SOLUTION OF MID-TERM QUESTION

CSE 2213 – Discrete Mathematics

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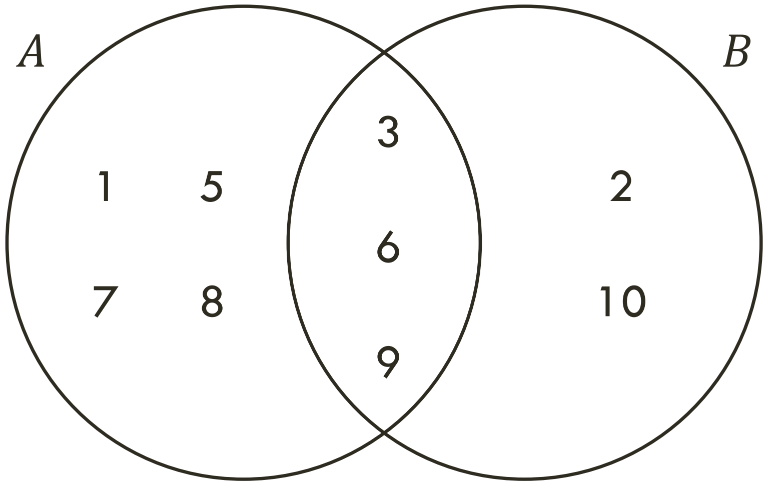
QUESTION 1(A-I) (MARKS: 1)

$A - B = \{1,5,7,8\},\ B - A = \{2,10\},\ A \cap B = \{3,6,9\}$

Find out A and B

From the Venn diagram, $A = \{1,3,5,6,7,8,9\}$
and $B = \{2,3,6,9,10\}$



QUESTION 1(A-II PART 1) (MARKS: 0.5)

If $A = \{1,2\}$, find out $\left|P\left(P\big(A \times P(A)\big)\right)\right|$

Note that the problem is to calculate the cardinality of the big power of power of power set, not the actual set.

$|P(A)| = 2^2 = 4$

$|A \times P(A)| = 2 \times 4 = 8$

$\left|P\big(A \times P(A)\big)\right| = 2^8 = 256$

$\left|P\left(P\big(A \times P(A)\big)\right)\right| = 2^{256}$

QUESTION 1(A-II PART 2) (MARKS: 1)

