



# AuE 8200: Machine Perception and Intelligence

Lecture: Image formation and vehicle visual perception

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# Outline

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- ➔ • Image Formation Steps
- Geometry
  - Pinhole camera model & Thin lens model
  - Perspective projection & Fundamental equation
- Radiometry
- Photometry
  - Color, human vision, & digital imaging
- Image Digitalization
  - Sampling, quantization & tessellations
- More on Digital Images
  - Neighbors, connectedness & distances



# Abstract Image

- An image can be represented by an image function whose general form is  $f(x,y)$ .
- $f(x,y)$  is a vector-valued function whose arguments represent a pixel location.
- The value of  $f(x,y)$  can have different interpretations in different kinds of images.

## Examples

Intensity Image

-  $f(x,y)$  = intensity of the scene

Range Image

-  $f(x,y)$  = depth of the scene from  
imaging system

Color Image

-  $f(x,y) = \{f_r(x,y), f_g(x,y), f_b(x,y)\}$

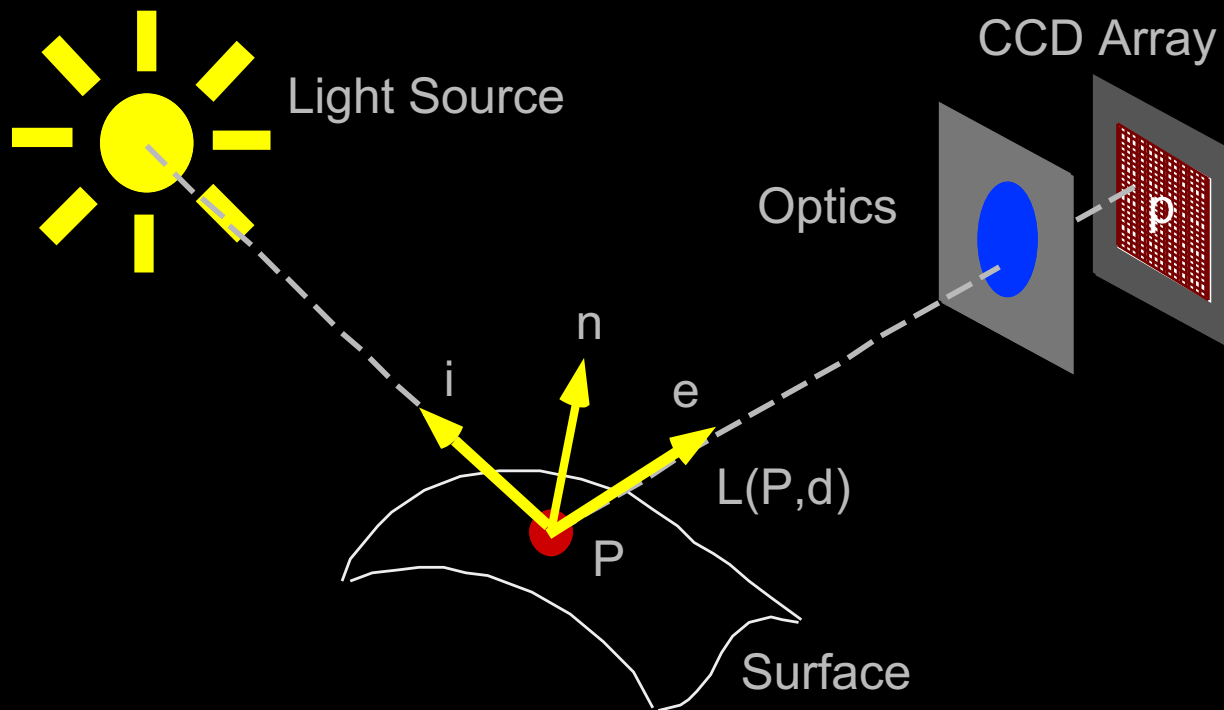
Video

-  $f(x,y,t)$  = temporal image sequence



# Basic Radiometry

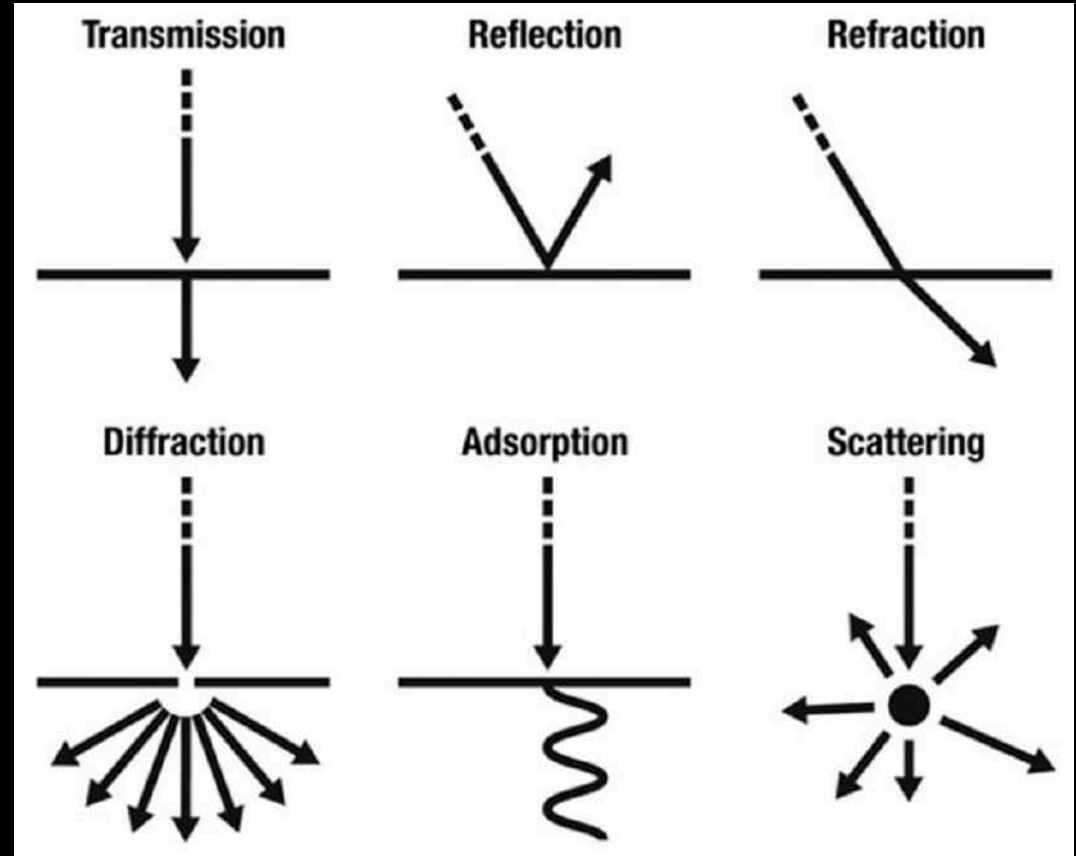
- Radiometry is the part of image formation concerned with the relation among the amounts of light energy
  - emitted from light sources,
  - reflected from surfaces,
  - registered by sensors,





# Light and Matter

- The interaction between light and matter can take many forms:
  - Transmission
  - Reflection
  - Refraction
  - Diffraction
  - Absorption
  - Scattering





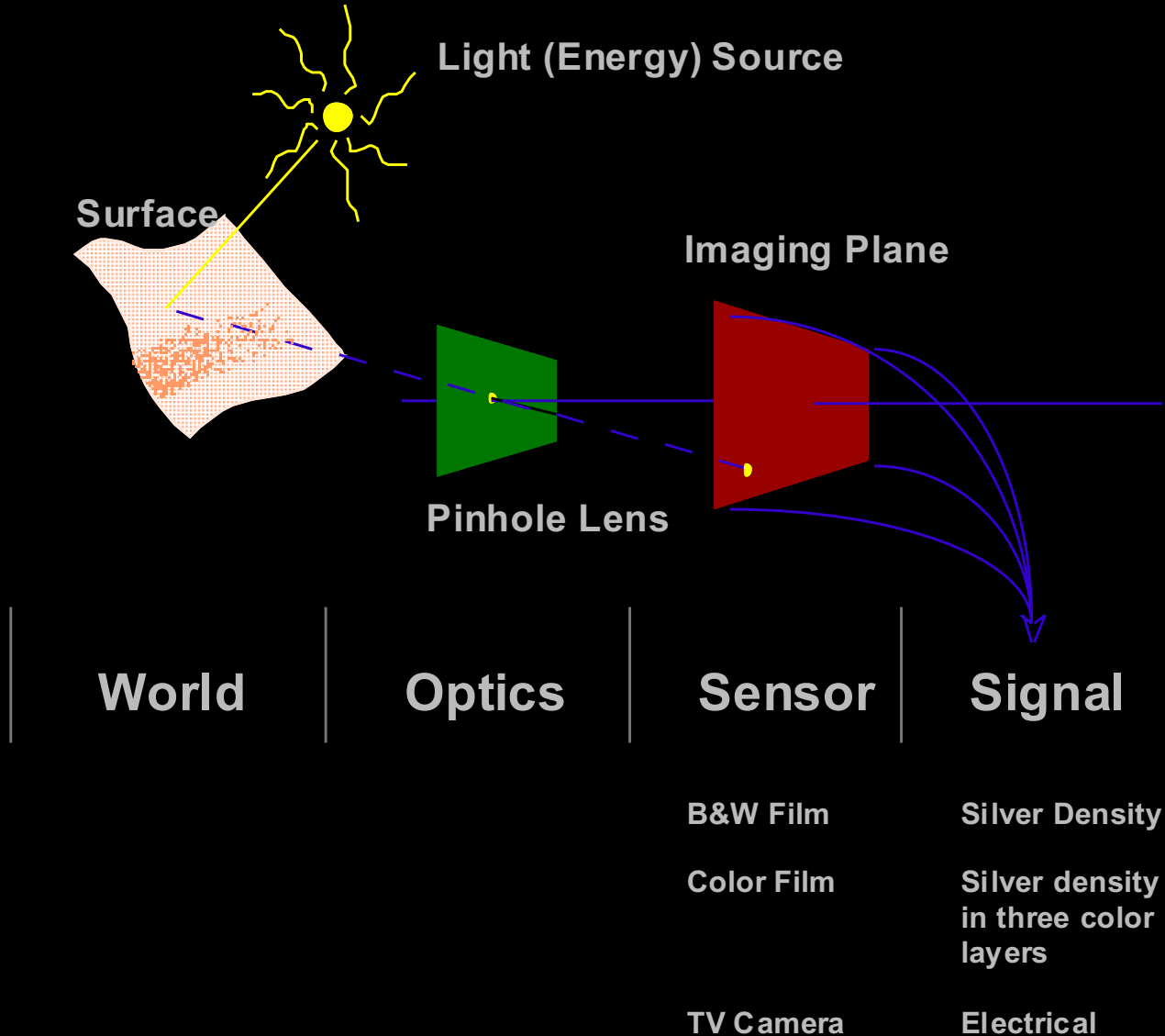
# Lecture Assumptions

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- Typical imaging scenario:
  - visible light
  - ideal lenses
  - Standard sensor (e.g. camera)
  - opaque objects
- The mechanism of image formation  
(from light → digital image)  
Retrieve useful information and processed to recover some of the characteristics of the 3D world which was imaged.

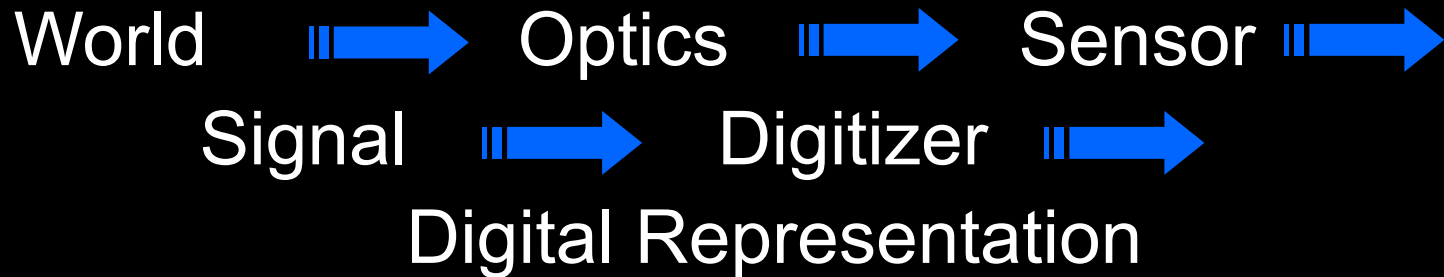


# Image Formation





# Steps



World	reality
Optics	focus {light} from world on sensor
Sensor	converts {light} to {electrical energy}
Signal	representation of incident light as continuous electrical energy
Digitizer	converts continuous signal to discrete signal
Digital Rep.	final representation of reality in computer memory





# Factors in Image Formation

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- Geometry
  - concerned with the relationship between points in the three-dimensional world and their images
- Radiometry
  - concerned with the relationship between the amount of light radiating from a surface and the amount incident at its image
- Photometry
  - concerned with ways of measuring the intensity of light
- Digitization
  - concerned with ways of converting continuous signals (in both space and time) to digital approximations



# Outline

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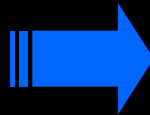
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# Geometry

- Geometry describes the projection of:

three-dimensional  
(3D) world



two-dimensional  
(2D) image plane.

- Typical Assumptions

- Light travels in a straight line

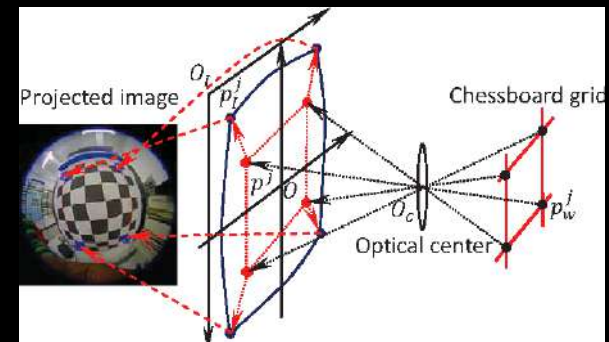
- Optical Axis: the axis perpendicular to the image plane and passing through the pinhole (also called the central projection ray)

- Each point in the image corresponds to a particular direction defined by a ray from that point through the pinhole.

