Advanced Statistics

F22 Data Science (Afternoon) Quiz 02

Name & id:	Marks: / 50

Note: Quiz has three questions.

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1. Let X be the random variable that denotes the lifespan (in hours) of a certain battery. The probability density function is given by:

$$f(x) = \begin{cases} \frac{10{,}000}{x3}\#, & x > 200 \\ 0, & elsewhere \end{cases}$$

Find the expected lifespan of this type of battery.

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx \qquad 2$$

$$= \int_{200}^{\infty} x f(x)\, dx$$

$$= \int_{200}^{\infty} x \times \frac{10000}{x^3}\, dx \qquad 2$$

$$= 10000 \int_{200}^{\infty} \frac{1}{x^2}\, dx \qquad 1$$

$$= 10000 \left[-\frac{1}{x}\right]_{200}^{\infty} \qquad 3$$

$$= 10000 \left(0 - \left(-\frac{1}{200}\right)\right) \qquad 3$$

$$= 10000 \times \frac{1}{200} \qquad 2$$

$$= 50 \text{ hours} \qquad 2$$

2. Two balls are drawn randomly from a box containing 4 blue balls, 3 red balls, and 2 green balls. Let X be the number of blue balls selected and Y be the number of red balls selected.
   - Find the joint probability distribution function f(x, y) for X and Y.
   - Calculate the probability that the number of blue and red balls selected satisfies $X+Y \leq 1$

(i) The possible pair of values (x, y) are $(0,0), (0,1), (1,0), (1,1), (0,2), (2,0)$ 1

$$f(x,y) = \frac{\binom{4}{x}\binom{3}{y}\binom{2}{2-x-y}}{\binom{9}{2}} \quad 1$$

$$f(0,0) = \frac{\binom{4}{0}\binom{3}{0}\binom{2}{2}}{\binom{9}{2}} = \frac{1}{36} \quad 2$$

$$f(0,1) = \frac{\binom{4}{0}\binom{3}{1}\binom{2}{1}}{\binom{9}{2}} = \frac{6}{36} \quad 2$$

$$f(1,0) = \frac{\binom{4}{1}\binom{3}{0}\binom{2}{1}}{\binom{9}{2}} = \frac{8}{36} \quad 2$$

$$f(1,1) = \frac{\binom{4}{1}\binom{3}{1}\binom{2}{0}}{\binom{9}{2}} = \frac{12}{36} \quad 2$$

$$f(0,2) = \frac{\binom{4}{0}\binom{3}{2}\binom{2}{0}}{\binom{9}{2}} = \frac{3}{36} \quad 2$$

$$f(2,0) = \frac{\binom{4}{2}\binom{3}{0}\binom{2}{0}}{\binom{9}{2}} = \frac{6}{36} \quad 2$$

$f(x,y)$	0	1	2	
0	1/36	6/36	3/36	10/36
1	8/36	12/36	0	20/36
2	6/36	0	0	6/36
	15/36	18/36	3/36	$\Sigma = 1$

(ii) $P(X+Y \leq 1) = f(0,0) + f(0,1) + f(1,0)$

$= \frac{1}{36} + \frac{6}{36} + \frac{8}{36}$

$= \frac{15}{36}$

/15

3. Let X and Y be the proportions of time that two machines in a factory are operational on a randomly selected day. The joint probability density function of X and Y is given by:

$F(x, y) = 3(x+2y) \qquad 0 \leq x \leq 1, 0 \leq y \leq 1$

- Find the marginal density function of X, g(x).
- Find the marginal density function of Y, h(y).

$$g(x) = \int_0^1 3(x+2y)\,dy \qquad 2$$

$$= 3\int_0^1 (x+2y)\,dy$$

$$= 3\left[xy + y^2\right]_0^1 \qquad 2$$

$$= 3(x + 1^2) \qquad 2$$

$$= 3(x+1) \qquad 1.5$$

$$h(y) = \int_0^1 3(x+2y)\,dx \qquad 2$$

$$= 3\int_0^1 (x+2y)\,dy$$

$$= 3\left[\frac{x^2}{2} + 2yx\right]_0^1 \qquad 2$$

$$= 3\left[\frac{(1)^2}{2} + 2(1)y\right] - \left[\frac{0}{2} + 2(0)(y)\right] \qquad 2$$

$$= 3\left(\frac{1}{2} + 2y\right) \qquad 1.5$$

Key

Advanced Statistics

F22 Data Science (Morning) **Quiz 02**

Name & id:	Marks: / 50

Note: Quiz has three questions.

