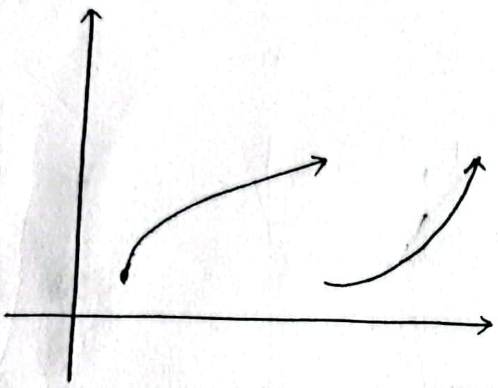
(Course Code → MATH 2183)

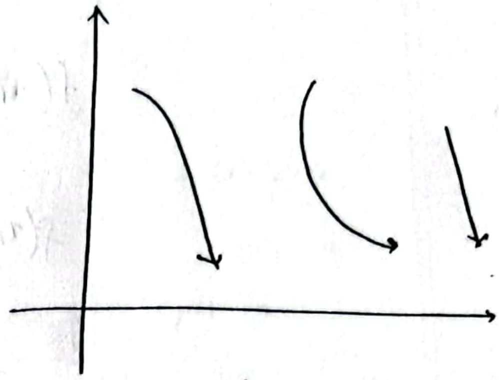
Class → 01

Date : 23.09.2023

(Ch → 4.1)

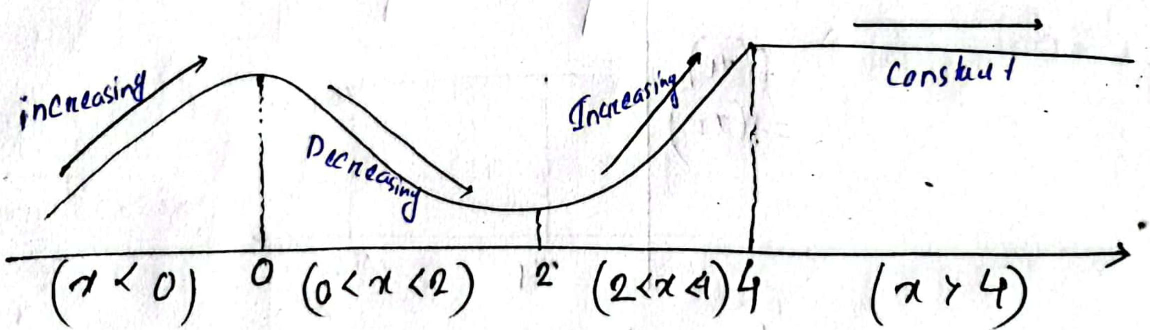
Increasing and Decreasing function:





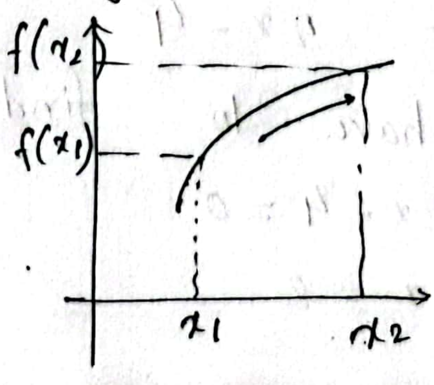
$f' = +ve$
(slope is +ve)

$f' = -ve$
(slope is -ve)



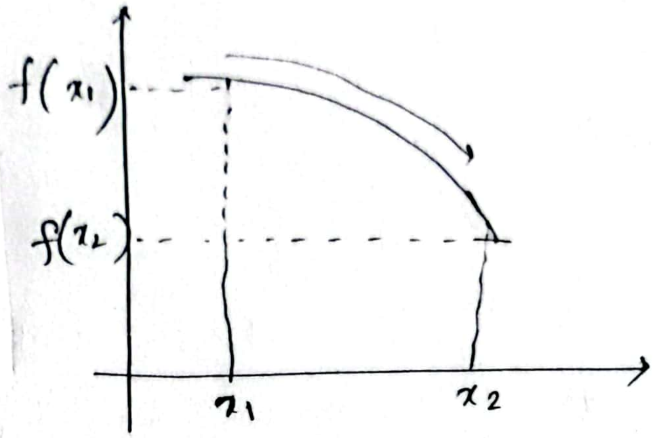
Defn: f is defined on an Interval,
x_1 and x_2 denote points in that interval.