- Various kinds of projections:

- perspective
- orthographic
- spherical
- oblique
- isometric

e.g.



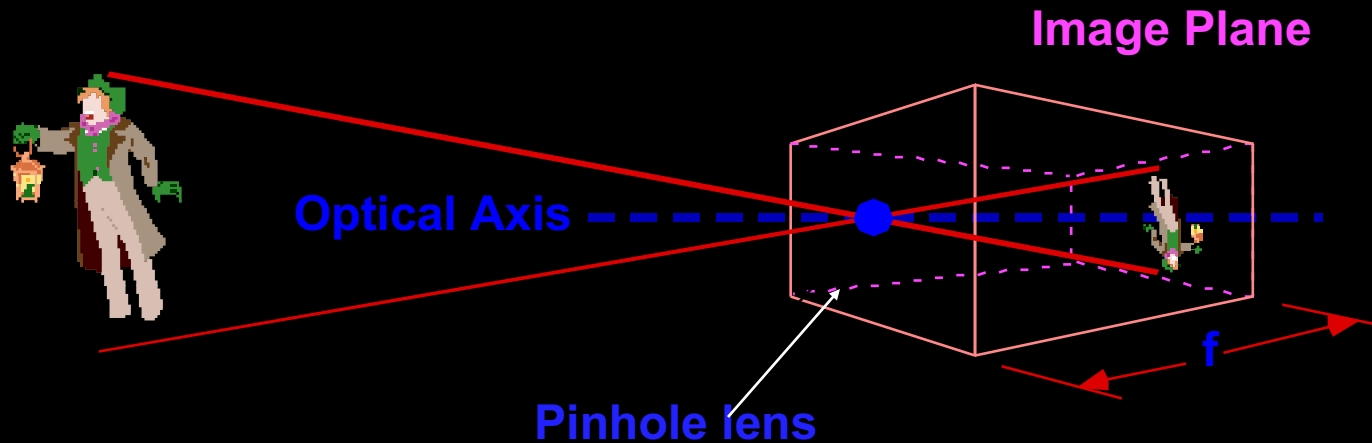


# Basic Optics

- Two models are commonly used:
  - Pin-hole camera
  - Optical system composed of lenses
- **Pin-hole** is the basis for most graphics and vision
  - Derived from physical construction of early cameras
  - Mathematics is very straightforward
- **Thin lens** model is first of the lens models
  - Mathematical model for a physical lens
  - Lens gathers light over area and focuses on image plane.



# Pinhole Camera Model

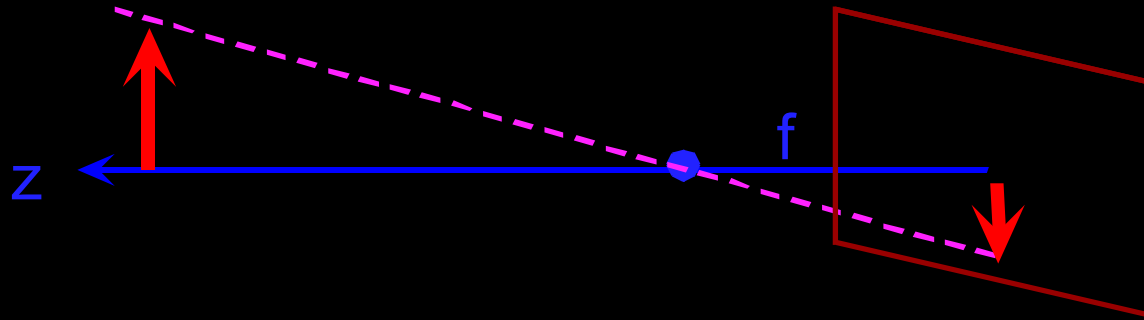


- World projected to 2D Image
  - Image inverted
  - Size reduced, (lost physical size)
  - Image is dim
  - No direct depth information
- $f$  called the focal length of the lens
- Known as perspective projection

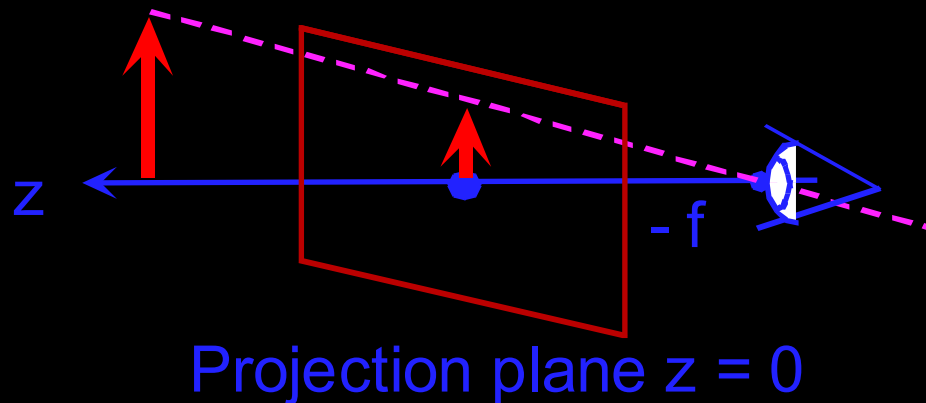


# Equivalent Geometry

- Consider case with object on the optical axis:



- More convenient with upright image:

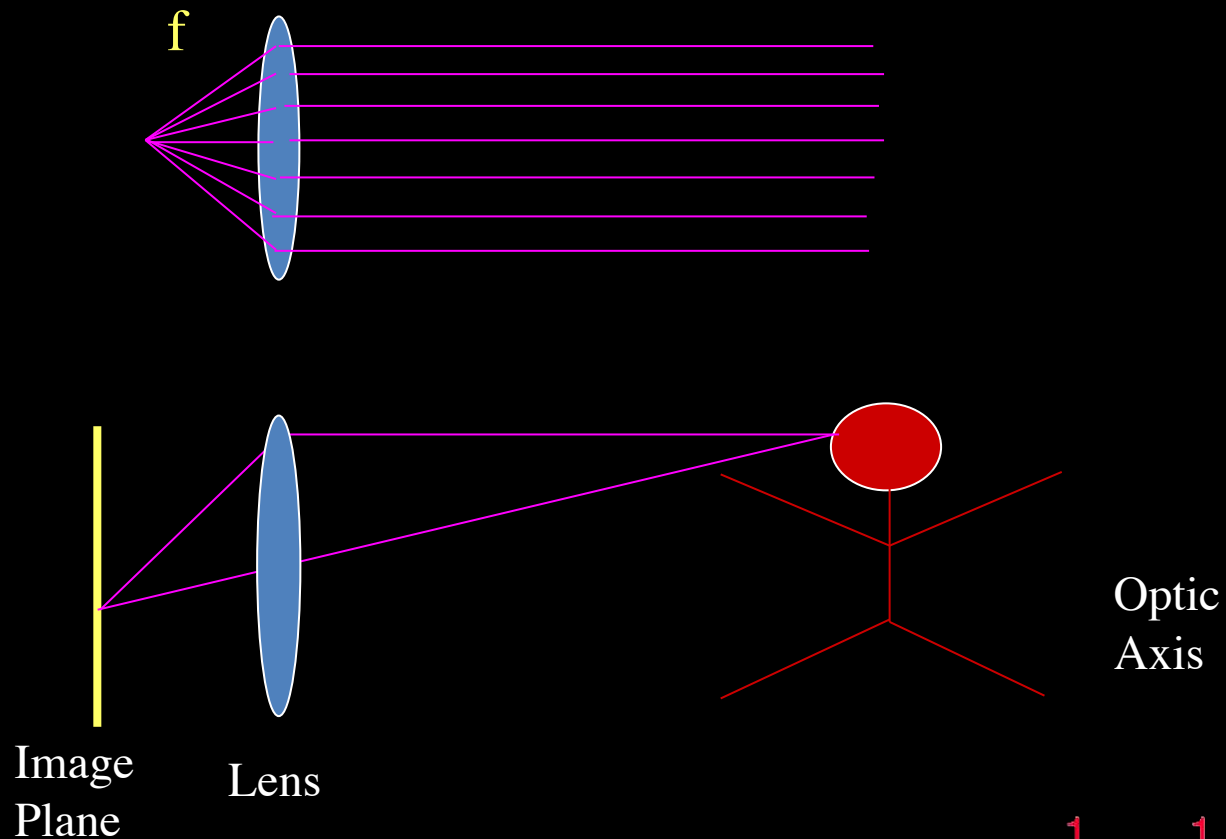


- Equivalent mathematically



# Thin Lens Model

- Rays entering parallel on one side converge at focal point.
- Rays diverging from the focal point become parallel.



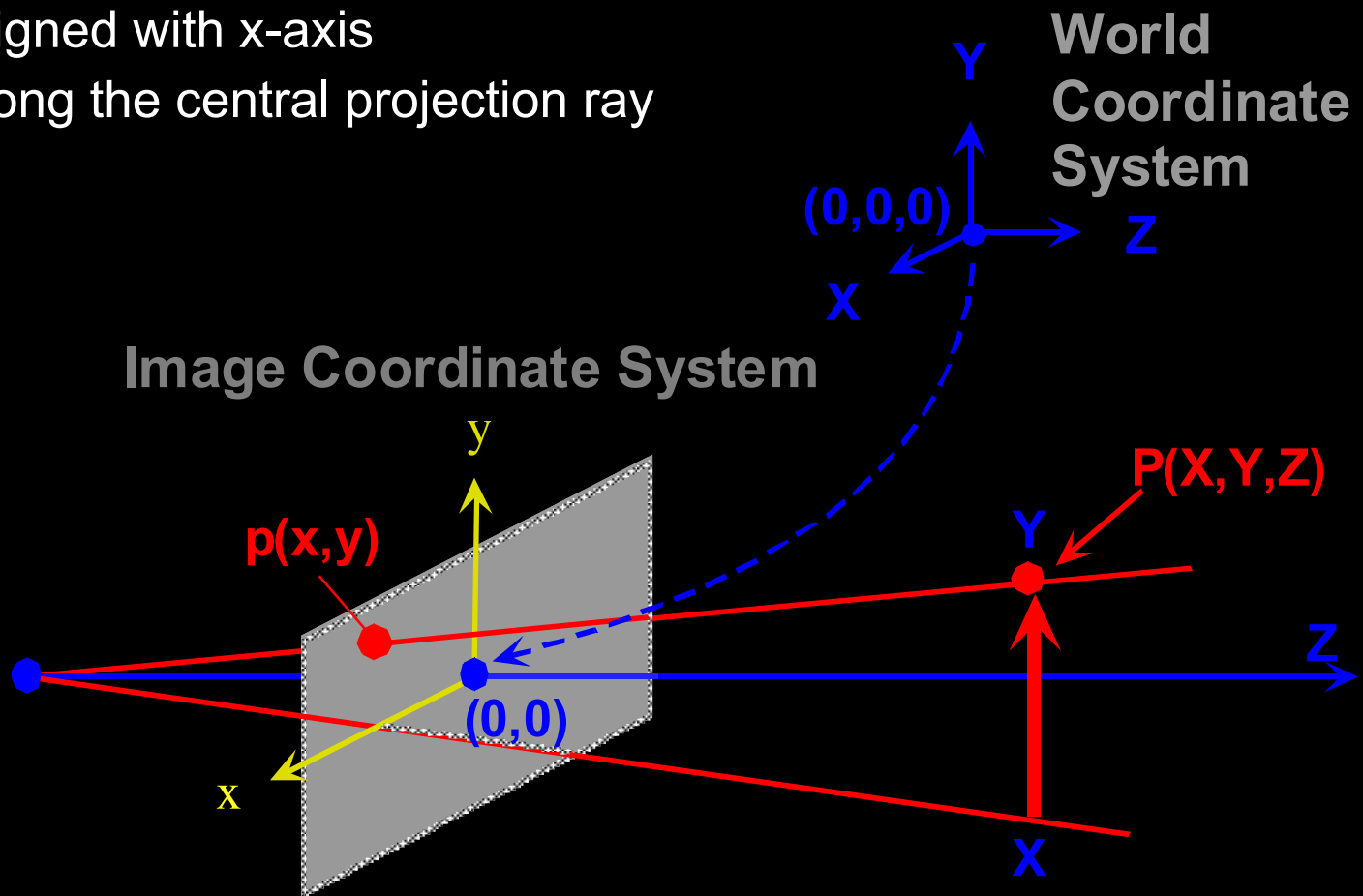
‘Thin Lens Law’

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$



# Coordinate Systems

- Simplified Case:
  - Origin of world and image coordinate systems coincide
  - Y-axis aligned with y-axis
  - X-axis aligned with x-axis
  - Z-axis along the central projection ray

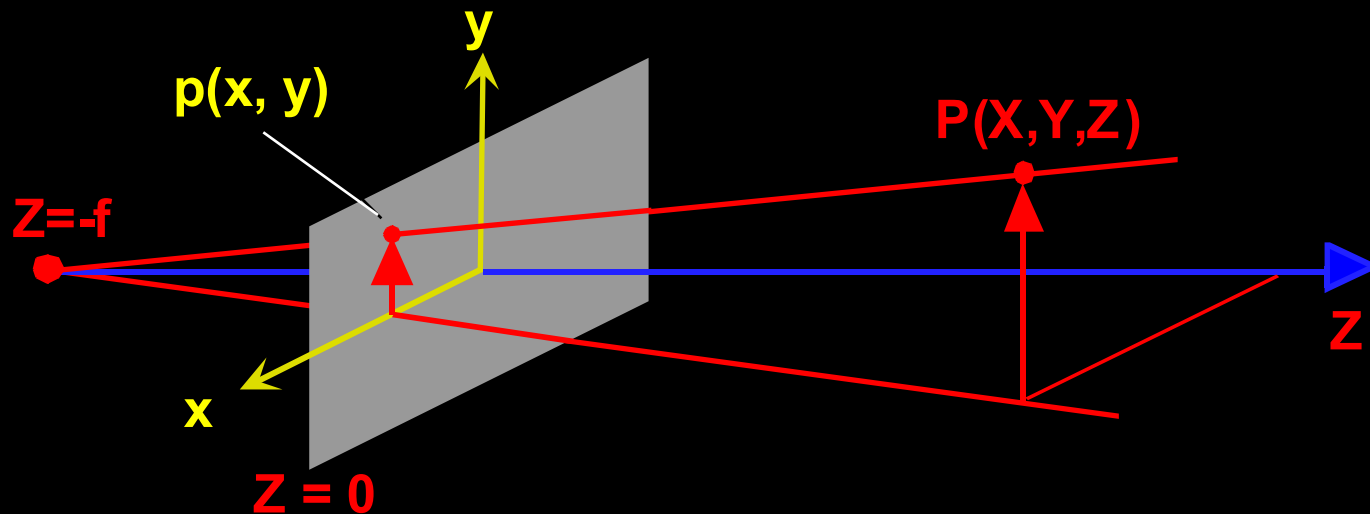






# Perspective Projection

- Compute the image coordinates of  $p$  in terms of the world coordinates of  $P$ .



