UIU QUESTION BANK

MID-TERM QUESTION SOLUTIONS

DIGITAL LOGIC DESIGN

CSE 1325

SOLUTION BY

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UPDATED TILL ***SPRING 2024***

nurulalamador.github.io/**UIUQuestionBank**

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Spring 2024

1. A) Suppose there is a number system with base x. In that number system the following equation is true. Find out the value of x/radix.

$$\frac{(126)_x}{(5)_x} = 21$$

Solution:

Given,

$$\frac{(126)_x}{(5)_x} = 21$$

or, $\dfrac{1 \times x^2 + 2 \times x^1 + 6 \times x^0}{5 \times x^0} = 21$

or, $\dfrac{x^2 + 2x + 6 \times 1}{5 \times 1} = 21$

or, $x^2 + 2x + 6 = 21 \times 5$

or, $x^2 + 2x + 6 - 105 = 0$

or, $x^2 + 2x - 99 = 0$

$\therefore x = 9 \quad$ or, $\quad x = -11$

Since the radix of a number system cannot be negative,
$\therefore$ The radix, $x = 9$.

1. B) Represent the numbers $(690)_{10}$ and $(1000)_8$ in BCD, and then show the steps necessary to form their sum in BCD.

Solution:

Here,

$$\begin{aligned}(1000)_8 &= 1 \times 8^3 + 0 \times 8^2 + 0 \times 8^1 + 0 \times 8 \\ &= 512 + 0 + 0 + 0 \\ &= (512)_{10}\end{aligned}$$

$\therefore (1000)_8 = (512)_{10}$

Now,

$(690)_{10} = (0110\ \ 1001\ \ 0000)_{BCD}$

$(512)_{10} = (0101\ \ 0001\ \ 0010)_{BCD}$

[P.T.O]

$$
\begin{array}{r}
690 \\
+\,512 \\
\hline
1202
\end{array}
\qquad
\begin{array}{r}
1 \\
\\
\\
\hline
0001
\end{array}
\quad
\begin{array}{r}
1 \\
0110 \\
+\,0101 \\
\hline
1100 \\
110 \\
\hline
1\ \ 0010
\end{array}
\quad
\begin{array}{r}
\\
1001 \\
+\,0001 \\
\hline
1010 \\
110 \\
\hline
1\ \ 0000
\end{array}
\quad
\begin{array}{r}
\\
0000 \\
+\,0010 \\
\hline
0010
\end{array}
$$

$$
\begin{array}{cccc}
0001 & 0000 & 0000 & 0010 \\
(1) & (2) & (0) & (2)
\end{array}
$$

2. A) Convert the following expression to both SOM and POM.

$$F(A,B,C,D) = A'B + BD' + B'C'D + ACD'$$

Solution:

Simplifying the expression,

$$
\begin{aligned}
F(A,B,C,D) &= A'B + BD' + B'C'D + ACD' \\
&= A'BCD + A'BCD' + A'BC'D + A'BC'D' + ABCD' + ABC'D' + AB'C'D \\
&\quad + A'B'C'D + AB'CD' \\
&= \Sigma_m(7,6,5,4,14,12,9,1,10) \\
&= \Pi_M(0,2,3,8,11,13,15)
\end{aligned}
$$

∴ Sum of Minterms, SOM $= \Sigma_m(1,4,5,6,7,9,10,12,14)$

$$
\begin{aligned}
&= A'B'C'D + A'BC'D' + A'BC'D + A'BCD' + A'BCD + AB'C'D \\
&\quad + AB'CD' + ABC'D' + ABCD'
\end{aligned}
$$

∴ Product of Maxterms, POM $= \Pi_M(0,2,3,8,11,13,15)$

$$
\begin{aligned}
&= (A+B+C+D)(A+B+C'+D)(A+B+C'+D') \\
&\quad (A'+B+C+D)(A'+B+C'+D')(A'+B'+C+D')(A'+B'+C'+D')
\end{aligned}
$$

2. B) Simplify the following function to 4 literals

$$F(A,B,C,D) = (AB + C')(AB + D') + (C + D)(A + B) + BD'$$

Solution:

$$
\begin{aligned}
F(A,B,C,D) &= (AB + C')(AB + D') + (C + D)(A + B) + BD' \\
&= AB + ABD' + ABC' + C'D' + AC + BC + AD + BD + BD' \\
&= A(B + BD' + BC' + C + D) + C'D' + B(C + D + D') \\
&= A(B + BC' + C + D) + C'D' + B(C + 1) \quad \begin{bmatrix} \because B + BD' = B \\ \because D + D' = 1 \end{bmatrix} \\
&= A(B + C + D) + C'D' + B(1) \quad \begin{bmatrix} \because B + BC' = B \\ \because D + D' = 1 \end{bmatrix} \\
&= AB + AC + AD + C'D' + B \\
&= B + BA + AC + ACD + AC'D + C'D' \quad [\because AD = ACD + AC'D] \\
&= B + AC(1 + D) + C'(AD + D') \quad [\because B + BA = B] \\
&= B + AC(1) + C'(D' + DA) \\
&= B + AC + C'(D' + A) \quad [\because D' + DA = D' + A] \\
&= B + AC + C'D' + AC' = B + A(C + C') + C'D'
\end{aligned}
$$

$$= B + A(1) + C'D' \quad [\because C + C' = 1]$$
$$= A + B + C'D'$$

$\therefore F(A, B, C, D) = A + B + C'D'$

3. For the following function:

$$F(A, B, C, D) = \Pi_M(0,1,3,7,8,9,11) + \Sigma_d(12,13,14,15)$$

(i) Find the sum-of-product (SOP) form.
(ii) Find the product-of-sum (POS) form.
(iii) Which of the forms of between (i) and (ii) will you choose to implement the function and why?

Solution:

Given,

$$F(A, B, C, D) = \Pi_M(0,1,3,7,8,9,11) + \Sigma_d(12,13,14,15)$$
$$= \Sigma_m(2,4,5,6,10) + \Sigma_d(12,13,14,15)$$

i) Simplified SOP form:

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	(0)	(1)	(3)	(2) 1
$\bar{A}B$	(4) 1	(5) 1	(7)	(6) 1
AB	(12) X	(13) X	(15) X	(14) X
$A\bar{B}$	(8)	(9)	(11)	(10) 1

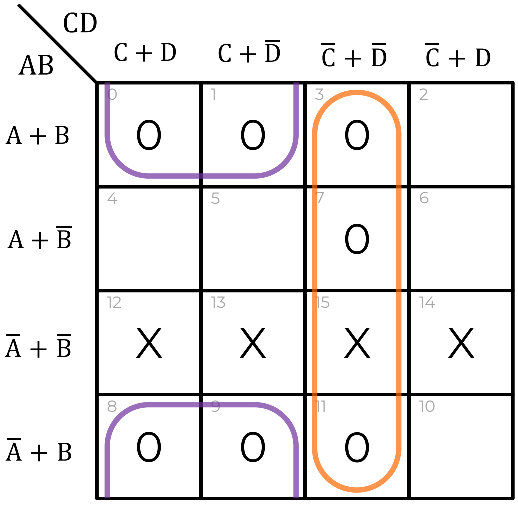
$$F(A, B, C, D) = B\bar{C} + C\bar{D}$$

Here, Literals $= 4$
Number of terms $= 2$
Distinct compliments $= 2$

$\therefore$ Gate cost in SOP, $G_{SOP} = 4 + 2 + 2 = 8$

ii) Simplified POS form:

[P.T.O]



$$F(A,B,C,D) = (B+C)(\bar{C}+\bar{D})$$

Here, Literals = 4
Number of terms = 2
Distinct compliments = 2

∴ Gate cost in POS, $G_{POS} = 4 + 2 + 2 = 8$

iii) Since $G_{POS} = G_{SOP}$,
∴ Both of the forms, SOP and POS, can be implemented since both have same cost.

4. Simplify the following Boolean function by finding:

(i) all prime implicants
(ii) essential prime implicants and
(iii) apply the selection rule:

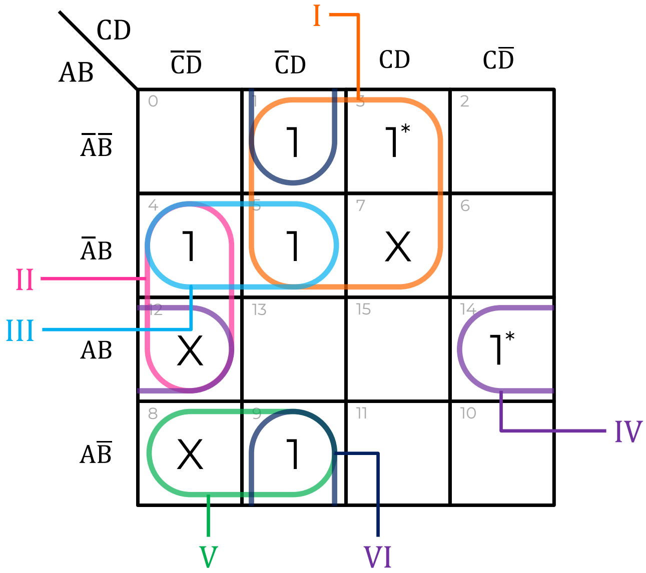
$$F(A,B,C,D) = \Sigma_m(1,3,4,5,9,14) + \Sigma_d(7,8,12)$$

Solution:

Given,

$$F(A,B,C,D) = \Sigma_m(1,3,4,5,9,14) + \Sigma_d(7,8,12)$$

Now,



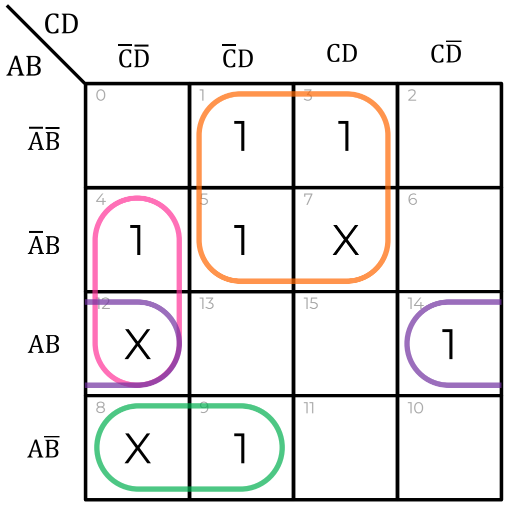
i) <u>**Prime Implicants, PI:**</u>

I, II, III, IV, V, VI $(\bar{A}D, B\bar{C}\bar{D}, \bar{A}B\bar{C}, AB\bar{D}, A\bar{B}\bar{C}, \bar{B}\bar{C}D)$

ii) <u>**Essential Prime Implicants, EPI:**</u>

I, IV $(\bar{A}D, AB\bar{D})$

iii) Applying Selection Rule:



$$F(A,B,C,D) = \bar{A}D + B\bar{C}\bar{D} + AB\bar{D} + A\bar{B}\bar{C}$$

5. A three-variable logic function that is equal to 1 if any two or all three of its variables are equal to 1 is called a three input majority function.

(i) Draw the truth table of the majority function.
(ii) Draw the k-map of the function.
(iii) Find the simplified product-of-sum (POS) form.
(iv) Draw the circuit diagram of the expression found in (iii).

