Problem Set 6

 # 1: (a) Consider the simple random walk generated from flipping a fair coin, explained in the lecture. Show that if the experiment is carried out for 5 coin flips there are 32 possible paths. Show that the probability of any one of these paths being the realized path is $\frac{1}{32}$.

(b) Now suppose that the coin is not fair, but instead that the probability of heads is p with $0 < p < 1$. What it the probablity of the path that starts with a head and the alternates between heads and tails each flip? Work out the probability for any path. What is the critical statistic you need from the path to calculate this probablility?

(c) Carry out the same calculations as in (b), but for a random walk where the coin is flipped n times for any integer $n > 0$.

 # 2: (a) Consider the sequence of Brownian differences

$$\begin{aligned} & \{W(i+1) - W(i)\}_{i=0}^{\infty} \\ = & \{W(1) - W(0), W(2) - W(1), W(3) - W(2), \dots\} \end{aligned}$$

where $W(t)$ is Brownian motion.

(i) Calculate the autocorrelation function of this time series.

(ii) Would you expect this series to exhibit volatility clustering?

(iii) Would you expect this series to have fat tails?

(b) Answer questions (i)-(iii) from part (a) for the series of overlapping Brownian differences

$$\begin{aligned} & \{W(i+2) - W(i)\}_{i=0}^{\infty} \\ = & \{W(2) - W(0), W(3) - W(1), W(4) - W(2), \dots\} \end{aligned}$$

Hint: For part (b)(i) it is helpful to note that a difference like

$$W(2) - W(0)$$

can be written as

$$\begin{aligned} & W(2) - W(1) + W(1) - W(0) \\ = & (W(2) - W(1)) + (W(1) - W(0)) \end{aligned}$$

which writes the difference as a sum of 2 independent normal random variables.