Exm 51 :

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 5e^{3x}$$

For complementary function we choose,

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0$$

$$\Rightarrow (D^2 + 6D + 9)y = 0$$

So, A.E :

$$D^2 + 6D + 9 = 0$$

$$\Rightarrow D^2 + 3D + 3D + 9 = 0$$

$$\Rightarrow D = -3, -3$$

$$\therefore y_c = c_1 e^{-3x} + x c_2 e^{-3x}$$

For the resting part, we need to find Partial integral, as follows,

$$y_p = \frac{1}{D^2+6D+9} \cdot 5e^{3x}$$

$$= \frac{5}{36} e^{3x}$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{-3x} + x c_2 e^{-3x} + \frac{5}{36} e^{3x}$$

Exm 52:

$$\frac{d^2y}{dx^2} - 6 \cdot \frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$$

$\therefore$ A.E : $D^2 - 6D + 9 = 0$

$\Rightarrow D^2 - 3D - 3D + 9 = 0$

$\Rightarrow D = 3, 3$

$$\therefore y_c = c_1 e^{3x} + x c_2 e^{3x}$$

Now, partial integral :

$$y_p = \frac{1}{D^2 - 6D + 9} \cdot (6e^{3x} + 7e^{-2x} - \log 2)$$

$$= \frac{1}{D^2 - 6D + 9} \cdot 6e^{3x} + \frac{1}{D^2 - 6D + 9} \cdot 7e^{-2x} - \frac{1}{D^2 - 6D + 9} \cdot \log$$

$$= \frac{x}{2D - 6} \cdot 6e^{3x} + \frac{1}{25} \cdot 7e^{-2x} - \frac{1}{D^2 - 6D + 9} \cdot \log 2 \cdot$$

$$= \frac{x^2}{2} \cdot 6e^{3x} + \frac{7}{25} \cdot e^{-2x} - \frac{\log 2}{9}$$

$$= 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \frac{\log 2}{9}$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{3x} + x c_2 e^{3x} + 3x^2 e^{3x} + \frac{7}{25} e^{-2x} - \frac{\log 2}{9}$$

Exercise :

1. $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$

$\therefore$ A.E : $D^2 - 3D + 2 = 0$

$\Rightarrow D^2 - 2D - D + 2 = 0$

$\Rightarrow D = 1, 2$

$\therefore y_c = c_1 e^x + c_2 e^{2x}$

Now, partial integral,

$y_p = \frac{1}{D^2 - 3D + 2} \cdot e^{3x}$

$= \frac{1}{9 - 9 + 2} \cdot e^{3x}$

$= \frac{1}{2} e^{3x}$

$\therefore y = y_c + y_p$

$= c_1 e^x + c_2 e^{2x} + \frac{1}{2} e^{3x}$

3. $\frac{d^2y}{dx^2} + 2 \cdot \frac{dy}{dx} + 2y = \sinh x$

A.E : $D^2 + 2D + 2 = 0$

$\Rightarrow D = \frac{-2 \pm \sqrt{4-8}}{2}$

$= -1 \pm i$

$\therefore y_c = e^{-x}(c_1 \cos x + c_2 \sin x)$

Now, partial integral,

$y_p = \frac{1}{D^2+2D+2} \cdot \sinh x$

$= \frac{1}{D^2+2D+2} \cdot \frac{e^x - e^{-x}}{2}$

$= \frac{1}{2}\left[\frac{1}{D^2+2D+2} \cdot e^x - \frac{1}{D^2+2D+2} \cdot e^{-x}\right]$

$= \frac{1}{2}\left[\frac{1}{5}e^x - e^{-x}\right]$

$= \frac{1}{10}e^x - \frac{1}{2}e^{-x}$

$\therefore y = y_c + y_p$

$= e^{-x}(c_1 \cos x + c_2 \sin x) + \frac{1}{10}e^x - \frac{1}{2}e^{-x}$

4. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$

$\therefore$ A.E: $D^2 + 4D + 5 = 0$

$\Rightarrow D = \frac{-4 \pm \sqrt{4^2 - 20}}{2}$

$= -2 \pm i$

$\therefore y_c = e^{-2x}(c_1 \cos x + c_2 \sin x)$

Now, partial integral,

$y_p = \frac{1}{D^2 + 4D + 5} \cdot (-2\cosh x)$

$= \frac{1}{D^2 + 4D + 5} \cdot -2 \cdot \frac{e^x + e^{-x}}{2}$

$= -\left[\frac{1}{D^2 + 4D + 5} \cdot e^x + \frac{1}{D^2 + 4D + 5} \cdot e^{-x}\right]$

$= -\left[\frac{1}{10} e^x + \frac{1}{2} e^{-x}\right]$

$= -\frac{1}{10} e^x - \frac{1}{2} e^{-x}$

$\therefore y = y_c + y_p$

$= e^{-2x}(c_1 \cos x + c_2 \sin x) - \frac{e^x}{10} - \frac{1}{2} e^{-x}$

5. $(D^3 - 2D^2 - 5D + 6)y = e^{3x}$

$\therefore$ A.E:

$$D^3 - 2D^2 - 5D + 6 = 0$$

$$\Rightarrow D^3 - D^2 - D^2 + D - 6D + 6 = 0$$

$$\Rightarrow D^2(D-1) - D(D-1) - 6(D-1) = 0$$

$$\Rightarrow (D-1)(D^2 - D - 6) = 0$$

$$\Rightarrow (D-1)(D^2 - 3D + 2D - 6) = 0$$

$$\Rightarrow D = 1, 3, -2$$

$$\therefore y_c = c_1 e^{-2x} + c_2 e^{x} + c_3 e^{3x}$$

Now,

$$y_p = \frac{1}{D^3 - 2D^2 - 5D + 6} \cdot e^{3x}$$

$$= \frac{x}{3D^2 - 4D - 5} \cdot e^{3x}$$

$$= \frac{x}{10} e^{3x}$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{-2x} + c_2 e^{x} + c_3 e^{3x} + x \frac{e^{3x}}{10}.$$