Solution:

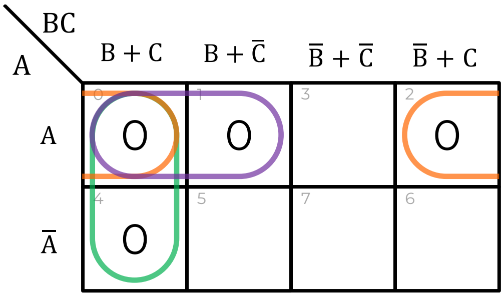
(i) The truth table has been drawn below:

Index	A	B	C	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

(ii) From (i),

$$F(A,B,C) = \Pi_M(0,1,2,4)$$

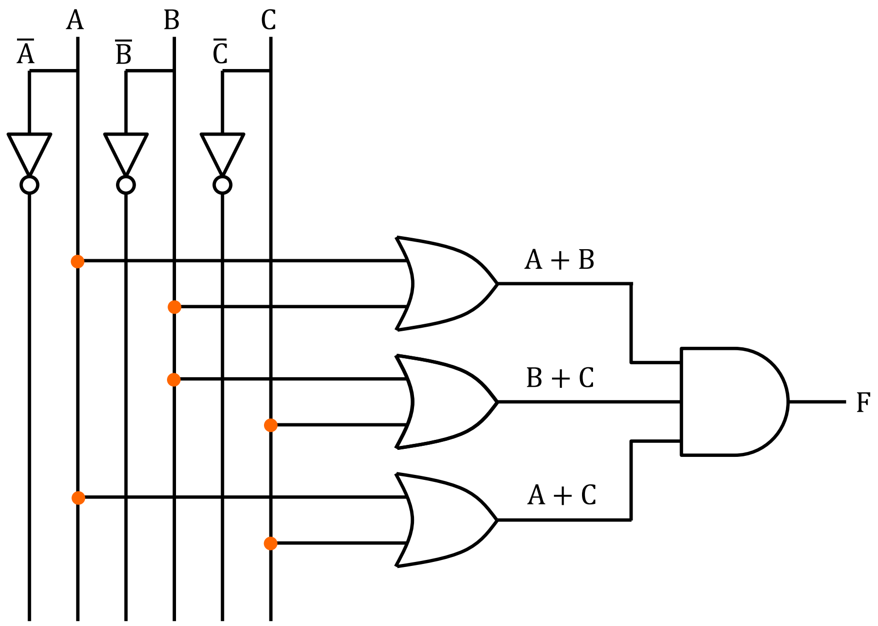
The k-map of the function has been drawn below:



(iii) From k-map of (ii),
Simplified product-of-sum (POS) form,

$$F(A, B, C) = (A + B)(B + C)(A + C)$$

(iv) The circuit diagram has been drawn below:



6. Suppose you decided to adopt a kitten. The kitten needs to be fed once every three hours. However, due to your busy schedule throughout the day, you require an alarm system to remind you to feed the kitten. Using all the knowledge you gained from your DLD course you decided to build a circuit for an alarm system and connect it with your table clock. The system takes the binary value of the time from the clock as 4 bit input. Considering that the first alarm is set at 3 o'clock, the system subsequently rings every three-hour interval. Remember the alarm clock only provides the numbers from 1 to 12 as inputs. Map the other inputs of your circuits as don't care conditions. The circuit outputs 1 for the alarm to be rung.

You have to **(i)** Show the truth table **(ii)** Find the simplified expression for the output bit in Sum-of-Product form **(iii)** Implement the simplified expression in PLA.

Few example inputs and outputs are given below:

Input : 0011, Output: 1, 3 o'clock so the alarm should ring
Input : 0101, Output: 0, 5 o'clock so the alarm should not ring
Input : 0110, Output: 1, 6 o'clock so the alarm should ring
Input : 0000, Output: X(don't care)

Solution:

(i) The truth table has been shown below:

Index	A	B	C	D	F
0	0	0	0	0	X
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	X
14	1	1	1	0	X
15	1	1	1	1	X

(ii) From (i),

$$F(A,B,C,D) = \Sigma_m(3,6,9,12) + \Sigma_m(0,13,14,15)$$

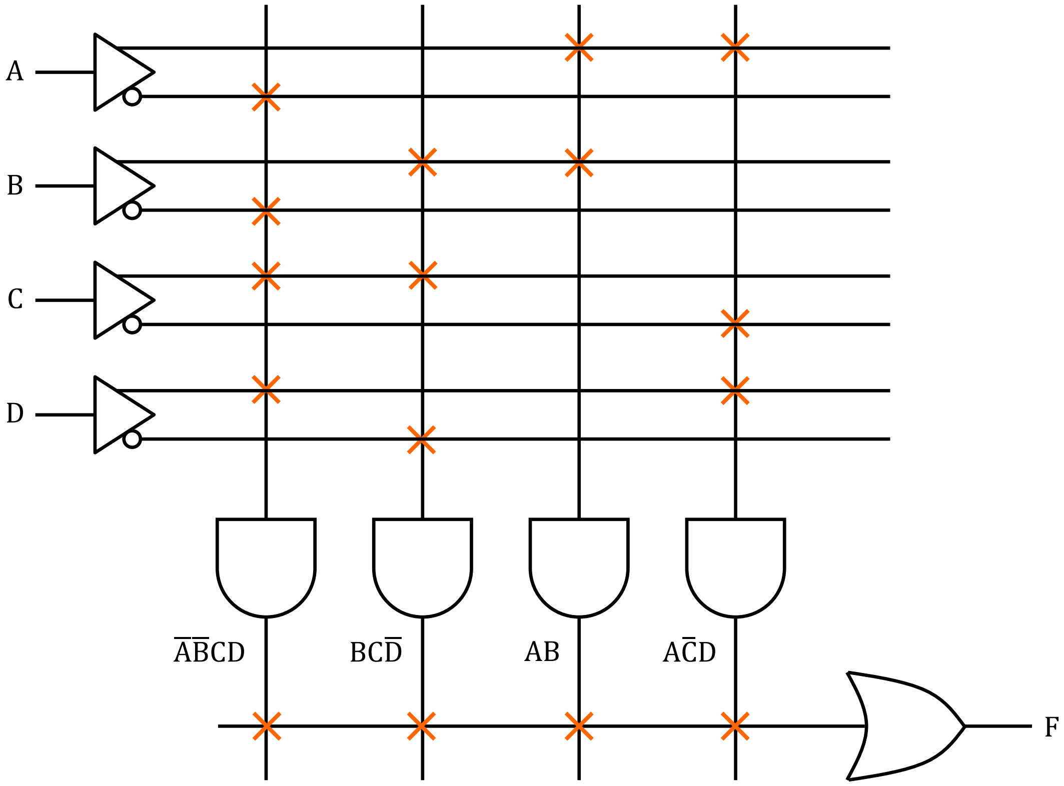
Simplified expression in SOP,

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	X		1	
$\bar{A}B$				1
AB	1	X	X	X
$A\bar{B}$		1		

$$F(A,B,C,D) = \bar{A}\bar{B}CD + BC\bar{D} + AB + A\bar{C}D$$

(iii) The simplified expression has been implemented in PLA below:

[P.T.O]


A
B
C
D
$\overline{A}\overline{B}CD$
$BC\overline{D}$
AB
$A\overline{C}D$
F

Fall 2023

1. A) Convert the Hexadecimal number $E4D6.B$ to octal, decimal and binary numbers.

Solution:

$(E4D6.B)_{16} = (?)_8$

$$\begin{array}{cccccc}
\text{E} & 4 & \text{D} & 6 & & \text{B} \\
\downarrow & \downarrow & \downarrow & \downarrow & . & \downarrow \\
1110 & 0100 & 1101 & 0110 & & 1011
\end{array}$$

$$\begin{array}{ccccccccc}
001 & 110 & 010 & 011 & 010 & 110 & & 101 & 100 \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & . & \downarrow & \downarrow \\
1 & 6 & 2 & 3 & 2 & 6 & & 5 & 4
\end{array}$$

$\therefore (E4D6.B)_{16} = (162326.54)_8$

$(E4D6.B)_{16} = (?)_{10}$

$$\begin{aligned}
(E4D6)_{16} &= E \times 16^3 + 4 \times 16^2 + D \times 16^1 + 6 \times 16^0 \\
&= 14 \times 4096 + 4 \times 256 + 13 \times 16 + 6 \times 1 \\
&= (58582)_{10}
\end{aligned}$$

$$\begin{aligned}
(0.B)_{16} &= B \times 16^{-1} \\
&= 11 \times 0.0625 \\
&= (0.6875)_{10}
\end{aligned}$$

$\therefore (E4D6.B)_{16} = (58582.6875)_{10}$

$(E4D6.B)_{16} = (?)_2$

$$\begin{array}{cccccc}
\text{E} & 4 & \text{D} & 6 & & \text{B} \\
\downarrow & \downarrow & \downarrow & \downarrow & . & \downarrow \\
1110 & 0100 & 1101 & 0110 & & 1011
\end{array}$$

$\therefore (E4D6.B)_{16} = (1110010011010110.1011)_2$

1. B) Perform the following conversion by using base 2 instead of base 10 as the intermediate base for the conversion: $(103)_4$ to octal.

Solution:

Here,
We will use 2 length binary as intermediate base since $4 = 2^2$

$(103)_4 = (?)_8$

$$\begin{array}{ccc}
1 & 0 & 3 \\
\downarrow & \downarrow & \downarrow \\
01 & 00 & 11
\end{array}$$

$$\begin{array}{cc}
010 & 011 \\
\downarrow & \downarrow \\
2 & 3
\end{array}$$

$\therefore (103)_4 = (23)_8$

2. A) Using Boolean identities, reduce the given Boolean expression:

$$F(A,B,C,D) = A'B(D' + C'D) + B(A + A'CD)$$

Solution:

$$\begin{aligned}
F(A,B,C,D) &= A'B(D' + C'D) + B(A + A'CD) \\
&= A'B(D' + DC') + B[A + A'(CD)] \\
&= A'B(D' + C') + B(A + CD) \quad \begin{bmatrix} \because D' + D'C' = D' + C' \\ \because A + A'(CD) = A + CD \end{bmatrix} \\
&= A'BD' + A'BC' + AB + BCD \\
&= B(A'D' + A'C' + A + CD) \\
&= B(A + A'D' + A'C' + CD) \\
&= B(A + D' + A'C' + CD) \quad [\because A + A'D = AD] \\
&= B(A + A'C' + D' + CD) \\
&= B(A + C' + D' + C) \quad \begin{bmatrix} \because A + A'C = A + C' \\ D' + DC = D' + C \end{bmatrix} \\
&= B(A + D' + 1) \quad [\because C' + C = 1] \\
&= B(1) \\
&= B
\end{aligned}$$

2. B) Convert the following expression into both canonical SoP and canonical PoS forms using Boolean algebra:

$$(A + B)(A + C)(AB'C)$$

Solution:

Simplifying the expression,

$$\begin{aligned}
(A + B)(A + C)(AB'C) &= (A + AC + AB + BC)(AB'C) \\
&= (A + C + AB + BC)(AB'C) \quad [\because A + AC = A + C] \\
&= (A + B + B + C)(AB'C) \quad \begin{bmatrix} \because A + AB = A + B \\ \because C + BC = A + B \end{bmatrix} \\
&= (A + B + C)(AB'C) \quad [\because B + B = B] \\
&= AB'C + ABB'C + AB'C \\
&= AB'C + 0 + AB'C \quad [\because BB' = 0] \\
&= AB'C \\
&= \Sigma_m(5) \\
&= \Pi_M(0,1,2,3,4,6,7)
\end{aligned}$$

$\therefore$ Canonical SoP $= \Sigma_m(5)$
$= AB'C$

$\therefore$ Canonical PoS $= \Pi_M(0,1,2,3,4,6,7)$
$= (X + Y + X)(X + Y + X')(X + Y' + X)(X + Y' + X')(X' + Y + X)$
$(X' + Y' + X)(X' + Y' + X')$

3. For the following function: **(i)** Find all the Prime implicants, **(ii)** Find all the essential prime implicants and **(iii)** Find a simplified expression in Sum of Product (SOP).

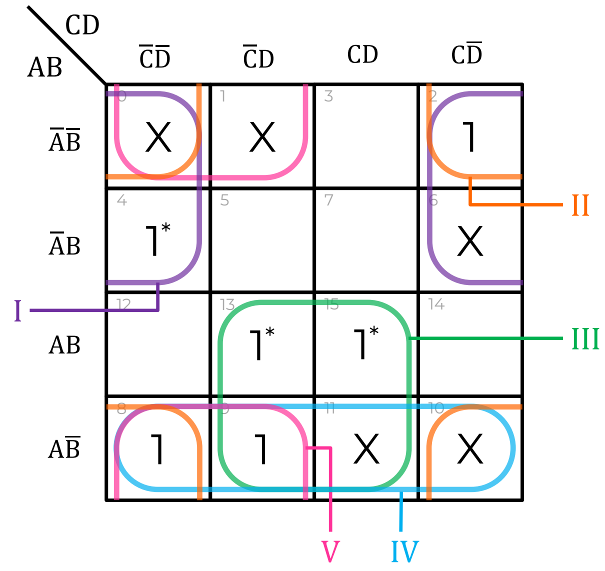
$$F(A,B,C,D) = \Pi_M(3,5,7,12,14) + \Sigma_d(0,1,6,10,11)$$

Solution:

Given,

$$F(A,B,C,D) = \Pi_M(3,5,7,12,14) + \Sigma_d(0,1,6,10,11)$$
$$= \Sigma_m(2,4,8,9,13,15) + \Sigma_d(0,1,6,10,11)$$

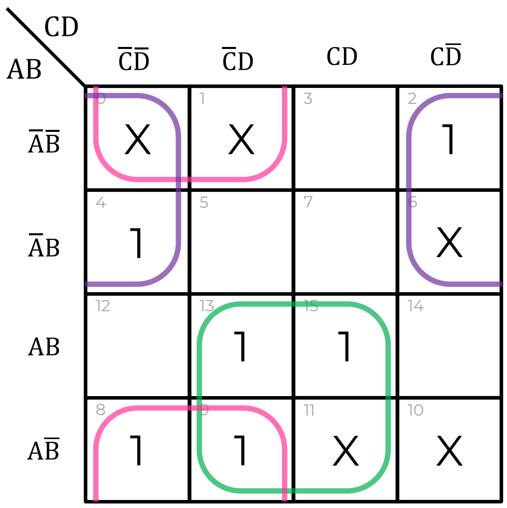
Now,



i) **<u>Prime Implicants, PI:</u>**
I, II, III, IV, V $(\bar{A}\bar{D}, \bar{B}\bar{D}, AD, A\bar{B}, \bar{B}\bar{C})$

ii) **<u>Essential Prime Implicants, EPI:</u>**
I, III $(\bar{A}\bar{D}, AD)$

iii) Simplified expression in Sum of Product:



$$F(A,B,C,D) = \bar{A}\bar{D} + AD + \bar{B}\bar{C}$$

4. Consider the following non-canonical function. Find the simplified expression in Sum of Product (SOP) and Product of Sum (POS) using k-map.

