Retain in Memory

Solar constant is the amount of solar energy emitted by the sun, falling per second per unit area of a surface held perpendicular to the rays of sun at a mean distance of the earth from the sun.

The value of solar constant is 1388 Js^{-1} m^{-2} = 1388 Wm^{-2}

Curiosity Questions

Q. 1. How has the Physics of heat been utilised by fire fighters ?

Ans. The fire fighters have to enter burning buildings to save lives and property. For this, the knowledge of Physics of heat for them plays an important role because

(*i*) knowing the types of fumes and radiations emitting from the present fire will help them to decide the type of spray (only water spray or fine mist spray) to be used to control the fire.

(*ii*) the type of clothes to be used during the fire fighting so that they are not harmed by fire.

(*iii*) the type of instruments to be used during spray and their placing to control the fire.

(*iv*) the best technique to be used to bring out the person trapped in fire.

Q. 2. Can temperature be assigned to a vacuum ?

Ans. According to classical physics, temperature is defined in terms of average kinetic energy of particles of the object. Since there are no particles in vacuum, hence temperature in a pure vacuum is undefined.

SOLVED EXAMPLES

TYPE I. TEMPERATURE SCALES

Formulae used

1. If T_C, T_F and T_K are temperature values of a body on Celsius scale, Fahrenheit scale and Kelvin scale, then

$$\frac{T_C - 0}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273{\cdot}15}{100}$$

2. If triple point of water is chosen as the reference point, then

$$T_K = 273{\cdot}16\left(\frac{P}{P_{tr}}\right)$$

where P is the pressure at unknown temperature T and P_{tr} is the pressure at triple point.

Units used. All temperatures are in degrees. P and P_{tr} are usually in cm. of mercury column.

Example 1 **At what temperature, if any, do the following pairs of scales give the same reading**

(a) Celsius and Fahrenheit ?

(b) Fahrenheit and Kelvin ?

(HP Board 2007)

Solution. (*a*) Let $T_C = T_F = \theta$

As, $$\frac{T_C - 0}{100} = \frac{T_F - 32}{180}$$

$$\therefore \quad \frac{\theta - 0}{100} = \frac{\theta - 32}{180} \qquad \therefore \quad \theta = -40°$$

(*b*) Let, $T_F = T_K = \theta$

As, $$\frac{T_F - 32}{180} = \frac{T_K - 273{\cdot}15}{100}$$

$$\therefore \quad \frac{\theta - 32}{180} = \frac{\theta - 273{\cdot}15}{100}; \quad \theta = 574{\cdot}6°$$

Example 2 **A Celsius and Farenhite thermometer are put in an oil bath. The reading on Farenhite thermometer is 3/2 times the reading on celsius thermometer. What is the temperature of bath on Celsius, Farenheit and Kelvin's scales.**

Solution. Let reading on Celsius scale be x. Therefore, reading on Farenheit scale = $3\,x/2$.

As, $$\frac{T_C - 0}{100} = \frac{T_F - 32}{180}$$

So, $$\frac{x}{5} = \frac{(3\,x/2) - 32}{9}$$

or $9x = 7.5x - 160$ or $1.5x = -160$

or $x = \frac{-160}{1.5} = -106.7°C$

Temperature on Celsius scale, $x = -106.7°C$

Temperature on Farenheit scale

$$= \frac{3}{2}x = \frac{3}{2}\left(-\frac{160}{1.5}\right) = -160°F$$

Temperature on Kelvin's scale $= x + 273.15$

$= -106.7 + 273.15 = 166.45$ K

Example 3 **A thermometer has wrong calibration. It records the melting point of ice – 5°C. It reads 55°C instead of 50°C. Find the temperature of boiling point of water on the given scale.**

Solution. Let n be the number of divisions between upper fixed point θ_2° and lower fixed point θ_1°.

Then $$\frac{C}{100} = \frac{\theta_2 - \theta_1}{n} \quad ...(i)$$

In first case : $\theta_2 = -5°C, C = 0°C$

$$\frac{0}{100} = \frac{-5-\theta_1}{n} \text{ or } -5-\theta_1 = 0 \text{ or } \theta_1 = -5°C$$

In second case, $\theta_2 = 55°C, C = 50°C$

Then $$\frac{50}{100} = \frac{55-\theta_1}{n} \text{ or } \frac{1}{2} = \frac{55-(-5)}{n} = \frac{60}{n}$$

or $n = 120$

We know that the boiling point of water on celsius scale is 100°C. From (*i*)

$$\frac{100}{100} = \frac{\theta_2 - (-5)}{120} \text{ or } \theta_2 = 115°C$$

Thus boiling point on faulty thermometer is 115°C.

TYPE II. THERMAL EXPANSION

Formulae used

1. (*i*) Coeff. of linear expansion $\alpha = \frac{\Delta L}{L(\Delta T)}$

(*ii*) Coeff. of area expansion $\beta = \frac{\Delta S}{S(\Delta T)}$

(*iii*) Coeff. of volume expansion $\gamma = \frac{\Delta V}{V(\Delta T)}$

2. $\beta = 2\alpha ; \gamma = 3\alpha$

3. In case of liquids, $\gamma_r = \gamma_a + \gamma_g$

4. Variation of density with temperature is given by

$$\rho' = \rho(1 - \gamma\,\Delta T)$$

5. Thermal stress $= Y\alpha\,\Delta T = Y\,\Delta L/L$

Units used

α, β, γ in K^{-1} ; L, ΔL in metre, S, ΔS in m^2 ; V, ΔV in m^3 ; ΔT in°C or K, ρ and ρ' in kg/m^3.

