Maze-Generating Algorithms

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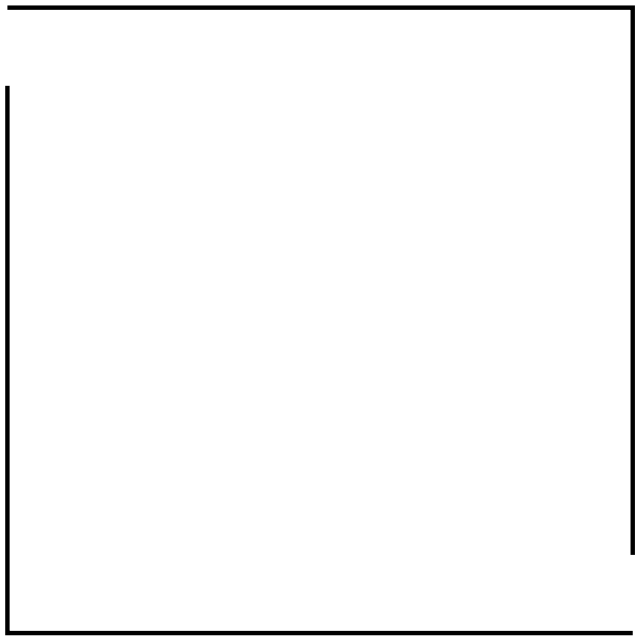
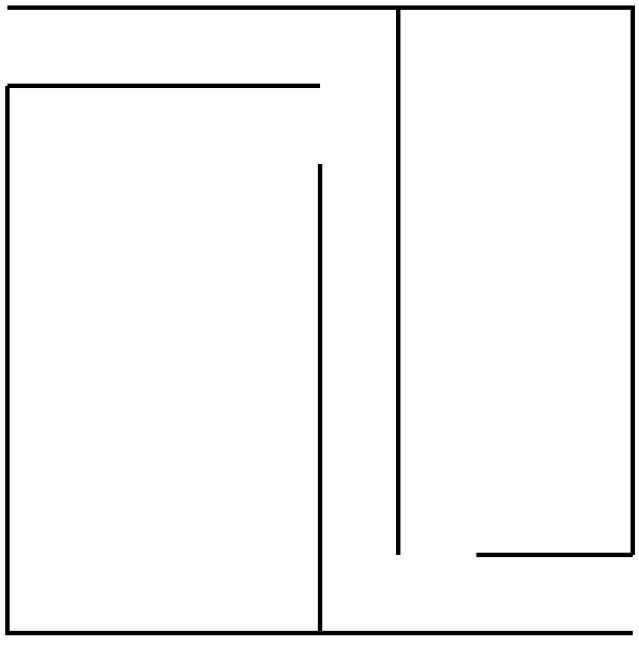
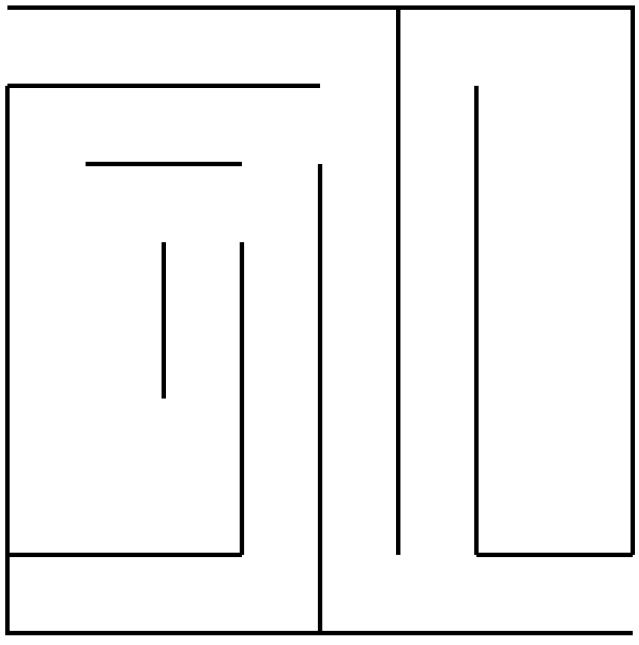
KAIST

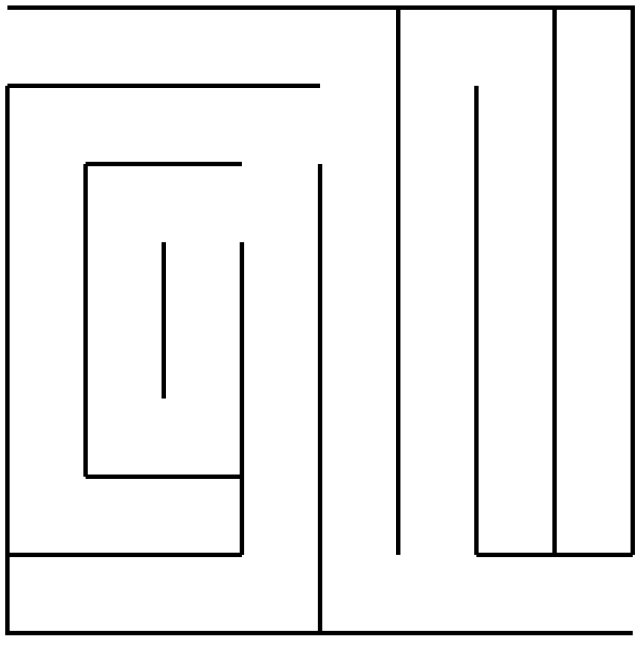
Physical Mathematics Conference 2009

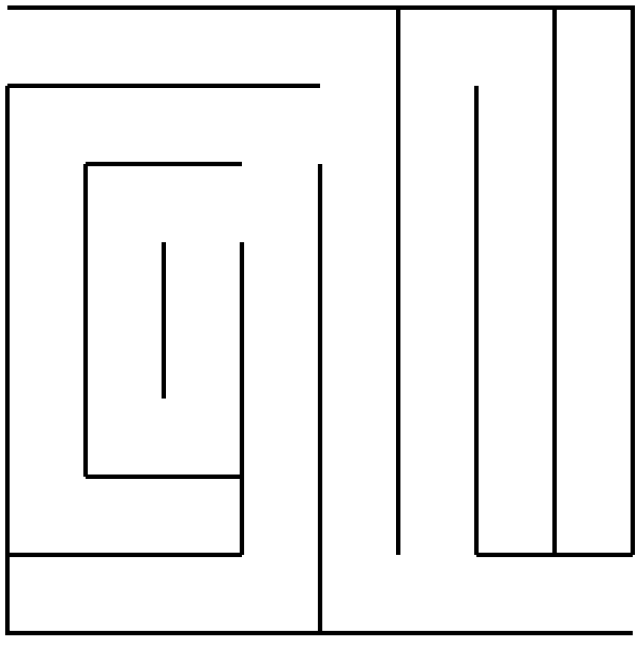
1. Maze Basics
2. General Idea of Maze Generating
3. Perfect Maze Generating
   - Depth-First Search
   - Kruskal's Algorithm



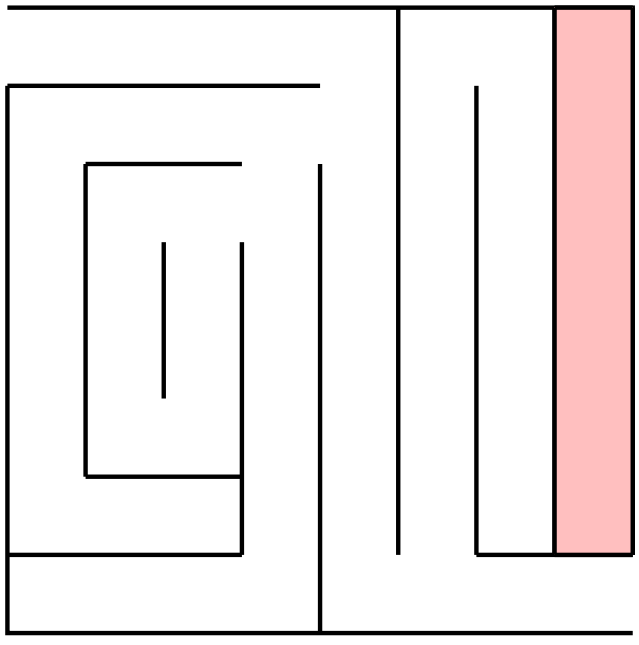








A Maze



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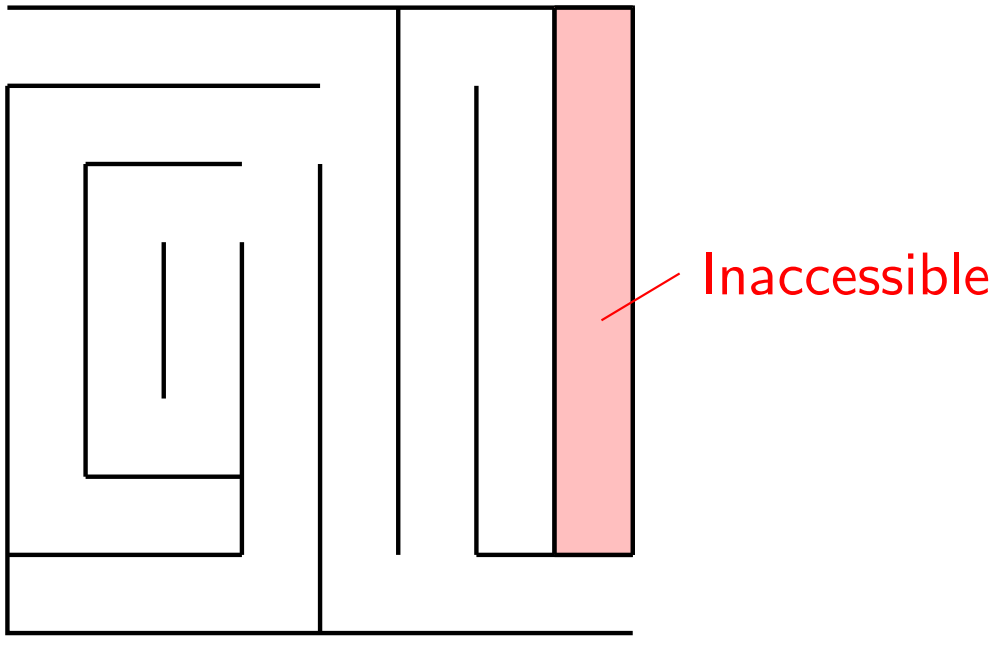

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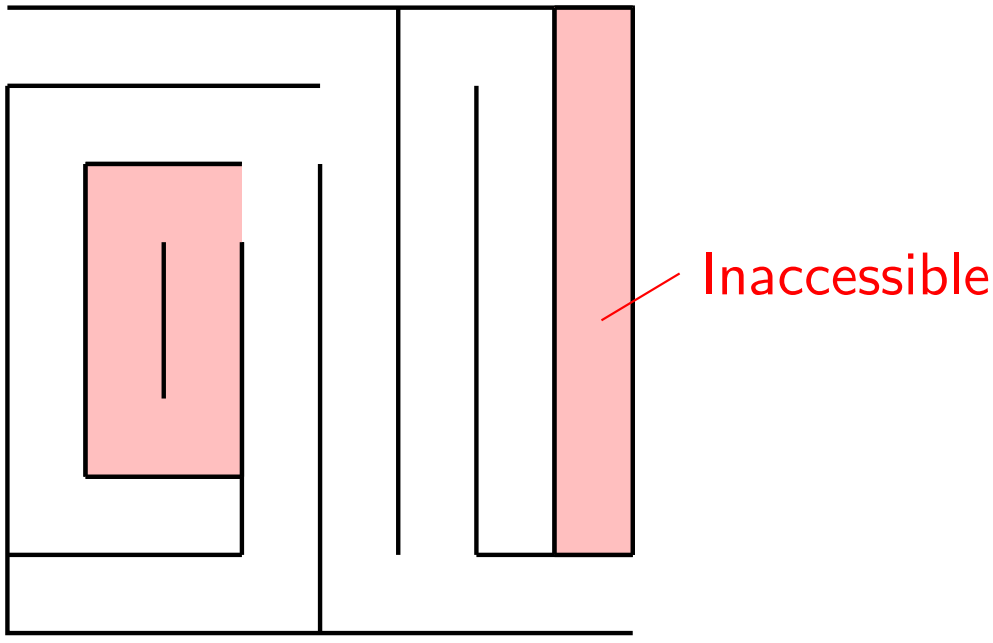

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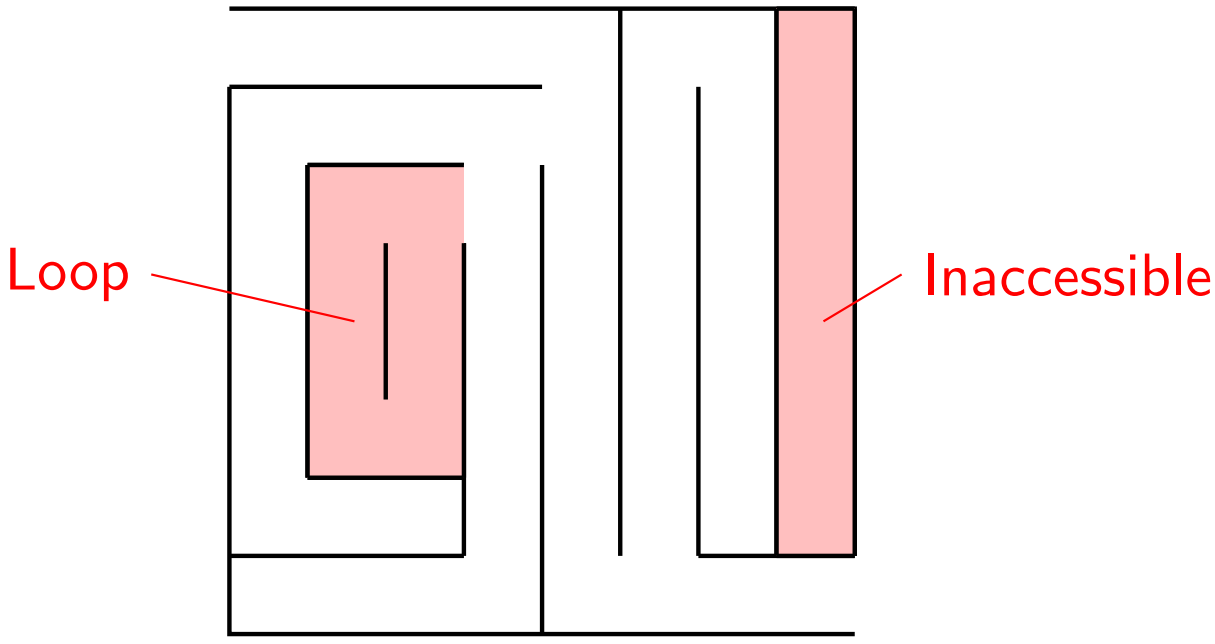

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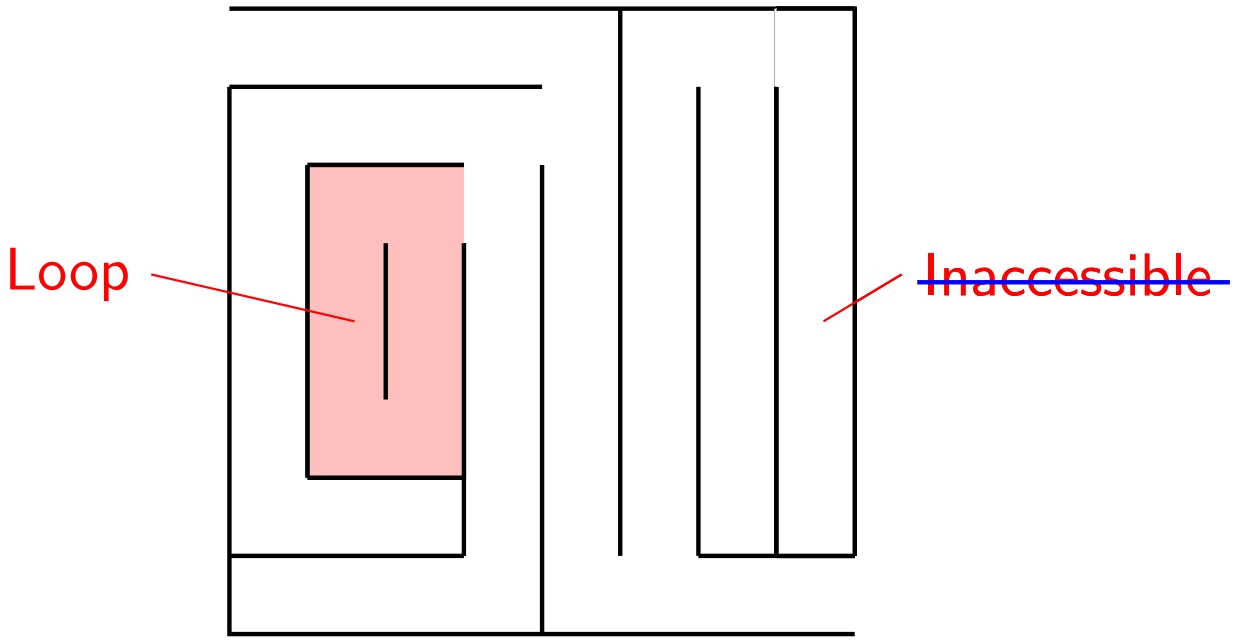

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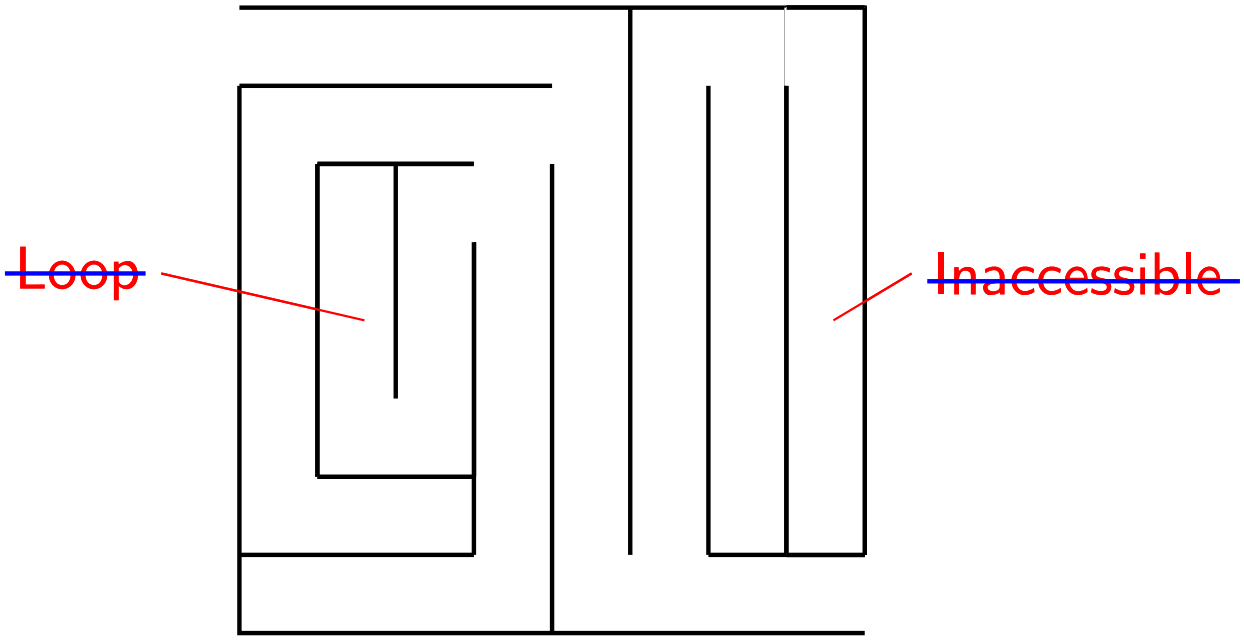
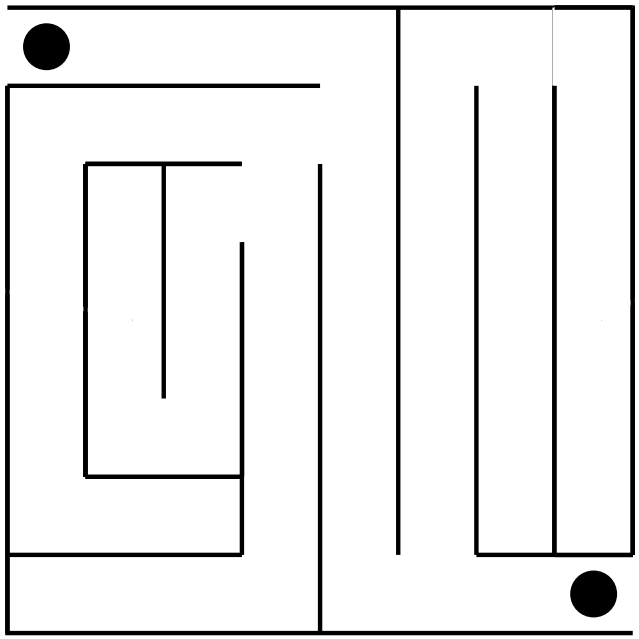


A Perfect Maze



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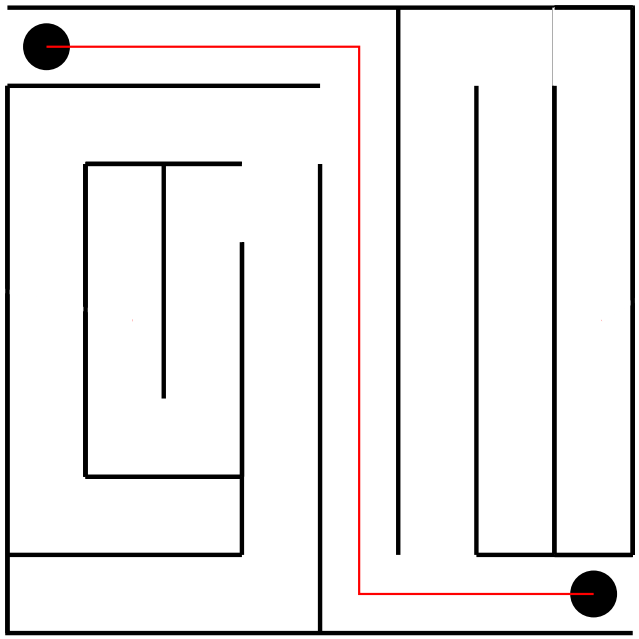

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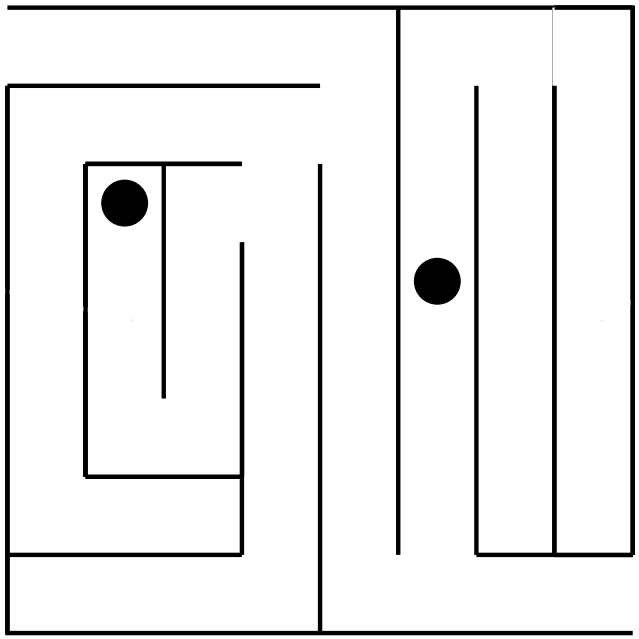

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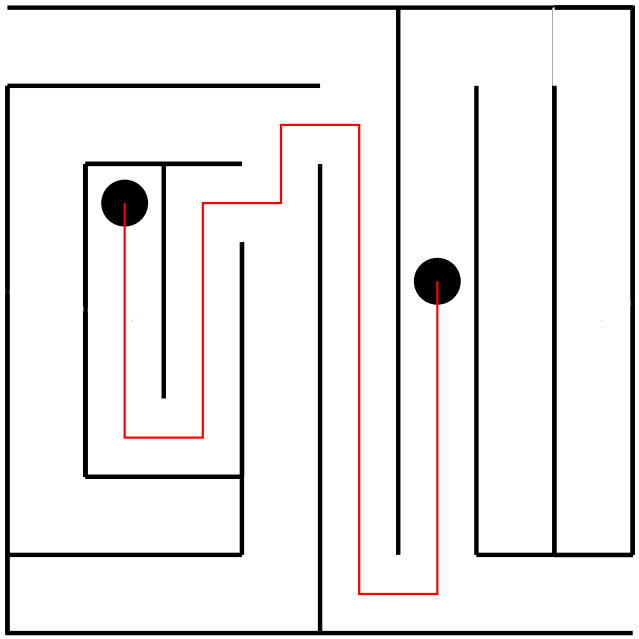

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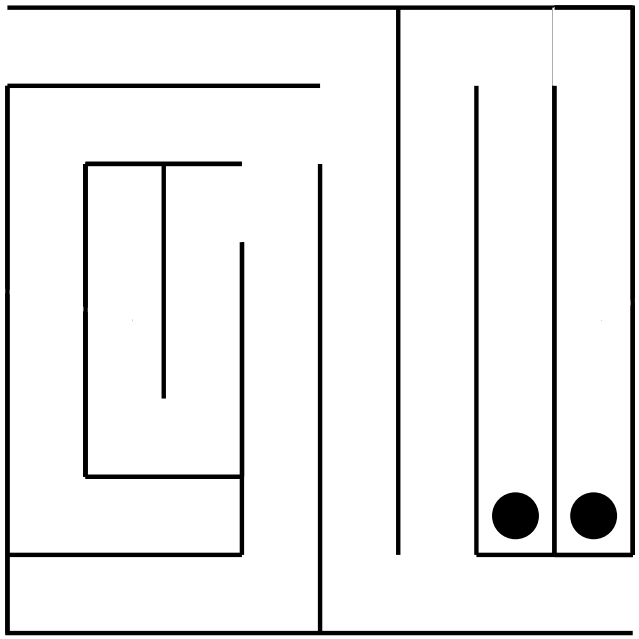
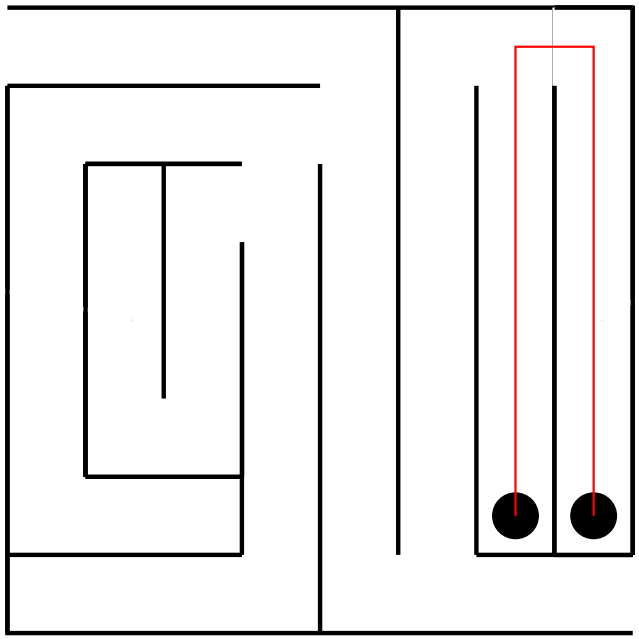


A Perfect Maze

Between any two points,



A Perfect Maze

Between any two points,
Only one path exists!