- Look at projections in  $x$ - $z$  and  $y$ - $z$  planes

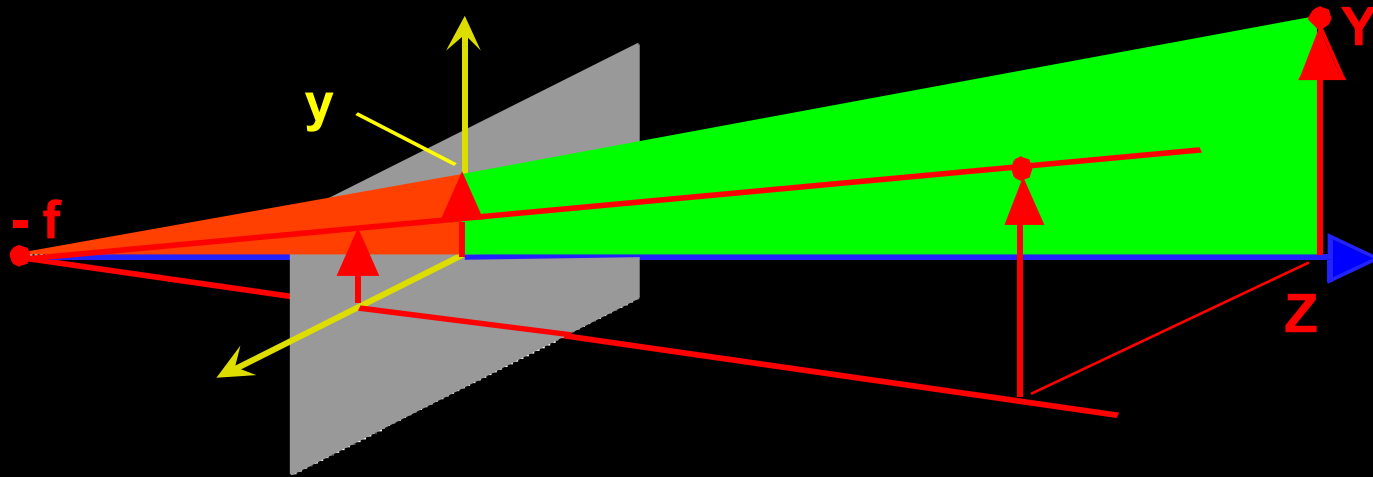


- $$\frac{x}{f} = \frac{X}{Z+f}$$

$$x = \frac{fX}{Z+f}$$



# Y-Z Projection



- By similar triangles:

$$\frac{y}{f} = \frac{Y}{Z+f}$$

$$y = \frac{fY}{Z+f}$$



# Perspective Equations

- Given point  $P(X,Y,Z)$  in the 3D world
- The two equations:

$$x = \frac{fX}{Z+f}$$

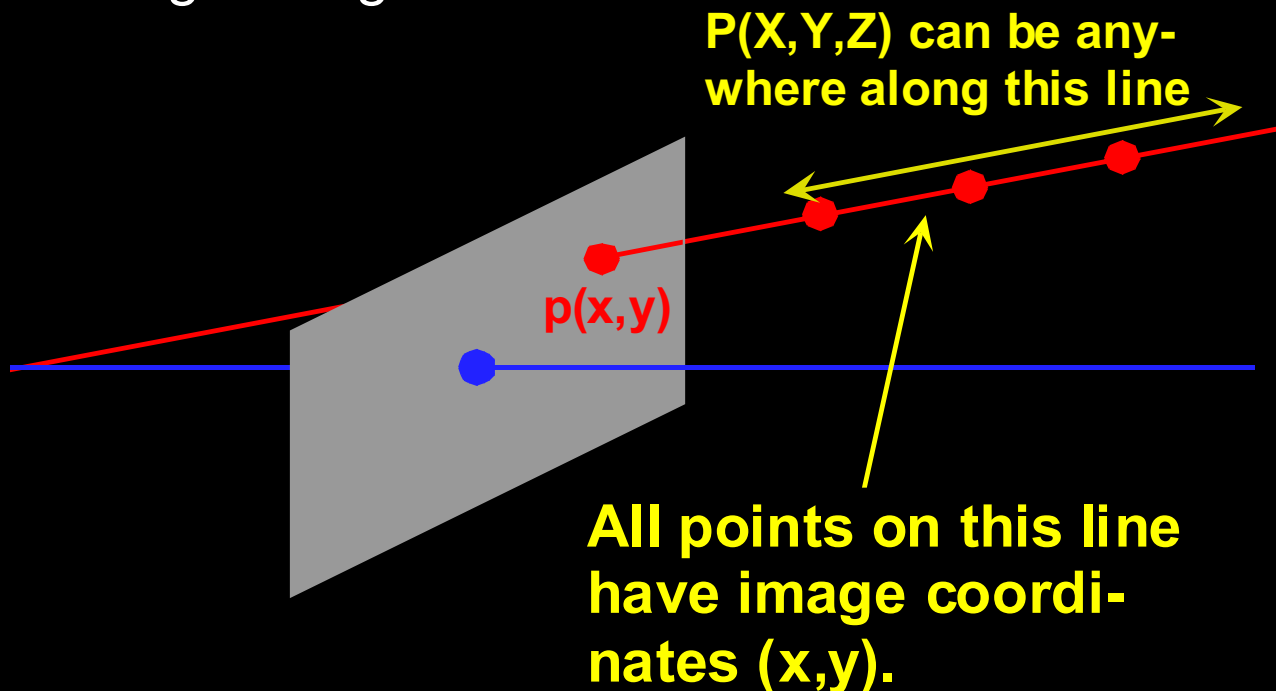
$$y = \frac{fY}{Z+f}$$

- Transform world coordinates  $(X,Y,Z)$  into image coordinates  $(x,y)$



# Reverse Projection

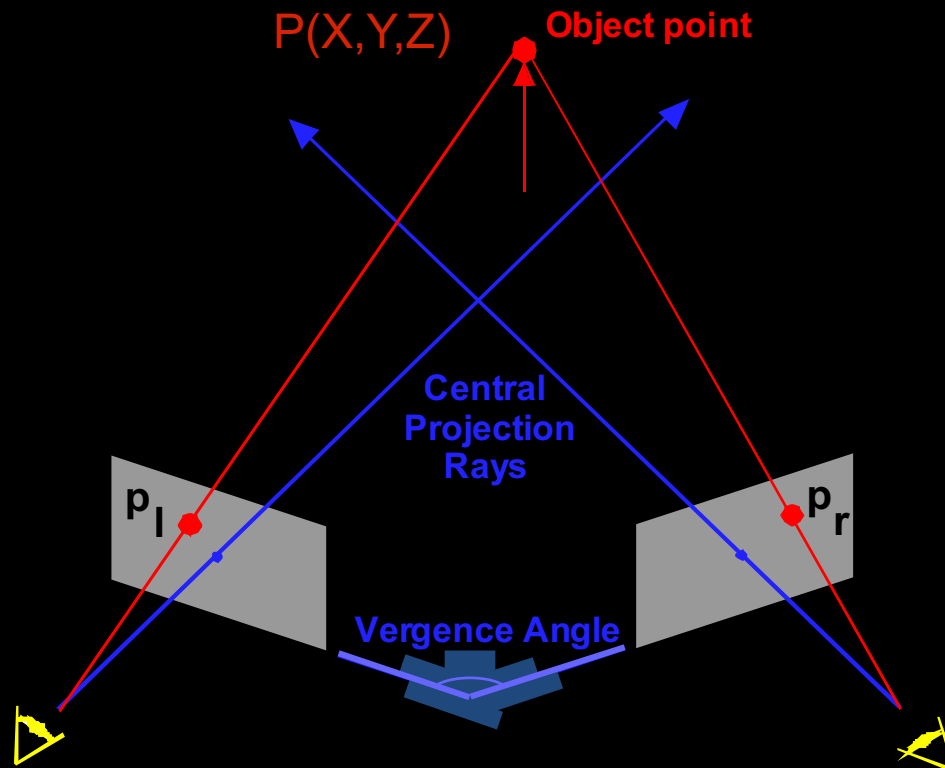
- Given a center of projection and image coordinates of a point, it is not possible to recover the 3D depth of the point from a single image.



In general, at least two images of the same point taken from two different locations are required to recover depth.



# Stereo Geometry



- Depth obtained by triangulation
- Correspondence problem:  $p_l$  and  $p_r$  must correspond to the left and right projections of  $P$ , respectively.



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# Radiometry

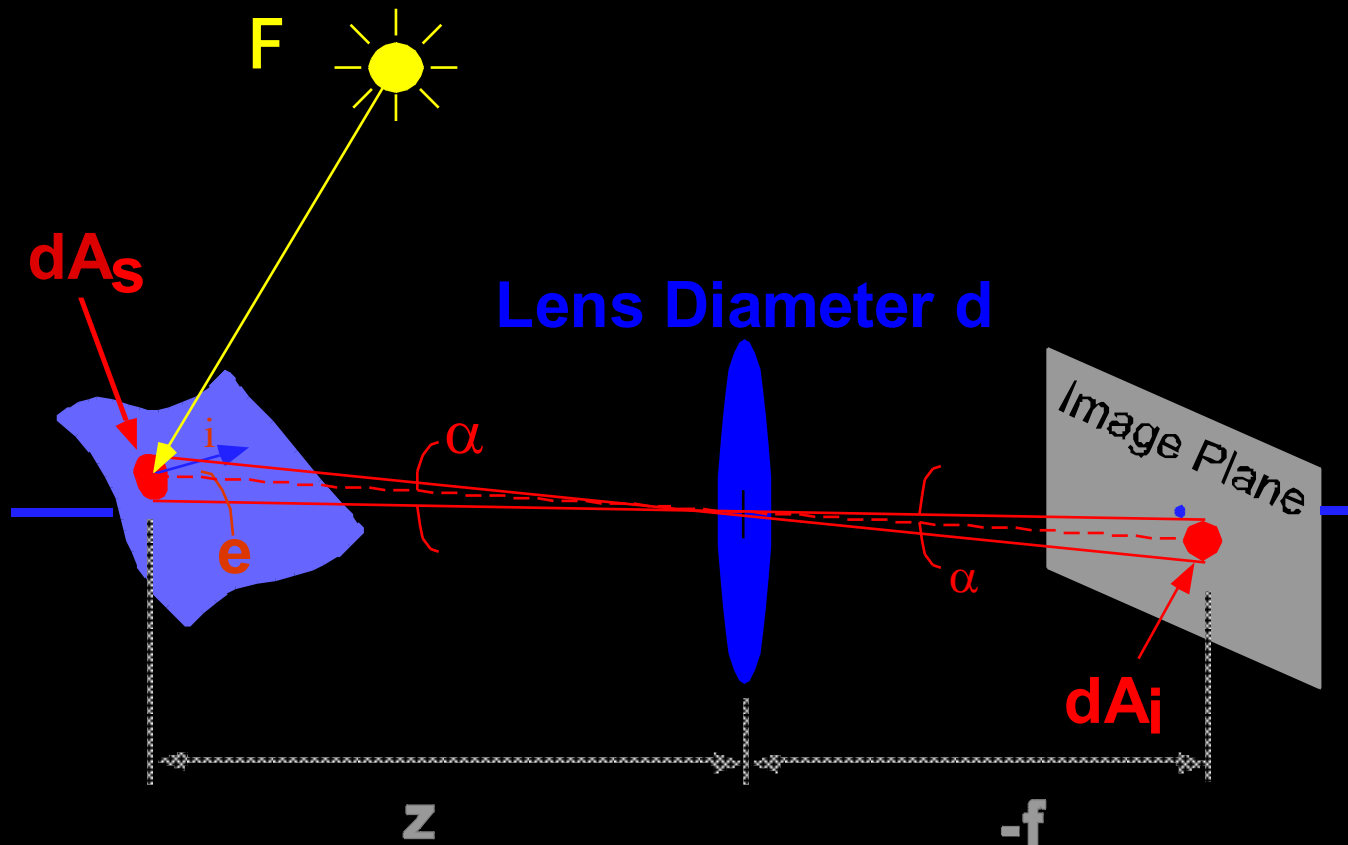
- **Image**: two-dimensional array of 'brightness' values.
- **Geometry**: where in an image a point will project.
- **Radiometry**: what the brightness of the point will be.
  - **Brightness**: informal notion used to describe both scene and image brightness.
  - **Image brightness**: related to energy flux incident on the image plane: => **Irradiance**
  - **Scene brightness**: brightness related to energy flux emitted (radiated) from a surface: => **Radiance**





# Radiometry and Geometry

- Goal: Relate the radiance of a surface to the irradiance in the image plane of a simple optical system.