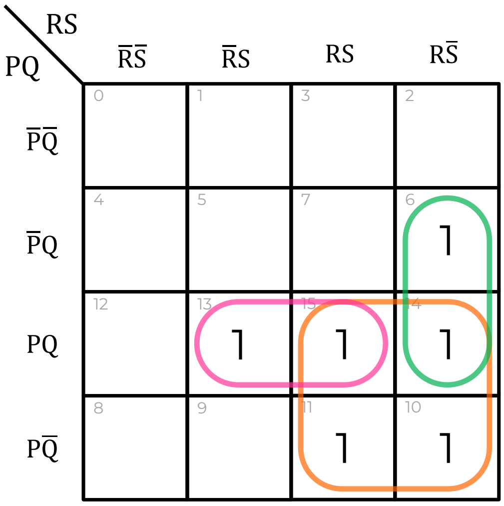
$$F(P,Q,R,S) = PQ'R + P'QRS' + PQR'S + PQR$$

Solution:

Given,

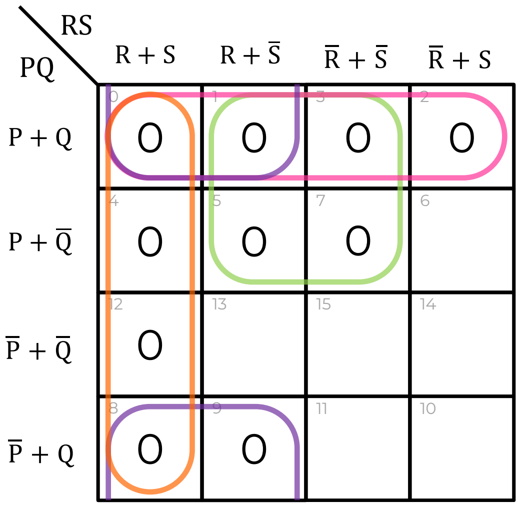
$$\begin{aligned} F(P,Q,R,S) &= PQ'R + P'QRS' + PQR'S + PQR \\ &= PQ'RS + Q'RS' + P'QRS' + PQR'S + PQRS + PQRS' \\ &= \Sigma_m(11,10,6,13,15,14) \\ &= \Pi_M(0,1,2,3,4,5,7,12,8,9) \end{aligned}$$

Now,
For simplified SOP,



$$F(P,Q,R,S) = QR\bar{S} + PQS + PR$$

For simplified POS,



$$F(P,Q,R,S) = (R+S)(P+Q)(Q+R)(P+\bar{S})$$

5. Optimize the following Boolean functions using K-map in **(i)** Sum of Product (SOP) form, **(ii)** Product of Sum (POS) form **(iii)** Between minimized SOP and POS, which one do you think will be easy to implement and why?

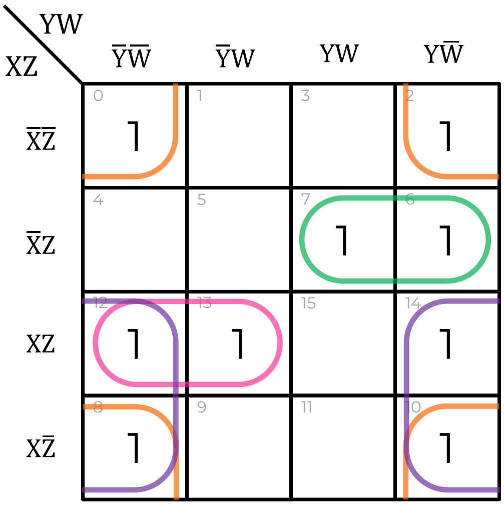
$$G(X,Z,Y,W) = \Sigma_m(0,2,6,7,8,10,12,13,14)$$

Solution:

Given,

$$G(X,Z,Y,W) = \Sigma_m(0,2,6,7,8,10,12,13,14)$$
$$= \Pi_M(1,3,4,5,9,11,15)$$

i) Simplified SOP form:

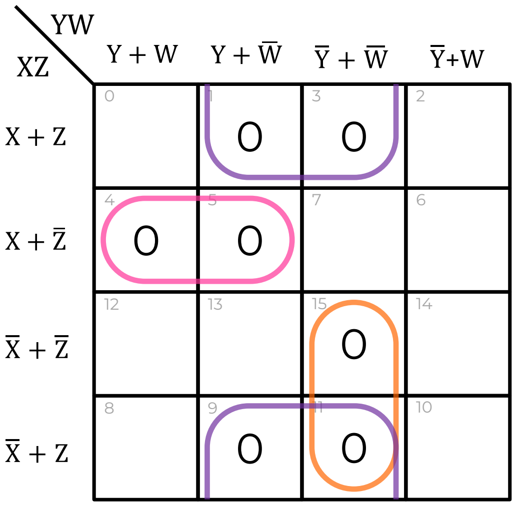


$$G(X,Z,Y,W) = \bar{Z}\bar{W} + \bar{X}ZY + XZ\bar{Y} + X\bar{W}$$

Here, Literals $= 10$
Number of terms $= 4$
Distinct compliments $= 4$

∴ Gate cost in SOP, $G_{SOP} = 10 + 4 + 4 = 18$

ii) Simplified POS form:



$$G(X,Z,Y,W) = (Z + \bar{W})(X + \bar{Z} + Y)(\bar{X} + \bar{Y} + \bar{W})$$

Here, Literals $= 8$
Number of terms $= 3$
Distinct compliments $= 4$

∴ Gate cost in POS, $G_{POS} = 8 + 3 + 4 = 15$

iii) Since $G_{POS} < G_{SOP}$,
∴ Implementing gate in minimized POS will be easier.

6. Your best friend's birthday is coming up in two weeks and you want to gift him something special! You want to design a box that would open its lid and dispense chocolates to your friend for the next two weeks. Knowing DLD, you figured you could make a circuit that would control the box's lid. However, you want to give him chocolates on weekdays only, on weekends the box should not open.

Now, design a combinational circuit that opens the box on weekdays for two weeks. Consider that the week starts on Saturday, and Thursday and Friday (your friend is also in UIU!) are the weekend. **Start your cases by mapping binary 0 to Saturday and continue for 14 days.** For all unmapped cases, consider "don't care" as the output. A table with sample input-output cases is given below for your reference.

Input	Case	Output	Reason
0000	1st Saturday	1	Weekday, so the box should open.
0001	1st Sunday	1	
0101	1st Thursday	0	Weekend, so the box shouldn't open.
0111	2nd Saturday	1	Weekday, so the box should open.
1111	-	X	It is not needed for the mapping

In your design process, show the following steps:

(i) Show the entire truth table.

(ii) Find the simplified expression for the output bits in the Product-of-Sum (POS) form using k-map.

(iii) Draw the circuit diagram using basic gates.

Solution:

(i) The truth table has been shown below:

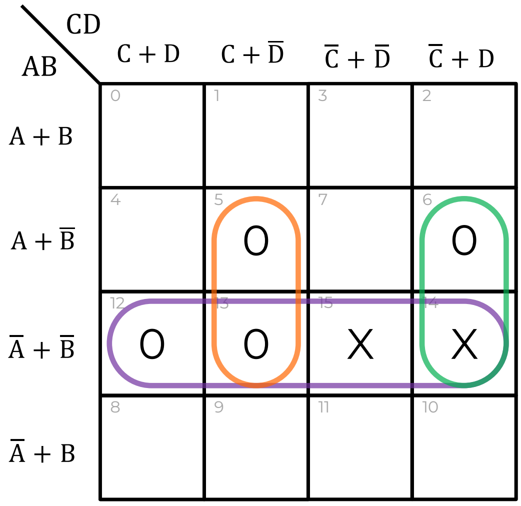
Index	A	B	C	D	F
0	0	0	0	0	1
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	X

15	1	1	1	1	X

(ii) From table of (i),

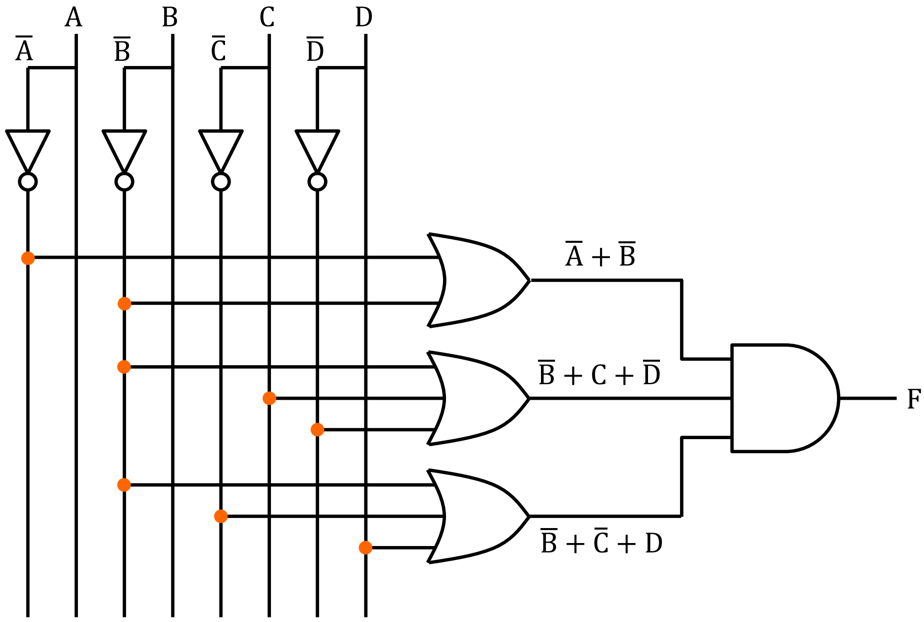
$$F(A,B,C,D) = \Pi_M(5,6,12,13) + \Sigma_d(14,15)$$

Simplified expression in POS,



$$F(A,B,C,D) = (\bar{A} + \bar{B})(\bar{B} + C + \bar{D})(\bar{B} + \bar{C} + D)$$

(iii) The circuit diagram has been drawn below:



Summer 2023

1. A) Find the value of the radix r for this statement where $x = 2r + 3$.

$$(234)_r + (12)_x = (66)_x$$

Solution:

Given,

$$(234)_r + (12)_x = (66)_x$$
or, $\left(2 \times r^2 + 3 \times r^1 + 4 \times r^0\right) + (1 \times x^1 + 2 \times x^0) = (6 \times x^1 + 6 \times x^0)$
or, $\left(2r^2 + 3r + 4 \times 1\right) + (x + 2 \times 1) = (6x + 6 \times 1)$
or, $2r^2 + 3r + x + 6 - 6x - 6 = 0$
or, $2r^2 + 3r - 5x = 0$
or, $2r^2 + 3r - 5(2r + 3) = 0$
or, $2r^2 + 3r - 10r - 15 = 0$
or, $2r^2 - 7r - 15 = 0$
$\therefore r = -1.5$ or, $r = 5$

Since the radix of a number system cannot be negative,
$\therefore$ The radix, $r = 5$.

