2021

AuE 835-AUTOMOTIVE ELECTRONICS INTEGRATION

FALL-2021 EXAM

SIDDHARTH THORAT

Fundamental Questions:

1. What problems do ABS, TRC and VSC address respectively? What are the common features in these three systems?

S.NO	ABS	TRC	VSC
1.	It has shorter stopping distances.	The TRC addresses to limit the wheel slippage during the vehicle intense acceleration.	The VSC addresses when it detects the loss in steering control on the road surface. It overcomes by applying automatic brake to control the stability of the vehicle.
2.	It avoids wheel lock on the wet surfaces.	It ensures that all wheels have equal amount of engine torque to regain the traction.	It overcomes when there is a poor steering on the road surface (Slippery, mud, sudden turn).
3.	The ABS allow you to steer car around even though intense braking is applied.	The TRC activates when the vehicle struggles to speed up on low-friction road surfaces.	The VSC estimates the skids direction, applying the brakes to each wheel of the vehicle disproportionately and brings back the vehicle in the actual direction.
4.	It reduces the braking distance.	The TRC addresses during the wheels lose traction due to hitting a muddy patch on a road surface.	It ensures that a good amount of traction and control to maintain on the road surface.

<u>Common feature between the ABS, TRC, VSC</u>:

- The common between the three sensor features enables the vehicle to allow optimal accelerative traction on any surface by measuring wheel spin, by controlling the vehicle and by employing throttle to trim power and slow the rotating wheel.

- The most common of these sensors which are mounted on the wheel speed sensors are sensor malfunctions. They can be either knocked out of alignment or become corroded or get damaged by the hazardous road surfaces

- Mostly, The TCS and VSC are switched ON by the same button in the vehicle. These sensors' ABS, TCS, VSC provide safety and comfort to the driver and it ensures that the vehicle is smooth and controlled while moving on the road surface. They are continuously monitored on the on-board diagnostics in the vehicle.

2. Design a Hall effect-based sensor to measure the angle of a steering wheel. Briefly explain how the steering angle is calculated from the sensor measurements.

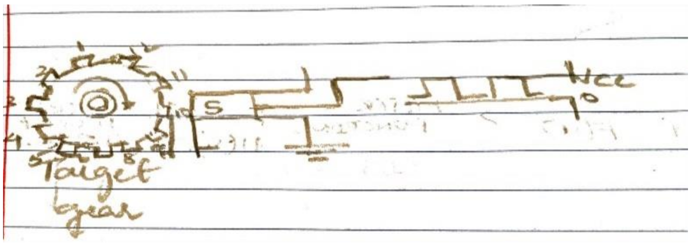
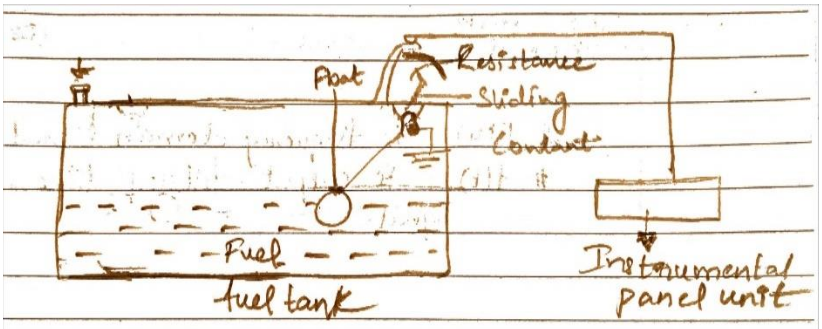


Figure-1

- The hall effect sensor is used to measure the steering wheel angle by as the wheel rotates, each tooth of wheel is assigned with a number 1-12 as shown in the figure. As the wheel is rotated it takes number of turns where the sensor senses the number assigned to teeth and analyzes the steering angle (0deg, 90deg,180deg) made.

3. Design a new sensor to accurately measure the fuel level in a gas tank even when the vehicle is on a slope. Briefly explain how the fuel level is calculated from the sensor measurements.



- The resistor-based fuel level sensor attached to the two ends in the gas tank is used to measure the fuel in a gas tank. The sensor is connected to the float which moves upwards or downwards depending on the fuel level in the gas tank. The fuel level is **calculated** by, As the float moves, the resistance of the sensor changes. As the resistance of the fuel sensor changes, the position of the needle changes proportional to the current flowing in the coil.

