# MAXIMUM POWER TRANSFER

Section 4.8

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# OUR GOAL

- In many practical situations, a circuit is designed to provide power to a load
- There are applications in areas such as communications where it is desirable to maximize the power delivered to a load
- We need to find the equation to maximize power delivered to a load


SOLUTION

# POWER CALCULATION

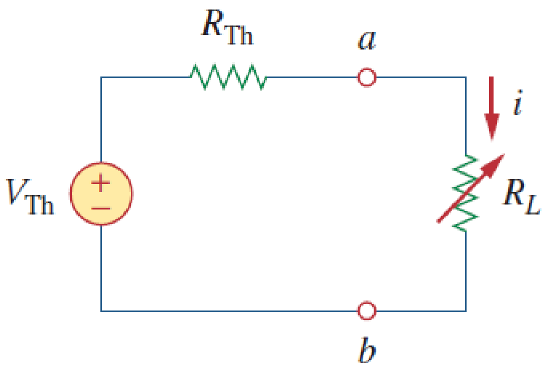



**Figure 4.48**
The circuit used for maximum power transfer.

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

# POWER CALCULATION

For a given circuit, $V_{Th}$ and $R_{Th}$ are fixed. By varying the load resistance $R_L$, the power delivered to the load varies as sketched in Fig. 4.49. We notice from Fig. 4.49 that the power is small for small or large values of $R_L$ but maximum for some value of $R_L$ between 0 and $\infty$. We now want to show that this maximum power occurs when $R_L$ is equal to $R_{Th}$. This is known as the *maximum power theorem.*

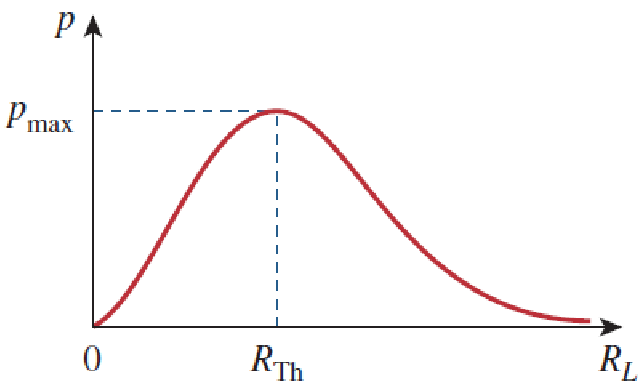



**Figure 4.49**
Power delivered to the load as a function of $R_L$.

# MAXIMUM POWER TRANSFER

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

# PROOF OF MAXIMUM POWER TRANSFER

To prove the maximum power transfer theorem, we differentiate $p$ in Eq. (4.21) with respect to $R_L$ and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{\text{Th}}^2 \left[ \frac{(R_{\text{Th}} + R_L)^2 - 2R_L(R_{\text{Th}} + R_L)}{(R_{\text{Th}} + R_L)^4} \right]$$

$$= V_{\text{Th}}^2 \left[ \frac{(R_{\text{Th}} + R_L - 2R_L)}{(R_{\text{Th}} + R_L)^3} \right] = 0$$

This implies that

$$0 = (R_{\text{Th}} + R_L - 2R_L) = (R_{\text{Th}} - R_L) \tag{4.22}$$

which yields

$$\boxed{R_L = R_{\text{Th}}} \tag{4.23}$$

showing that the maximum power transfer takes place when the load resistance $R_L$ equals the Thevenin resistance $R_{\text{Th}}$. We can readily confirm that Eq. (4.23) gives the maximum power by showing that $d^2p/dR_L^2 < 0$.

The maximum power transferred is obtained by substituting Eq. (4.23) into Eq. (4.21), for

$$\boxed{p_{\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}} \tag{4.24}$$

Equation (4.24) applies only when $R_L = R_{\text{Th}}$. When $R_L \neq R_{\text{Th}}$, we compute the power delivered to the load using Eq. (4.21).

# MATH PROBLEM PRACTICE:

Find the value of $R_L$ for maximum power transfer in the circuit of Fig. 4.50. Find the maximum power.