1. B) Using Boolean algebra convert the following function into Sum-of-Products (SOP) and Product-of-Sum (POS)

$$\bar{X} + X(X + \bar{Y})(XY + \bar{Z})$$

Solution:

Simplifying the expression,

$$\begin{aligned}
\bar{X} + X(X + \bar{Y})(XY + \bar{Z}) &= \bar{X} + (XX + X\bar{Y})(XY + \bar{Z}) \\
&= \bar{X} + (X + X\bar{Y})(XY + \bar{Z}) \quad [\because XX = X] \\
&= \bar{X} + (X)(XY + \bar{Z}) \quad [\because X + X\bar{Y} = X] \\
&= \bar{X} + XXY + X\bar{Z} \\
&= \bar{X} + XY + X\bar{Z} \quad [\because XX = X] \\
&= \bar{X}YZ + \bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + XYZ + XY\bar{Z} + X\bar{Y}\bar{Z} \\
&= \Sigma_m(3,2,1,0,7,6,4) \\
&= \Pi_M(5)
\end{aligned}$$

$\therefore$ Sum of products, SOP $= \Sigma_m(0,1,2,3,4,6,7)$
$= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$

$\therefore$ Product of Sums, POS $= \Pi_M(5)$
$= (\bar{X} + Y + \bar{X})$

2. A) Perform BCD addition between the two numbers $(11101100)_2$ and $(10100100)_2$ using their BCD representations. You need to show the detailed steps of number system conversion if needed.

Solution:

Here,

$$
\begin{aligned}
(11101100)_2 &= 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
&= 128 + 64 + 32 + 0 + 8 + 4 + 0 + 0 \\
&= (236)_{10}
\end{aligned}
$$

$\therefore (11101100)_2 = (236)_{10}$

$$
\begin{aligned}
(10100100)_2 &= 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 \\
&= 128 + 0 + 32 + 0 + 0 + 4 + 0 + 0 \\
&= (164)_{10}
\end{aligned}
$$

$\therefore (11101100)_2 = (164)_{10}$

Now,

$(236)_{10} = (0010\ \ 0011\ \ 0110)_{BCD}$
$(164)_{10} = (0001\ \ 0110\ \ 0100)_{BCD}$

$$
\begin{array}{r}
\\ \\ \\ 236 \\ +\,164 \\ \hline 400
\end{array}
\qquad
\begin{array}{r}
1 \\ 0010 \\ +\,0001 \\ \hline 0100 \\ \\ \\ \\ 0100 \\ (4)
\end{array}
\quad
\begin{array}{r}
1 \\ 0011 \\ +\,0110 \\ \hline 1010 \\ 110 \\ \hline 1\ \ 0000 \\ \\ 0000 \\ (0)
\end{array}
\quad
\begin{array}{r}
\\ 0110 \\ +\,0100 \\ \hline 1010 \\ 110 \\ \hline 1\ \ 0000 \\ \\ 0000 \\ (0)
\end{array}
$$

2. B) Prove that $AB + B\bar{C}\bar{D} + \bar{A}BC + \bar{C}D = B + \bar{C}D$

Solution:

$$
\begin{aligned}
AB + B\bar{C}\bar{D} + \bar{A}BC + \bar{C}D &= AB + \bar{A}BC + B\bar{C}\bar{D} + \bar{C}D \\
&= B(A + \bar{A}C) + \bar{C}(B\bar{D} + D) \\
&= B(A + C) + \bar{C}(B + D) \qquad \left[\begin{aligned} &\because A + \bar{A}C = A + C \\ &\because D + B\bar{D} = B + D \end{aligned}\right] \\
&= AB + BC + B\bar{C} + \bar{C}D \\
&= AB + B(C + \bar{C}) + \bar{C}D \\
&= AB + B + \bar{C}D \qquad [\because C + \bar{C} = 1] \\
&= B + \bar{C}D \qquad [\because B + AB = B]
\end{aligned}
$$

$\therefore AB + B\bar{C}\bar{D} + \bar{A}BC + \bar{C}D = B + \bar{C}D$ (Proved)

3. Find the optimized product-of-sums (POS) of the following function considering don't care conditions. In your solution, you have to show **(i)** all prime implicants, **(ii)** essential prime implicants, and **(iii)** apply the selection rule.

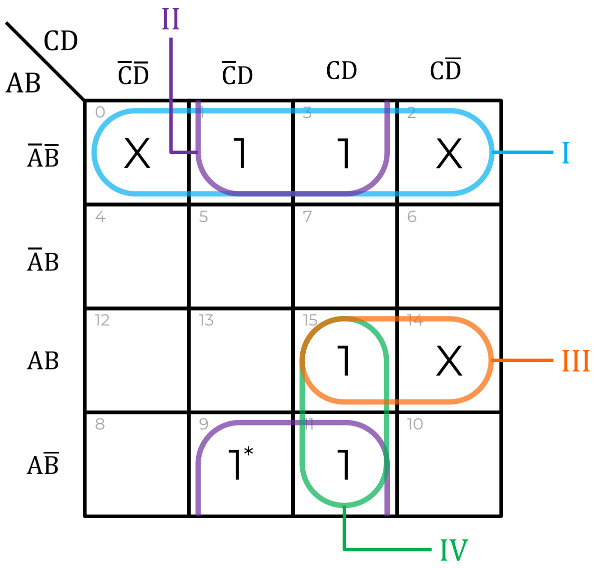
$$F(A, B, C, D) = \Sigma_m(1,3,9,11,15) + \Sigma_d(0,2,14)$$

Solution:

Given,

$$F(A,B,C,D) = \Sigma_m(1,3,9,11,15) + \Sigma_d(0,2,14)$$

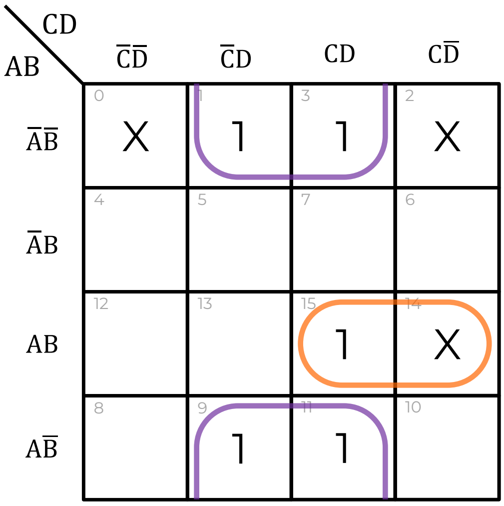
Now,



i) **<u>Prime Implicants, PI:</u>**
I, II, III, IV $(\bar{A}\bar{B}, \bar{B}D, ABC, ACD)$

ii) **<u>Essential Prime Implicants, EPI:</u>**
II $(\bar{A}\bar{D}, \bar{B}D)$

iii) Applying Selection Rule:



$$F(A,B,C,D) = \bar{B}D + ABC$$

4. Optimize the following Boolean function F in
i) Simplified sum-of-products (SOP) and
ii) Simplified product-of-sums (POS) form.
iii) Between minimized SOP and POS, which one will be easy to implement and why?

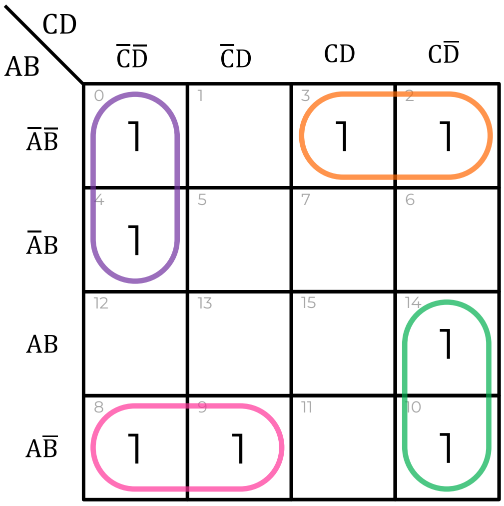
$$F(A,B,C,D) = \Sigma_m(0,2,3,4,8,9,10,14)$$

Solution:

Given,

$$F(A,B,C,D) = \Sigma_m(0,2,3,4,8,9,10,14)$$
$$= \Pi_M(1,5,6,7,11,12,13,15)$$

i) Simplified SOP form:

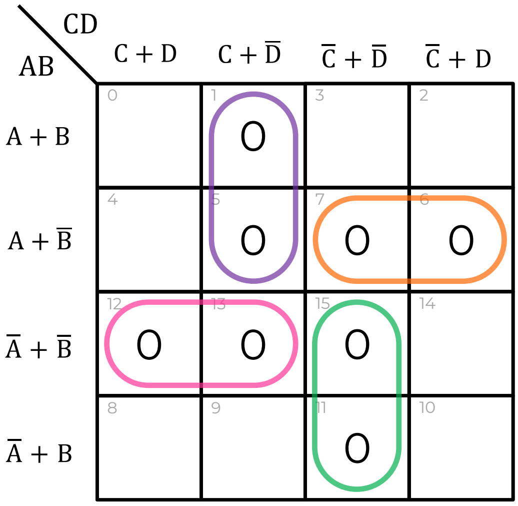


$$F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + AC\bar{D}$$

Here, Literals = 12
Number of terms = 4
Distinct compliments = 4

∴ Gate cost in SOP, $G_{SOP} = 12 + 4 + 4 = 20$

ii) Simplified POS form:



$$F(A,B,C,D) = (A + C + \bar{D})(A + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + \bar{C} + \bar{D})$$

Here, Literals = 12
Number of terms = 4
Distinct compliments = 4

∴ Gate cost in POS, $G_{POS} = 12 + 4 + 4 = 20$

iii) Since $G_{POS} = G_{SOP}$,
∴ Implementing both minimized SOP and POS has the same difficulty.

5. Optimize the following function using K-map. You have to show the minimized sum-of-product (SOP) form.

$$F(A,B,C,D) = (A' + B).(A' + C').(A' + B' + D).(A + C + D).(C + D).(A + C' + D)$$

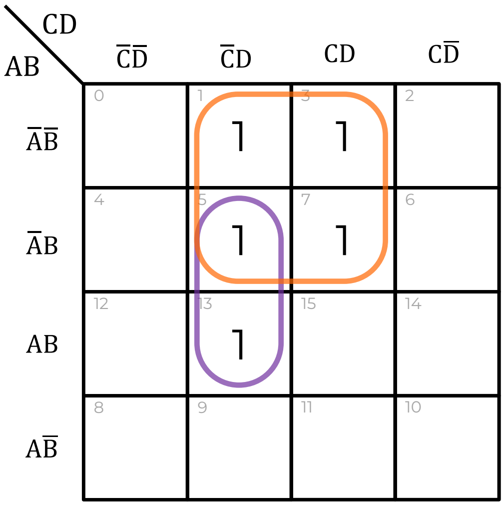
Solution:

Given,

$$\begin{aligned}F(A,B,C,D) &= (A' + B)(A' + C')(A' + B' + D)(A + C + D)(C + D)(A + C' + D)\\ &= (A' + B + C + D)(A' + B + C + D')(A' + B + C' + D)(A' + B + C' + D')(A' \\ &\quad + B' + C' + D)(A' + B' + C' + D')(A' + B' + C + D)(A + B + C + D)(A + B' + C \\ &\quad + D)(A + B + C' + D)(A + B' + C' + D)\\ &= \Pi_M(8,9,10,11,14,15,12,0,4,2,6)\\ &= \Pi_M(0,2,4,6,8,9,10,11,12,14,15)\\ &= \Sigma_m(1,3,5,7,13)\end{aligned}$$

Now,

For simplified SOP,



$$F(A,B,C,D) = B\bar{C}D + \bar{A}D$$

6. Design a system that takes decimal numbers in *Excess3* encoding and determines if that decimal number is even (output will be '1') or odd (output will be '0'). Note that zero is an even number. Following are some example & inputs corresponding outputs.

Input: Decimal number in *Excess3* encoding	Output
0110	0
1001	1
0011	1
0010	'x'

(i) Show the entire truth table and write it in SOP (sum of product) form.

(ii) Simplify the expression form (i) and draw the circuit diagram using basic gates.