Example 4 **How much should the temperature of a brass rod be increased so as to increase its length by 1% ? Given α for brass is 0·00002 °C^{-1}.**

Solution. Here, $\Delta T = ?$; $\frac{\Delta L}{L} = \frac{1}{100}$;

$$\alpha = 0.00002\ °C^{-1}$$

As, $$\Delta L = \alpha L \Delta T$$

$$\therefore \quad \Delta T = \frac{\Delta L}{L\alpha} = \frac{1}{100 \times 0.00002}$$

$$= \frac{10^5}{2 \times 10^2} = 500°C$$

Example 5 **Railway lines are laid with gaps to allow for expansion. If the gap between steel rails 60 m long be 3·60 cm at 10°C, then at what temperature will the lines just touch ? Coefficient of linear expansion of steel = 11 × 10^{-6} °C^{-1}.**

Solution. We know that the rail expands in both the directions, so the gap between two rails will be filled by the expansion of half length of each rail. For this we take the expansion of one rail in one direction to fill the gap.

Here, $l = 60$ m ; $\Delta l = 3.60$ cm $= 3.6 \times 10^{-2}$ m ;

$\theta_1 = 10°C$, $\theta_2 = ?$; $\alpha = 11 \times 10^{-6}\ °C^{-1}$

$$\alpha = \frac{\Delta l}{l.(\theta_2 - \theta_1)} \text{ or } \theta_2 - \theta_1 = \frac{\Delta l}{l\alpha}$$

$$\text{or } \theta_2 = \theta_1 + \frac{\Delta l}{l\alpha} = 10 + \frac{3.60 \times 10^{-2}}{60 \times 11 \times 10^{-6}}$$

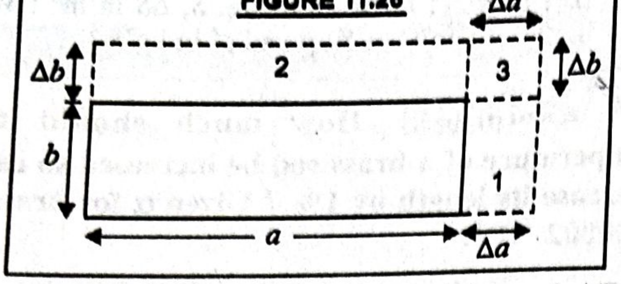
$$= 10 + 54.54 = 64.54°C$$

Example 6 **Show that the coefficient of area expansions $(\Delta A/A)/\Delta T$ of a rectangular sheet of the solid is twice its linear expansitivity α.**

(NCERT Solved Example)

Solution. Consider a rectangular sheet of a solid material of length a and breadth b as shown in Fig. 11.26. Area of sheet $A = ab$. When the temperature of sheet is raised by ΔT, let increase in length of sheet be Δa and increase in breadth of sheet be Δ b. If α is the coefficient of linear expansion, then

FIGURE 11.26



$$\Delta a = \alpha\, a\, \Delta T \quad \text{and} \quad \Delta b = \alpha\, b\, \Delta T$$

The increase in the area of sheet is

$$\Delta A = (a + \Delta a)(b + \Delta b) - ab$$
$$= a\,\Delta b + b\,\Delta a + \Delta a\,\Delta b$$
$$= a\,\alpha\, b\,\Delta T + b\,\alpha\, a\,\Delta T + \alpha^2 ab\,(\Delta T)^2$$
$$= \alpha\, ab\,\Delta T\,(2 + \alpha\,\Delta T)$$
$$= \alpha\, A\,\Delta T\,(2 + \alpha\,\Delta T)$$

Since, α is a small quantity, hence the product ΔT for small change in temperature is very very ll, can be neglected in comparison to 2.

$$\therefore \quad \Delta A = \alpha\, A\,\Delta T\, 2$$

or $$\left(\frac{\Delta A}{A\,\Delta T}\right) = 2\alpha \quad \text{(proved)}$$

Example 7 **A blacksmith fixes iron ring on im of the wooden wheel of a bullock cart. The neter of the rim and the ring are 5.243 m and l m respectively at 27°C. To what temperature ld the ring be heated so as to fit the rim of the l ? Coefficient of linear expansion of iron = × 10^{-5} K^{-1}.** **(NCERT Solved Example)**

Solution. Here, $L_{T_1} = 5{\cdot}231$ m ;

$L_{T_2} = 5{\cdot}243$ m ; $T_1 = 27$°C, $T_2 = ?$

As, $$\alpha = \frac{L_{T_2} - L_{T_1}}{L_{T_1}(T_2 - T_1)} \quad \therefore \quad T_2 - T_1 = \frac{L_{T_2} - L_{T_1}}{L_{T_1} \times \alpha}$$

r $$T_2 = \frac{L_{T_2} - L_{T_1}}{L_{T_1} \times \alpha} + T_1$$

$$= \frac{5{\cdot}243 - 5{\cdot}231}{5{\cdot}231 \times 1{\cdot}2 \times 10^{-5}} + 27$$

$$= 191{\cdot}1 + 27 = 218{\cdot}1 \approx \mathbf{218°C}$$

Example 8 **What should be the lengths of steel and copper rods at 0°C that the length of steel rod is 5 cm longer than copper at all temperature ? Given $\alpha_{Cu} = 1{\cdot}7 \times 10^{-5}$ °C^{-1} and $\alpha_{steel} = 1{\cdot}1 \times 10^{-5}$ °C^{-1}.**

Solution. Let l be the length of copper rod at 0°C. The length of steel rod at 0°C $= l + 5$.

If ΔT is the rise in temperature, then as per question

increase in length of copper rod = increase in length of steel rod

$$\therefore \quad l \times 1{\cdot}7 \times 10^{-5} \times \Delta T = (l + 5) \times 1{\cdot}1 \times 10^{-5} \times \Delta T$$

or $1{\cdot}7\, l = 1{\cdot}1\, l + 5{\cdot}5$ or $0{\cdot}6\, l = 5{\cdot}5$

or $l = 5{\cdot}5/0{\cdot}6 = 9{\cdot}17$ cm

Thus length of copper rod, l = **9·17 cm**

Length of steel rod $= l + 5 = 9{\cdot}17 + 5$
$= \mathbf{14{\cdot}17}$ **cm.**

Example 9 **A tooth cavity is filled with a copper having coefficient of linear expansion $1{\cdot}7 \times 10^{-5}$ °C^{-1} and bulk modulus $1{\cdot}4 \times 10^{11}$ Nm^{-2}. The temperature of the tooth 35°C. Calculate the thermal stress developed inside the tooth cavity when hot milk at temperature of 60°C is drunk.**

