Replication Codes Description: Porter (1983)

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1 Abstract

- This documents may help you understand the replication codes for the paper by R.H. Porter in 1983 about the cartel stability.
- Most of the codes are written by Matlab, and the others are written by Stata.
- You just need to run *mlepn.m* to obtain the final result.

2 Files

2.1 Stata

- **datageneration.do** This do-file generates necessary variables used in the later process. This file also generates necessary values to replicate Table 2 Summary Statistics. (import: "*Data/jec.txt*"; output: "*Data/jecnew.dta*")
- **probit.do** This do-file runs Probit regression to provide the initial value of $\{w_1, ..., w_T\}$ for iterations. But when using this file to obtain the initial value, the final result of MLE will be quite different from the value on Table 3 as well as the result by the code provider. (import: "*Data/jecnew.dta*"; output: "*Data/wght0.csv*")
- **tsls.do** This do-file runs two-stage least squares regression on demand side and supply side. The result of this regression consists the first column of Table 3 on page 309. But I don't why this estimation result is quite different from the value on Table 3. So I do not use this result as the initial value in MLE.

2.2 Matlab

- **tsls.m** (script): The file is much similar with *tsls.do* to run two-stage least squares regression on demand side and supply side. The result of this regression consists the first column of Table 3 on page 309. And it also provides the initial value in MLE to search the best parameters. (import: "*Data/jec.txt*"; output: "*Data/param.mat*")

- **mlepn.m** (script): This file contains the codes of maximum log likelihood estimation. (import: "*Data/jec.txt*", "*Data/param.mat*"; output: "*Data/jecnew.mat*", results on the second column of Table 3)
  - **L1.m** (function): This function provides the maximum log likelihood function (set negative to find its minimum value). (import: "*Data/jecnew.mat*", "*Data/wght0.csv*"; output: the formula of maximum log likelihood function)
    - **lmda.m** (function): This function computes the weights for the expected log maximum likelihood in the later iterations (update λ_t and w_t series). (import: "*Data/jecnew.mat*"; output: the value of λ_t)
  - **L0.m** (function): This file is similar with *mlepn.m* which contains the codes of log maximum likelihood estimation, but we here let the coefficient of I_t (that is *po*) equal to 0. (import: "*Data/param.mat*", "*Data/jecnew0.mat*"; output: the optimal value of the log likelihood function L_0).
    - **LL0.m** (function): This file is similar with *L1.m* whichprovides the maximum likelihood function (set negative to find its minimum value), but we here let the coefficient of I_t (that is *po*) equal to 0 and do not need to calculate λ_t and w_t series. (import: "*Data/jecnew.mat*"; output: the formula of log maximum likelihood function)

3 Math Appendix: Maximum Likelihood Function

Reference: [DM] Russell Davidson and James G. MacKinnon (1999). Econometric Theory and Methods.

3.1 Full-Information Maximum Likelihood Function

$$\log L(I_1, ..., I_T) = -\frac{gn}{2}\log 2\pi - \frac{n}{2}\log|\mathbf{\Sigma}| + n\log|det\mathbf{\Gamma}| - \frac{1}{2}(\boldsymbol{y}. - \boldsymbol{X}.\boldsymbol{\beta}.)^T(\mathbf{\Sigma}^{-1}\otimes\boldsymbol{I_n})(\boldsymbol{y}. - \boldsymbol{X}.\boldsymbol{\beta}.)$$

(Davidson & MacKinnon, 12.80)

```
GM=[1 -beta(nb1d+1+nb1s+1);-beta(nb1d+1) 1];
SG=[(beta(nb-2)) beta(nb);beta(nb) (beta(nb-1))];
B=[beta(1:nb1d)' 0 0 0 0 0; beta(nb1d+1+1) 0 beta(nb1d+1+2:nb1d+1+nb1s)']';
betadot=beta(1:nb1d+1+nb1s+1);
ydot=[Y(:,1);Y(:,2)];
W(:,size(W,2))=pn; %po replaced with the new weights
Z2(:,size(Z2,2))=pn; %po replaced with the new weights
Xdot=blkdiag([Z1 lngr],[Z2 lnQ]);
%The negative log likelihood
kost=-(G*T/2)*log(2*pi)-(T/2)*log(det(SG))+T*(log(abs(det(GM))));
f=-kost+.5*((ydot-Xdot*betadot)'*kron(inv(SG),eye(T))*(ydot-Xdot*betadot));
```

