Basic Circuit Laws and their Applications

Section 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

Md. Shafqat Talukder Rakin

Lecturer, Department of CSE,

United International University

Email id : shafqat@cse.uiu.ac.bd

Courtesy: Rifat Bin Rashid

Resistance

- R is resistance in ohms (Ω)
- *l* is length of the conductor (m)
- A is cross-sectional area of conductor (m^2)
- ρ is Resistivity of the material
- Notice the sign convention followed in the figure

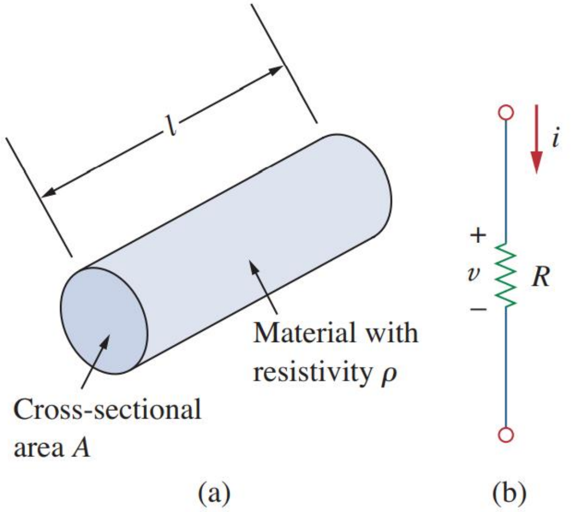


Figure 2.1
(a) Resistor, (b) Circuit symbol for resistance.

$$R = \rho \frac{\ell}{A}$$

Ohm's Law

Ohm's law states that the voltage v across a resistor is directly proportional to the current i flowing through the resistor.

That is,

$$v \propto i \tag{2.2}$$

$$\boxed{v = iR}$$

Branches and Nodes

A **branch** represents a single element such as a voltage source or a resistor.

A **node** is the point of connection between two or more branches.

A **loop** is any closed path in a circuit.

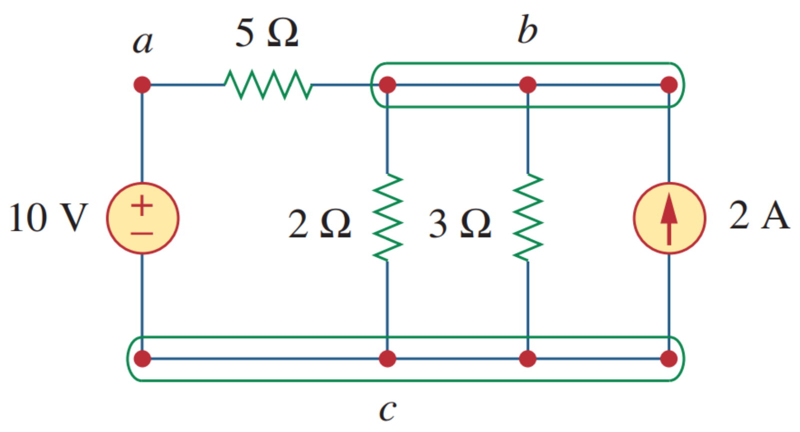


Figure 2.10
Nodes, branches, and loops.

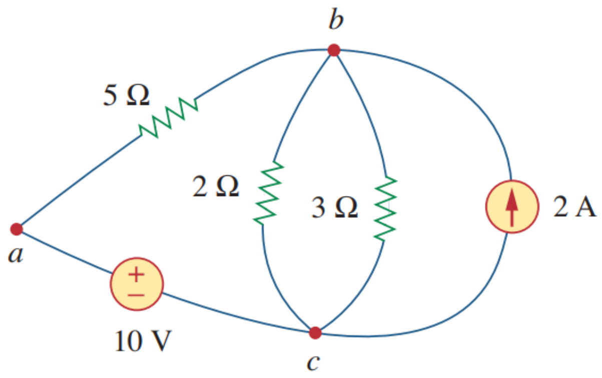


Figure 2.11
The three-node circuit of Fig. 2.10 is redrawn.