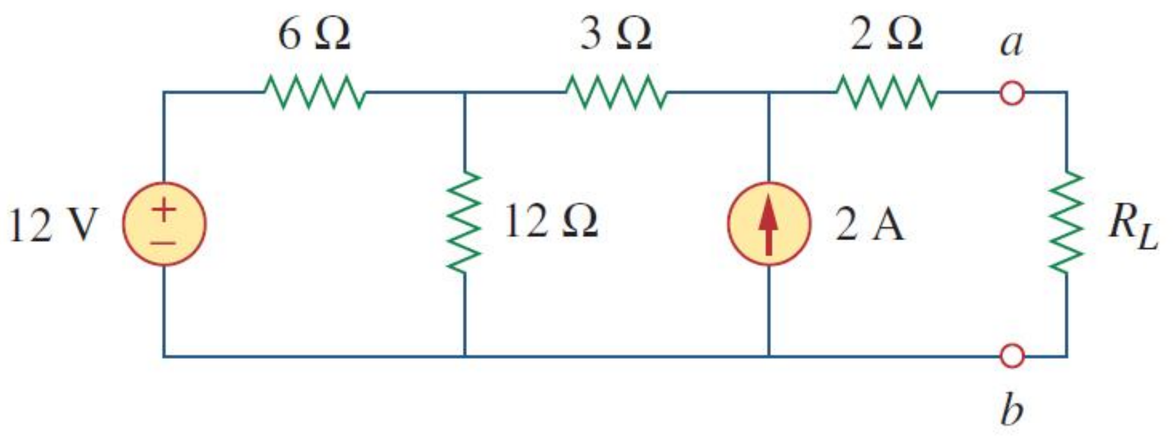



**Figure 4.50**
For Example 4.13.

***Reference: Sadiku_Example_4.13***

# MATH PROBLEM PRACTICE:

**Solution:**

We need to find the Thevenin resistance $R_{Th}$ and the Thevenin voltage $V_{Th}$ across the terminals *a-b*. To get $R_{Th}$, we use the circuit in Fig. 4.51(a) and obtain

$$R_{Th} = 2 + 3 + 6 \parallel 12 = 5 + \frac{6 \times 12}{18} = 9\ \Omega$$

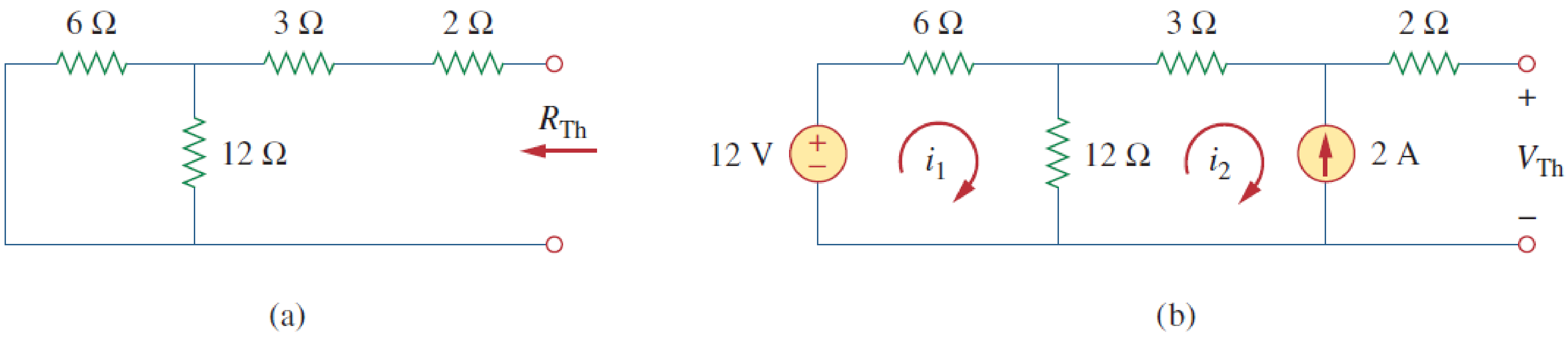



**Figure 4.51**
For Example 4.13: (a) finding $R_{Th}$, (b) finding $V_{Th}$.

***Reference: Sadiku_Example_4.13***

# MATH PROBLEM PRACTICE:

To get $V_{\text{Th}}$, we consider the circuit in Fig. 4.51(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2 \text{ A}$$