3.2 The covariance matrix of $\hat{\boldsymbol{\beta}}_{\cdot}^{\boldsymbol{ML}}$

$$\hat{Var}(\hat{\boldsymbol{\beta}}_{\cdot}^{\boldsymbol{ML}}) = (\boldsymbol{X}_{\cdot}^{T}(\hat{\boldsymbol{B}}_{\boldsymbol{ML}}, \hat{\boldsymbol{\Gamma}}_{\boldsymbol{ML}})(\hat{\boldsymbol{\Sigma}}_{\boldsymbol{ML}}^{\boldsymbol{-1}} \otimes \boldsymbol{I_n})\boldsymbol{X}_{\cdot}(\hat{\boldsymbol{B}}_{\boldsymbol{ML}}, \hat{\boldsymbol{\Gamma}}_{\boldsymbol{ML}}))^{-1} \text{ (DM, 12.89)}$$

where

$$\boldsymbol{X}_{\cdot}(\hat{\boldsymbol{B}}_{\boldsymbol{ML}}, \hat{\boldsymbol{\Gamma}}_{\boldsymbol{ML}}) = \begin{bmatrix}\boldsymbol{Z}_{\cdot} & \hat{\boldsymbol{Y}}_{\cdot}(\hat{\boldsymbol{B}}_{\boldsymbol{ML}}, \hat{\boldsymbol{\Gamma}}_{\boldsymbol{ML}})\end{bmatrix} \text{ (DM, 12.55)}$$

where

$$\hat{\boldsymbol{Y}}_{\cdot}(\hat{\boldsymbol{B}}, \hat{\boldsymbol{\Gamma}})\hat{\boldsymbol{\Gamma}} = \boldsymbol{W}\hat{\boldsymbol{B}} \text{ (DM, 12.68)}$$

$$\hat{\boldsymbol{Y}}_{\cdot}(\hat{\boldsymbol{B}}, \hat{\boldsymbol{\Gamma}}) = \boldsymbol{W}\hat{\boldsymbol{B}}\hat{\boldsymbol{\Gamma}}^{-1} \text{ (DM, 12.70)}$$

Thus

$$\boldsymbol{X}_{\cdot}^{n}ew = \begin{bmatrix}\boldsymbol{Z}_{\cdot} & \boldsymbol{W}\hat{\boldsymbol{B}}\hat{\boldsymbol{\Gamma}}^{-1}\end{bmatrix}$$

```
Ydot_new=W*B*(inv(GM));
        %Y*GM=W*B+U -> Y=W*B*(GM^-1)+V
Xdot_new=blkdiag([Z1 Ydot_new(:,2)],[Z2 Ydot_new(:,1)]);
        %Xt=[Zt Yt], since y=X*beta+u=Z*beta1+Y*beta2+u
se=sqrt(diag(inv(Xdot_new'*(kron(inv(SG),eye(T)))*Xdot_new)));
        %Calculate the standard errors of estimaterd coefficients
se=[se;sqrt(diag(2.*kron(SG,SG)./T))];
        %Add estimated standard errors to the vector se
se(length(se)-1)=[];
        % [shat1 shat12; shat12 shat2] -> [shat1 shat12 shat12 shat2]
        % Delete one of the shat12 to make the vector dimension=37
```

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4 Index of *beta*

Table 1: Index of *beta1* & *beta2*

beta1		***beta2***	
Index	**Coefficient of**	**Index**	**Coefficient of**
1	C	1	C
2	L	2-5	DM1-4
3-14	month1-12	6	po
15	lngr	7-18	month1-12
		19	lnQ

Table 2: Index of *beta3*

beta3		
Index	**Coefficient of**	**Equation**
1	C	Demand
2	L	
3-14	month1-12	
15	lngr	
16	C	Supply
17-28	month1-12	
29-32	DM1-4	
33	po	
34	lnQ	
35	shat1	Standard error
36	shat2	
37	shat12	