7. $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = e^{3x}$

For complementary function,

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$$

$\Rightarrow (D^2 - 6D + 9)y = 0$

$\therefore$ A.E : $D^2 - 6D + 9 = 0$

$\Rightarrow D^2 - 3D - 3D + 9 = 0$

$\Rightarrow D = 3, 3$

$\therefore y_c = c_1 e^{3x} + x c_2 e^{3x}$

Now, partial integral :

$$y_p = \frac{1}{D^2 - 6D + 9} \cdot e^{3x}$$

$$= \frac{x}{2D - 6} \cdot e^{3x}$$

$$= \frac{x^2}{2} \cdot e^{3x}$$

$\therefore y = y_c + y_p$

$$= c_1 e^{3x} + x c_2 e^{3x} + x^2 \frac{e^{3x}}{2}$$

8. $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = e^{-x}$

$\therefore$ A.E: $D^3 + 3D^2 + 3D + 1 = 0$

$\Rightarrow D^3 + D^2 + 2D^2 + 2D + D + 1 = 0$

$\Rightarrow D^2(D+1) + 2D(D+1) + 1(D+1) = 0$

$\Rightarrow (D+1)(D^2 + 2D + 1) = 0$

$\Rightarrow (D+1)(D^2 + D + D + 1) = 0$

$\Rightarrow D = -1, -1, -1$

$\therefore y_c = c_1 e^{-x} + x c_2 e^{-x} + x^2 c_3 e^{-x}$

Now,

$$y_p = \frac{1}{D^3 + 3D^2 + 3D + 1} \cdot e^{-x}$$

$$= \frac{x}{3D^2 + 6D + 3} \cdot e^{-x}$$

$$= \frac{x^2}{6D + 6} \cdot e^{-x}$$

$$= \frac{x^3}{6} \cdot e^{-x}$$

$\therefore y = y_c + y_p$

$= c_1 e^{-x} + x c_2 e^{-x} + x^2 c_3 e^{-x} + \frac{x^3}{6} \cdot e^{-x}$

9. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = e^x \cosh 2x$

$\therefore$ A.E: $D^2 - D - 6 = 0$

$\Rightarrow D^2 - 3D + 2D - 6 = 0$

$\Rightarrow D = 3, -2$

$\therefore y_c = c_1 e^{-2x} + c_2 e^{3x}$

Now,

$$y_p = \frac{1}{D^2 - D - 6} \cdot e^x \cosh 2x$$

$$= \frac{1}{D^2 - D - 6} \cdot e^x \left(\frac{e^{2x} + e^{-2x}}{2}\right)$$

$$= \frac{1}{2} \frac{1}{D^2 - D - 6} \cdot \left(e^{3x} + e^{-x}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^2 - D - 6} \cdot e^{3x} + \frac{1}{D^2 - D - 6} \cdot e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{2D - 1} \cdot e^{3x} + \frac{1}{-4} \cdot e^{-x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{5} e^{3x} - \frac{1}{4} e^{-x}\right]$$

$$= \frac{x}{10} e^{3x} - \frac{1}{8} e^{-x}$$

$\therefore y = y_c + y_p$

$$= c_1 e^{-2x} + c_2 e^{3x} + \frac{x}{10} e^{3x} - \frac{1}{8} e^{-x}$$

10. $(D-2)(D+1)^2 y = e^{2x} + e^x$

$\therefore$ A.E : $(D-2)(D+1)^2 = 0$

$\Rightarrow (D-2)(D+1)(D+1) = 0$

$\Rightarrow D = 2, -1, -1$

$\therefore y_c = c_1 e^{2x} + c_2 e^{-x} + x c_3 e^{-x}$

Now,

$$y_p = \frac{1}{(D-2)(D+1)^2} \cdot (e^{2x} + e^x)$$

$$= \frac{1}{D^3 + 2D^2 + D - 2D^2 - 4D - 2} \cdot (e^{2x} + e^x)$$

$$= \frac{1}{D^3 - 3D - 2} \cdot (e^{2x} + e^x)$$

$$= \frac{1}{D^3 - 3D - 2} \cdot e^{2x} + \frac{1}{D^3 - 3D - 2} \cdot e^x$$

$$= \frac{x}{3D^2 - 3} \cdot e^{2x} + \frac{1}{-4} \cdot e^x$$

$$= \frac{x}{9} e^{2x} - \frac{1}{4} e^x$$

$\therefore y = y_c + y_p$

$$= c_1 e^{2x} + c_2 e^{-x} + x c_3 e^{-x} + \frac{x}{9} e^{2x} - \frac{1}{4} e^x$$

