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10/24/2021

# AuE 835-AUTOMOTIVE ELECTRONICS INTEGRATION ASSIGNMENT-3

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**PROBLEM-1: Fundamental Questions:**

1. Design a multi-sensor fusion system for an automotive application. Present sensing objective, selection of sensors, reasons for fusion, and process of fusion including prediction and correction.
  - Multi-sensor fusion system for unmanned vehicles under extreme conditions.

**Sensing Objective:**

- The sensing objective is to aim the solution for the insufficient accuracy in “Simultaneous localization and mapping” of the vehicle robot using a single sensor in extreme environment scenes. According to the characteristics of the sensors, a method of fusing the data of lidar and inertial sensors is proposed. The vehicle robot system is designed, and the positioning principle of lidar and inertial sensor is developed. Two data fusion algorithms, weighted fusion and Kalman filter, are mainly studied, and the experiments prove that the Kalman filter algorithm has higher positioning accuracy.

**Selection of sensors:**

- The lidar and inertial sensors are selected for the model.

**Lidar sensor:**

- The lidar sensor is a sensor which emits a laser beam, and then evaluates the received signal reflected from the object with the transmitted signal to obtain the information about the object.

**Inertial sensor:**

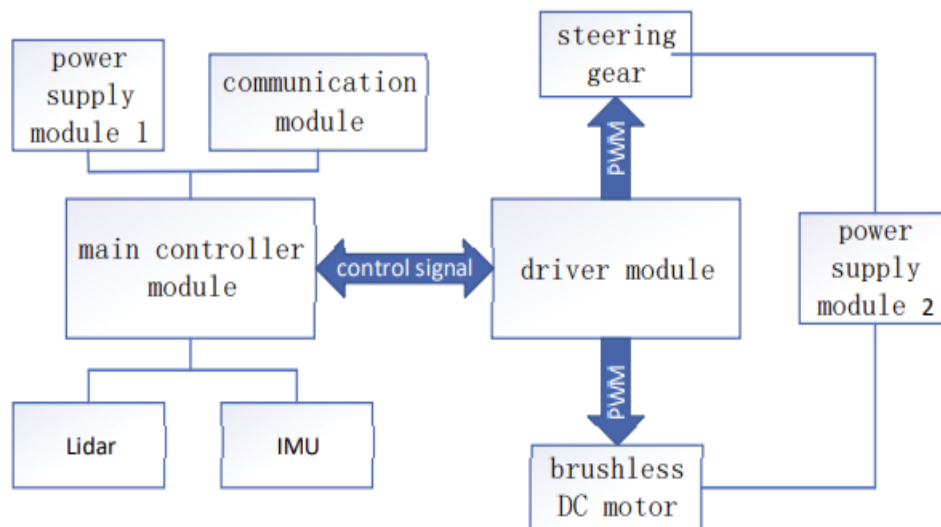
- The inertial measurement unit (IMU) is a sensor which measures the angular velocity and acceleration of the object in space through the built-in three-axis gyroscope and accelerometer to estimate the position and the angle of the object. It has advantages of high updated frequency and estimation accuracy in the shorter period of time.

**Reasons for fusion:**

- The reason for the fusion of lidar and inertial sensors is to obtain the information more accurate, efficient and to get adaptable results.

**Process of fusion:**

- After the data from the lidar and IMU fused by the upper computer installed with the ROS operating system, the upper processor communicates the data with the lower processor through serial port communication to control the brushless DC motor and steering gear. In addition, the robot car is fitted out with a router, which is accessible for remote control. Intelligent vehicle system includes modules such as power supply module, main controller module, sensor module, driver module, communication module, etc.



**Figure-1: Structural block diagram of unmanned vehicle**

- The primary framework of the software is the “ROS robot operating system” which is based on drivers of lidar and IMU attitude sensors where they are added to obtain the information of the surrounding environment and the overall performance information of the vehicle. After the process of the collected data, the map is constructed through the lidar (SLAM) algorithm, the vehicle can be controlled to reach the desired location through route planning and autonomous navigation. We can also use SSH software to attain the remote control of the entire system.
- The results represent that the combined navigation of lidar and inertial sensor can effectively improve the accuracy of the system in extreme environment compared. To get the information accurately and effectively the Kalman filter is used to process the fusion data and even it reduces the random error compared with fusion algorithm. So, sensor data fusion using Kalman filter can make use of the advantages of sensors, improve the positioning accurateness. It includes two steps: prediction and correction. All through prediction, the impact of uncertain system dynamics generated during the measurement process is considered to update the estimation error and to update the estimation error by getting new information from the sensor measurement through correction.