a) f increasing if $f(x_1) < f(x_2)$; $x_1 <$

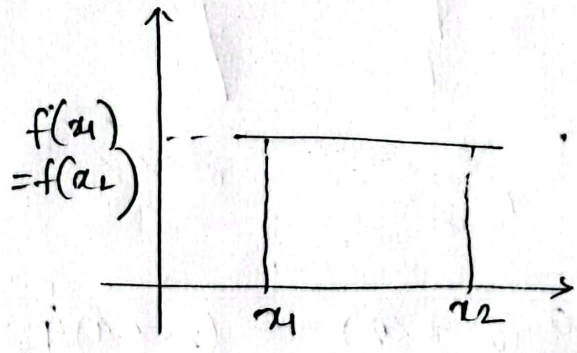


$x_1 < x_2$
$f(x_1) < f(x_2)$

b) f is decreasing. $f(x_1) > f(x_2)$; $x_1 < x_2$



c) constant $x_1 = x_2$, $f(x_1) = f(x_2)$



(First Derivative Test)

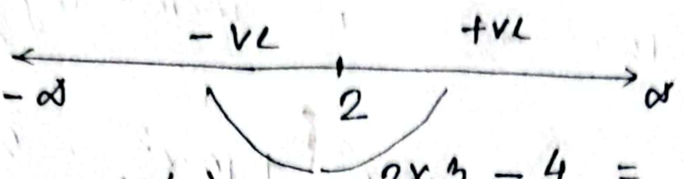
Problem (1): Find the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.

Solⁿ: $f'(x) = 2x - 4$

Now, we have to find critical point,

$$2x - 4 = 0.$$

$$x = 2$$



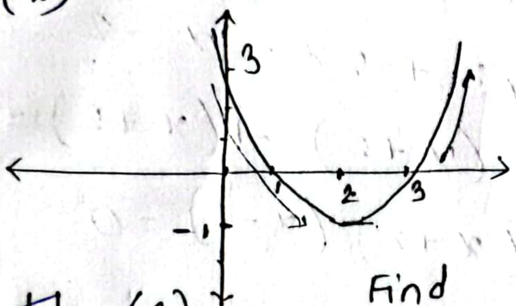
$x > 2$; $f'(x) = 2 \times 3 - 4 = 2 > 0$

$x < 2$; $f'(x) = 2 \times 0 - 4 = -4 < 0$

$f(x)$ is increasing $\rightarrow [2, \infty)$

$f(x)$ is decreasing $\rightarrow (-\infty, 2]$

[Ans.]

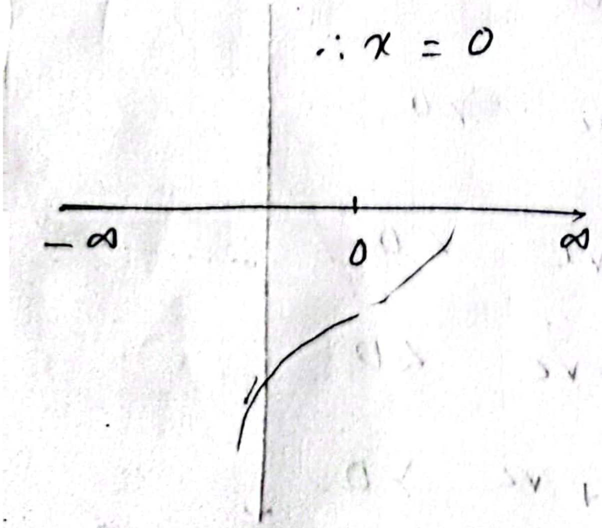


<u>Problem (2):</u> Find the intervals on which $f(x) = x^3$ is increasing and decreasing.

