We examine the free group via the set that it's defined over. In the language of categories $\mathscr{F}^A$ is the free categoy of $A$, where the objects are like : $A \to G$, and the morphisms are group homomorphisms $\sigma$ like

$$\begin{tikzcd} A \arrow[r, "j_1"] \arrow[dr, "j_2"] & G \arrow[d, "\sigma"] \\ & H \end{tikzcd}$$

Now we define

**Definition 0.1.** $F(A)$ is an initial object in $\mathscr{F}^A$.

**Claim.** *The maps $\{a\} \to \langle a \rangle$ are initial in the category.*

This defined $F(A)$ up to isomorphism, but why do they exist? In particular, define the resolution of the first cancelation relation among the words by $r : W(A) \to W(A)$, and define moreover $R : W(A) \to W(A)$ by $w \longmapsto Rr^{\lfloor n/2 \rfloor}(w)$. Then $F(A) = (R(W(A)), \cdot)$, where $\cdot$ denotes concatonation.

**Fact.** *This is a group, and this is easy to check for yourself.*