1 More about Ideals

We begin with a discussion of prime and maximal ideals. All rings here are commutative with 1.

Definition 1.1. For some ideal $I \neq \langle 1 \rangle$, I is a *prime* ideal if R/I is an integral domain.
I is *maximal* if R/I is a field.

Example 1.2. In $R[x]$, $(x-a)$ is prime iff R is an integral domain, and maximal iff R is a field. This comes directly from the fact that $R[x]/\langle x-a \rangle \cong R$

You might notice that the definitions above aren't the ones that are normally given for prime and maximal ideals. The normal definitions are that I being prime iff $\forall ab \in I$, either $a \in I$ or $b \in I$, and I is maximal iff $\forall$ ideals J of I, $I \subseteq J \Rightarrow I = J$ or $J = R$. The equivalence of these definitions is an easy exercise and can be verified quickly.