Computational Problem Set 3

 # 1: We'll start by using the options module for some simple option calculations.

(a) An excellent exercise to start with is to check calculations that have been done in lecture and also in problem sets. These resources are excellent tools for checking your work.

(i) Redo the example from the delta hedging lecture using the options module. For convenience, here is the specification of that problem:
Suppose we have a 6 month call on a stock currently trading at \$60 with a volatility of 20%. Suppose the strike price of the call is \$50 and the risk free rate is 5%. What is the delta of the call? Suppose we have a call position on 200 shares of this stock. What position in the underlying stock must we take to have a delta neutral position? What is the P&L of this position 1 month later if the stock now has the same volatility and a price of \$70? \$50?

(ii) Redo problem 5 from problem set 10. The problem is restated here:
Suppose you hold a position of 50 call options on a stock currently trading at \$110 with a volatility of 15%. Suppose the calls expire in 1 year and have a strike price of \$105. Suppose the risk free interest rate is 4%. What position must you take alongside your option position so as to have a total delta neutral position? Suppose you take this position and hold it for 3 months. What is your P&L if the stock's volatililty has not changed and it is then trading at \$100? \$120?
Notice that in both scenarios, the position lost a bit of money, in spite of the fact that it is delta hedged? Do you have any theories as to why that might be? Redo the entire exercise, except for 1 detail: Suppose the proposed moves in the stock price happen 1 month after entering the position, and check the P&L for both scenarios at that time. Now calculate the theta of the initial position and consider all these results in light of that.

(b) For the following option positions, and given market data, calculate the option premium as well as the Greeks (delta, gamma, vega, theta). Assume the the spot price of the underlying is \$110, and that the interest rate is 3% in all scenarios. Also, for each option, assuming that the position is on 100 shares of the underlying stock, determine what position you need to take in the underlying in order to have a delta neutral position. In order to get oriented to the options module, I recommend carrying out these calculations 2 ways: (i) by calling the pricing functions (like BSCall, BSCall_Delta etc) directly, and (ii) by instantiating an option object. As part of (ii) you should also make plots of the option price and the greeks.

Option Type	Strike	Expiration	Implied Vol
Call	$100	3 months	20%
Call	$125	1 month	30%
Call	$110	2 months	15%
Put	$100	2 months	20%
Put	$130	6 weeks	35%

 # 2: In this exercise we will consider market positions consisting of a single option.

(a) Consider a portfolio consisting of 1 long call position. In particular, we do not reduce the delta by adding a position on the underlying. This is a case where we are seeking the delta exposure, and this represents a directional trade on the underlying. Assume that the underlying stock is trading for $160 per share and you enter a long call position on 100 shares at the money and expiring in 2 months. Also assume that the implied volatility for this option is 15% and that the risk free rate is 6% for all purposes.

(i) How much does it cost to establish this option position? Compare this with the cost to take the equivalent position in outright share purchases, and note the leverage offered by options. Can you see the advantages of options from an investor's point of view? What might be some of the disadvantages?

(ii) Calculate the delta and also plot the option price as a function of the underlying price. What is the position (long or short) on the underlying? Use the delta to estimate the P&L of the position if the share price increases by $10 in 1 month? Check how good your estimate is by fully repricing the option. Do the same under the scenario that the stock declines by $10. Calculate an updated delta for each of these positions. What considerations would you take into account on deciding what to do with the position at this juncture?

(iii) Calculate the theta on the opening date of the position. On the basis of this number, estimate what will happen if you simply hold this position and the share price doesn't move at all for a month. Check your prediction by fully repricing the option one month later. If you find your prediction is a bit off, recall that the theta gives the time decay of the option at a particular point in time. Try calculating the theta for a few times between 0 and 1 month and check if that is more consistent with how the price changed. What are the implications when considering whether to hold the position?

(iv) Calculate the vega on the opening date. What additional considerations does this suggest an investor should think about?

(v) Consider, instead, taking a position in an out of the money option. Suppose the strike price of the option you buy is \$170. Consider the advantages and disadvanteges of this position in comparison to the at the money option from the point of view of the payoff, the price, the delta, theta, and gamma. Use both numerical values and plots in your considerations.

(vi) Now consider entering into an in the money option with a strike price of \$150. Answer the same questions as in (v).