# Radiometry Final Result

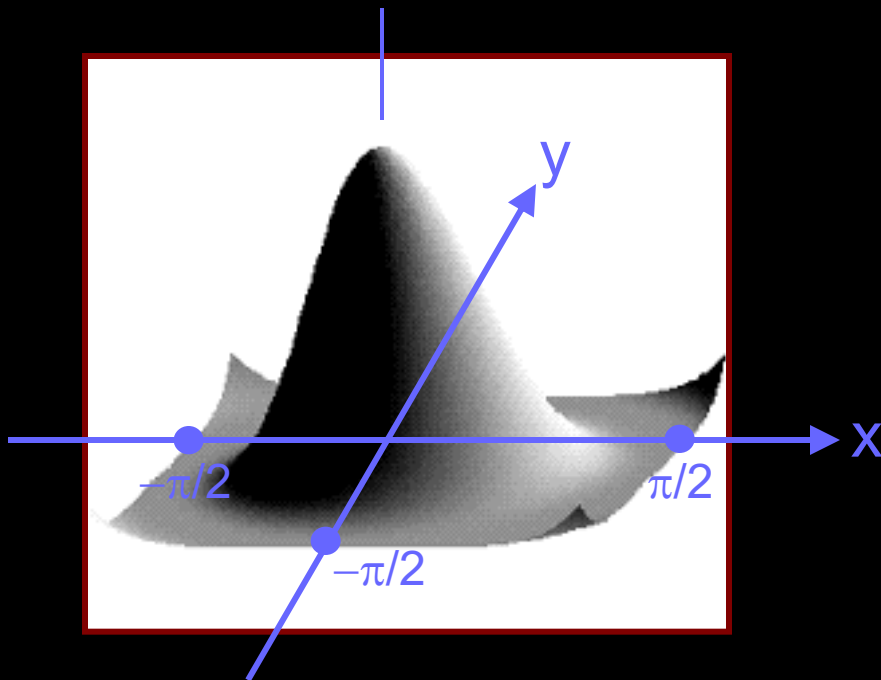
$$E_i = L_s \frac{\pi}{4} \left[ \frac{d}{-f} \right]^2 \cos^4 \alpha$$

- Image irradiance is proportional to:
  - Scene radiance  $L_s$
  - Focal length of lens  $f$
  - Diameter of lens  $d$ 
    - $f/d$  is often called the **f-number** of the lens
  - Off-axis angle  $\alpha$

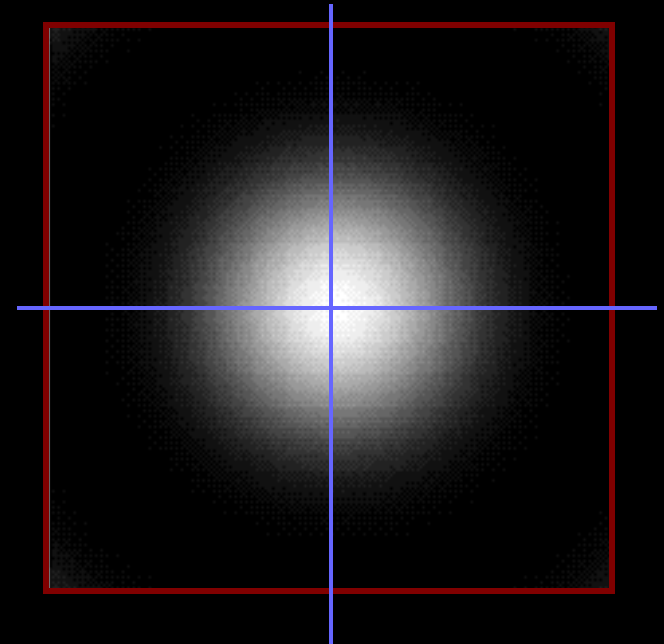


# $\cos^4(\alpha)$ Light Falloff

Lens Center



Top view shaded by height





# Outline

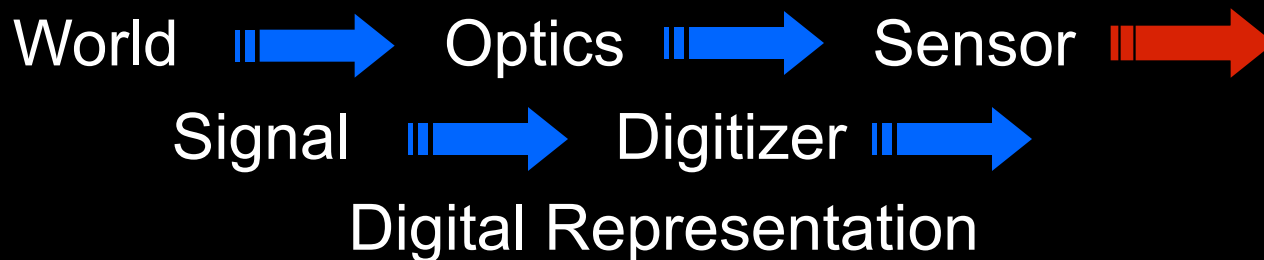
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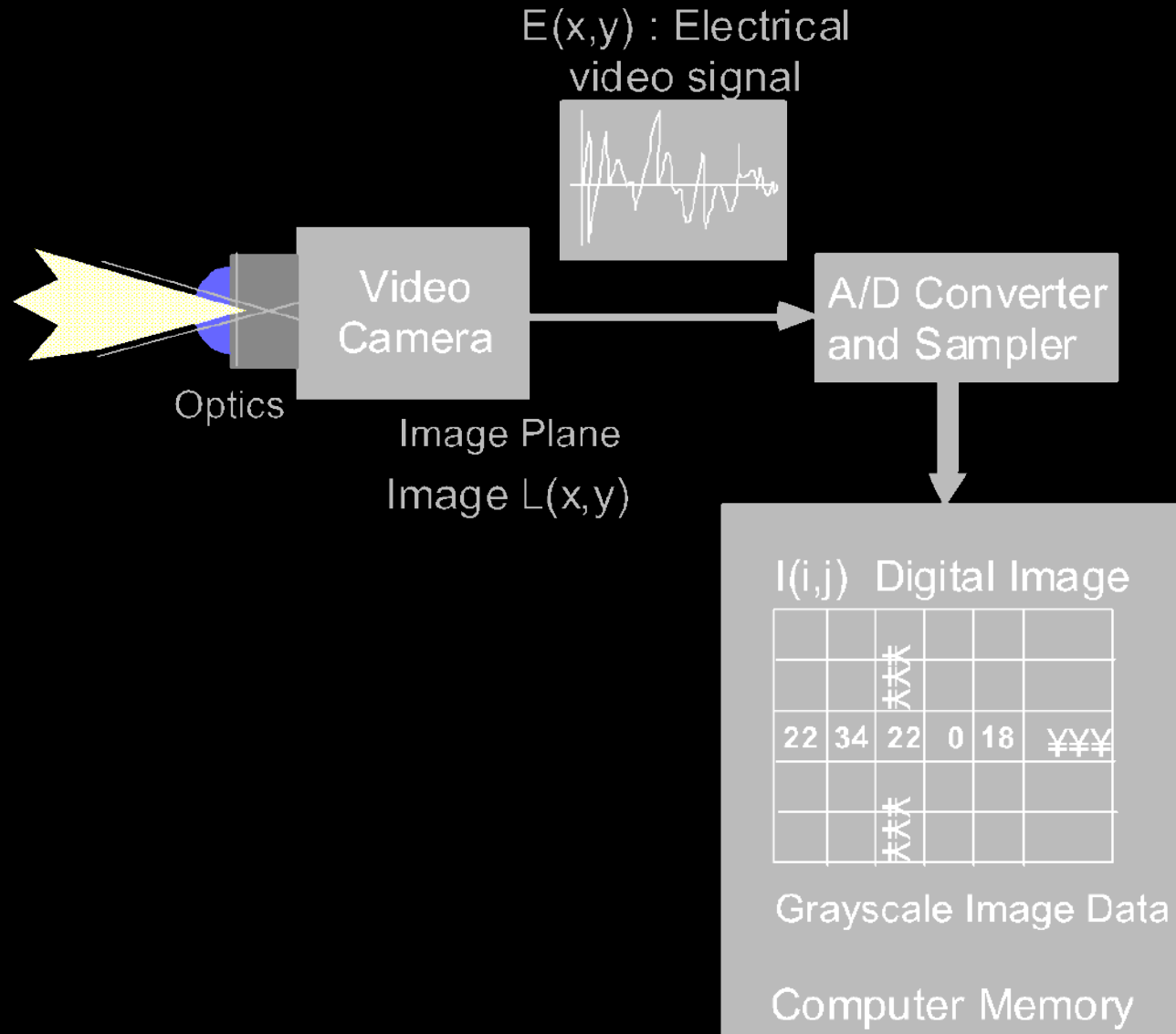
# Photometry

- Photometry:  
Concerned with mechanisms for converting light energy into electrical energy.



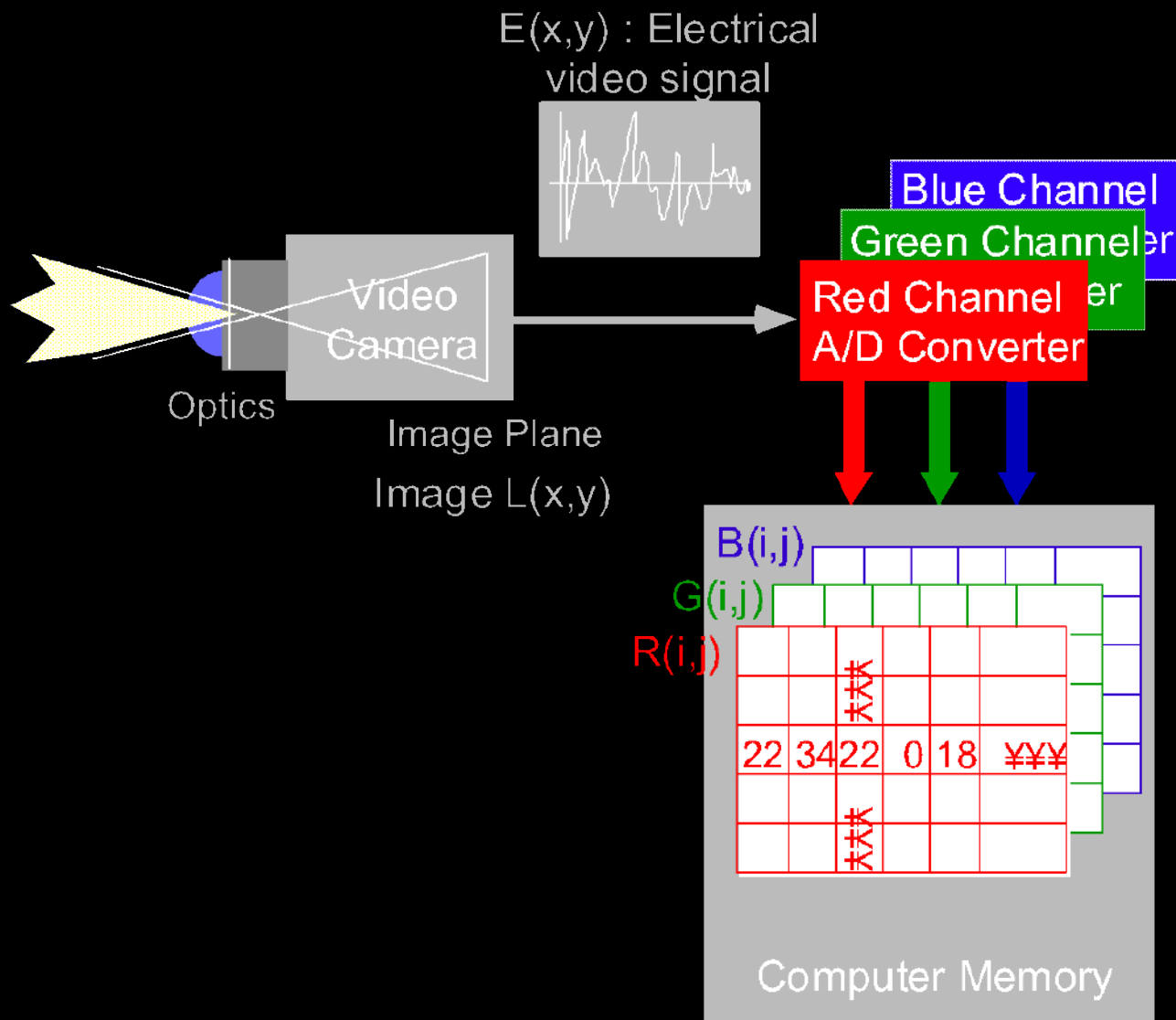


# Image/Video Generating





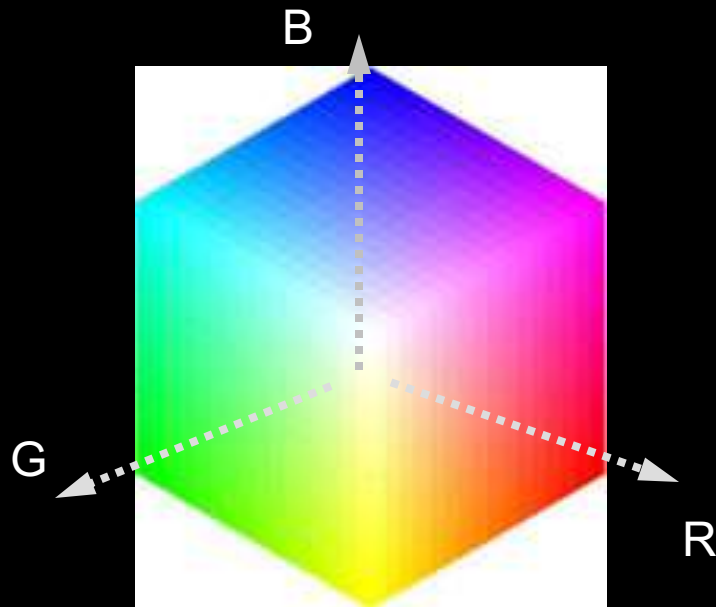
# Color Image/Video Generating



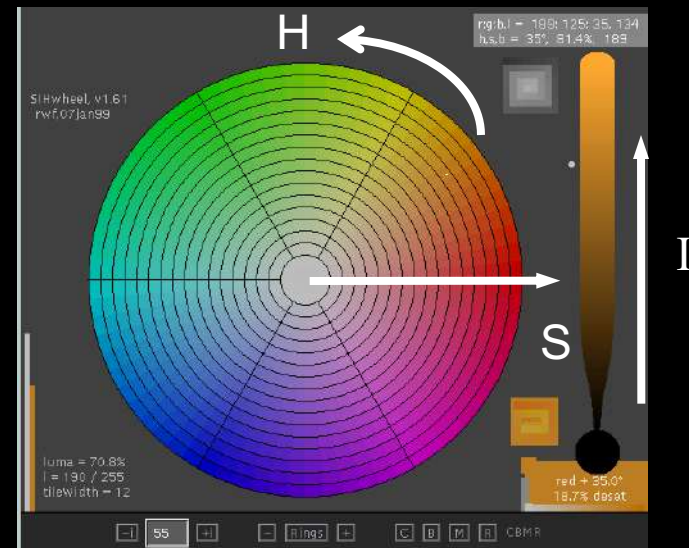


# Color Representation

- Color Cube and Color Wheel



HSI  
(hue, saturation, intensity)



