Interest Rate Summary

Future Values

Let $P(t)$ be the future value, at time t, of an investment of X made now $t = 0$. Let $r(t)$ be the interest rate for term t. $P(t)$ is given by the following expressions under the different compounding conventions for r.

Annual Compounding:

$$P(t) = (1 + r(t))^t X$$

Periodic Compounding with Frequency m:

$$P(t) = \left(1 + \frac{r(t)}{m}\right)^{mt} X$$

Continuous Compounding:

$$P(t) = e^{r(t)t} X$$

Present or Discounted Values

Let PV be the present value at time 0 of a payment of X made at time $t > 0$. Let $r(t)$ be the interest rate for term t. Under different compounding conventions for $r(t)$ we have for

Annual Compounding:

$$PV = \frac{X}{(1 + r(t))^t}$$

Periodic Compounding with Frequency m:

$$PV = \frac{X}{\left(1 + \frac{r(t)}{m}\right)^{mt}}$$

Continuous Compounding:

$$PV = e^{-r(t)t} X$$

Discount Factors

The discount factor $d(t)$ is the present value of \$1 paid at time t. It can be expressed in terms of different interest rates as follows:

Annual Compounding:

$$d(t) = \frac{1}{(1 + r(t))^t}$$

Periodic Compounding with Frequency m:

$$d(t) = \frac{1}{\left(1 + \frac{r(t)}{m}\right)^{mt}}$$

Continuous Compounding:

$$d(t) = e^{-r(t)t}$$

Interest Rate Conversions

To make conversions between different interest rates, equate the future value of \$1 as calculated with 2 different interest rates, to convert between them.

Periodic Compounding Conversions:

Let $r_m(t)$ and $r_k(t)$ be periodically compounded interest rates with frequencies m and k respectively for a term t. Equating the future value of \$1 at time t calculated with the 2 rates gives

$$\left(1 + \frac{r_m(t)}{m}\right)^{mt} = \left(1 + \frac{r_k(t)}{k}\right)^{kt}$$

Solving this equation for $r_m(t)$ gives

$$r_m(t) = m\left(1 + \frac{r_k(t)}{k}\right)^{k/m} - m$$

Continuous to/from Periodic Conversions:

Letting $r_c(t)$ be a continuously compounded interest rate with term t and $r_m(t)$ be a periodically compounded interest rate with frequency m, then equating the future value of \$1 under the two rates gives

$$e^{r_c(t)t} = \left(1 + \frac{r_m(t)}{m}\right)^{mt}.$$

We may solve this for $r_c(t)$ to get

$$r_c(t) = m \log\left(1 + \frac{r_m(t)}{m}\right)$$

as the relation between the two rates.