

Quaternions

Quaternions are a number system that go beyond the complex number system in such a way that a complex number is represented by only one imaginary part while a quaternion number is represented by three imaginary parts and one real. A general example of a quaternion number is :- $w + xi + yj + zk$; where: w, x, y, z are real numbers and i, j, k are imaginary. They can also be described as a combination of a scalar and a vector where the real part is the scalar and the imaginary parts represent each component of a three dimensional vector.

In quaternions the i, j, k components have the magnitude of square root of one and follow the equation $i^2 = j^2 = k^2 = ijk = -1$.although their magnitude is same they are unit vectors each mutually perpendicular to one other. Mathematical operations like addition and subtraction of quaternions are similar to those of complex numbers: where the real and imaginary parts are added/subtracted separately. Quaternion multiplication on the other hand is noncommutative and associative. It follows a bunch of rules such as : $ij = k, jk = i, ki = j$. These are similar to cross product rules of unit vectors i, j, k in the real plane.

Quaternions are chiefly employed for rotation of vectors in 3D planes as they are more accurate and less ambiguous than euler angles. Euler angles despite being easier to visualize have the possibility of gimbal lock occurring. Gimbal lock occurs when one plane rotates by approximately 90 degrees which makes it parallel to one of the other two planes. This results in a loss of degree of freedom in 3D as both the planes start rotating together. Quaternions eliminate the possibility of gimbal lock making them more convenient to use. One drawback of quaternions is that they are difficult to visualize for humans when compared to euler angles.