- For [color spaces](#), please read
  - Color Cube <http://www.morecrayons.com/palettes/webSmart/>
  - Color Wheel <http://r0k.us/graphics/SIHwheel.html>
  - [http://www-viz.tamu.edu/faculty/parke/ends489f00/notes/sec1\\_4.html](http://www-viz.tamu.edu/faculty/parke/ends489f00/notes/sec1_4.html)





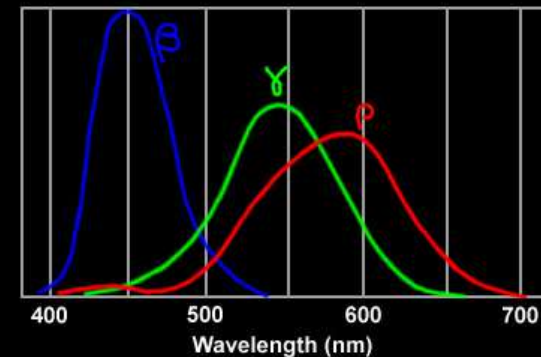
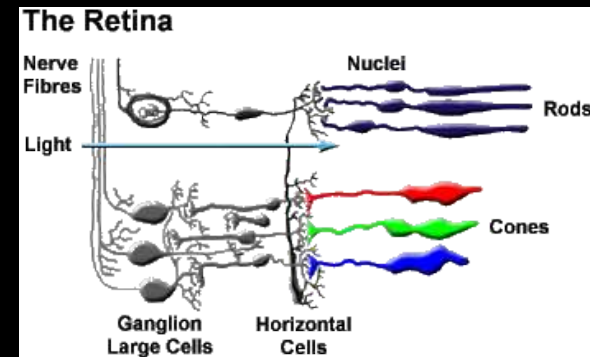
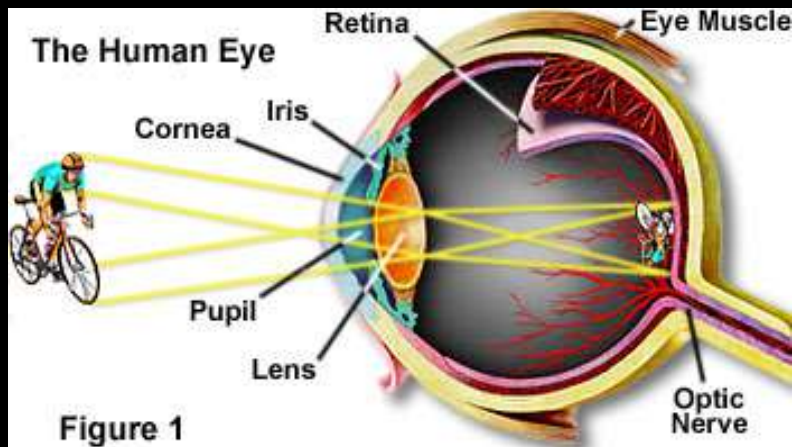
# Digital Color Cameras

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- Three CCD-chips cameras
  - R, G, B separately, AND digital signals instead analog video
- One CCD Cameras
  - RGB “Bayer” Color and MicroLenses
  - <http://www.siliconimaging.com/RGB%20Bayer.htm>



# Human Eyes & Color Perception



- Visit a cool site with Interactive Java tutorial:
  - [Human Vision and Color Perception](#)
- Another site about human color perception:
  - [Color Vision](#)



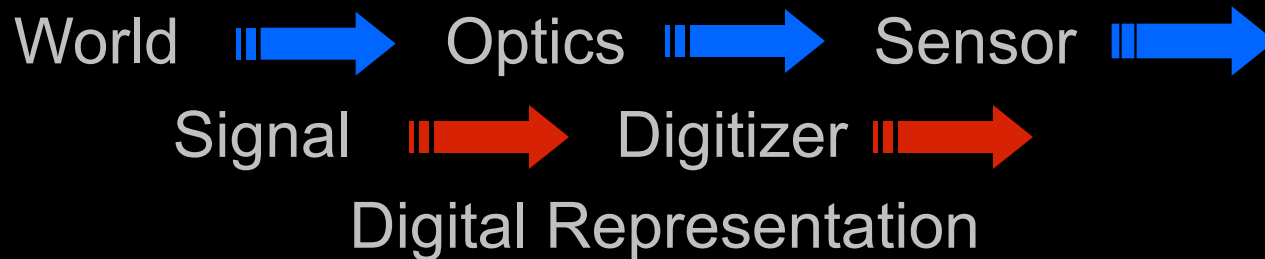
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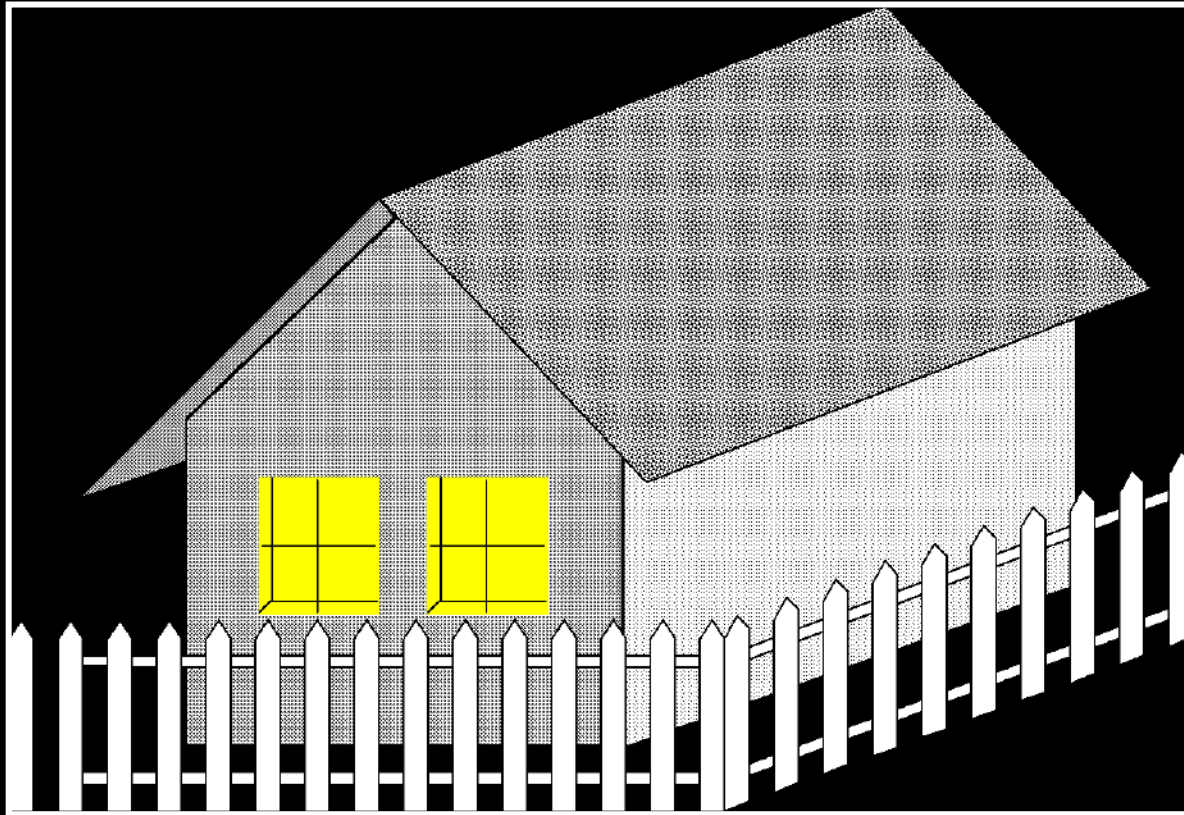
# Image Digitization



- Digitization: conversion of the continuous (in space and value) electrical signal into a digital signal (digital image)
- Three decisions must be made:
  - Spatial resolution (how many samples to take)
  - Signal resolution (dynamic range of values- quantization)
  - Tessellation pattern (how to 'cover' the image with sample points)

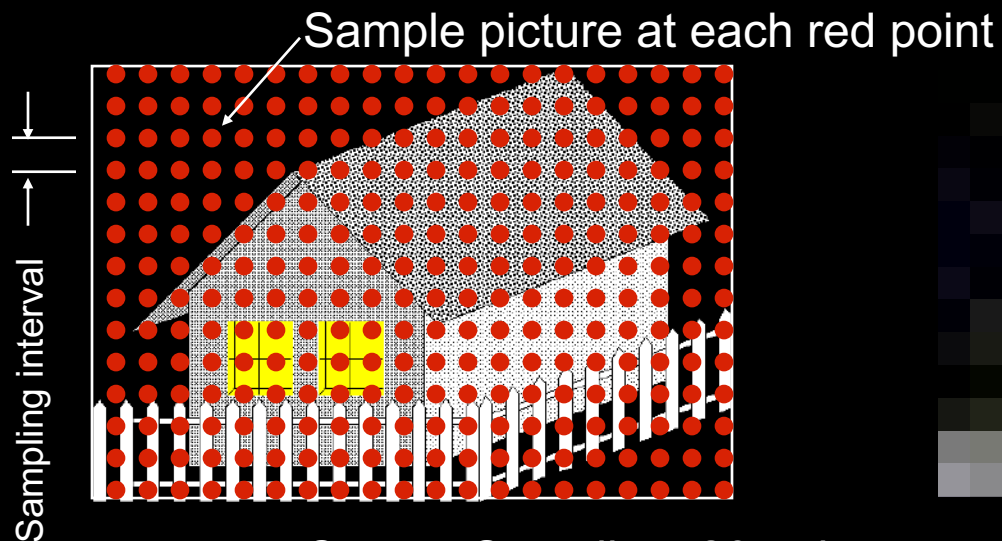


# Digitization: Spatial Resolution

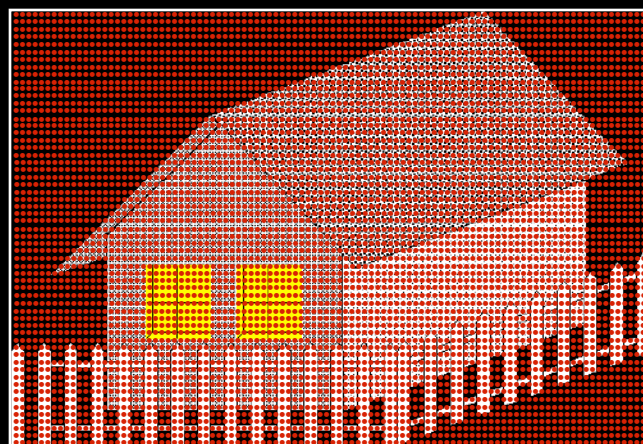


- Let's digitize this image
  - Assume a square sampling pattern
  - Vary density of sampling grid

# Spatial Resolution



Coarse Sampling: 20 points per row by 14 rows



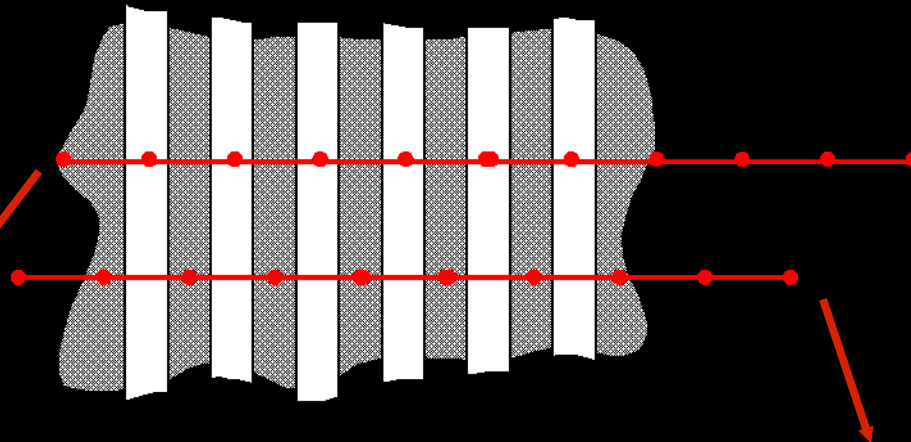
Finer Sampling: 100 points per row by 68 rows



# Effect of Sampling Interval

- Look in vicinity of the picket fence:

Sampling Interval: 



100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100
100	100	100	100	100	100

White Image!

No evidence  
of the fence!

40	40	40	40	40	40
40	40	40	40	40	40
40	40	40	40	40	40
40	40	40	40	40	40

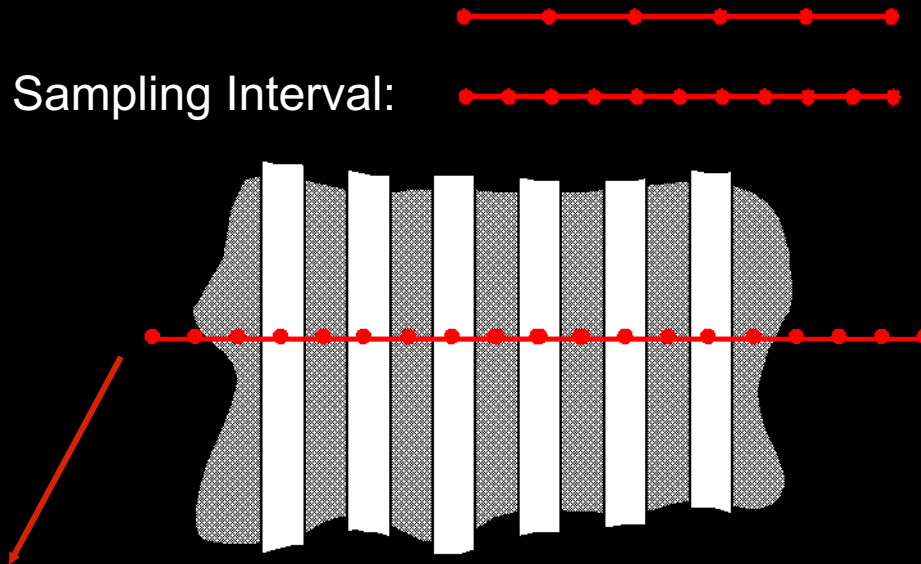
Dark Gray Image!