Solution:

(i) The truth table has been shown below:

Index	A	B	C	D	F
0	0	0	0	0	X
1	0	0	0	1	X
2	0	0	1	0	X
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	X
14	1	1	1	0	X
15	1	1	1	1	X

Function in SOP form:

$$F(A,B,C,D) = \Sigma_m(3,5,7,9,11) + \Sigma_m(0,1,2,13,14,15)$$
$$= \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}CD + \Sigma_m(0,1,2,13,14,15)$$

(ii) From (i),

$$F(A,B,C,D) = \Sigma_m(3,5,7,9,11) + \Sigma_m(0,1,2,13,14,15)$$

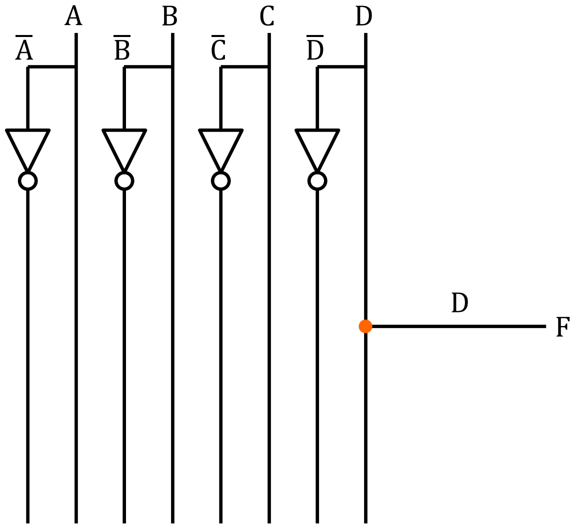
Simplified expression in SOP,

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: X	1: X	3: 1	2: X
$\bar{A}B$	4:	5: 1	7: 1	6:
AB	12:	13: X	15: X	14: X
$A\bar{B}$	8:	9: 1	11: 1	10:

$$F(A,B,C,D) = D$$

Now,

The circuit diagram has been drawn below:


A
B
C
D
$\overline{A}$
$\overline{B}$
$\overline{C}$
$\overline{D}$
D
F

1. a) Determine the radix r from the following equation: $(405)_r = (261)_{10}$

Solution:

Given,

$$(405)_r = (261)_{10}$$
$$\text{or, } 4 \times r^2 + 0 \times r^1 + 5 \times r^0 = 2 \times 10^2 + 6 \times 10^1 + 1 \times 10^0$$
$$\text{or, } 4r^2 + 0 + 5 \times 1 = 261$$
$$\text{or, } 4r^2 + 5 = 261$$
$$\text{or, } 4r^2 = 261 - 5$$
$$\text{or, } r^2 = \frac{256}{4}$$
$$\text{or, } r = \sqrt{64}$$
$$\therefore r = 8$$

$\therefore$ The radix, $r = 8$.

1. b) Encode the following numbers $(157)_8$ and $(234)_8$ to BCD binary format and perform BCD addition.

Solution:

Here,

$$(157)_8 = 1 \times 8^2 + 5 \times 8^1 + 7 \times 8^0 = 64 + 40 + 7 = (111)_{10}$$

$$(234)_8 = 2 \times 8^2 + 3 \times 8^1 + 4 \times 8^0 = 128 + 24 + 4 = (156)_{10}$$

$\therefore (157)_8 = (111)_{10}$

$\therefore (234)_8 = (156)_{10}$

Now,

$$(111)_{10} = (0001\ \ 0001\ \ 0001)_{BCD}$$
$$(156)_{10} = (0001\ \ 0101\ \ 0110)_{BCD}$$

$$\begin{array}{r} 111 \\ +\,156 \\ \hline 267 \end{array} \qquad \begin{array}{r} 0001 \\ +\,0001 \\ \hline 0010 \end{array} \qquad \begin{array}{r} 0001 \\ +\,0101 \\ \hline 0110 \end{array} \qquad \begin{array}{r} 0001 \\ +\,0110 \\ \hline 0111 \end{array}$$

$$\begin{array}{ccc} 0010 & 0110 & 0111 \\ (2) & (6) & (7) \end{array}$$

2. a) Prove the identity of the following Boolean equation, using algebraic manipulation:

$$\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}$$

Solution:

$$
\begin{aligned}
\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} &= \bar{X}\bar{Y} + \bar{Y}Z + XZ + XYZ + XY\bar{Z} + Y\bar{Z} \qquad [\because XY = XYZ + XY\bar{Z}] \\
&= \bar{X}\bar{Y} + Z(\bar{Y} + X + XY) + \bar{Z}(XY + Y) \\
&= \bar{X}\bar{Y} + Z(\bar{Y} + X + XY) + \bar{Z}(Y + YX) \\
&= \bar{X}\bar{Y} + Z(\bar{Y} + X) + \bar{Z}(Y) \qquad \begin{bmatrix} \because X + XY = X \\ \because Y + YX = Y \end{bmatrix} \\
&= \bar{X}\bar{Y} + \bar{Y}Z + XZ + Y\bar{Z} \\
&= \bar{X}\bar{Y} + X\bar{Y}Z + \bar{X}\bar{Y}Z + XZ + Y\bar{Z} \qquad [\because \bar{Y}Z = X\bar{Y}Z + \bar{X}\bar{Y}Z] \\
&= \bar{X}\bar{Y} + \bar{X}\bar{Y}Z + X\bar{Y}Z + XZ + Y\bar{Z} \\
&= \bar{X}(\bar{Y} + \bar{Y}Z) + X\bar{Y}Z + XZ + Y\bar{Z} \\
&= \bar{X}(\bar{Y} + \bar{Y}Z) + X(\bar{Y}Z + Z) + Y\bar{Z} \\
&= \bar{X}(\bar{Y} + \bar{Y}Z) + X(Z + Z\bar{Y}) + Y\bar{Z} \\
&= \bar{X}(\bar{Y}) + X(Z) + Y\bar{Z} \qquad \begin{bmatrix} \because \bar{Y} + \bar{Y}Z = \bar{Y} \\ \because Z + Z\bar{Y} = Z \end{bmatrix} \\
&= \bar{X}\bar{Y} + XZ + Y\bar{Z}
\end{aligned}
$$

$\therefore \bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}$ (Proved)

2. b) Prove that the dual of $A\bar{B} + \bar{A}D$ is also its complement.

Solution:

Let,

$F = A\bar{B} + \bar{A}B$

Now,

$$
\begin{aligned}
\text{Dual of } F, F_D &= (A + \bar{B})(\bar{A} + B) \\
&= (A \cdot \bar{A} + A \cdot B + \bar{A} \cdot \bar{B} + B \cdot \bar{B}) \\
&= (0 + AB + \bar{A}\bar{B} + 0) \\
&= AB + \bar{A}\bar{B}
\end{aligned}
$$

$$
\begin{aligned}
\text{Complement of } F, \bar{F} &= \overline{A\bar{B} + \bar{A}B} \\
&= \overline{A\bar{B}} \cdot \overline{\bar{A}B} \\
&= \left(\bar{A} + \bar{\bar{B}}\right) \cdot \left(\bar{\bar{A}} + \bar{B}\right) \\
&= (\bar{A} + B)(A + \bar{B}) \\
&= (A \cdot \bar{A} + \bar{A} \cdot \bar{B} + A \cdot B + B \cdot \bar{B}) \\
&= (0 + \bar{A}\bar{B} + AB + 0) \\
&= AB + \bar{A}\bar{B}
\end{aligned}
$$

Since $F_D = \bar{F}$,

$\therefore$ Dual of $A\bar{B} + \bar{A}B$ is also its complement. (Proved)

3. a) Convert the following expressions into canonical sum-of-products and canonical product-of sums forms:

$$\bar{X} + X(X + \bar{Y})(Y + \bar{Z})$$

Solution:

Simplifying the expression,

$$
\begin{aligned}
\bar{X} + X(X+\bar{Y})(Y+\bar{Z}) &= \bar{X} + (XX + X\bar{Y})(Y+\bar{Z}) \\
&= \bar{X} + (X + X\bar{Y})(Y+\bar{Z}) \quad [\because XX = X] \\
&= \bar{X} + (X)(Y+\bar{Z}) \quad [\because X + X\bar{Y} = X] \\
&= \bar{X} + XY + X\bar{Z} \\
&= \bar{X}YZ + \bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + XYZ + XY\bar{Z} + X\bar{Y}\bar{Z} \\
&= \Sigma_m(3,2,1,0,7,6,4) \\
&= \Pi_M(5)
\end{aligned}
$$

$\therefore$ Canonical SOP $= \Sigma_m(0,1,2,3,4,6,7)$

$= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$

$\therefore$ Canonical POS $= \Pi_M(5)$

$= (\bar{X} + Y + \bar{Z})$

3. b) Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C) \cdot (\bar{A} + B) \cdot (B + C) = B \cdot C$$

Solution:

Given, $A \cdot B = 0$

$A + B = 0$

Now,

$$
\begin{aligned}
(A + C) \cdot (\bar{A} + B) \cdot (B + C) &= (A \cdot \bar{A} + A \cdot B + \bar{A} \cdot C + B \cdot C)(B + C) \\
&= (0 + 0 + \bar{A}C + BC)(B + C) \quad \begin{bmatrix} \because A \cdot \bar{A} = 0 \\ \because A \cdot B = 0 \end{bmatrix} \\
&= (\bar{A}C + BC)(B + C) \\
&= \bar{A}BC + \bar{A}C + BC + BC \\
&= \bar{A}BC + \bar{A}C + BC \quad [\because BC + BC = BC] \\
&= C(\bar{A}B + \bar{A} + B) \\
&= C(\bar{A} + B) \quad [\because \bar{A} + \bar{A}B = \bar{A}] \\
&= C(\bar{A} + B) \cdot 1 \\
&= C(\bar{A} + B)(A + B) \quad [\because 1 = A + B] \\
&= C(A \cdot \bar{A} + \bar{A} \cdot B + A \cdot B + B \cdot B) \\
&= C(0 + \bar{A}B + 0 + B) \quad \begin{bmatrix} \because A \cdot \bar{A} = 0 \\ \because A \cdot B = 0 \end{bmatrix} \\
&= C(B + B\bar{A}) \\
&= C(B) \quad [\because B + B\bar{A} = B] \\
&= B \cdot C
\end{aligned}
$$

$\therefore (A + C) \cdot (\bar{A} + B) \cdot (B + C) = B \cdot C$ (Proved)

4. You have to design a combinational circuit that will take a 4-bit binary number as input and check whether the bit pattern is palindrome or not. The output will be 1 if the input bit pattern is palindrome, otherwise output will be 0. (A pattern is palindrome if the reverse of that pattern is similar to the original pattern.)

You have to:

i) Show the truth table.

ii) Find the simplified expression for the output bit in Product-of-Sum (POS) form.
iii) Draw the circuit diagram using basic gates.

Few example inputs and outputs are given below:

Input	Output	Reason
0000	1	Reverse pattern: 0000 = Input
0001	0	Reverse pattern: 1000 ≠ Input
0110	1	Reverse pattern: 0110 = Input
1101	0	Reverse pattern: 1011 ≠ Input

Solution:

(i) The truth table has been shown below:

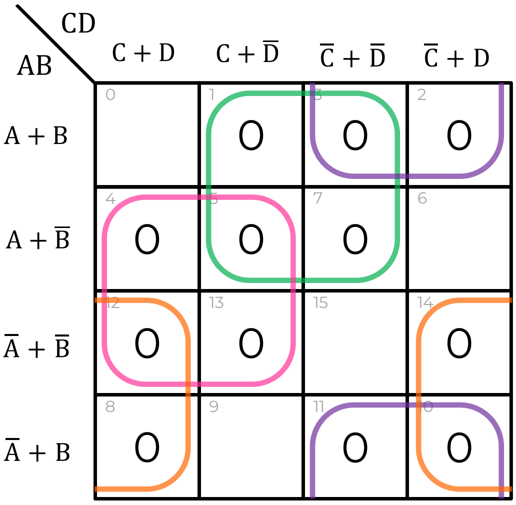
Index	A	B	C	D	F
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	0
13	1	1	0	1	0
14	1	1	1	0	0
15	1	1	1	1	1

(ii) From table of (i),

$$F(A,B,C,D) = \Pi_M(1,2,3,4,5,7,8,10,11,12,13,14)$$

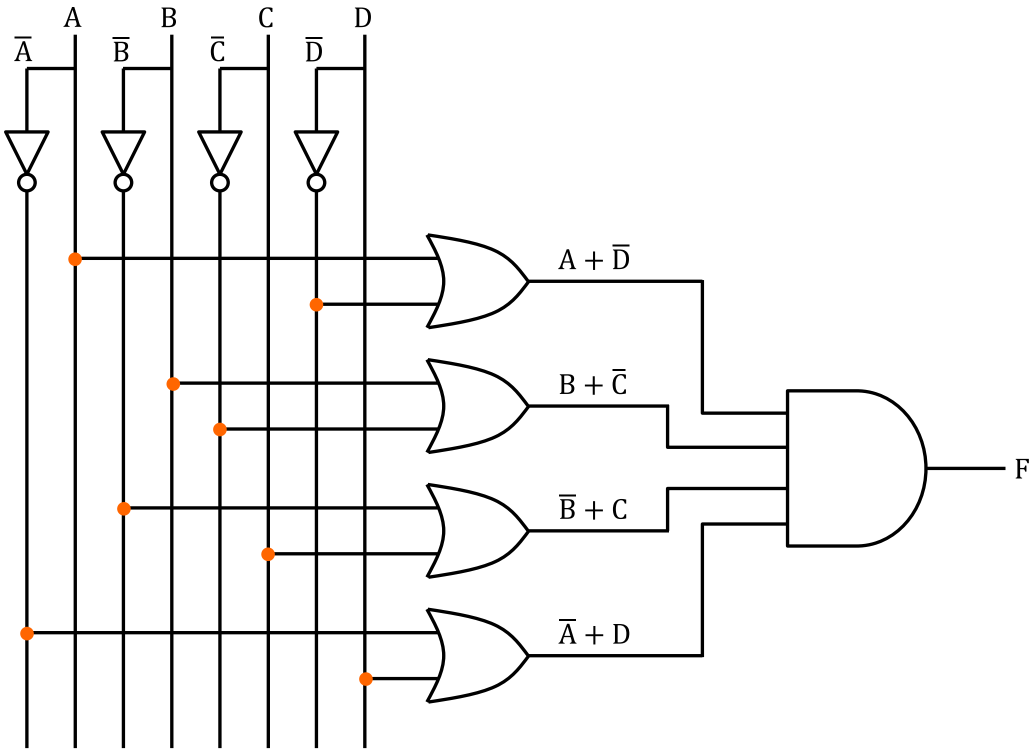
Simplified expression in POS,

[P.T.O]



$$F(A,B,C,D) = (A+\bar{D})(B+\bar{C})(\bar{B}+C)(\bar{A}+D)$$

(iii) The circuit diagram has been drawn below:



5. You have to design a combinational circuit that will take a 4-bit binary number as input and detect which bit is major in count. If the number of 0's is greater than the number of 1's, then the output will be 0. If the number of 1's is greater than the number of 0's, then the output will be 1. For all other cases, consider don't care as output.