Assume mazes are

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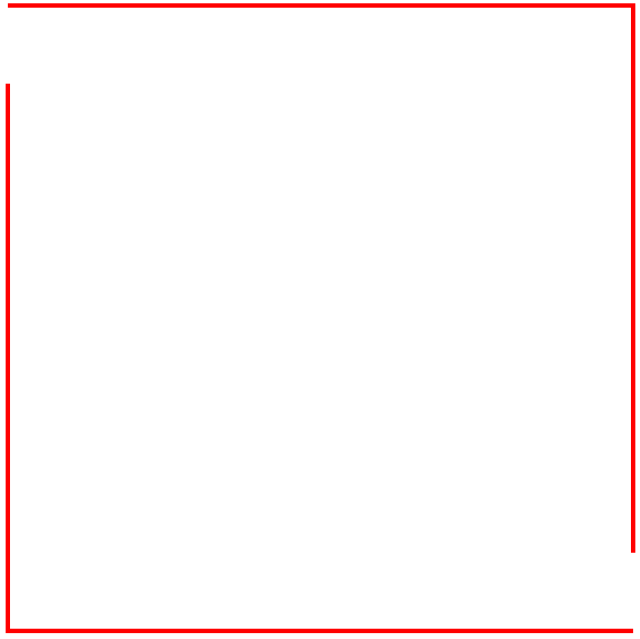
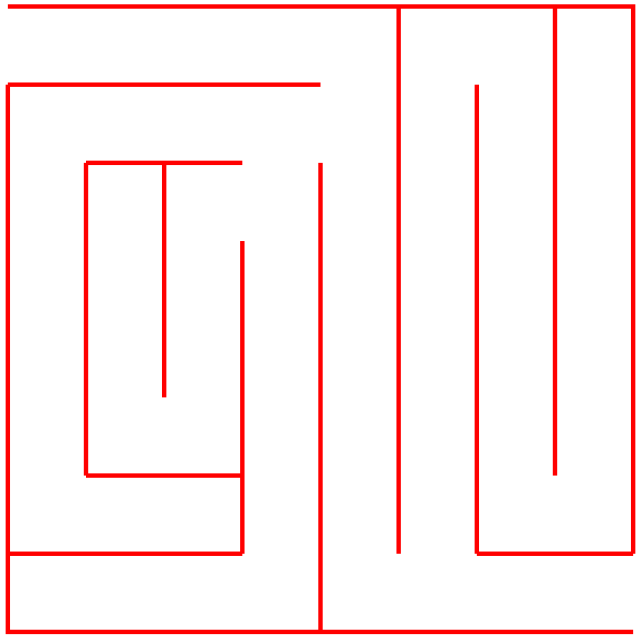


In 2-Dimensional Rectangular Grid

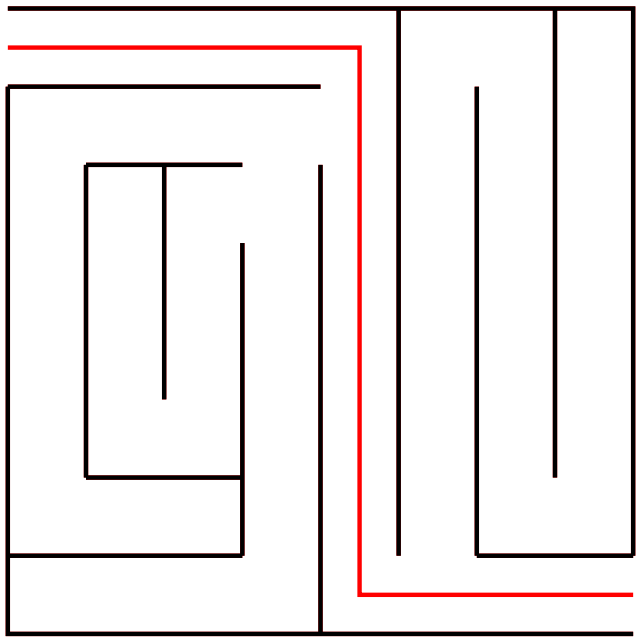
Assume mazes are



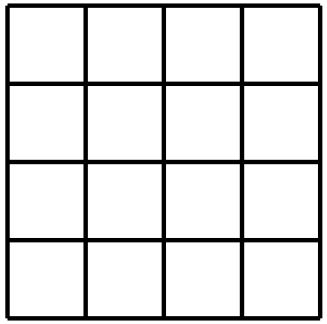
In 2-Dimensional Rectangular Grid

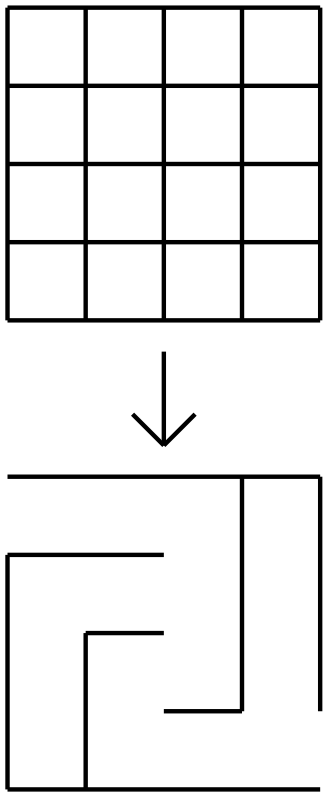
Perfect

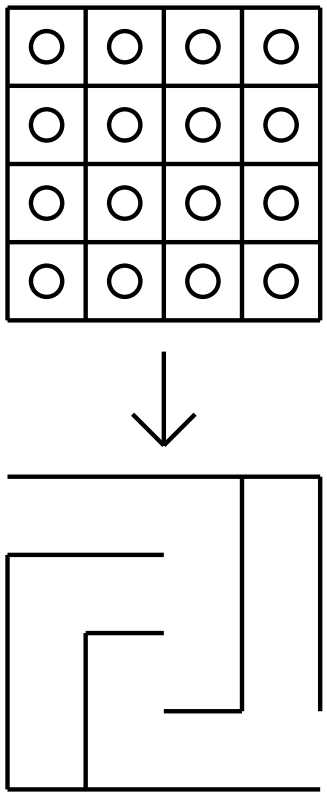
Assume mazes are

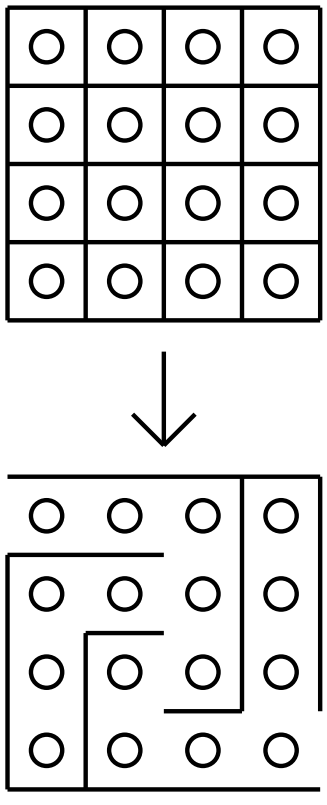


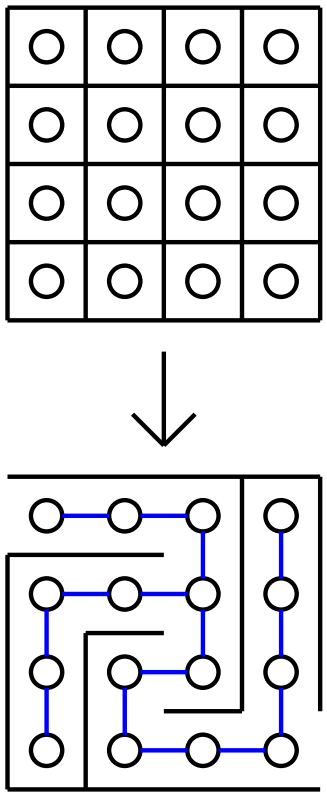
In 2-Dimensional Rectangular Grid
Perfect
Orthogonal Path

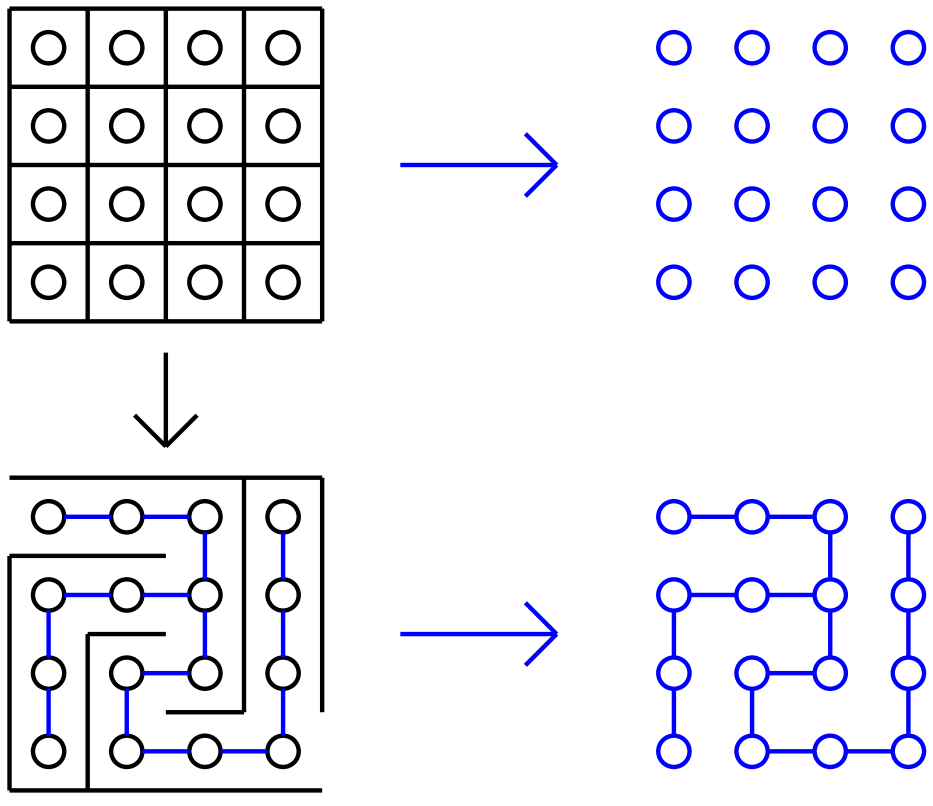


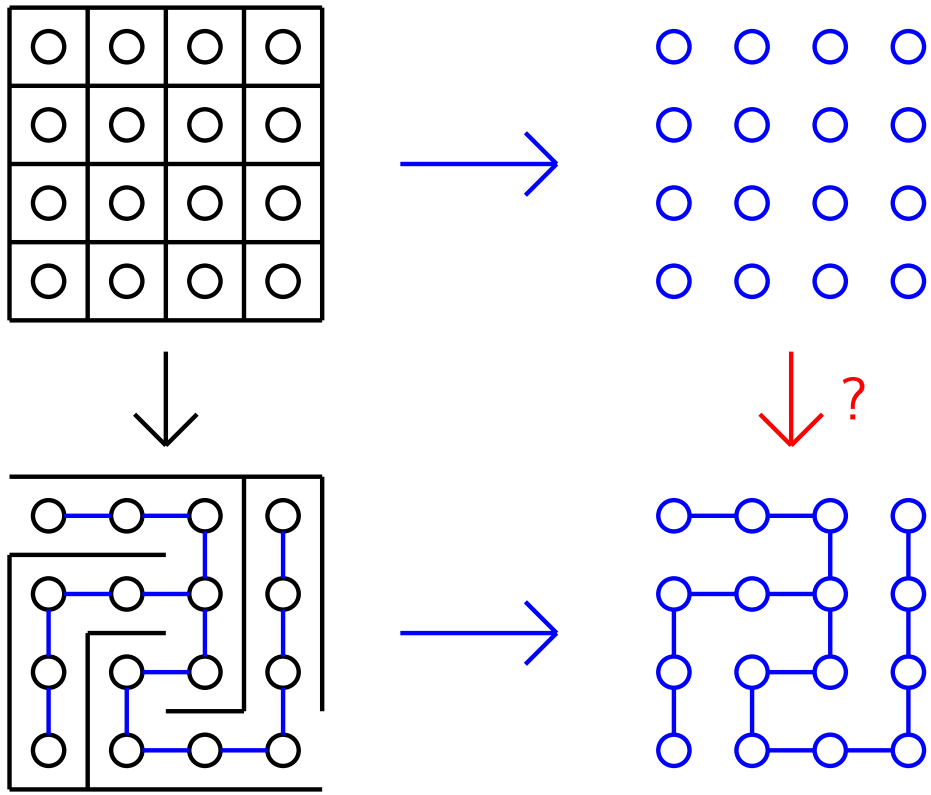






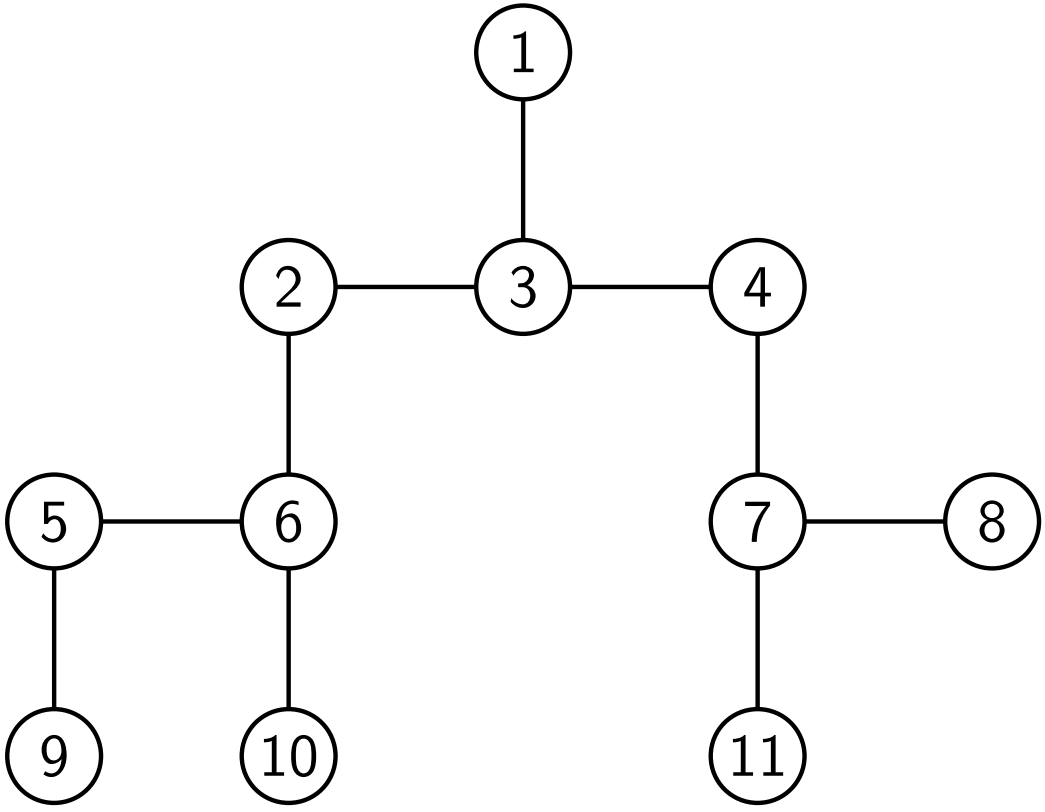






How can the 'perfect' graph be made?

- DFS : One of the searching algorithms



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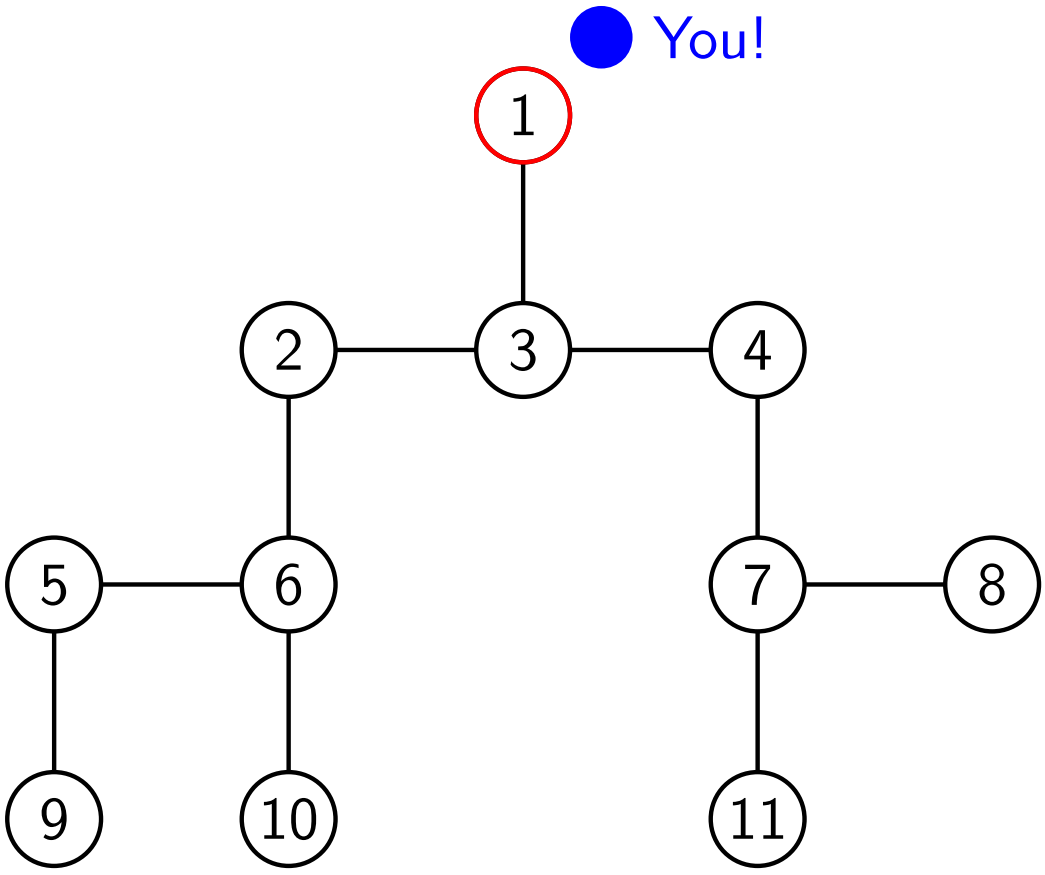

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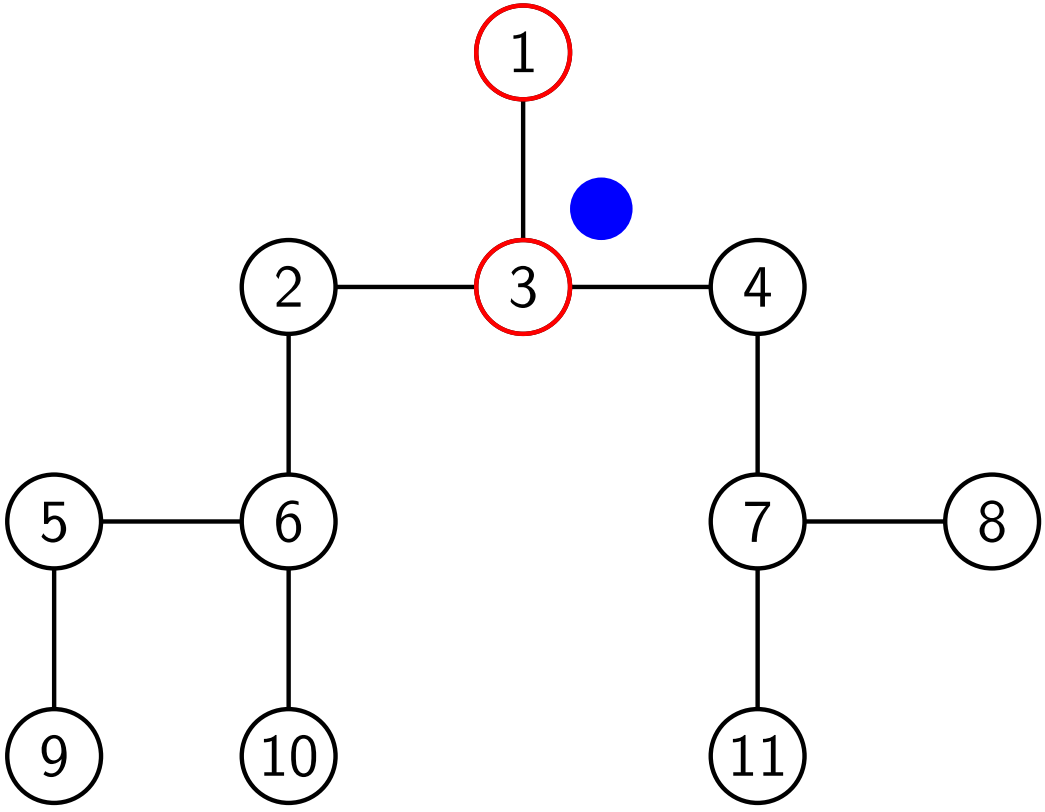

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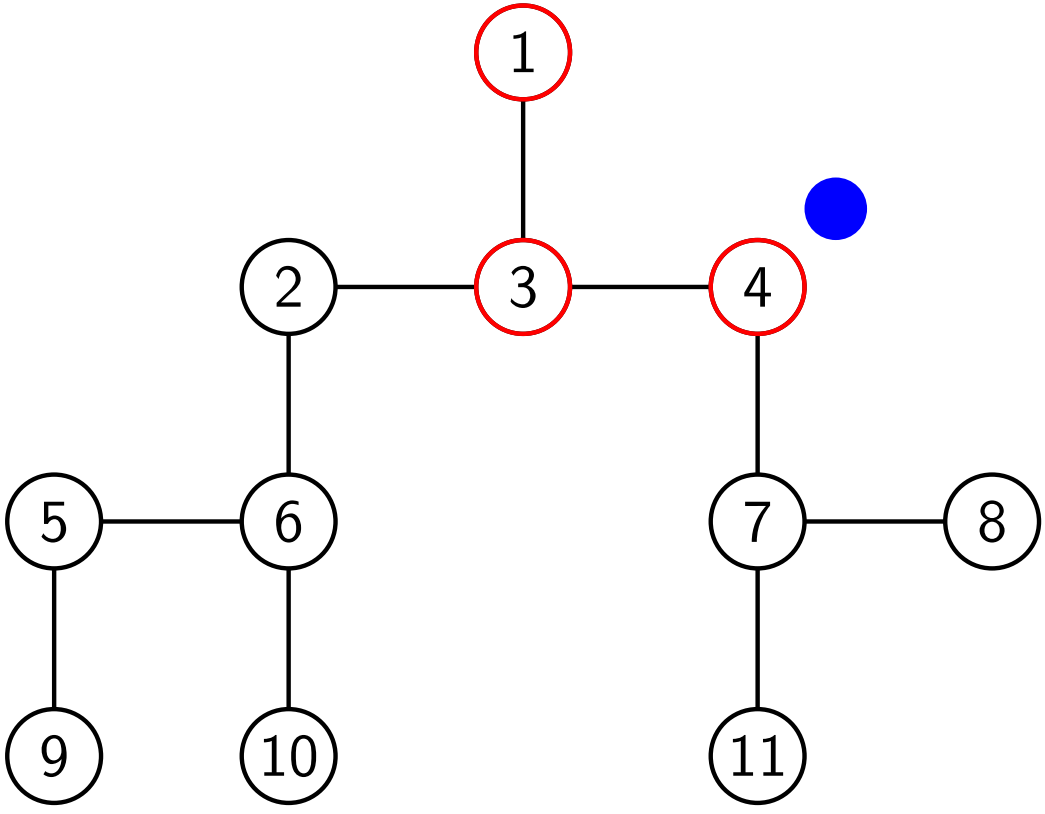

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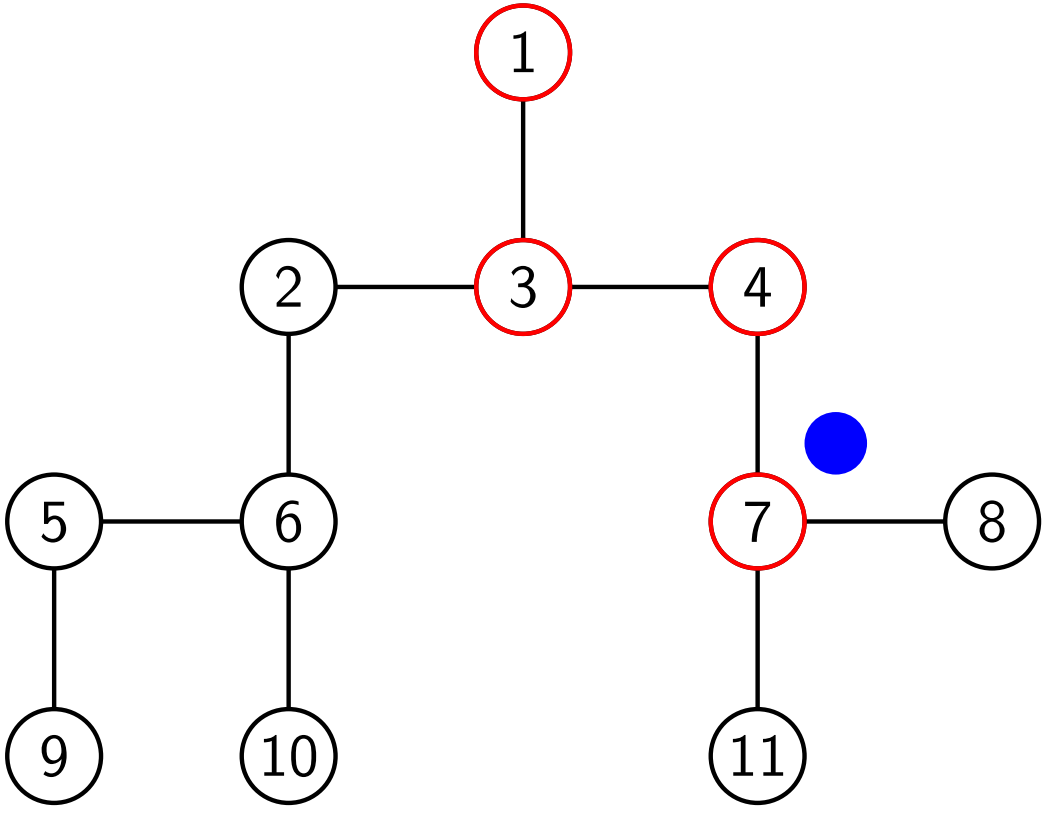

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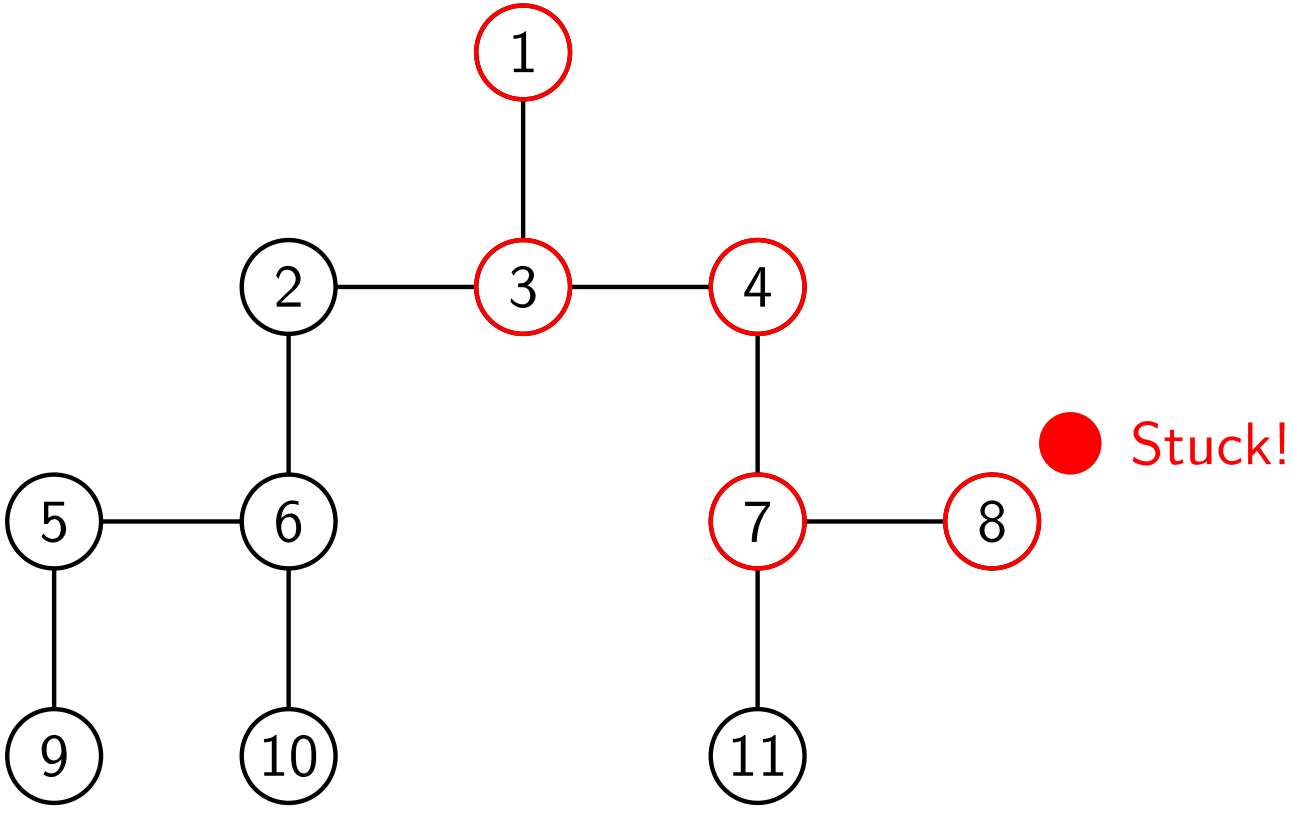

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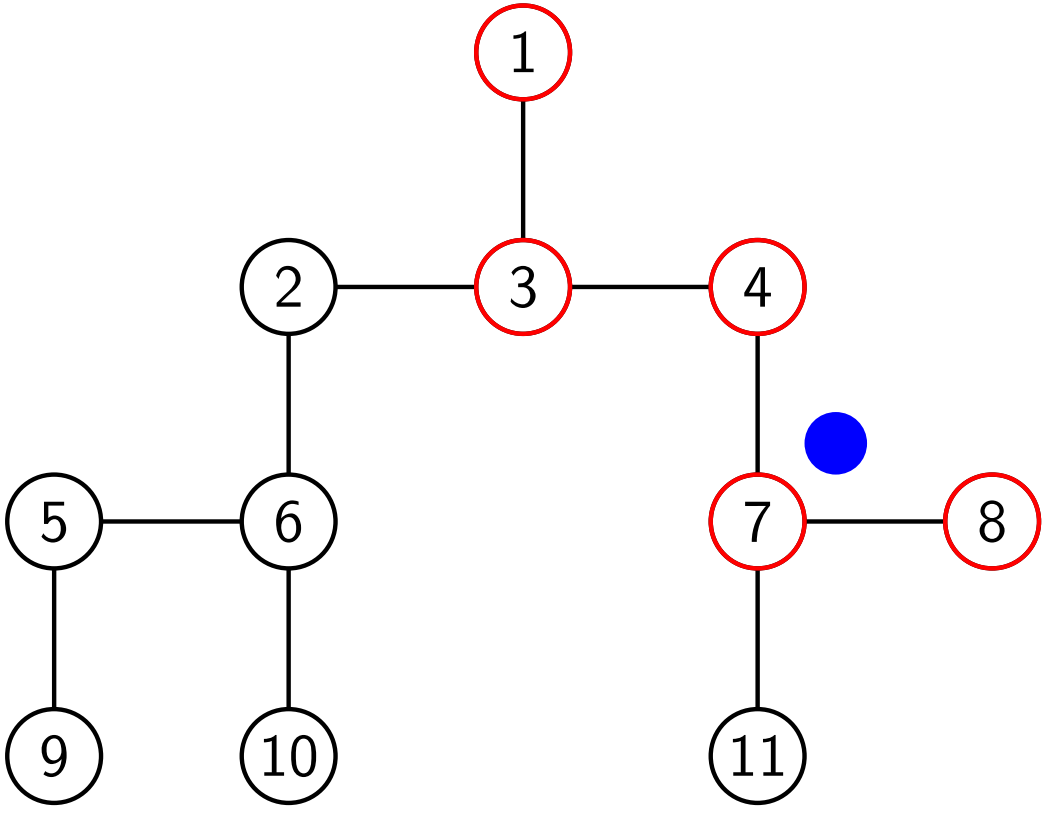

