Problem Set 2: Solutions

 # 1:

a) The allocation of +200 to IBM stock indicates a long position in 200 shares of IBM.

The allocation of -1000 invested at the Euro risk free rate r_f indicates a debt of 1000 Euros borrowed at time 0 at this risk free rate.

b) The allocation of -650 indicates the portfolio is short 650 shares of GE stock.

The second term represents a holding of cash equivalent to $9000 invested at the USD risk free rate at time 0.

c) The first 2 terms represent long positions in 3000 shares of IBM and 5000 shares of GE.

The third term represents a debt of 5000 Euros borrowed at time 0 at the Euro risk free rate.

The last term represents a debt $12000 borroed at time 0 at the USD risk free rate.

d) The first term represents a long position in 8000 shares of GE.

The second term represents being short 2000 shares of IBM.

The third term represents a holding of Euros equivalent to 5000 Euros invested at the Euro risk free rate at time 0.

The last term represents a US dollar debt equivalent to $15,000 borrowed at the USD risk free rate at time 0.

 # 2:

We are given that we start with an initial allocation of Euros worth $100,000. We must first convert this to Euro. We are given an initial Euro/USD exchange rate of $1.25/Euro, so, in Euros, this is

$$\frac{100,000}{1.25} = 80,000 \text{ Euros}$$

The value function for this holding of Euros, plus the holding of \$50,000 of US cash, invested at the corresonding risk free rates is

$$V(t) = 80,000e^{r_f t}E(t) + 50,000e^{r_d t}$$

where $E(t)$ is the Euro/USD exchange rate. Since we are given $r_f = 3\% = 0.03$ and $r_d = 6\% = 0.06$, we can write

$$V(t) = 80,000e^{0.03t}E(t) + 50,000e^{0.06t}$$

To compute $V(5)$ we are given that $E(5) = 1.18$ so we may evaluate

$$\begin{aligned} V(5) &= 80,000e^{0.03(5)}E(5) + 50,000e^{0.06(5)} \\ &= 80,000e^{0.03(5)}(1.18) + 50,000e^{0.06(5)} \\ &= \$177,170 \end{aligned}$$

 # 3:

We let $S_A(t), S_B(t)$ and $S_C(t)$ denote the share prices of companies A, B, and C respectively. The value function, given the stated allocations, is

$$V(t) = 1000S_A(t) + 1000S_B(t) - 2000S_C(t).$$

With the given share prices of $S_A(t) = \$45$, $S_B(t) = \$60$, and $S_C(t) = \$55$, the portfolio value is

$$\begin{aligned} V(t) &= 1000(45) + 1000(60) - 2000(55) \\ &= -\$5000. \end{aligned}$$

In order to reduce the short position in C from 2000 shares to 500 shares, we must purchase 1500 shares of C on the market and close out this much of the short position. Purchasing these shares will cost

$$1500 \times \$55 = \$82,500.$$

If we do not want to contribute any further funds to the portfolio the only way to raise this money is by selling off some of the A and B stocks. We want to liquidate x shares of A and y shares of B to raise \$82,500, so x and y solve the equation

$$x(45) + y(60) = 82,500$$

where $0 \leq x, y \leq 1000$. There are many ways to do this. For simplicity, lets say we sell off all of the B stock, so choose $y = 1000$. Then x solves the equation

$$x(45) + 60,000 = 82,500.$$

One may then solve this equation for x to get $x = 500$. Thus one solution is to sell 1000 shares of B, 500 shares of A, and use the proceeds to buy 1500 shares of C, and close out 1500 short positions in shares of C.

 # 4:

We can take a short position on the JPY/USD exchange rate by borrowing Japanese yen (this assumes that USD is our home currency).

Now introduce the following notation:

$$\begin{aligned} r_d &= \text{USD risk free interest rate} \\ r_f &= \text{JPY risk free interest rate} \\ J(t) &= \text{JPY/USD exchange rate} \\ S_{\text{XYZ}}(t) &= \text{XYZ share price.} \end{aligned}$$

With this notation, and the allocations we have been given, the value of the portfolio is

$$V(t) = -2,500,000 e^{r_f t} J(t) + 30,000 e^{r_d t} - 500 S_{\text{XYZ}}(t).$$

We are given the following numerical data:

$$\begin{aligned} r_d &= 7\% = 0.07 \\ r_f &= 5\% = 0.05 \\ J(2) &= 0.01 \\ S_{\text{XYZ}}(2) &= \$12. \end{aligned}$$

With this data, the value of the portfolio at time 2 is

$$\begin{aligned} V(2) &= -2,500,000 e^{0.05(2)}(0.01) + 30,000 e^{0.07(2)} - 500(12) \\ &= \$879 \end{aligned}$$

 # 5: The holding of the Eurozone stock would be represented in a portfolio value function by the term

$$200 S_{\mathrm{EZ}}(t) E(t).$$

With the given allocations, the value function is

$$V(t) = 200 S_{\mathrm{EZ}}(t) E(t) + 100 S_{\mathrm{IBM}}(t) - 12,000 e^{r_d t}$$

where $S_{\mathrm{IBM}}(t)$ is the share price of IBM and r_d is the USD risk free interest rate.

With the given data:

$$\begin{aligned} r_d &= 6\% = 0.06 \\ S_{\mathrm{EZ}}(3) &= 80 \text{ Euros} \\ E(3) &= \$1.28/\text{Euro} \\ S_{\mathrm{IBM}}(3) &= \$120 \end{aligned}$$

the value of the portfolio is

$$\begin{aligned} V(3) &= 200(80)(1.28) + 100(120) - 12,000 e^{0.06(3)} \\ &= \$18,113 \end{aligned}$$