You have to:

i) Show the truth table.

ii) Find the simplified expression for the output bit in Sum-of-Product (SOP) form.

iii) Draw the circuit diagram using basic gates.

Few example inputs and outputs are given below:

Input	Output	Reason
0001	0	Number of 0's: 3 > Number of 1's: 1

0101	x	Number of 0's: 2 = Number of 1's: 2
1010	x	Number of 0's: 2 = Number of 1's: 2
1101	1	Number of 0's: 1 < Number of 1's: 3

Solution:

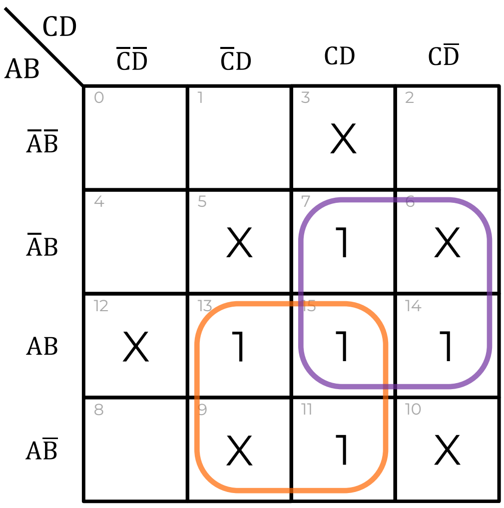
(i) The truth table has been shown below:

Index	A	B	C	D	F
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	x
4	0	1	0	0	0
5	0	1	0	1	x
6	0	1	1	0	x
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	x
10	1	0	1	0	x
11	1	0	1	1	1
12	1	1	0	0	x
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

(ii) From table of (i),

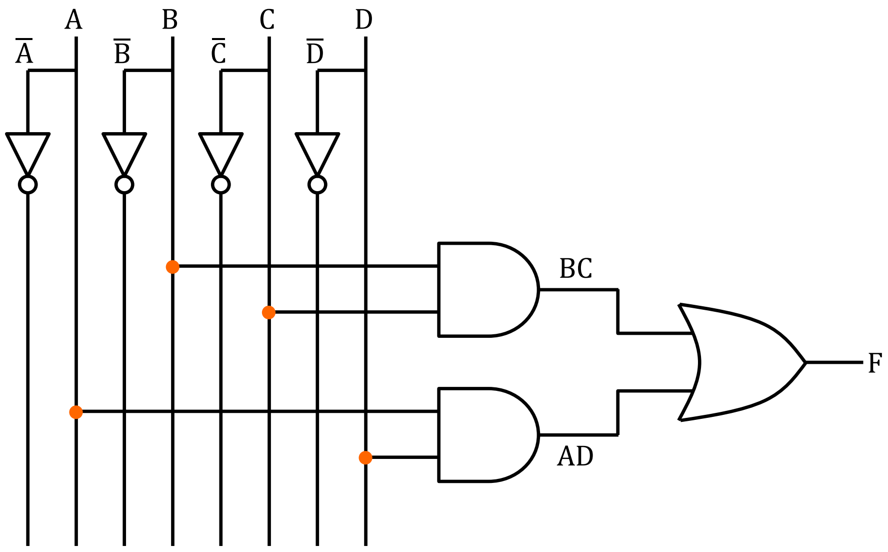
$$F(A,B,C,D) = \Sigma_m(7,11,13,14,15) + \Sigma_d(3,5,6,9,10,12)$$

Simplified expression in SOP,



$$F(A,B,C,D) = BC + AD$$

(iii) The circuit diagram has been drawn below:



6. Find the optimized sum-of-products (SOP) of the following function considering don't care conditions. In your solution, you have to show **(i)** all prime implicants, **(ii)** essential prime implicants, and **(iii)** apply the selection rule.

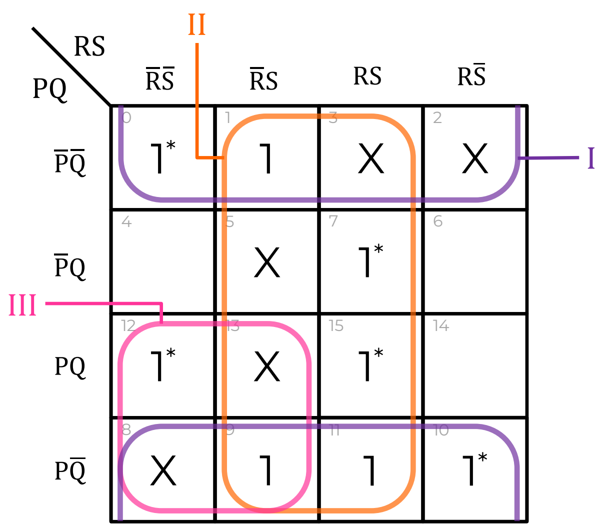
$$F(P,Q,R,S) = \Sigma_m(0,1,7,9,10,11,12,15) + \Sigma_d(2,3,5,8,13)$$

Solution:

Given,

$$F(P,Q,R,S) = \Sigma_m(0,1,7,9,10,11,12,15) + \Sigma_d(2,3,5,8,13)$$

Now,



i) **Prime Implicants, PI:**

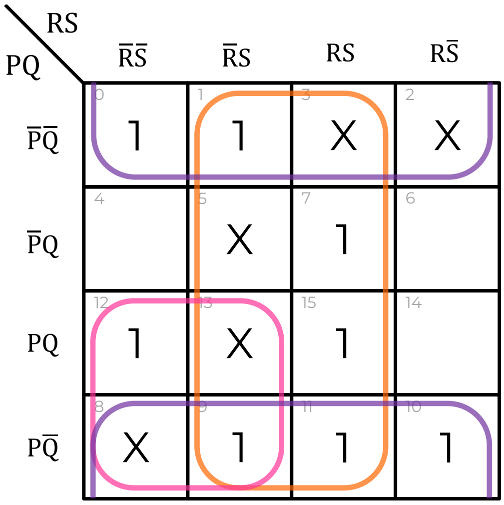
I, II, III $(\bar{Q}, S, P\bar{R})$

ii) **Essential Prime Implicants, EPI:**

I, II, III $(\bar{Q}, S, P\bar{R})$

iii) Applying Selection Rule:

[P.T.O]



$$F(P,Q,R,S) = \bar{Q} + S + P\bar{R}$$

7. Optimize the following function in **i)** simplified sum-of-products (SOP) and **ii)** simplified product-of-sums (POS) form. Between simplified SOP and POS, which one should you implement? Justify your answer.

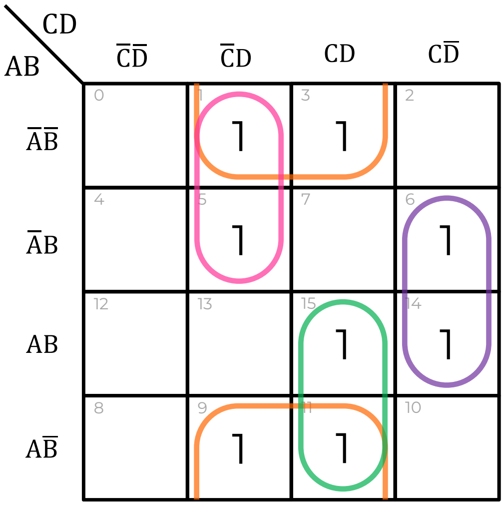
$$F(A,B,C,D) = \Pi_M(0,2,4,7,8,10,12,13)$$

Solution:

Given,

$$F(A,B,C,D) = \Pi_M(0,2,4,7,8,10,12,13)$$
$$= \Sigma_m(1,3,5,6,9,11,14,15)$$

i) Simplified SOP form:



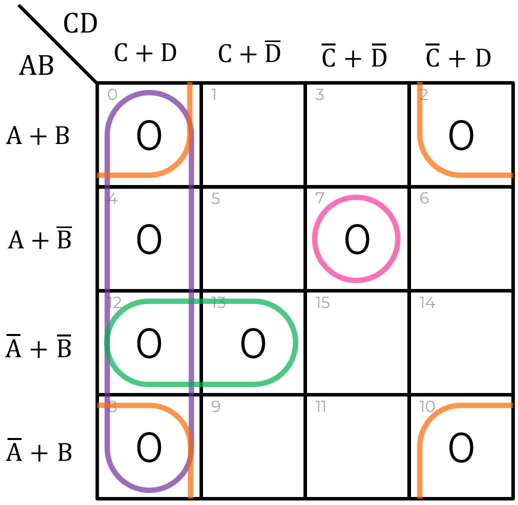
$$F(A,B,C,D) = \bar{A}\bar{C}D + \bar{B}D + BC\bar{D} + ACD$$

Here, Literals $= 11$
Number of terms $= 4$
Distinct compliments $= 4$

$\therefore$ Gate cost in SOP, $G_{SOP} = 11 + 4 + 4 = 19$

ii) Simplified POS form:

[P.T.O]



$$F(A,B,C,D) = (B+D)(C+D)(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+\bar{B}+C)$$

Here, Literals $= 11$
Number of terms $= 4$
Distinct compliments $= 4$

∴ Gate cost in POS, $G_{POS} = 11 + 4 + 4 = 19$

iii) Since $G_{POS} = G_{SOP}$,
∴ Both of simplified SOP and simplified POS can be implemented.

8. **Optimize the following function using K-map. You have to show your answer in simplified product-of-sum (POS) as well as simplified sum-of products (SOP) form.**

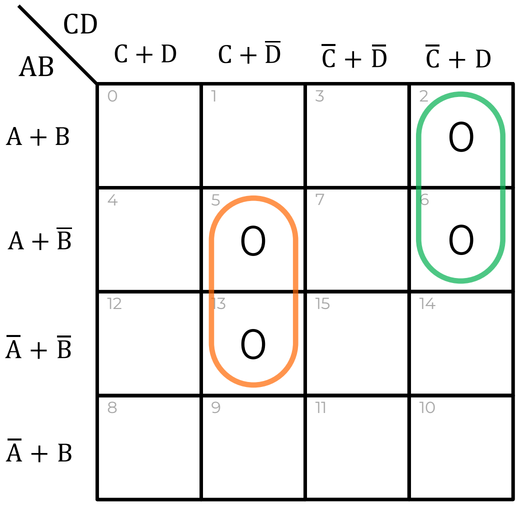
$$(\bar{A}+\bar{B}+C+\bar{D}).(A+B+\bar{C}+D).(A+\bar{B}+C+\bar{D}).(A+\bar{B}+\bar{C}+D)$$

Solution:

Given,

$$\begin{aligned} F(A,B,C,D) &= (\bar{A}+\bar{B}+C+\bar{D})(A+B+\bar{C}+D)(A+\bar{B}+C+\bar{D})(A+\bar{B}+\bar{C}+D) \\ &= \Pi_M(13,2,5,6) \\ &= \Sigma_m(0,1,3,4,7,8,9,10,11,12,14,15) \end{aligned}$$

Now,
For simplified POS,



$$F(A,B,C,D) = (\bar{B}+C+\bar{D})(A+\bar{C}+D)$$

For simplified SOP,

$AB \backslash CD$	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0: 1	1: 1	3: 1	2:
$\bar{A}B$	4: 1	5:	7: 1	6:
AB	12: 1	13:	15: 1	14: 1
$A\bar{B}$	8: 1	9: 1	11: 1	10: 1

$$F(A,B,C,D) = \bar{C}\bar{D} + \bar{B}\bar{C} + CD + AC$$

Fall 2022

1. a) Find the value of the radix r for the statement: $\sqrt{(224)_r} = (13)_r$

Solution:

Given,

$$\sqrt{(224)_r} = (13)_r$$

or, $2 \times r^2 + 2 \times r^1 + 4 \times r^0 = \left(1 \times r^1 + 3 \times r^0\right)^2$

or, $2r^2 + 2r + 4 = (r+3)^2$

or, $2r^2 + 2r + 4 = r^2 + 6r + 9$

or, $2r^2 + 2r + 4 - r^2 - 6r - 9 = 0$

or, $r^2 - 4r - 5 = 0$

or, $r = \sqrt{64}$

$\therefore r = 5$ or, $r = -1$

Since the radix of a number system cannot be negative,

$\therefore$ The radix, $r = 5$.