# Effect of Sampling Interval

- Look in vicinity of picket fence:



40	100	40	100	40
40	100	40	100	40
40	100	40	100	40
40	100	40	100	40

What's the difference between this attempt and the last one?

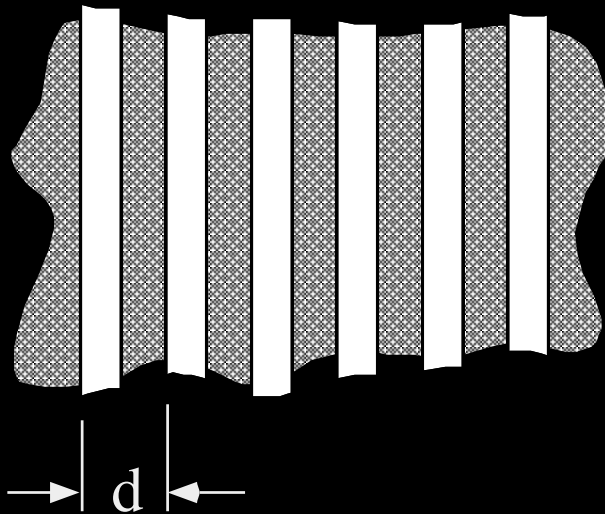
Now we've got a fence!



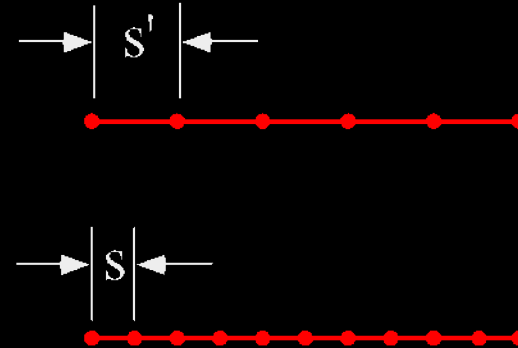


# The Missing Fence Found

- Consider the repetitive structure of the fence:



Sampling Intervals



Case 1:  $s' = d$

The sampling interval is equal to the size of the repetitive structure

No Fence

Case 2:  $s = d/2$

The sampling interval is one-half the size of the repetitive structure

Fence



# The Sampling Theorem

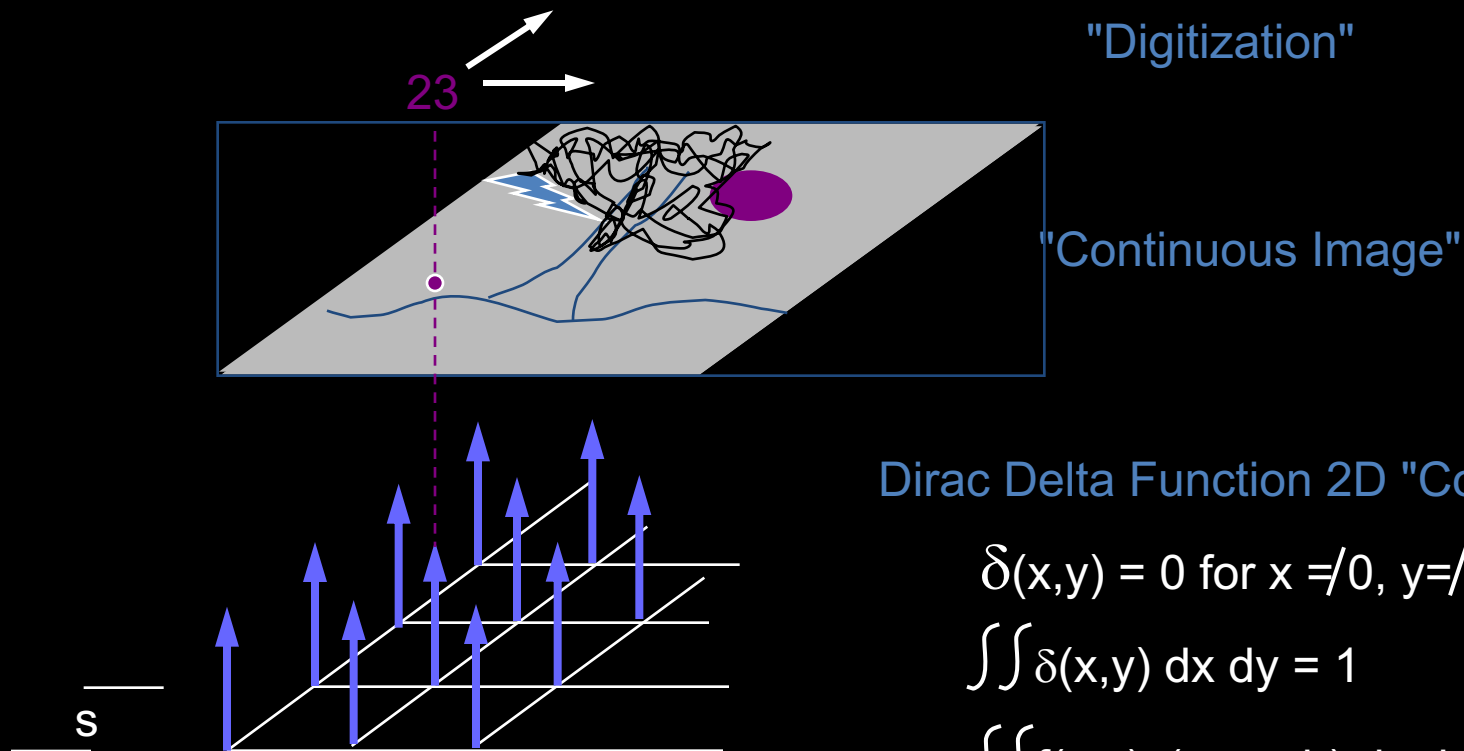
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- IF: the size of the smallest structure to be preserved is  $d$
- THEN: the sampling interval must not be larger than  $d/2$
- Can be shown to be true mathematically
- Repetitive structure has a certain frequency
  - To preserve structure must sample at twice the frequency
  - Holds for images, audio CDs, digital television....
- Leads naturally to Fourier analysis



# 2D Sampling

- Rough Idea: Ideal Case



Dirac Delta Function 2D "Comb"

$$\delta(x,y) = 0 \text{ for } x \neq 0, y \neq 0$$

$$\iint \delta(x,y) dx dy = 1$$

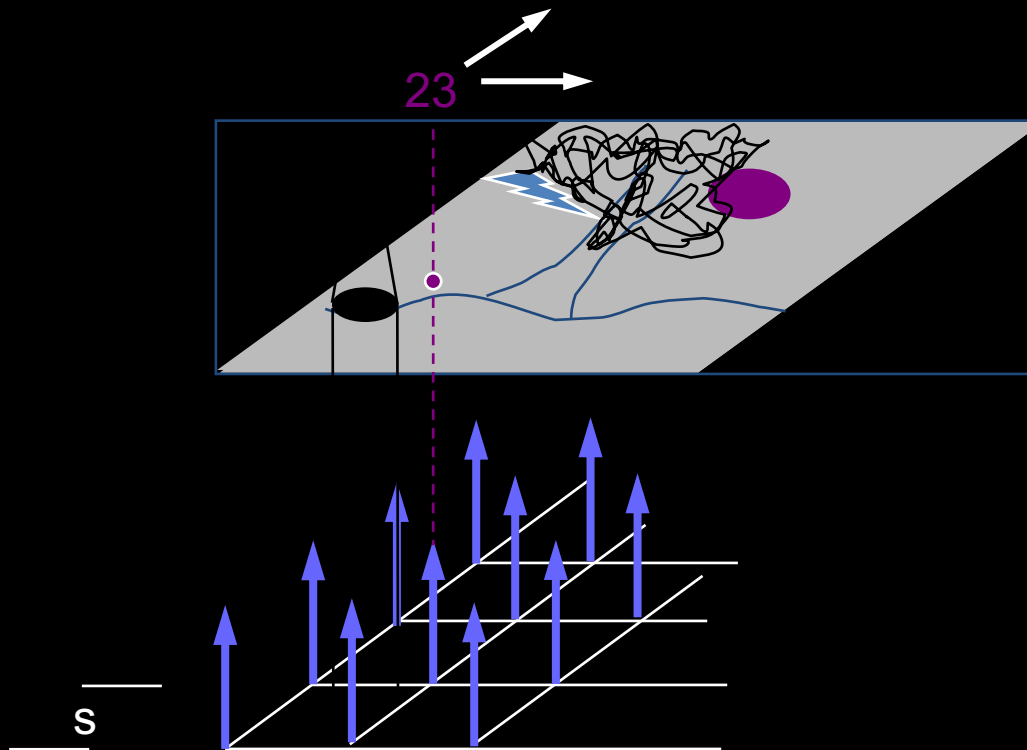
$$\iint f(x,y) \delta(x-a,y-b) dx dy = f(a,b)$$

$$\delta(x-ns,y-ns) \text{ for } n = 1 \dots 32 \text{ (e.g.)}$$



# 2D Sampling

- Rough Idea: Actual Case
  - Can't realize an ideal point function in real equipment
  - "Delta function" equivalent has an area
  - Value returned is the average over this area





# Mixed Pixel Problem



Reflected energy



# Image Quantization

- $I(x,y)$  = continuous signal:  $0 \leq I \leq M$
- Want to quantize to  $K$  values  $0, 1, \dots, K-1$
- $K$  usually chosen to be a power of 2:

$K$	#Levels	#Bits
2	2	1
4	4	2
8	8	3
16	16	4
32	32	5
64	64	6
128	128	7
256	256	8

- Mapping from input signal to output signal is to be determined.
- Several types of mappings: uniform, logarithmic, etc.

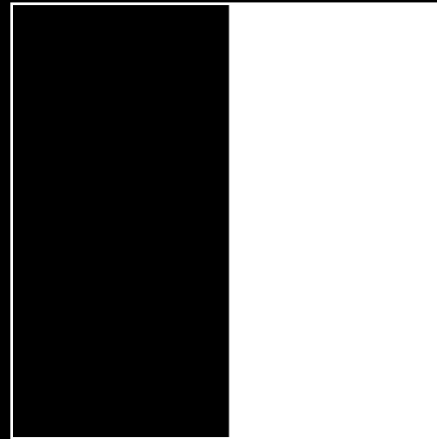


# Choice of K

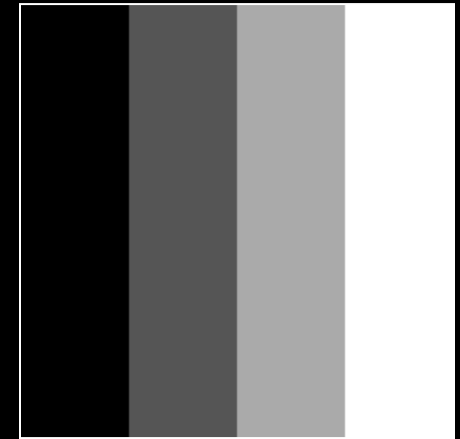
Original



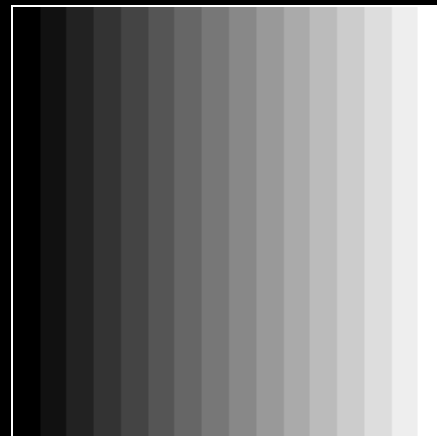
Linear Ramp



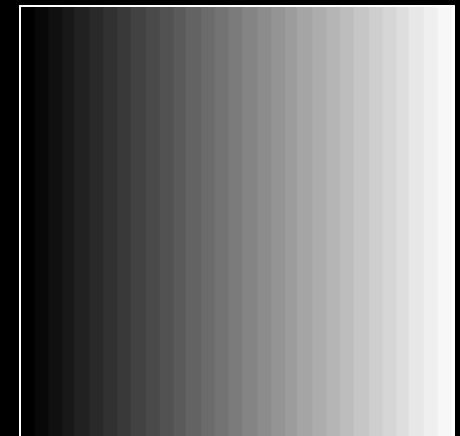
$K=2$



$K=4$



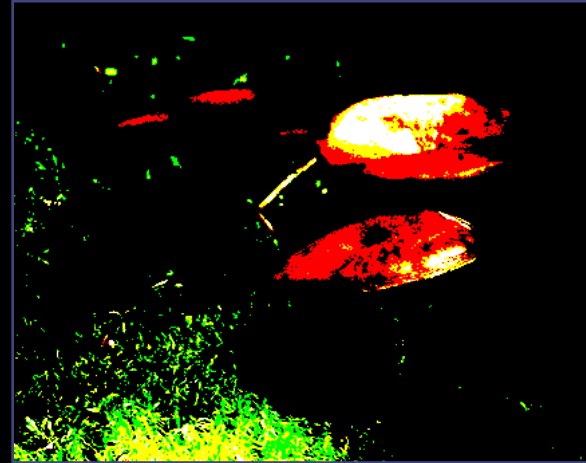
$K=16$



$K=32$



# Choice of K



$K=2$  (each color)



