $1 + r < d < u$ or $d < u < 1 + r$. Construct the arbitrage in the cases a) $r = 0.05$, $d = 1.1$, $u = 1.2$, and b) $r = 0.1$, $d = 0.9$, $u = 1.05$.

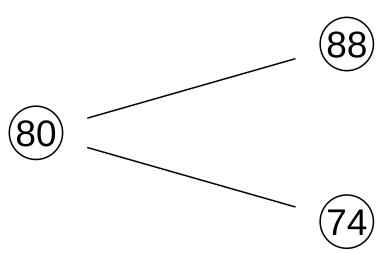
**# 3:** Verify that the delta hedged portfolio in the binomial model takes the same value in both states of the world at time 1 in the general case.

**# 4:** Prove that the risk neutral probabilities $\tilde{P} = (\tilde{p}, \tilde{q})$ make a probability distribution by confirming that $\tilde{p} + \tilde{q} = 1$.

**# 5:** Explain why, in the binomial model, $1 + r$ is the gross return of the riskless asset. Then show with a direct computation that

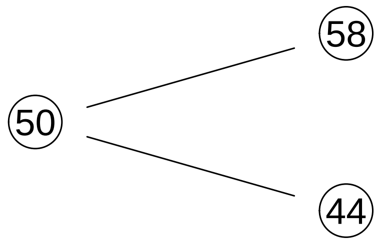
$$1 + r = E^{\tilde{P}}\left[\frac{S_1}{S_0}\right] = E^{\tilde{P}}[\text{Gross Return of } S]$$

**# 6:** Consider the following 1 step binomial model of a stock, with values

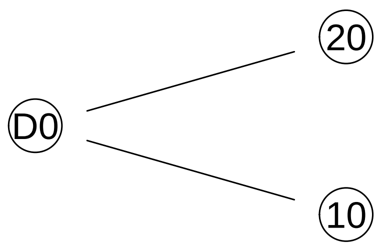



Price a put on this stock, expiring at time 1 and with strike price \$82. Assume the risk free interest rate is 5%.

# **7:** Consider a 1 step binomial model for a stock with prices given as in the following diagram:




Suppose the risk free rate is 3%. Calculate the up and down jump factors $u$ and $d$. Verify that the arbitrage conditions between $u$, $d$ and $r$ are satisfied. Calculate the risk neutral probabilities $\tilde{p}$ and $\tilde{q}$. Consider a contingent derivative in the model with prices




Calculate $D_0$, the fair price (arbitrage price) of the derivative at time 0. Do this using 2 different procedures: 1) using a riskless portfolio, 2) by computing a risk neutral expectation, and ensure that your 2 prices are the same.