There are **5 (Five)** questions. Answer **all 5 (Five)** questions. All questions are of values indicated on the right-hand margin.

Q1. a. Use equivalence laws to prove the following propositions are equivalent or not: [2.5]

$$(p \rightarrow q) \vee (r \rightarrow s) \equiv (p \wedge r) \rightarrow (q \vee s)$$

Q1. b. Consider at the following statement: [1.5+1+1]

"If everything goes right, Rony or Jahid will become a member of the committee."

    i.   Determine whether an inclusive or, or an exclusive or, is intended in the above sentence. Justify your answer.

    ii.   Let p be "Everything goes right", q be "Rony will become an executive member", and r be "Jahid will become an executive member". Represent the abovementioned statement using these propositions and appropriate logical connectives.

    iii.   Write down the contrapositive of statement provided in Q1. (b).

Q2. a. Let T(x): x is a top scorer; M(x): x gets Man of the Match prize; W(x): x works hard;   [3x1=3]
C(x): x is complacent; H(x): x is haughty; R(x): x remains content with whatever he performs;
Translate the following statements using quantifiers.

    i.   Every top scorer gets Man of the Match prize.

    ii.   Not all hardworking player becomes top scorer.

    iii.   Any complacent and haughty player remains content with whatever he performs.

Q2. b. Determine the truth values of the following propositions. Here, the domain of each variable consists of all real numbers ($\mathbb{R}$).   [3x1=3]

    i. $\neg \forall x \ (x^5 > 0)$

    ii. $\neg \exists x \ ((-x^2 + 2) = (x^2 + 1))$

    iii. $\forall x \exists y \ (x^2 + y^2 < 0)$

Q3. a. Show that, for any two sets A and B, $|P \ (A \times B) \ | = 2^{|A||B|}$ [2]

Q3. b. Given that A and B are two mutually exclusive/disjoint sets such that,   [2+2]

$$A - B = \{1, 2\}$$
$$A \cup B = \{1, 2, x, y\},$$
$$\text{Universal Set} = \{1, 2, x, y, a\}$$

    i.   Represent the set operations mentioned above in a Venn Diagram.

    ii.   Find P (B x (A U B)')

Q4. a. Determine whether the functions are one-to-one or not from $\mathbb{R}$ to $\mathbb{R}$. Here, $\mathbb{R}$ = set of real numbers   [1.5x2=3]

    i. $p(x) = |1 - 3x|$

    ii. $f(x) = (3x - 5)^3$

Q4. b. Given $f(x) = \dfrac{4x}{5-x}$, Determine the following [1.5x2=3]

      i. $f^{-1}(x)$

      ii. $f \circ f^{-1}$

Q5. a. Using proof by contrapositive technique, provide a proof for the following claim. [3]

For any two integers p and q, p + q ≥ 15 implies that p ≥ 8 or q ≥ 8

Suppose p, q ∈ ℤ (set of integers)

Q5. b. Using proof by contradiction technique, provide a proof for the following claim. [3]

If $x^2 - 2x + 9$ is even, then x is odd.

Suppose x ∈ ℤ (set of integers)