- As the vehicle is in the slope, the fuel level may vary so the resistor sensor at ends in the gas tank measures the fuel level in the gas tank.

4. What type of motors and what type of batteries do electrical vehicles commonly use and why

MOTORS:

- The different types of Motors used in electric vehicles are as follows:

a) DC Series Motor
b) Brushless DC Motor
c) Permanent Magnet Synchronous Motor (PMSM)
d) Three Phase AC Induction Motors
e) Switched Reluctance Motors (SRM)

- **Three phase Ac induction motors** is most commonly used in the electric vehicles as
    - Its maintenance free.
    - High starting torque and more acceleration.
    - High efficiency
    - Better regulation and Absence of commutators
    - It can get better grip at rougher terrains.

BATTERIES:

- The different types of batteries used in electric vehicles are as follows:

a) lithium-ion
b) Nickel-metal hydride
c) Lead-acid
d) Ultracapacitors

- The lithium-ion battery is most commonly used battery in the electric vehicle as it is a rechargeable battery which has higher energy density and low self-discharge than other batteries mentioned. The li-ion battery has high power to the weight ratio.

5. What are the advantages and disadvantages of LIN bus compared to CAN bus? Give two real examples in vehicles which are using LIN bus.

S, NO	Advantages of LIN bus compared to CAN bus	Disadvantages of LIN bus compared to CAN bus
1.	Single wire operation.	It has Low band.
2.	Data rate which helps to maintain reliability of network.	The LIN bus has Less effective interface compared to CAN bus.
3.	Broadcast serial network can have one master and sixteen slave nodes.	It has lower Bandwidth.
4.	The crystal or resonator is not required which lowers the cost significantly.	It has single master for media access control.
5.	They provide accurate latency time which make more predictable.	It has less data byte per frames compared to CAN bus.

- The real examples in vehicles which are using LIN bus are

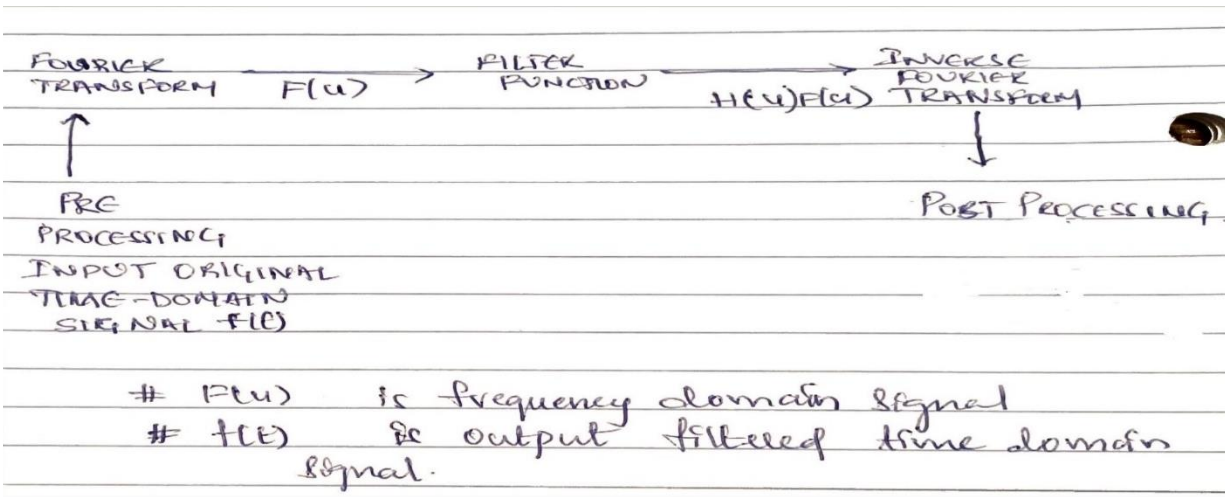
    i. Sunroof
    ii. Cooling fan motors
    iii. Rain sensors
    iv. Position motors

6. Why is frequency-domain filter needed? What is the general process of a frequency-domain filter?

- Smoothing and sharpening is attained in the frequency domain by reducing the high frequency components with respect to frequency rather than time. There are three types of frequency domain filters:

    i. Low pass filter
    ii. High pass filter
    iii. Band pass filter

- The general process of a frequency domain filter is shown in the flowchart above:



7. What are the differences between the open-loop control and closed-loop control in terms of requirements and performances?