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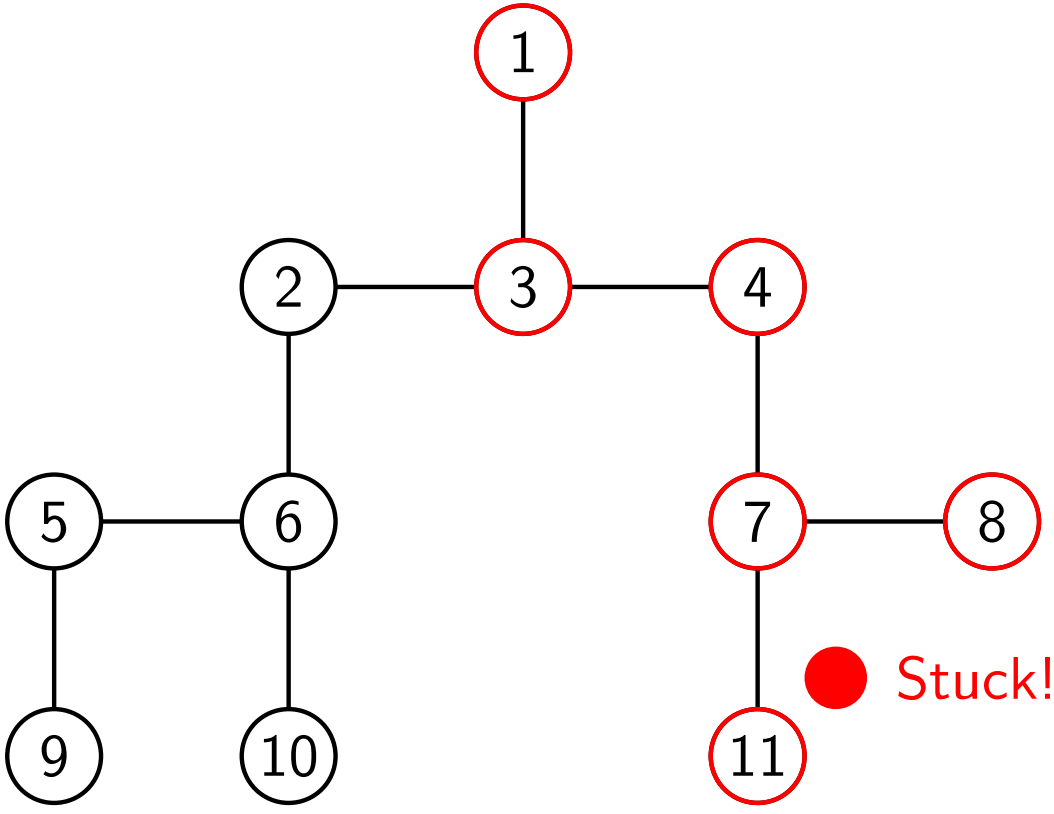

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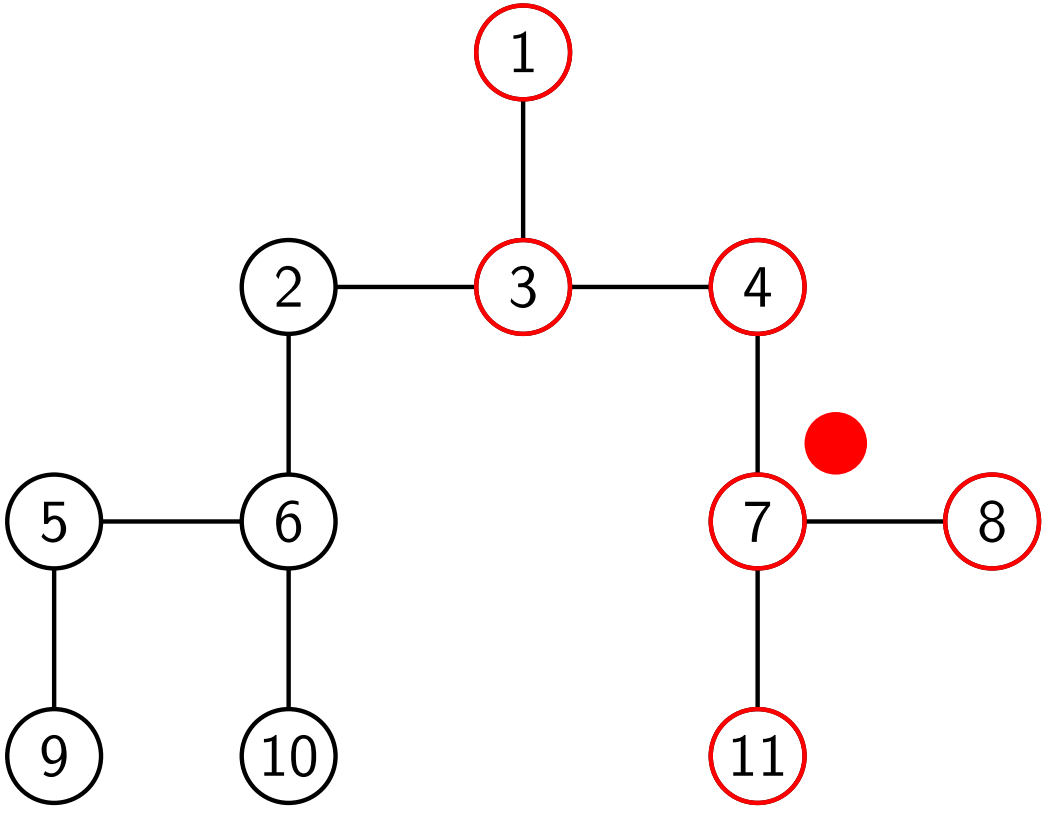

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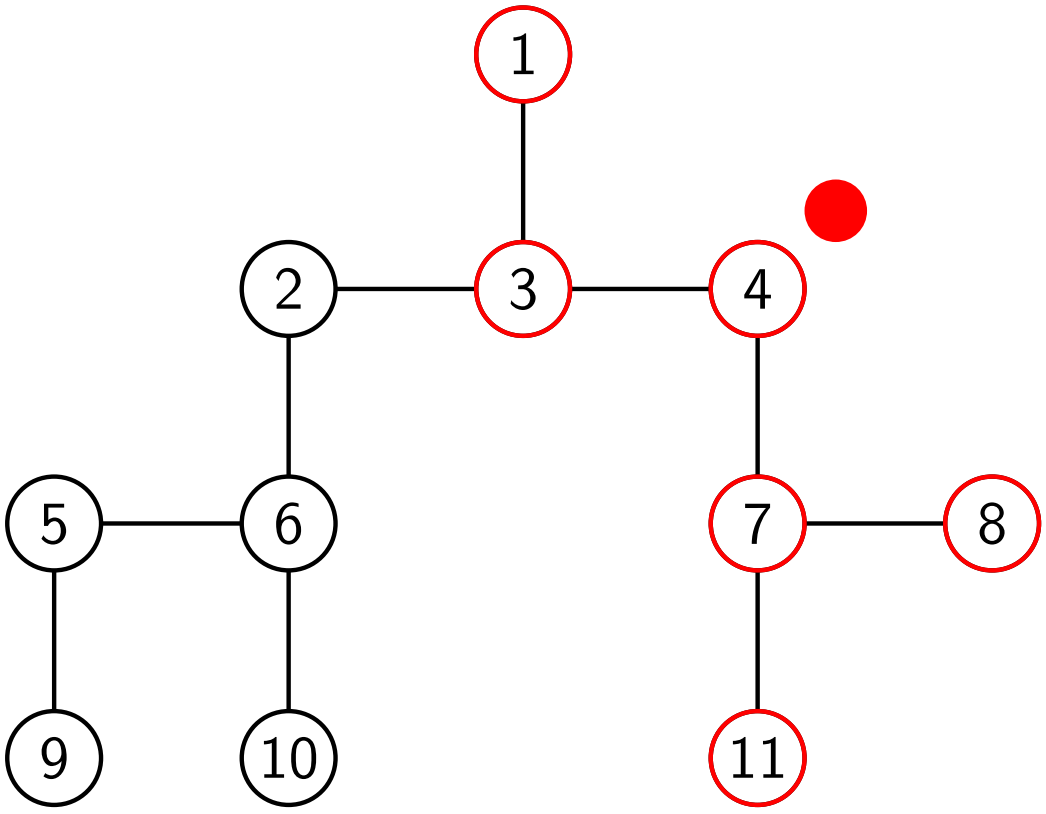

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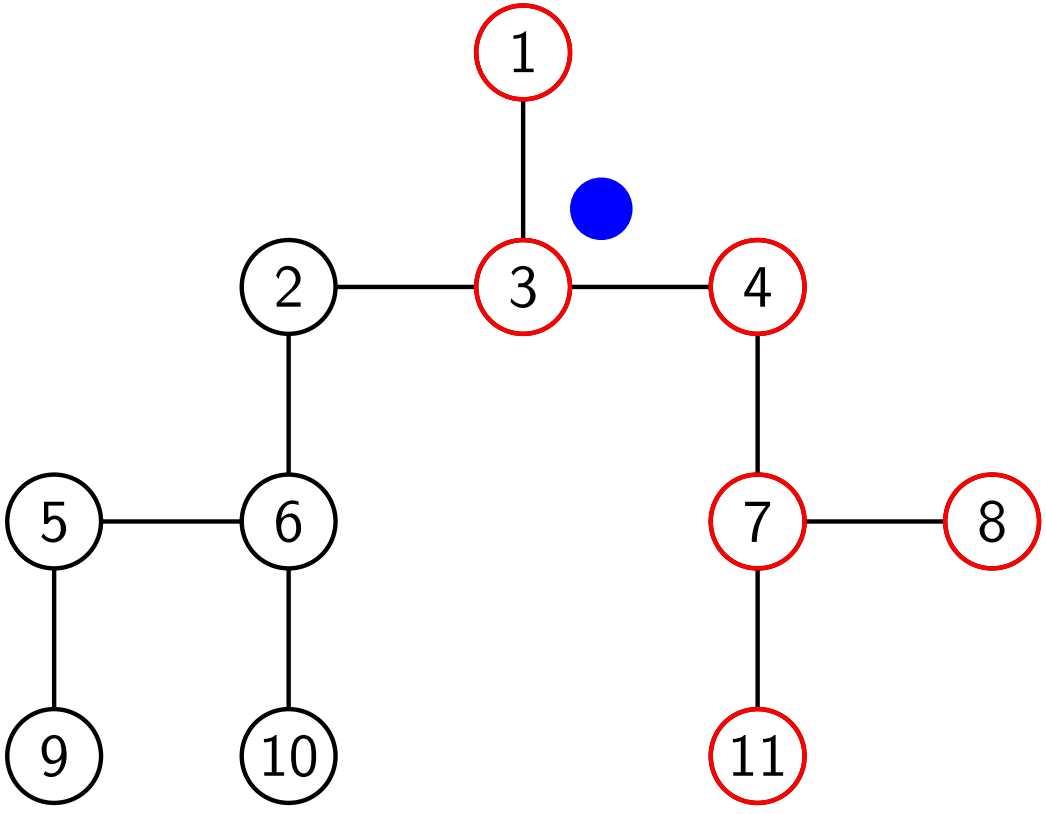

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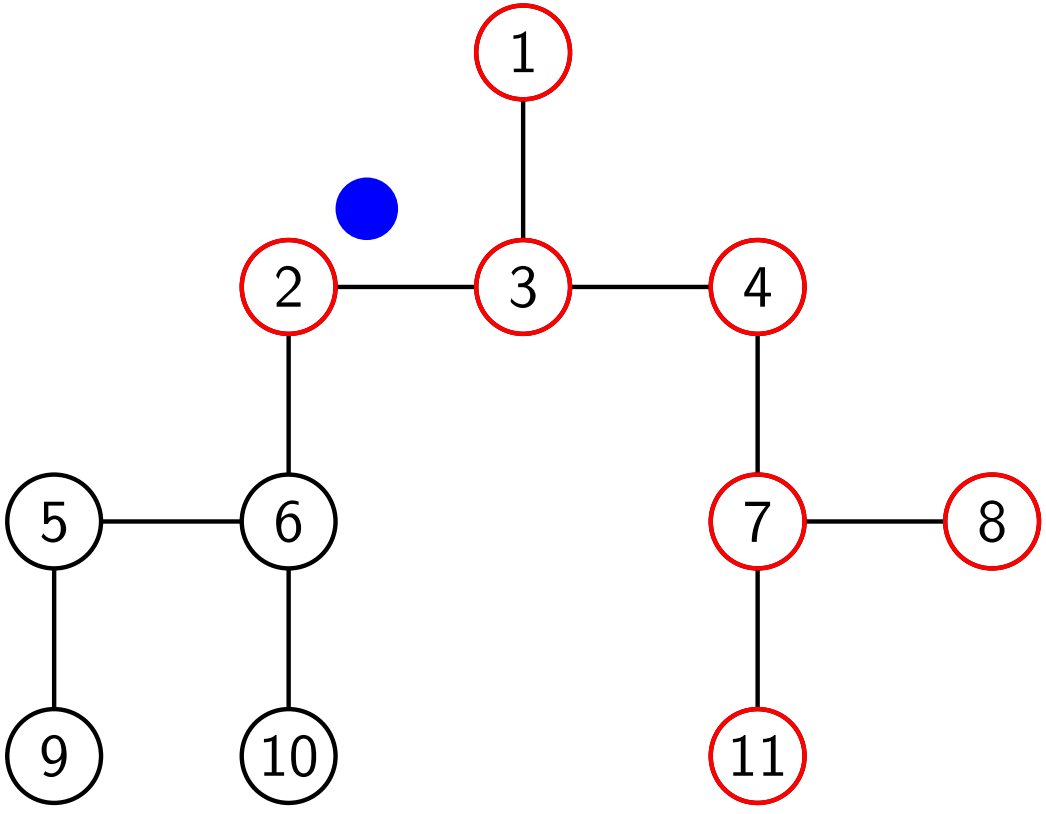

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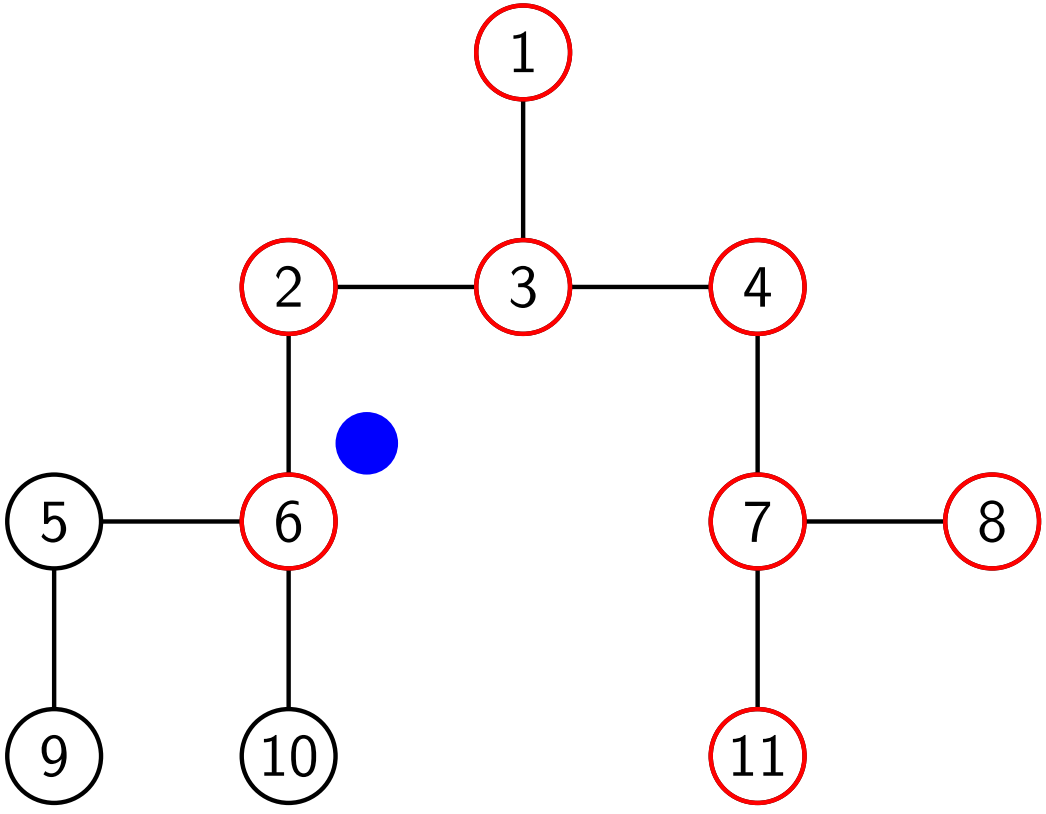

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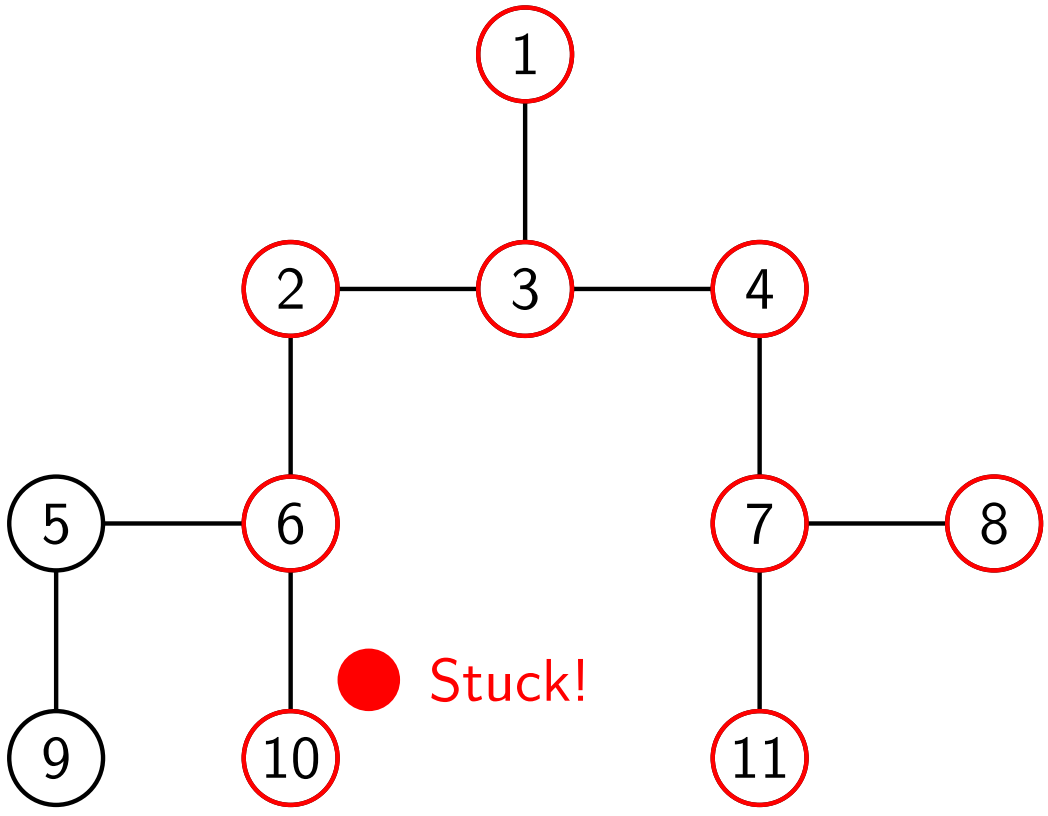

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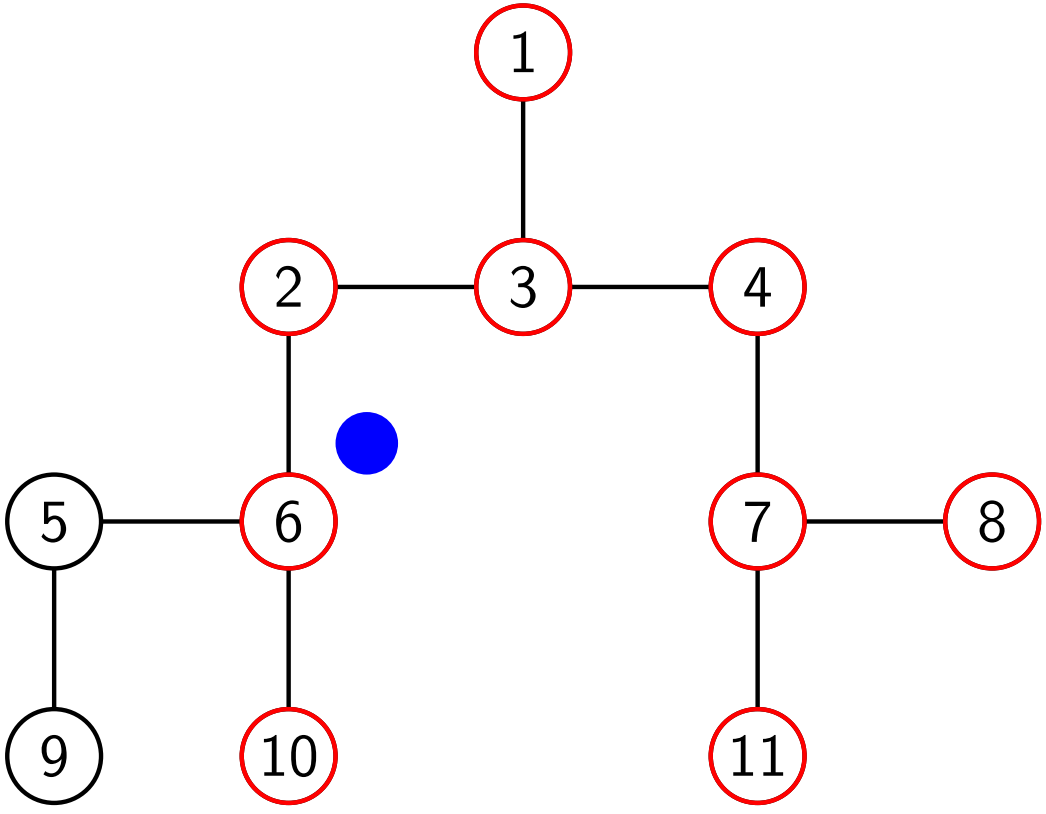

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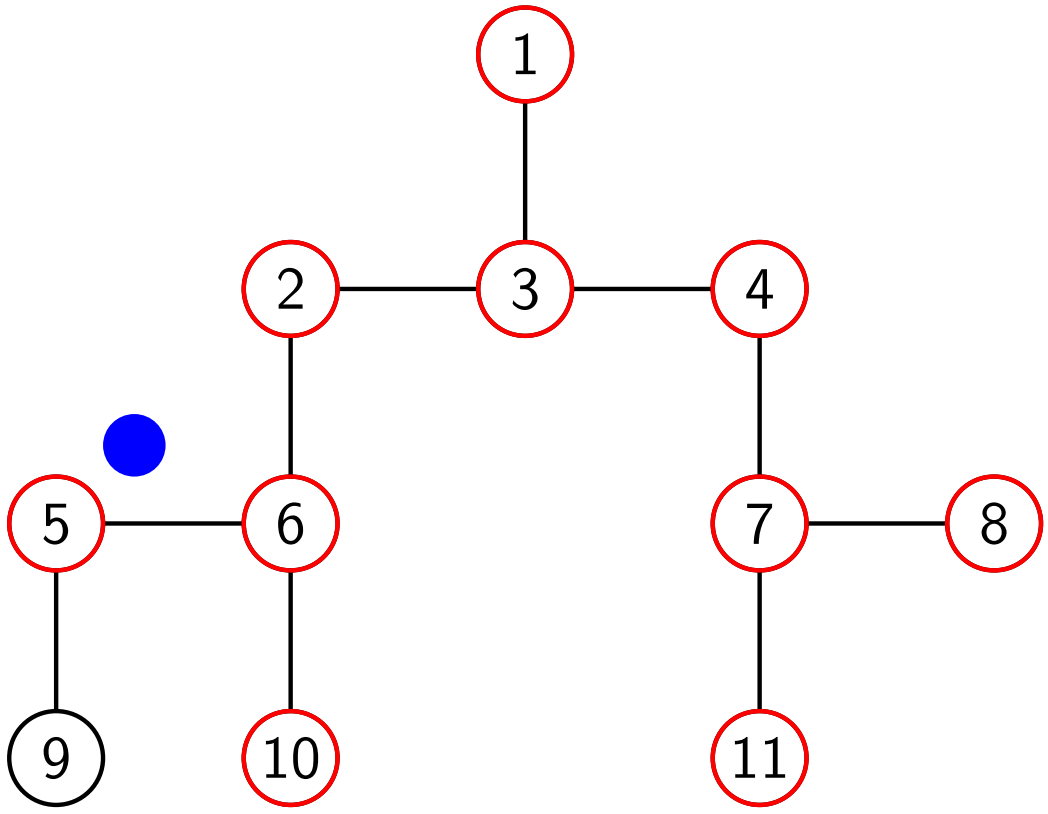

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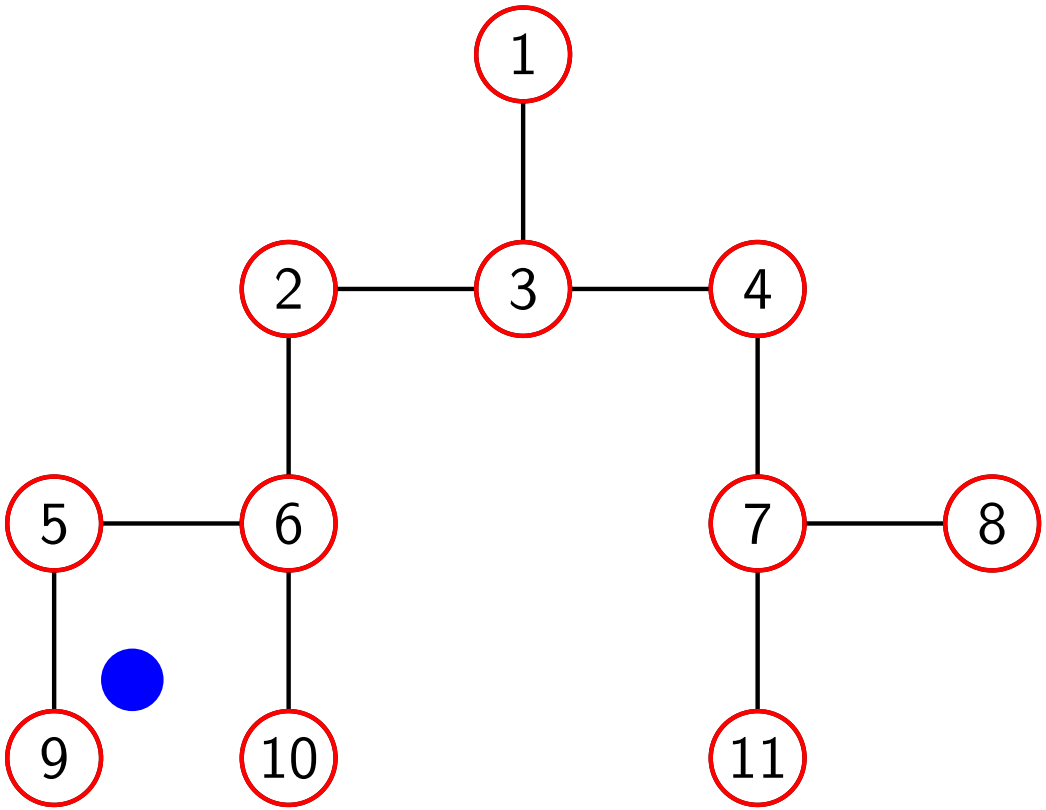

End!

- Go deeper as you can.

