

MPS¹ Protocol – BAN logic

Sign Up Real Protocol

$$M_1: A \rightarrow S: K_a \parallel S(K_a, N_a), E_{k_s}(A, N_a) \parallel S(A, N_a)$$

$$M_2: S \rightarrow A: E_{k_a}(N_a, N_s) \parallel E_{k_s^-}(S(N_a, N_s))$$

$$M_3: A \rightarrow S: E_{k_s}(N_s, g_a, p_a) \parallel E_{k_a^-}(S(N_s, g_a, p_a))$$

$$M_4: S \rightarrow A: E_{k_a}(S(N_s, g_a, p_a)) \parallel E_{k_s^-}(S(S(N_s, g_a, p_a)))$$

Sign Up Idealized Protocol

$$M_1: A \rightarrow S: \{A, N_a\}_{k_s}$$

$$M_2: S \rightarrow A: \{N_a, \#(N_a), N_s\}_{k_a} \parallel \{S(N_a, \#(N_a), N_s)\}_{k_s^-}$$

$$M_3: A \rightarrow S: \{N_s, \#(N_s), g_a, p_a\}_{k_s} \parallel \{S(N_s, \#(N_s), g_a, p_a)\}_{k_a^-}$$

$$M_4: S \rightarrow A: \{S(N_s, \#(N_s), g_a, p_a)\}_{k_a} \parallel \{S(S(N_s, \#(N_s), g_a, p_a))\}_{k_s^-}$$

Sign Up Protocol Analysis

Objectives

$$S \models (g_a, p_a)$$

Diffie Hellman Parameters

$$A \models S \models (g_a, p_a)$$

DH Parameters confirmation

Assumptions

$$A \stackrel{k_s}{\Rightarrow} S$$

Hardcoded Server's Public Key

$$S \stackrel{k_a}{\Rightarrow} A$$

Alice's Public Key sent in plaintext in the first message

$$S \models A \Rightarrow g_a, p_a$$

After M_1 :

$$S \triangleleft \{A, N_a\}_{k_s}$$

After M_2 :

$$\frac{A \stackrel{k_s}{\Rightarrow} S, A \triangleleft \{N_a, \#(N_a), N_s\}_{k_s^-}}{A \models S \mid \sim (N_a, \#(N_a), N_s)}$$

$$\frac{A \models S \mid \sim (N_a, \#(N_a), N_s), A \models \#(N_a)}{A \models S \models (N_a, \#(N_a), N_s)}$$

¹ Magherini – Pochiero – Sieni (MPS)

After M_3 :

$$\begin{array}{c}
 \frac{S \stackrel{k_a}{\models} A, S \triangleleft \{N_s, \#(N_s), g_a, p_a\}_{k_a^-}}{S \models A \mid \sim (N_s, \#(N_s), g_a, p_a)} \\
 \frac{S \models A \mid \sim (N_s, \#(N_s), g_a, p_a), S \models \#(N_s)}{S \models A \models g_a, p_a} \\
 \frac{S \models A \Rightarrow g_a, p_a, S \models A \models (N_s, \#(N_s), g_a, p_a)}{S \models (g_a, p_a)}
 \end{array}$$

After M_4 :

$$\begin{array}{c}
 \frac{A \stackrel{k_s}{\models} S, A \triangleleft \{S(N_s, \#(N_s), g_a, p_a)\}_{k_s^-}}{A \models S \mid \sim (S(N_s, \#(N_s), g_a, p_a))} \\
 \frac{A \models S \mid \sim (S(N_s, \#(N_s), g_a, p_a)), A \models \#(N_s)}{A \models S \models g_a, p_a}
 \end{array}$$

Authentication Real Protocol

$$\begin{array}{l}
 M_1: A \rightarrow S: E_{k_s}(A, N_a) \parallel E_{k_a^-}(S(A, N_a)) \\
 M_2: S \rightarrow A: E_{k_a}(N_a, N_s, K_{as}) \parallel E_{k_s^-}(S(N_a, N_s, K_{as})) \\
 M_3: A \rightarrow S: E_{k_{as}}(N_s)
 \end{array}$$