(b) Now, consider the same circumstances as in (a) but now suppose you enter into a put position instead of a call? What fundamental difference in outlook can we infer from an investor's choice to enter into this position? Carry out the same analyses as in (a) for this put position.

 # 3: In this problem we study volatility trading. Recall the example treated in lecture, reproduced here for convenience:

Suppose we have a stock currently trading at \$100. Suppose 1 year at the money call options are currently priced with a volatility of 10%. Suppose the risk free interest rate is 5%. Suppose we have a forecast that the volatility of this stock will increase over the next 6 months, but we have no view on the stock price itself (it may go up or down). How do we implement an option position on 1000 shares to profit from our forecast? Now suppose in 6 months the volatility of the stock has increased to 40%, and consider 3 different scenarios for the stock price in 6 months: \$90, \$100, and \$110. Check the P&L for the position in each case. For simplicity assume we did not change the delta hedge.

(a) (i) As a twist on the procedure carried out in class, take the view of a naive investor who does not know about delta neutrality or the importance of monitoring risk when trading. Take a positition purely in options and calculate the P&L of your position. To decide which option to use, try a range of strikes at levels that are at the money, in the money, and out of the money. For each option, plot the vega, and then, recalling that the vega is the senstivity of the position to implied volatility, choose a strike for which you will get the maximum benefit if the volatility goes up. Now compute the P&L of your position for the scenarios proposed in the problem, and compare to the results we found in lecture. Do these seem like sensible results?

(ii) Now, carry through the same exercise, but now delta hedging the position. For this task, you are really just reimplementing the calculations done in lecture, so the main point here is to get practice using the Python tools. Make sure you check your results against what we got in the lecture. You might want to do this exercise before (i).

(iii) In fact, the problem as stated is not very realistic. A trader would rarely if ever hold a volatility position for 6 months. For one thing, in this particular case, time decay of the option would severely cripple the profitability of the trade. For the strategy just described, try to estimate how much the position has lost, due to time decay of the option alone.

(iv) For a more realistic example, suppose that instead we implement the described volatility position but with an investment horizon of 3 days. And suppose the three scenarios described are realized within 3 trading days. Now calculate the P&L for each one, and compare.

(b) Now we will consider taking the opposite position: shorting (ie selling) volatility. It is very common when companies report their quarterly earnings for implied volatility to peak just before the report is released, and immediately crash as soon as the report is public, when the results turn out to be within expectations. Consider how to set up a portfolio to take advantage of this phenomenon. Anticipating a drop in implied volatility the day the report is published, we want to set up a portfolio that is short implied volatility, ie we want to sell volatility. This means we need to sell options.

To be specific, suppose that company AAA's stock is currently trading at \$80 per share. For simplicity, suppose that the implied volatility for 1 month options on company AAA's stock is 35% across all strikes. Also, suppose the risk free rate is 6%. In order to profit from a drop in volatility, that is, to short volatility, we need to sell options. The portfolio we will construct in this part is the exact reverse of what we built in part (a). We will sell options (so our position will have a negative vega), and then, in order to delta hedge, we will need to take a long position in the stock.

(i) Plot the vega of a few different 1 month options, with different strikes, and consider what would be a good strike price to use in order to set up a volatility play. Choose 3 different strikes, one for each of at the money, in the money, and out of the money. Suppose you enter into a short position (ie

sell) options on 500 shares of company AAA. In order to isolate the effects of volatility, you should make your total position delta neutral. To do this, calculate the delta of the option position to determine how much offsetting stock you must have to offset your exposure to the underlying. The only difference between this and a long option position is now you will buy the stock, not short it.
Having now, on the day before the company's report, set up a delta neutral option position on 500 shares of company AAA which is short implied volatility, hold the position until after the report is released, and consider what the P&L of your position is under a number of different scenarios. First, assume that, as expected, the volatility drops to 20% after the report and that the stock price does not change. Compute the P&L of your 3 positions. Can you explain the results you observe?
(ii) Now, for the one of your 3 positions that was the most profitable consider what happens with the same volatility change, but if the share price jumps to \$81, \$83, \$85, \$90, \$79, \$77, \$75, \$70. Can you reconcile these results with the fact that this was set up as a delta neutral position?