**Equations:**

From kalman filter model of process -

$$y_k = Ay_{k-1} + Bu_k + w_k$$

$$z_k = Hy_k + v_k$$

where,

$k$  = Discrete Time

$y$  = System data to be estimated

$u$  = System input

$z$  = System measurement

$A, B, H$  = Constant matrices

$w$  = Process noise  $w \sim \text{approx } N(0, Q)$

$v$  = measurement noise with  $v \sim \text{approx } (0, R)$

→ In the Calculation method, the kalman filter adapts recursive form. the state estimation  $y(t)$  at time ' $t$ ' is obtained by algorithm to measured value  $z(t)$  at time ' $t$ '.

→ The kalman filter can applied to linear dynamic systems. The state equation and measurement equation of discrete system are

$$x_k = \phi_{k/k-1} x_{k-1} + \Gamma_k w_{k-1}$$

$$z_k = H_k x_k + v_k$$

where,

$x_k$  = Estimated  $n$ -dimensional state matrix at time ' $k$ '.

b.  $z_k$  is  $n$ -dimensional measurement matrix at time " $k$ ".

c.  $\Phi_{k|k-1}$  is transition matrix from time " $k$ " to " $k+1$ ".

d.  $w_k$  is the system noise matrix at time " $k$ ".

e.  $v_k$  is  $n$ -dimensional measurement noise matrix at time " $k$ ".

f.  $w_k, v_k$  are non correlated zero-mean noise sequences.  
In Kalman filtering,

$$E[w_k] = 0$$

$$E[w_j w_k^T] = Q_j \delta_{jk}$$

$$E[v_k] = 0$$

$$E[v_j v_k^T] = R_j \delta_{jk}$$

$$E[w_j v_k^T] = 0$$

where

a.  $Q_k$  is variance matrix of noise vector  $w_k$ .

b.  $R_k$  is variance matrix of measured noise vector  $v_k$ .

c.  $\delta_{jk}$  is Kronecker function.

a. Prediction Estimation

b. Correction Estimation

} Kalman filter estimations

a. Prediction Estimation

$$\hat{x}_{k|k-1} = \Phi_{k|k-1} \hat{x}_{k-1|k-1}$$

$$P_{k|k-1} = \Phi_{k|k-1} P_{k-1|k-1} \Phi_{k|k-1}^T + \Gamma_{k|k-1} \Gamma_{k|k-1}^T$$

b. Filtering (Correction) Estimation -

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (z_k - H_k \hat{x}_{k|k-1})$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1} (I - K_k H_k)^T + K_k R_k K_k^T$$

∴ (or)

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

where,

a.  $K_k$  is filter gain matrix at time  $k$

b.  $P_{k|k-1}$  is prediction estimation error covariance matrix from  $k-1$  to time  $k$

c.  $P_{k|k}$  is optimal filter value error covariance matrix at time  $k$ .

2. Design a closed-loop control system for an automotive application. Present the control objective, selection of actuators, selection of sensors, and controller design process.
- The below is Closed loop control system for Fuel Control System.

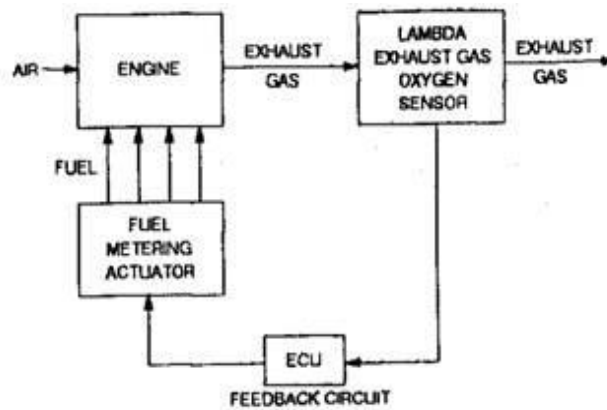


Figure: Closed loop fuel control system

### Control Objective:

- Systems using the open-loop principle can only follow the program set-up later testing a sample engine in a mechanical condition. If the engine doesn't maintain the related condition, then the timing and fuel settings become inaccurate. Thus, the problem can be identified by emphasizing the job of adjusting the ignition timing of an older type of engine with the support of a strobe light. Nevertheless, the timing may be set to the angle proposed by the manufacturer, but it may not be correct setting for the actual engine being tuned, because the proposed setting is applicable for new engine and the older engine may not be in a required condition.
- Thus, these problems can be minimized by using a closed-loop control system. This control system can work autonomously to control ignition and fuel systems or can be applied to the combined ignition and fuel system to give a full engine management.



**Selection of actuators:**

- Linear actuator:
  - The linear actuator acts as a **feedback actuator** which incorporates the positional feedback making to adaptable to applications where there is a closed-loop system is needed.
- Fuel Metering Actuator:
  - The fuel metering actuator is the actuator which electronically controls the solenoid valve, When the Fuel metering Actuator is opened, the maximum amount of fuel is being supplied to the fuel injection pump. Its is located at rear of high-pressure fuel injection pump.

**Selection of sensors:**

- Knock Sensor:
  - The knock sensor is the sensor which identifies the high frequency engine vibration characteristics of knocking and it transmits a signal to the ECU to generate maximum energy yield by starting the ignition as quick as possible.
- Lambda Sensor:
  - The lambda is the sensor which ensures the fuel mixture has right amount of oxygen for efficient and environment friendly combustion.

**Controller Design Process:**

- When the sensor signals about the oxygen content is below the specific limit, a voltage pulse from the Lambda sensor directs fuel system to decrease the fuel supply to reduce the mixture. Immediately after this fuel adjustment the Lambda sensor recognizes the presence of oxygen in the gas due to which the voltage output decreases. As a result, the output from the sensor changes to zero.
- When the controller receives no feedback signal, it directs the fuel system to enrich the mixture. Thus, mixture control is achieved by alternating the mixture strength between the rich and poor limits.
- This allows the controller to keep the air-fuel ratio within a range. Ever since modern sensors work to a small tolerance with respect to the air-fuel ratio, it's possible to control exhaust emission accurately as early as possible.



3. Explain when a control system can be called marginally stable, asymptotically stable, and exponentially stable

➤ Marginally Stable:

- The control system is marginally stable when any of two poles of the control system (open loop or closed loop) transfer function is present on the imaginary axis.

➤ Asymptotically stable:

- The control system is asymptotically stable if the solutions that start close enough not only remain close enough but also eventually converge to the equilibrium.

➤ Exponentially stable:

- The control system is exponentially stable if and only if the poles of its transfer function lie strictly within the unit circle centered on the origin of the complex plane.

4. Compare the difference between model-based control and non-model-based control in terms of controller design and control performance.

| S.NO | MODEL BASED CONTROL SYSTEM  | NON-MODAL BASED CONTROL SYSTEM   |
|------|---|--|
| 1.   | Closed loop gives high performance.                                       | The closed loop does not give high performance.  |
| 2.   | It is a method to efficiently design complex computer-controlled systems. | They are more appropriate due to less dependency on dynamic models.  |
| 3.   | Gives higher quality of results and have less errors.                     | Gives good quality of results but has more errors.   |
| 4.   | There is no need of real plant simulation.                                | Does not require any information of parameters neither the manipulation of actuators and no mathematical model for manipulation is needed. |
| 5.   | The serious reduction in design time can be achieved by using simulation. | They have lower computational costs and also robustness to uncertainties.  |
| 6.   | Controller is not restricted to PID form.                                 | The controller is restricted to PID form.  |
| 7.   | Makes full use of available models and                                    | Makes less use of available models.  |
| 8.   | Generates PID controllers for some model types.                           | They are simply called motion based techniques where it purely depends on system states.   |

5. What is the format of a PID controller and what do proportional control, integral control and derivative control do respectively? Describe one approach to tune the parameters in a PID controller.