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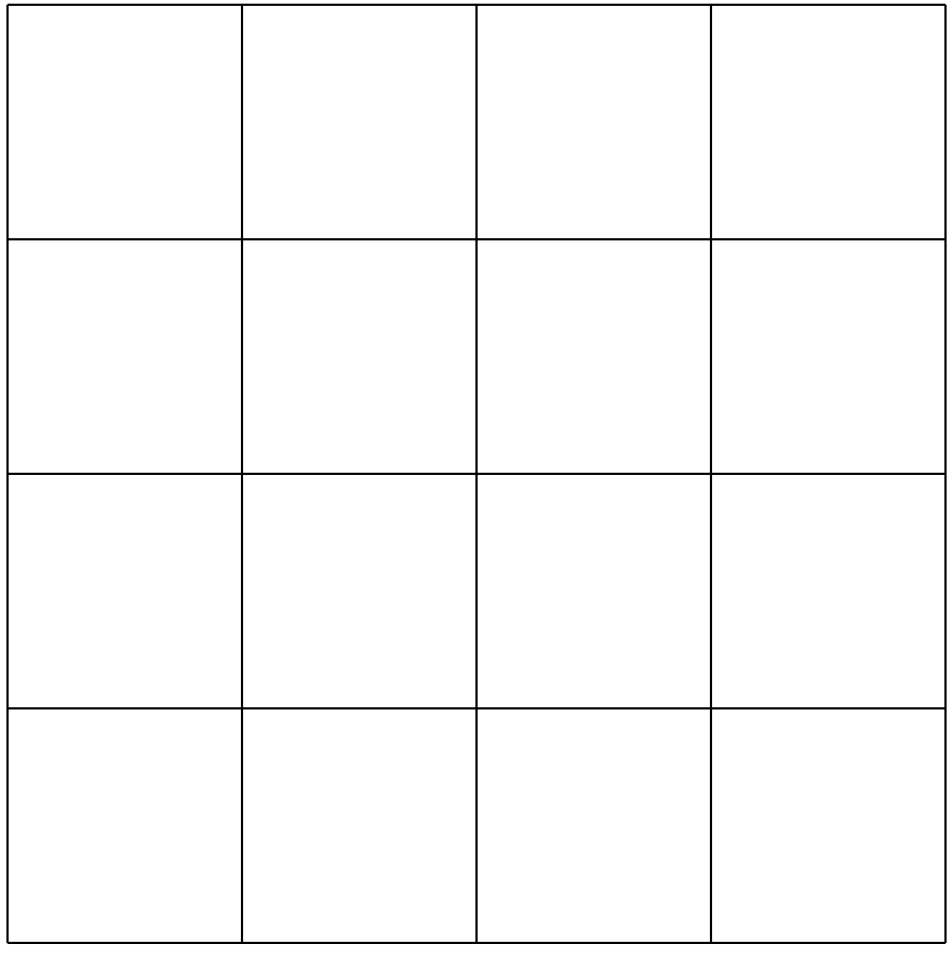
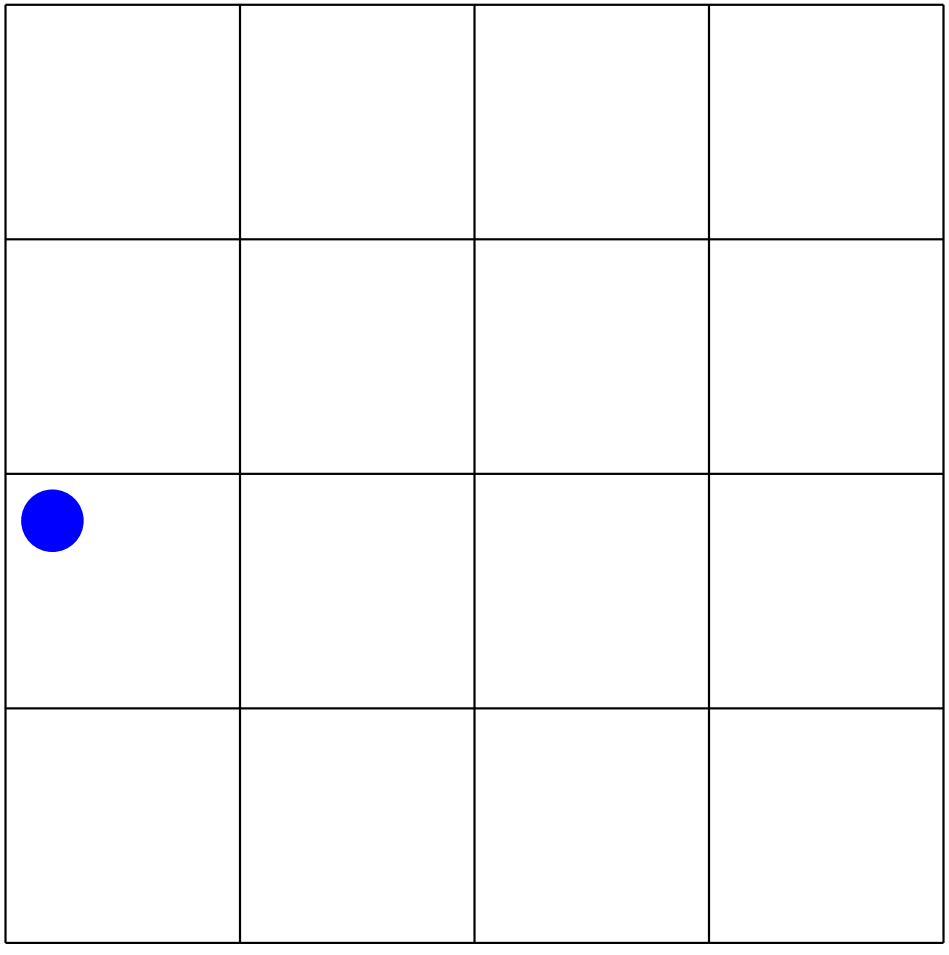
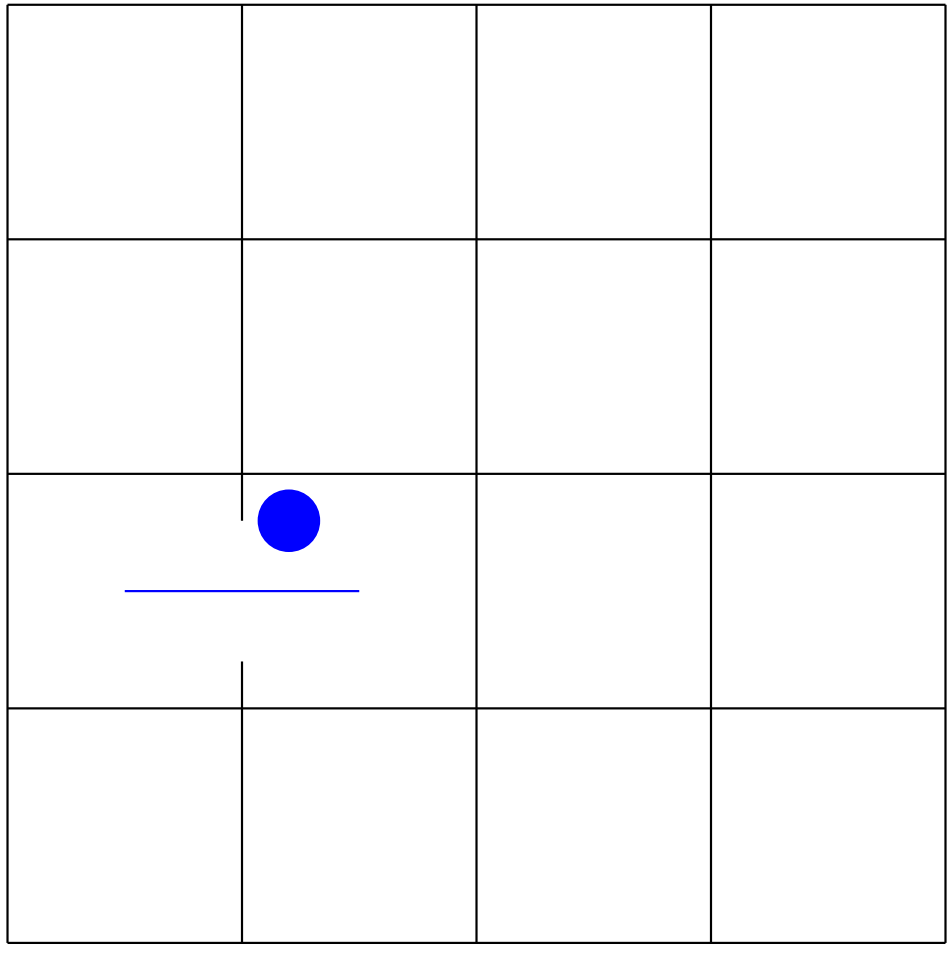
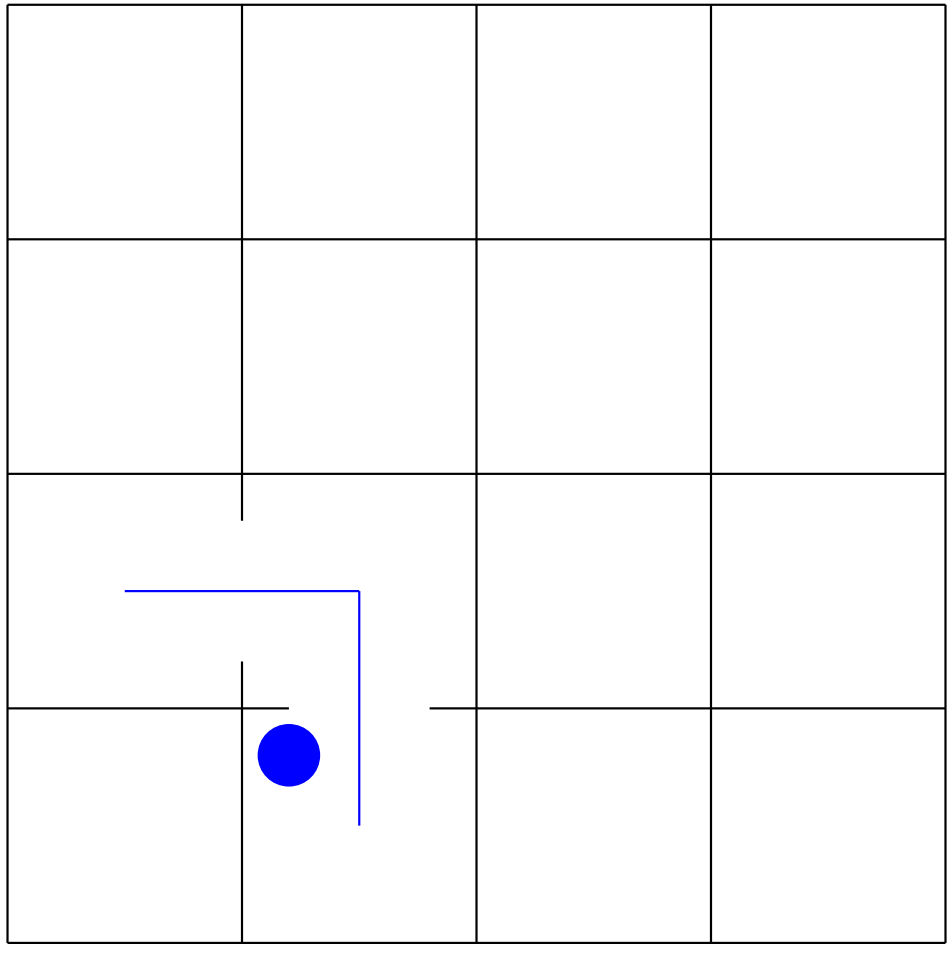
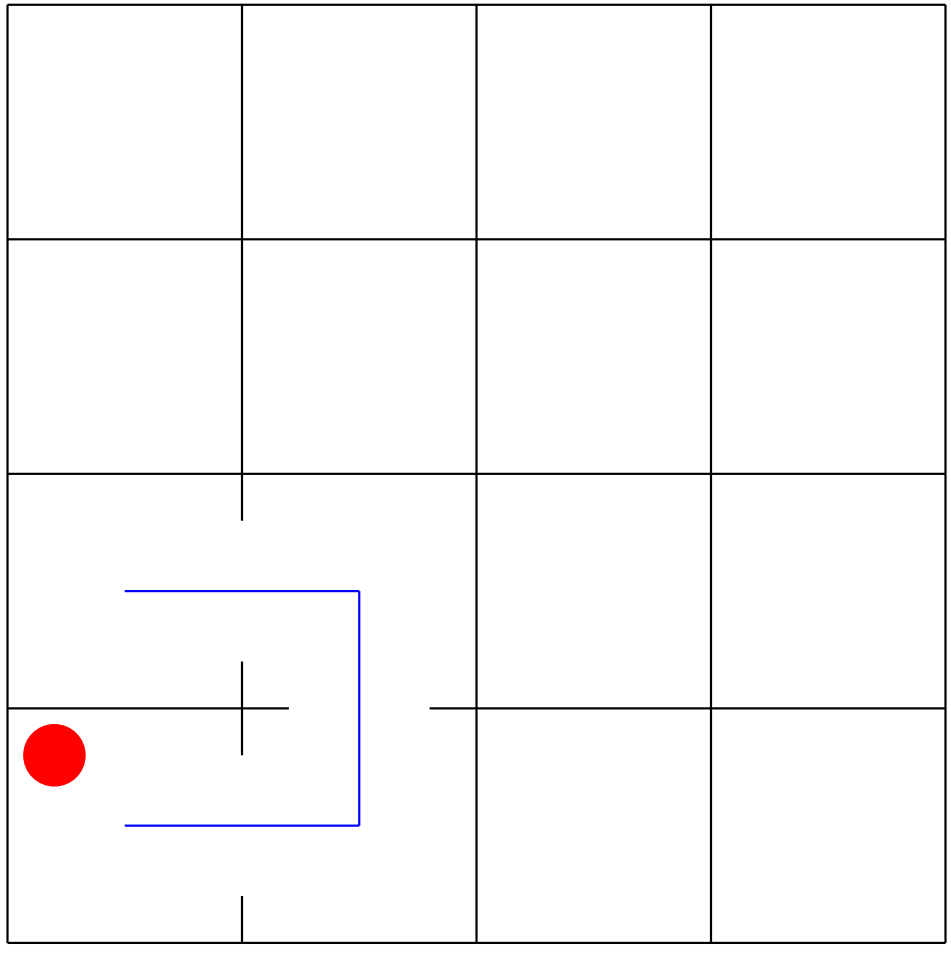
- Backtrack to possible branch when you are stuck.