1. b) Perform BCD addition between the two numbers $(23D)_{16}$ and $(F67)_{16}$ using their BCD representations. You need to show the detailed steps of number system conversion if needed

Solution:

Here,

$$\begin{aligned}(23D)_{16} &= 2 \times 16^2 + 3 \times 16^1 + D \times 16^0 \\ &= 2 \times 256 + 3 \times 16 + 13 \times 1 \\ &= 512 + 48 + 13 \\ &= (573)_{10}\end{aligned}$$

$\therefore (23D)_{16} = (573)_{10}$

$$\begin{aligned}(F67)_{16} &= F \times 16^2 + 6 \times 16^1 + 7 \times 16^0 \\ &= 15 \times 256 + 6 \times 16 + 7 \times 1 \\ &= 3840 + 96 + 7 \\ &= (3943)_{10}\end{aligned}$$

$\therefore (F67)_{16} = (3943)_{10}$

Now,

$(3943)_{10} = (0011\ \ 1001\ \ 0100\ \ 0011)_{BCD}$

$(573)_{10} = (0101\ \ 0111\ \ 0011)_{BCD}$

$$\begin{array}{r}3943 \\ +\ \ 573 \\ \hline 4516\end{array}
\qquad
\begin{array}{cccc}
1 \leftarrow & 1 \leftarrow & & \\
\begin{array}{r}0011 \\ + \\ \hline 0100 \\ \\ \\ \end{array} &
\begin{array}{r}1001 \\ +\,0101 \\ \hline 1111 \\ 110 \\ \hline 1\ \ 0101\end{array} &
\begin{array}{r}0100 \\ +\,0111 \\ \hline 1011 \\ 110 \\ \hline 1\ \ 0001\end{array} &
\begin{array}{r}0011 \\ +\,0011 \\ \hline 0110 \\ \\ \\ \end{array} \\
0100 & 0000 & 0001 & 0110 \\
(4) & (5) & (1) & (6)
\end{array}$$

2. a) Prove the identity of the following Boolean equation, using algebraic manipulation:

$$\overline{W}X(\overline{Z}+\overline{Y}Z)+X(W+\overline{W}YZ)=X$$

Solution:

$$\begin{aligned}
W\overline{X}(\overline{Z}+\overline{Y}Z)+X(W+\overline{W}YZ) &= \overline{W}X(\overline{Z}+Z\overline{Y})+X[W+\overline{W}(YZ)] \\
&= \overline{W}X(\overline{Z}+\overline{Y})+X(W+YZ) \quad \begin{bmatrix} \because \overline{Z}+Z\overline{Y}=\overline{Z}+\overline{Y} \\ \because W+\overline{W}(YZ)=W+YZ \end{bmatrix} \\
&= \overline{W}X\overline{Z}+\overline{W}X\overline{Y}+XW+XYZ \\
&= X(\overline{W}\overline{Z}+\overline{W}\overline{Y}+W+YZ) \\
&= X(\overline{W}\overline{Z}+W+\overline{Y}+YZ) \quad [\because W+\overline{W}\overline{Y}=W+\overline{Y}] \\
&= X(\overline{W}\overline{Z}+W+\overline{Y}+Z) \\
&= X(Z+\overline{Z}\overline{W}+W+\overline{Y}) \\
&= X(Z+\overline{W}+W+\overline{Y}) \quad [\because Z+\overline{Z}\overline{W}=Z+\overline{W}] \\
&= X(Z+1+\overline{Y}) \\
&= X(1) \\
&= X
\end{aligned}$$

$\therefore \overline{W}X(\overline{Z}+\overline{Y}Z)+X(W+\overline{W}YZ)=X$ (Proved)

2. b) Convert the following expression into both canonical SOP & canonical POS forms.

$$F(X,Y,Z)=(XY+Z)(Y+ZX)(X+YZ)$$

Solution:

Simplifying the expression,

$$\begin{aligned}
(XY+Z)(Y+ZX)(X+YZ) &= (X+Z)(Y+Z)(Y+X)(Y+Z)(X+Y)(X+Z) \\
&= (X+Z)(Y+Z)(X+Y) \\
&= (X+Y+Z)(X+\overline{Y}+Z)(\overline{X}+Y+Z)(X+Y+\overline{Z}) \\
&= \Pi_M(0,2,4,1) \\
&= \Sigma_m(3,5,6,7)
\end{aligned}$$

$\therefore$ Canonical SOP $=\Sigma_m(3,5,6,7)$

$=\overline{X}YX+X\overline{Y}Z+XY\overline{Z}+XYZ$

$\therefore$ Canonical POS $=\Pi_M(0,1,2,4)$

$=(X+Y+Z)+(X+Y+\overline{Z})+(X+\overline{Y}+Z)+(\overline{X}+Y+Z)$

3. a) Find the **(i)** dual and **(ii)** complement of the following function.

$$F(X,Y,Z)=X(\overline{Y}Z+Y\overline{Z})+\overline{X}(\overline{Y}+Z)(Y+\overline{Z})$$

Solution:

Given,

$F(X,Y,Z)=X(\overline{Y}Z+Y\overline{Z})+\overline{X}(\overline{Y}+Z)(Y+\overline{Z})$

[P.T.O]

(i) Dual of $F, F_D = [X + (\bar{Y} + Z) \cdot (Y + \bar{Z})][\bar{X} + (\bar{Y} \cdot Z) + (Y \cdot \bar{Z})]$

$$= (X + Y\bar{Y} + \bar{Y}\bar{Z} + YZ + Z\bar{Z})(\bar{X} + \bar{Y}Z + Y\bar{Z})$$
$$= (X + \bar{Y}\bar{Z} + YZ)(\bar{X} + \bar{Y}Z + Y\bar{Z})$$

(Ii) Complement of $F, \bar{F} = \overline{X(\bar{Y}Z + Y\bar{Z}) + \bar{X}(\bar{Y} + Z)(Y + \bar{Z})}$

$$= \left(\overline{X(\bar{Y}Z + Y\bar{Z})}\right) \cdot \left(\overline{\bar{X}(\bar{Y} + Z)(Y + \bar{Z})}\right)$$
$$= \left(\bar{X} + \left(\overline{\bar{Y}Z + Y\bar{Z}}\right)\right)\left(\bar{\bar{X}} + \left(\overline{\bar{Y} + Z}\right) + \left(\overline{Y + \bar{Z}}\right)\right)$$
$$= \left(\bar{X} + \left(\overline{\bar{Y}Z} \cdot \overline{Y\bar{Z}}\right)\right)\left(X + \left(\bar{\bar{Y}} \cdot \bar{Z}\right) + \left(\bar{Y} \cdot \bar{\bar{Z}}\right)\right)$$
$$= \left(\bar{X} + \left(\left(\bar{\bar{Y}} + \bar{Z}\right) \cdot \left(\bar{Y} + \bar{\bar{Z}}\right)\right)\right)\left(X + \left(\bar{\bar{Y}} \cdot \bar{Z}\right) + \left(\bar{Y} \cdot \bar{\bar{Z}}\right)\right)$$
$$= \left(\bar{X} + (Y + \bar{Z})(\bar{Y} + Z)\right)\left(X + (Y\bar{Z}) + (\bar{Y}Z)\right)$$
$$= \left(\bar{X} + (Y\bar{Y} + YZ + \bar{Y}\bar{Z} + Z\bar{Z})\right)(X + Y\bar{Z} + \bar{Y}Z)$$
$$= (\bar{X} + YZ + \bar{Y}\bar{Z})(X + Y\bar{Z} + \bar{Y}Z)$$

3. a) **Convert the following expressions into canonical sum-of-products and canonical product-of sums forms:**

$$\bar{X} + X(X + \bar{Y})(Y + \bar{Z})$$

Solution:

Simplifying the expression,

$$\bar{X} + X(X + \bar{Y})(Y + \bar{Z}) = \bar{X} + (XX + X\bar{Y})(Y + \bar{Z})$$
$$= \bar{X} + (X + X\bar{Y})(Y + \bar{Z}) \quad [\because XX = X]$$
$$= \bar{X} + (X)(Y + \bar{Z}) \quad [\because X + X\bar{Y} = X]$$
$$= \bar{X} + XY + X\bar{Z}$$
$$= \bar{X}YZ + \bar{X}Y\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}\bar{Y}\bar{Z} + XYZ + XY\bar{Z} + X\bar{Y}\bar{Z}$$
$$= \Sigma_m(3,2,1,0,7,6,4)$$
$$= \Pi_M(5)$$

$\therefore$ Canonical SOP $= \Sigma_m(0,1,2,3,4,6,7)$

$$= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}\bar{Z} + XY\bar{Z} + XYZ$$

$\therefore$ Canonical POS $= \Pi_M(5)$

$$= (\bar{X} + Y + \bar{X})$$

3. b) **Simplify the following Boolean Expression (using algebraic manipulation) to an expression containing a minimum number of literals.**

$$F(A, B, C, D) = (AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD)(\overline{AC})$$

Solution:

$$F(A, B, C, D) = (AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD)(\overline{AC})$$
$$= (AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD)(\bar{A} + \bar{C})$$
$$= (AB + \bar{A}\bar{B})(\bar{A}\bar{C}\bar{D} + \bar{A}CD + \bar{C}\bar{C}\bar{D} + C\bar{C}D)$$
$$= (AB + \bar{A}\bar{B})(\bar{A}\bar{C}\bar{D} + \bar{C}\bar{D} + \bar{A}CD + \bar{C}\bar{D} + 0)$$
$$= (AB + \bar{A}\bar{B})(\bar{C}\bar{D}(\bar{A} + 1) + \bar{A}CD + \bar{C}\bar{D})$$

$$= (AB + \bar{A}\bar{B})(\bar{C}\bar{D} + \bar{A}CD + \bar{C}\bar{D})$$
$$= (AB + \bar{A}\bar{B})(\bar{C}\bar{D} + \bar{A}CD)$$
$$= AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + A\bar{A}BCD + \bar{A}\bar{A}\bar{B}CD$$
$$= AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD + 0 \quad [\because A\bar{A} = 0]$$
$$= AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}CD$$

4. Find the optimized sum-of-products (SOP) of the following function considering don't care conditions. In your solution, you have to show **(i)** all prime implicants, **(ii)** essential prime implicants, and **(iii)** apply the selection rule.

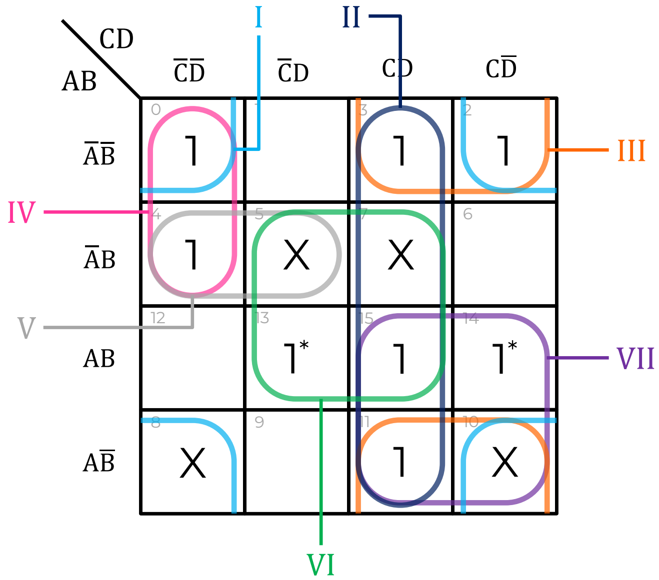
$$F(A, B, C, D) = \Sigma_m(0,2,3,4,8,11,13,14,15) + \Sigma_d(5,7,8,10)$$

Solution:

Given,

$$F(A, B, C, D) = \Sigma_m(0,2,3,4,11,13,14,15) + \Sigma_d(5,7,8,10)$$

Now,

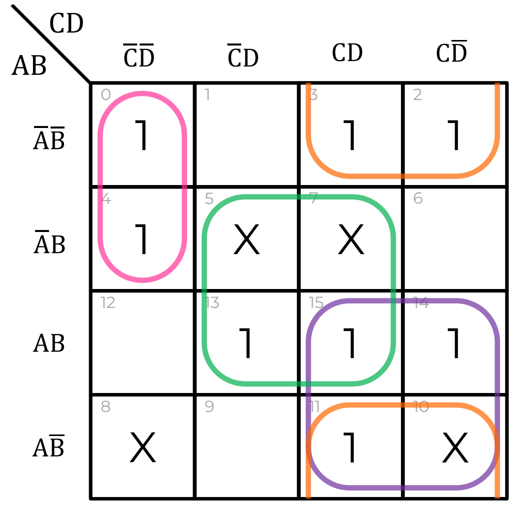


i) <u>**Prime Implicants, PI:**</u>
I, II, III, IV, V, VI, VII $(\bar{B}\bar{D}, CD, \bar{B}C, \bar{A}\bar{C}\bar{D}, \bar{A}B\bar{C}, BD, AC)$

ii) <u>**Essential Prime Implicants, EPI:**</u>
VI, VII (BD, AC)

iii) Applying Selection Rule:

[P.T.O]



$$F(A,B,C,D) = \bar{A}\bar{C}\bar{D} + \bar{B}C + BD + AC$$

5. Optimize the following function in **i)** simplified sum-of-products (SOP) and **ii)** simplified product-of-sums (POS) form. Between simplified SOP and POS, which one should you implement? Justify your answer.