11. $(D-1)^3 y = 16e^{3x}$

$\therefore$ A.E: $(D-1)^3 = 0$

$\Rightarrow D = 1, 1, 1$

$\therefore y_c = c_1 e^x + x c_2 e^x + x^2 c_3 e^x$

Now,

$$y_p = \frac{1}{(D-1)^3} \cdot 16e^{3x}$$

$$= \frac{1}{8} \cdot 16e^{3x}$$

$$= 2e^{3x}$$

$\therefore y = y_c + y_p$

$= c_1 e^x + x c_2 e^x + x^2 c_2 e^x + 2e^{3x}$

Chp - 3.16

1. $(D^2+5D+4)y = 3-2x$

$\therefore$ A.E : $D^2+5D+4=0$

$\Rightarrow D^2+4D+D+4=0$

$\Rightarrow D=-4,-1$

$\therefore y_c = c_1e^{-x}+c_2e^{-4x}$

Now,

$$y_p = \frac{1}{D^2+5D+4}\cdot(3-2x)$$

$$= \frac{1}{4\left(\frac{D^2+5D}{4}+1\right)}\cdot(3-2x)$$

$$= \frac{1}{4}\left[1+\frac{D^2+5D}{4}\right]^{-1}\cdot(3-2x)$$

$$= \frac{1}{4}\left[1-\frac{D^2+5D}{4}\right](3-2x)$$

$$= \frac{1}{4}\left[(3-2x)-\frac{1}{4}\left(D^2(3-2x)+5D(3-2x)\right)\right]$$

$$= \frac{1}{4}\left[3-2x-\frac{1}{4}\left(0+5(-2)\right)\right]$$

$$= \frac{1}{4}\left[3-2x+\frac{10}{4}\right]$$

$$= \frac{1}{4}\left[3-2x+\frac{5}{2}\right]$$

$$= \frac{1}{4}\left[\frac{11}{2}-2x\right]$$

$$= \frac{1}{8}(11-4x)$$

$\therefore y = y_c + y_p$

$= c_1 e^{-x} + c_2 e^{-4x} + \frac{1}{8}(11 - 4x)$

3. $(2D^2 + 3D + 4)y = x^2 - 2x$

$\therefore$ A.E: $2D^2 + 3D + 4 = 0$

$\Rightarrow D = \frac{-3 \pm \sqrt{-23}}{4}$

$= -\frac{3}{4} \pm i\frac{\sqrt{23}}{4}$

$\therefore y_c = e^{-3/4x}\left(c_1 \cos\frac{\sqrt{23}}{4}x + c_2 \sin\frac{\sqrt{23}}{4}x\right)$

Now,

$y_p = \frac{1}{2D^2 + 3D + 4}(x^2 - 2x)$

$= \frac{1}{4}\left[1 + \frac{2D^2 + 3D}{4}\right]^{-1}(x^2 - 2x)$

$= \frac{1}{4}\left[1 - \frac{2D^2 + 3D}{4} + \left(\frac{2D^2 + 3D}{4}\right)^2 - \cdots\right](x^2 - 2x)$

$= \frac{1}{4}\left[1 - \frac{D^2}{2} - \frac{3}{4}D + D^4 + 3D^3 + \frac{9}{4}D^2\right](x^2 - 2x)$

$= \frac{1}{4}\left[1 + \frac{D^2}{16} - \frac{3}{4}D\right](x^2 - 2x)$

$= \frac{1}{4}\left[x^2 - 2x + \frac{1}{16}D^2(x^2 - 2x) - \frac{3}{4}D(x^2 - 2x)\right]$

$= \frac{1}{4}\left[x^2 - 2x + \frac{1}{8} - \frac{3}{2}x + \frac{3}{2}\right]$

$= \frac{1}{4}\left[x^2 - \frac{7}{2}x + \frac{13}{8}\right]$

$$= \frac{1}{32}\left[8x^2 - 28x + 13\right]$$

$$\therefore y = y_c + y_p$$

$$= e^{-3/4x}\left(c_1 \cos\frac{\sqrt{23}}{4}x + c_2 \sin\frac{\sqrt{23}}{4}x\right) +$$

$$\frac{1}{32}\left(8x^2 - 28x + 13\right)$$

4. $(D^2 - 4D + 3)y = x^3$

$\therefore$ A.E: $D^2 - 4D + 3 = 0$

$$\Rightarrow D^2 - 3D - D + 3 = 0$$

$$\Rightarrow D = 3, 1$$

$$\therefore y_c = c_1 e^x + c_2 e^{3x}$$

Now,

$$y_p = \frac{1}{D^2 - 4D + 3} \cdot x^3$$

$$= \frac{1}{3}\left[1 + \frac{D^2 - 4D}{3}\right]^{-1} \cdot x^3$$

$$= \frac{1}{3}\left[1 - \frac{D^2 - 4D}{3} + \left(\frac{D^2 - 4D}{3}\right)^2 - \left(\frac{D^2 - 4D}{3}\right)^3\right] x^3$$

$$= \frac{1}{3}\left[1 - \frac{D^2 - 4D}{3} + \frac{D^4 - 8D^3 + 16D^2}{9} - \frac{D^6 - 3D^4 \cdot 4D + 3D^4 - 64D^3}{27}\right]$$

$$= \frac{1}{3}\left[x^3 - \frac{1}{3}D^2x^3 + \frac{4}{3}Dx^3 - \frac{8}{9}D^3x^3 + \frac{16}{9}D^2x^3 - \frac{1}{9}D^2x^3 + \frac{16}{27}D^3x^3\right]$$

$$= \frac{1}{3}\left[x^3 - 2x + 4x^2 - \frac{48}{9} + \frac{96}{9}x + \frac{240}{27}\right]$$

$$= \frac{x^3}{3} + \frac{4}{3}x^2 + \frac{78}{27}x + \frac{80}{27}$$

$$= \frac{1}{27}(9x^3 + 36x^2 + 78x + 80)$$

$\therefore y = y_c + y_p$

$$= c_1 e^x + c_2 e^{3x} + \frac{1}{27}(9x^3 + 36x^2 + 78x + 80)$$

5. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$

$\therefore$ A.E: $D^3 - D^2 - 6D = 0$

$\Rightarrow D^3 + 2D^2 - 3D^2 - 6D = 0$

$\Rightarrow D^2(D+2) - 3D(D+2) + 0(D+2) = 0$

$\Rightarrow (D+2)(D^2 - 3D) = 0$

$\Rightarrow (D+2)D(D-3) = 0$

$\Rightarrow D = 0, -2, 3$

$\therefore y_c = c_1 + c_2 e^{-2x} + c_3 e^{3x}$

Now,

$$y_p = \frac{1}{D^3 - D^2 - 6D} \cdot (1 + x^2)$$

$$= -\frac{1}{6D}\left[1 - \frac{D^2 - D}{6}\right]^{-1} \cdot (1 + x^2)$$

$$= -\frac{1}{6D}\left[1 + \frac{D^2-D}{6} + \left(\frac{D^2-D}{6}\right)^2 + \cdots\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1 + \frac{D^2}{6} - \frac{1}{6}D + \frac{1}{36}(D^4 - 2D^3 + D^2)\right](1+x^2)$$

$$= -\frac{1}{6D}\left[(1+x^2) + \frac{1}{6}D^2(1+x^2) - \frac{1}{6}D(1+x^2) + \frac{1}{36}D^2(1+x^2)\right]$$

$$= -\frac{1}{6D}\left[1 + x^2 + \frac{1}{6}\cdot 2 - \frac{1}{6}\cdot 2x + \frac{1}{36}\cdot 2\right]$$

$$= -\frac{1}{6D}\left[1 + x^2 + \frac{1}{3} - \frac{1}{3}x + \frac{1}{18}\right]$$

$$= -\frac{1}{6D}\left[x^2 - \frac{1}{3}x + \frac{25}{8}\right]$$

$$= -\frac{1}{6}\left[\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^2}{2} + \frac{25}{18}x\right]$$

$$= -\frac{1}{6}\left[\frac{x^3}{3} - \frac{x^2}{6} + \frac{25}{18}x\right]$$

$$= -\frac{1}{108}(6x^3 - 3x^2 + 25x)$$

$\therefore\ y = y_c + y_p$

$$= c_1 + c_2 e^{-2x} + c_3 e^{3x} - \frac{1}{108}(6x^3 - 3x^2 + 25x)$$

6. $\frac{d^4y}{dx^4} + 4y = x^4$

$\therefore$ A.E : $D^4 + 4 = 0$

$\Rightarrow D^4 = -4$

$\Rightarrow D^2 = \pm 2i$

$\Rightarrow D^2 = 1 \pm 2i + i^2$

$\Rightarrow D^2 = (1 \pm i)^2$

$\Rightarrow D = \pm (1 \pm i)$

$\therefore y_c = e^x(c_1 \cos x + c_2 \sin x) + e^{-x}(c_3 \cos x + c_4 \sin x)$

Now,

$$y_p = \frac{1}{D^4+4} \cdot x^4$$

$$= \frac{1}{4}\left[1 + \frac{D^4}{4}\right]^{-1} \cdot x^4$$

$$= \frac{1}{4}\left[1 - \frac{D^4}{4}\right] x^4$$

$$= \frac{1}{4}\left(x^4 - \frac{1}{4} D^4 x^4\right)$$

$$= \frac{1}{4}\left(x^4 - \frac{1}{4} \cdot 24\right)$$

$$= \frac{1}{4}(x^4 - 6)$$

$\therefore y = y_c + y_p$

$$= e^x(c_1 \cos x + c_2 \sin x) + e^{-x}(c_3 \cos x + c_4 \sin x) + \frac{1}{4}(x^4 - 6)$$

8. $D^2(D^2+4)y = 96x^2$

$\therefore$ A.E: $D^2(D^2+4) = 0$

$\Rightarrow D = 0, 0, \pm 2i$

$\therefore y_c = c_1 + xc_2 + c_3\cos 2x + c_4 \sin 2x$

Now,

$$y_p = \frac{1}{D^2(D^2+4)} \cdot 96x^2$$

$$= \frac{1}{4D^2}\left[1+\frac{D^2}{4}\right]^{-1} \cdot 96x^2$$

$$= \frac{1}{4D^2}\left[1-\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2\right] \cdot 96x^2$$

$$= \frac{1}{4D^2}\left[96x^2 - \frac{1}{4} \cdot 96D^2x^2\right]$$

$$= \frac{1}{4D^2}\left[96x^2 - 48\right]$$

$$= \frac{1}{4D^2} \cdot 48\left[2x^2-1\right]$$

$$= \frac{12}{D^2}(2x^2-1)$$

$$= \frac{12}{D}\left(\frac{2x^3}{3}-x\right).$$

$$= 12\left(\frac{2}{3}\frac{x^4}{4} - \frac{x^2}{2}\right)$$

$$= 12\left(\frac{x^4}{6} - \frac{x^2}{2}\right)$$

$$= 2x^4 - 6x^2$$

$\therefore y = c_1 + xc_2 + c_3\cos 2x + c_4 \sin 2x + 2x^4 - 6x^2$

Exm 54:

$(D^2+4)y = \sin 3x$

$\therefore$ A.E: $D^2+4=0$

$\Rightarrow D = \pm 2i$

$\therefore y_c = c_1 \cos 2x + c_2 \sin 2x$

Now,

$$y_p = \frac{1}{D^2+4} \cdot \sin 3x$$

$$= \frac{1}{(-3^2)+4} \sin 3x$$

$$= -\frac{1}{5} \sin 3x$$

$\therefore y = y_c + y_p$

$= c_1 \cos 2x + c_2 \sin 2x - \frac{1}{5} \sin 3x$

Exm 55:

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \cos 2x$$

$\therefore$ A.E: $D^2+D+1=0$

$$\Rightarrow D = \frac{-1 \pm \sqrt{-3}}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\therefore y_c = e^{-x/2}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

Now,

$$y_p = \frac{1}{D^2+D+1} \cdot \cos 2x$$

$$= \frac{1}{-4+D+1} \cdot \cos 2x$$

$$= \frac{1}{D-3} \cdot \cos 2x$$

$$= \frac{D+3}{D^2-9} \cos 2x$$

$$= \frac{D+3}{-4-9} \cos 2x$$

$$= -\frac{1}{13}(D+3)\cos 2x$$

$$= -\frac{1}{13}\left[D(\cos 2x) + 3\cos 2x\right]$$

$$= -\frac{1}{13}\left[-2\sin 2x + 3\cos 2x\right]$$

$$= \frac{1}{13}\left[2\sin 2x - 3\cos 2x\right]$$

$$\therefore y = y_c + y_p$$

$$= e^{-x/2}\left[c_1 \cos\frac{\sqrt{3}}{2}x + c_2 \sin\frac{\sqrt{3}}{2}x\right]$$

$$+ \frac{1}{13}\left[2\sin 2x - 3\cos 2x\right]$$

Exm 56 :

$$(D^2+4)y = \cos 2x$$

$\therefore$ A.E : $D^2+4=0$

$$\Rightarrow D = \pm 2i$$

$$y_c = c_1\cos 2x + c_2\sin 2x$$

Now,

$$y_p = \frac{1}{D^2+4}\cdot\cos 2x$$

$$= \frac{x}{2D}\cos 2x$$

$$= \frac{x}{2}\cdot\frac{\sin 2x}{2}$$

$$= \frac{x}{4}\sin 2x$$

$$\therefore y = y_c + y_p$$

$$= c_1\cos 2x + c_2\sin 2x + \frac{x}{4}\sin 2x$$

Exercise :

2. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = \sin t$

$\therefore$ A. E: $D^2 + 2D + 3 = 0$

$\Rightarrow D = \frac{-2 \pm \sqrt{-8}}{2}$

$= -1 \pm \sqrt{2}i$

$\therefore y_c = e^{-t}(c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t)$

Now,

$x_p = \frac{1}{D^2+2D+3} \cdot \sin t$

$= \frac{1}{(-1^2)+2D+3} \cdot \sin t$

$= \frac{1}{2D+2} \cdot \sin t$

$= \frac{1}{2(D+1)} \cdot \sin t$

$= \frac{1}{2} \cdot \frac{D-1}{D^2-1} \sin t$

$= \frac{1}{2} \cdot \frac{D-1}{(-1^2)-1} \sin t$

$= \frac{D-1}{-4} \sin t$

$= -\frac{1}{4}(\cos t - \sin t)$

$\therefore y = y_c + y_p$

$= e^{-t}(c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t) - \frac{1}{4}(\cos t - \sin t)$

3. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \sin 2t$

$\therefore$ A.E : $D^2 + 2D + 5 = 0$

$\Rightarrow D = \frac{-2 \pm 4i}{2}$

$= -1 \pm 2i$

$\therefore x_c = e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$

Now,

$x_p = \frac{1}{D^2 + 2D + 5} \cdot \sin 2t$

$= \frac{1}{(-4) + 2D + 5} \sin 2t$

$= \frac{1}{1 + 2D} \cdot \sin 2t$

$= \frac{1}{2} \frac{D - 1/2}{D^2 - 1/4} \cdot \sin 2t$

$= \frac{1}{2} \cdot \frac{D - 1/2}{(-2^2) - 1/4} \cdot \sin 2t$

$= \frac{1}{2} \cdot \frac{D - 1/2}{-\frac{17}{4}} \sin 2t$

$= -\frac{2}{17}[2\cos 2t - \frac{1}{2}\sin 2t]$

$\therefore x = x_c + x_p$

$= e^{-t}(c_1 \cos 2t + c_2 \sin 2t) - \frac{2}{17}(2\cos 2t - \frac{1}{2}\sin 2t)$

5. $(D^3+1)y = 2\cos^2 x$

A.E :

$$D^3+1=0$$

$$\Rightarrow (D+1)(D^2-D+1)=0$$

$$\Rightarrow D+1=0 \quad , \quad D^2-D+1=0$$

$$\Rightarrow D=-1 \qquad \Rightarrow D = \frac{1\pm\sqrt{3}i}{2}$$

$$= \frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$\therefore y_c = c_1 e^{-x} + e^{1/2 x}\left(c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right)$$

Now,

$$y_p = \frac{1}{D^3+1} \cdot 2\cos^2 x$$

$$= \frac{1}{D^3+1} \cdot (1+\cos 2x)$$

$$= \frac{1}{D^3+1} \cdot e^0 + \frac{1}{D \cdot D^2+1} \cdot \cos 2x$$

$$= 1 + \frac{1}{D(-4)+1} \cdot \cos 2x$$

$$= 1 + \frac{1}{1-4D} \cos 2x$$

$$= 1 + \frac{1+4D}{1-16D^2} \cos 2x$$

$$= 1 + \frac{1+4D}{1-16(-4)} \cdot \cos 2x$$

$$= 1 + \frac{1+4D}{1+64} \cdot \cos 2x$$

$$= 1 + \frac{1}{65}(\cos 2x - 8\sin 2x)$$

$$\therefore y = y_c + y_p$$

$$= c_1 e^{-x} + e^{x/2}\left(c_2 \cos\frac{\sqrt{3}}{2}x + c_3 \sin\frac{\sqrt{3}}{2}x\right) + 1 + \frac{1}{65}(\cos 2x - 8\sin 2x)$$

7. $(D^4 + 2a^2D^2 + a^4)y = 8\cos ax$

$\therefore$ A.E : $D^4 + 2a^2D^2 + a^4 = 0$

$$\Rightarrow (D^2 + a^2)^2 = 0$$

$$\Rightarrow D^2 = -a^2, -a^2$$

$$\Rightarrow D = \pm ai, \pm ai$$

$$\therefore y_c = c_1 \cos ax + c_2 \sin ax + x(c_3 \cos ax + c_4 \sin ax)$$

$$\therefore y_p = \frac{1}{D^4 + 2a^2D^2 + a^4} \cdot 8\cos ax$$

$$= 8 \cdot \frac{x}{4D^3 + 4a^2D} \cdot \cancel{8}\cos ax$$

$$= 8x^2 \frac{1}{12D^2 + 4a^2} \cos ax$$

$$= 8x^2 \frac{1}{-12a^2 + 4a^2} \cdot \cos ax$$

$$= 8x^2 \cdot \frac{1}{-8a^2} \cdot \cos ax$$

$$= -\frac{x^2}{a^2} \cos ax$$

$\therefore y = y_c + y_p$

$= c_1 \cos ax + c_2 \sin ax + x(c_3 \cos ax + c_4 \sin ax)$

$- \frac{x^2}{a^2} \cos ax$

8. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 2x$

$\Rightarrow \therefore$ A.E: $D^2 + 3D + 2 = 0$

$\Rightarrow D^2 + 2D + D + 2 = 0$

$\Rightarrow D = -2, -1$

$\therefore y_c = c_1 e^{-2x} + c_2 e^{-x}$

Now,

$y_p = \frac{1}{D^2 + 3D + 2} \cdot \sin 2x$

$= \frac{1}{(-4) + 3D + 2} \cdot \sin 2x$

$= \frac{1}{3D - 2} \cdot \sin 2x$

$= \frac{3D + 2}{9D^2 - 4} \cdot \sin 2x$

$= \frac{3D + 2}{9(-4) - 4} \sin 2x$

$= \frac{3D + 2}{-40} \sin 2x$

$= -\frac{1}{40} (3.2 \cos 2x + 2 \sin 2x)$

$\therefore y = c_1 e^{-2x} + c_2 e^{-x} - \frac{1}{40} (6 \cos 2x + 2 \sin 2x)$

9. $\frac{d^2y}{dx^2} + y = \sin 3x \cos 2x$

$\therefore$ A.E: $D^2 + 1 = 0$

$\Rightarrow D^2 = -1$

$\Rightarrow D = \pm i$

$\therefore y_c = c_1 \cos x + c_2 \sin x$

Now,

$$y_p = \frac{1}{D^2+1} \cdot \sin 3x \cos 2x$$

$$= \frac{1}{2} \cdot \frac{1}{D^2+1} \cdot 2 \sin 3x \cos 2x$$

$$= \frac{1}{2} \cdot \frac{1}{D^2+1} \cdot [\sin(3x+2x) + \sin(3x-2x)]$$

$$= \frac{1}{2} \cdot \frac{1}{D^2+1} \cdot (\sin 5x + \sin x)$$

$$= \frac{1}{2} \cdot \left[\frac{1}{D^2+1} \cdot \sin 5x + \frac{1}{D^2+1} \sin x\right]$$

$$= \frac{1}{2} \left[\frac{1}{-25+1} \cdot \sin 5x + \frac{x}{2D} \sin x\right]$$

$$= \frac{1}{2} \left[\frac{1}{-24} \cdot \sin 5x + \frac{x}{2} (-\cos x)\right]$$

$$= -\frac{1}{4} \left[\frac{1}{12} \sin 5x + x \cos x\right]$$

$\therefore y = y_c + y_p$

$$= c_1 \cos x + c_2 \sin x - \frac{1}{4}\left[\frac{1}{12} \sin 5x + x \cos x\right]$$

11. $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$

$\therefore$ A.E : $D^2+4=0$

$\Rightarrow D = \pm 2i$

$\therefore y_c = c_1\cos 2x + c_2 \sin 2x$

Now,

$y_p = \frac{1}{D^2+4} \cdot (e^x + \sin 2x)$

$= \frac{1}{D^2+4} \cdot e^x + \frac{1}{D^2+4} \sin 2x$

$= \frac{1}{5} e^x + \frac{x}{2D} \sin 2x$

$= \frac{1}{5} e^x + \frac{1}{2} x \left(-\frac{\cos 2x}{2}\right)$

$= \frac{1}{5} e^x - \frac{x\cos 2x}{4}$

$\therefore y = y_c + y_p$

$= c_1 \cos 2x + c_2 \sin 2x + \frac{1}{5} e^x - \frac{x\cos 2x}{4}$

13. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = e^{2x} + \cos x$

$\therefore$ A.E : $D^2 - 4D + 3 = 0$

$\Rightarrow D^2 - 3D - D + 3 = 0$

$\Rightarrow D = 3, 1$

$\therefore y_c = c_1 e^x + c_2 e^{3x}$

Now,

$$y_p = \frac{1}{D^2-4D+3} \cdot (e^{2x} + \cos x)$$

$$= \frac{1}{D^2-4D+3} \cdot e^{2x} + \frac{1}{D^2-4D+3} \cdot \cos x$$

$$= -e^{2x} + \frac{1}{2-4D} \cos x$$

$$= -e^{2x} - \frac{4D+2}{16D^2-4} \cos x$$

$$= -e^{2x} - \frac{4D+2}{-16-4} \cdot \cos x$$

$$= -e^{2x} + \frac{1}{20} [4(-\sin x) + 2\cos x]$$

$$= -e^{2x} - \frac{1}{5} \sin x + \frac{1}{10} \cos x$$

$\therefore$ $y = y_c + y_p$

$$= c_1 e^x + c_2 e^{3x} - e^{2x} - \frac{1}{5} \sin x + \frac{1}{10} \cos x$$

15. $(D^3 - 3D^2 + 4D - 2) y = e^x + \cos x$

$\therefore$ A.E: $D^3 - 3D^2 + 4D - 2 = 0$

$$\Rightarrow D^3 - D^2 - 2D^2 + 2D + 2D - 2 = 0$$

$$\Rightarrow D^2(D-1) - 2D(D-1) + 2(D-1) = 0$$

$$\Rightarrow (D-1)(D^2 - 2D + 2) = 0$$

$\therefore$ $D - 1 = 0$ $\qquad$ $D^2 - 2D + 2 = 0$

$\Rightarrow D = 1$ $\qquad$ $\Rightarrow D = 1 \pm 2i$

$\therefore y_c = c_1 e^x + e^x (c_2 \cos 2x + c_3 \sin 2x)$

Now,

$$y_p = \frac{1}{D^3 - 3D^2 + 4D - 2} \cdot (e^x + \cos x)$$

$$= \frac{1}{D^3 - 3D^2 + 4D - 2} \cdot e^x + \frac{1}{D^3 - 3D^2 + 4D - 2} \cdot \cos x$$

$$= \frac{x}{3D^2 - 6D + 4} \cdot e^x + \frac{1}{D \cdot D^2 - 3D^2 + 4D - 2} \cos x$$

$$= xe^x + \frac{1}{-D + 3 + 4D - 2} \cos x$$

$$= xe^x + \frac{1}{3D + 1} \cos x$$

$$= xe^x + \frac{1 - 3D}{1 - 9D^2} \cos x$$

$$= xe^x + \frac{1 - 3D}{1 + 9} \cos x$$

$$= xe^x + \frac{1}{10} (\cos x - 3D \cdot \cos x)$$

$$= xe^x + \frac{1}{10} (\cos x + 3 \sin x)$$

$\therefore y = y_c + y_p$

$$= c_1 e^x + e^x (c_2 \cos 2x + c_3 \sin 2x) + xe^x + \frac{1}{10} (\cos x + 3 \sin x)$$