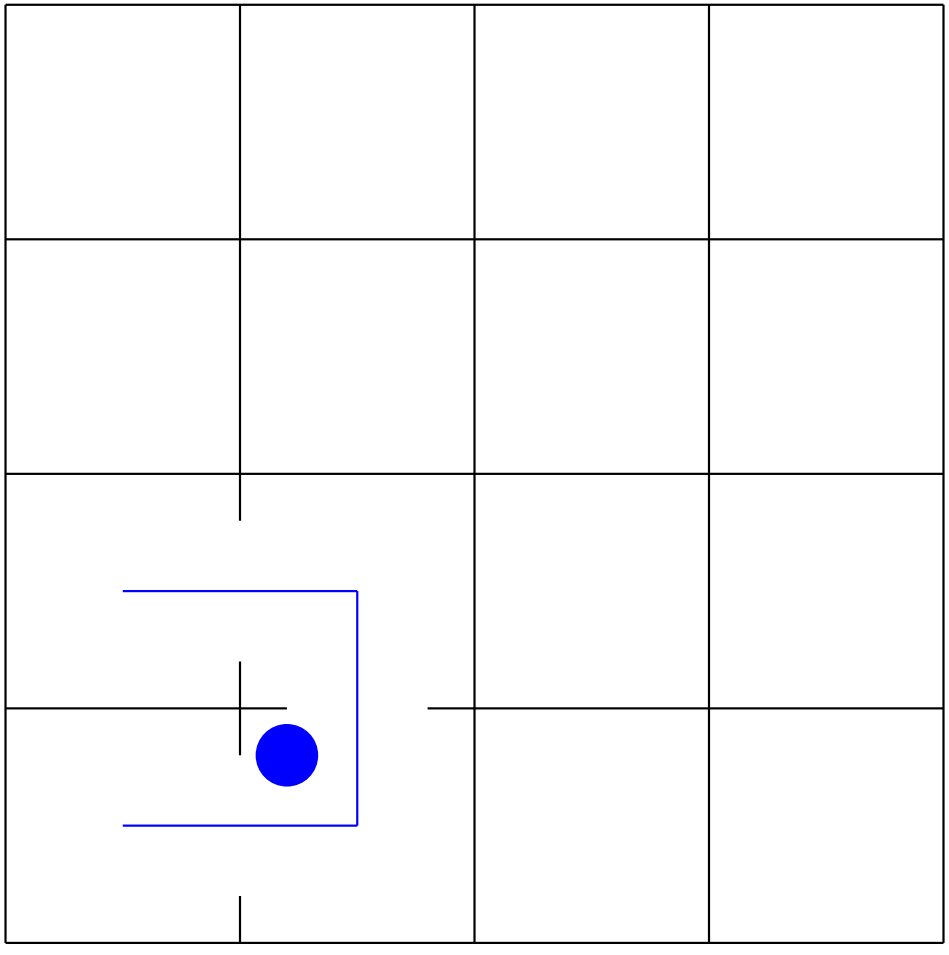


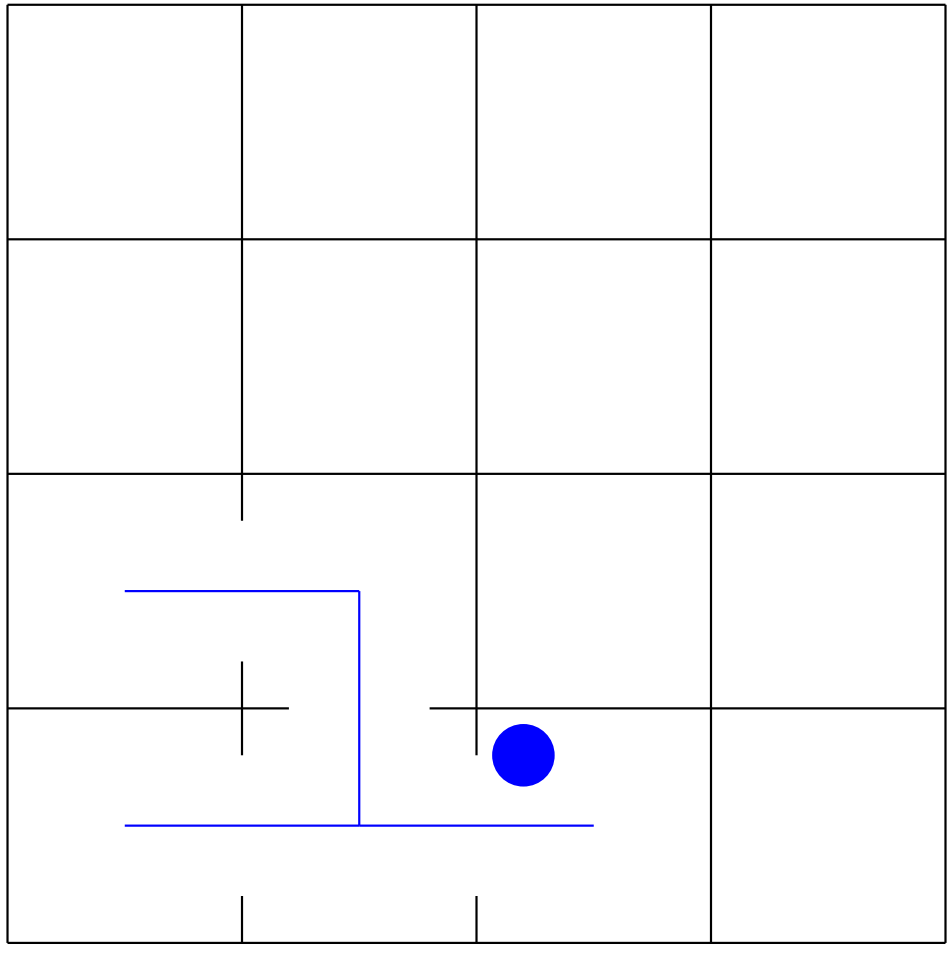


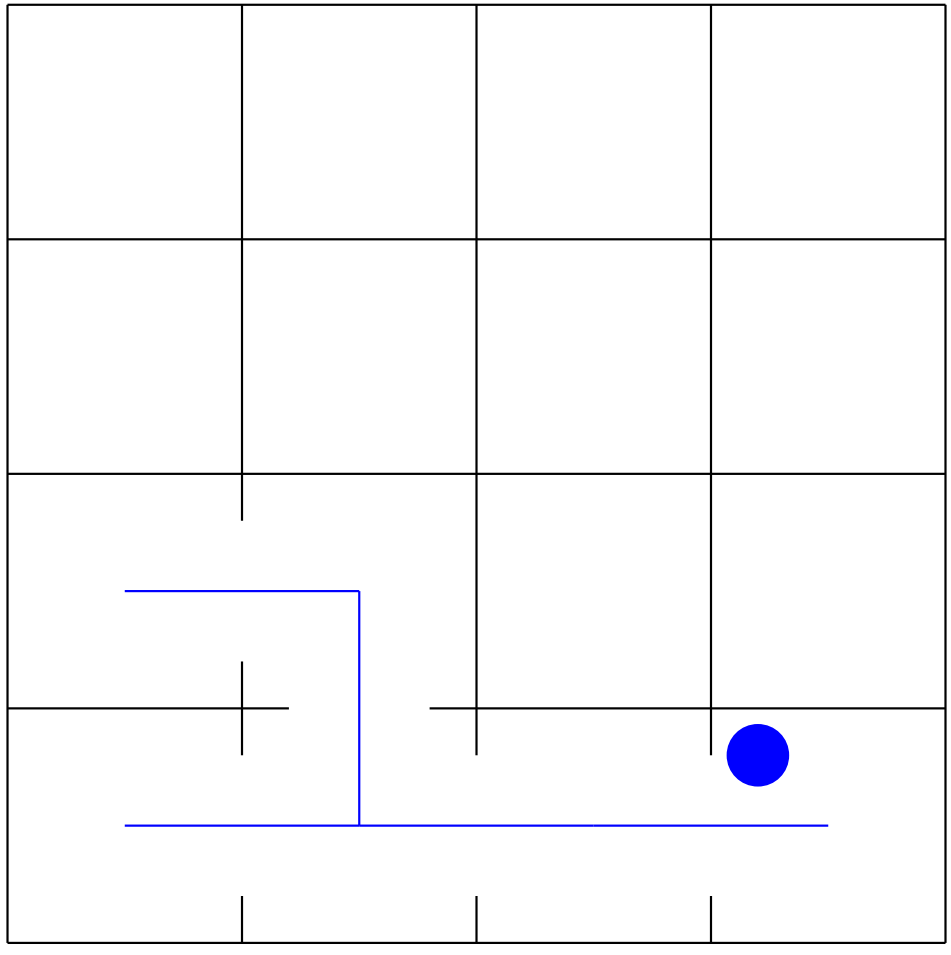


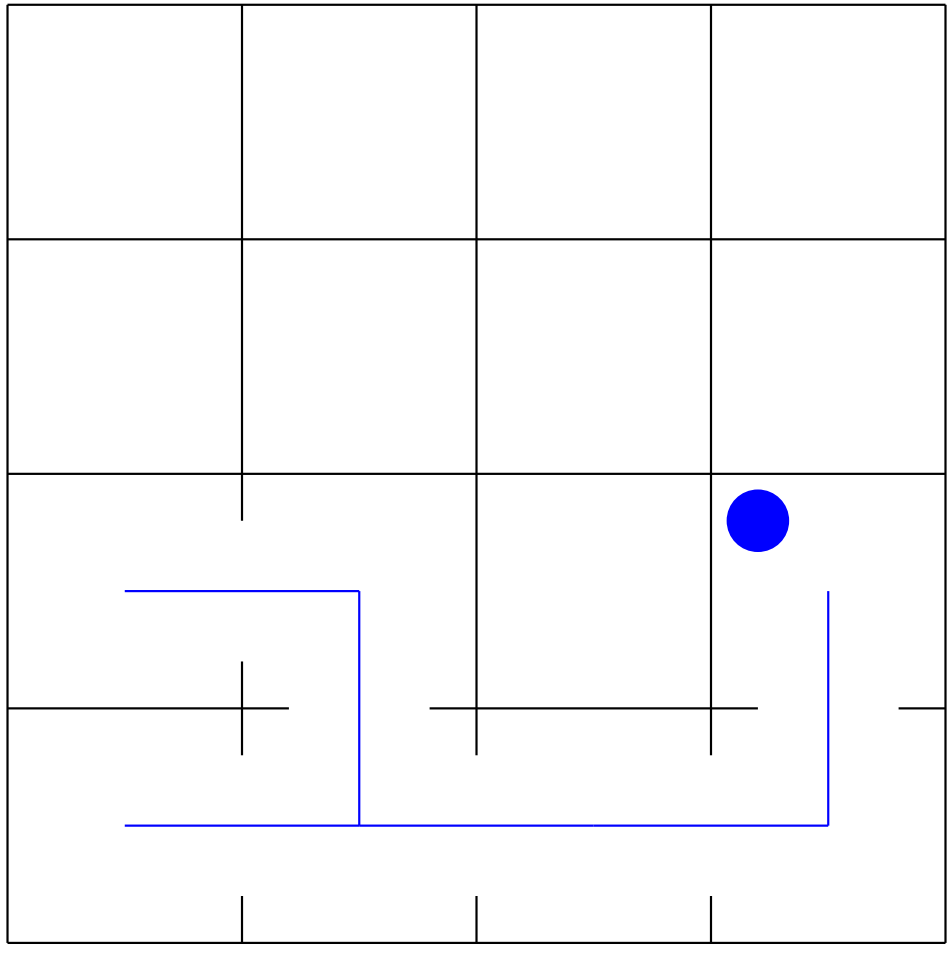


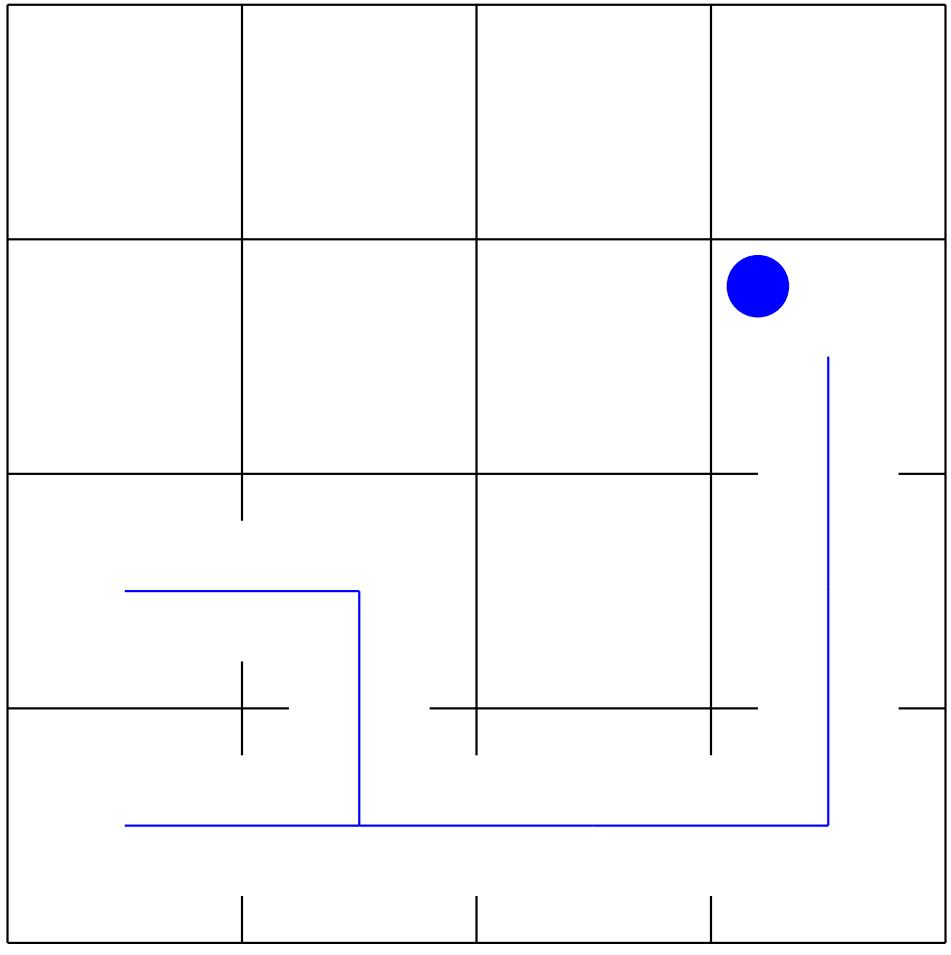


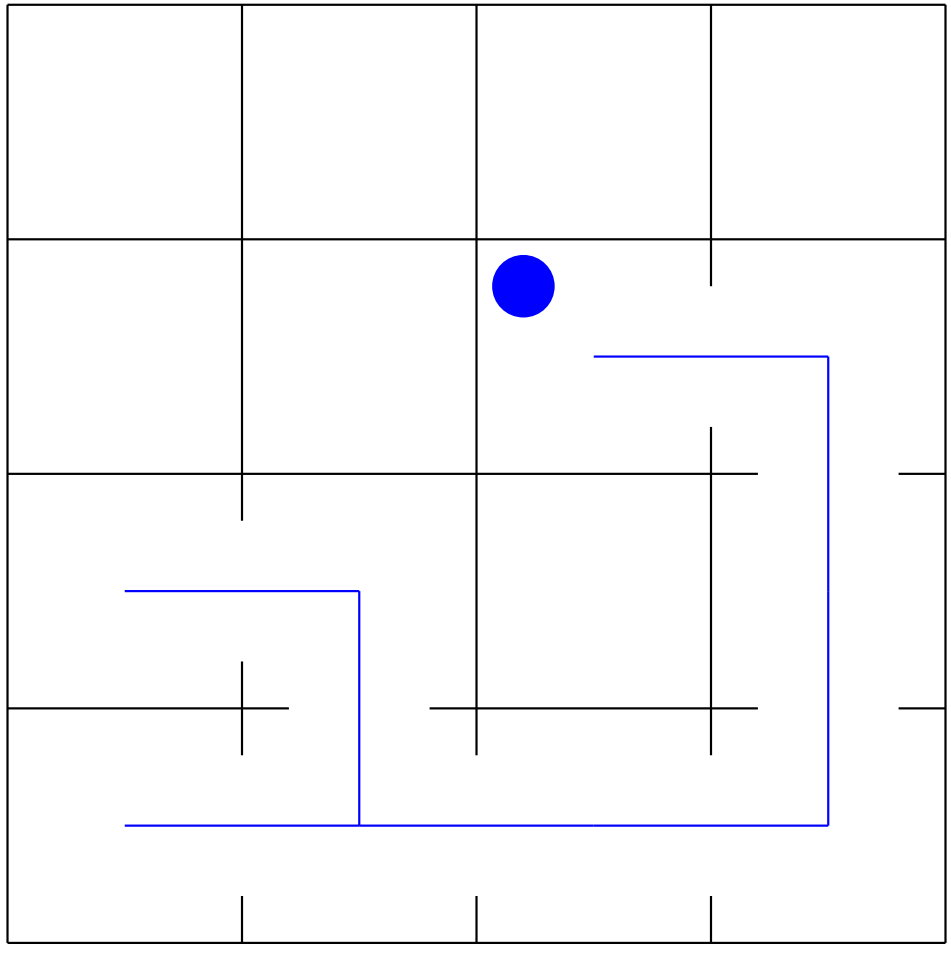


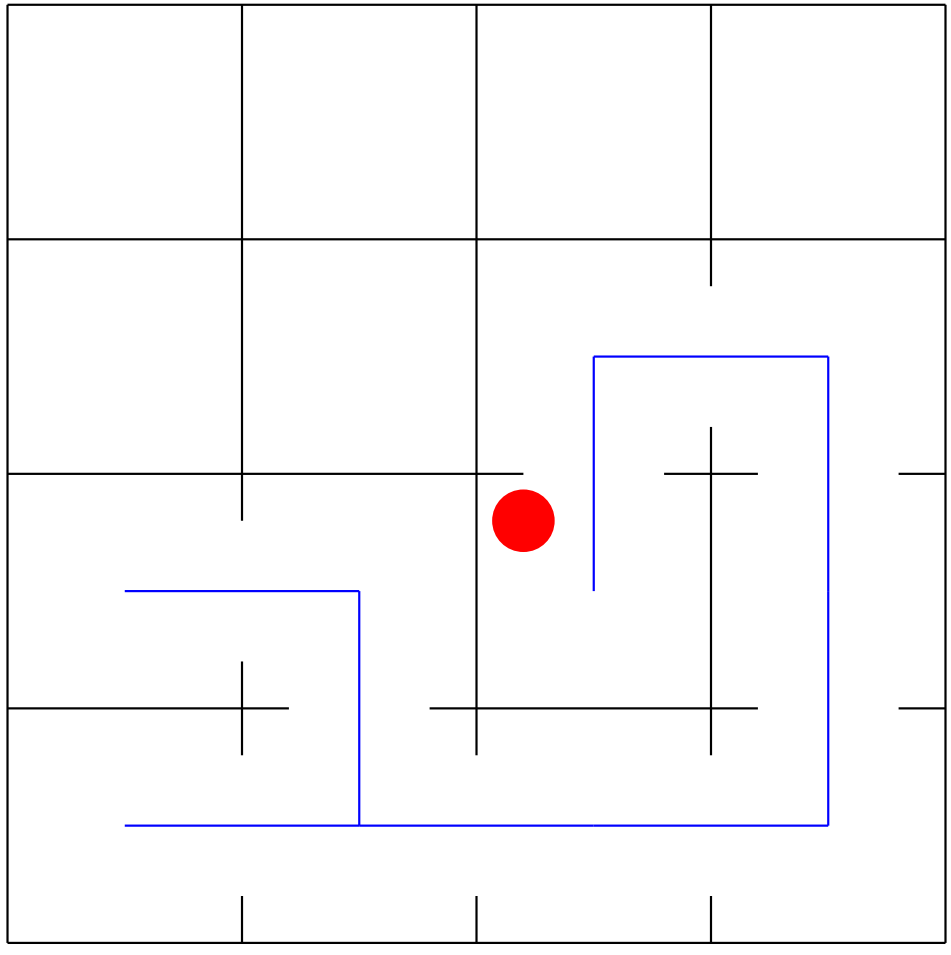


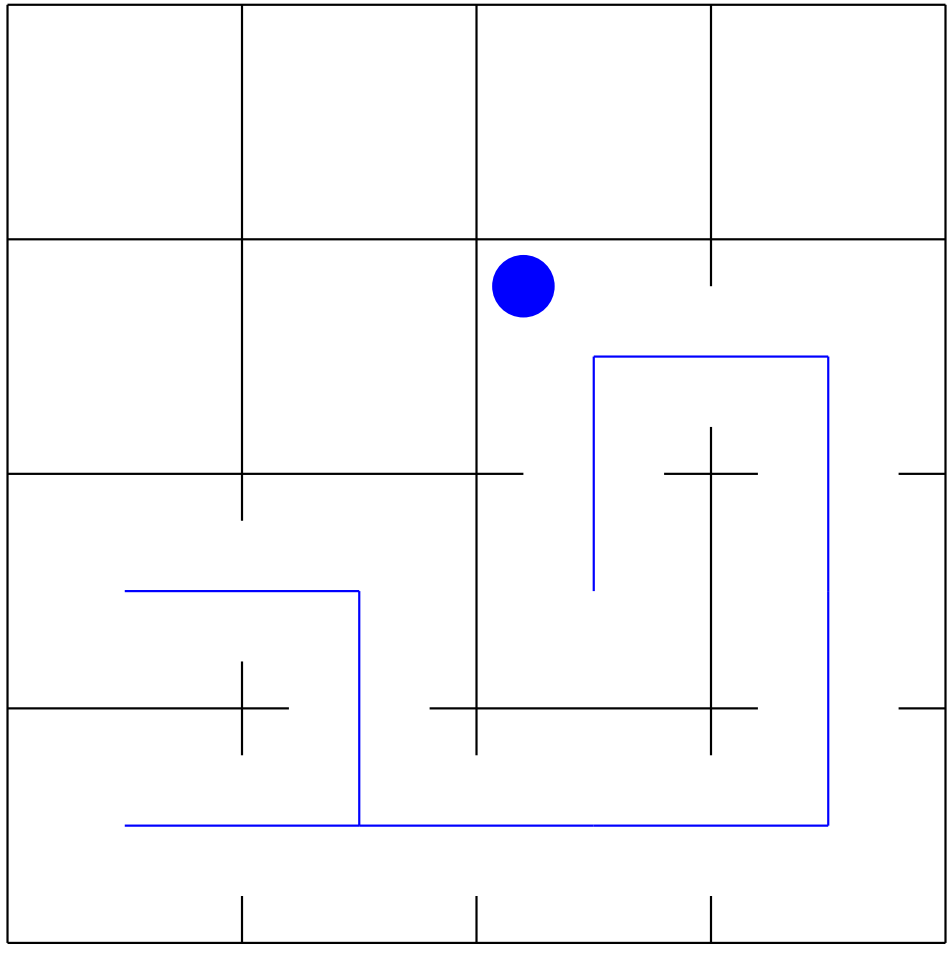


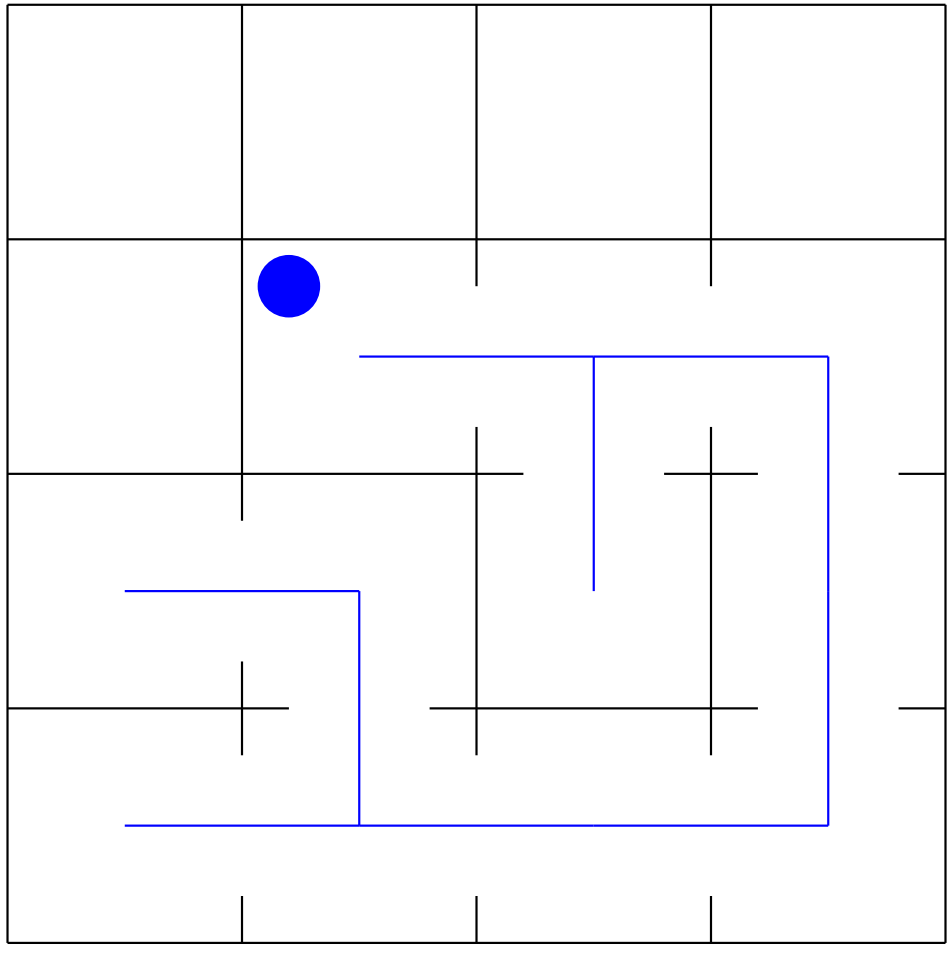


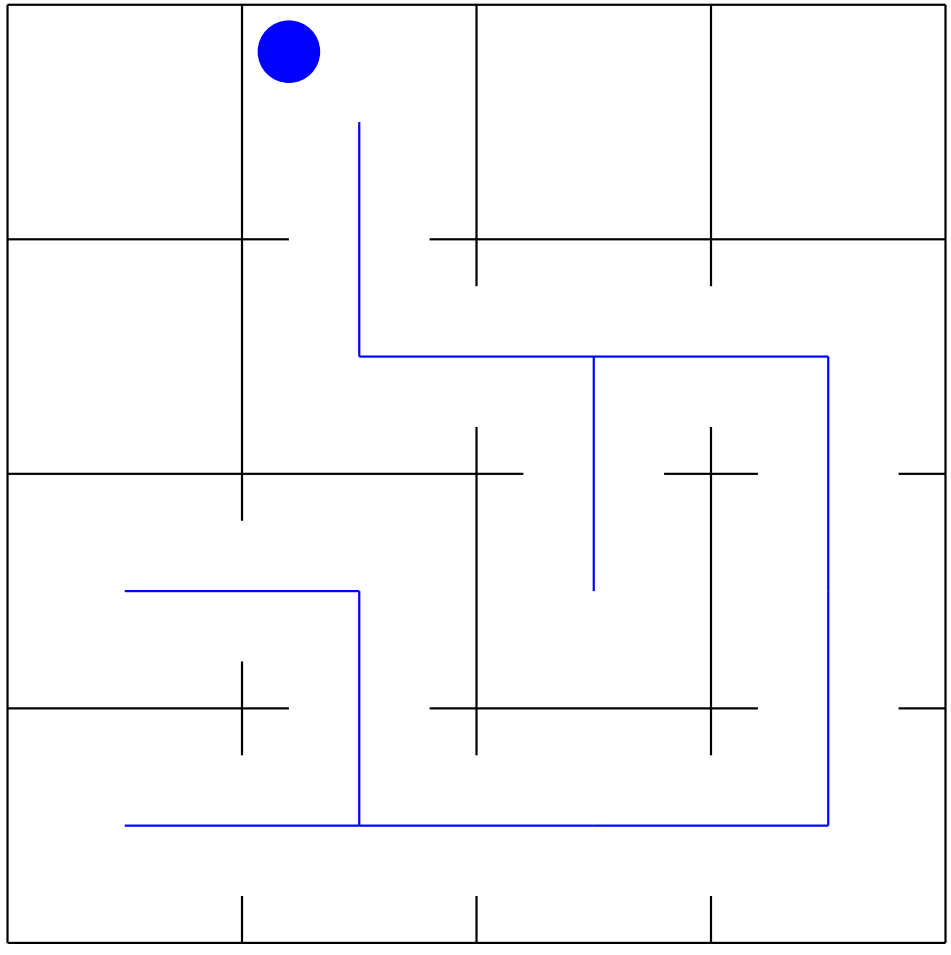


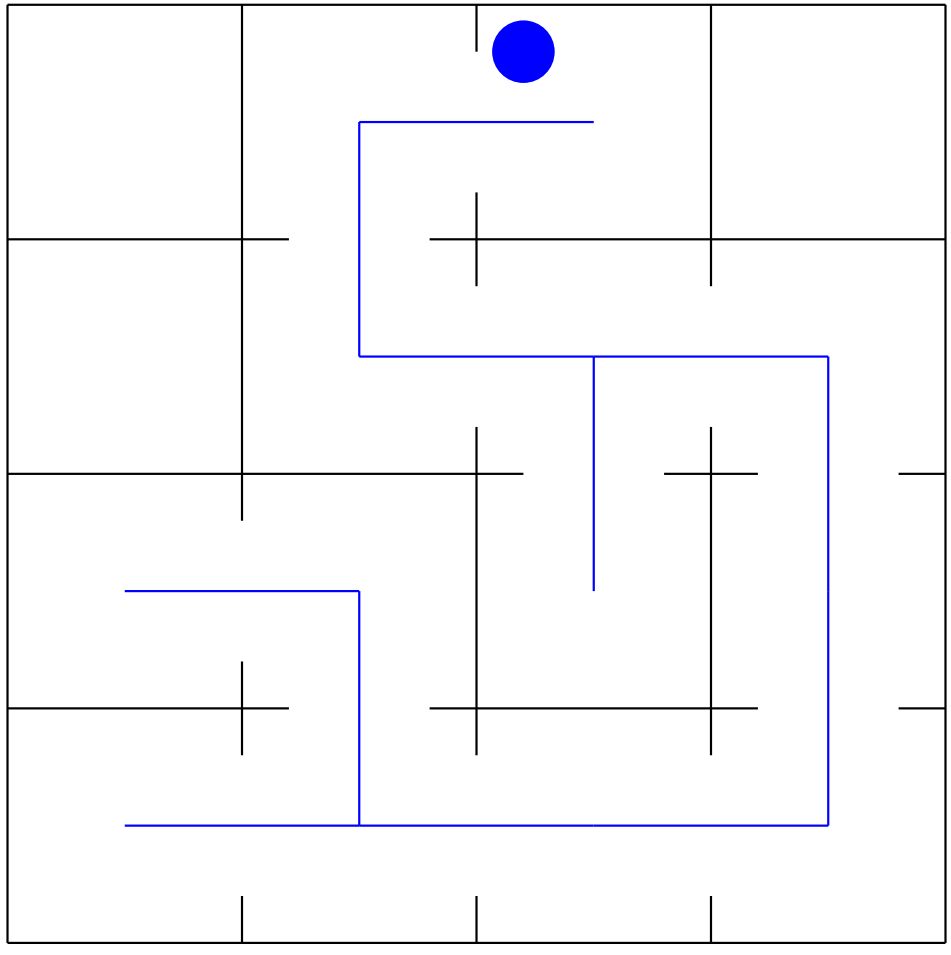


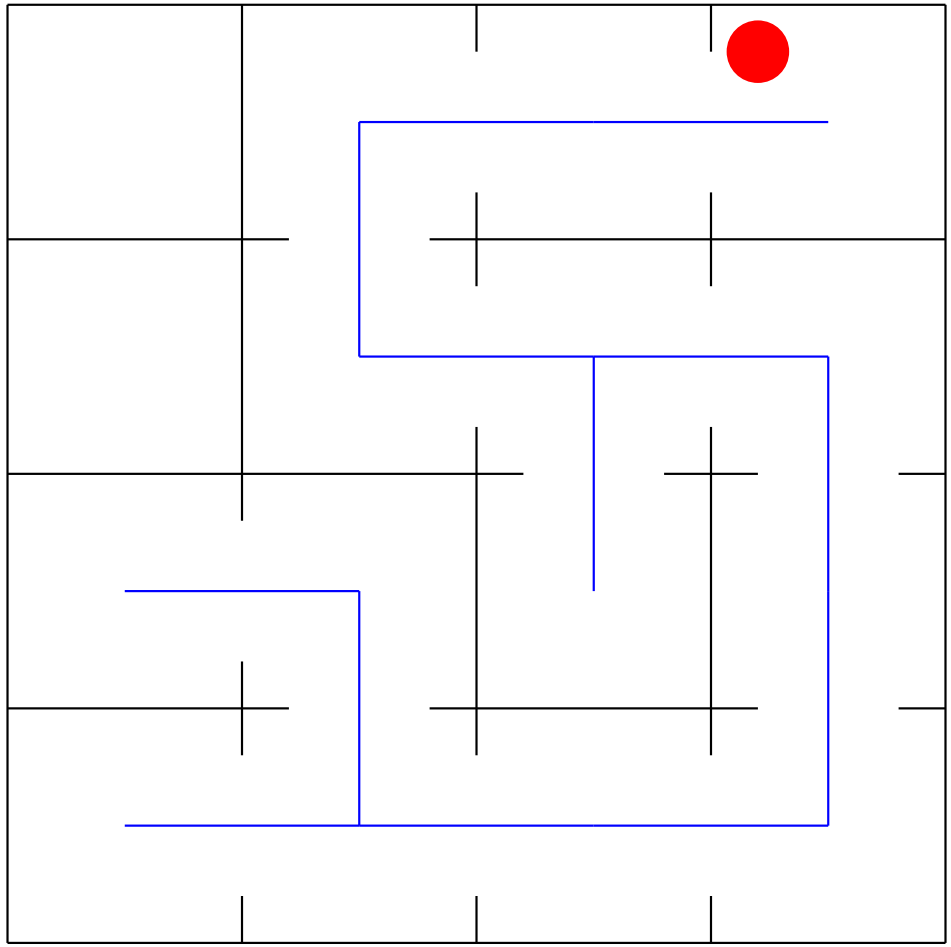


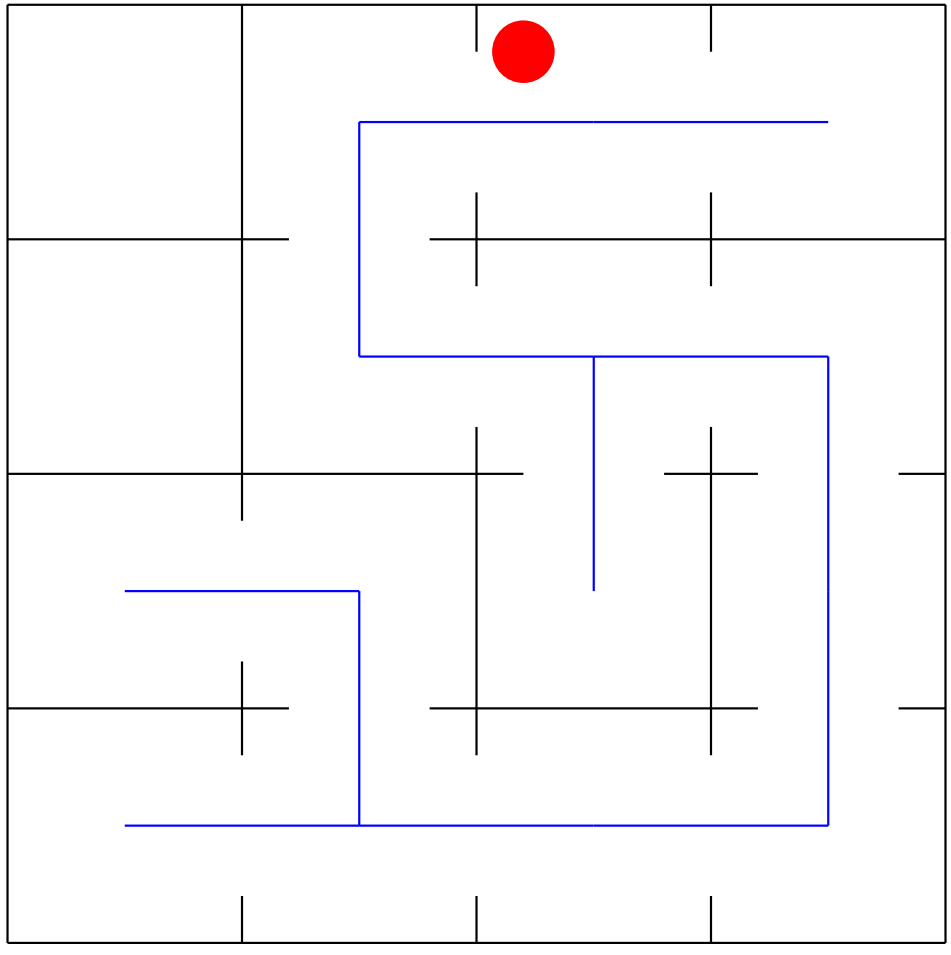


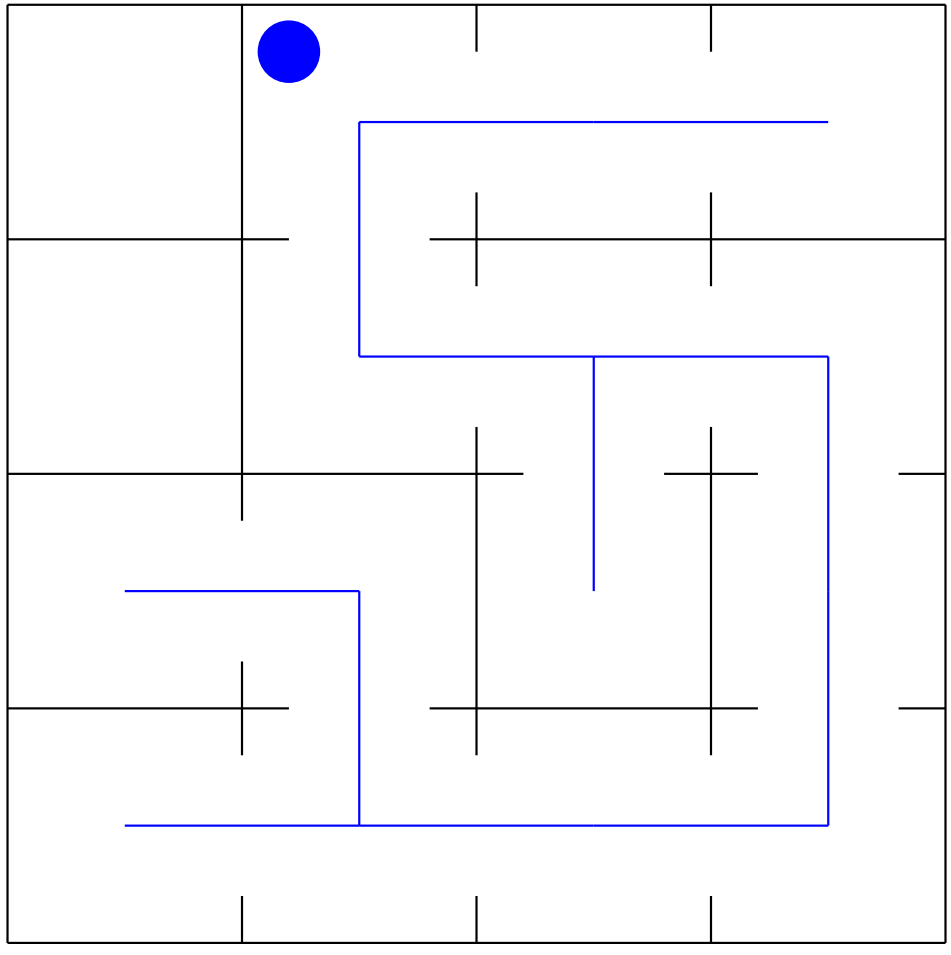


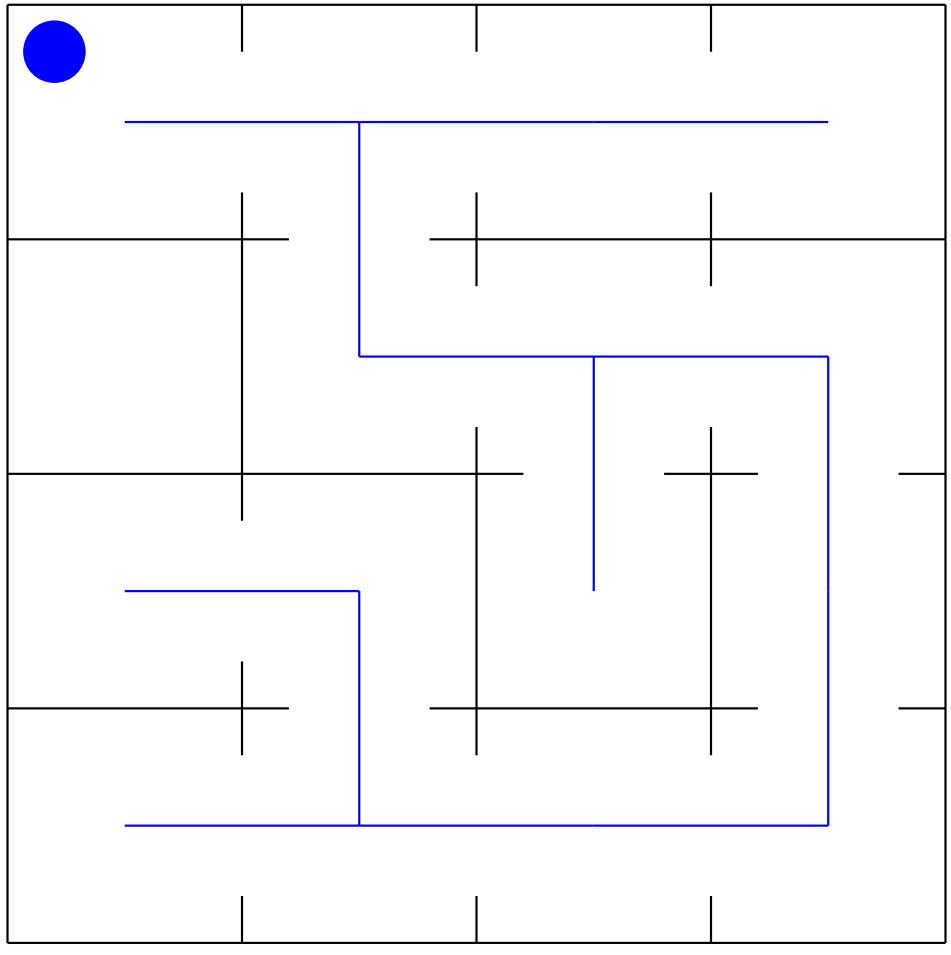


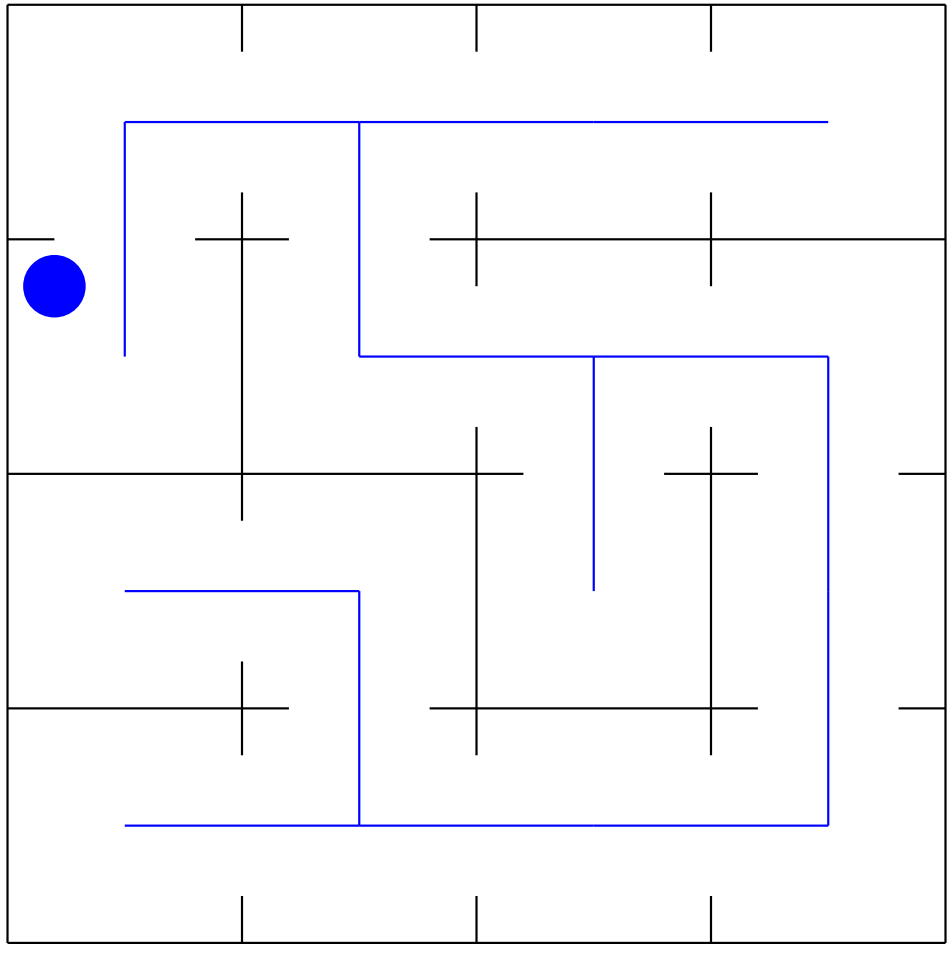


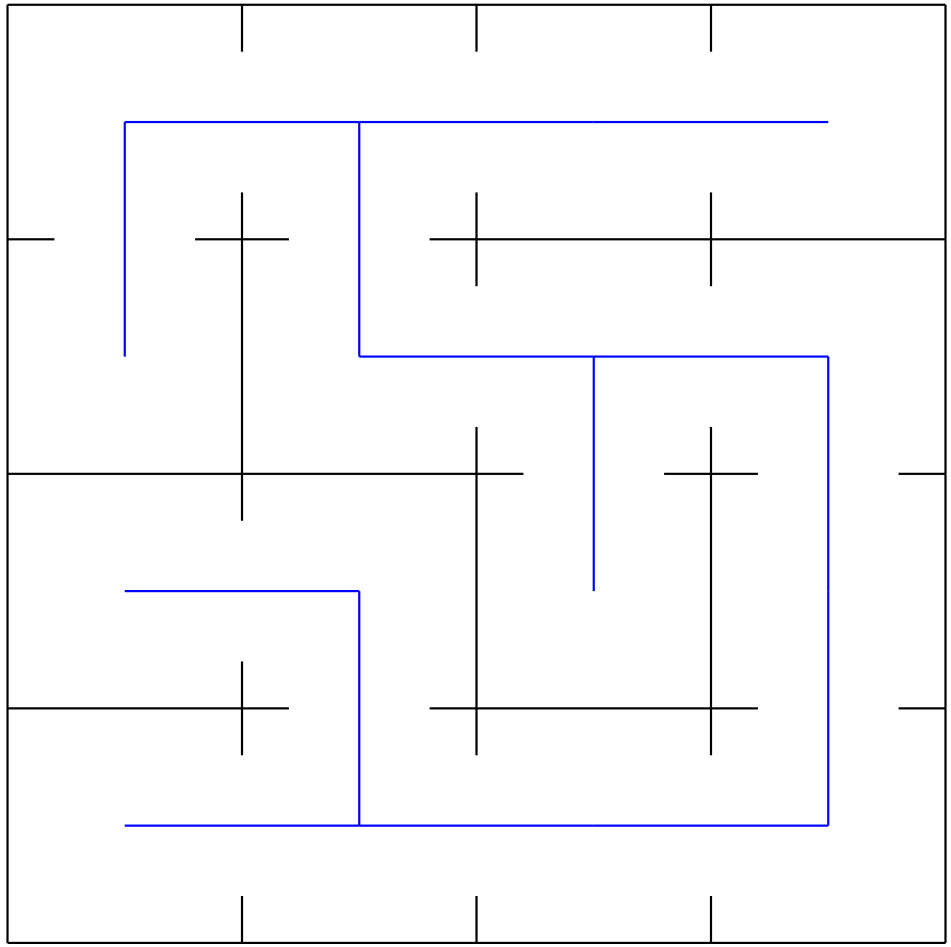






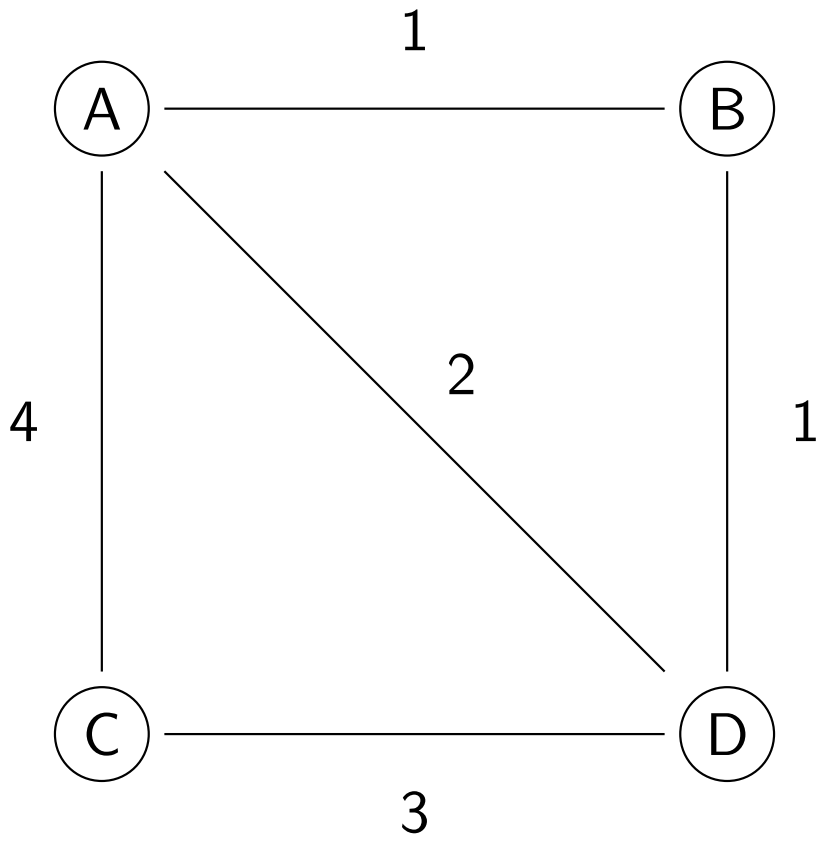




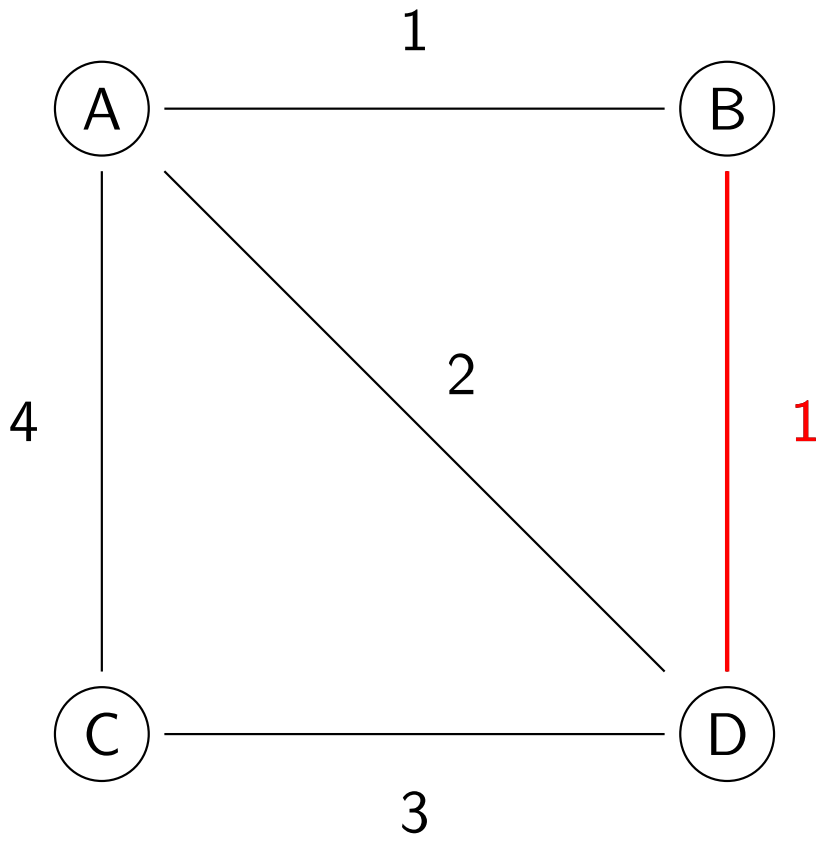


Perfect Maze!

- Minimum Spanning Tree

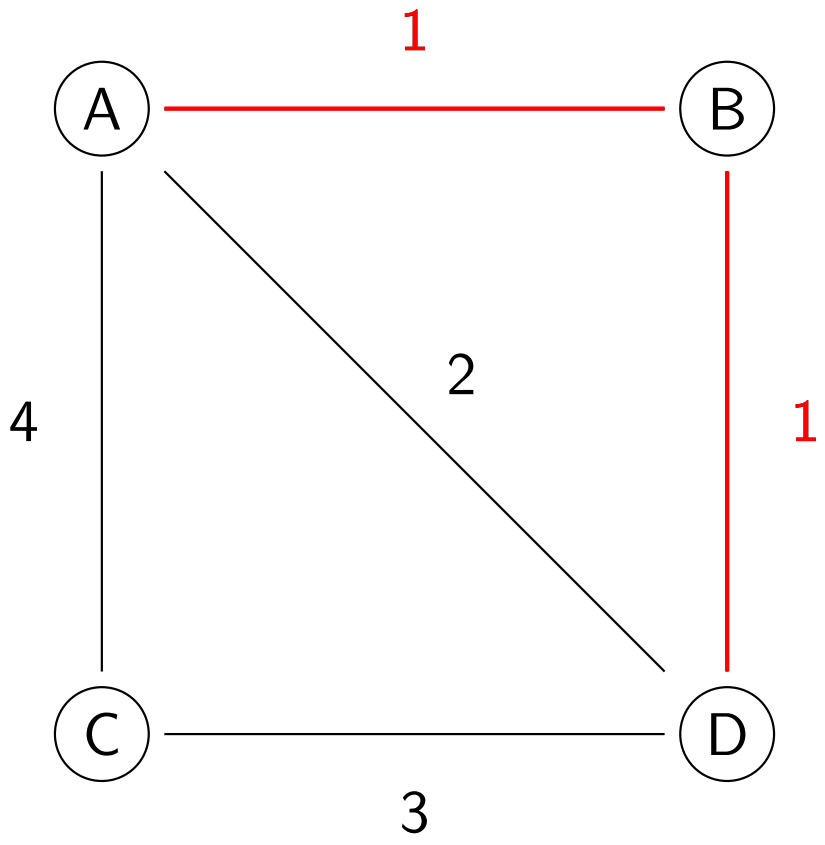


- Minimum Spanning Tree



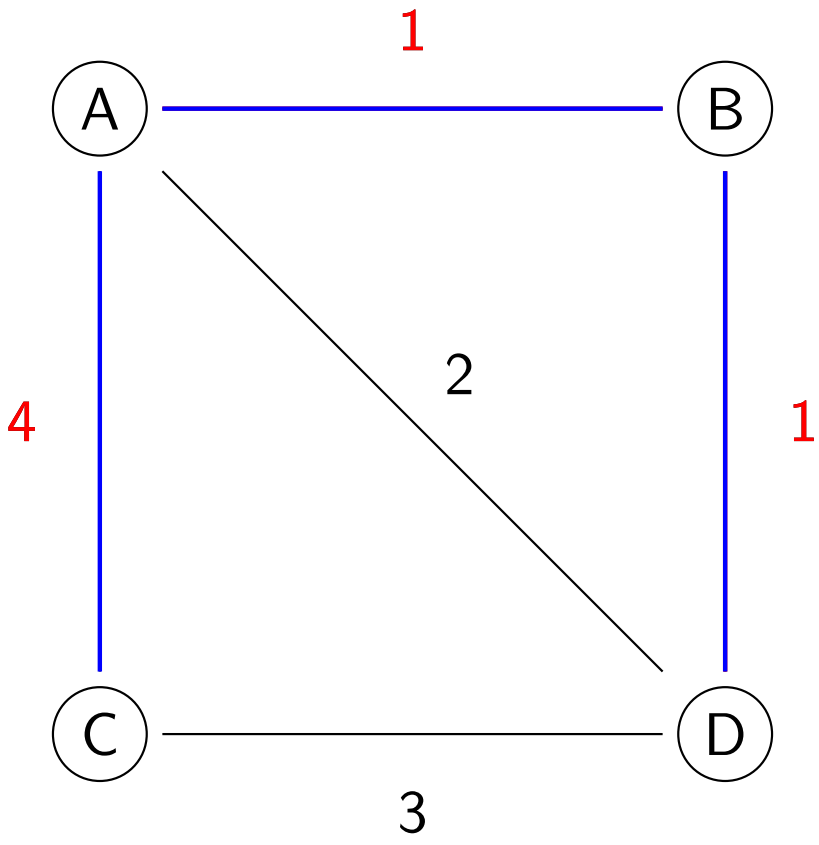
Not Spanning Tree

- Minimum Spanning Tree



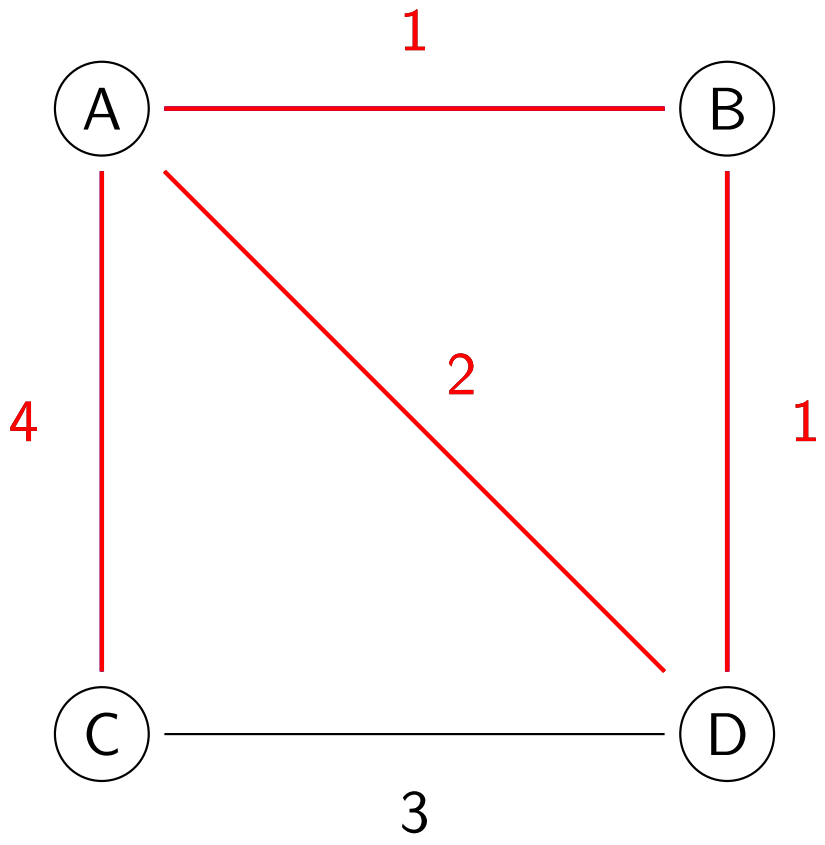
Not Spanning Tree

- Minimum Spanning Tree



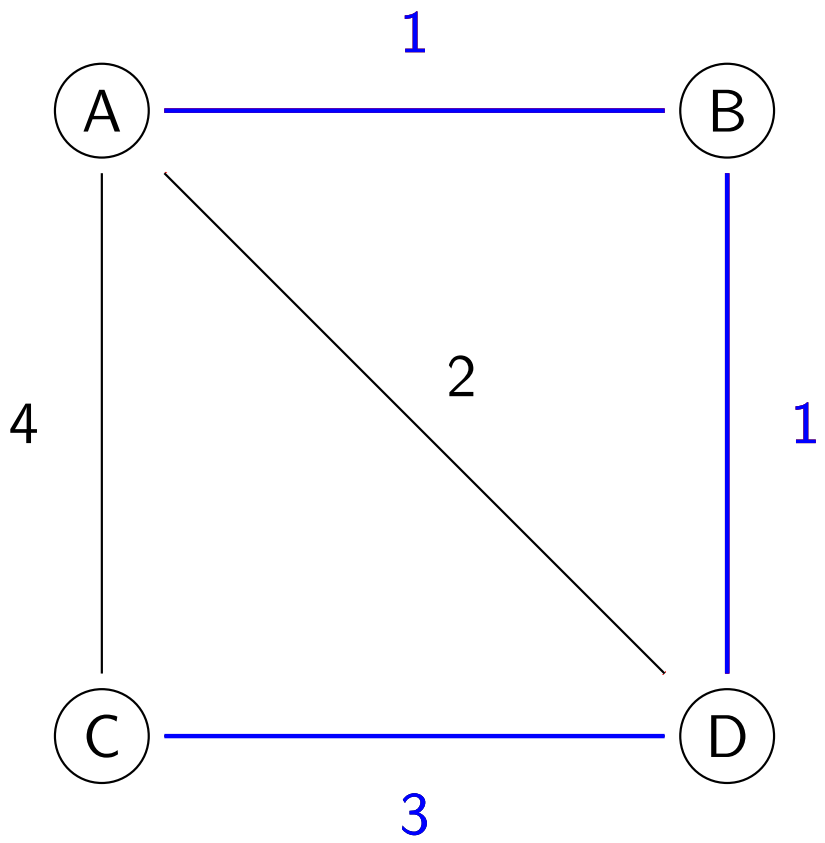
Spanning Tree
With Not Minimum Weights

- Minimum Spanning Tree



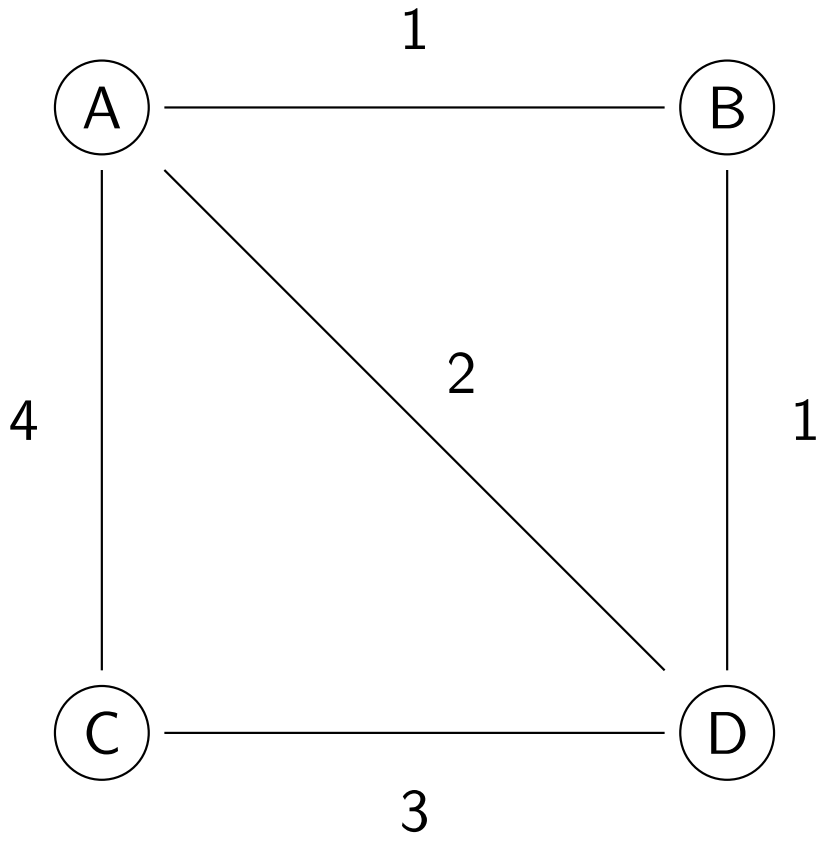
Not Spanning Tree(Cycle Exists)

- Minimum Spanning Tree

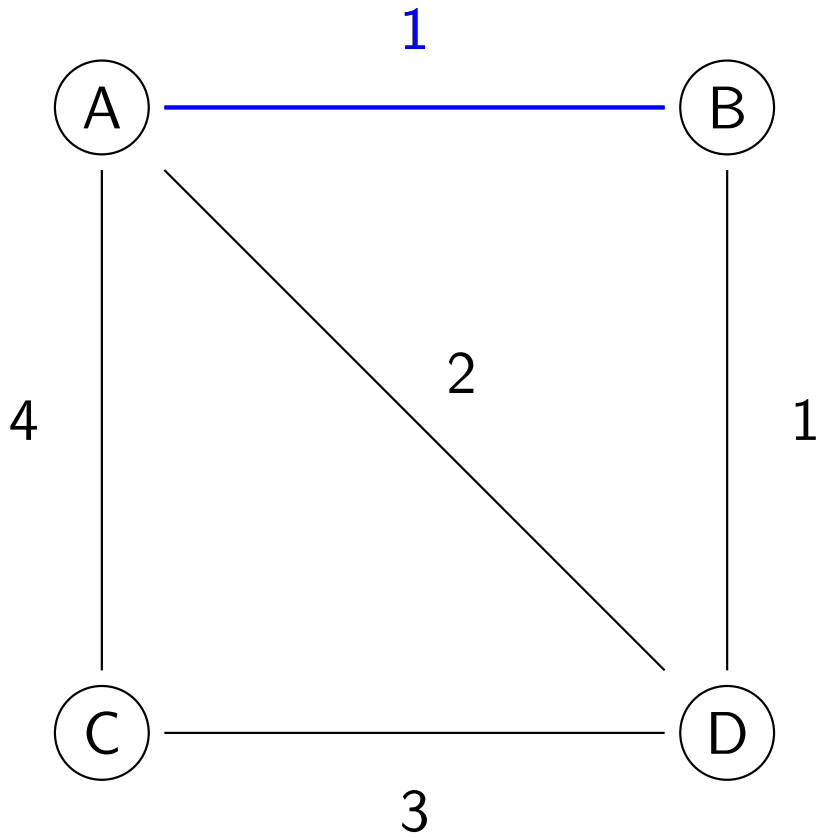


Spanning Tree
With Minimum Weights

- Kruskal's Algorithm : Finding Minimum Spanning Tree



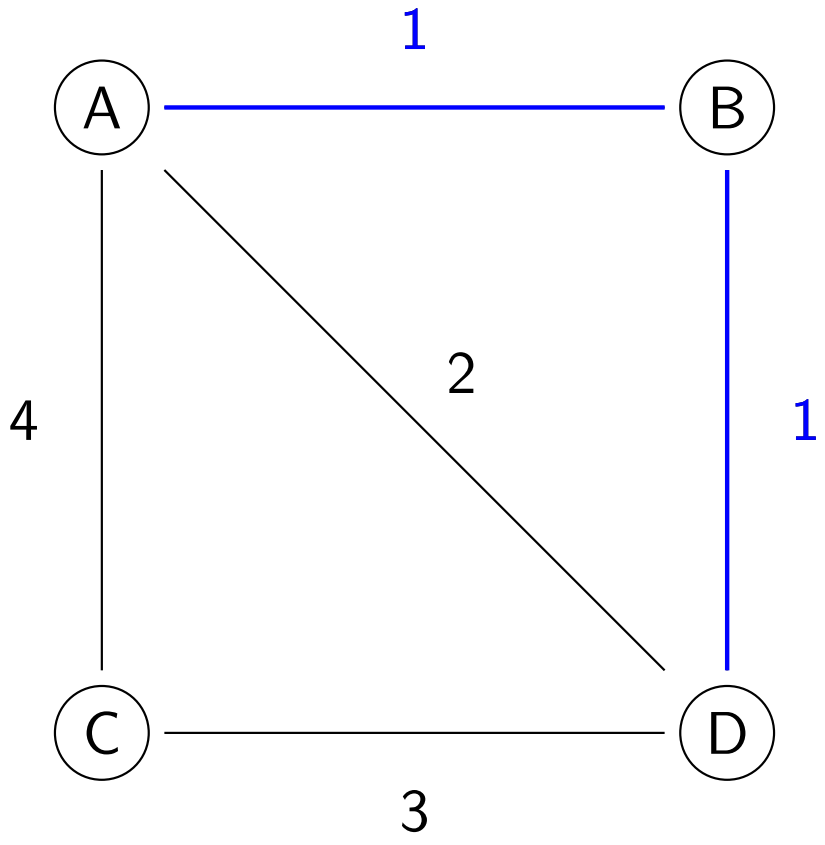
- Kruskal's Algorithm : Finding Minimum Spanning Tree



Subgraph S

1

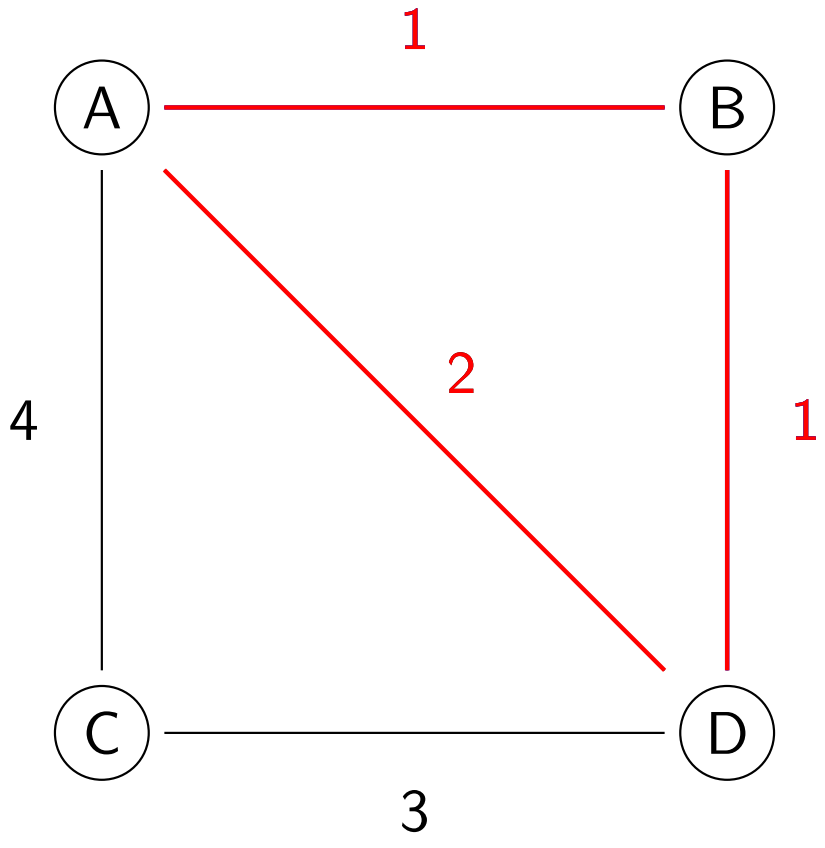
- Kruskal's Algorithm : Finding Minimum Spanning Tree



Subgraph S

1 — 1

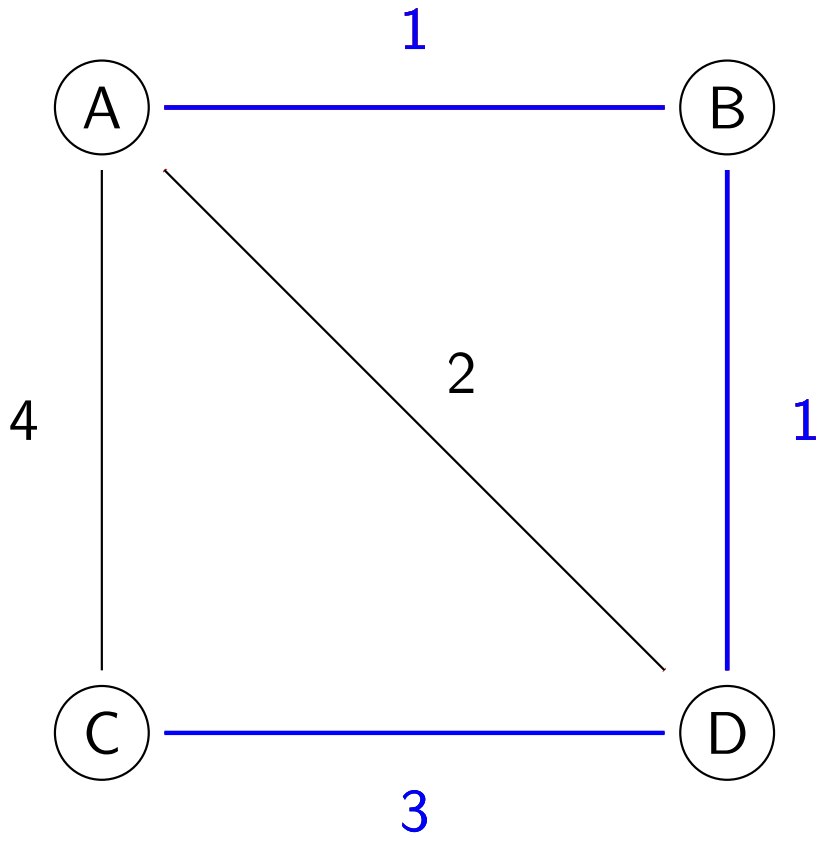
- Kruskal's Algorithm : Finding Minimum Spanning Tree



Subgraph S (Cycle Exists)

$1 \text{ —— } 1 \text{ —— } 2$

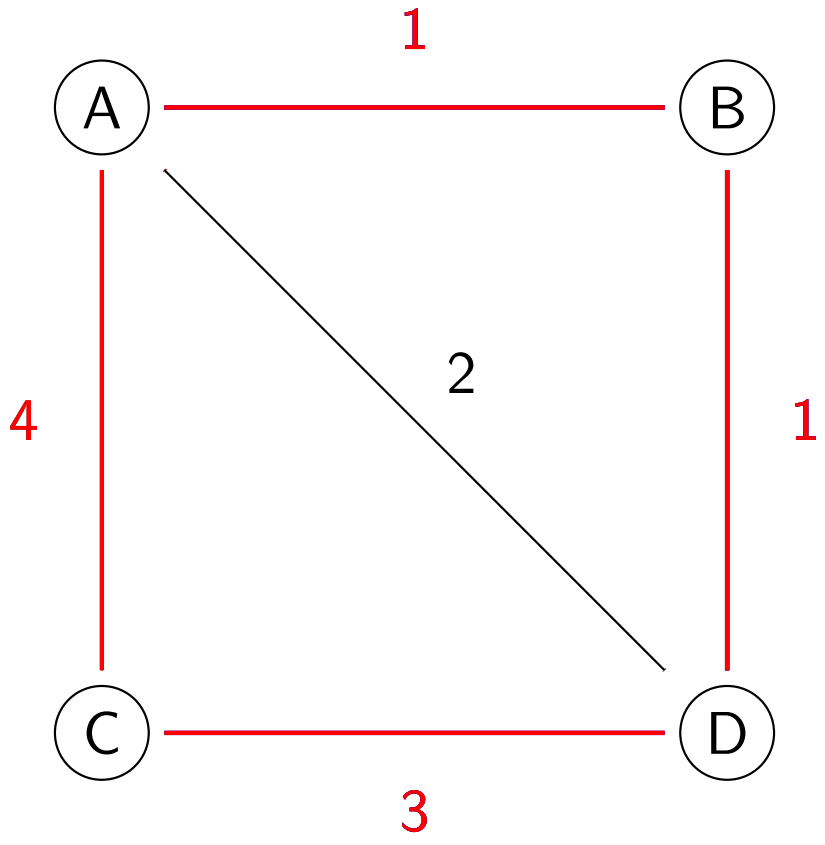
- Kruskal's Algorithm : Finding Minimum Spanning Tree



Subgraph S

1 — 1 — ~~2~~ — 3

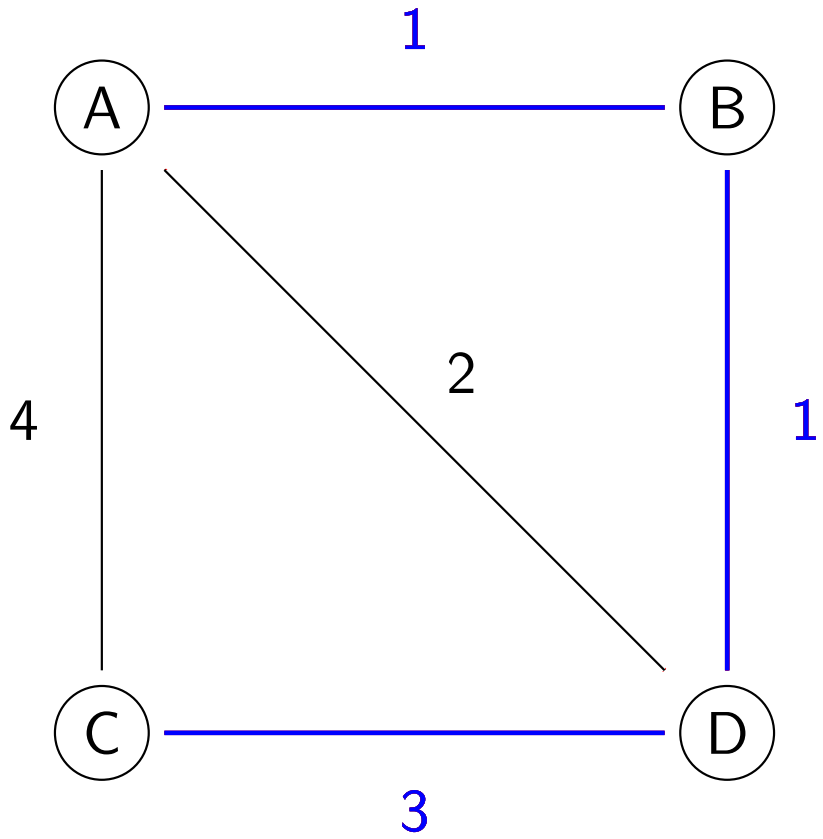
- Kruskal's Algorithm : Finding Minimum Spanning Tree



Subgraph S (Cycle Exists)

1 — 1 — ~~2~~ — 3 — 4

- Kruskal's Algorithm : Finding Minimum Spanning Tree



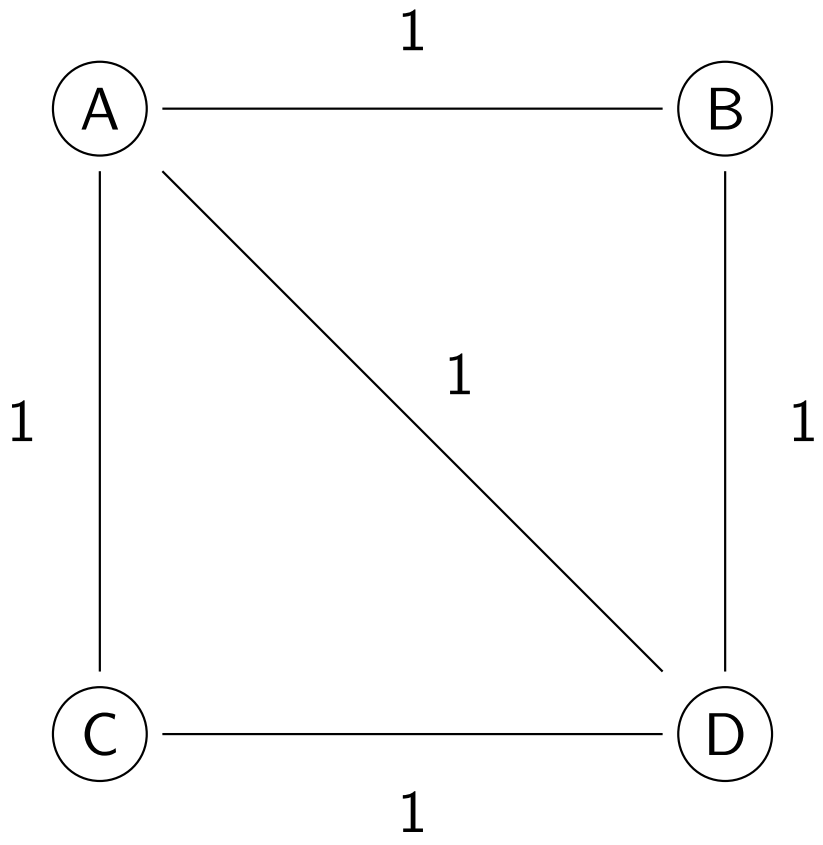
Subgraph S is minimum spanning tree!