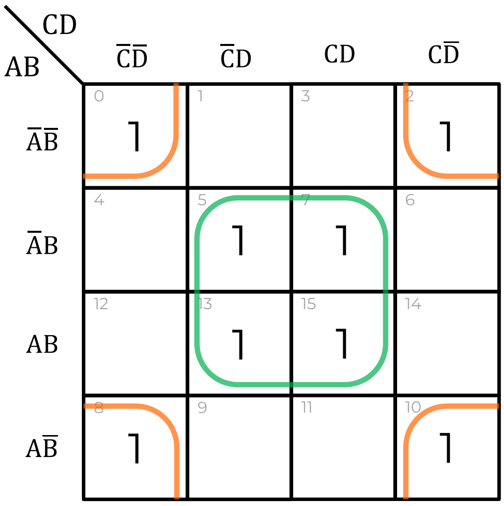
$$F(A,B,C,D) = \Pi_M(1,3,4,6,9,11,12,14)$$

Solution:

Given,

$$F(A,B,C,D) = \Pi_M(1,3,4,6,9,11,12,14)$$
$$= \Sigma_m(0,2,5,7,8,10,13,15)$$

i) Simplified SOP form:



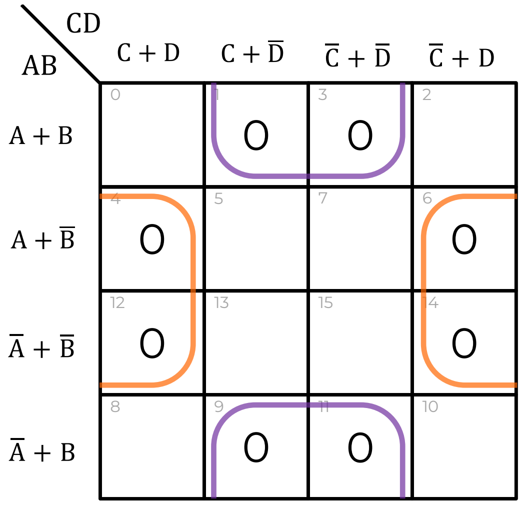
$$F(A,B,C,D) = \bar{B}\bar{D} + BD$$

Here, Literals = 4
Number of terms = 2
Distinct compliments = 2

$\therefore$ Gate cost in SOP, $G_{SOP} = 4 + 2 + 2 = 8$

ii) Simplified POS form:

[P.T.O]



$$F(A,B,C,D) = (B + \bar{D})(\bar{B} + D)$$

Here, Literals = 4
Number of terms = 2
Distinct compliments = 2

∴ Gate cost in POS, $G_{POS} = 4 + 2 + 2 = 8$

iii) Since $G_{POS} = G_{SOP}$,
∴ Both of simplified SOP and simplified POS can be implemented.

6. Optimize the following function using K-map. You have to show your answer in simplified sum-of products (SOP) form.

$$F(A,B,C,D) = (\bar{A} + \bar{B} + \bar{D})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{D})(B + \bar{C} + \bar{D})$$

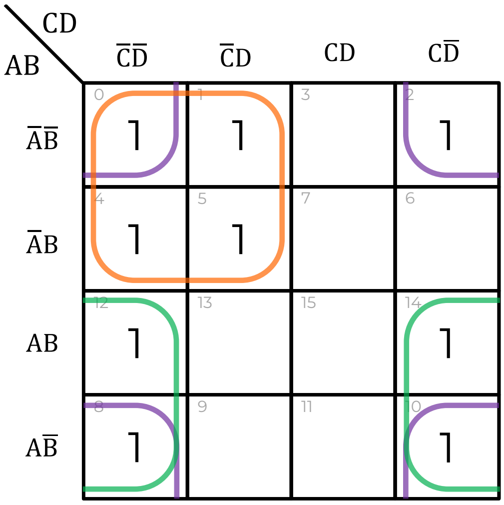
Solution:

Given,

$$\begin{aligned} F(A,B,C,D) &= (\bar{A} + \bar{B} + \bar{D})(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{D})(B + \bar{C} + \bar{D}) \\ &= (\bar{A} + \bar{B} + C + \bar{D})(\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + \bar{B} + \bar{C} + D)(A + \bar{B} + \bar{C} + \bar{D})(\bar{A} + B + C \\ &\quad + \bar{D})(\bar{A} + B + \bar{C} + \bar{D})(A + B + \bar{C} + \bar{D}) \\ &= \Pi_M(13,15,6,7,9,11,3) \\ &= \Sigma_m(0,1,2,4,5,8,10,12,14) \end{aligned}$$

Now,
For simplified SOP,

[P.T.O]



$$F(A,B,C,D) = (\bar{B} + C + \bar{D})(A + \bar{C} + D)$$

4. Design a combinational logic circuit named FCT that will take a 4-bit binary number as an input. If the corresponding decimal of the input is zero or has exactly two factors, the output of the circuit will be HIGH. For the numbers having more than three factors, the output of the circuit will be LOW.

(i) Show the truth table **(ii)** Find the simplified expression for the output bit in Sum-of-Products form **(iii)** Draw the circuit diagram using basic gates.

Few example inputs and outputs are given below:

Input : 0001, Output: 0, Reason: 1 has one factor (1)
Input : 0011, Output: 1, Reason: 3 has two factors (1,3)
Input : 0100, Output: x, Reason: 4 has three factors (1,2,4)
Input : 0110, Output: 0, Reason: 6 has four factors (1,2,3,6)

Solution:

(i) The truth table has been shown below:

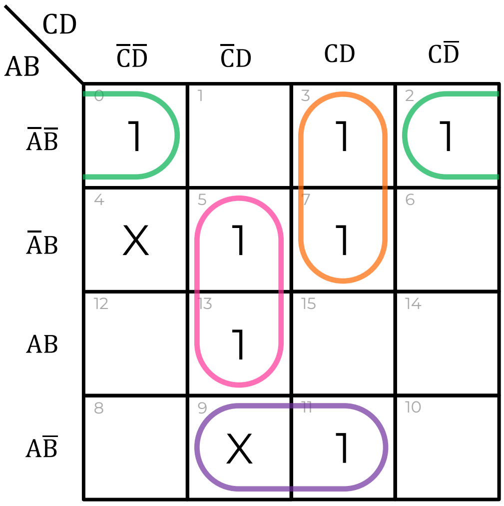
Index	A	B	C	D	F
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	x
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	x
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	0
13	1	1	0	1	1

14	1	1	1	0	0
15	1	1	1	1	0

(ii) From table of (i),

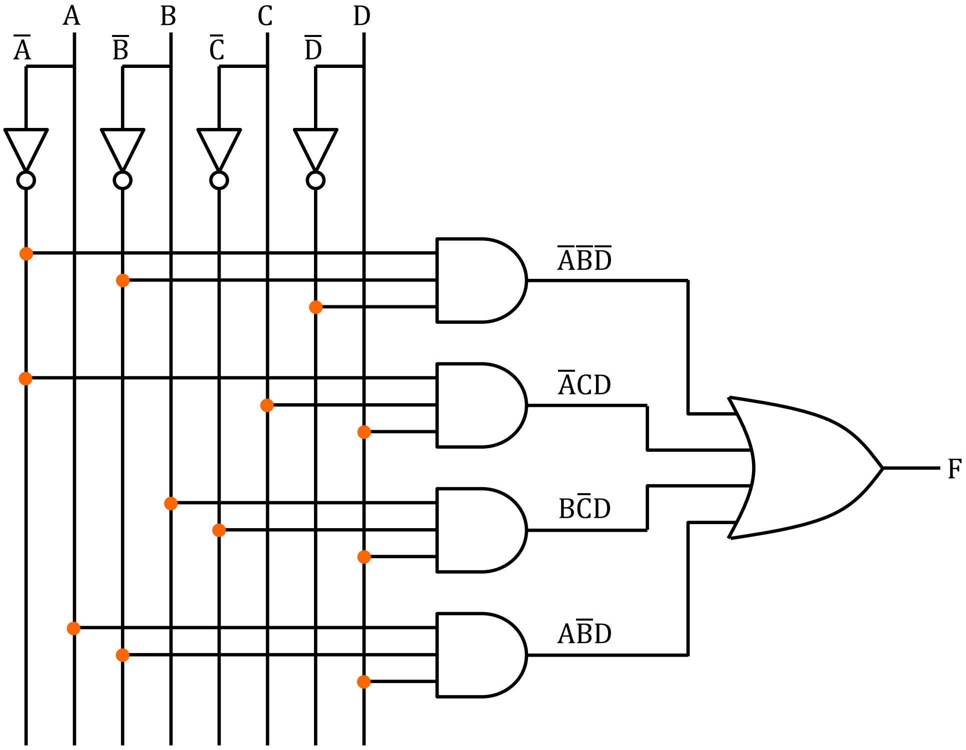
$$F(A,B,C,D) = \Sigma_m(0,2,3,5,7,11,13) + \Sigma_d(4,9)$$

Simplified expression in POS,



$$F(A,B,C,D) = \bar{A}\bar{B}\bar{D} + \bar{A}CD + B\bar{C}D + A\bar{B}D$$

(iii) The circuit diagram has been drawn below:



5. Design a combinational logic circuit named PD that will take a 4-bit binary number as an input. The output will be 1 if the corresponding decimal of the input is

- an even number but not divisible by 5
- divisible by 3 but not divisible by 5

For all other inputs, the output will be 0. **(i)** Show the truth table **(ii)** Find the simplified expression for the output bit in Sum-of-Products form **(iii)** Draw the circuit diagram using basic gates.

Few example inputs and outputs are given below:

Input : 0010, Output: 1, Reason: even number
Input : 0111, Output: 0, Reason: neither even number nor divisible by 3
Input : 1001, Output: 1, Reason: divisible by 3
Input : 1010, Output: 0, Reason: even number but divisible by 5

Solution:

(i) The truth table has been shown below:

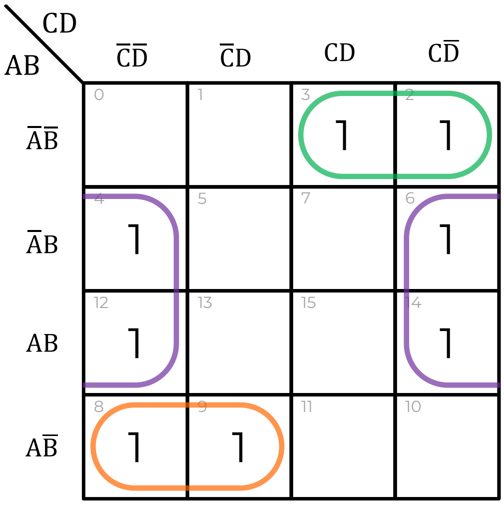
Index	A	B	C	D	F
0	0	0	0	1	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	0
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	0

(ii) From table of (i),

$$F(A,B,C,D) = \Sigma_m(2,3,4,6,8,9,12,14)$$

Simplified expression in SOP,

[P.T.O]



$$F(A,B,C,D) = \bar{A}\bar{B}C + B\bar{D} + A\bar{B}\bar{C}$$

(iii) The circuit diagram has been drawn below: