Assignment-04

1. Express $w = (-6,9,7)$ as a linear combination of the vectors $v_1 = (-1,2,2)$, $v_2 = (1,3,0)$, and $v_3 = (5,3,-1)$.

2. Find the solution of the following system of linear equations by graphing method.

$$x - y = 3$$
$$x + y = 1$$
$$2x + 3y = 6$$

3. Solve the following linear system by the row operation and write the solution in the vector form.

$$x_1 - 3x_2 - x_4 + x_5 = 0$$
$$3x_2 + x_3 - x_5 = 2$$

4. Find the nature of the solution of the following system of linear equations for the different values of "k".

   a. $2x + y = 3, x - 3y = k$
   b. $x + y = -1,\ kx - y = 1$

5. Determine the values of "λ" for the following system has no solution or unique solution or infinitely many solutions.

$$x + 2y = 1$$
$$3x + (\lambda^2 - 3)y = \lambda$$

6. Solve the following system by using the Gaussian and Gauss-Jordan methods. Verify that both of the methods give the same solution.

$$-2x_2 + 3x_3 + 5x_4 = 6$$
$$-2x_1 + x_2 + 3x_3 + x_5 = 3$$
$$2x_1 + x_2 + 2x_3 - 3x_4 - 2x_5 = 0$$
$$x_1 - 3x_2 + 2x_3 + x_4 + x_5 = 2$$

7. For the matrices $A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \\ 2 & 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & 4 & 6 \\ 1 & 1 & 5 & 3 \\ 7 & 5 & 3 & 2 \end{bmatrix}$ find $r_2(AB)$, $c_3(BA)$, $(AB)_{12}$, and $(BA)_{32}$.

8. Consider $A = \begin{bmatrix} -3 & 1 \\ 4 & -2 \end{bmatrix}$ and $p(x) = x^2 - 3x + 2$. Determine whether the matrix A is a root of the polynomial $p(A)$.

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9. If $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}$, and $C = \begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}$, solve $A^T A - CX = BX$ for X.

10. For matrix $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$ find $\mathrm{tr}(A^{-1} - 2I)$ and $\det(A^T + A^2)$.

11. If $(I - 5Y^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$ find Y, where I is an identity matrix.

12. For $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ verify the identities $(A^{-1})^{-1} = A$, $(A^T)^{-1} = (A^{-1})^T$, $\left(\frac{1}{5}A\right)^{-1} = 5A^{-1}$, and $\mathrm{tr}(A^{-1}) = \frac{\mathrm{tr}(A)}{\det(A)}$.

13. For $A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 5 & 0 \\ -2 & 4 & 6 \end{bmatrix}$, $u = \begin{bmatrix} 3 \\ -2 \\ 4 \end{bmatrix}$, and $v = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$ verify the formula $Au.v = u.A^T v$.

14. Solve the following linear system by (i) matrix inversion (co-factor approach) (ii) matrix inversion (row-operation approach) (iii) determinant method (Cramer rule).

$$5x + 15y + 56z = 35$$
$$-4x - 11y - 41z = -26$$
$$-x - 3y - 11z = -7$$

15. Find all the eigenvalues and corresponding eigenvectors of the matrix $A = \begin{bmatrix} -3 & 3 \\ 4 & -2 \end{bmatrix}$. Verify your result and sketch the eigen-space.