16. $(D^3 - 4D^2 + 13D)y = 1 + \cos 2x$

$\therefore$ A.E : $D^3 - 4D^2 + 13D = 0$

$\Rightarrow D(D^2 - 4D + 13) = 0$

$\therefore D = 0 \;,\; D^2 - 4D + 13 = 0$

$\Rightarrow D = \dfrac{4 \pm \sqrt{16-52}}{2} = 2 \pm 3i$

$\therefore y_c = c_1 + e^{2x}[c_2 \cos 3x + c_3 \sin 3x]$

Now,

$$y_p = \frac{1}{D^3 - 4D^2 + 13D} \cdot (1 + \cos 2x)$$

$$= \frac{1}{D^3 - 4D^2 + 13D} \cdot e^0 + \frac{1}{D^3 - 4D^2 + 13D} \cdot \cos 2x$$

$$= x \cdot \frac{1}{3D^2 - 8D + 13} \cdot e^0 + \frac{1}{D(-4) - 4(-4) + 13D} \cos 2x$$

$$= x \cdot \frac{1}{13} \cdot e^0 + \frac{1}{16 + 9D} \cos 2x$$

$$= \frac{1}{13} x + \frac{16 - 9D}{256 - 81D^2} \cdot \cos 2x$$

$$= \frac{1}{13} x + \frac{16 - 9D}{256 - 81(-4)} \cdot \cos 2x$$

$$= \frac{1}{13} x + \frac{16 - 9D}{580} \cdot \cos 2x$$

$$= \frac{1}{13} x + \frac{16 \cos 2x + 18 \sin x}{580}$$

$\therefore y = y_c + y_p$

$= c_1 + e^{2x}[c_2 \cos 3x + c_3 \sin 3x] + \frac{1}{13} x + \dfrac{16 \cos 2x + 18 \sin x}{580}$

19. $y'' + y' - 2y = -6\sin 2x - 18\cos 2x$

$\therefore$ A.E: $D^2 + D - 2 = 0$

$\Rightarrow D^2 + 2D - D - 2 = 0$

$\Rightarrow D = 1, -2$

$\therefore y_c = c_1 e^{x} + c_2 e^{-2x}$

Now,

$$y_p = \frac{1}{D^2+D-2}\{-6\sin 2x - 18\cos 2x\}$$

$$= \frac{1}{D^2+D-2}(-6\sin 2x) - \frac{1}{D^2+D-2}\cdot 18\cos 2x$$

$$= -6\cdot\frac{1}{D-6}\sin 2x - 18\cdot\frac{1}{D-6}\cdot\cos 2x$$

$$= -6\,\frac{D+6}{D^2-36}\sin 2x - 18\,\frac{D+6}{D^2-36}\cdot\cos 2x$$

$$= -6\,\frac{D+6}{-4-36}\sin 2x - 18\cdot\frac{D+6}{-4-36}\cos 2x$$

$$= \frac{6}{40}(2\cos 2x + 6\sin 2x) + \frac{18}{40}(-2\sin 2x + 6\cos 2x)$$

$$= \frac{1}{20}(6\cos 2x + 18\sin 2x - 18\sin 2x + 54\cos 2x)$$

$$= \frac{1}{20}\cdot 60\cos 2x$$

$$= 3\cos 2x$$

$\therefore y = y_c + y_p$

$= c_1 e^{x} + c_2 e^{-2x} + 3\cos 2x$

1. $(D^2-5D+6)y = e^x \sin x$

$\therefore$ A.E: $D^2-5D+6=0$

$\Rightarrow D^2-3D-2D+6=0$

$\Rightarrow D = 3, 2$

$\therefore y_c = c_1e^{2x}+c_2e^{3x}$.

Now,

$$y_p = \frac{1}{D^2-5D+6} \cdot e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2-5(D+1)+6} \cdot \sin x$$

$$= e^x \frac{1}{D^2+2D+1-5D-5+6} \cdot \sin x$$

$$= e^x \cdot \frac{1}{D^2-3D+2} \cdot \sin x$$

$$= e^x \cdot \frac{1}{1-3D} \sin x$$

$$= e^x \frac{1+3D}{1-9D^2} \sin x$$

$$= e^x \frac{1+3D}{1+9} \sin x$$

$$= e^x \frac{1}{10}(\sin x + 3\cos x)$$

$$\therefore y = c_1e^{2x}+c_2e^{3x}+\frac{1}{10}e^x(\sin x + 3\cos x)$$

2. $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x \cos 2x$

$\therefore$ A.E: $D^3 - D^2 + 3D + 5 = 0$

$\Rightarrow D^3 + D^2 - 2D^2 - 2D + 5D + 5 = 0$

$\Rightarrow D^2(D+1) - 2D(D+1) + 5(D+1) = 0$

$\Rightarrow (D+1)(D^2 - 2D + 5) = 0$

$\Rightarrow D = -1, \ 1 \pm 2i$

$\therefore y_c = c_1 e^{-x} + e^x(c_2 \cos 2x + c_3 \sin 2x)$

$\therefore$ Now,

$y_p = \frac{1}{D^3 - D^2 + 3D + 5} \cdot e^x \cos 2x$

$= e^x \frac{1}{(D+1)^3 - (D+1)^2 + 3(D+1) + 5} \cdot e^x \cos 2x$

$= e^x \cdot \frac{1}{D^3 + 2D^2 + 4D + 8} \cdot \cos 2x$

$= e^x, x \frac{1}{3D^2 + 4D + 4} \cos 2x$

$= xe^x \cdot \frac{1}{3(-4) + 4D + 4} \cdot \cos 2x$

$= xe^x \cdot \frac{1}{-8 + 4D} \cdot \cos 2x$

$= xe^x \cdot \frac{4D+8}{(4D+8)(4D-8)} \cos 2x$

$= xe^x \frac{4D+8}{16D^2 - 64} \cos 2x$

$$= xe^x \frac{4D+8}{16(-4)-64} \cdot \cos 2x$$

$$= xe^x \frac{4D+8}{-128} \cos 2x$$

$$= -xe^x \frac{(-8\sin 2x + 8\cos 2x)}{-128}$$

$$= -xe^x \frac{(-\sin 2x + \cos 2x)}{16}$$

$\therefore\ y = y_c + y_p$

$$= c_1 e^{-x} + e^x(c_2 \cos 2x + c_3 \sin 2x) - xe^x \frac{(-\sin 2x + \cos 2x)}{16}$$