Solving for $i_1$, we get $i_1 = -2/3$. Applying KVL around the outer loop to get $V_{\text{Th}}$ across terminals $a$-$b$, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{\text{Th}} = 0 \qquad \Rightarrow \qquad V_{\text{Th}} = 22 \text{ V}$$

For maximum power transfer,

$$R_L = R_{\text{Th}} = 9\ \Omega$$

and the maximum power is

$$p_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44 \text{ W}$$

***Reference: Sadiku_Example_4.13***

# MATH PROBLEM PRACTICE:

**4.67** The variable resistor $R$ in Fig. 4.133 is adjusted until it absorbs the maximum power from the circuit.
(a) Calculate the value of $R$ for maximum power.
(b) Determine the maximum power absorbed by $R$.

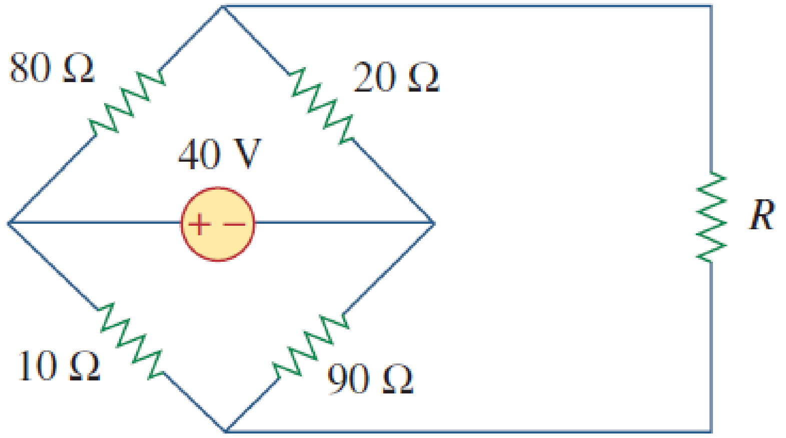


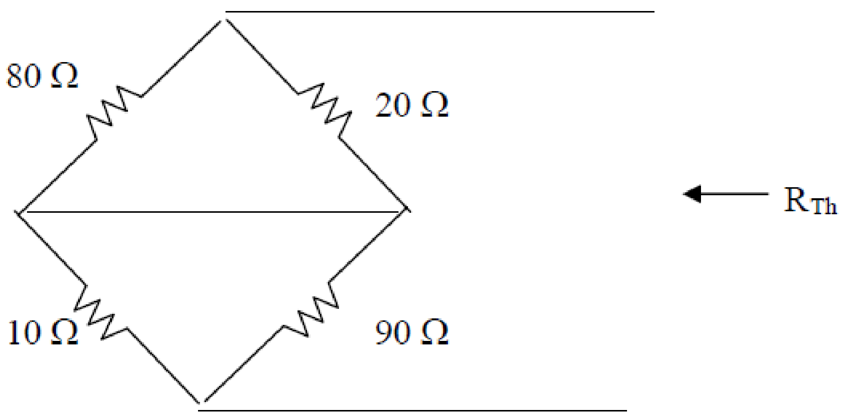

**Figure 4.133**
For Prob. 4.67.

***Reference: Sadiku_Exercise_4.67***

# MATH PROBLEM PRACTICE:

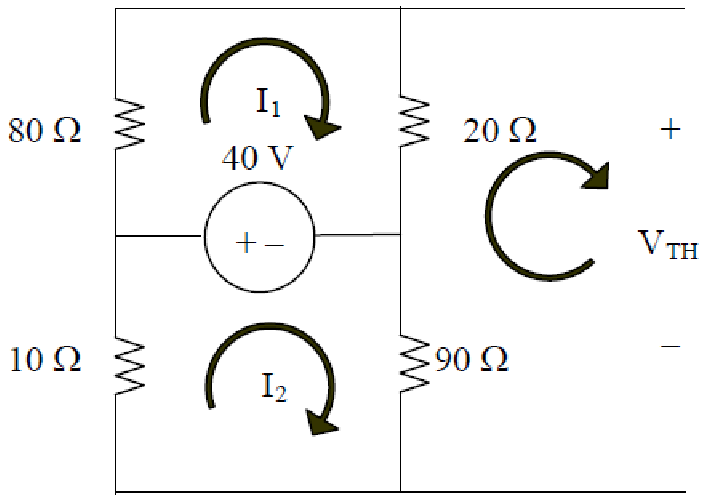
**Chapter 4, Solution 67.**

We first find the Thevenin equivalent. We find $R_{Th}$ using the circuit below.




$$R_{Th} = 20//80 + 90//10 = 16 + 9 = 25\ \Omega$$

We find $V_{Th}$ using the circuit below. We apply mesh analysis.




$$(80+20)i_1 - 40 = 0 \longrightarrow i_1 = 0.4$$

$$(10+90)i_2 + 40 = 0 \longrightarrow i_2 = -0.4$$

$$-90i_2 - 20i_1 + V_{Th} = 0 \longrightarrow V_{Th} = -28\text{ V}$$

(a) $R = R_{Th} = \mathbf{25\ \Omega}$

(b) $P_{max} = \dfrac{V_{Th}^2}{4R_{Th}} = \dfrac{(28)^2}{100} = \underline{7.84\text{ W}}$

***Reference: Sadiku_Exercise_4.67***

# MATH PROBLEM PRACTICE:

***4.68** Compute the value of $R$ that results in maximum power transfer to the 10-Ω resistor in Fig. 4.134. Find the maximum power.

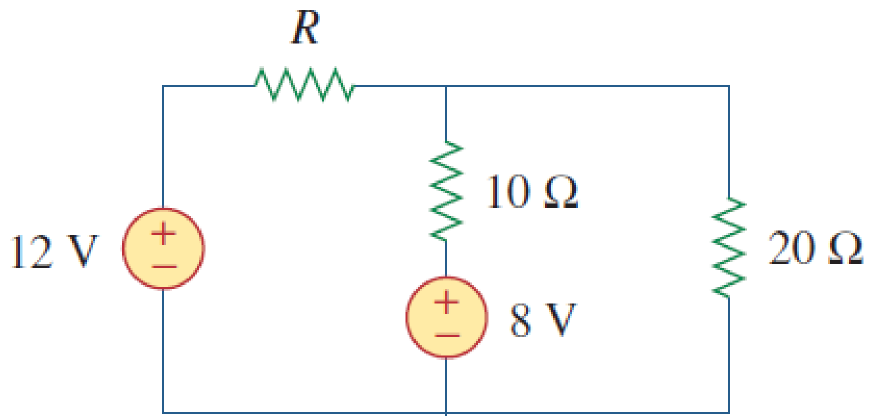


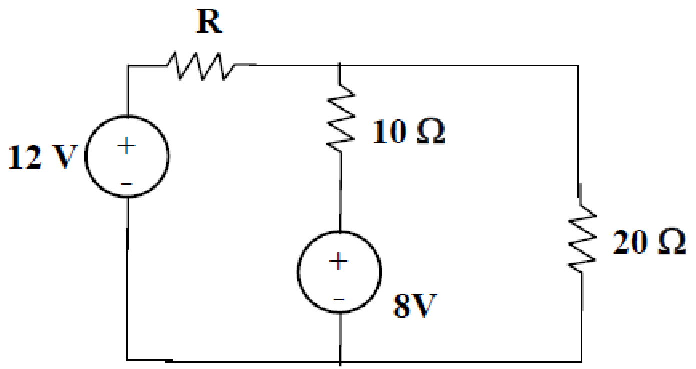

**Figure 4.134**
For Prob. 4.68.

***Reference: Sadiku_Exercise_4.68***

# MATH PROBLEM PRACTICE:

**Chapter 4, Solution 68.**

This is a challenging problem in that the load is already specified. This now becomes a "minimize losses" style problem. When a load is specified and internal losses can be adjusted, then the objective becomes, reduce $R_{Thev}$ as much as possible, which will result in maximum power transfer to the load.




