United International University
Department of Computer Science and Engineering

CSE 2213/CSI 219: Discrete Mathematics
Mid-term Examination : Spring 2022
Total Marks: 30 Time: 1 hour and 45 minutes

Any examinee found adopting unfair means will be expelled from the trimester / program as per UIU disciplinary rules.

Answer all the 5 questions. Numbers to the right of the questions denote their marks.

1. (a) My favorite football team qualifies for final whenever I buy a ticket. [1.5]

   i. Find inverse and contrapositive of the above sentence.

   ii. Suppose, p = My favorite football team qualifies for final, q = I buy a ticket
   Translate the given sentence into a proposition.

   (b) Find the truth table of the following compound proposition and comment on whether the proposition is a tautology, contradiction, or neither of them. [2.5]

$$\neg((x \rightarrow \neg y) \vee \neg(z \longleftrightarrow \neg z))$$

   (c) Using propositional laws prove that, [2]

$$(p \longrightarrow q) \wedge (p \longrightarrow r) \equiv p \longrightarrow (q \wedge r)$$

2. (a) Consider the following predicates- [3]

$D(x) : x$ is a student of discrete mathematics course.
$T(x) : x$ is an instructor of discrete mathematics course.
$A(x) : x$ got an A in their final exam.
$C(x, y) : x$ receives counseling from y regarding discrete mathematics course.

   Represent the following statements using the above predicates, quantifiers and logical connectives. The domain of all variables consists of all people associated with UIU.

   i. All students of discrete mathematics course get A on their final exam.

   ii. Some students that receive counseling from an instructor, get A in their final exam.

   iii. All students of this course receive counseling from some course instructor.

   (b) Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers. [3]

   i. $\forall x \exists y(xy = 1)$

   ii. $\forall x \forall y(((x < 0) \wedge (y < 0)) \longrightarrow (xy > 0))$

   iii. $\exists x \exists y((x^2 > y) \wedge (x < y))$

3. (a) Consider the Venn diagram in Figure 1: [3]

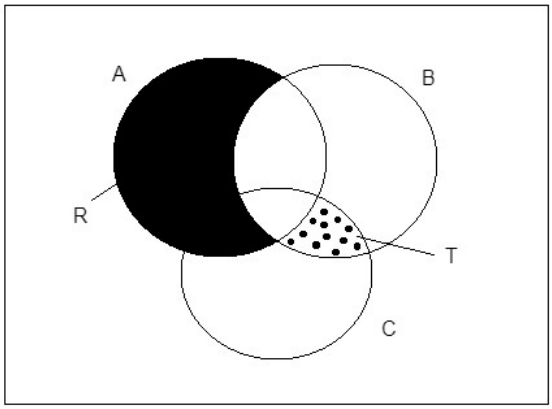

Figure 1: Venn Diagram for Question 3(a)

Here, the three circles represent the sets A, B, and C respectively. The shaded portion on the left represents set R. And the dotted portion on the right represents the set T. Here, $A = \{1, 2, 3, 4, 5\}$, $B = \{5, 2, 1, 0, 9\}$, and $C = \{0, 1, 7, 5, 8\}$.

Find $R \cup T$. You must use the values given in the Set definitions above.

(b) Given that, $A = \{x, y\}$, $B = \{a, b, c\}$, $C = \{1, 2, 3\}$, and $D = \{x, y, z\}$. [1.5 × 2 = 3]

i. Find A × B × C; ii. What is the value of $|P(A \times B \times C \times D)|$?

4. (a) Find $f \circ g$, $g \circ f$ and $f \circ f$, where $f(x) = x^2 + 1$ and $g(x) = \sqrt{x} - 1$ [3]

(b) Provide a counterexample for each of the following functions to show that they are **not** one-to-one: [3]

i. $f : R \to R, f(x) = 2x^2 - 5x + 1$

ii. $f : Z \to Z, f(x) = x^2 + 1$

iii. $f : Z \to Z, f(x) = \left|\frac{x}{2}\right| + x^4$

5. (a) Prove the following statement: [3]

"If mn is even, then m is even or n is even."

(b) Prove the following statement using a direct proof: [3]

"The sum of two rational numbers is rational."