

Use the power of the internet and of LLMs to identify a physical system that solves differential equations inherently, through its physical properties, without executing instructions as a traditional processor does

Prompt:

“What physical systems naturally solve differential equations through their internal dynamics, without needing instruction execution?”

LLM Response Sketch:

Here are a few examples of “analog” physical systems that **just do** differential-equation solving in their physics—no CPU instructions needed:

- **RC (Resistor–Capacitor) Networks**

- A grid of resistors and capacitors naturally obeys $C \frac{dV}{dt} + \frac{V}{R} = I$, so it solves first-order ODEs (e.g. low-pass filters, diffusion).

- **Mass–Spring–Damper Mechanical Systems**

- A mass on a spring with damping follows $m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t)$, so it analog-computes second-order dynamics.

- **Optical Waveguides & Lenses**

- Light propagation through graded-index media or free space solves the paraxial wave equation (a PDE), focusing and diffracting in real time.

- **Thermal (Heat-Diffusion) Materials**

- Temperature spread in a metal plate follows $\frac{\partial T}{\partial t} = \alpha \nabla^2 T$; the plate’s temperature field is the solution.

- **Memristor/Crossbar Arrays**

- Applying voltages to a resistive crossbar computes weighted sums and, with the right peripheral circuits, can emulate time-dependent neural dynamics.

- **Chemical Reaction Networks**

- Concentrations evolving under reaction-rate laws directly solve sets of coupled ODEs (used in DNA computing and synthetic biology).

Each of these systems “embodies” the mathematics of the differential equations in its device physics, so you simply apply inputs and read outputs, and the hardware’s natural response is your solution.

Then, I validated via Web Research

Searched: “Analog computer differential equations”

Got that: Analog computers use op-amp integrators and summers to directly implement ODEs

RC circuits satisfy $\frac{dV}{dt} + \frac{1}{RC}V = 0$, i.e. exponential decay ODEs, right out of Kirchhoff's laws.

I researched the Specific technologies

Mechanical integrators: gear-and-wheel setups (differential analysers) that integrate motion.

Memristor crossbars: state-dependent resistance follows nonlinear ODEs.

Optical analog: light in photonic circuits governed by wave equations.

Selected a Physical solver

Choice: An **analog electronic circuit** built from op-amp integrators, summers and comparators.

Why it works:

- An RC integrator node inherently solves $\frac{dV}{dt} = \frac{1}{RC}(V_{\text{in}} - V)$
- Cascading integrators and summers lets you build higher-order ODE solvers directly in hardware.

LLM Prompt Used:

“How can I design an op-amp + RC network to solve a first-order differential equation $x' = a x + b$?”

I learnt that a simple way is to use an op-amp in an integrator setup (a feedback capacitor C and input resistors R). You sum two inputs at its (inverting) node, one resistor carrying a scaled version of the output voltage (to generate “a·x”) and another carrying a fixed voltage (to generate “b”), so that the op-amp’s output voltage naturally satisfies

$$\frac{dx}{dt} = a x + b$$

by integration.