<u>Soln:</u> $f'(x) = 3x^2$

NOW, $3x^2 = 0$

$\therefore x = 0$



$x > 0$; $f'(x) = 3 > 0$

$x < 0$; $f'(x) = 3 > 0$

f is increasing $(-\infty, \infty)$

[Ans.]

Problem: Find the intervals on which $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$ is increasing / decreasing.

soln: $f'(x) = 12x^3 + 12x^2 - 24x$

Now, $f'(x) = 0$

$\Rightarrow 12x^3 + 12x^2 - 24x = 0$

$\Rightarrow 12x(x^2 + x - 2) = 0$

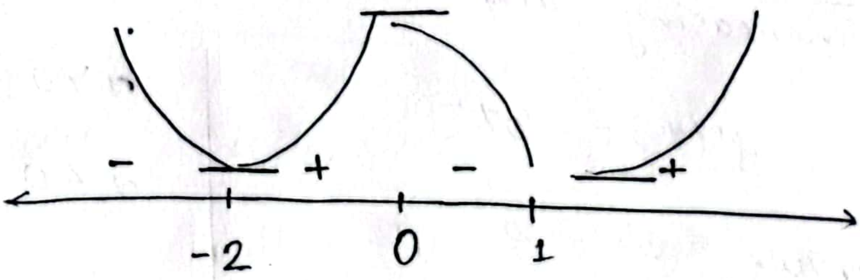
$\therefore x = 0 \quad ; \quad x^2 + x - 2 = 0$

$\Rightarrow x^2 + 2x - x - 2 = 0$

$\Rightarrow x(x+2) - 1(x+2) = 0$

$\therefore (x+2)(x-1) = 0$

$\therefore \quad x = 0, 1, -2$

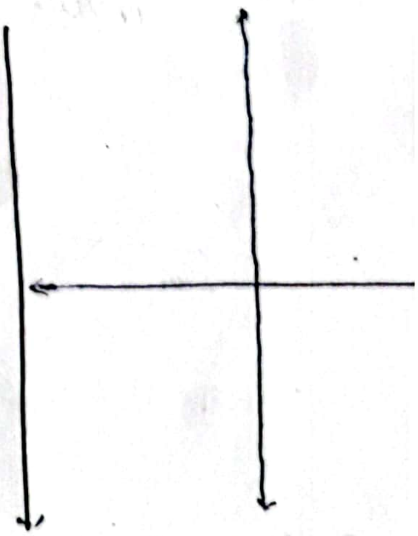


$\Leftarrow x > 1 \; ; \quad f'(x) = +ve > 0$

$-3 \Leftarrow x < -2 \; ; \quad f'(x) = -ve < 0$

$r.5 \Leftarrow 0 < x < 1 \; ; \quad f'(x) = -ve < 0$

$1 \Leftarrow -2 < x < 0 \; ; \quad f'(x) = +ve > 0$



$x = 0 \; ; \; y = 2$

f is increasing on $(1, +\infty)$

f is decreasing on $(-\infty, -2]$

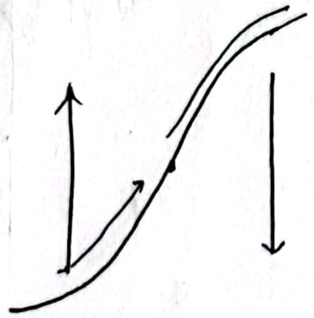
f is increasing on $(-2, 0)$

f is decreasing on $(0, 1)$

[Ans.]

Concavity

(Reveals the direction of graph)

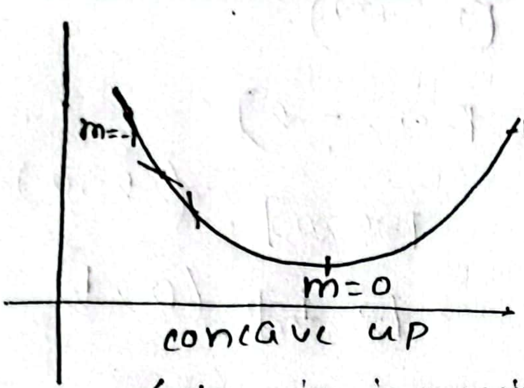


The graph is increasing But on the left side it has an upward curvature and on the right side, it has a downward curvature.