S. NO	Open-Loop Control System	Closed-Loop Control System
1.	The Open-loop control system shows the signal path from input to output signifies a linear path with no feedback loop in the system	The Closed-loop control system shows the signal path from input to output with feedback loop in the system
2.	The process entered into an open-loop control system may vary due to external disturbances (noise, without the operator noticing).	The constant feedback allows certain variable to remain as stated during the tests in the system.
3.	The open-loop control system does not have feedback control to let the controller know if any changes done in the system.	The closed-loop control system has a feedback control to let the controller know if any changes done in the system.
4.	The accuracy depends on the calibration.	It is accurate due to the feedback in the system.
5.	The response in the system is fast and the optimization is not possible.	The response in the system is low and the optimization is possible.

8. What are the advantages and disadvantages of model-based controls compared to non-model-based controls? What is the key reason why people commonly use non-model based PID control in practice?

S, NO	Advantages of model-based control compared to non-model bused control	Disadvantages of model-based control compared to non-model based control
1.	The model based has high performance and stability. Also, easy to analyze.	The physical models are complicated and not always available.
2.	Gives higher quality of results and have less errors.	The LIN bus has Less effective interface compared to CAN bus.
3.	There is no need of real plant simulation.	It has lower Bandwidth.
4.	The Controller is not restricted to PID form.	It has single master for media access control.
5.	They provide accurate latency time which make more predictable.	It has less data byte per frames compared to CAN bus.

- The key reason why people commonly use non-model based PID control in practice as
  - They have lower computational costs and also robustness to uncertainties.
  - It requires less models to complete the process.
  - It provides improved performance and gives accurate results.
  - They are more appropriate due to less dependency on dynamic models.

Computational Questions:

1. Sensing Single Processing

An IMU can fuse a gyroscope, an accelerometer and a magnetometer to estimate the roll, pitch and yaw angles which are represented by a variable vector $y = [\phi, \theta, \psi]^T$, where ϕ, θ and ψ are roll, pitch and yaw angles respectively. The gyroscope can measure three angular rates $[\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ with noises and assume that the noise for each measurement is independent and has a Gaussian distribution with a mean of 0 and a variance of $\sigma_\phi, \sigma_\theta$ and σ_ψ respectively. The sampling time is T.

(1) The accelerometer measures the roll and pitch angles $[\phi, \theta]^T$ with noises and assume that the noise for each measurement is independent and has a Gaussian distribution with a mean of 0 and a variance of m_ψ and m_θ respectively. Write down the process model and Kalman Filter process to fuse the gyroscope and accelerometer to estimate the angles of roll, pitch and yaw.

$\varphi_{k+1} = \varphi_k + T\cdot\varphi'$
$\theta_{k+1} = \theta_k + T\cdot\theta'$
$\psi_{k+1} = \psi_k + T\cdot\psi'$

[By comparing the coefficients in equations]

$$\begin{bmatrix}\varphi\\ \theta\\ \psi\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}\varphi\\ \theta\\ \psi\end{bmatrix} + \begin{bmatrix}T&0&0\\0&T&0\\0&0&T\end{bmatrix}\begin{bmatrix}\varphi'\\ \theta'\\ \psi'\end{bmatrix}$$

$\varphi_k = \varphi_{k-1} + \vartheta$
$\theta_k = \theta_{k-1} + \vartheta$
$0\cdot\psi_k = 0\cdot\psi_{k-1} + 0\cdot\vartheta$

[by comparing the coefficients in equations]

$$\begin{bmatrix}\varphi\\ \theta\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{bmatrix}\varphi\\ \theta\\ \psi\end{bmatrix} + \vartheta$$

(2) The magnetometer measures the yaw angle ψ with noise and assume that the noise has a Gaussian distribution with a mean of 0 and a variance of m_ψ. Write down the process model and the Kalman Filter process to fuse the gyroscope, accelerometer and magnetometer to estimate the angles of roll, pitch and yaw.

2) .

