

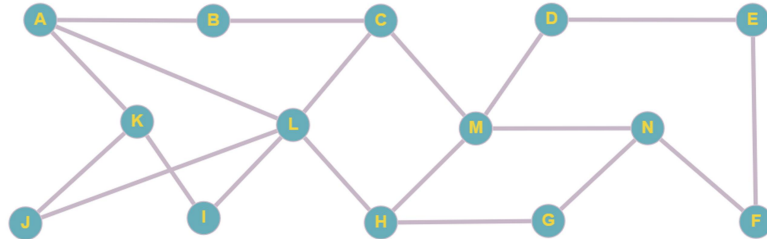


**[Any examinee found adopting unfair means will be expelled from the trimester/program as per UIU disciplinary rules.]**

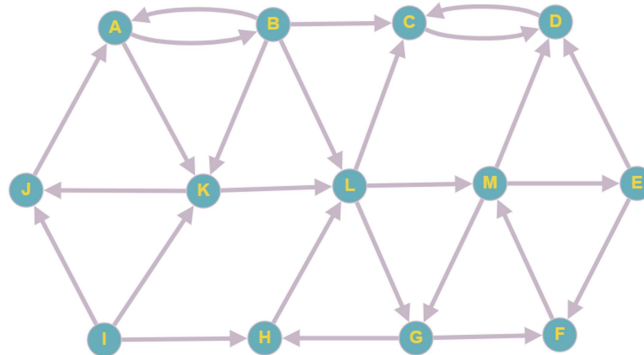
There are **5 (Five)** questions. Answer **all 5 (Five)** questions. All questions are of values indicated on the right-hand margin.

**Q1. a.** A complete graph  $K_n$  and a wheel graph  $W_{(n-1)}$  have the same number of edges  $n$ . What is the value of  $n$ ? [2]

**Q1. b.** Using two-coloring algorithm, find out whether the following graph is bipartite or not. If bipartite, show the graph in bipartite form. [4]



**Q1. c.** Find out the strongly connected components of the following graph. [3]



**Q2. a.** How many strings (considering only lowercase alphabets) are there of length **five** or less that begin and end with the letter "a"? (Do not consider empty string). [3]

**Q2. b.** Suppose that a software company has 7 front-end developers and 9 back-end developers. A standard team is formed by having at least one front-end developer and at least one back-end developer. How many ways are there to form a standard team of seven members so that it must have more front-end developers than back-end developers? [3]

**Q2. c.** In a game of UNO, there are cards of 4 colors- red, green, blue and yellow. There are 25 cards for each color (there are some special cards, but we will not be considering them now). A player has dealt 7 cards in a round. Now using the pigeonhole or generic pigeonhole principle, do the following: [1+2]

- Explain why there is no guarantee that a player will get at least 2 blue cards.
- Minimum how many cards should be picked to ensure that he gets at least 3 cards of the same color?

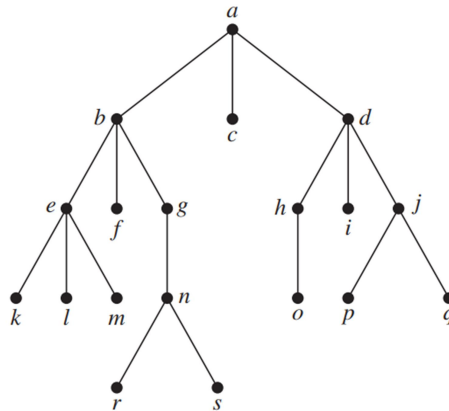
**Q3.** Use mathematical induction to prove the following summation formula for all positive integer values of  $n$ . [5]

$$\frac{1}{2 \times 5} + \frac{1}{5 \times 8} + \frac{1}{8 \times 11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

**Q4. a.** Form a binary search tree for the words: *mathematics, physics, geography, radiology, technology, meteorology, geology, criminology, psychology, neurology, chemistry, terminate* (use alphabetical order). [2]

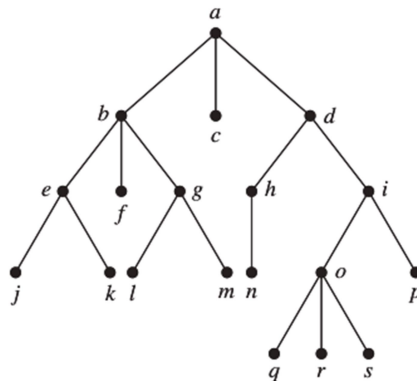
**Q4. b.** Is the tree you constructed in Q4. (a) A balanced tree? Explain with proper justification. What is the maximum limit of the number of leaves in this tree? [1+1]

**Q4. c.** Traverse the following tree using post-order technique. You must show all the steps in the process. [2.5]



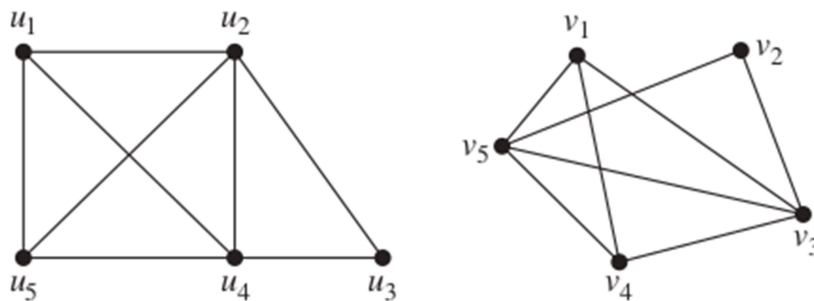
**Q4. d.** Represent the expression  $x + ((x*y + x)/y)$  using binary tree. Write the expression in prefix notation. Evaluate the prefix expression that you created using values of  $x = 4$ , and  $y = 3$ . [1+1 +0.5]

**Q5. a.** Is the rooted tree in the figure a full  $m$ -ary tree for some positive integer  $m$ ? If yes, what is the possible value of  $m$ ? If not, how can you make it a full  $m$ -ary tree? [1+2]



**Q5. b.** A full  $m$ -ary tree has 136 vertices. Among them, 109 are leaves. Calculate the values of  $m$ , and the number of edges in the tree. [2]

**Q5. c.** Find the degree sequences of both the graphs. Determine whether the given pair of graphs is isomorphic. [3]



Good Luck