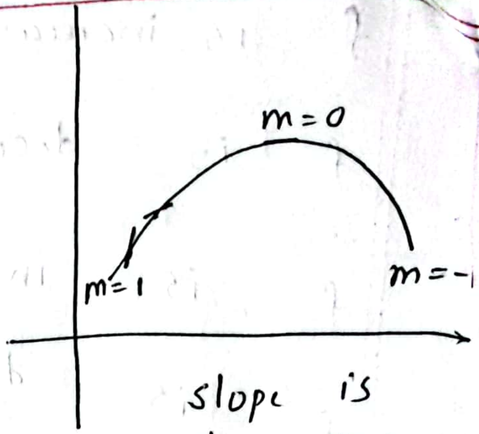
Theorem: Let, f be twice differentiable on an open interval.

a) If $f''(x) > 0$, f is concave up.

b) If $f''(x) < 0$, f is concave down.



concave up
(slope is increasing)

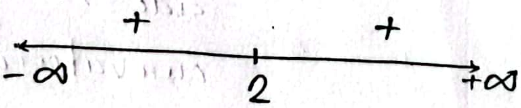


slope is decreasing
(concave down)

Problem: check the concavity of $f(x) = x^2 - 4x + 3$

Soln: $f'(x) = 2x - 4$

$f''(x) = 2$



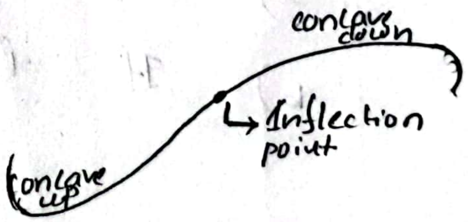
$f''(x) > 0$ on the interval $(-\infty, \infty)$

concave up.

Inflection points:

where the concavity changes.

$f''(x) = 0$



Problem: $f(x) = x^3 - 3x^2 + 1$. Use the first and 2nd derivatives of f to determine the intervals on which f is increasing, decreasing, concave up and concave down. Locate all inflection points and confirm that your conclusions are consistent with the graph.