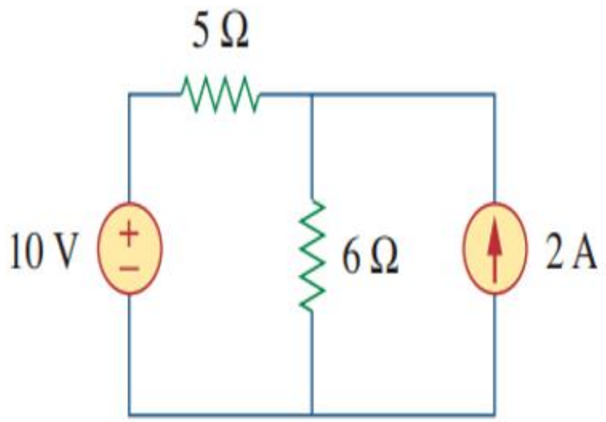


Figure 2.12
For Example 2.4.

Find the nodes for this circuit

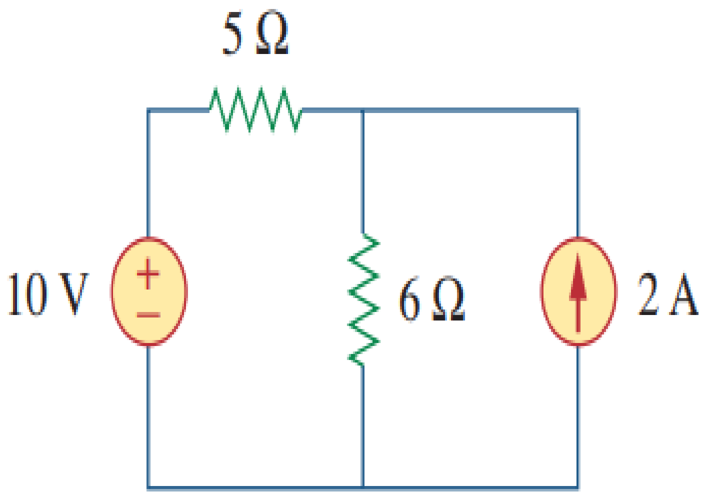


Figure 2.12
For Example 2.4.

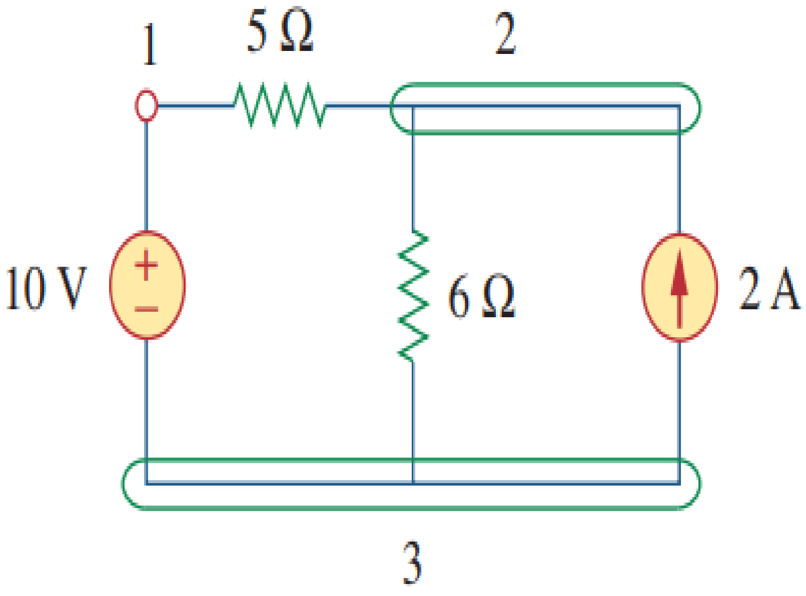


Figure 2.13
The three nodes in the circuit of Fig. 2.12.

Series and Parallel Circuits

Two or more elements are in **series** if they exclusively share a single node and consequently carry the same current.

Two or more elements are in **parallel** if they are connected to the same two nodes and consequently have the same voltage across them.