Removing the 10 ohm resistor and solving for the Thevenin Circuit results in:

$$R_{Th} = (Rx20/(R+20)) \text{ and a } V_{oc} = V_{Th} = 12x(20/(R+20)) + (-8)$$

As R goes to zero, $R_{Th}$ goes to zero and $V_{Th}$ goes to 4 volts, which produces the maximum power delivered to the 10-ohm resistor.

$$P = vi = v^2/R = 4x4/10 = 1.6 \text{ watts}$$

Notice that if R = 20 ohms which gives an $R_{Th}$ = 10 ohms, then $V_{Th}$ becomes -2 volts and the power delivered to the load becomes 0.1 watts, much less that the 1.6 watts.

It is also interesting to note that the internal losses for the first case are $12^2/20 = 7.2$ watts and for the second case are = to 12 watts. This is a significant difference.

***<u>Reference: Sadiku_Exercise_4.68</u>***

# *Fall 2023*

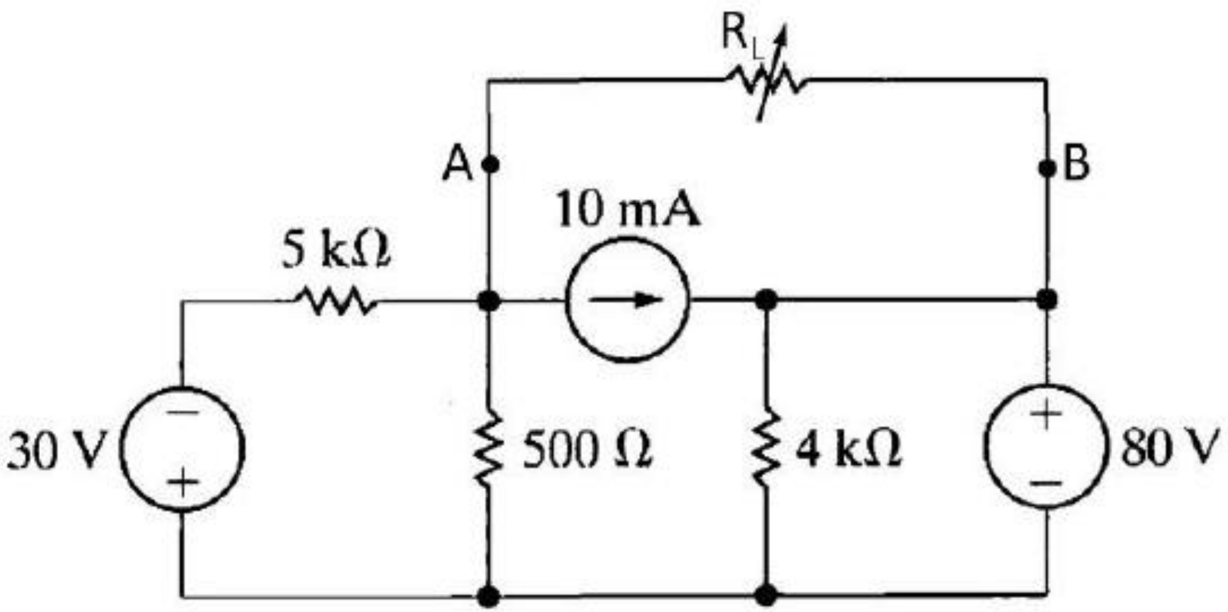
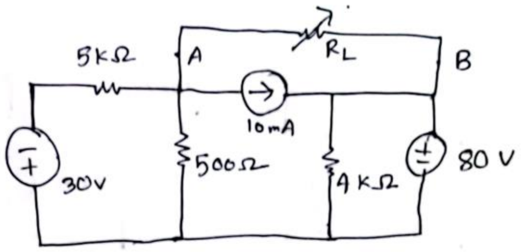