➤ The format of a PID controller:

$$u(t) = K_p e(t) + K_i \int e(t) dt + K_d \frac{de}{dt}$$

Where,

- a.  $u(t)$  = proportional–integral–derivative controller variable
- b.  $K_p$  = proportional gain
- c.  $e(t)$  = error value
- d.  $K_i$  = integral gain
- e.  $de$  = change in error value
- f.  $dt$  = change in time

#### A. Proportional control:

- The proportional control involves the correcting a target proportional to the difference. Therefore, the target value is never achieved due to the difference approaches to zero, so too does the applied correction or the output of a proportional controller is the multiplication product of the error signal and the proportional gain. Hence it helps in reducing the steady-state error and makes the system more stable.
- The proportional controller produces an output which proportional to error signal.
- The proportional term is given by

$$P_{out} = K_p e(t).$$

#### B. Integral control:

- The integral control is the sum of the instantaneous error over time and gives the collected offset that should have been corrected previously.
- The integral controller produces an output, which is integral of the error signal.
- The integral term is given by

$$I_{out} = K_i \int_0^t e(\tau) d\tau.$$

C. Derivative Control:

- The derivative control is determined by the slope of the error over time and multiplying this rate of change by the derivative gain  $K_d$ .
- The derivative term is given by

$$D_{\text{out}} = K_d \frac{de(t)}{dt}.$$

- Where  $K_p$ ,  $K_i$ ,  $K_d$  are proportional gain constants, integral gain constants, derivative gain constants.

**PID controller Tuning Methods:**

- Tuning approaches for PID controller are

- Manual tuning
- Ziegler–Nichols
- Tyres Luyben
- Software tools
- Cohen–Coon
- Åström–Hägglund

B. Ziegler–Nichols approach for tuning the PID controller:

- In this method for tuning the PID controller, the  $K_i$  and  $K_d$  gains are first set to zero then the proportional gain is raised until it reaches the ultimate gain. The  $K_u$  at which the output of the loop starts to oscillate constantly. The  $K_u$  and the oscillation period  $T_u$  are used to set the gains.

**Ziegler–Nichols method**

| Control Type      | $K_p$     | $K_i$         | $K_d$        |
|-------------------|-----------|---------------|--------------|
| <b><i>P</i></b>   | $0.50K_u$ | —             | —            |
| <b><i>PI</i></b>  | $0.45K_u$ | $0.54K_u/T_u$ | —            |
| <b><i>PID</i></b> | $0.60K_u$ | $1.2K_u/T_u$  | $3K_uT_u/40$ |

- When applied to the standard proportional–integral–derivative controller form, only the integral and the derivative gains  $K_i$  and  $K_d$  are the dependent on oscillation period  $T_u$ . But these gains apply to the ideal, parallel form of the PID controller.

**PROBLEM-2: Sensor Calibration:**

1. What does sensor calibration do? If a polynomial function is used as the calibration function, how is the order of the polynomial function determined.
    - The sudden change in the working environment of the sensors give undesired output values. Hence, the expected output differs from the measured output. This difference between the expected output and measured output is called Sensor Calibration. It plays a crucial role in increasing the performance of the sensor and used to measure the Structural errors caused by sensors.
- The order of the polynomial function determined by the rail and error method but for the here in the given statement there no method to determine the function. Usually, the polynomial function is calibrated till the linear degree satisfies the nth order of differential equation ( $y=A x+ B$ ).
  - Pick a polynomial form at least several orders lower than the number of data points.
  - Start with linear and add order until trends are matched.

2. For this task,  $N$  pairs of a sensor input and output  $(x_i, y_i)$  are given, where  $i=1, 2, \dots, N$ . Here,  $x_i$  and  $y_i$  are the measured and physical variable of the sensor respectively. If a special fourth-order polynomial function  $y = a_4x^4 + a_2x^2 + a_0$  is selected to calibrate the sensor, use the least-square approach to find the parameters  $a_4, a_2, a_0$  and write down the calibration process.

$$y = f(x)$$

$$\Rightarrow a_4x^4 + a_2x^2 + a_0$$

The fitting Curve should have the Least Square Error

$$\pi = \sum_{i=1}^n \{ y_i - f(x_i) \}^2 \quad (\because \text{minimum})$$

$$\pi = \sum_{i=1}^n \{ y_i - [a_0 + a_1x_i + a_2x_i^2 + a_3x_i^3 + a_4x_i^4] \}^2$$

$$(\because [a_1, a_3] = 0) \text{ from the } f(x)$$

$$\pi = \sum_{i=1}^n \{ y_i - [a_0 + a_2x_i^2 + a_4x_i^4] \}^2 \quad (\because \text{minimum})$$

Here

The partial differential derivative should be equated to zero to obtain the error.