Solution. Here, $\alpha = 1{\cdot}7 \times 10^{-5}$ °C^{-1},

$B = 1{\cdot}4 \times 10^{11}$ Nm^{-2} ;

$\Delta T = 60 - 35 = 25$°C

$$\text{Thermal stress} = B\,\gamma\,\Delta T = B\,(3\,\alpha)\,\Delta T$$
$$= (1{\cdot}4 \times 10^{11}) \times (3 \times 1{\cdot}7 \times 10^{-5}) \times 25$$
$$= \mathbf{1{\cdot}78 \times 10^8\ Nm^{-2}}$$

Example 10 **A one litre flask contains some mercury. It is found that at different temperatures, the volume of air inside the flask remains the same. What is the volume of the mercury in this flask ? Given α for glass = 9×10^{-6} °C^{-1} and γ for mercury = $1{\cdot}8 \times 10^{-4}$ °C^{-1}**

Solution. For glass $\gamma_g = 3\,\alpha_g = 3 \times 9 \times 10^{-6}$
$= 27 \times 10^{-6}$ °C^{-1}

Volume of glass flask, V = 1 litre = 1000 cm^3

Let V_m be the volume of mercury in the flask and V_a be the volume of air in the flask.

Volume of glass flask = volume of mercury + volume of air

$$V = V_m + V_a$$

As V_a remains constant, when temperature is raised by ΔT, so

$$\Delta V = \Delta V_m$$

or $$V \gamma_g \Delta T = V_m \gamma_m \Delta T$$

or $$V_m = \frac{\gamma_g V}{\gamma_m} = \frac{(27\times10^{-6})\times1000}{1{\cdot}8\times10^{-4}} = \mathbf{150\ cm^3}$$

TYPE III. SPECIFIC HEAT, LATENT HEAT AND CALORIMETRY

Formulae used

1. $\Delta Q = m\, s\, \Delta T$, where s is specific heat of the substance.
2. $C = M \times s$, where C is molar specific heat of the substance and M is molecular weight of the substance.
3. In the method of mixtures,

   Heat gained = Heat lost

   i.e., mass × sp. heat × rise in temperature

   = mass × sp. heat × fall in temperature
4. For change of state, $\Delta Q = m\, L$

   where L is latent heat of the substance

Units used

ΔQ in joule, m in kg, s in J kg^{-1} K^{-1}, ΔT in K ; molar specific heat C in J mole^{-1} K^{-1}, L in J kg^{-1}

Example 11 **Calculate the heat required to convert 3 kg of ice at – 12°C kept in a calorimeter to steam at 100°C at atmospheric pressure. Given, specific heat capacity of ice = 2100 J kg^{-1} K^{-1} specific heat capacity of water = 4186 J kg^{-1} K^{-1} Latent heat of fusion of ice = 3·35 × 10^5 J kg^{-1} and latent heat of steam = 2·256 × 10^6 J kg^{-1}.**

Solution. Here, $m = 3$ kg,

$$s_{ice} = 2100 \text{ J kg}^{-1}\text{ K}^{-1},$$
$$s_w = 4186 \text{ J kg}^{-1}\text{ K}^{-1},$$
$$L_{ice} = 3{\cdot}35\times10^5 \text{ J kg}^{-1},$$
$$L_{steam} = 2{\cdot}256\times10^6 \text{ J kg}^{-1}$$

Heat required to convert ice at – 12°C into ice at 0°C

$$Q_1 = m s_{ice}\,\Delta T_1 = 3\times2100\times[0-(-12)] = 75600 \text{ J}$$

Heat required to melt the ice at 0°C into water at 0°C

$$Q_2 = m\,L_{ice} = 3\times(3{\cdot}35\times10^5) = 1005000 \text{ J}$$

Heat required to convert water at 0°C into water at °C

$$Q_3 = m s_w\,\Delta T_2 = 3\times4186\times(100-0) = 1255800 \text{ J}$$

Heat required to convert water at 100°C into steam

$$Q = m\,L_{steam} = 3\times(2{\cdot}256\times10^6) = 6768000 \text{ J}$$

Total heat required to convert 3 kg of ice at – 12°C into steam at 100°C is

$$Q = Q_1 + Q_2 + Q_3 + Q_4 = 75600 + 1005000 + 1255800 + 6768000 = \mathbf{9{\cdot}1\times10^6\ J}$$

Example 12 **A sphere of aluminium of mass 0·047 kg placed for sufficient time in a vessel containing boiling water, so that the sphere is at 100°C. It is then immediately transferred to 0·14 kg copper calorimeter containing 0·25 kg of water at 20°C. The temperature of water rises and attains a steady state at 23°C. Calculate the specific heat capacity of aluminium. Specific heat capacity of copper = 0·386 × 10^3 J kg^{-1} K^{-1}. Specific heat capacity of water = 4·18 × 10^3 J kg^{-1} K^{-1}.**

(NCERT Solved Example)

Solution. Here, $m_{Al} = 0{\cdot}047$ kg ; $T_1 = 100°$ C ; $m_{Cu} = 0{\cdot}14$ kg ; $m_w = 0{\cdot}25$ kg ; $T_0 = 20°$C ; $T_2 = 23°$C, $s_{Cu} = 0{\cdot}386\times10^3$ J kg^{-1} K^{-1}.

