Problem Set 1

 # 1: Compute the future value of \$50,000 invested at an annually compounded interest rate of 6% for 8 years. Do the same for a semiannually compounded interest rate of 5.5% and a continuously compounded interest rate of 5%. In all cases, calculate the gross and net returns. All interest rates are annual rates.

 # 2: Calculate the present value of a contract that pays \$100,000 every 6 months for the next year and then \$200,000 every year for the 2 years following that. Assume the prevailing semiannually compounded risk free interest rates have the following term structure:

$$\begin{aligned} r(0.5) &= 2\% \\ r(1) &= 3\% \\ r(2) &= 3.5\% \\ r(3) &= 4.2\% \end{aligned}$$

 # 3: Price a 2 year bond on its issue date with a \$10,000 face value and paying a 6% coupon, with semiannual payments. Assume the semiannually compounded risk free rate is 4% with a flat term structure.

 # 4: Prove the formula for converting from a periodically compounded interest rate with a compounding frequency of k to one with a compounding frequency of m,

$$r_m(t) = m\left(1 + \frac{r_k(t)}{k}\right)^{k/m} - m$$

from the condition that the future value of \$1 is the same under both interest rates.

 # 5: Suppose the semiannually compounded interest rate is 7%. Compute the equivalent annually compounded and continuously compounded interest rate.

 # 6: Suppose we invest \$100,000 at a semiannually compounded interest rate of 6% for 1 year. What are the gross and net returns? Do the same if this was a monthly compounded rate and a continuously compounded rate. Compare the three net returns to the interest rate. Can you explain the ordering of these 4 quantities?

 # 7: Assuming the following term structure of discount factors

$$\begin{aligned} d(1) &= 0.966 \\ d(2) &= 0.916 \\ d(3) &= 0.84 \\ d(4) &= 0.763 \end{aligned}$$

Compute the corresponding term structure of annually compounded and continuously compounded interest rates. Compute the price of a 4 year bond with a \$100,000 face value paying a 5% coupon making annual payments.

 # 8: (a) Construct a spot curve out to 2.5 years using the following data for 5 bonds.

Bond	Face Value	Coupon	Maturity	Price
6 month T-bill (no coupons)	\$10,000	0%	6 months	\$9910
1 year semiannual coupon bond	\$10,000	7%	1 year	\$10,050
1.5 year semiannual coupon bond	\$50,000	4%	1.5 years	\$46,500
2 year semiannual coupon bond	\$100,000	5%	2 years	\$96,500
2.5 year semiannual coupon bond	\$100,000	7%	2.5 years	\$98,000

(b) Use the curve you built in part (a) to price a semiannual coupon bond maturing in 15 months, with a \$10,000 face value and a 3% coupon (Use piecewise linear interpolation for the curve).

 # 9: (a) Iterate the defining formula for the SONIA compounded index $I(t)$ on pg. 5 of the lecture to justify the formula

$$I(t+m) = I(t) \times \left(1 + \frac{S(t)\tau(t)}{365}\right) \times \cdots \times \left(1 + \frac{S(t+m-1)\tau(t+m-1)}{365}\right)$$

Hint: Start with small values $m = 1, 2, 3$. Note that $m = 1$ is just the defining formula for $I(t)$.

For instance, the first 2 cases of the defining formula are

$$I(t+1) = I(t) \times \left(1 + \frac{S(t)\tau(t)}{365}\right)$$
$$I(t+2) = I(t+1) \times \left(1 + \frac{S(t+1)\tau(t+1)}{365}\right)$$

Substitution of the first formula into the second yields the identity for the case $m = 2$. Ultimately, this can be proved by induction by assuming it is true form m and using the defining formula to extend that to the case $m+1$.

(b) Demonstrate that SONIA compounded in arrears over a period is the simple term rate which, it were earned over that period would be the same return earned by investing every day at the daily SONIA rate, compounding daily.

(c) Download daily SONIA interest rates from the Bank of England website (www.bankofengland.co.uk/markets/sonia-benchmark) to calculate SONIA compounded in arrears on January 17, 2019 For the 1 week term ending on that date.