

7A.

$$\text{Entropy}(S) = -P_{+} \log_2 P_{+} - P_{-} \log_2 P_{-}$$

$$\text{Entropy}(S) = -\left(\frac{4}{10} \log_2 \frac{4}{10} + \frac{6}{10} \log_2 \frac{6}{10}\right) = 0.9709 \quad \text{Total Entropy}$$

Age:

	Yes	No	Total
Young	2	2	4
Presbyopic	1	2	3
Prepresbyopic	1	2	3

Young calculation: $2/4 = 0.5 \quad 2/4 = 0.5$
 $-(0.5 \cdot \log_2(0.5) + 0.5 \log_2(0.5)) = 1.0$

Presbyopic: $P_{\text{yes}} = 1/3 = 0.333, \quad P_{\text{No}} = 2/3 = 0.667$

$$-(0.333 \cdot \log_2(0.333) + 0.667 \cdot \log_2(0.667)) = 0.9183$$

Prepresbyopic: $-(0.333 \cdot \log_2(0.333) + 0.667 \cdot \log_2(0.667)) = 0.9183$

Weighted Average Entropy Age: $\left(\frac{4}{10}\right) \cdot 1 + \left(\frac{3}{10}\right) \cdot 0.9183 + \left(\frac{3}{10}\right) \cdot 0.9183$
 $= 0.951$

Information Gain:

$$H(S) - H(\text{Age}) = 0.9709 - 0.951 = \boxed{0.02}$$

Spectacle Prescription

$$H(\text{Myope}) = -\left(\frac{4}{8}\right)\log_2\left(\frac{4}{8}\right) - \left(\frac{4}{8}\right)\log_2\left(\frac{4}{8}\right) = 1.0$$

$$H(\text{Hypermetrope}) = -\left(\frac{0}{2}\right)\log_2\left(\frac{0}{2}\right) - \left(\frac{2}{2}\right)\log_2\left(\frac{2}{2}\right) = 0$$

$$H(\text{spectacle prescription}) = \frac{8}{10} \cdot 1.0 + \frac{2}{10} \cdot 0 = 0.8$$

$$\text{Information Gain} = 0.9709 - 0.8 = \boxed{0.1709}$$

Astigmatism

Yes: 2 Yes, 2 NO

NO: 2 Yes, 4 NO

$$H(\text{Yes}) = -\left(\frac{2}{4}\log_2\frac{2}{4} + \frac{2}{4}\log_2\frac{2}{4}\right) = 1.0$$

$$H(\text{NO}) = -\left(\frac{2}{6}\log_2\frac{2}{6} + \frac{4}{6}\log_2\frac{4}{6}\right) = 0.918$$

$$\text{Weighted Entropy} = H(\text{Astigmatism}) = \left(\frac{4}{10} \cdot 1.0\right) + \left(\frac{6}{10} \cdot 0.918\right) = 0.951$$

$$\text{Information Gain} = 0.9709 - 0.951 = \boxed{0.0199}$$

Tear Production Rate

	Yes	No	Total
Reduced	1	5	6
Normal	3	1	4

$$H(\text{reduced}) = -\left(\frac{1}{6}\log_2\frac{1}{6} + \frac{5}{6}\log_2\frac{5}{6}\right) = 0.651$$

$$H(\text{normal}) = -\left(\frac{3}{4}\log_2\frac{3}{4} + \frac{1}{4}\log_2\frac{1}{4}\right) = 0.811$$

$$H(\text{tear production rate}) = \left(\frac{4}{10} \cdot 0.811\right) + \left(\frac{6}{10} \cdot 0.651\right) = 0.714$$

$$\text{Information Gain} = 0.9709 - 0.714 = \boxed{0.257}$$

Decision Tree