Heat lost by aluminium,

$$Q_1 = m_{Al} s_{Al}(T_1 - T_2) = 0{\cdot}047\times s_{Al}\times(100-23) = 0{\cdot}047\times s_{Al}\times77 \text{ J}$$

Heat taken by copper calorimeter and water is

$$Q_2 = m_{Cu} s_{Cu}(T_2 - T_0) + m_w s_w (T_2 - T_0) = 0{\cdot}14\times(0{\cdot}386\times10^3)\times(23-20) + 0{\cdot}25\times(4{\cdot}18\times10^3)(23-20)$$
$$= 162{\cdot}12 + 3135 = 3297{\cdot}12 \text{ J}$$

In the steady state, heat lost = heat gained

$$\therefore \quad 0{\cdot}047\times s_{Al}\times77 = 3297{\cdot}12$$

or $$s_{Al} = \frac{3297{\cdot}12}{0{\cdot}047\times77} = \mathbf{911\ J\ kg^{-1}\ K^{-1}}$$

Example 13 **When 0·15 kg of ice at 0°C is mixed with 0·30 kg of water at 50°C in a container, the resulting temperature is 6·7°C. Calculate the heat of fusion of ice. (s_{water} = 4186 J kg^{-1} K^{-1})**

(NCERT Solved Example)

Solution. Heat lost by water

$= m_w s_w (T_1 - T_2) = 0{\cdot}30 \times 4186 \times (50 - 6{\cdot}7)$
$= 54376{\cdot}14$ J

Heat taken by ice $= m_l L + m_l s_w (T_2 - T_0)$
$= 0{\cdot}15 \times L + 0{\cdot}15 \times 4186 \times (6{\cdot}7 - 0)$
$= 0{\cdot}15\, L + 4206{\cdot}93$ J

Heat lost = heat gained

$\therefore \quad 54376{\cdot}14 = 0{\cdot}15\, L + 4206{\cdot}93$

or $\quad L = 3{\cdot}34 \times 10^5 \text{ J kg}^{-1}$

Example 14 **How many grams of ice at $-14°$C are needed to cool 200 gram of water form 25°C to 10°C ? Take specific heat of ice $= 0{\cdot}5$ cal g^{-1} °C^{-1} and latent heat of ice $= 80$ cal g^{-1}.**

Solution. Here, $m_{ice} = ?$ $m_w = 200$ g ;
$s_{ice} = 0{\cdot}5$ cal g^{-1} °C^{-1}, $L_{ice} = 80$ cal g^{-1}

Heat lost by water in cooling from 25°C to 10°C is

$Q_1 = m_w \times s_w \times \Delta T_1 = 200 \times 1 \times (25 - 10)$
$= 3000$ cal.

Heat gained by ice at -14°C to change into water at 10°C is

$Q_2 = m_{ice}\, s_{ice}\, \Delta T_2 + m_{ice}\, L_{ice} + m_{ice} \times s_w \times \Delta T_3$
$= m \times 0{\cdot}5 \times [0 - (-14)] + m \times 80 + m \times 1 \times (10 - 0)$
$= 97$ m cal

As heat lost = heat gained, so $Q_1 = Q_2$

or $\quad 3000 = 97$ m or $m = \dfrac{3000}{97} = 31$ g

Example 15 **How much metres can a 50 kg man climbs by using the energy from a slice of a bread which produces 420 kJ heat ? Assuming that the human body efficiency working is 30%. Use $g = 10$ m/s^2.**

Solution. Let h be the height climbed by man.

Increase in PE of man $= mgh = 50 \times 10 \times h$ J

Heat produced ; $H = 420$ kJ $= 420 \times 1000$ J
$= 4{\cdot}2 \times 10^5$ J

efficiency of man = 30%, so heat energy utilized by man in doing work is

$$= \frac{30}{100} \times 4{\cdot}2 \times 10^5 = 12{\cdot}6 \times 10^4 \text{ J}$$

Now, increase in PE = heat energy utilized

$50 \times 10 \times h = 12{\cdot}6 \times 10^4$

or $\quad h = \dfrac{12{\cdot}6 \times 10^4}{50 \times 10} = 252$ m

Example 16 **A geyser heats water flowing at the rate of 3 kg per minute from 27°C to 77°C. If the geyser operates on a gas burner, what is the rate of consumption of fuel if the heat of combustion is 4×10^4 J/g ? Given specific heat of water is $4{\cdot}2 \times 10^3$ J/kg/K.**

Solution. Here, $m = 3$ kg/min ;
$\Delta T = 77 - 27 = 50°\text{C} = 50$ K

Heat sphent to raise the temperature ΔT of water is

$Q = mc\,\Delta T$
$= 3 \text{ kg/min} \times (4{\cdot}2 \times 10^3) \text{ J kg}^{-1}\text{K}^{-1} \times 50 \text{ K}$
$= 63 \times 10^4$ J/min

Rate of consumption of fuel

$$= \frac{Q}{\text{heat of compustion of fuel}}$$

$$= \frac{63 \times 10^4 \text{ J/min}}{4 \times 10^4 \text{ Jg}^{-1}} = 15{\cdot}75 \text{ g min}^{-1}$$

TYPE IV. SPECIFIC HEAT OF GASES

Formulae used. **1.** $C_p - C_v = \dfrac{R}{J}$,

where $R = \dfrac{PV}{T}$ = gas constant for one gram mole of the gas.

2. $c_p - c_v = \dfrac{r}{J}$, where $r = \dfrac{Pv}{T} = \dfrac{R}{M}$ = gas constant for one gram of the gas and M is the molecular weight of the gas in gram.

3. For monoatomic gases, $C_v = \dfrac{3}{2}R$; $C_p = \dfrac{5}{2}R$

4. For diatomic gases, $C_v = \dfrac{5}{2}R$, $C_p = \dfrac{7}{2}R$

5. For triatomic gases (non linear molecule)
$C_v = 3\,R$, $C_p = 4\,R$

6. For triatomic gases (linear molecule)

$$C_v = \frac{7}{2}R, \; C_p = \frac{9}{2}R$$

Units used. C_p, C_v in J mole^{-1} K^{-1} ;
c_p, c_v in cal. g^{-1} K^{-1}

Standard Values. $R = 8{\cdot}31$ J mole^{-1} K^{-1}
$= 1{\cdot}98$ cal. mol^{-1} K^{-1} ; $J = 4{\cdot}18$ J cal^{-1}.