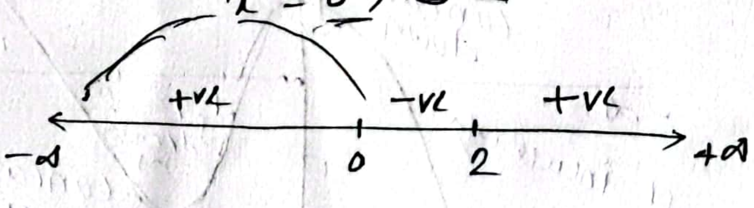
soln: $f'(x) = 3x^2 - 6x$

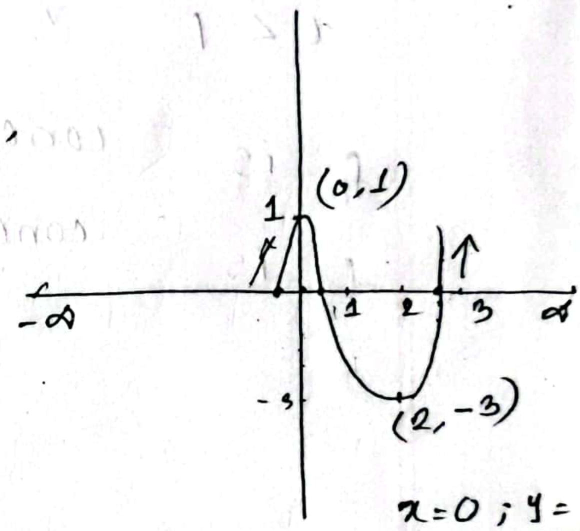
Now, Find critical points,

$3x^2 - 6x = 0$

$\Rightarrow 3x(x-2) = 0$

$x = 0, 2$





$x = 0;\ y =$

$y = 0;\ x = 2.87$

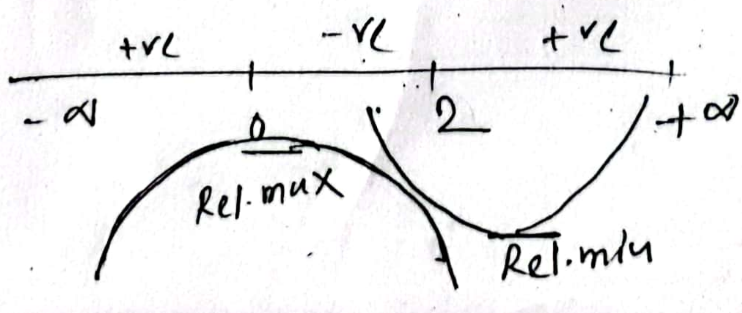
0.65

-0.5

$x > 2$; $f'(x) = +ve > 0$; increasing $(2, \infty)$

$x < 0$, $f'(x) = +ve > 0$; increasing $(-\infty, 0]$

$0 < x < 2$; $f'(x) = -ve < 0$; decreasing $[0 < x < 2]$ $[0, 2]$



$x = 0$;

$f(x) = 1$ (Rel. max)

$x = 2$;

$f(x) = -3$ (Rel. min

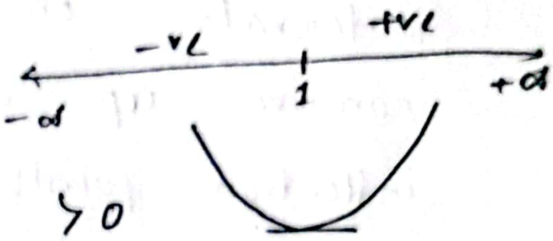
(Ch- nd

$f''(x) = 6x - 6 = 6(x-1)$

Now, Find inflection points.

$6(x-1) = 0$

$\therefore x = 1$

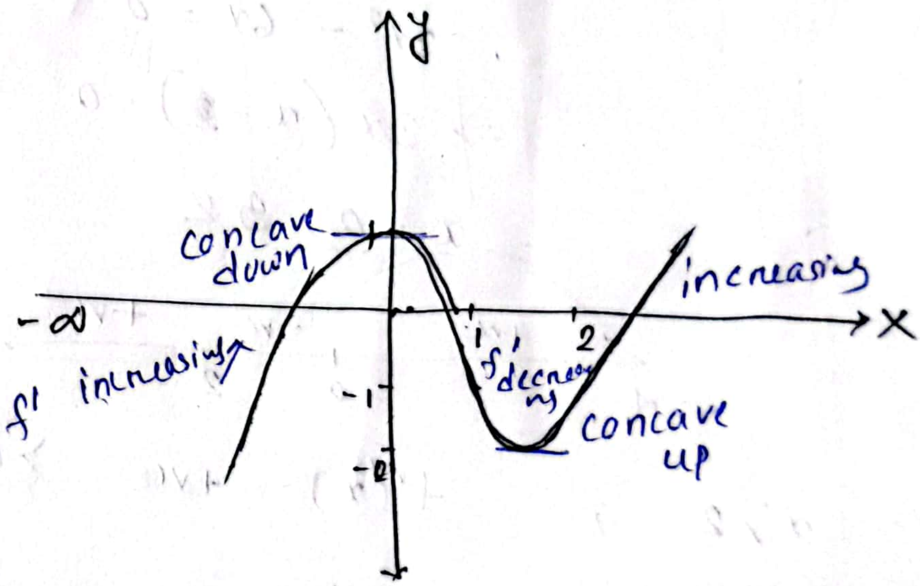


$x > 1$; $f''(x) = +ve > 0$

$x < 1$; $f''(x) = -ve < 0$

f is concave up. $[1, +\infty)$

f is concave down $(-\infty, 1]$



consistent with the graph.

[Ans.]