Kirchhoff's Current Law (KCL)

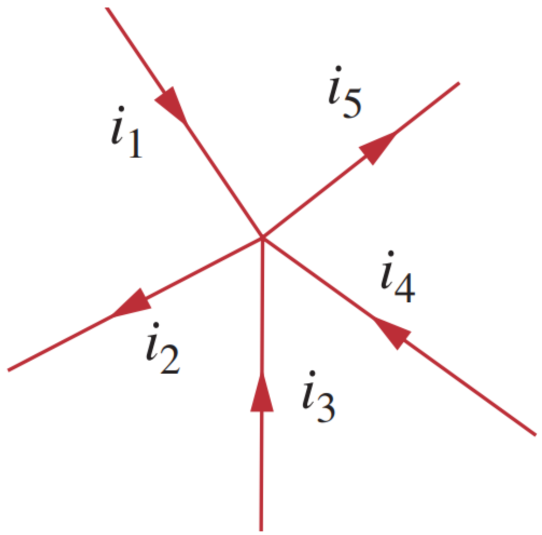


Figure 2.16
Currents at a node illustrating KCL.

The sum of the currents entering a node is equal to the sum of the currents leaving the node.

$$i_1 + i_3 + i_4 = i_2 + i_5$$

Applying KCL to Current Sources in Parallel

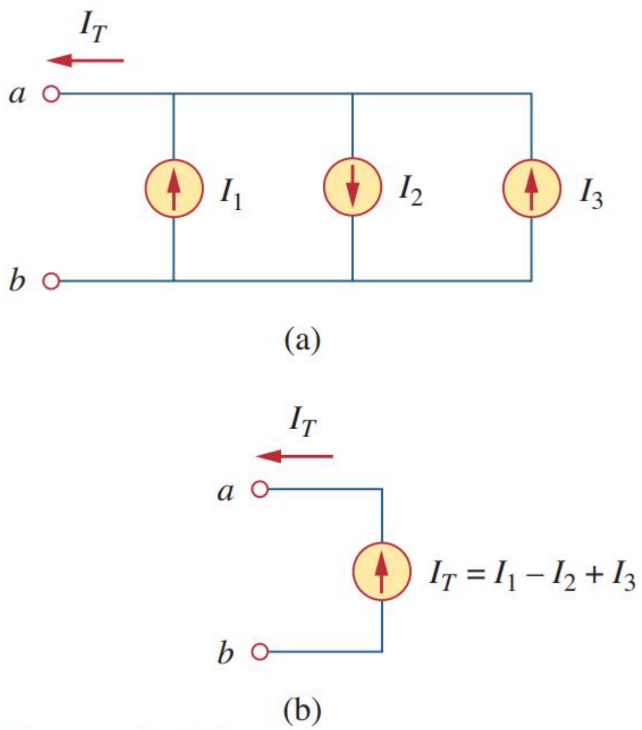
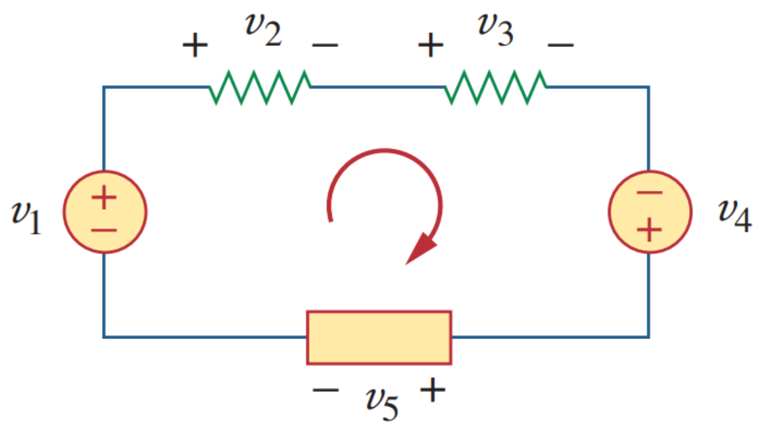


Figure 2.18
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Kirchhoff's Voltage Law (KVL)

Kirchhoff's voltage law (KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.



