Foundations of Statistical Modeling

Prof. Dr. Stefan Kettemann
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Exercise sheet 2, submit on TEAMS by Wednesday March 6th, 2024
Your name: SIFAEL SEBASTIAN NDANDALA

CONSTRUCTOR UNIVERSITY

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1. Products and Projections of RV-Functions and Data Value Spaces [5 points]

a) Say, the universe Ω consists of you, two dice with 6 sides and numbers ranging from 1 to 6, one coinsand a table. Define the RV-function X and the Data Value Space S, describing the throwing of both dices and one coin, using the mathematically precise formalism. How many elements does S have?
b) Next, you want to throw one of the dice and the coin. Formulate precisely how you generate the corresponding RV function and DVS from X and S of a).

a/ Suppose $X_1 = \text{Dice}_1$, $X_2 = \text{Dice}_2$, $X_3 = \text{Coin}$

Then $X_1 : \Omega \to S_1$, $X_2 : \Omega \to S_2$, $X_3 : \Omega \to S_3$

The product RV can be written as: $X : \Omega \to S_1 \times S_2 \times S_3$

The RV can the be written as $X = X_1(\omega) \otimes X_2(\omega) \otimes X_3(\omega)$

RV Formally: $X = \bigotimes_{i=1}^{N=3} X_i$ where $X_i : \Omega \to S_i$

DVS Formally: $S = \prod_{i=1}^{N=3} S_i$ Element in $S = 6 \times 6 \times 2$

$S = \{\{1,2,\dots,6\}, \{1,2,\dots,6\}, \{H,T\}\}$

b/ $X = \bigotimes_{i=1}^{N=2} X_i$.

Given a dice and coin, $i = 2$

$X_1 : \Omega \to S_1$, $X_2 : \Omega \to S_2$.

RV: $X = X_1(\omega) \otimes X_2(\omega)$

The Data Values in S is given by $S_1 \times S_2$

$S = \{\{1,2,3,4,5,6\}, \{H,T\}\}$

2. Formalisation Exercise [5 points]

Consider a handwriting recognition system which recognises handwritten numbers as shown here. The raw input to this system is a grayscale photographic image of handwritten numbers. Assume that this raw input is delivered by grayscale images of size 200 by 200 pixels. Define the components of the corresponding data generating scenario. Define the RV-function and the data value space S, which you need to measure the gray scale, as defined in the interval $[0, 1]$, where 1 corresponds to black and 0 to white, of each pixel as well as its exact location on the image.

The size of the image $= 200 \times 200 = 40000$ with each event with possibility $[0,1]$.

Therefore the RV function: $X = \bigotimes_{i=1}^{N=40000} X_i$ where $X_i : \Omega \to S_i$, $s_i \in [0,1]$

The Data value space: $S = \prod_{i=1}^{n=40000} S_i$, where $S_i \in [0,1]$

The values will be $S = \{[0,1]_1, [0,1]_2, \dots, [0,1]_{40000}\}$