$S = \{x \in Z^+ \mid x \text{ is a divisor of 40 and 10 and less than 11}\}$

Divisors of 40 = 1, 2, 4, 5, 8, 10, 20, 40

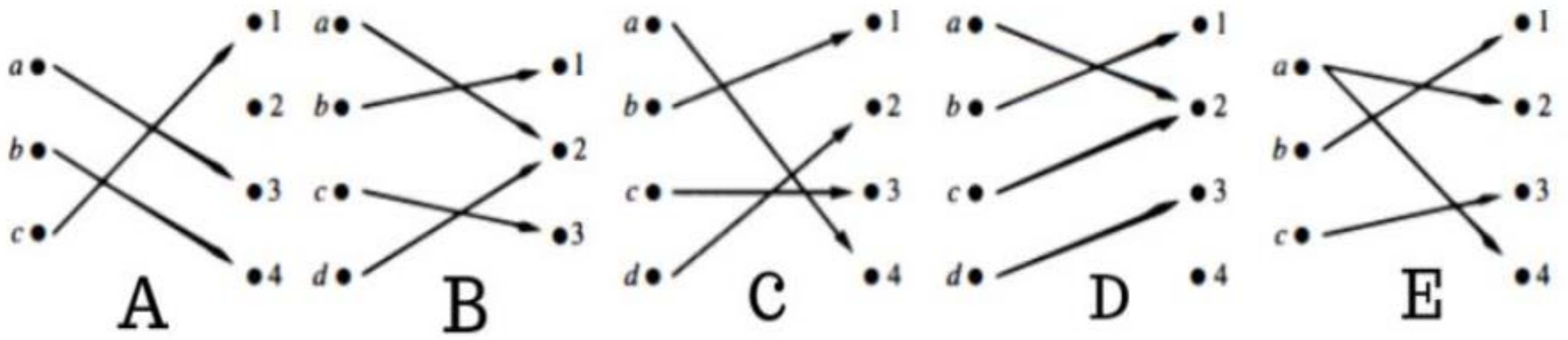
Divisors of 10 = 1, 2, 5, 10

Common divisors of 40 and 10 (and also <11) are 1, 2, 5, 10

$S = \{1,2,5,10\}$

$$P(S) = \left\{\begin{matrix} \{1\},\{2\},\{5\},\{10\},\{1,2\},\{1,5\},\{1,10\},\{2,5\},\{2,10\},\{5,10\}, \\ \{1,2,5\},\{1,2,10\},\{1,5,10\},\{2,5,10\},\{1,2,5,10\},\emptyset \end{matrix}\right\}$$

QUESTION 1(B)



A does not have an inverse function, because it is not onto.

B is not a one-to-one function, because $B(a) = B(d) = 2$.

C is one-to-one and onto function, because of unique outputs and codomain $=$ range.

D is not an onto function, because 4 is not an output for any input.

E is not a one-to-one function, because it is not even a function.

QUESTION 2(A)

The answer is directly given in 05-proposition.pptx, slide no. 12 (p, q, r replaced with r, s, t).

QUESTION 2(B)

There is a student in UIU who hasn't learnt C and C++ but is learning Python.

=> There is someone who is a student of UIU, hasn't learnt C & C++, and is learning Python.

=> $\exists x\big(P(x) \land \neg Q(x) \land R(x)\big)$

It is not that every student in UIU has learnt C and C++ programming but some UIU students are learning python.

=> $\neg\Big(\forall x\big(P(x) \to Q(x)\big)\Big) \land \Big(\exists x\big(P(x) \land R(x)\big)\Big)$

QUESTION 3(A)

You have to define p, q, r for each given proposition.

You can be a member of UIU Programming Club only if you are a student of UIU and you have been admitted into the CSE department.

=> $p \rightarrow (q \land r)$

A necessary condition for you have shown up on Interview is you got the job.

=> If you have shown up on Interview, you got the job.

=> $p \rightarrow q$

QUESTION 3(B)

$\forall x \exists y \left((x+1)^2 = y \right)$

True, because $(x+1)^2$ is a real number, so such y exists.

$\exists x \left((-x+1)^2 = x^2 \right)$

True, because such x exists. The value of x is $\frac{1}{2}$.

$\neg \forall x (x^3 > 0)$

True, because cubes of negative real numbers are negative.

QUESTION 4(A)

$P(x) \equiv 1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Basic: To prove $P(1)$

L.H.S. $= 1.2 = 2$

R.H.S. $= \frac{1.2.3}{3} = 2$

So $P(1)$ is true.

QUESTION 4(A) CONTD.

$P(x) \equiv 1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Induction: To prove $P(k) \rightarrow P(k+1)$

Let $P(k)$ be true, i.e. $1.2 + 2.3 + 3.4 + \cdots + k(k+1) = \frac{k(k+1)(k+2)}{3}$

We have to prove that $1.2 + 2.3 + 3.4 + \cdots + (k+1)(k+2) = \frac{(k+1)(k+2)(k+3)}{3}$

To prove this, we add $(k+1)(k+2)$ to both sides of $P(k)$.

$$1.2 + 2.3 + 3.4 + \cdots + k(k+1) + (k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$
$$= \frac{k(k+1)(k+2)+3(k+1)(k+2)}{3} = \frac{(k+1)(k+2)(k+3)}{3}$$

QUESTION 4(B)

To prove that, if $n^3 - 1$ is even, then n is odd.

It is sufficient to prove that, if n is even, then $n^3 - 1$ is odd.

Let $n = 2k$, where k is an integer.

$\therefore n^3 - 1 = (2k)^3 - 1 = 8k^3 - 1 = 2(4k^3) - 1$, which is odd.