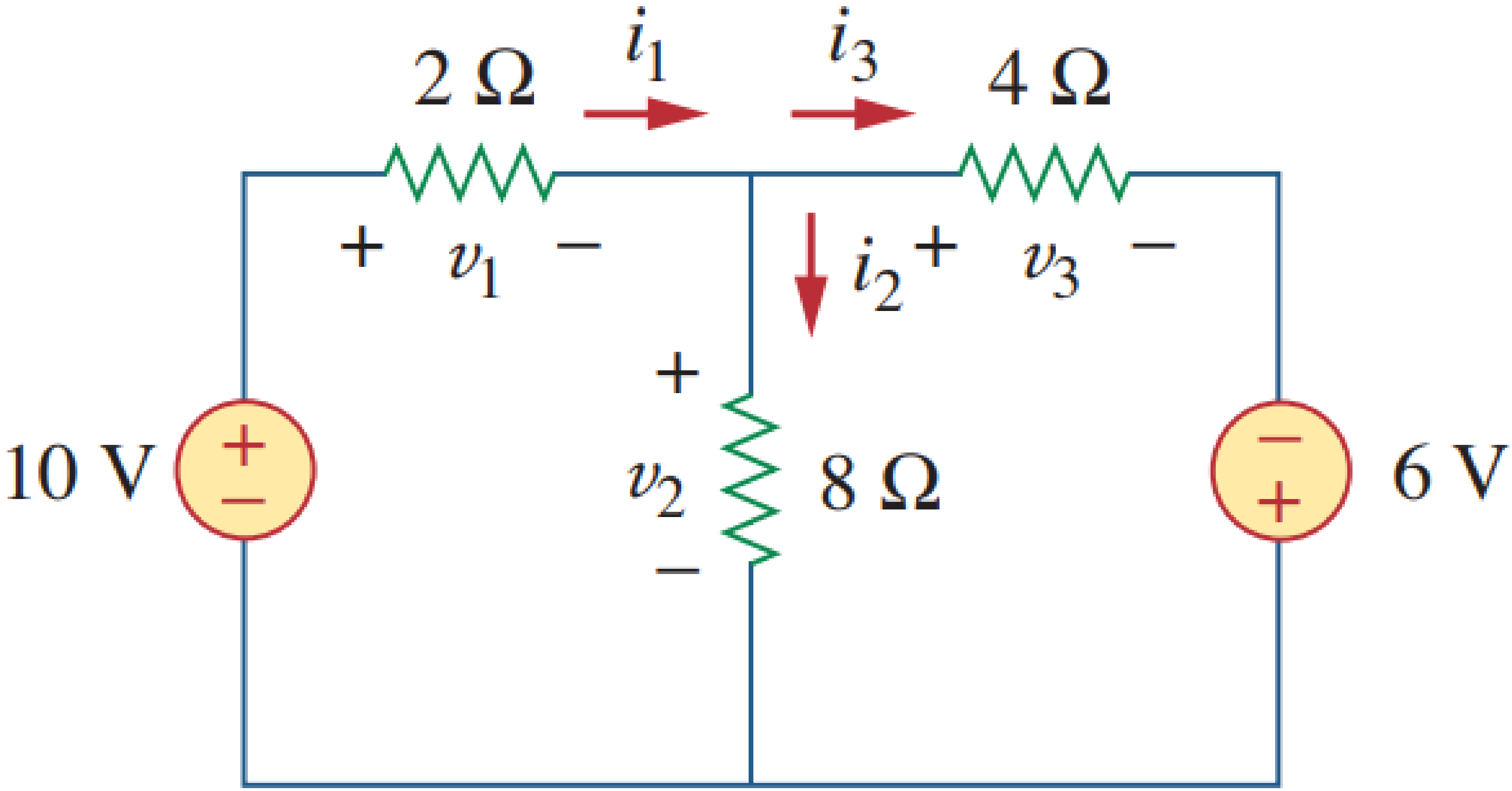
In the above example,

$$-v_1 + v_2 + v_3 - v_4 + v_5 = 0$$

KVL Rules:

- Assign currents in all the branches with suitable direction.
- Name all the nodes and terminals.
- Add voltage polarity (+/-) to all the elements according to assigned current direction.
- Apply KVL by collecting voltages with appropriate signs along the independent loops.

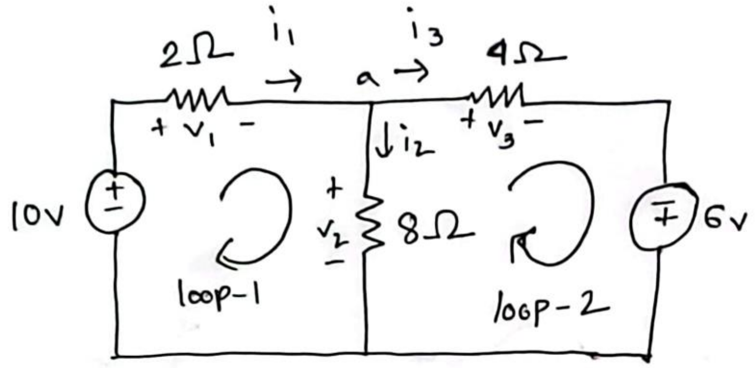
Math Problem Practice:



Find the currents and voltages in the circuit shown in Fig. 2.28.

Answer: $v_1 = 6$ V, $v_2 = 4$ V, $v_3 = 10$ V, $i_1 = 3$ A, $i_2 = 500$ mA, $i_3 =$ 2.5 A

Reference: Sadiku_Practice_Problem_2.8



At node a, KCL gives,

$$i_1 = i_2 + i_3 \Rightarrow i_1 - i_2 - i_3 = 0 \quad \cdots (1)$$

Applying KVL at loop-1 gives,

$$2i_1 + 8i_2 - 10 = 0 \quad \cdots (2)$$

Applying KVL at loop-2 gives,

$$4i_3 - 6 - 8i_2 = 0 \quad \cdots (3)$$

Solving (1), (2), (3) we get,

$$i_1 = 3A, \; i_2 = 0.5A, \; i_3 = 2.5A$$

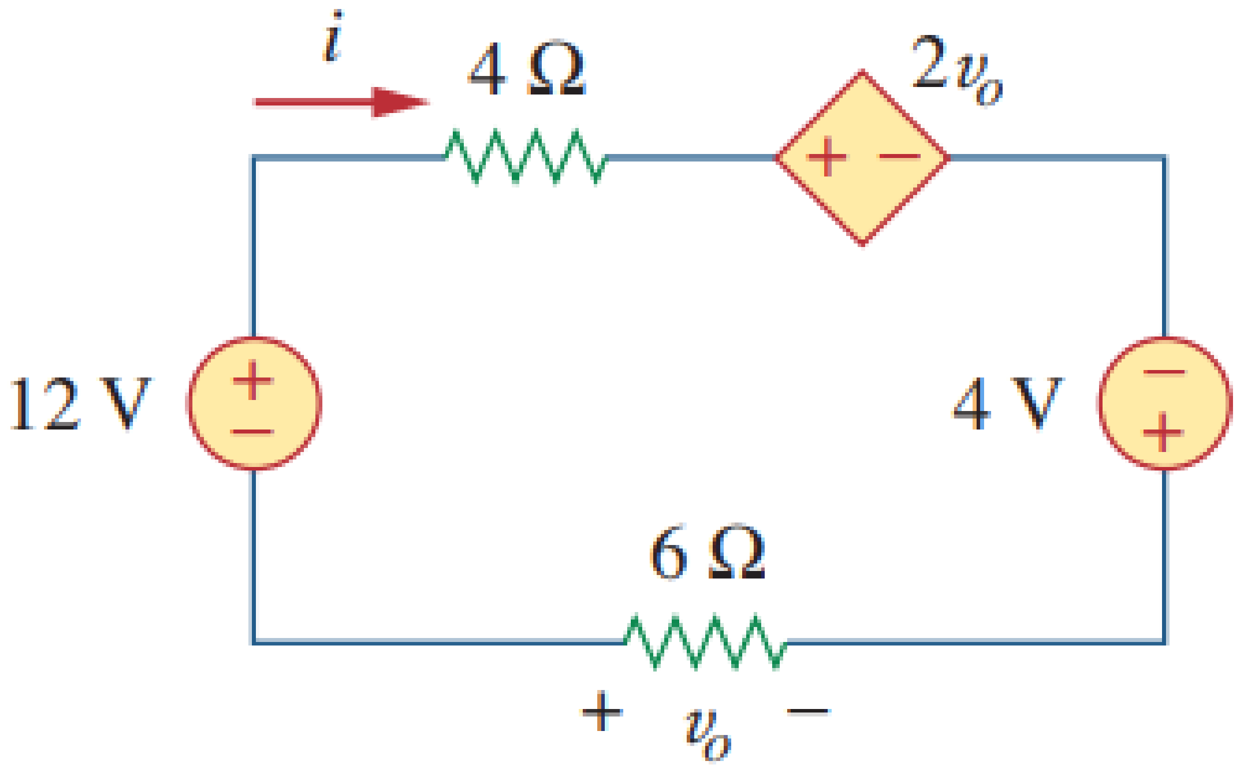
Applying ohm's law we get,