To find the coefficients of  $a_0, a_2, a_4$

$$2) \frac{\partial \pi}{\partial a_0} = 2 \cdot \sum_{i=1}^n [y_i - (a_0 + a_2x_i^2 + a_4x_i^4)] = 0$$

①

Similarly,

$$\frac{\partial \pi}{\partial a_2} = 2 \sum_{i=1}^n \left[ y_i - (a_0 + a_2 x_i^2 + a_4 x_i^4) \right] x_i^2 \quad (2)$$

Similarly,

$$\frac{\partial \pi}{\partial a_4} = 2 \sum_{i=1}^n x_i^4 \left[ y_i - (a_0 + a_2 x_i^2 + a_4 x_i^4) \right] \quad (3)$$

By Re-arranging the equations (1), (2), (3)

We get,

$$\sum_{i=1}^n y_i = a_0 \sum_{i=1}^n 1 + a_2 \sum_{i=1}^n x_i^2 + a_4 \sum_{i=1}^n x_i^4$$

$$\sum_{i=1}^n x_i^2 y_i = a_0 \sum_{i=1}^n x_i^2 + a_2 \sum_{i=1}^n x_i^4 + a_4 \sum_{i=1}^n x_i^6$$

$$\sum_{i=1}^n x_i^4 y_i = a_0 \sum_{i=1}^n x_i^4 + a_2 \sum_{i=1}^n x_i^6 + a_4 \sum_{i=1}^n x_i^8$$

$$A = \begin{bmatrix} 0 & \varepsilon \eta_1^2 & \varepsilon \eta_1^4 \\ \varepsilon \eta_1^2 & \varepsilon \eta_1^4 & \varepsilon \eta_1^6 \\ \varepsilon \eta_1^4 & \varepsilon \eta_1^6 & \varepsilon \eta_1^8 \end{bmatrix}$$

$$P = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$B = \begin{bmatrix} \varepsilon y_1 \\ \varepsilon(\eta_1^2, y_1^2) \\ \varepsilon(\eta_1^4, y_1^4) \end{bmatrix}$$

**PROBLEM-3: Controls :**

1. In an adaptive cruise control system, the vehicle is controlled by acceleration  $a(t)$ . Assuming that the front vehicle speed  $v_d$  is a known constant, use a state space model to describe the relationship between the system input acceleration  $a(t)$  and system output  $y = [d(t), v_r(t)]^T$ , where  $d(t)$  is the inter-vehicle distance and  $v_r(t) = v_d - v(t)$  is the inter-vehicle speed.
2. Is the system completely controllable? Justify your answer.
3. Is the system completely observable? Justify your answer.
4. To make the vehicle follow the front vehicle and keep the inter-vehicle distance as a constant  $d_d$ , we need to make  $d(t)$  approach  $d_d$  and  $v_r(t)$  approach 0 at the same time. Design a full state feedback controller  $u(t) = -K * (x(t) - x_d(t))$  to achieve this goal and analyze the conditions of the controller gains  $K$  for making the controlled system asymptotically stable/
5. For the controller above, determine the value of  $K$  to minimize the following cost function, which is to achieve the control goal using minimum amount of fuel?

Solutions from 1 to 5 is written in the paper below:



### Problem-3

Given,

- $v_d$  = front vehicle speed,  $v_d$  is constant known
- use state space model
- $a(t)$  = acceleration
- $y = [d(t), v_r(t)]^T$  is system output
- $d(t)$  = inter-vehicle distance
- $v_r(t) = v_d - v(t)$  is inter vehicle speed.

From above given data,

- \*  $a(t)$  is input
- \*  $y(t)$  is output.