$1 \text{ —— } 1 \text{ —— } \cancel{2} \text{ —— } 3 \text{ —— } \cancel{4}$

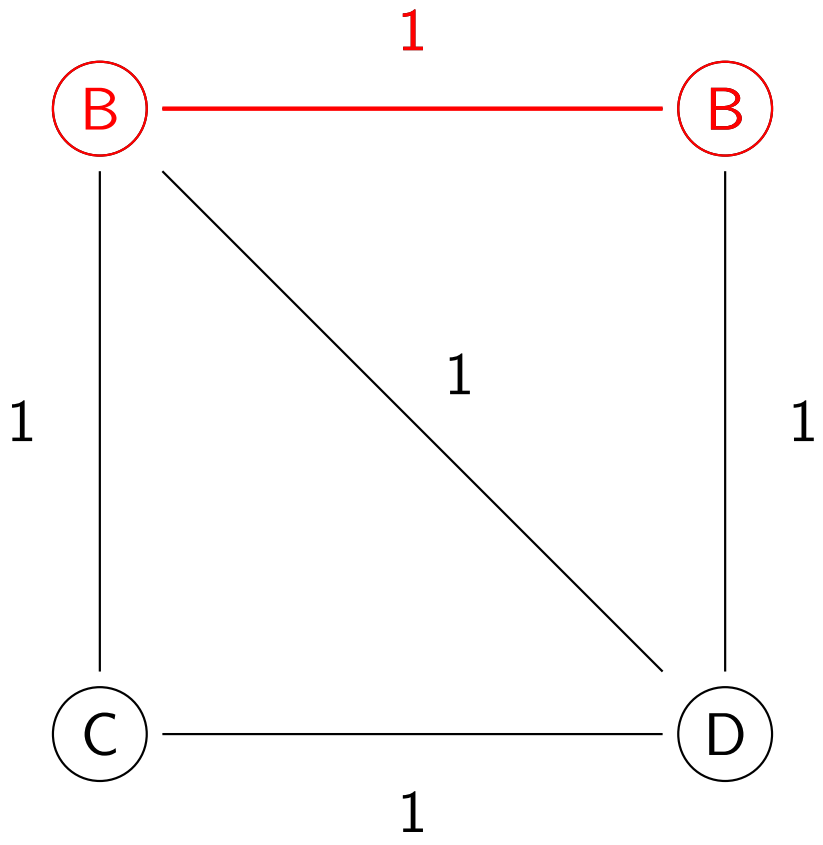
- If all weights are same, we can ignore order of the weights.

- If all weights are same, we can ignore order of the weights.
- Just check a node makes cycles or not, for random order.

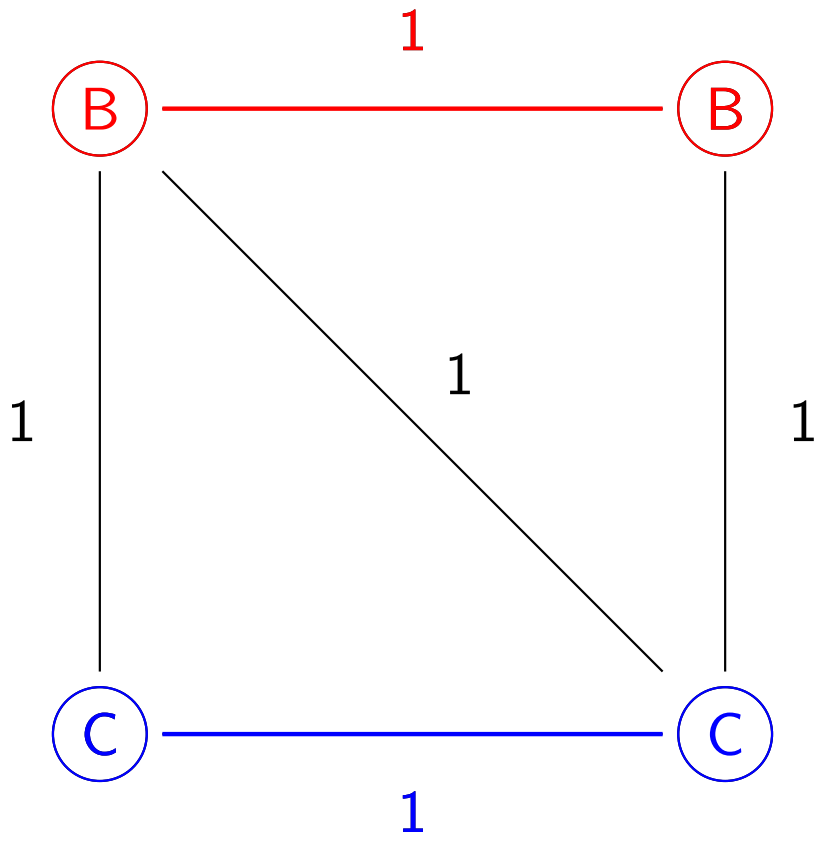
- 'Randomized' Kruskal's Algorithm



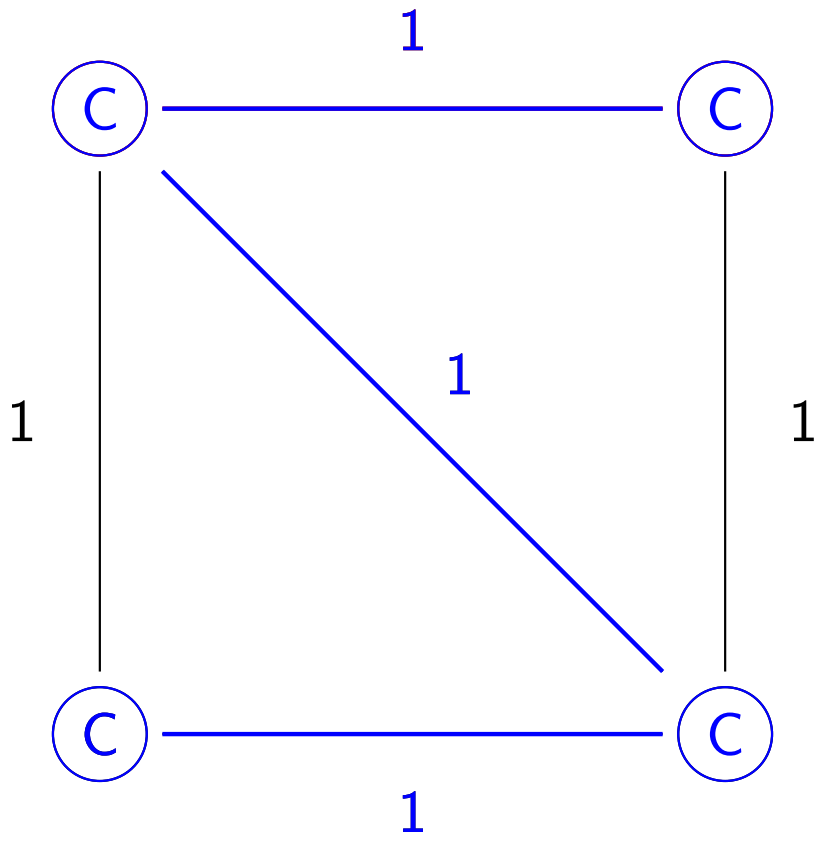
- 'Randomized' Kruskal's Algorithm



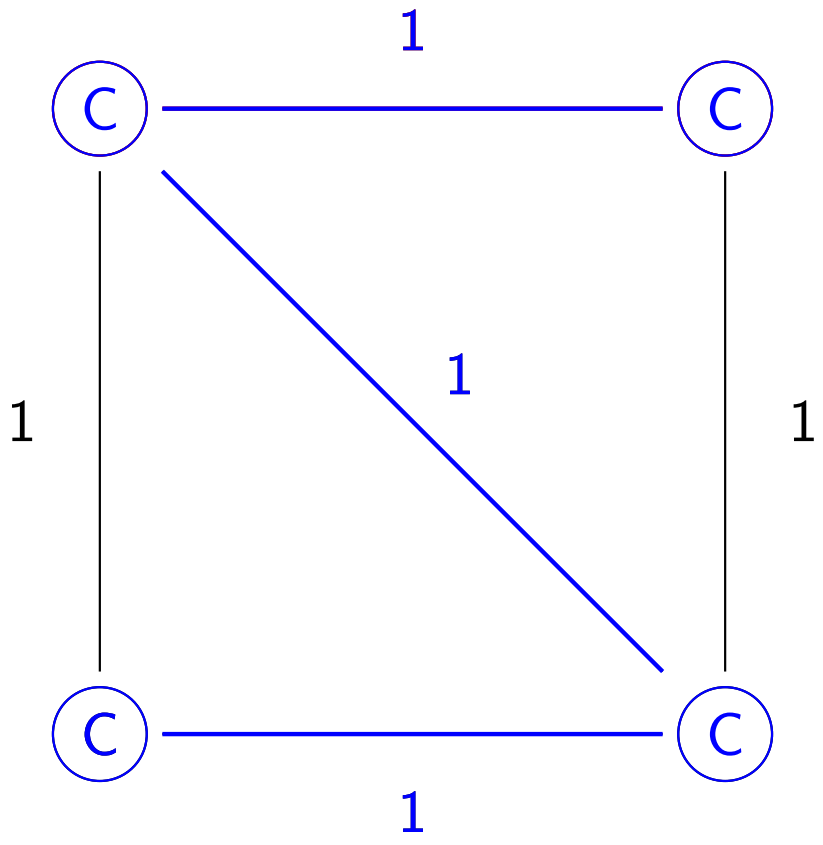
- 'Randomized' Kruskal's Algorithm



- 'Randomized' Kruskal's Algorithm

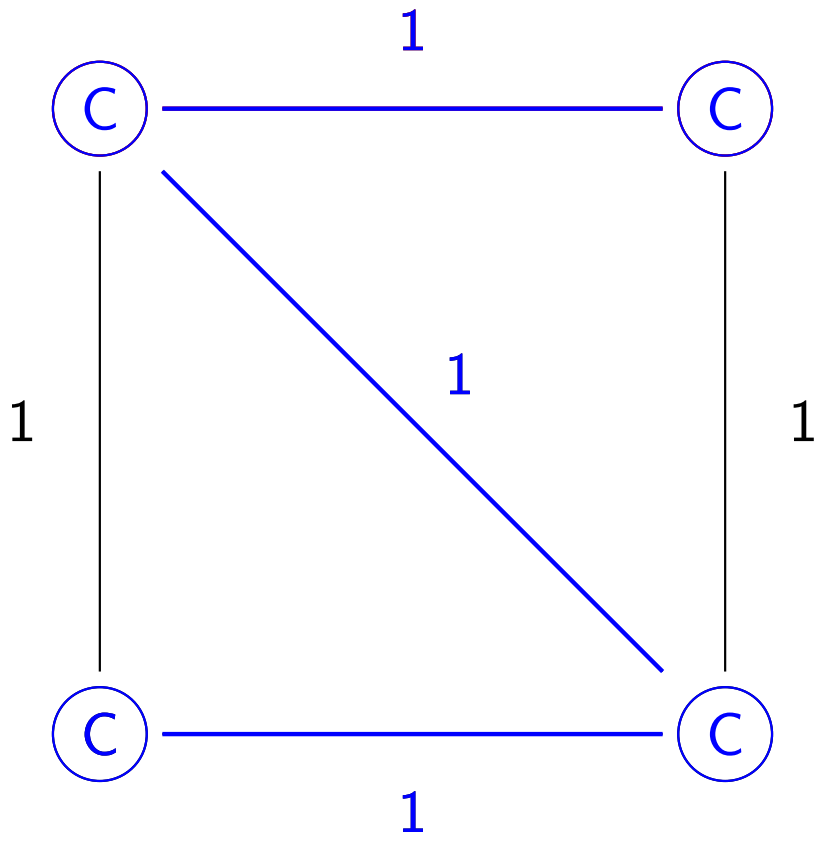


- 'Randomized' Kruskal's Algorithm

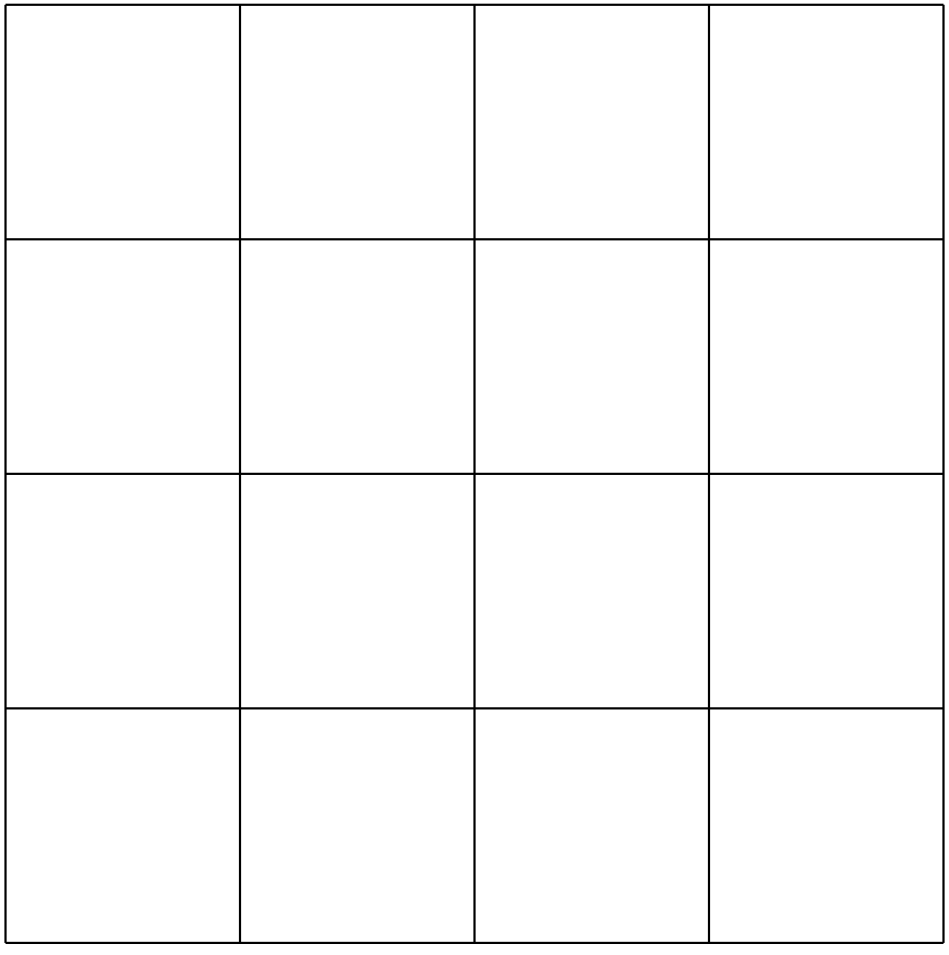


Minimum Spanning Tree

- 'Randomized' Kruskal's Algorithm



Minimum Spanning Tree
Also 'Perfect' Graph!



A	B	C	D
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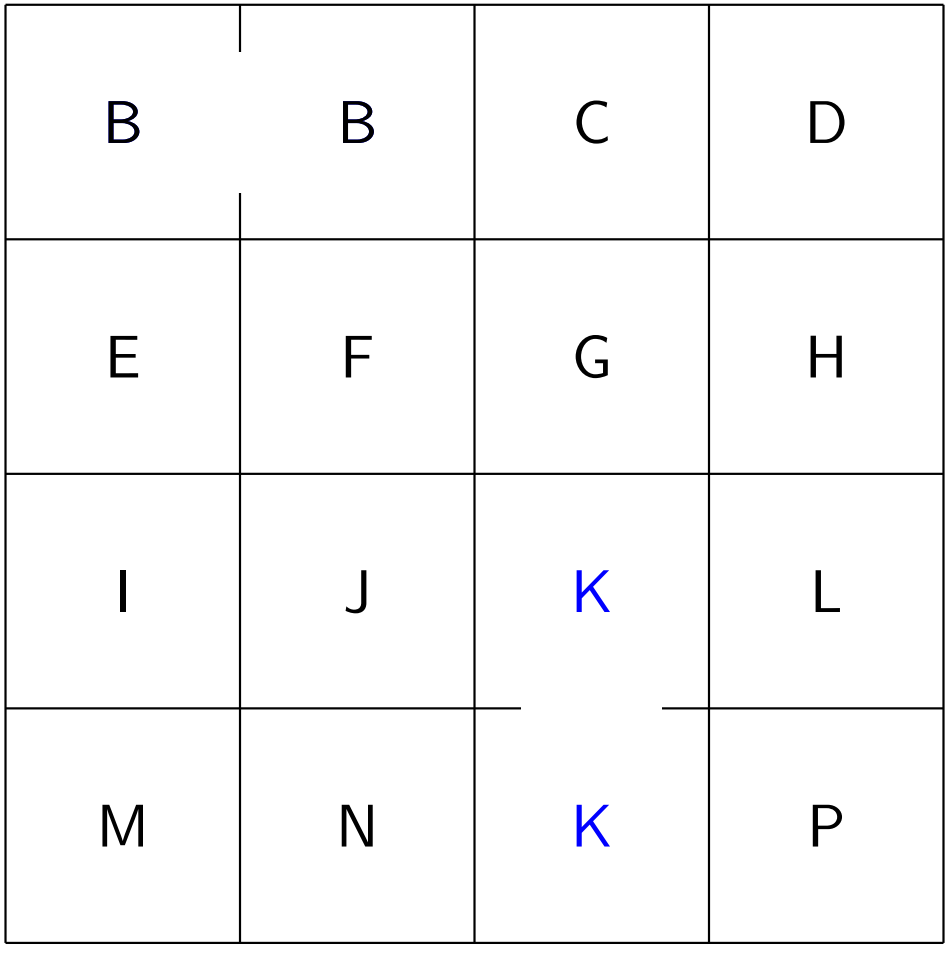
I	J	K	L
M	N	O	P