Example 17 **Calculate the difference between the two principal specific heats of 2 gram of helium gas at S.T.P. Given atomic weight of helium = 4 and J = 4·186 J cal^{-1} and R = 8·31 J mol^{-1} K^{-1}.**

Solution. No. of moles in 2 g of helium

$$n=\frac{2}{4}=\frac{1}{2}$$

Gas constant for 2 g of helium, $R' = nR = \frac{1}{2}R$

$$\therefore\ C_p - C_v = \frac{R'}{J} = \frac{R}{2J} = \frac{8\cdot31}{2\times4\cdot186}$$

$$= \mathbf{0\cdot993\ cal\ g^{-1}\ K^{-1}}$$

Example 18 **Calculate the specific heat capacity at constant volume for a gas. Given specific heat capacity at constant pressure is 6·85 cal mol^{-1} K^{-1}, R = 8·31 J mol^{-1} K^{-1}, J = 4·18 J cal^{-1}.**

(NCERT Exercise – Supplementary Textual Material)

Solution. Here, C_p = 6·85 cal mol^{-1} K^{-1} ;
R = 8·31 J mol^{-1} K^{-1}

$$\therefore\ R = \frac{8\cdot31}{4\cdot18}\ \text{cal mol}^{-1}\ \text{K}^{-1}$$

$$= 1\cdot988\ \text{cal mol}^{-1}\ \text{K}^{-1}$$

$$C_V = C_p - R = 6\cdot85 - 1\cdot988$$

$$= \mathbf{4\cdot862\ cal\ mol^{-1}\ K^{-1}}$$

Example 19 **Specific heat of argon at constant pressure is 0·125 cal. g^{-1} K^{-1}, and at constant volume 0·075 cal. g^{-1} K^{-1}. Calculate the density of argon at N.T.P. Given J = 4·18 × 10^7 erg cal^{-1} and normal pressure = 1·01 × 10^6 dyne cm^{-2}.** **(NCERT Exercise – Supplementary Textual Material)**

Solution. Here c_p = 0·125 cal. g^{-1} K^{-1} ;
c_v = 0·075 cal. g^{-1} K^{-1} ;
J = 4·18 × 10^7 erg cal.$^{-1}$
Normal pressure, P = 1·01 × 10^6 dyne/cm^2
Normal temperature, T = 273°K ;
We have to calculate density, ρ = ?

As, gas constant for 1 g of gas $r = \frac{Pv}{T} = \frac{P}{\rho T}$

where, v = volume occupied by 1 gram of gas

Now, $c_p - c_v = \frac{r}{J}$

or $r = (c_p - c_v) J$

$$\frac{P}{\rho T} = (0\cdot125 - 0\cdot075)\ 4\cdot18\times10^7$$

or $$\frac{1\cdot01\times10^6}{\rho\times273} = 0\cdot05\times4\cdot18\times10^7$$

$$\therefore\ \rho = \frac{1\cdot01\times10^6}{273\times0\cdot05\times4\cdot18\times10^7}$$

$$= \mathbf{1\cdot77\times10^{-3}\ g\ cm^{-3}}$$

Example 20 **The difference between the two specific heat capacities (at constant pressure and volume) of a gas is 5000 J kg^{-1} K^{-1} and the ratio of these specific heat capacities is 1·6. Find the two specific heat capacities, *i.e.*, C_V and C_p.**

(NCERT Exercise – Supplementary Textual Material)

Solution. Given,

$$C_p - C_V = 5000\ \text{J kg}^{-1}\ \text{K}^{-1} \quad ...(i)$$

and $C_p/C_V = 1\cdot6$ or $C_p = 1\cdot6\ C_V$

From (*i*),

$$1\cdot6\ C_V - C_V = 5000$$

or $$0\cdot6\ C_V = 5000$$

or $$C_V = \frac{5000}{0\cdot6} = \mathbf{8333\cdot3\ J\ kg^{-1}\ K^{-1}}$$

$$C_p = 1\cdot6\ C_V = 1\cdot6\times\frac{5000}{0\cdot6}$$

$$= \mathbf{13333\cdot3\ J\ kg^{-1}\ K^{-1}}$$

TYPE V. THERMAL CONDUCTIVITY

Formulae used

1. $\frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$

where $\frac{\Delta Q}{\Delta t}$ = rate of conduction of heat,

$\frac{\Delta T}{\Delta x}$ = temperature gradient = rate of fall of temperature with distance,

A = area of the hot surface, K = coefficient of thermal conductivity.

2. If heat so conducted is used in changing the state of m gram of the substance, then

$$\Delta Q = m\,L = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t,$$

where L is latent heat of the substance.

3. If heat so conducted is used in increasing the temp. of the substance through range $\Delta\theta$, then

$$\Delta Q = s\,m\,\Delta\theta = KA\left(\frac{\Delta T}{\Delta x}\right)\Delta t$$

where m is mass of the substance and s is specific heat of the substance.

Units used. ΔQ in J, Δt in s, A in m^2, ΔT in K or °C. Δx in m and K in W m^{-1} K^{-1}. mass m is in gram L is in joule/gram and s is in joule/gram/°C.

Example 21 **A metal rod of length 20 cm and diameter 2 cm is covered with a non conducting substance. One of its ends is maintained at 100°C, while the other end is put at 0°C. It is found that 25 g of ice melts in 5 min. Calculate the coefficient of thermal conductivity of the metal. Latent heat of ice = 80 cal. g^{-1}.**

Solution. Here, length of the rod,

$\Delta x = 20$ cm $= 20 \times 10^{-2}$ m,

Diameter = 2 cm,

radius $= r = 1$ cm $= 10^{-2}$ m,

Area of cross section

$A = \pi r^2 = \pi (10^{-2})^2 = 10^{-4}\pi$ sq. m,

$\Delta T = 100 - 0 = 100$°C,

Mass of ice melted, $m = 25$ g ; $L = 80$ cal. g^{-1}

$\therefore$ Heat conducted,

$\Delta Q = m\,L = 25 \times 80 = 2000$ cal.

$= 2000 \times 4{\cdot}2$ J

$\Delta t = 5$ min $= 300$ s

$$\text{As, } \frac{\Delta Q}{\Delta t} = KA\frac{\Delta T}{\Delta x}$$

$$\therefore \quad K = \frac{\Delta Q/\Delta t}{A\Delta T/\Delta x} = \frac{\Delta Q\,\Delta x}{\Delta t.A\Delta T}$$

$$= \frac{2000\times 4{\cdot}2\times 20\times 10^{-2}}{300\times 10^{-4}\,\pi\times 100}$$

$$= \mathbf{1{\cdot}78\ J\ s^{-1}\ m^{-1}\ {}^\circ C^{-1}}$$

Example 22 **A cubical ice box of side 50 cm has a thickness of 5·0 cm. If 5 kg of ice is put in the box, estimate the amount of ice remaining after 4 hours. The outside temperature is 40°C and coefficient of thermal conductivity of the material of the box = 0·01 Js^{-1} m^{-1} °C^{-1}. Heat of fusion of ice = 335 Jg^{-1}.**

Solution. Here, $l = 50$ cm $= 0{\cdot}50$ m ;

$\Delta x = 5$ cm $= 5 \times 10^{-2}$ m

As a cubical box has six faces, so total area

$A = 6 \times l^2 = 6 \times (0{\cdot}5)^2 = 1{\cdot}50$ m^2

$\Delta T = T_1 - T_2 = 40° - 0° = 40$°C ;

$\Delta t = 4$ hr $= 4 \times 60 \times 60$ s ;

$L = 335$ Jg^{-1} $= 335 \times 10^3$ J kg^{-1} ;

$K = 0{\cdot}01$ J s^{-1} m^{-1} °C^{-1}

Amount of heat conducted into the box is

$$Q = \frac{KA(\Delta T)t}{\Delta x}$$

$$= \frac{0{\cdot}01\times(1{\cdot}5)\times(40)\times(4\times 60\times 60)}{5\times 10^{-2}}$$

$$= \frac{172800}{5} = 34560\text{ J}$$

Let m be the mass of ice melted in 4 hr, then heat spent, $Q = mL = m \times (335 \times 10^3)$ J

$\therefore\ m \times 335 \times 10^3 = 34560$

$$\text{or} \quad m = \frac{34560}{335\times 10^3} = 0{\cdot}103\text{ kg}$$

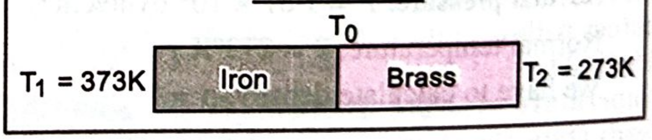
Mass of the ice left in box after 4 hr

$= 5 - 0{\cdot}103 = \mathbf{4{\cdot}897\ kg}$

Example 23 **An iron bar ($L_1 = 0{\cdot}1$ m, $A_1 = 0{\cdot}02$ m^2, $K_1 = 79$ Wm^{-1} K^{-1}) and a brass bar ($L_2 = 0{\cdot}1$m, $A_2 = 0{\cdot}02$$m^2$, $K_2 = 109$ Wm^{-1} K^{-1}) are soldered end to end as shown in Fig. 11.27. The free ends of iron bar and brass bar are maintained at 373 K and 273 K respectively. Obtain expressions for and hence compute (*i*) the temperature of the junction of the two bars, (*ii*) the equivalent thermal conductivity of the compound bar, and (*iii*) the heat current through the compound bar.**

(NCERT Solved Example)

FIGURE 11.27



Solution. Let T_0 be the temperature of the junction of the two bars and A be the area of cross-section of each bar.

Example 27 **A pan filled with hot food cools from 94°C to 86°C in 2 minutes when the room temperature is at 20°C. How long will it take to cool from 71°C to 69°C ? Here cooling takes place according to Newton's law of cooling.**

(NCERT Solved Example)

Solution. The average temperature of 94°C and 86°C is, $T = \dfrac{94+86}{2} = 90°\,C$

The average temperature of 71°C and 69°C,

$$T = \frac{71+69}{2} = 70°\,C$$

As, $\dfrac{dT}{dt} = -K(T - T_0)$

For first condition

$$\frac{(94-86)}{2\,\text{min}} = -K(90-20)$$

or $$\frac{8}{2\,\text{min}} = -K \times 70 \quad ...(i)$$

For second condition

$$\frac{71-69}{dt} = -K(70-20)$$

or $$\frac{2}{dt} = -K \times 50 \quad ...(ii)$$

Dividing (*i*) by (*ii*), we get

$$\frac{8/2}{2/dt} = \frac{70}{50} \text{ or } dt = \frac{70}{50 \times 2} = 0{\cdot}7 \text{ min} = 42 \text{ s}$$

Example 28 **A body cools in 7 minutes from 60°C to 40°C. What will be its temperature after the next 7 minutes ? The temperature of the surroundings is 10°C. Assume that Newton's law of cooling holds good throughout the process.**

Solution. In Ist case, here, $T_1 = 60°C$, $T_2 = 40°C$, $T_0 = 10°C$, $t = 7$ min $= 7 \times 60$ s

$dT = T_1 - T_2 = 60 - 40 = 20°C$, $dt = 7 \times 60$ s

Average temperature, $T = \dfrac{T_1 + T_2}{2} = \dfrac{60+40}{2}$

$= 50°C$

According to Newton's law of cooling

$$\frac{dT}{dt} = -K\,(T - T_0) \text{ or } \frac{20}{7 \times 60} = -K\,[50-10]$$

or $$-K = \frac{1}{7 \times 3 \times 40}$$

In second case, $T_1 = 40°C$, $T_2 = ?$, $t = 7 \times 60$ s

$dT = (40 - T_2)°C$, $dt = 7 \times 60$ s

$$T = \frac{40+T_2}{2}$$

According to Newton's law of cooling

$$\frac{dT}{dt} = -K\,[T - T_0]$$

$$\frac{40-T_2}{7 \times 60} = -K\left[\frac{40+T_2}{2} - 10\right]$$

$$= \frac{1}{7 \times 3 \times 40}\left[\frac{40+T_2-20}{2}\right]$$

or $4\,(40 - T_2) = 20 + T_2$

On solving ; $T_2 = 28°C$

TYPE VII. STEFAN'S LAW AND WIEN'S DISPLACEMENT LAW

Formula used

1. *Stefan's law* : $E = \sigma T^4$

where σ is Stefan's constant

$\sigma = 5{\cdot}67 \times 10^{-8}$ W m^{-2} K^{-4}.

2. When black body at T K is in a black enclosure at T_0 K, then net loss of heat energy/sec/area,

$$E = \sigma(T^4 - T_0^4).$$

3. *Wien's displacement law* $\lambda_m = \dfrac{b}{T}$

where b = Wien's constant $= 2{\cdot}898 \times 10^{-3}$ mK.

Units used. E is in W m^{-2}, λ_m in *m* and T is in *K*.

Example 29 **Calculate the temperature (in K) at which a perfect black body radiates energy at the rate of 5·67 W cm^{-2}. Given $\sigma = 5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4}.** **(CBSE Delhi 2005)**

Solution. Here, $E = 5{\cdot}67$ W cm^{-2}

$= 5{\cdot}67 \times 10^4$ Wm^{-2},

$\sigma = 5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4}

According to Stefan's law, $E = \sigma T^4$

or $$T = \left(\frac{E}{\sigma}\right)^{1/4} = \left(\frac{5{\cdot}67 \times 10^4}{5{\cdot}67 \times 10^{-8}}\right)^{1/4} = \mathbf{1000\ K}$$

Example 30 **Due to change in main voltage, the temperature of an electric bulb rises from 3000 K to 4000 K. What is the percentage rise in electric power consumed ?**

Solution. When temperature, $T_1 = 3000$ K, Then

$$E_1 = \sigma T_1^4 = \times\sigma(3000)^4 \quad ...(i)$$

When temperature, $T_2 = 4000$ K, Then

$$E_2 = \sigma T_2^4 = \times\sigma(4000)^4 \quad ...(ii)$$

% rise in electric power is

$$= \frac{E_2 - E_1}{E_1}\times 100 = \left(\frac{E_2}{E_1} - 1\right)\times 100$$

$$= \left(\frac{(4000)^4}{(3000)^4} - 1\right)\times 100 = \left(\frac{256}{81} - 1\right)\times 100 = 216\%$$

Example 31 Determine the surface area of the filament of a 100 W incandescent lamp radiating out its labelled power at 3000 K. Given $\sigma = 5{\cdot}7\times10^{-8}$ Wm^{-2} K^{-4}, and emissivity $\in$ of the material of the filament = 0·3.

(NCERT Example – Supplementary Textual Material)

Solution. Here, $A = ?$, $P = 100$ W,

$T = 3000$ K ; $\in = 0{\cdot}3$;

$\sigma = 5{\cdot}7\times10^{-8}$ Wm^{-2} K^{-4}.

$P = EA = (\sigma \in T^4)$ A

$$\text{or } A = \frac{P}{\sigma \in T^4} = \frac{100}{(5{\cdot}7\times10^{-8})\times0{\cdot}3\times(3000)^4}$$

$$= 7{\cdot}25\times10^{-5}\ \text{m}^2$$

Example 32 An indirectly heated filament is radiating maximum energy of wavelength $2{\cdot}16\times10^{-5}$ cm. Find the net amount of heat energy lost per sec per unit area, if temp. of surrounding air is 13°C. Given $b = 0{\cdot}288$ cm K, $\sigma = 5{\cdot}77\times10^{-5}$ erg s^{-1} cm^{-2} K^{-4}.

Solution. Here, $\lambda_m = 2{\cdot}16\times10^{-5}$ cm,

$E = ?$; $T_0 = 13°C = 13 + 273 = 286$ K

$b = 0{\cdot}288$ cm K,

$\sigma = 5{\cdot}77\times10^{-5}$ erg s^{-1} cm^{-2} K^{-4}

From Wien's displacement law,

$$T = \frac{b}{\lambda_m} = \frac{0{\cdot}288}{2{\cdot}16\times10^{-5}} = 13333{\cdot}3\ \text{K}$$

As $E = \sigma(T^4 - T_0^4)$

$\therefore$ $E = 5{\cdot}77\times10^{-5}\,[(13333{\cdot}3)^4 - (286)^4]$ erg s^{-1} cm^{-2}

$= 18{\cdot}24\times10^{8}$ Js^{-1} m^{-2}

Example 33 A room heater is made of 10 polished thin walled tubes of copper, each one metre long and 5 cm in diameter. If hot water at 70°C circulates constantly through the tubes, calculate the amount of heat radiated in an hour in a room where the average temperature is 15°C. Emissivity of copper $= 4\times10^{-2}$ cal/degree/sec/sq. metre.

Solution. Here, no of tubes, $n = 10$

Length of each tube, $l = 1$ metre

Radius of each tube,

$$r = \frac{5}{2}\ \text{cm} = \frac{5}{200}\ \text{m} = \frac{1}{40}\ \text{m}$$

Surface area of each tube $= 2\pi r l$

and total area of all the tubes $A = 2\pi r l n$

$$= 2\times\frac{22}{7}\times\frac{1}{40}\times1\times10\ \text{m}^2 = \frac{11}{7}\text{m}^2$$

Temperature of hot water = 70°C

Average room temperature = 15°C

$\therefore$ Difference of temperatures,

$$\theta = 70 - 15 = 55°C$$

Emissivity, $\in = 4\times10^{-2}$ cals/sec/degree/sq metre

Time, $t = 1$ hour = 3600 secs

Assuming that heat energy radiated/sec/area is directly proportional to temp. diff. θ,

the quantity of heat (Q) radiated $= A.\theta.t.\in$

$$\therefore \quad Q = \frac{11}{7}\times55\times3600\times4\times10^{-2}$$

$$= 1{\cdot}245\times10^4\ \text{calories}$$

Example 34 Calculate the maximum amount of heat which may be lost per second by radiation by a sphere 14 cm in diameter at a temperature of 227°C, when placed in an enclosure at 27°C. Given Stefan's constant $= 5{\cdot}7\times10^{-8}$ W m^{-2} K^{-4}.