$$V_1 = 2i_1 = 2 \times 3 = 6V \qquad V_2 = 8i_2 = 8 \times 0.5 = 4V$$

$$V_3 = 4i_3 = 4 \times 2.5 = 10V$$

Math Problem Practice:

Determine v_o and i in the circuit shown in Fig.



Reference: Sadiku_Example_2.6

Math Problem Practice:

Determine v_o and i in the circuit shown in Fig. 2.23(a). **Example 2.6**

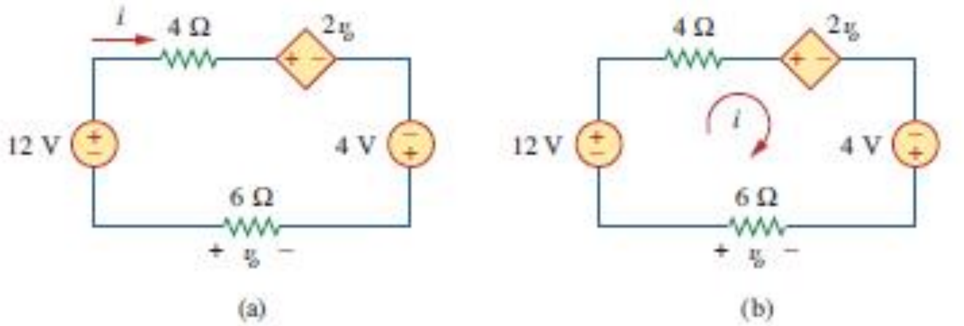


Figure 2.23
For Example 2.6.

Solution:
We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \textbf{(2.6.1)}$$

Applying Ohm's law to the 6-Ω resistor gives

$$v_o = -6i \qquad \textbf{(2.6.2)}$$

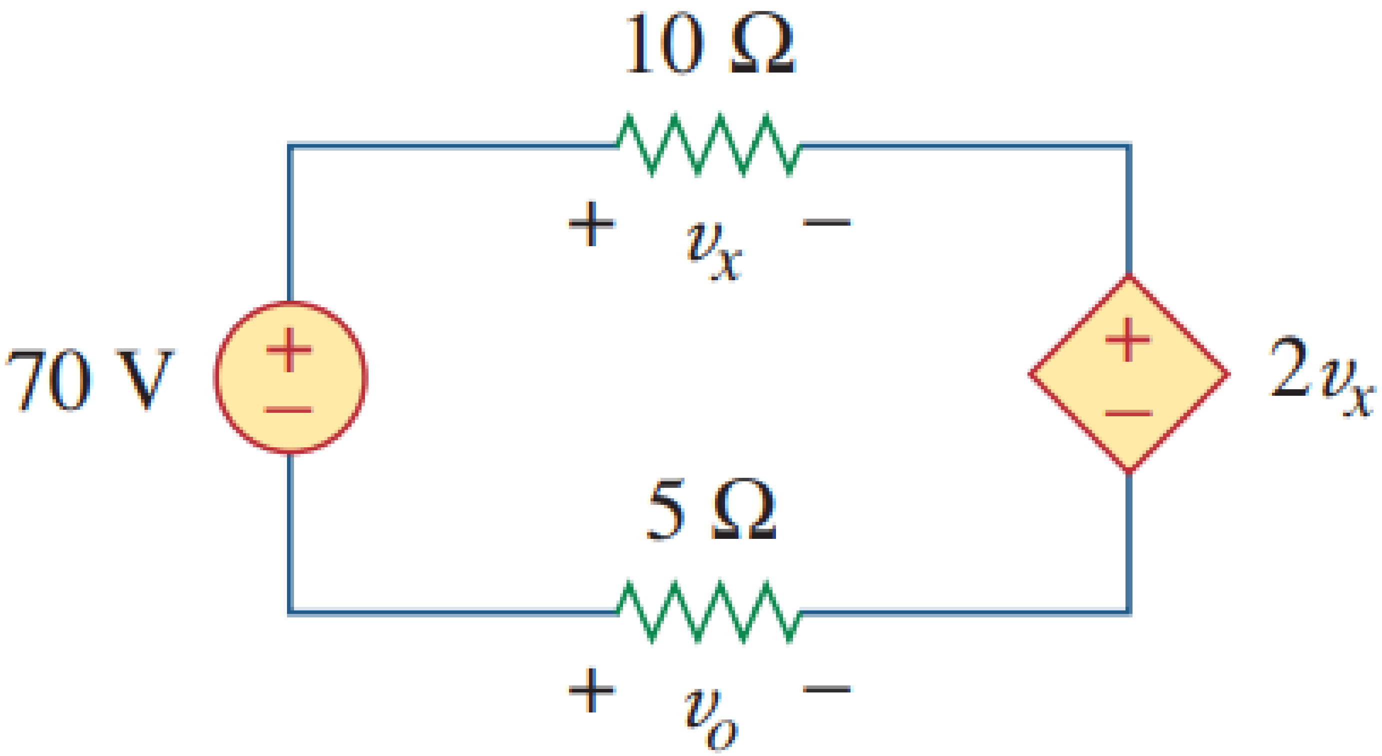
Substituting Eq. (2.6.2) into Eq. (2.6.1) yields