Exercise (4.1):

Problem:

Interval	sign $f'(x)$	sign $f''(x)$
$x<1$	$-$	$+$
$1<x<2$	$+$	$+$
$2<x<3$	$+$	$-$
$3<x<4$	$-$	$-$
$4<x$	$-$	$+$

Soln:

a) f is increasing $(1,3)$

b) f is decreasing $(-\infty,1)$, $[3,+\infty)$

c) f is concave up $(-\infty,2)$, $(4,+\infty)$

d) f is concave down $(2,4)$

e) Points of inflection at $(2,4)$

Problem:

Interval	sign $(f'(x))$	sign $(f''(x))$
$x<1$	$+$	$+$
$1<x<3$	$+$	$-$
$3<x$	$+$	$+$

a) f is increasing $(-\infty, +\infty)$

b) f is decreasing nowhere

c) f concave up $(-\infty, 1)$; $(3, +\infty)$

d) f concave down $(1, 3)$

e) Inflection points, at $x = 1, 3$

[Ans.]

Problem: (15) $f(x) = x^2 - 3x + 8$

$f'(x) = 2x - 3$

Now, $2x - 3 = 0$

$\therefore x = 3/2$

$f' < 0$ $f' > 0$

$-$ $+$

$3/2$

a) f is increasing $\left[\frac{3}{2}, +\infty\right)$

b) f is decreasing $(-\infty, 3/2]$

Then, $f''(x) = 2$

$-\infty$ 2 ∞

c) concave up $(-\infty, +\infty)$

d) Nowhere   e) none (No inflection points)

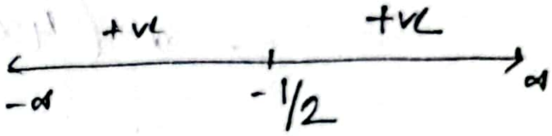
Problem: (17) $f(x) = (2x+1)^3$

$f'(x) = 3(2x+1)^2 \cdot 2 = 6(2x+1)^2$

Now, $6(2x+1)^2 = 0$

$\Rightarrow 2x = -1$

$\therefore x = -1/2$



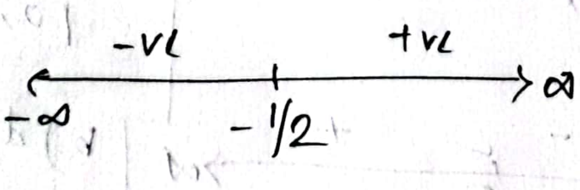
a) f increasing $(-\infty, \infty)$

b) Nowhere

c) $f''(x) = 12(2x+1) \cdot 2 = 24(2x+1)$

Now, $24(2x+1) = 0$

$\therefore x = -1/2$



concave up : $(-1/2, \infty)$

d) concave down : $(-\infty, -1/2)$

e) Inflection point : $x = -1/2$

Problem: (20) $f(x) = x^4 - 5x^3 + 9x^2$

$f'(x) = 4x^3 - 15x^2 + 18x$

Now, $4x^3 - 15x^2 + 18x = 0$

$\Rightarrow x(4x^2 - 15x + 18) = 0$; $x = 0$

(complex number)