Solution. Here, temperature of sphere,

$$T = 227°C = 227 + 273 = 500\ \text{K}$$

Temperature of surroundings,

$$T_0 = 27°C = 27 + 273 = 300\ \text{K}$$

Radius of the sphere, $r = 7$ cm $= 0{\cdot}07$ metre

Area of the sphere, $A = 4\pi r^2$

$$\therefore \quad A = 4\times\frac{22}{7}\times(0{\cdot}07)^2 = 6{\cdot}16\times10^{-2}\ \text{m}^2$$

Taking the sphere as a black body, we know, net energy lost per sec. per unit area is given by

$$E = \sigma [T^4 - T_0^4]$$

$\therefore$ Total energy lost from area A per sec. (W)

$= AE = A\sigma [T^4 - T_0^4]$

$= 6{\cdot}16 \times 10^{-2} \times 5{\cdot}7 \times 10^{-8} [500^4 - 300^4]$

$= 6{\cdot}16 \times 10^{-2} \times 5{\cdot}7 \times 10^{-8} \times 100^4 [5^4 - 3^4]$

$= 6{\cdot}16 \times 5{\cdot}7 \times 10^{-2} \times 544 \text{ Js}^{-1}$

As, total heat lost per sec. $(H) = \dfrac{W}{J}$

$$\therefore\ H = \frac{6{\cdot}16 \times 5{\cdot}7 \times 10^{-2} \times 544}{4{\cdot}2} = 45{\cdot}48 \text{ cal/sec}$$

Example 35 **A man, the surface area of whose skin is 2 m², is sitting in a room where air temperature is 20°C. If his skin temperature is 28°C and emissivity of his skin equals 0·97, find the rate at which his body loses heat.**

(Given $\sigma = 5{\cdot}67 \times 10^{-8}$ Wm^{-2} K^{-4})

(NCERT Example – Supplementary Textual Material)

Solution. Here, $A = 2$ m²,

$T_0 = (20 + 273)$ K $= 293$ K,

$T = (28 + 273)$ K $= 301$ K, $\epsilon = 0{\cdot}97$

The rate of loss of heat by body is

$= \epsilon A \sigma (T^4 - T_0^4)$

$= 0{\cdot}97 \times 2 \times (5{\cdot}67 \times 10^{-8}) [(301)^4 - (293)^4]$

$\approx$ **92·2 W**

Example 36 **Light from the moon, is found to have a peak (or wavelength of maximum emission) at $\lambda = 14\ \mu$m. Given that the Wien's constant b equals $2{\cdot}8988 \times 10^{-3}$ mK, estimate the temperature of the moon.**

(NCERT Example – Supplementary Textual Material)

Solution. Here, $\lambda_m = 14\ \mu\text{m} = 14 \times 10^{-6}$ m ;

$T = ?$

As $\lambda_m T = b$

or $$T = \frac{b}{\lambda_m} = \frac{2{\cdot}8988 \times 10^{-3} \text{ mK}}{14 \times 10^{-6} \text{ m}} = 207 \text{ K}$$

Example 37 **The surface temperature of the hot body is 1127°C. Find the wavelength at which it radiates maximum energy. Given Wien's constant = 0·2898 cm K. To which spectrum region this wavelength belongs ?**

Solution. Here, $T = 1127 + 273 = 1400$ K,

$b = 0{\cdot}2898$ cm K.

From Wien's law,

$$\lambda_m = \frac{b}{T} = \frac{0{\cdot}2898 \text{ cm K}}{1400 \text{ K}} = \mathbf{2{\cdot}07 \times 10^{-4}\ cm}$$

This wavelength belongs to infrared region of electromagnetic spectrum.

TYPE VIII. TYPICAL EXAMPLES

Example 38 **The height of Niagra falls is 50 m. Calculate the difference in temperature of water at the top and at the bottom of fall, if $J = 4{\cdot}2$ J cal^{-1}.**

Solution. Here, $h = 50$ m, $g = 9{\cdot}8$ ms^{-2}, $J = 4{\cdot}2$ J cal^{-1}. Let M be the mass of water that falls and θ be the rise in temp. of water at the bottom. As

$$J = \frac{W}{H} = \frac{Mgh}{Ms\theta} \quad \therefore \quad \theta = \frac{gh}{sJ}$$

specific heat of water, $s = 1000$ cal kg^{-1} °C^{-1}.

$$\theta = \frac{9{\cdot}8 \times 50}{1000 \times 4{\cdot}2} = 0{\cdot}117°\text{C}$$

Example 39 **Calculate heat of combustion of coal, when 0·5 kg of coal on burning raises the temperature of 50 litres of water from 20°C to 90°C.**

Solution. Here, mass of coal, $M = 0{\cdot}5$ kg ;

mass of water, $m = (50 \times 1000)$ cc $\times$ 1 g/cc

$= 50 \times 10^3$ g

$\Delta T = 90° - 20° = 70°$C

specific heat of water $s = 10^3$ cal. kg^{-1} °C^{-1}

Total heat produced $= m\, s\, \Delta T$

$= (50 \times 10^3) \times 1 \times 70$ cal. $= 35 \times 10^5$ cal,

Heat of combustion of coal

$$= \frac{35 \times 10^5}{M} = \frac{35 \times 10^5}{0{\cdot}5} = \mathbf{7 \times 10^6\ cal,\ kg^{-1}}$$

Example 40 **The heat of combustion of ethane gas at 373 k cal per mole. Assume that 50% of heat is useful, how many litres of ethane measured at S.T.P. must be burnt to convert 60 kg of water at 20°C to steam at 100°C ? One mole of a gas occupies 22·4 litre at S.T.P. Latent heat of steam = $2{\cdot}25 \times 10^6$ J kg^{-1}.**