$$-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8 \text{ A}$$

and $v_o = 48$ V.

Reference: Sadiku_Example_2.6

Math Problem Practice:

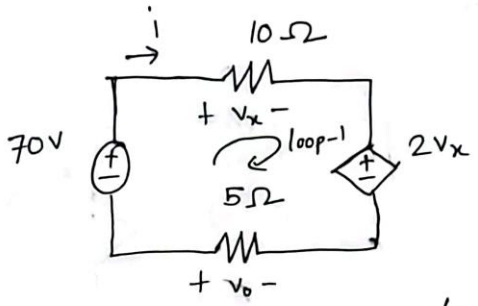


Find v_x and v_o in the circuit of Fig. 2.24.

Answer: 20 V, −10 V.

Reference: Sadiku_Practice_Problem_2.6

Math Problem Practice:



Applying KVL at loop-1 we get,

$$10i + 2v_x + 5i - 70 = 0$$

$$\Rightarrow 15i + 2v_x = 70 \quad \cdots (1)$$

Using Ohm's law we get,

$$V_x = 10i$$

$$\Rightarrow 10i - V_x = 0 \quad \cdots (2)$$

Solving (1) and (2) we get,

$$i = 2A, \quad V_x = 20V$$

Again, $V_o = -5i = -5 \times 2$

$$\therefore V_o = -10V$$

Reference: Sadiku_Practice_Problem_2.6

Math Problem Practice:

Practice Problem 2.7

Find v_o and i_o in the circuit of Fig. 2.26.

Answer: 12 V, 6 A.

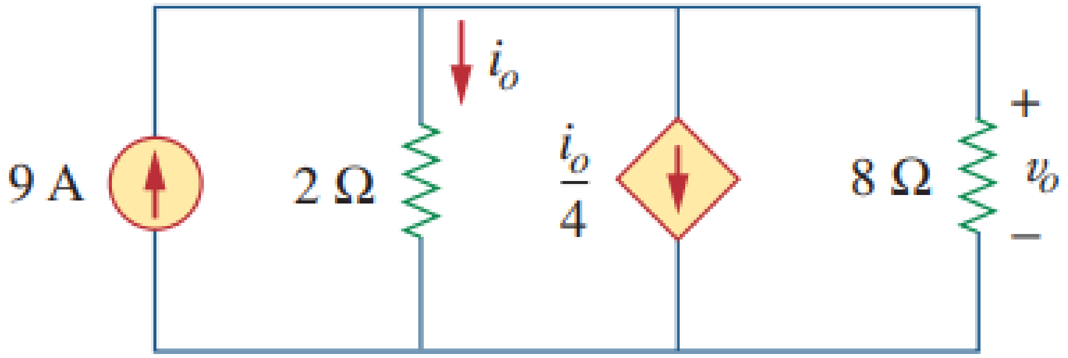
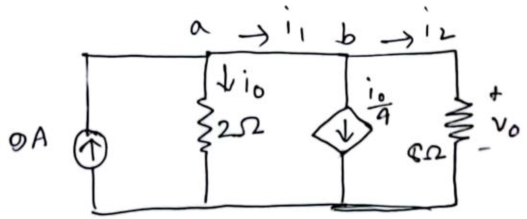


