Computational Problem Set 2

 # 1: (a) Use the curve factory function in the fixed income module to redo the LIBOR bootstrapping exercises from class and from Problem Set 5. For reference, the data from problem set 5 is displayed below:

Type	Tenor	Rate/Price
Deposits	1W	2.0
	1M	2.2
	2M	2.27
	3M	2.36
Futures	6M	97.4
	9M	97.0
Swaps	1Y	3.0
	2Y	3.6
	3Y	3.95
	4Y	4.2

(b) Check that the program is correctly constructing the curve by recalculating the benchmark rates used to construct the curve for both the curves you built in (a).

(c) Using the curve you built in (a), price an FRA (from the lender's perspective), expiring in 3 years, on a 6 month $100,000 deposit with a contract rate of 2%. What is the fair contract rate for such an FRA.

(d) Using the curve in (a) determine the fair swap rate for a 2 year swap making quarterly payments. Value such a swap if the contract swap rate is 3%.

 # 2: Test the spot rate and forward rate functions for curve objects for consistency. The basic relationship for testing (for a compounding frequency of m) is

$$\left(1+\frac{L(0,T_1)}{m}\right)^{mT_1}\left(1+\frac{L(0,T_1,T_2)}{m}\right)^{m(T_2-T_2)}=\left(1+\frac{L(0,T_2)}{m}\right)^{mT_2}. \tag{1}$$

Here $L(0,T)$ is the spot interest rate for maturity T and $L(0,T_1,T_2)$ is the forward rate spanning a term from T_1 to T_2.

You can use any combination of tenors you desire, and it would be instructive to try a few, but to get started, try testing a 5 year spot rate against a 2 year spot rate and a rate 2 years forward maturing in 5 years. That is test the above relationship for $L(0,5)$ against $L(0,2)$ and $L(0,2,5)$. Do this for annual, semiannual, and monthly compounding.

You should also work out relationships similar to (1) for simple compounding and continuous compounding and do similar tests for these compounding conventions.

You can carry out these tests for any curve you want, but I will give a suggested curve built from the following continuously compounded rates:

Tenor (years)	Spot Rate
0.5	1.65
1.0	2.0
2.0	2.7
3.0	3.1
5.0	3.85
7.0	4.2
10.0	4.3

 # 3: Explore forward rates as "break even" rates. Suppose you hold a long bond position in a 5 year bond, with a 6% coupon and annual coupon payments. Consider the following 2 strategies for this 1 bond portfolio for the following year:

Bond Strategy 1: Hold the bond for 1 year.

Bond Strategy 2: Sell the bond at its current market price and invest the proceeds at the prevailing spot rate for 1 year.

Suppose the present yield curve appropriate for pricing this bound is as determined by the following (continuously compounded) spot rates:

Tenor (years)	Spot Rate
1.0	1.5
2.0	2.1
3.0	2.5
5.0	2.8
7.0	3.1
10.0	3.3

Now suppose that in 1 year it turns out that the realized spot yield curve is characterized by the following rates:

Tenor (years)	Spot Rate
1.0	1.7
2.0	2.1
4.0	2.4
6.0	2.7
9.0	3.0

Compute the 1 year forward interest rates corresponding to the coupon payment dates of the bond implied by the yield curve on the original date, and compare them to the corresponding realized spot rates 1 year hence. Based on this comparison, theorize which of the 2 proposed bond strategies should be more profitable. Test your theory out by calculating the value of the portfolio in 1 years time under the two different strategies. (Important: make sure to take into account that in 1 years time you will have just received a coupon payment).

Now, suppose that instead, the prevailing yield curve in 1 year is characterized by the following spot rates:

Tenor (years)	Spot Rate
1.0	3.0
2.0	3.4
4.0	3.7
6.0	4.0
9.0	4.2

Carry out the same analysis as for the previous hypothesis.