Authentication Idealized Protocol

$$\begin{array}{l}
 M_1: A \rightarrow S: \{A, N_a\}_{k_s} \parallel \{S(A, N_a)\}_{k_a^-} \\
 M_2: S \rightarrow A: \left\{ N_a, \#(N_a), N_s, (A \stackrel{k_{as}}{\leftrightarrow} S) \right\}_{k_a} \parallel \left\{ S(N_a, \#(N_a), N_s, (A \stackrel{k_{as}}{\leftrightarrow} S)) \right\}_{k_s^-} \\
 M_3: A \rightarrow S: \left\{ N_s, \#(N_s), (A \stackrel{k_{as}}{\leftrightarrow} S) \right\}_{k_{as}}
 \end{array}$$

Authentication Analysis

Objectives

$A \models \#(N_s),$	$S \models A \models \#(N_s)$	# Session ID
$A \models A \stackrel{k_{as}}{\leftrightarrow} S$	$A \models \#(A \stackrel{k_{as}}{\leftrightarrow} S)$	# Session Key
$S \models A \models A \stackrel{k_{as}}{\leftrightarrow} S$		# Session Key confirmation

Assumptions

$$S \models A \xleftrightarrow{k_{as}} S$$

Session Key

$$A \models \rightarrow S$$

Hardcoded Server's Public Key

$$S \xrightarrow{k_a} A$$

Alice's Public Key

$$A \models S \Rightarrow N_s$$

Nonce Authority

$$A \models S \Rightarrow A \xleftrightarrow{k_{as}} S$$

Session Key Authority

After M_1 :

$$\frac{S \xrightarrow{k_a} A, S \triangleleft \{A, N_a\}_{k_a^-}}{S \models A \mid \sim (A, N_a)}$$

After M_2 :

$$\frac{A \xrightarrow{k_s} S, A \triangleleft \{N_a, \#(N_a), N_s, (A \xleftrightarrow{k_{as}} S)\}_{k_s^-}}{A \models S \mid \sim (N_a, \#(N_a), N_s, (A \xleftrightarrow{k_{as}} S))}$$

$$\frac{A \models S \mid \sim (N_a, \#(N_a), N_s, (A \xleftrightarrow{k_{as}} S)), A \models \#(N_a)}{A \models S \models (N_a, \#(N_a), N_s, (A \xleftrightarrow{k_{as}} S))}$$

$$\frac{A \models S \models N_s, A \models S \Rightarrow N_s}{A \models N_s}$$

$$\frac{A \models S \models (A \xleftrightarrow{k_{as}} S), A \models S \Rightarrow (A \xleftrightarrow{k_{as}} S)}{A \models (A \xleftrightarrow{k_{as}} S)}$$

$$\frac{A \models N_s, A \models \#(N_a)}{A \models \#(N_s)}$$

$$\frac{A \models A \xleftrightarrow{k_{as}} S, A \models \#(A \xleftrightarrow{k_{as}} S)}{A \models \#(A \xleftrightarrow{k_{as}} S)}$$

After M_3 :

$$\frac{S \models A \xleftrightarrow{k_{as}} S, S \triangleleft \{N_s, \#(N_s), (A \xleftrightarrow{k_{as}} S)\}_{k_{as}}}{S \models A \mid \sim (N_s, \#(N_s), (A \xleftrightarrow{k_{as}} S))}$$

$$\frac{S \models A \mid \sim (N_s, \#(N_s), (A \xleftrightarrow{k_{as}} S)), S \models \#(N_s)}{S \models A \models \#(N_s)}$$

$$\frac{S \models A \mid \sim (N_s, \#(N_s), (A \xleftrightarrow{k_{as}} S)), S \models \#(N_s)}{S \models A \models A \xleftrightarrow{k_{as}} S}$$

Online Key Exchange Real Protocol

$$M_1: A \rightarrow S: E_{k_{as}}(A, B, N_{sa})$$

$$M_2: S \rightarrow A: E_{k_{as}}(N_{sa}, g_b, p_b, k_b, E_{k_{bs}}(A, k_a, N_{sb}, N_{sa}))$$

$$M_3: A \rightarrow B: E_{k_b}(Y_A, E_{k_{bs}}(A, k_a, N_{sb}, N_{sa})) \parallel E_{k_a^-}(S(Y_A, E_{k_{bs}}(A, k_a, N_{sb}, N_{sa})))$$

$$M_4: B \rightarrow A: E_{k_a}(Y_B, N_{sa}, E_{k_{ab}}(N_b)) \parallel E_{k_b^-}(S(Y_B, N_{sa}, E_{k_{ab}}(N_b)))$$