$f''(x) = 12x^2 - 30x + 18$

$= 6(2x^2 - 5x + 3)$

$= 6(2x^2 - 3x - 2x + 3)$

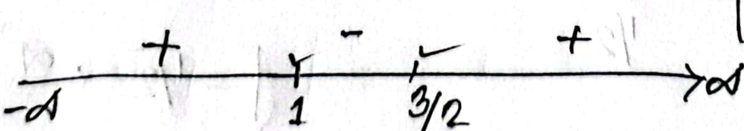
$= 6\{x(2x - 3) - 1(2x - 3)\}$

$= 6(2x - 3)(x - 1)$

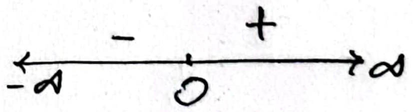
2 | 72
2 | 36
2 | 18
3 | 9
3

Now, $f''(x) = 0$

$x = 3/2$; $x = 1$



a) increasing



$[0, +\infty)$

b) decreasing

$(-\infty, 0]$

c) concave up, $f''(x) > 0$

$(-\infty, 1]$, $[3/2, +\infty)$

d) concave down, $f''(x) < 0$

$[1, 3/2]$

e) Inflection point $1, 3/2$

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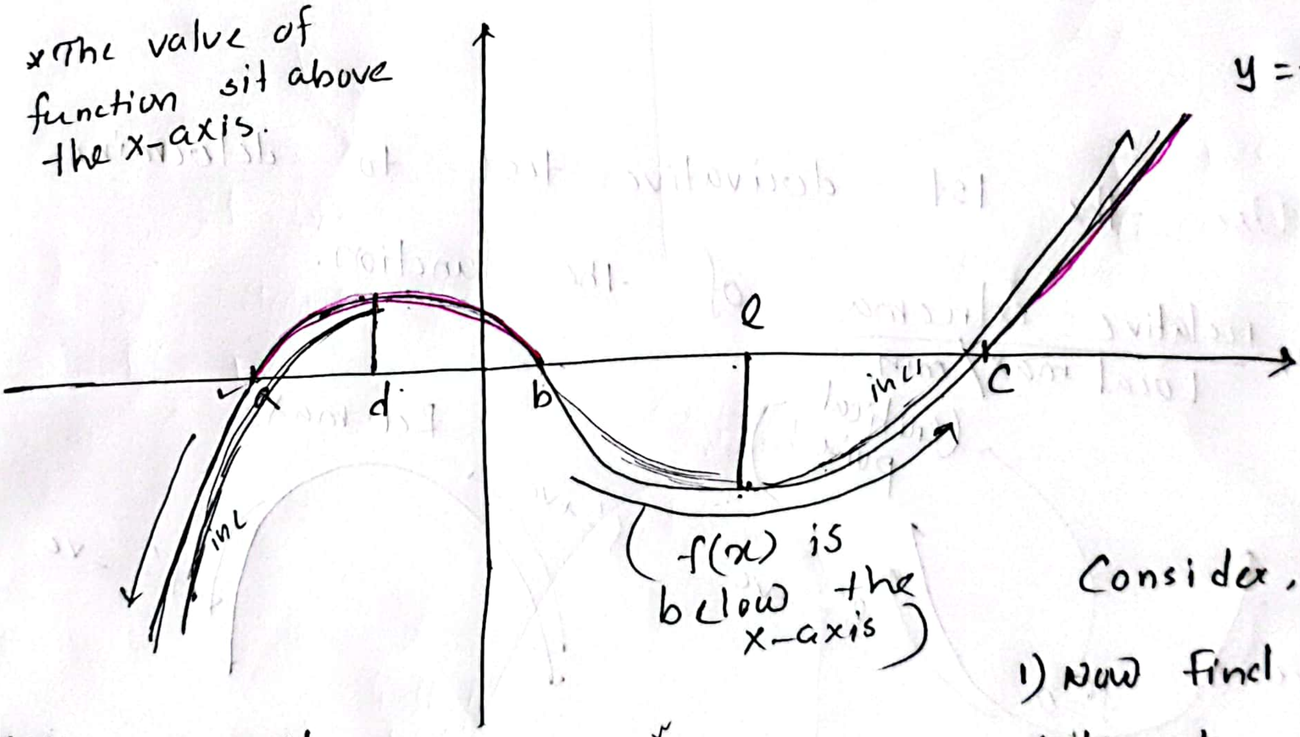
* Increasing, Decreasing function: (Example)

* The value of function sit above the x-axis.