3. $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = e^{2x}\sin x$

$\therefore$ A.E: $D^2 - 7D + 10 = 0$

$\Rightarrow D^2 - 5D - 2D + 10 = 0$

$\Rightarrow D = 5, 2$

$\therefore\ y_c = c_1 e^{2x} + c_2 e^{5x}$

Now,

$$y_p = \frac{1}{D^2 - 7D + 10} \cdot e^{2x} \sin x$$

$$= e^{2x} \frac{1}{(D+2)^2 - 7(D+2) + 10} \cdot \sin x$$

$$= e^{2x} \cdot \frac{1}{D^2+4D+4-7D-14+10} \sin x$$

$$= e^{2x} \cdot \frac{1}{D^2-3D} \cdot \sin x$$

$$= e^{2x} \cdot \frac{1}{-1-3D} \cdot \sin x$$

$$= -e^{2x} \frac{1-3D}{1-9D^2} \sin x$$

$$= -e^{2x} \frac{1-3D}{10} \sin x$$

$$= -e^{2x} \frac{\sin x - 3\cos x}{10}$$

$$= e^{2x} \frac{3\cos x - \sin x}{10}$$

$\therefore$ $y = y_c + y_p$

$$= c_1 e^{2x} + c_2 e^{5x} + e^{2x} \frac{3\cos x - \sin x}{10}$$

8. $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = \dfrac{e^{-x}}{x^2}$

$\therefore$ A.E :

$$D^2+2D+1=0$$

$$\Rightarrow (D+1)^2 = 0$$

$$\Rightarrow D = -1, -1$$

$\therefore$ $y_c = c_1 e^{-x} + x c_2 e^{-x}$

Now,

$$y_p = \frac{1}{D^2+2D+1} \cdot \frac{e^{-x}}{x^2}$$

$$= \frac{1}{(D+1)^2} \cdot \frac{e^{-x}}{x^2}$$

$$= e^{-x} \frac{1}{(D-1+1)^2} \cdot \frac{1}{x^2}$$

$$= e^{-x} \cdot \frac{1}{D^2} \cdot \frac{1}{x^2}$$

$$= e^{-x} \cdot \frac{1}{D} \cdot \left(-\frac{1}{x}\right)$$

$$= -e^{-x} \ln x$$

$\therefore$ $y = y_c + y_p$

$$= c_1 e^{-x} + x c_2 e^{-x} - e^{-x} \ln x$$

11. $\frac{d^2y}{dx^2} - 4y = x\sin hx$

$\therefore$ A.E: $D^2 - 4 = 0$

$$\Rightarrow (D+2)(D-2) = 0$$

$$\Rightarrow D = \pm 2$$

$\therefore$ $y_c = c_1 e^{-2x} + c_2 e^{2x}$

Now,

$$y_p = \frac{1}{D^2-4} \cdot x\sin h x$$

$$= \frac{1}{D^2-4} \cdot \frac{x(e^x - e^{-x})}{2}$$

$$= \frac{1}{2}\left[\frac{1}{D^2-4} \cdot xe^x - \frac{1}{D^2-4} xe^{-x}\right]$$

$$= \frac{1}{2}\left[e^x \frac{1}{(D+1)^2-4} \cdot x - e^{-x} \frac{1}{(D-1)^2-4} x\right]$$

$$= \frac{1}{2}\left[e^x \frac{1}{D^2+2D+1-4} \cdot x - e^{-x} \frac{1}{D^2-2D+1-4} \cdot x\right]$$

$$= \frac{1}{2}\left[e^x \frac{1}{D^2+2D-3} \cdot x - e^{-x} \frac{1}{D^2-2D-3} \cdot x\right]$$

$$= \frac{1}{2}\left[-\frac{e^x}{3} \cdot \frac{1}{1-\frac{D^2+2D}{3}} \cdot x + \frac{e^{-x}}{3} \cdot \frac{1}{1-\frac{D^2-2D}{3}} \cdot x\right]$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{3} \cdot \left(1-\frac{D^2+2D}{3}\right)^{-1} x + \frac{e^{-x}}{3}\left(1-\frac{D^2-2D}{3}\right)^{-1} x\right]$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{3}\left(1+\frac{D^2+2D}{3}\right) x + \frac{e^{-x}}{3}\left(1+\frac{D^2-2D}{3}\right) x\right]$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{3}\left(x+\frac{D^2}{3}x+\frac{2}{3}Dx\right) + \frac{e^{-x}}{3}\left(x+\frac{D^2}{3}x-\frac{2}{3}Dx\right)\right]$$

$$= \frac{1}{2}\left[\frac{e^{-x}}{3}\left(x+\frac{2}{3}\right) - \frac{e^{x}}{3}\left(x-\frac{2}{3}\right)\right]$$

$$= \frac{1}{6}\left[xe^{-x} - xe^{x} + \frac{2}{3}e^{-x} + \frac{2}{3}e^{x}\right]$$

$$= \frac{1}{6}\left[2 \cdot \frac{-x(e^x - e^{-x})}{2} + \frac{2}{3} \cdot 2 \cdot \frac{e^x + e^{-x}}{2}\right]$$

$$= \frac{1}{6}\left[-2x \sin h x + \frac{4}{3} \cos h x\right]$$

$$= \frac{1}{3}\left[-x\sin hx + \frac{2}{3}\cos hx\right]$$

$$\therefore\ y = y_c + y_p$$

$$= c_1 e^{-2x} + c_2 e^{2x} + \frac{1}{3}\left(-x\sin hx + \frac{2}{3}\cos hx\right).$$