B	B	C	D
E	F	G	H
I	J	K	L
M	N	O	P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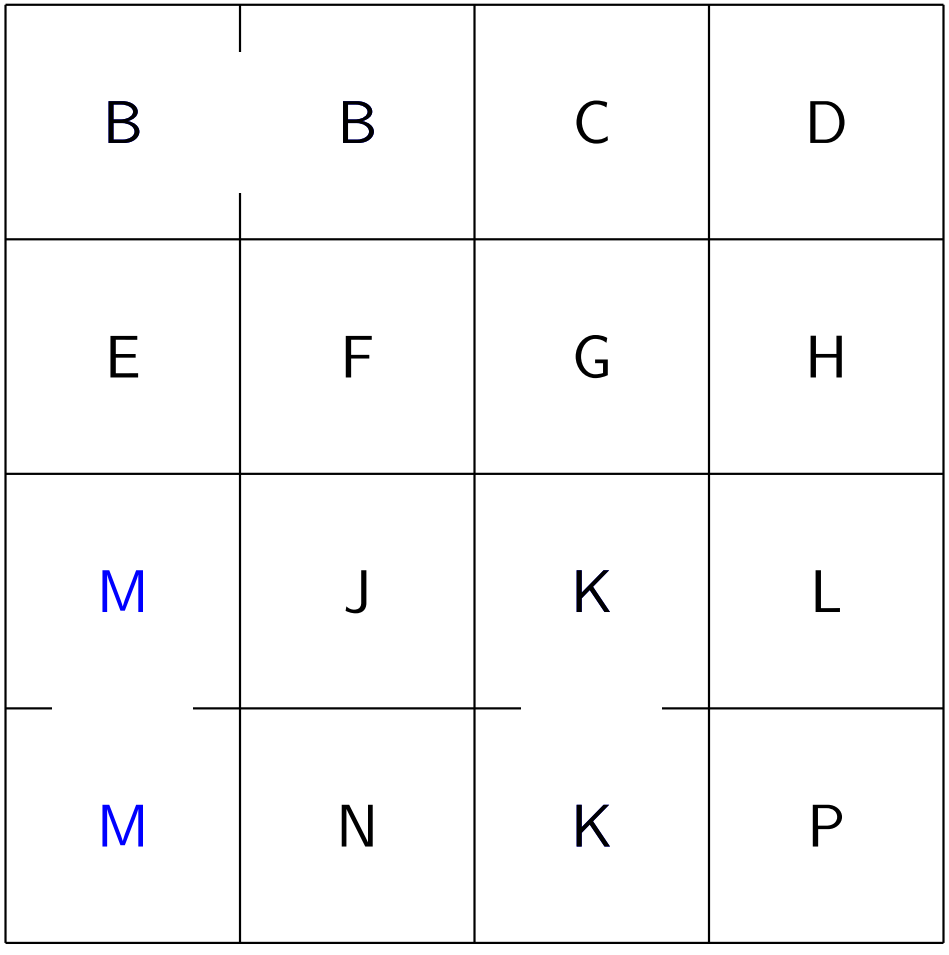
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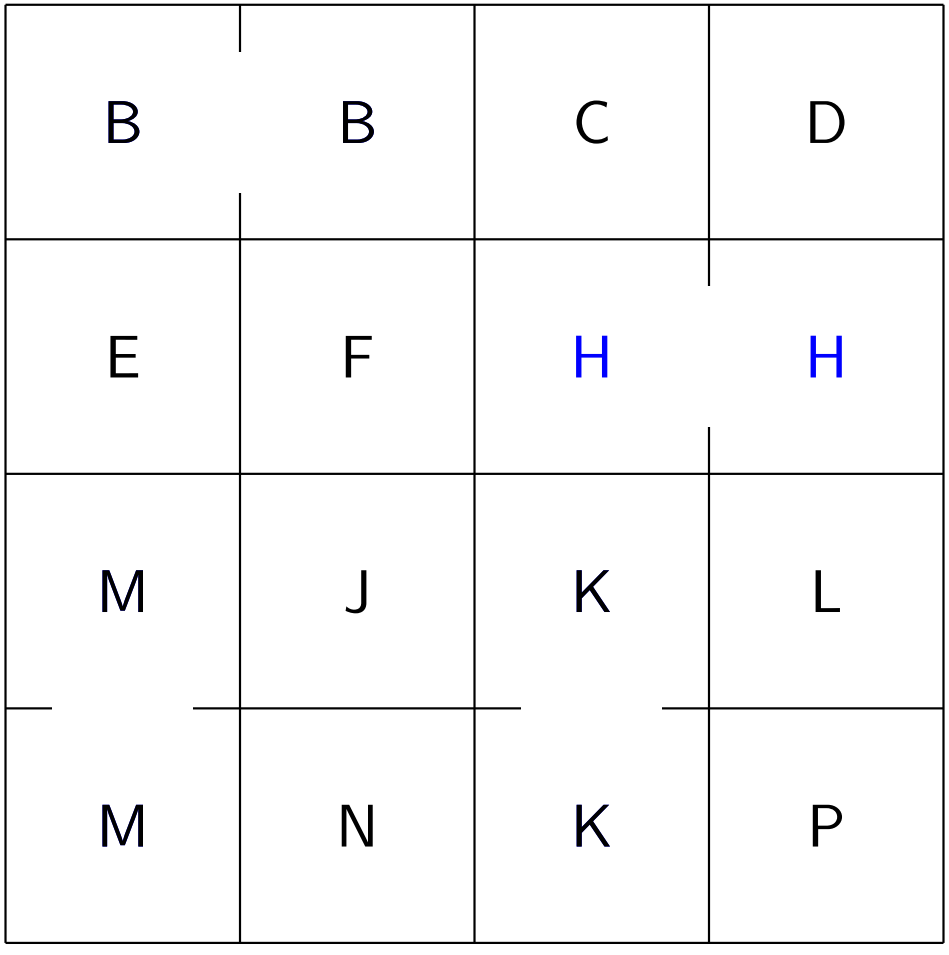
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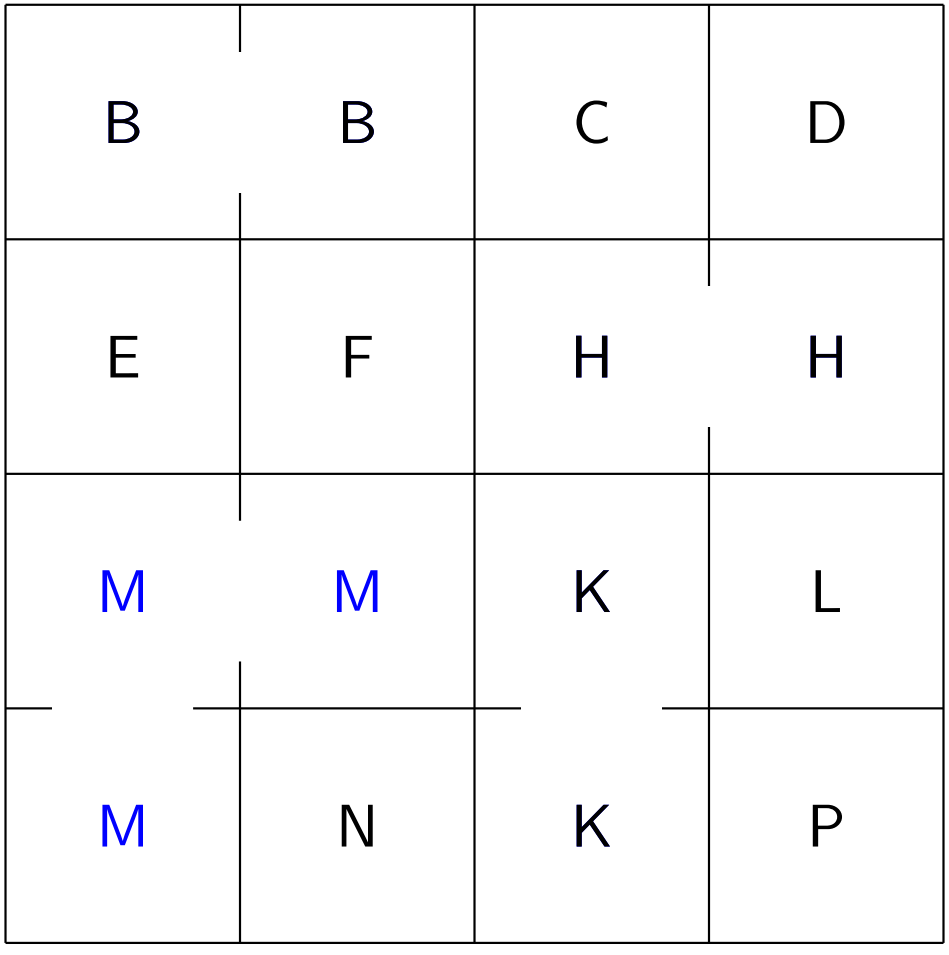
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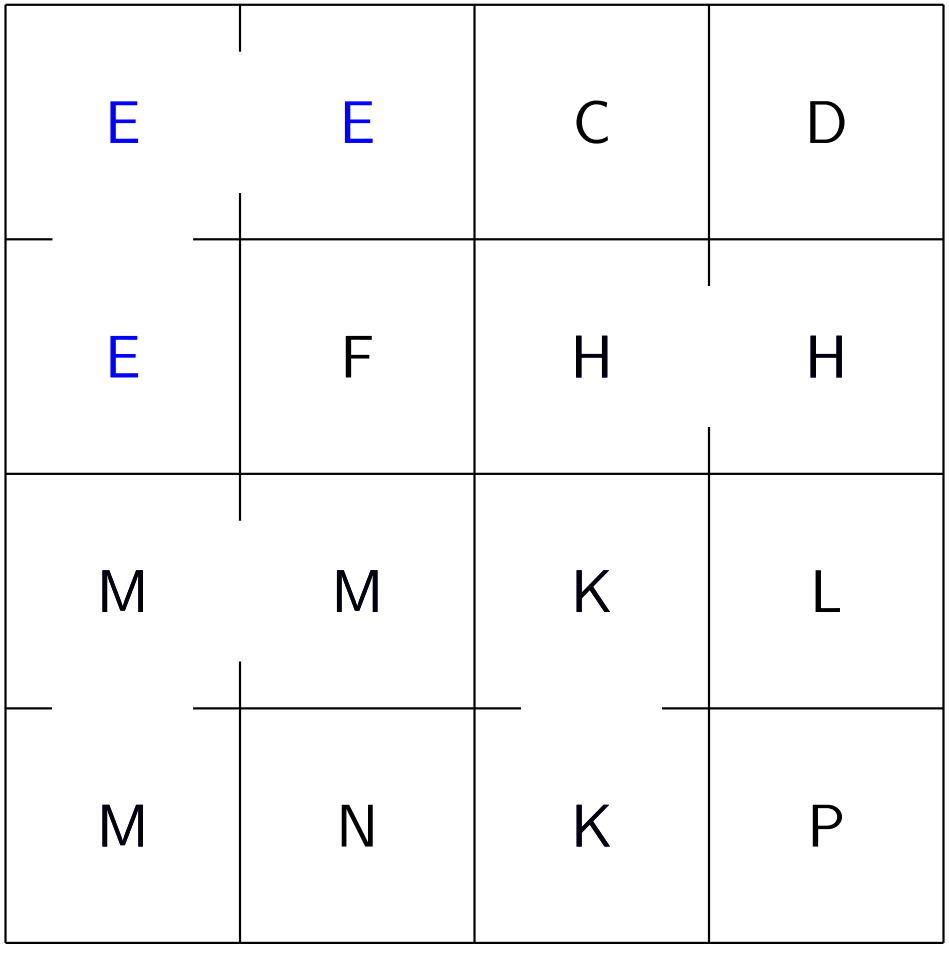
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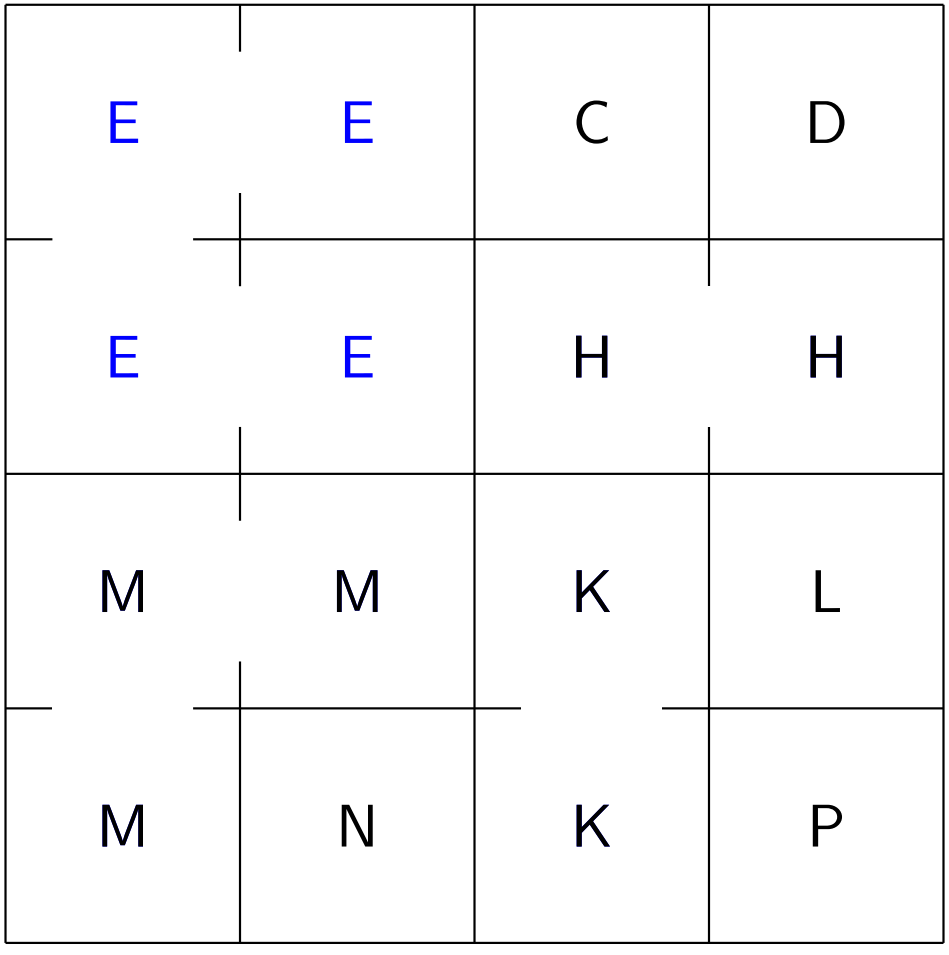
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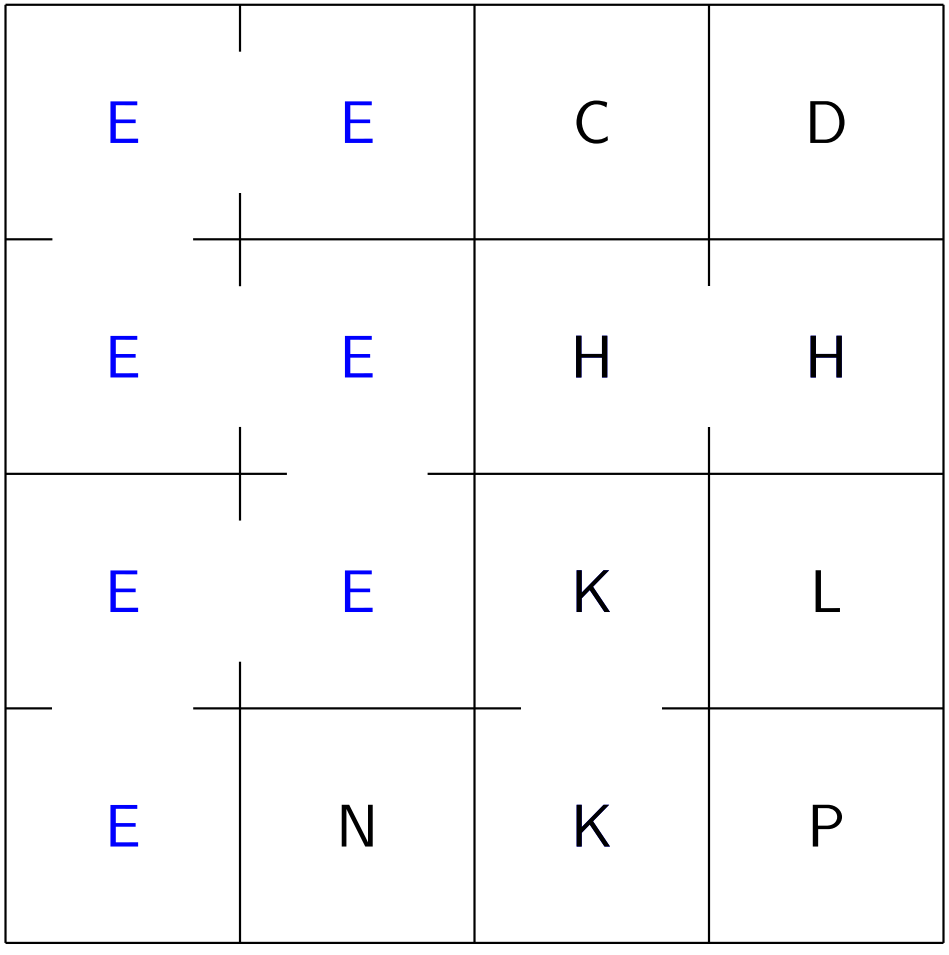
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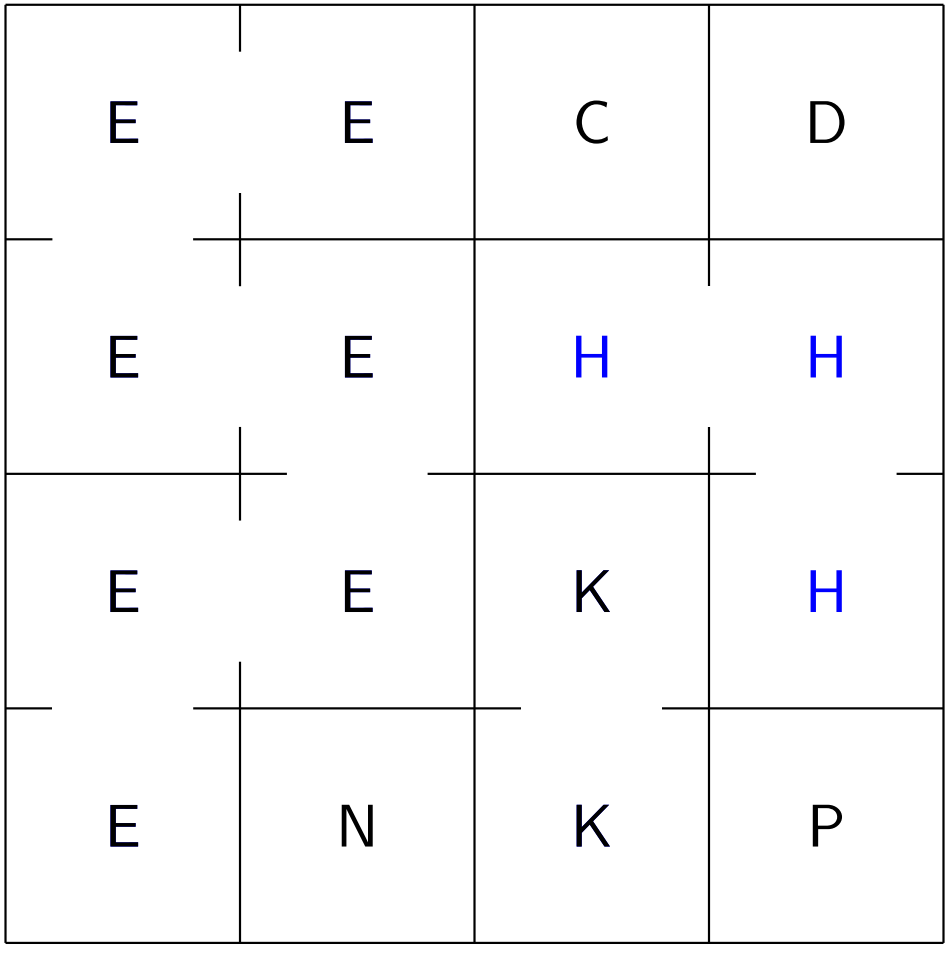
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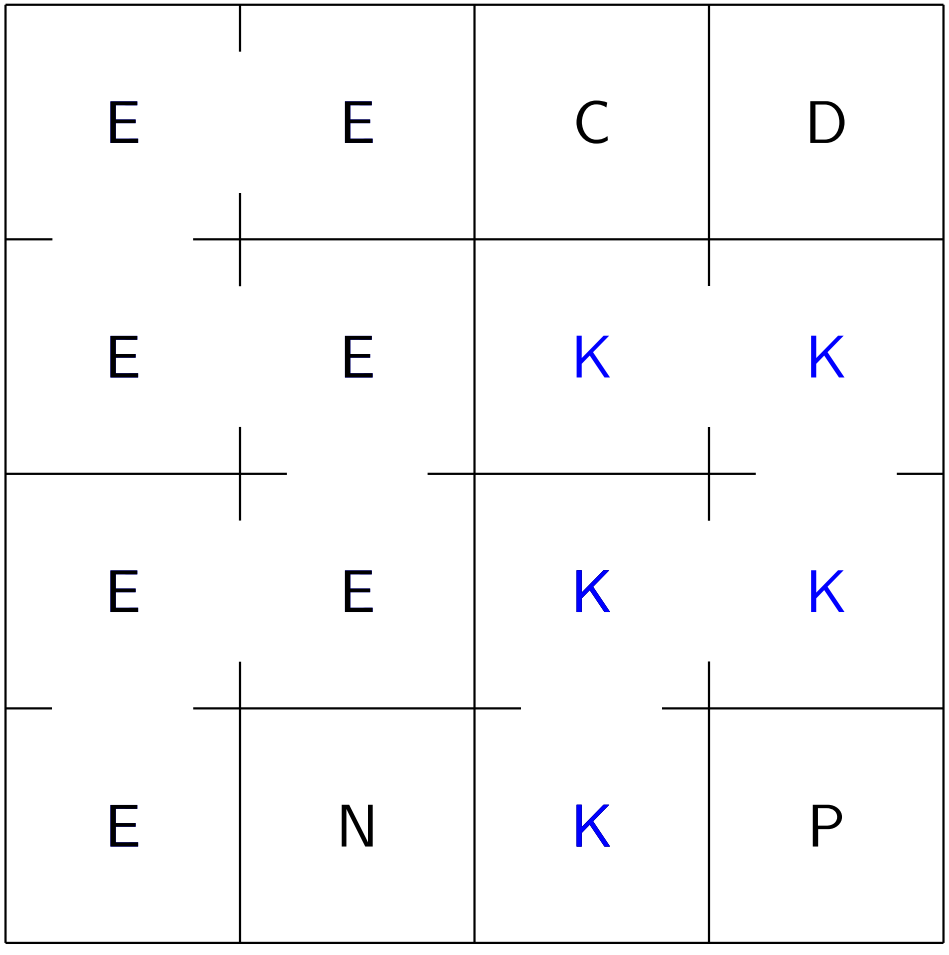
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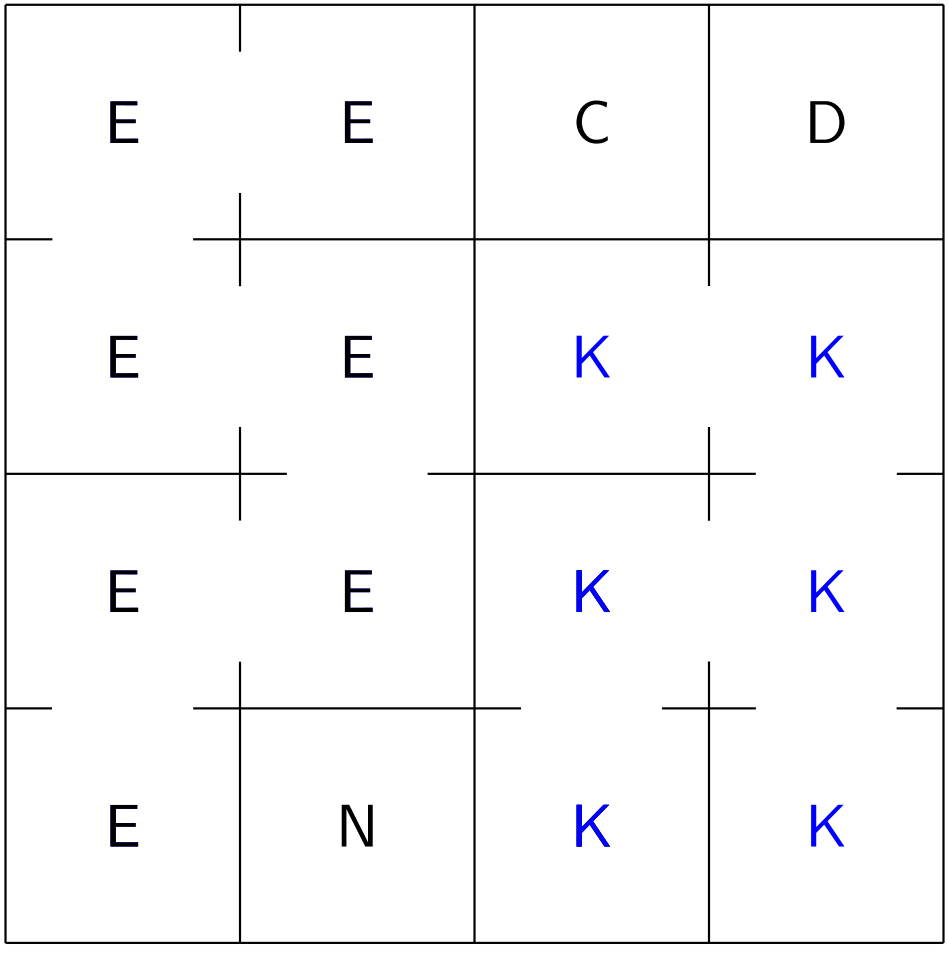
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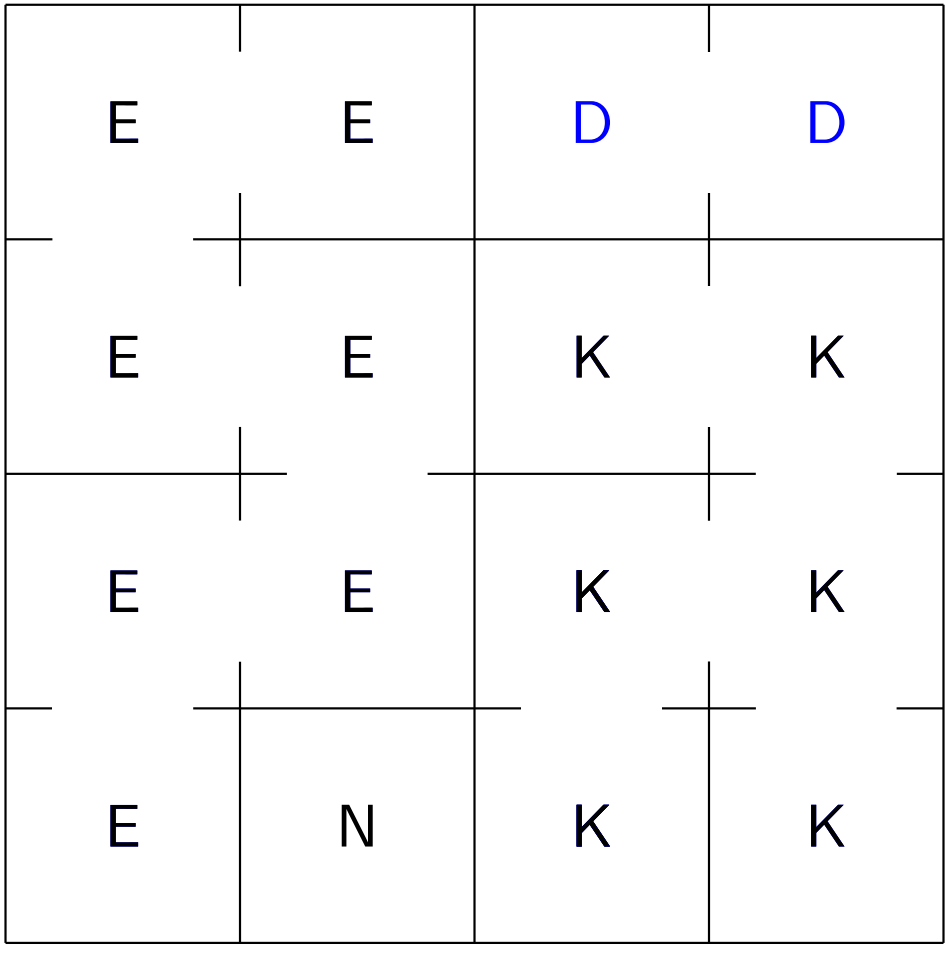
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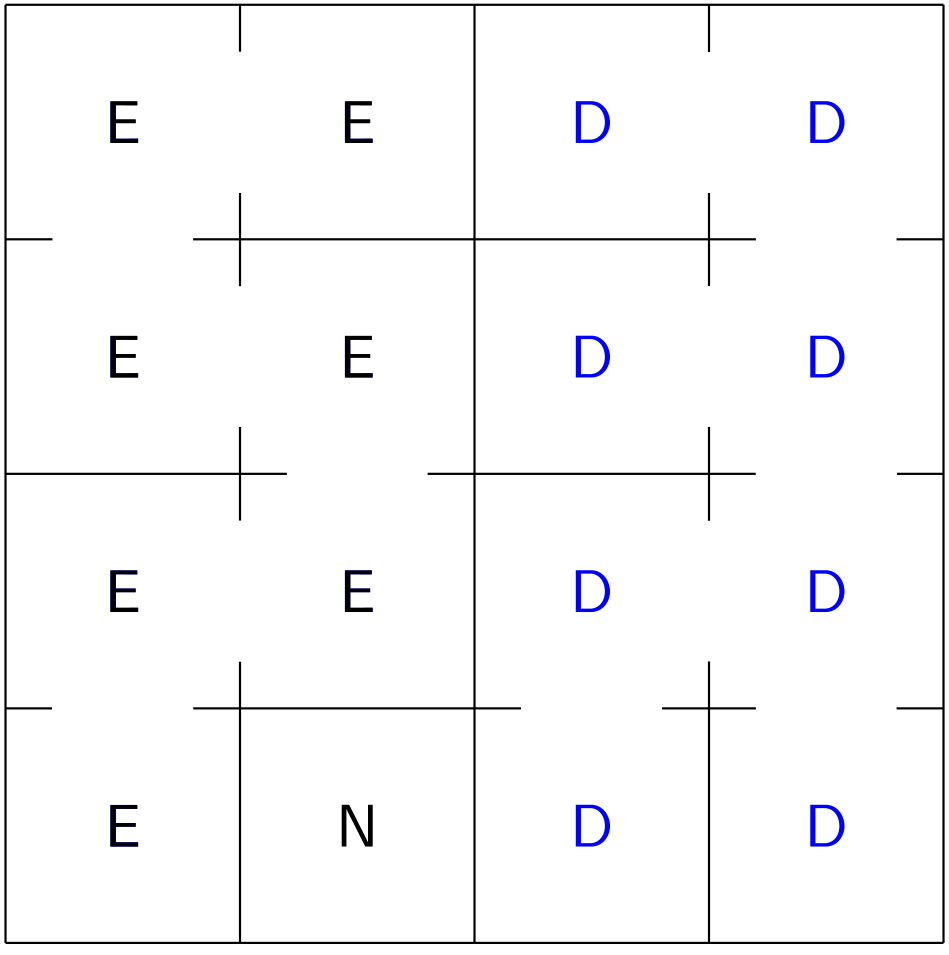
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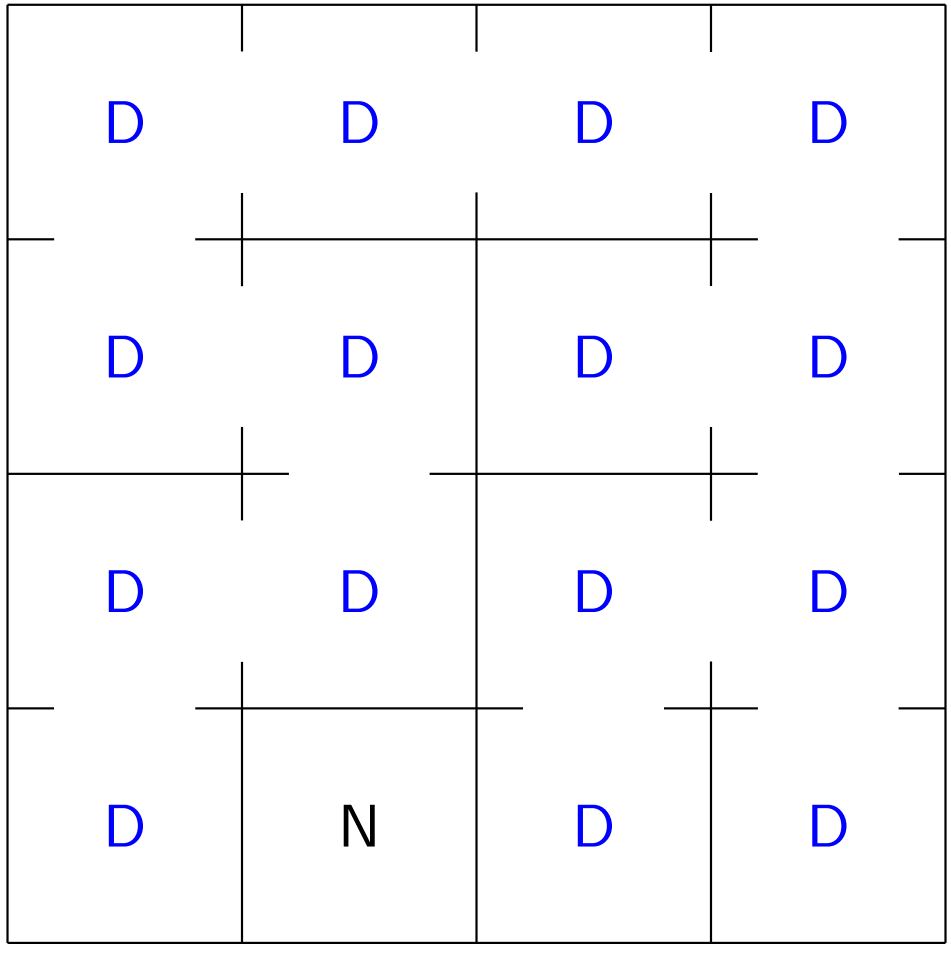
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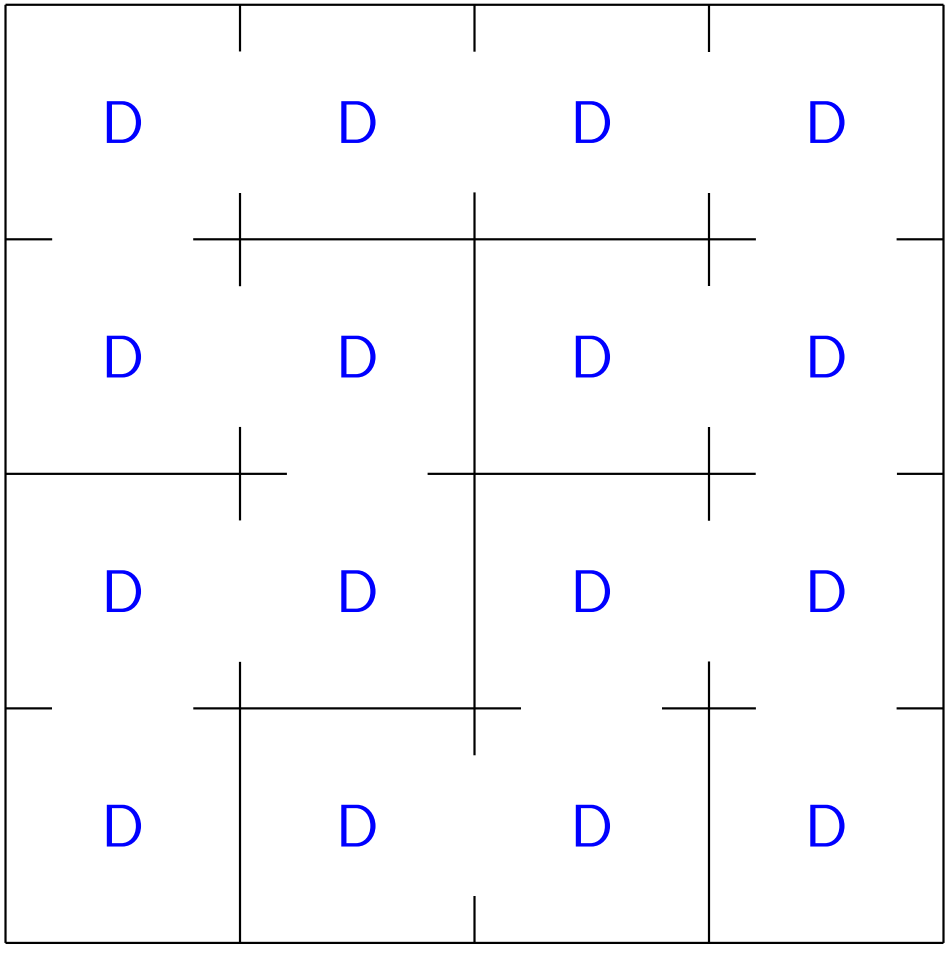
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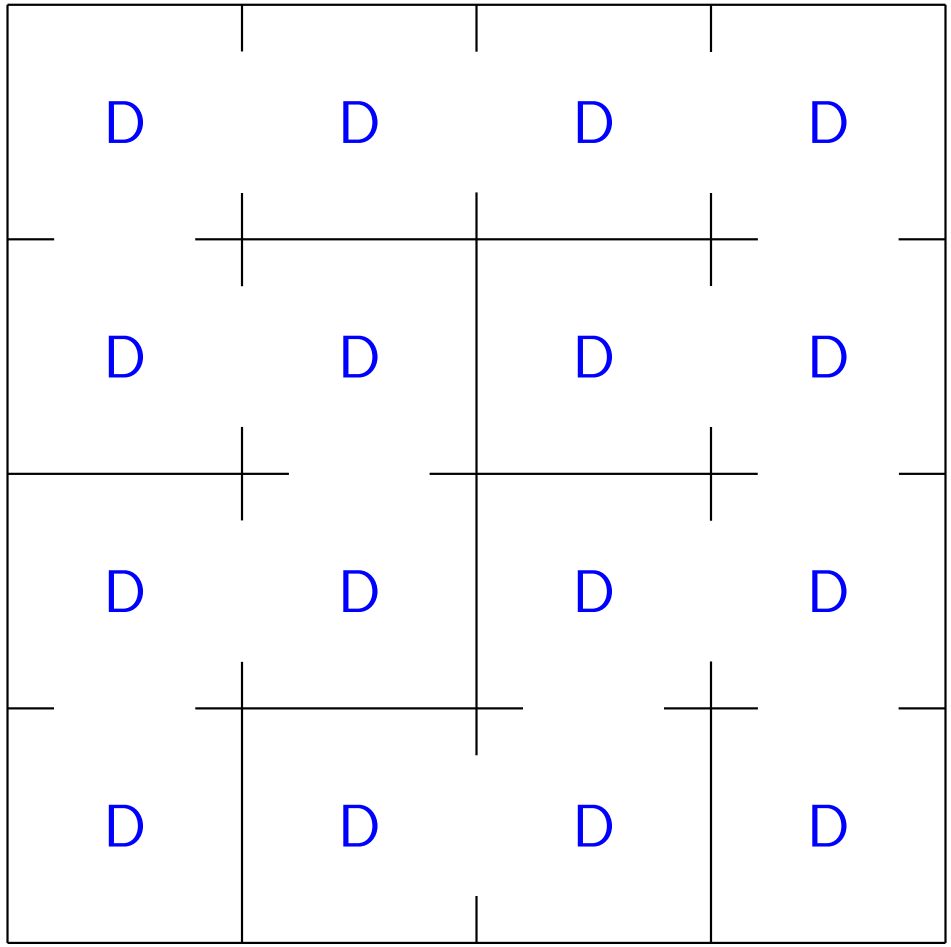

Perfect Maze!