Consider, $y = f(x)$

1) Now find on which intervals, $f(x)$ is positive

+ve

$a < x < b$

$x > c$

−ve

$x < a$

$b < x < c$

$x = a; f(x) = 0$

$x = b; f(a) = 0$

$= c; f(a) = 0$

3) when is the function increasing/decreasing

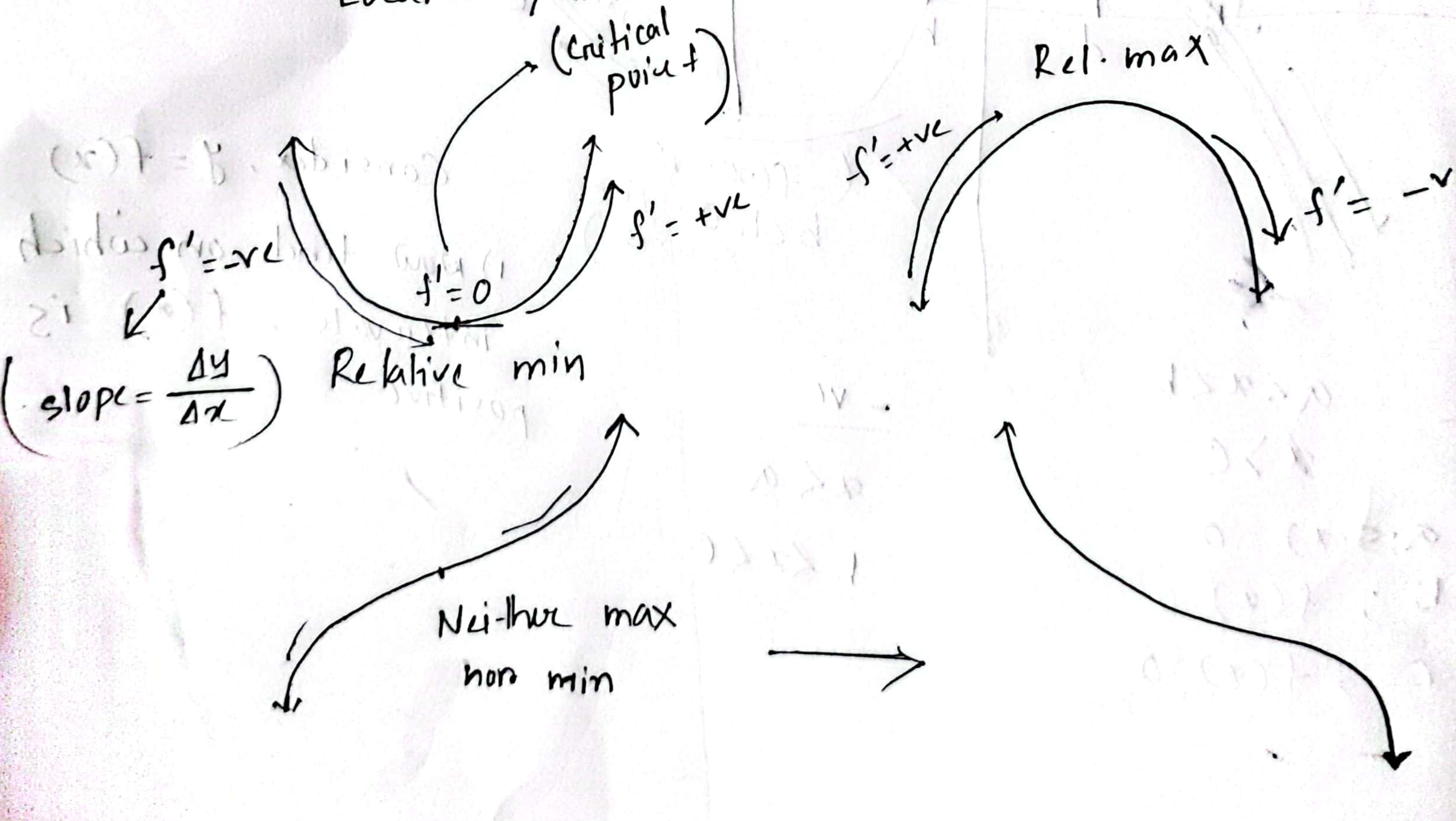
* $f(x)$ increasing

$x < d$ or $x > c$

* $f(x)$ decreasing

~~$x > d$~~ . $d < x < e$

* Use the 1st derivative test to determine the $\dfrac{\text{relative extrema}}{\text{Local max/min}}$ of the function.



* **Concavity**

1) concave up $\longrightarrow$ $f'' = +ve$ (f' ↑)

2) concave down $\longrightarrow$ $f'' = -ve$ (f' ↓) slope is decreasing