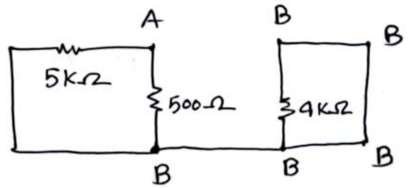

Figure 2.

ii) For any value of $R_L$, what will be the maximum power delivered to this resistance?
iii) If $R_L$=1kΩ, then would maximum power be achieved? If not, then what should you do to achieve maximum power?

# Fall 2023




For $R_{TH}$,



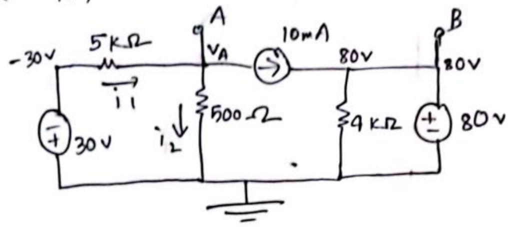

So, 5KΩ and 500Ω are in parallel.

$$\therefore R_{TH} = \left(\frac{1}{5000} + \frac{1}{500}\right)^{-1}$$

$$\therefore R_{TH} = 454 \cdot 545\ \Omega$$



For $V_{TH}$,



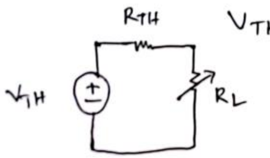

At $V_A$ node,

$$i_1 = i_2 + 10\,mA$$

$$\Rightarrow \frac{-30 - V_A}{5000} = \frac{V_A}{500} + 10\times10^{-3}$$

$$\Rightarrow -30 - V_A = 10\,V_A + 50$$

$$\Rightarrow V_A = \frac{-80}{11}\ V$$




$$V_{TH} = V_A - V_B$$

$$= \frac{-80}{11} - 80$$

$$= -87 \cdot 2727\,V$$

# *Fall 2023*

ii) Max power

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{[illegible]}{4.18} \ W$$

iii) $P_{max}$ will not occur for $R_L = 1 \ k\Omega$

# *Summer 2023*

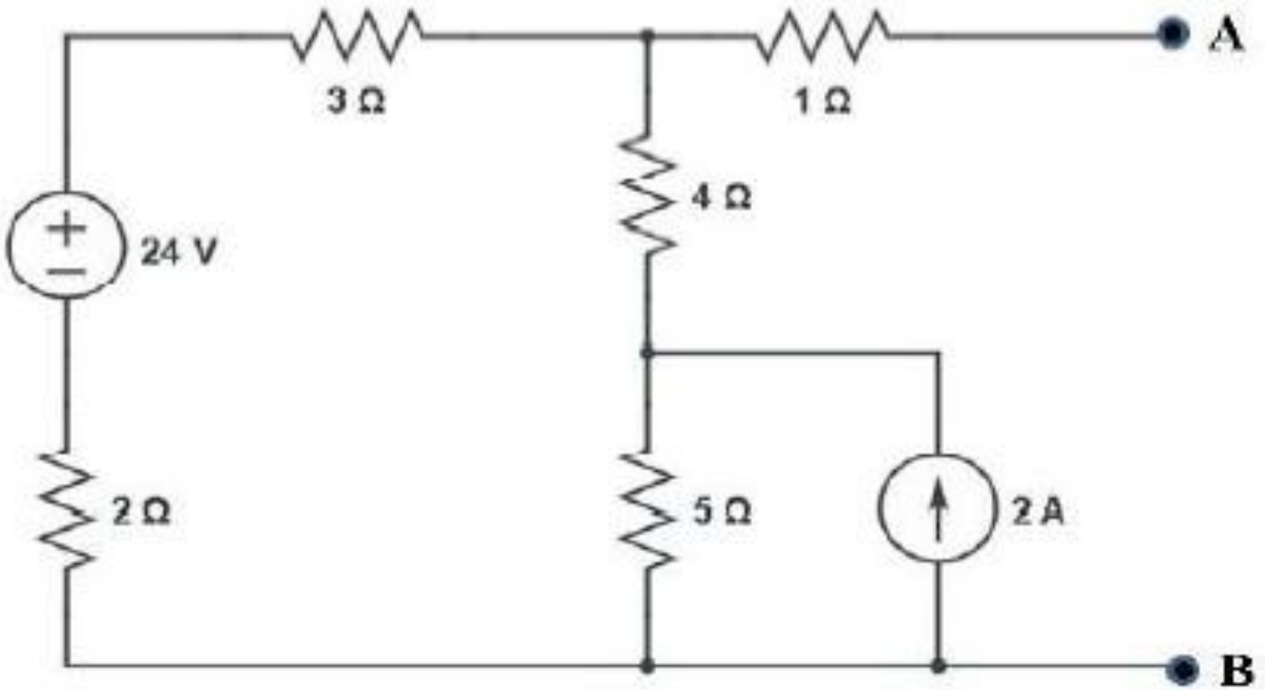
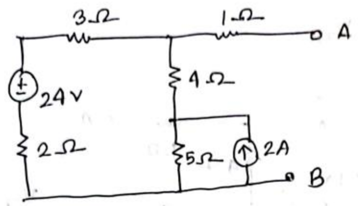