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1. Let X be the random variable representing the time (in days) until a certain machine component fails. The probability density function is:

$$f(x) = \begin{cases} \frac{15{,}000}{x4}\#, & x > 50 \\ 0, & elsewhere \end{cases}$$

Find the expected time until failure for this machine component.

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx \qquad 2$$

$$= \int_{50}^{\infty} x\, f(x)\, dx$$

$$= \int_{50}^{\infty} x \times \frac{15000}{x^4}\, dx \qquad 3$$

$$= 15000 \int_{50}^{\infty} \frac{1}{x^3}\, dx \qquad 1$$

$$= 15000 \left[\frac{-1}{2x^2} \right]_{50}^{\infty} \qquad 3$$

$= 15000\left(0 - \left(\frac{-1}{2\times 50^2}\right)\right)$ 3

$= 15000 \times \frac{1}{2\times 2500}$ 2

$= 15000 \times \frac{1}{5000}$

$= 3$ days 7

/20

2. A box contains 5 yellow balls, 4 black balls, and 3 white balls. Two balls are randomly selected from the box. Let X be the number of yellow balls selected and Y be the number of black balls selected.

- Find the joint probability distribution function f(x, y) for X and Y.
- Calculate the probability that the number of yellow and black balls selected satisfies X+Y≤1

(i) The possible pair of values (x, y) are $(0,0)$, $(0,1)$, $(1,0)$, $(1,1)$, $(0,2)$, $(2,0)$

$$f(x,y) = \frac{\binom{5}{x}\binom{4}{y}\binom{3}{2-x-y}}{\binom{12}{2}}$$

$$f(0,0) = \frac{\binom{5}{0}\binom{4}{0}\binom{3}{2}}{\binom{12}{2}} = \frac{1}{22}$$ 2

$$f(0,1) = \frac{\binom{5}{0}\binom{4}{1}\binom{3}{1}}{\binom{12}{2}} = \frac{2}{11}$$ 2

$$f(1,0) = \frac{\binom{5}{1}\binom{4}{0}\binom{3}{1}}{\binom{12}{2}} = \frac{5}{22}$$ 2

$$f(1,1) = \frac{\binom{5}{1}\binom{4}{1}\binom{3}{0}}{\binom{12}{2}} = \frac{10}{33}$$ 2

$$f(0,2) = \frac{\binom{5}{0}\binom{4}{2}\binom{3}{0}}{\binom{12}{2}} = \frac{6}{66} = \frac{1}{11}$$ 2

$$f(2,0) = \frac{\binom{5}{2}\binom{4}{0}\binom{3}{0}}{\binom{12}{2}} = \frac{5}{33}$$ 2

$f(x,y)$	$x=0$	$x=1$	$x=2$	
$y=0$	1/22	2/11	1/11	7/22
$y=1$	5/22	10/33	0	35/66
$y=2$	5/33	0	0	5/33

2

1

(ii) $P(X+Y \leq 1) = f(0,0) + f(0,1) + f(1,0)$ 1

$= \frac{1}{22} + \frac{2}{11} + \frac{5}{22}$ 2

$= \frac{5}{11}$ 1

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3. Let X and Y represent the proportions of time two hospital departments are in use during a day. Suppose their joint probability density function is:

$f(x,y)=2(x+y) 0 \leq x \leq 2, 0 \leq y \leq 2$

- Find the marginal density function of X, g(x).
- Find the marginal density function of Y, h(y).

(i) $g(x) = \int_0^2 2(x+y)\,dy$ 2

$= 2\int_0^2 (x+y)\,dy$

$= 2\left[xy + \frac{y^2}{2}\right]_0^2$ 2

$= 2\left(2x + \frac{4}{2}\right)$ 2

$= 2(2x+2)$

$= 4x+4$ 1.5

(ii) $h(y) = \int_0^2 2(x+y)\,dx$ 2

$= 2\int_0^2 (x+y\,dx$

$= 2\left[\frac{x^2}{2} + yx\right]_0^2$ 2

$= 2\left(\frac{4}{2} + 2y\right)$ 2

$= 2(2+2y)$

$= 4(1+y)$ 1.5