

Problem Set 9

1: Verify the 2 and 3 trial binomial probabilities, that were used to calculate a fair derivative price in the 2 and 3 step binomial models:

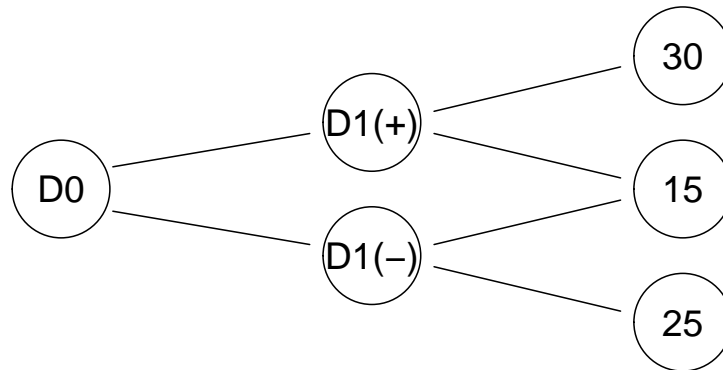
$$\begin{aligned}\text{bin}(0; 2, \tilde{p}) &= \tilde{q}^2 \\ \text{bin}(1; 2, \tilde{p}) &= 2\tilde{p}\tilde{q} \\ \text{bin}(2; 2, \tilde{p}) &= \tilde{p}^2 \\ \text{bin}(0; 3, \tilde{p}) &= \tilde{q}^3 \\ \text{bin}(1; 3, \tilde{p}) &= 3\tilde{p}\tilde{q}^2 \\ \text{bin}(2; 3, \tilde{p}) &= 3\tilde{p}^2\tilde{q} \\ \text{bin}(3; 3, \tilde{p}) &= \tilde{p}^3\end{aligned}$$

where $\tilde{q} = 1 - \tilde{p}$.

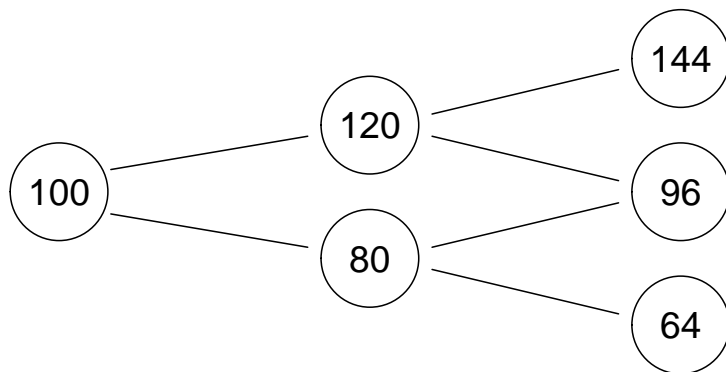
2: Derive the risk neutral expectation formula for the price of a derivative in the 3 step binomial model

$$D_0 = E^{\text{bin}(\cdot; 3, \tilde{p})} \left[\frac{D_3}{(1+r)^3} \right]$$

3: Consider a binomial model for an underlying asset for which the up and down jump factors are $u = 1.3$ and $d = 0.9$. Assume the risk free interest rate is $r = 4\%$. Price the derivative whose payoff is as specified in the following diagram:



4: Consider a 2 step binomial model, and a stock represented in this model as illustrated in the following binomial diagram:



Assume the risk free interest rate is 7%. Price the following derivatives in this binomial model: A call with strike price \$110, a put with strike \$110, and a forward contract with forward price \$110, all with the given stock as the underlying asset, and all expiring at time 2. Does put-call parity hold for these instruments?