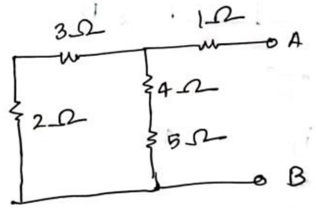

**Figure 2.**

**ii)** For any resistance connected right to **A-B** terminal, what will be the maximum power delivered to the resistance?
**iii)** If 10Ω resistance is connected between **A-B**, then would maximum power be achieved? If not then what should you do?

# Summer 2023




For $R_{TH}$ ,

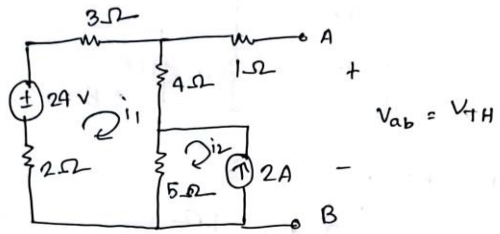



$$R_{TH} = 1 + \left( (3+2) \,||\, (4+5) \right)$$

$$= 4.214\ \Omega$$



For $V_{TH}$ ,




From mesh-2,

$$i_2 = -2A$$

For mesh-1,

$$3i_1 + 4i_1 + 5(i_1 - i_2) + 2i_1 - 24 = 0$$

$$\Rightarrow 14i_1 - 5i_2 - 24 = 0$$

$$\Rightarrow 14i_1 = 24 + 5 \times (-2)$$

$$\therefore i_1 = 1A$$

$$V_{TH} = 4i_1 + 5(i_1 - i_2)$$

$$= 4 \times 1 + 5(1 - (-2))$$

$$= 19V$$

# Summer 2023

ii) Max power

$$P_{max} = \frac{V_{TH}^2}{4\,R_{TH}} = \frac{19^2}{4 \times 4.214}$$

$$= 21.417\ W$$

iii) $P_{max}$ will not occur if

$R_L = 10\,\Omega$.

As we know for $P_{max}$,

$$R_L = R_{TH} = 4.214\,\Omega$$

# SHORT CIRCUIT REFERENCE

- What is SHORT CIRCUIT - Explained with Example | Basics of Electronics (youtube.com)
- What is Short Circuit ? Easiest Explanation | TheElectricalGuy (youtube.com)
- The Concept of Short Circuit (youtube.com)

# MATH TO PRACTICE FROM THE BOOK FOR EXAM

- **Maximum Power Transfer theorem:**
  - **Example:**
    - 4.13
  - **Problem:**
    - 4.67, 4.68
  - **N:B: Please note that Circuits with Independent sources only, for Thevenin's theorem and maximum power transfer!**

# THANK YOU