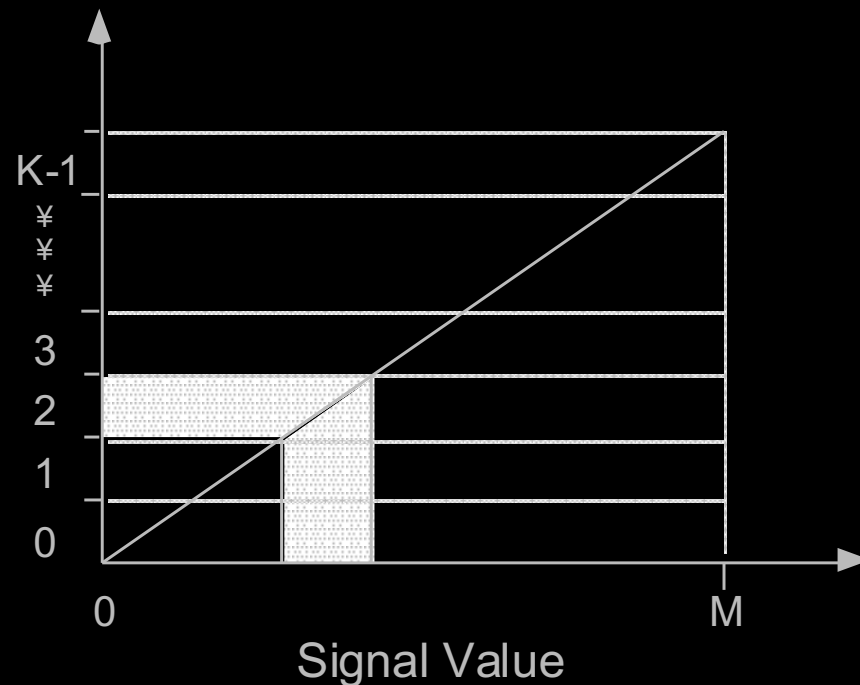
$K=4$  (each color)





# Choice of Function: Uniform

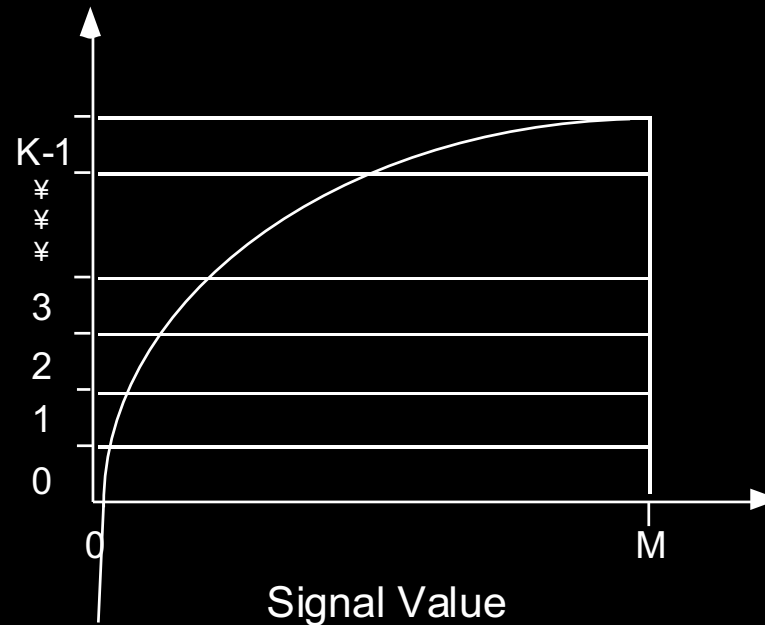
- Uniform quantization divides the signal range  $[0-M]$  into  $K$  equal-sized intervals.
- The integers  $0, \dots, K-1$  are assigned to these intervals.
- All signal values within an interval are represented by the associated integer value.
- Defines a mapping:





# Logarithmic Quantization

- Signal is  $\log I(x,y)$ .
- Effect is:

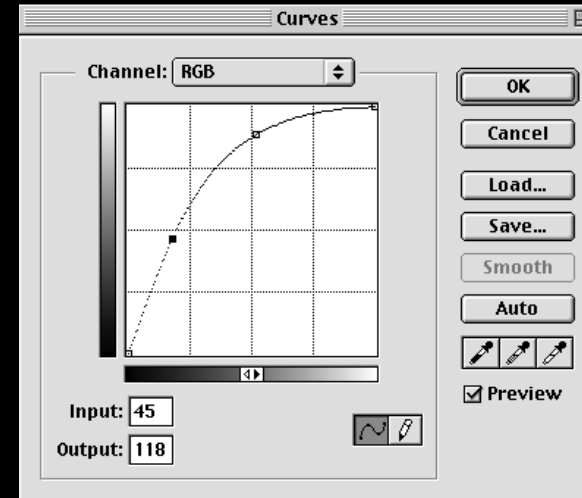


- Detail enhanced in the low signal values at expense of detail in high signal values.



# Logarithmic Quantization

Original



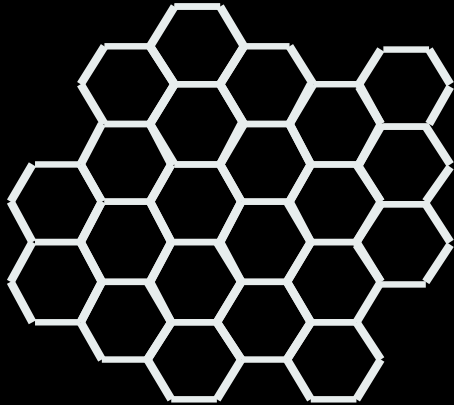
Quantization  
Curve

Logarithmic  
Quantization

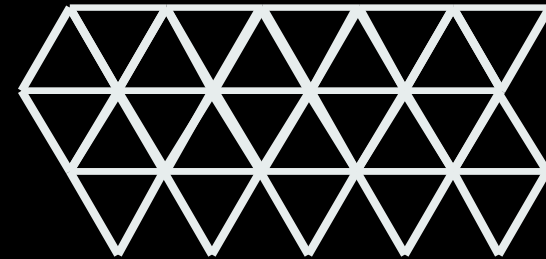




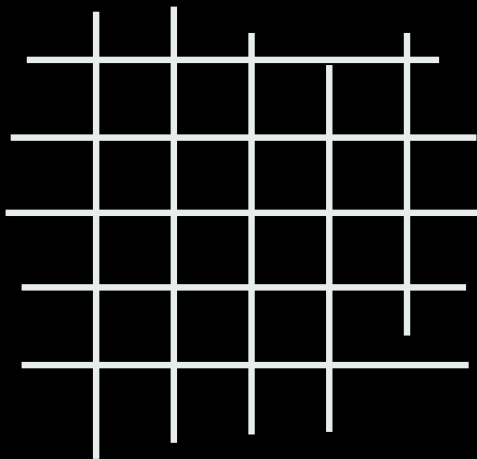
# Tessellation Patterns



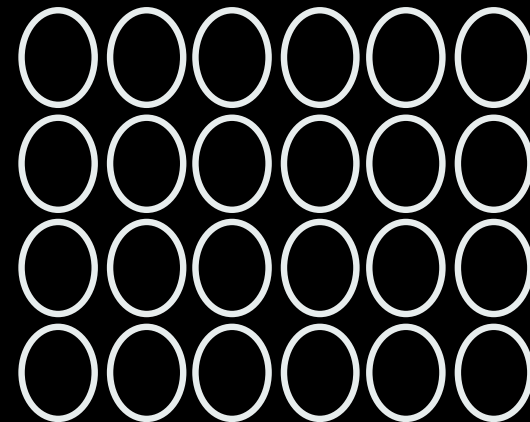
Hexagonal



Triangular



Rectangular



Typical



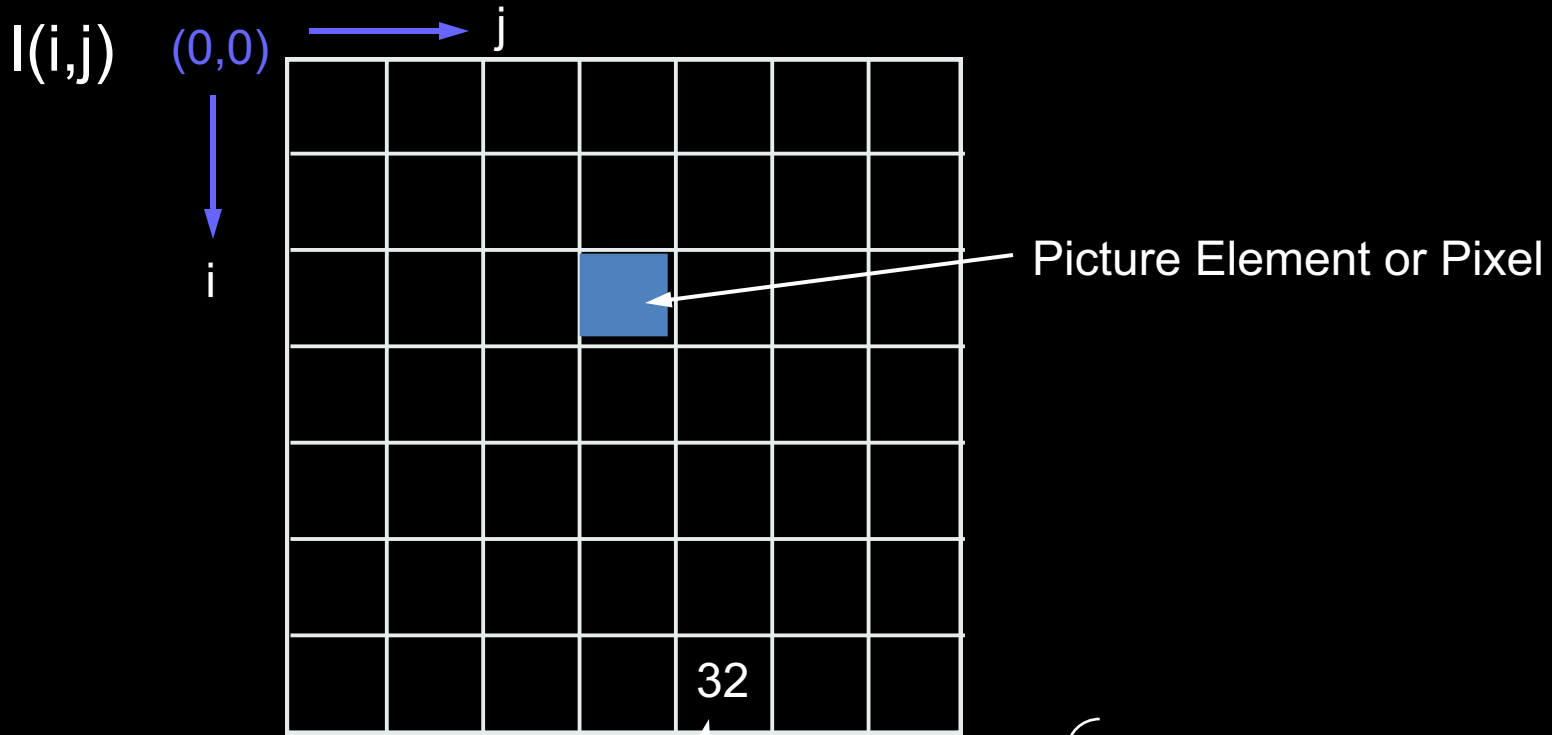
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  - Neighbors, connectedness & distances



# Digital Geometry



- Neighborhood
- Connectedness
- Distance Metrics

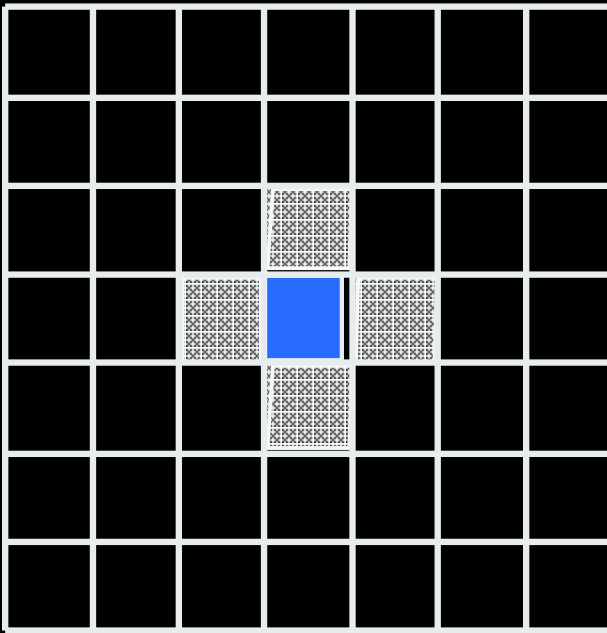
Pixel value  $I(i,j) =$

$\left\{ \begin{array}{l} 0,1 \text{ Binary Image} \\ 0 \sim K-1 \text{ Gray Scale Image} \\ \text{Vector: Multispectral Image} \end{array} \right.$

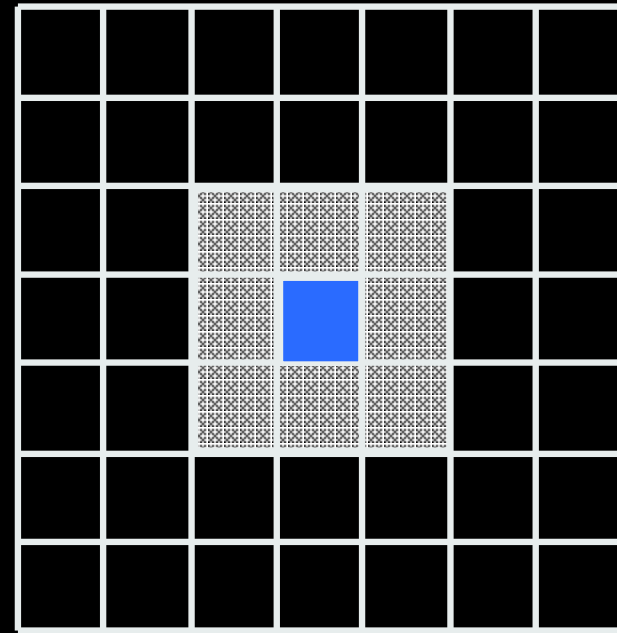


# Neighbor

- Consider the definition of the term 'neighbor'
- Two common definitions:



Four Neighbor

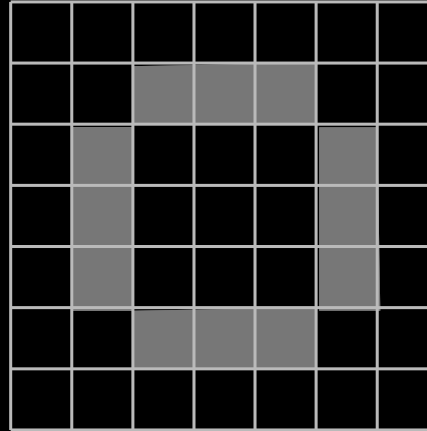


Eight Neighbor

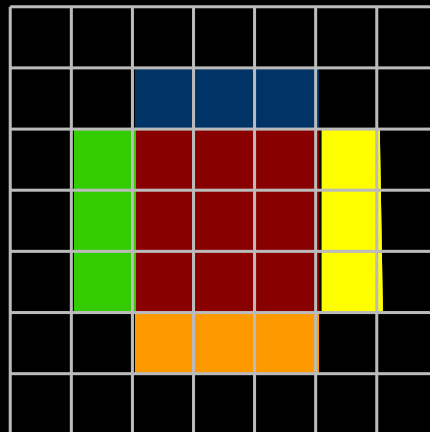
- Consider what happens with a closed circle.
- One would expect a closed curve to partition the plane into two connected regions.