$\phi_{k+1} = \phi_k + T\cdot\phi'$

$\theta_{k+1} = \theta_k + T\cdot\theta'$

$\varphi_{k+1} = \varphi_k + T\cdot\varphi'$ [by comparing the equations]

$$2) \begin{bmatrix} \varphi \\ \theta \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi \\ \theta \\ \varphi \end{bmatrix} + \begin{bmatrix} T & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} \phi' \\ \theta' \\ \varphi' \end{bmatrix}$$

$\phi_k = \phi_{k-1} + \upsilon$

$\theta_k = \theta_{k-1} + \upsilon$

$\varphi_k = \varphi_k + \upsilon$

$$\begin{bmatrix} \phi \\ \theta \\ \varphi \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi \\ \theta \\ \varphi \end{bmatrix} + \upsilon$$

(3) The magnetometer measures the yaw angle by sensing the voltage on the sensor. Assume a seventh-order polynomial function $\psi = a_7 v^7 + a_5 v^5 + a_2 v^2 + 10$ is used to map the sensed voltage to yaw angle. Given N pairs of data $(\psi_i, v_i), i = 1,2,\ldots N$, where ψ_i and v_i are the sensed voltage and corresponding yaw angle respectively, design a least-square approach to find the exact equations to calculate the parameters of a_7, a_5 \and a_2 give the detailed derivation process.

The 3×3 matrix can be defined as:

$$\underset{3\times3}{B} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}$$

Then the inverse matrix is:

$$\underset{3\times3}{B^{-1}} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix}^{-1} = \frac{1}{\det(B)} \begin{bmatrix} (ek-fh) & -(bk-ch) & (bf-ce) \\ -(dk-fg) & (ak-cg) & -(af-cd) \\ (dh-eg) & -(ah-bg) & (ae-bd) \end{bmatrix}$$

Where $det(B)$ is equal to:

$$det(B) = a(ek-fh) - b(dk-fg) + c(dh-eg)$$

Solution:

(3) $\varphi = 10 + a_2 v^2 + a_5 v^5 + a_7 v^7$

$$\pi = \sum_{i=1}^{n} (\varphi_i - P(x_i))^2 = \text{minimum}$$

$$\pi = \sum_{i=1}^{n} \left\{ \varphi_i - \left[a_0 + a_1 x_i + a_2 x_i^2 + \cdots + a_m x_i^m \right] \right\}^2$$

by differentiating on both sides

we get,

$$\frac{\delta \pi}{\delta a_0} = 2 \cdot \sum_{i=1}^{n} \varphi_i - \left[a_0 + a_2 v_i^2 + a_5 v_i^5 + a_7 v_i^7 \right] = 0 \quad (1)$$

$$\frac{\delta \pi}{\delta a_2} = 2 \cdot \sum_{i=1}^{n} v_i^2 \left[\varphi_i - \left[a_0 + a_2 v_i^2 + a_5 v_i^5 + a_7 v_i^7 \right] \right] = 0 \quad (2)$$

$$\frac{\delta \pi}{\delta a_5} = 2 \cdot \sum_{i=1}^{n} v^5 \left[\varphi_i - \left[a_0 + a_2 v_i^2 + a_5 v_i^5 + a_7 v^7 \right] \right] = 0 \quad (3)$$

$$\frac{\delta \pi}{\delta a_7} = 2 \cdot \sum_{i=1}^{n} v^7 \left[\varphi_i - \left[a_0 + a_2 v_i^2 + a_5 v_i^5 + a_7 v^7 \right] \right] = 0 \quad (4)$$

by re-arranging the above equations (1) (2) (3) (4)

$$\sum_{i=1}^{n} \varphi_i = a_0 + a_2 \sum_{i=1}^{n} v_i^2 + a_5 \sum_{i=1}^{n} v_i^5 + a_7 \sum_{i=1}^{n} v_i^7$$

$$\sum_{i=1}^{n} \varphi_i v_i^2 = a_0 \sum v_i^2 + a_2 \sum_{i=1}^{n} v_i^4 + a_5 \sum_{i=1}^{n} v_i^7 + a_7 \sum_{i=1}^{n} v_i^9$$