$$M_5: A \rightarrow B: E_{k_{ab}}(N_b)$$

$$M_x: A \rightarrow B: E_{k_{ab}}(data)$$

Online Key Exchange Idealized Protocol

$$M_1: A \rightarrow S: \{A, B, N_{sa}\}_{k_{as}}$$

$$M_2: S \rightarrow A: \left\{ N_{sa}, g_b, p_b, \xrightarrow{k_b} B, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sa}) \right\}_{k_{as}}$$

$$M_3: A \rightarrow B: \left\{ Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sb}) \right\}_{k_b} \parallel \left\{ S(Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb} \right\}_{k_{bs}}, \#(\xrightarrow{k_a} A)) \right\}_{k_a^-}$$

$$M_4: B \rightarrow A: \{Y_b, N_{sa}, \{N_b\}_{k_{ab}}\}_{k_a} \parallel \{S(Y_b, N_{sa}, \{N_b\}_{k_{ab}})\}_{k_b^-}$$

$$M_5: A \rightarrow B: \{N_b\}_{k_{ab}}$$

Online Key Exchange Analysis

Objectives

$$A \models A \xleftrightarrow{k_{ab}} B, \quad B \models A \xleftrightarrow{k_{ab}} B \quad \# \text{ Key Authentication}$$

$$A \models B \models A \xleftrightarrow{k_{ab}} B, \quad B \models A \models A \xleftrightarrow{k_{ab}} B \quad \# \text{ Key Confirmation}$$

Assumptions

$$A \models A \xleftrightarrow{k_{as}} S, \quad S \models A \xleftrightarrow{k_{as}} S$$

A-S Session keys (authentication protocol)

$$B \models B \xleftrightarrow{k_{bs}} S, \quad S \models B \xleftrightarrow{k_{bs}} S$$

B-S Session keys (authentication protocol)

$$S \models \#(N_{sa}), \quad A \models \#(N_{sa}), \quad B \models \#(N_{sb})$$

Freshness session ids (authentication protocol)

$$S \models (g_b, p_b)$$

Diffie Hellman's parameters

$$A \models B \Rightarrow Y_B, \quad B \models A \Rightarrow Y_A$$

Authority on Y parameters

After M_1 :

$$\frac{S \models A \xleftrightarrow{k_{as}} S, \quad S \triangleleft \{A, B, N_{sa}\}_{k_{as}}}{S \models A \mid \sim (A, B, N_{sa})}$$

$$\frac{S \models A \mid \sim (A, B, N_{sa}), \quad S \models \#(N_{sa})}{S \models A \models (A, B, N_{sa})}$$

After M_2 :

$$\frac{A \models A \xleftrightarrow{k_{as}} S, \quad A \triangleleft \left\{ N_{sa}, g_b, p_b, \xrightarrow{k_b} B, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sa}) \right\}_{k_{as}}}{A \models S \mid \sim (N_{sa}, g_b, p_b, \xrightarrow{k_b} B, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sa}))}$$

$$\frac{A \models S \mid \sim (X), \quad A \models \#(N_{sa})}{A \models S \models (X)}$$

$$\frac{A \models S \models \xrightarrow{k_b} B, \quad A \text{ trusts } S \text{ on } k_b}{A \models \xrightarrow{k_b} B}$$

$$\frac{A \models S \models (g_b, p_b), \quad A \text{ trusts } S \text{ on } (g_b, p_b)}{A \models (g_b, p_b)}$$

After M_3 :

$$\frac{B \models B \xleftrightarrow{k_{bs}} S, \quad B \triangleleft \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}}{B \models S \mid \sim (A, \xrightarrow{k_a} A, N_{sb}, N_{sa})}$$

$$\frac{B \models S \mid \sim (\xrightarrow{k_a} A), \quad B \models \#(N_{sb})}{B \models S \models (\xrightarrow{k_a} A)}$$

$$\frac{B \models S \models \xrightarrow{k_a} A, \quad B \text{ trusts } S \text{ on } k_a}{B \models \xrightarrow{k_a} A}$$

$$\begin{array}{c}
B \models \xrightarrow{k_a} A, \quad B \triangleleft \left\{ Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sb}) \right\}_{k_a^-} \\
\hline
B \models A \mid \sim (Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sb})) \\
\\
B \models A \mid \sim (Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sb})), \quad B \models \#(N_{sb}) \\
\hline
B \models A \models (Y_A, \left\{ A, \xrightarrow{k_a} A, N_{sb}, N_{sa} \right\}_{k_{bs}}, \#(N_{sb})) \\
\\
B \models A \models Y_A, \quad B \models A \Rightarrow Y_A \\
\hline
B \models Y_A \\
\\
B \models Y_A, \quad B \Rightarrow Y_B \\
\hline
B \models A \xleftrightarrow{k_{ab}} B
\end{array}$$