From Step-1:-

$$u(t) = a(t)$$

$$y(t) = \begin{bmatrix} d(t) \\ v_r(t) \end{bmatrix}$$

Considering the inter vehicle speed

$$v_r(t) = v_d - v(t).$$

- by diffn.

$$\frac{d}{dt} v_r(t) = \frac{d}{dt} v_d - \frac{d}{dt} v(t).$$

$$\Rightarrow \dot{v}_r(t) = -\dot{v}(t) \quad (\because v_d \text{ is constant})$$

⇒ From the known relation  
 $v(t) = a(t)$

$$\Rightarrow v_h(t) = -a(t) \quad \text{--- (1)}$$

$$\Rightarrow d(t) = v(t) \quad \text{--- (2)}$$

From step-2

$$u(t) = a(t)$$

$$y(t) = \begin{bmatrix} d(t) \\ v_h(t) \end{bmatrix}$$

$$x(t) = \begin{bmatrix} d(t) \\ v(t) \end{bmatrix}$$

From the equations of steady state flow

$$x(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

$$\Rightarrow y(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + 0 \cdot u(t) \quad \text{--- (3)}$$

From the equations (1) and (2)

$$i(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad \text{--- (4)}$$

By Comparing the equations (3) and (4) with steady state flow equations we get

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = [0]$$

By considering the state space model above obtained

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

The Controllability matrix is

$$T_c [A \ B] = [B \ AB]$$

$$\Rightarrow \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

The Rank of the controllability matrix is

$$T_c [A \ B] = [B \ AB] \Rightarrow \text{rank}[AB] = 2$$

$\therefore$  The rank is 2

Therefore, the System is Completely Controllable



By Considering the state Space Model  
above obtained-

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From the Observability matrix Equation.

$$T_o [A, C] = \begin{bmatrix} C \\ CA \end{bmatrix}$$

$$\Rightarrow T_o [A, C] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Therefore, the Rank obtained from observability matrix equation is 2

$$\boxed{\text{Rank } T_o [A, C] = 2}$$

$$4 \quad u(t) = -kx(t)$$

$$\ddot{x}(t) = \bar{A}(x(t))$$

$$\bar{A} = A - Bk$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix} [k_1, k_2]$$

$$\ddot{x}(t) = \left[ \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 \\ -1 \end{bmatrix} [k_1, k_2] \right] \cdot x(t)$$

$$= \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} \cdot x(t)$$

$$\det(\lambda I - \bar{A}) = 0$$

$$= \left| \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ k_1 & k_2 \end{bmatrix} \right| = 0$$

$$\text{by solving the } \det(\lambda I - \bar{A}) = 0$$

$$\Rightarrow \lambda^2 - \lambda k_2 - k_1 = 0$$

Rowth Array -

$$\begin{array}{c|cc} 2^1 & 1 & -k_1 \\ 2^2 & -k_2 & 0 \\ 2^3 & -k_3 & \end{array}$$

For the by observing that system must be stable, there should be no sign change in the first column.

Hence, the values of  $k_1, k_2 < 0$ .

from the given equation in the task.

$$\text{Min } J = \int_0^{\infty} \left( (d(t) - d_d)^T \cdot (d(t) - d_d) + (a(t))^T \cdot (a(t)) \right) dt$$

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad R = 16. \quad P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$$

from the equation-

$$A^T P + P A - P B R^{-1} B^T P$$

(by substituting the variables)

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}^T \cdot \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} =$$



(- Continuation of equation)

$$- \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot \left(\frac{1}{16}\right) \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}^T \cdot \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix}$$

by solving the above variable by

$$A^T P + P \cdot A - P \cdot B R^{-1} B^T \cdot P$$

we get,

$$P = \begin{bmatrix} 2.828 & 4 \\ 4 & 11.31 \end{bmatrix}$$

$$u(t) = -R^{-1} B^T \cdot P (x(t) - d_d)$$

$$u(t) = \left(\frac{1}{16}\right) \cdot \begin{bmatrix} 0 \\ -1 \end{bmatrix}^T \cdot \begin{bmatrix} 2.828 & 4 \\ 4 & 11.31 \end{bmatrix} (x(t) - d_d)$$

$$u(t) = \begin{bmatrix} -0.25 & -0.7 \end{bmatrix} x(t) - d_d$$

$$[K_1 \ K_2] = \begin{bmatrix} -0.25 & -0.7 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} K_1 = -0.25 \\ K_2 = -0.7 \end{matrix}$$