$$\sum_{i=1}^{n} \varphi_i v_i^5 = a_0 \sum_{i=1}^{n} v_i^5 + a_2 \sum_{i=1}^{n} v_i^7 + a_5 \sum_{i=1}^{n} v_i^{10} + a_7 \sum_{i=1}^{n} v_i^{12}$$

$$\sum_{i=1}^{n} \varphi_i v_i^7 = a_0 \sum_{i=1}^{n} v_i^7 + a_2 \sum_{i=1}^{n} v_i^9 + a_5 \sum_{i=1}^{n} v_i^{12} + a_7 \sum v_i^{14}$$

by re-arranging the equations -

$$\sum_{i=1}^{n} \varphi_i v_i^2 - a_0 \sum_{i=1}^{n} v_i^2 = a_2 \sum_{i=1}^{n} v_i^4 + a_5 \sum_{i=1}^{n} v_i^7 + a_7 \sum_{i=1}^{n} v_i^9$$

$$\sum_{i=1}^{n} \varphi_i v_i^5 - a_0 \sum_{i=1}^{n} v_i^5 = a_2 \sum_{i=1}^{n} v_i^7 + a_5 \sum_{i=1}^{n} v_i^{10} + a_7 \sum_{i=1}^{n} v_i^{12}$$

$$\sum_{i=1}^{n} \varphi_i v_i^7 - a_0 \sum_{i=1}^{n} v_i^7 = a_2 \sum_{i=1}^{n} v_i^9 + a_5 \sum_{i=1}^{n} v_i^{12} + a_7 \sum v_i^{14}$$

$$A = \begin{bmatrix} v_i^4 & v_i^7 & v_i^9 \\ v_i^7 & v_i^{10} & v_i^{12} \\ v_i^9 & v_i^{12} & v_i^{14} \end{bmatrix} \qquad X = \begin{bmatrix} a_2 \\ a_5 \\ a_7 \end{bmatrix}$$

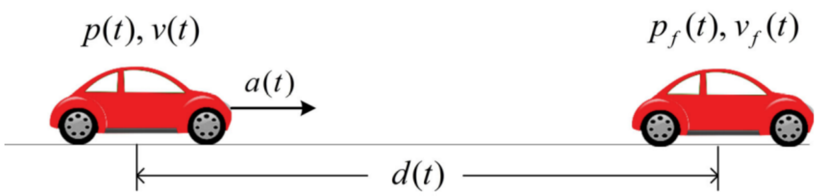
$$B = \begin{bmatrix} \sum_{i=1}^{n} v_i^2 \varphi_i - 10 \sum_{i=1}^{n} v_i^2 \\ \sum_{i=1}^{n} v_i^5 \varphi_i - 10 \sum_{i=1}^{n} v_i^5 \\ \sum_{i=1}^{n} v_i^7 \varphi_i - 10 \sum_{i=1}^{n} v_i^7 \end{bmatrix}$$

$$AX = B$$

$$\boxed{X = A^{-1} \cdot B}$$

2. <u>Controls:</u>

A vehicle is controlled by acceleration to follow another vehicle in front as shown below. The following vehicle's position, speed and acceleration are defined as $p(t)$, $v(t)$ and $a(t)$ respectively. Assume that the front vehicle is driving at a constant speed $v_f(t)$, and its position is represented by $p_f(t)$. The relative distance between the two vehicles is defined as $d(t)$. The control goal is to use acceleration to control the following vehicle to follow the front vehicle at a speed of $v_f(t)$, while keeping the distance at a constant of D.



(1) Assume that the vehicle has a lidar sensor to measure the relative distance.

a) Write the state space model.

b) Analyse if using acceleration as the control input is sufficient to accomplish the control goal.

c) Since the lidar sensor only measures relative distance but the system should have two states including both the relative distance and relative speed, design an estimator based on Kalman filter to estimate both states.

Note: (i) Assume Q and R are both known in the Kalman filter design; (ii) A continuous system x=Ax + Bu can be converted to a discrete system $x_k = (1+AT)\, x_{k-1} + TBu_k$ where k is discrete time and T is sampling time.

Solutions:

Controls.

(1)

a) write the state space model.

