1 Ideals

They have generating sets, which is nice.

Now let's just talk about commutative rings. Then we let $\langle a \rangle$ denote the ideal generated by a. If an ideal is generated by a single element, we say that it is a *principal ideal*. Moreover, if, for some ring R, every ideal is principal and R has no zero divisors, then we say that R is a *principal ideal domain*.

> **Example 1.1.** $\mathbb{Z}$ is a principal ideal domain.

let I_α be a faimily ideals indexed by some set Λ. Then

$$\sum_{\alpha \in \Lambda} I_\alpha$$

is also an ideal. Moreover, if the ideals are finitely generated, then the sum is as well.

> **Theorem 1.2.** $(R/\langle a \rangle)/\langle \bar{b} \rangle \cong R/\langle a, b \rangle$. *This is one of the isomorphism theorems. I think that it's the second one.*

> **Definition 1.3.** A commutative ring is *Noetherian* if every ideal is finitely generated.

> **Proposition 1.4.** *Assume R is a finite commutative ring. Then R is an integral domain iff it is a field. (Notice that in the infinite case, $\mathbb{Z}$ disproves this)*

This is far, far easier to check then the field condition, since inverses are – in general – hard to find.

So how else can we build ideals? Well we can take products and intersections of ideals that we already have.

Yeah, sorry, I'm tired. I can't typeset this today. If you were counting on these notes, sorry I let you down.