Bayesian Econometrics

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Outline

Conjungate Families

- A Conjugate family is a kind of prior distributions for which the posterior distribution can be easily calculated.
- Conjugate prior distributions play an important role in Bayesian methods, because their use can simplify the process of integration required for marginalization.
- When a prior and posterior are from a same family, the actualization process for parameters is simplified.

Conjungate Families

Binomial distribution

Theorem

Suppose that $X_1 \ldots X_n$ is a random sample from a Bernoulli distribution with parameter p, which is unknown. Also assume that the prior distribution for p is beta with parameters $\alpha(>0)$ and $\beta(>0)$. Then the posterior distribution for p, is beta with parameters $\alpha + \sum X_i$ and $\beta + n + \sum X_i$.

Conjungate Families
Binomial distribution

$$\begin{array}{rcl} \pi(p) & \propto & p^{\alpha-1}(1-p)^{\beta-1}, \\ L(p|X) & \propto & p^{\sum X_i}(1-p)^{n-\sum X_i} \end{array}$$

then,

$$\begin{array}{rcl} \pi(p|X) & \propto & \pi(p)L(p|X) \\ \pi(p|X) & \propto & p^{\alpha+\sum X_i-1}(1-p)^{\beta+n-\sum X_i-1}. \end{array}$$

We can see that: $p|x \sim beta(\alpha + \sum X_i; \beta + n - \sum X_i)$.

Conjungate Families

Poisson distribution

Theorem

Suppose that $X_1 \ldots X_n$ is a random sample from a Poisson distribution with unknown mean λ. Also assume that the prior distribution for λ is Gamma with parameters $\alpha (> 0)$ and $\beta (> 0)$. Then the posterior distribution for λ is Gamma with parameters $\alpha + \sum X_i$ and $\beta + n$.

Conjungate Families

Poisson distribution

$$\begin{array}{rcl} \pi(\lambda) & \propto & \lambda^{\alpha-1}e^{-\lambda\beta}, \\ L(\lambda|X) & \propto & e^{-n\lambda}\lambda^{\sum X_i}, \end{array}$$

then,

$$\begin{array}{rcl} \pi(\lambda|X) & \propto & \pi(\lambda)L(\lambda|X) \\ \pi(p|X) & \propto & \lambda^{\alpha+\sum X_i-1}e^{-\lambda(\beta+n)}. \end{array}$$

We can see that: $\lambda|x \sim Gamma(\alpha + \sum X_i; \beta + n)$.

Conjungate Families

Exponential distribution

Theorem

Suppose that $X_1 \dots X_n$ is a random sample from an Exponential distribution with unknown mean λ. Also assume that the prior distribution for λ is Gamma with parameters $\alpha(> 0)$ and $\beta(> 0)$. Then the posterior distribution for λ is Gamma with parameters $\alpha + n$ and $\beta + \sum X_i$.

Conjungate Families
Exponential distribution

$$\begin{aligned} \pi(\lambda) &\propto \lambda^{\alpha-1}e^{-\lambda\beta}, \\ L(\lambda|X) &\propto \lambda^{n}e^{-\lambda\sum X_i}, \end{aligned}$$

then,

$$\begin{aligned} \pi(\lambda|X) &\propto \pi(\lambda)L(\lambda|X) \\ \pi(p|X) &\propto \lambda^{\alpha+n-1}e^{-\lambda(\beta+\sum X_i)}. \end{aligned}$$

We can see that: $\lambda|x \sim Gamma(\alpha+n;\beta+\sum X_i)$.

Conjungate Families
Normal-Gamma

Theorem

Suppose that $X_1 \ \ldots \ X_n$ is a random sample from a Normal distribution with unknown parameters μ and precision τ. Also assume that the prior distributions for μ and τ are respectively Normal $N(\mu_0, \tau_0)$ and $Gamma(\alpha_0, \beta_0)$. The posterior distributions for both are respectively $N(\mu_1, \tau_1)$ and $Gamma(\alpha_1, \beta_1)$, where:
$\mu_1 = \frac{\tau_0\mu_0 + n\bar{X}}{\tau_0 + n}$; $\tau_1 = \tau_0 + n$;$\alpha_1 = \alpha_0 + n/2$ and
$\beta_1 = \beta_0 + \frac{1}{2}\sum(X_i - \bar{X})^2 + \frac{\tau_0 n(\bar{X} - \mu_0)}{2(\tau_0 - n)}$.