After M_4 :

$$\begin{array}{c}
A \models \xrightarrow{k_b} B, \quad A \triangleleft \left\{ Y_b, N_{sa}, \{N_b\}_{k_{ab}} \right\}_{k_b^-} \\
\hline
A \models B \mid \sim (Y_b, N_{sa}, \{N_b\}_{k_{ab}}) \\
\\
A \models B \mid \sim (Y_b, N_{sa}, \{N_b\}_{k_{ab}}), \quad A \models \#(N_{sa}) \\
\hline
A \models B \models (Y_b, N_{sa}, \{N_b\}_{k_{ab}}) \\
\\
A \models B \models Y_B, \quad A \models B \Rightarrow Y_B \\
\hline
A \models Y_B \\
\\
A \models Y_B, \quad A \Rightarrow Y_A \\
\hline
A \models A \xleftrightarrow{k_{ab}} B \\
\\
A \models A \xleftrightarrow{k_{ab}} B, \quad A \triangleleft \{N_b\}_{k_{ab}} \\
\hline
A \models B \mid \sim (N_b) \\
\\
A \models B \mid \sim (N_b), \quad A \models \#(N_{sa}) \\
\hline
A \models B \models A \xleftrightarrow{k_{ab}} B
\end{array}$$

After M_5 :

$$\begin{array}{c}
B \models A \xleftrightarrow{k_{ab}} B, \quad B \triangleleft \{N_b\}_{k_{ab}} \\
\hline
B \models A \mid \sim (N_b) \\
\\
B \models A \mid \sim (N_b), \quad B \models \#(N_b) \\
\hline
B \models A \models A \xleftrightarrow{k_{ab}} B
\end{array}$$

Offline Communication Real Protocol

$$M_1: A \rightarrow S: E_{k_{as}}(A, B, N_{sa})$$

$$M_2: S \rightarrow A: E_{k_{as}}(N_{sa}, k_b)$$

$$M_x: A \rightarrow S: E_{k_{as}}(B, E_{k_b}(data))$$

$$M_y: S \rightarrow B: E_{k_{bs}}(A, E_{k_b}(data))$$

Offline Communication Idealized Protocol

$$M_1: A \rightarrow S: \{A, B, N_{sa}\}_{k_{as}}$$

$$M_2: S \rightarrow A: \{N_{sa}, \xrightarrow{k_b} B, \#(N_{sa})\}_{k_{as}}$$

Offline Communication Analysis

Objective

$$A \models \xrightarrow{k_b} B$$

Bob's Public Key

Assumptions

$$A \models A \leftrightarrow^{k_{as}} S, \quad S \models A \leftrightarrow^{k_{as}} S$$

Session key (authentication protocol)

$$S \models \#(N_{sa}), \quad A \models \#(N_{sa})$$

Freshness session ids (authentication protocol)

After M_1 :

$$\frac{S \models A \leftrightarrow^{k_{as}} S, \quad S \triangleleft \{A, B, N_{sa}\}_{k_{as}}}{S \models A \mid \sim (A, B, N_{sa})}$$

$$\frac{S \models A \mid \sim (A, B, N_{sa}), \quad S \models \#(N_{sa})}{S \models A \models (A, B, N_{sa})}$$

After M_2 :

$$A \models A \leftrightarrow^{k_{as}} S, \quad A \triangleleft \{N_{sa}, \xrightarrow{k_b} B, \#(N_{sa})\}_{k_{as}}$$

$$A \models S \mid \sim (N_{sa}, \xrightarrow{k_b} B, \#(N_{sa}))$$

$$\frac{A \models S \mid \sim (X), \quad A \models \#(N_{sa})}{A \models S \models (X)}$$

$$\frac{A \models S \models \overset{k_b}{\rightarrow} B, \text{ } A \text{ trusts } S \text{ on } k_b}{A \models \overset{k_b}{\rightarrow} B}$$