A.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad -① $$
$$y(t) = Cx(t) + Du(t) \quad -② $$

$$u(t) = -a(t), \quad x(t) = \begin{bmatrix} d(t) \\ v_{rl}(t) \end{bmatrix}$$

$$y(t) = d(t)$$

$$d'(t) = v_f(t) - v(t) \Rightarrow 0 d(t) + 1 v_{rl}(t) + 0 \cdot a(t) \quad -③$$
$$v_{rl}(t) = v_f(t) - v(t)$$
$$v_{rl}'(t) = -a(t) \Rightarrow 0 \cdot d(t) + 0 \cdot v_{rl}(t) + 1 \cdot a(t) \quad -④$$

$$\dot{x}(t) = \begin{bmatrix} d'(t) \\ v_{rl}'(t) \end{bmatrix}$$

By comparing ① with equations ③ and ④

we get,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$y(t) = [d(t)] \Rightarrow ⑤$$

by comparing ⑤ with equation ②

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = [0]$$

b) CONTROLABILITY:-

$$T_c[A\ B] = [B \quad AB]$$

$$\Rightarrow B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$T_c[A\ B] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\because$ Rank of $T_c[A\ AB] = 2$

Therefore it is a completely a row matrix, it is completely controllable.

OBSERVABILITY:-

$$T_c[A\ C] = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad CA = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$T_c[A\ C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$r_c[C \; CA] = 2$

Therefore it is completely a column matrix,
it is completely observable

c) PROCESS MODEL

From the given equation –

$x_k = (I + AT)x_{k-1} + TBu_k$

$\dot{x} = Ax + Bu$

$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$x_k = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & T \\ 0 & 0 \end{bmatrix} \right) \cdot x_{k-1} + \begin{bmatrix} 0 \\ T \end{bmatrix} u_{k-1}$

$x_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ T \end{bmatrix} u(t)$

$z_k = [1 \; 0] \cdot y_k + v_k$

PREDICTION EQUATIONS –

$\hat{x}_k^- = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \cdot x_{k-1} + \begin{bmatrix} 0 \\ T \end{bmatrix} u_k$

$\hat{P}_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \cdot x_{k-1} \cdot P_{k-1} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}^T + Q$

CORRECTION EQUATIONS –

$K = P_k^- [1 \; 0]^T \left[[1 \; 0] \cdot P_k^- [1 \; 0]^T + R \right]^{-1}$

$\hat{x}_k = \hat{x}_k^- + K \left[z_k - [1 \; 0] \hat{x}_k^- \right]$

$P_k = \left[1 - K[1 \; 0] \right] P_k^-$

(2) Assume that the vehicle has a radar to measure both the relative distance and the relative speed.

a) Write the state space model.

b) Analyze if using a radar is sufficient to accomplish the control goal.

c) Design a full state feedback controller to accomplish the goal. Use Routh-Hurwitz stability criterion to analyze the region of controller parameters to make the controlled system asymptotically stable.
d) What controller gains are to be used in u(t) in order to minimize the following linear quadratic cost-function? The cost-function aims to achieve the goal as soon as possible while minimizing the energy cost.

$$\text{Min: } J_0^{\infty}(2d((t)\text{-}D)^2 + v_f(t)\text{-}(t)^2)\ 2a(t^2)\ )\ dt$$

e) When using the controller derived in d), analyze the exact stability type of the controlled system by calculating the eigenvalues rather than Routh-Hurwitz stability criterion.

a). write a State space model.

$\dot{x}(t) = Ax(t) + Bu(t)$

$y(t) = Cx(t) + Du(t)$

$\dot{d}(t) = v_f(t) - v(t)$

$\dot{v}_q(t) = -a(t)$

=

$x(t) = \begin{bmatrix} d(t) \\ v_q(t) \end{bmatrix}$

$y(t) = \begin{bmatrix} d(t) \\ v_q(t) \end{bmatrix}$

$u(t) = -a(t)$

$\dot{x}(t) = \begin{bmatrix} \dot{d}(t) \\ \dot{v}_q(t) \end{bmatrix}$

By comparing we get

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$y(t) = \begin{bmatrix} d(t) \\ v_q(t) \end{bmatrix}$

By comparing $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 0 \end{bmatrix}$.

From State space model

$\dot{x}(t) = Ax(t) + Bu(t) \Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$

$y(t) = Cx(t) + Bu(t)$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot x(t) + \begin{bmatrix} 0 \end{bmatrix} u(t)$

b) CONTROLABILITY EQUATIONS

$$\tau_c[A\ B] = [B\ \ AB]$$

$$\tau_c[A\ B] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\tau_c[A\ B] = 2$$

Therefore, it is a row matrix, the system is Controllable.