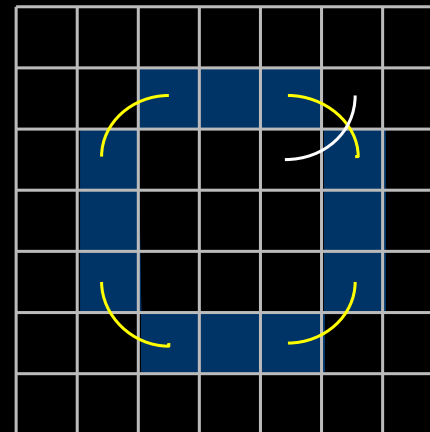
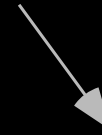
# Alternate Neighborhood Types



4-neighbor  
connectedness



8-neighbor  
connectedness



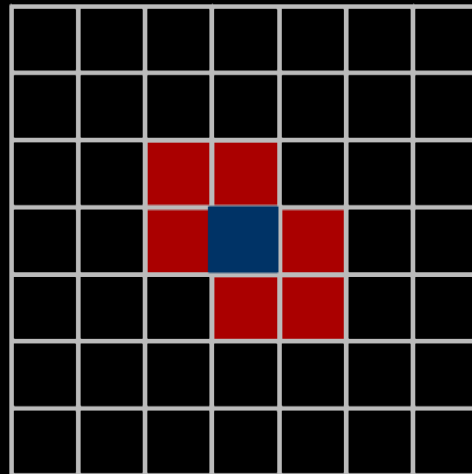
Neither neighborhood definition satisfactory!





# Possible Solutions

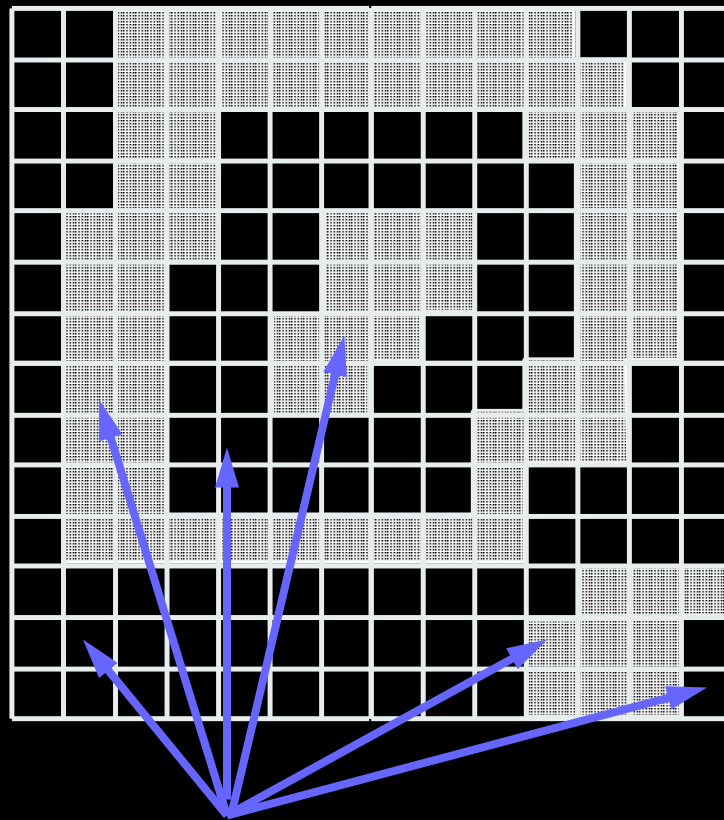
- Use 4-neighborhood for object and 8-neighborhood for background
  - requires a-priori knowledge about which pixels are object and which are background
- Use a six-connected neighborhood:





# Connected Components

- Binary image with multiple 'objects'
- Separate 'objects' must be labeled individually

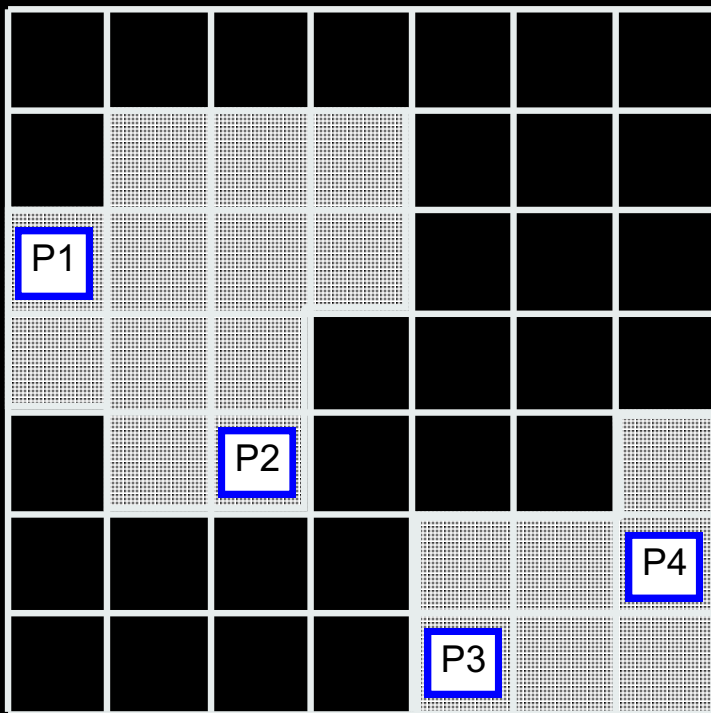


6 Connected Components



# Find Connected Components

- Two points in an image are 'connected' if a path can be found for which the value of the image function is the same all along the path.



$P_1$  connected to  $P_2$

$P_3$  connected to  $P_4$

$P_1$  not connected to  $P_3$  or  $P_4$

$P_2$  not connected to  $P_3$  or  $P_4$

$P_3$  not connected to  $P_1$  or  $P_2$

$P_4$  not connected to  $P_1$  or  $P_2$



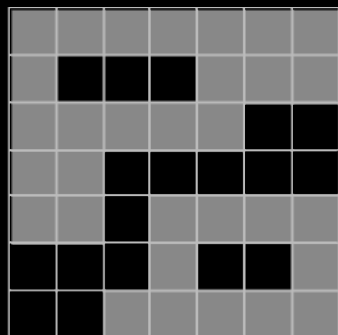
# Algorithm Steps

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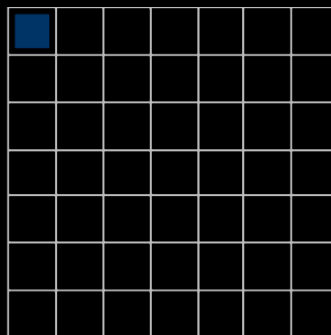
- Pick any pixel in the image and assign it a label
- Assign same label to any neighbor pixel with the same value of the image function
- Continue labeling neighbors until no neighbors can be assigned this label
- Choose another label and another pixel not already labeled and continue
- If no more unlabeled image points, stop.

Who's my neighbor?

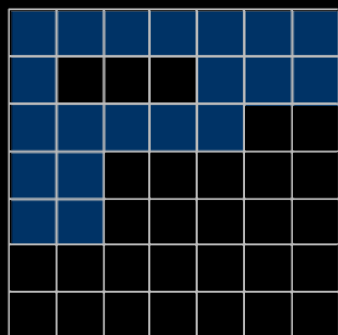
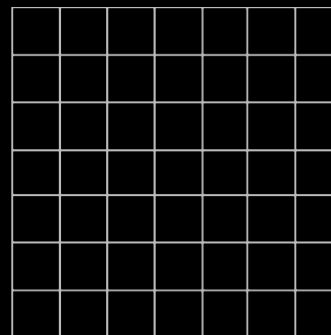
# Algorithm Example: Region Growing



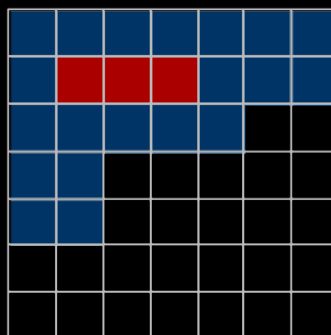
Image



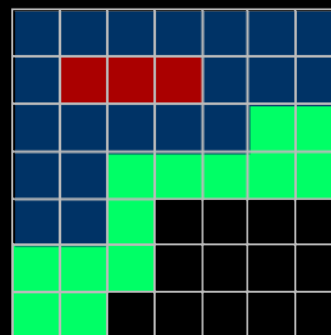
'Label' Image



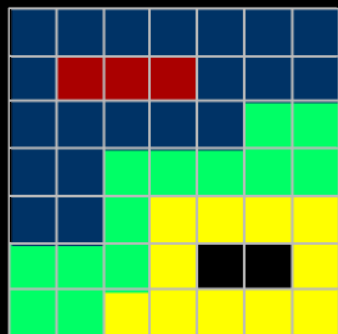
Lab. Im. - 1st Component



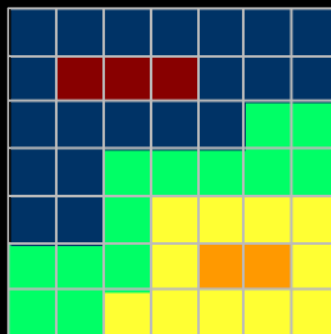
Lab. Im. - 2nd Component



Lab. Im. - 3rd Component



Lab. Im. - 4th Component



Final Labeling



# Algorithm Example: CCL

CCL (Connected Component Labelling)

1	0	0	0	1
1	1	1	0	0
1	0	0	0	0
0	1	0	0	1
0	0	1	1	0

(a)

1	0	0	0	2
1	1	1	0	0
1	0	0	0	0
0	3	0	0	5
0	0	4	4	0

(b)

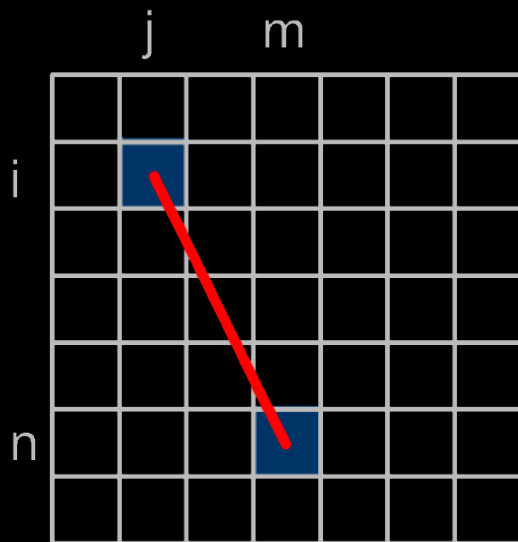
1	0	0	0	2
1	1	1	0	0
1	0	0	0	0
0	1	0	0	1
0	0	1	1	0

(c)



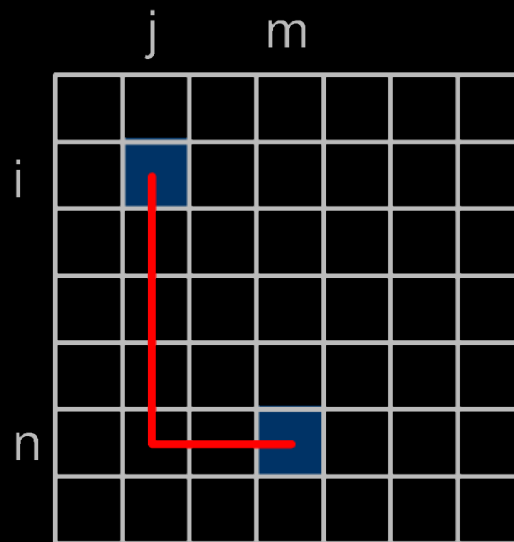
# Digital Distances

- Alternate distance metrics for digital images



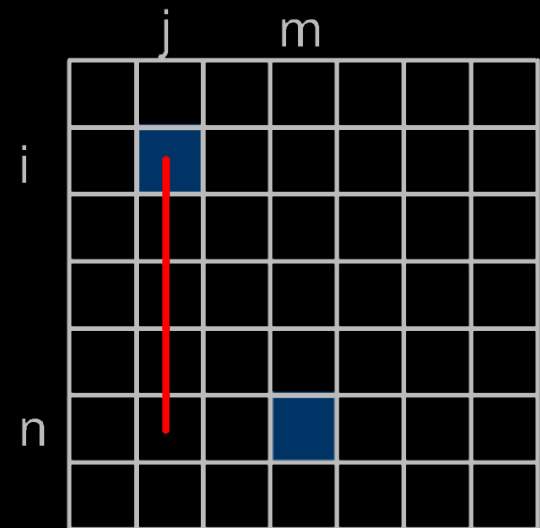
Euclidean Distance

$$= \sqrt{(i-n)^2 + (j-m)^2}$$



City Block Distance

$$= |i-n| + |j-m|$$



Chessboard Distance

$$= \max[ |i-n|, |j-m| ]$$



# Summary

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- Image formation steps
- Optical pinhole model
- Perspective projection
- Image sampling and resolution
- Image digitalization
- Color cube: RGB, HSI, ...
- Digital image geometry: connectivity
- Digital image: distance





# Acknowledgements

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The City College of New York (CCNY),

The City University of New York (CUNY)