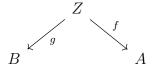
We discuss a little bit more before moving on to "abstract nonsense".

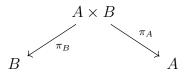
Recall that we have the universal property of the product, where something is a product if maps to projections factor through the product. Diagramatically, create a category whose morphisms are the  $\sigma$  such that the below diagram commutes,

$$\begin{array}{c}
Z_1 \\
\downarrow \sigma \\
B \xleftarrow{g_1} Z_2 \xrightarrow{f_2} A
\end{array}$$

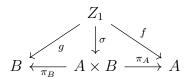
and the objects are things of the type



Then notice that, in this cattegory, the product



Is final, with morphisms



Where  $\sigma(z) = (f(z), g(z))$ . Here it is clear that the product map is unique, so this truly is final.

**Definition 0.1.** We say that a category C has products if  $\forall A, B \in \text{Obj}(C)$ , then  $C_{A,B}$  has a final object. in SET, final objects are disjoint unions.

## 1 Groups

**Definition 1.1** (Group). G is a set with a closed binary operation

$$(\cdot):G\times G\to G \quad \cdot (g,h)\mapsto g\times h$$

such that

- $1. \cdot is associative$
- 2.  $\exists e_G \in G$  such that  $\forall g \in G \ g \cdot e = e \cdot g = g$
- 3. We get inverses:  $\forall g \in G$ , there exists  $g^{-1} \in G$  such that  $g \cdot g^{-1} = g^{-1} \cdot g = e_G$ .

**Example 1.2.** The trivial group. Yep.

**Example 1.3.**  $(\mathbb{Z}, +)$ ,  $(\mathbb{C}, +)$ ,  $(\pm 1, \cdot)$ . Notice that all of these groups are abelian.

**Example 1.4.** Matrix groups:  $(GL_n(F), \cdot)$ , the multiplicative group of invertible  $n \times n$  matrices over a field. Notice that matrix multiplication doesn't commute: a trivial fact.

**Proposition 1.5.** Both the identity and  $g^{-1}$  are unique.

*Proof.* Trivial. Suppose  $\exists e_1, e_2$  both identities for the sake of contradiction. But then their product blah blah. And likewise for inverses.

**Notation.** By associativity, we can justify that

$$g^n = \underbrace{g \cdot g \cdot \ldots \cdot g}_{n \text{times}}$$

As always, a group is called *Abelian* if the binary operation is commutative.

**Example 1.6.** The Commutator group for a non-abelian group is such that the binary op. is

$$[a, b] = aba^{-1}b^{-1}$$

Likewise the commutator subgroup is a subgroup generated by all commutators of elements from G.

**Definition 1.7.** The order of an element  $g \in G$  is finite if  $\exists n \in \mathbb{N}$  such that  $g^n = \text{Id}$ . The order of the element is the least such n. An element has infinite order otherwise.

**Lemma 1.8.** If  $g^n = e$  for some n > 0, then |g| divides n.

*Proof.* Trivial exercise in number theory.