OBSERVABILITY EQUATIONS :-

$$\tau_c[A\ C] = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$\tau_c[A\ C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\tau_c[A\ C] = 2$$

Therefore, it is a column matrix, the system is observable

$$= \left| \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \right| = 0.$$

$$= \begin{vmatrix} \lambda & -1 \\ k_1 & \lambda+k_2 \end{vmatrix} = 0.$$

$$\lambda(\lambda+k_2) + k_1 = 0$$

$$\lambda^2 + \lambda k_2 + k_1 = 0.$$

$$\Rightarrow \begin{array}{c|cc|} \lambda^2 & 1 & k_1 \\ \lambda^1 & k_2 & 0 \\ \lambda^0 & k_1 & 0 \end{array} \quad k_1 k_2 > 0.$$

Therefore, To be the system stable the k_1 and k_2 should be greater than zero ($k_1, k_2 >$

d) From the given equation,

$$\text{Min } J = \int_0^{\infty} \left(2(d(t) - D)^2 + (v_f(t) - v(t))^2 + 2\dot{a}(t)^2 \right)$$

$$= \int_0^{\infty} \Big[2\big((d(t) - d)(d(t) - d)^T \big) + \big((v_f(t) - v_l(t))^T \cdot v_f(t) - v_l(t) \big) + 2(a(t))^2 \Big] \, dt$$

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad R = 2$$

$$u(t) = -R^{-1} B^T \cdot P(x(t)) \, , \quad P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$$

$$A^T \cdot P + P \cdot A - PBR^{-1} \cdot B^T \cdot P + Q = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^T \cdot \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \frac{1}{2} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \cdot \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

by solving above matrices.

$$\Rightarrow \begin{bmatrix} -\frac{P_2^2}{2} & P_1 - \frac{P_2 P_3}{2} \\ P_1 - \frac{P_2 P_3}{2} & 2P_2 - \frac{P_3^2}{2} \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$2 - \frac{P_2^2}{2} = 0$$

$$P_2 = 2$$

$$2P_2 - \frac{P_3^2}{2} + 1 = 0$$

$$2P_2 - \frac{P_3^2}{2} + 1 = 0$$

$$P_3 = \sqrt{10}$$

$$P_1 - \frac{P_2 P_3}{2} = 0$$

$$P_1 - \frac{2(\sqrt{10})}{2} = 0.$$

$$P_1 = \sqrt{10}.$$

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \Rightarrow P = \begin{bmatrix} \sqrt{10} & 2 \\ 2 & \sqrt{10} \end{bmatrix}$$

$$u(t) = -R^{-1} B^T P [x(t)]$$

$$u(t) = -\frac{1}{2} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{10} & 2 \\ 2 & \sqrt{10} \end{bmatrix} x(t)$$

$$u(t) = \begin{bmatrix} -1 & -\frac{1}{2}\sqrt{10} \end{bmatrix} \cdot x(t)$$

$$u(t) = -\begin{bmatrix} 1 & \frac{\sqrt{10}}{2} \end{bmatrix} \cdot x(t)$$

by equating k_1, k_2 values in the equation -

$$\lambda^2 + \lambda k_2 + k_1 = 0$$

$$\lambda^2 + \frac{\sqrt{10}}{2}\lambda + 1 = 0$$

from $\frac{-b \pm \sqrt{b^2 - 4ac}}{2b}$.

$$\lambda = \frac{-\frac{\sqrt{10}}{2} \pm \sqrt{10/4 - 4(1)(1)}}{2(1)}$$

$$\lambda = \frac{-\frac{\sqrt{10}}{2} + \sqrt{\frac{6}{4}}\, i}{2}$$

$$\lambda = \frac{-\sqrt{\frac{10}{2}} + \frac{\sqrt{6}}{2}\, i}{2}$$

$$\lambda = -\sqrt{\frac{10}{4}} + \frac{\sqrt{6}}{4}\, i$$

by comparing from above

$$\lambda = a_i + i b_i$$

$a_i < 0$ and $b_i \neq 0$.

Therefore, the system should be asymptotically stable.