Figure 2.26
For Practice Prob. 2.7.

Reference: Sadiku_Practice_Problem_2.7

Math Problem Practice:



Applying KCL at a, we get,

$$9 = i_o + i_1 \quad \cdots (1)$$

Applying KCL at b, we get,

$$i_1 = \frac{i_o}{4} + i_2 \quad \cdots (2)$$

Using Ohm's law we get,

$$v_o = 8 i_2 \quad \cdots (3), \qquad v_o = 2 i_o \quad \cdots (4)$$

$$(2) \Rightarrow i_1 = \frac{i_o}{4} + \frac{v_o}{8} \quad \cdots (5)$$

$$(1) \Rightarrow 9 = i_o + \frac{i_o}{4} + \frac{v_o}{8}$$

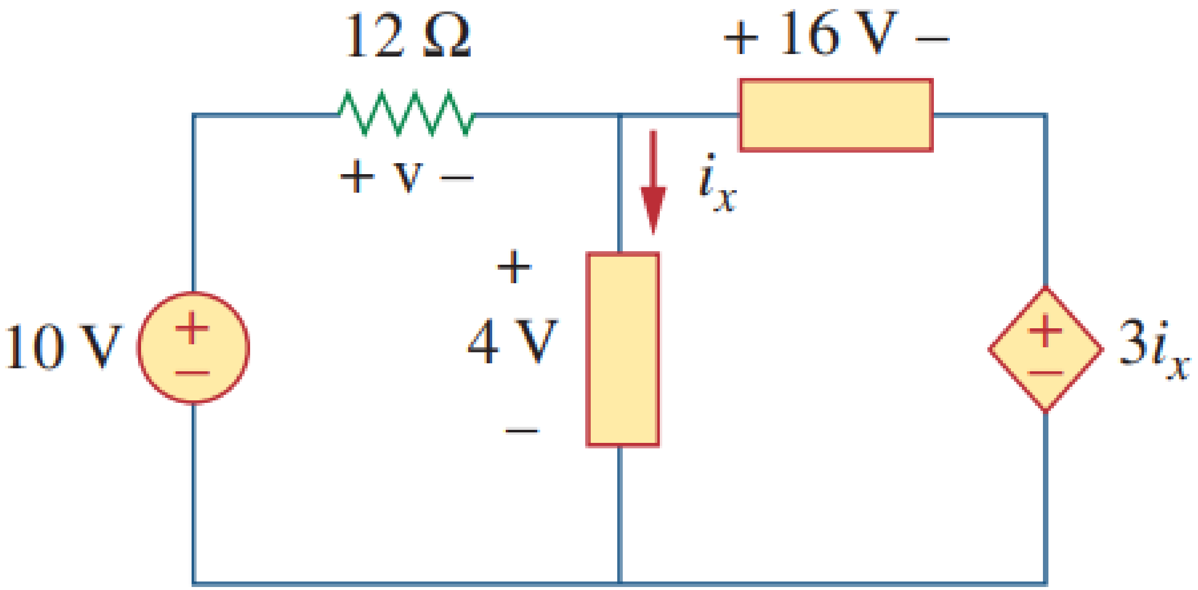
$$\Rightarrow 9 = \frac{5 i_o}{4} + \frac{v_o}{8} \quad \cdots (6)$$

Solving (4) and (6) we get, $i_o = 6A$, $v_o = 12V$

Reference: Sadiku_Practice_Problem_2.7

Math Problem Practice:

2.15 Calculate v and i_x in the circuit of Fig. 2.79.



Reference: Sadiku_Exercise_Problem_2.15

Math Problem Practice:

Step 2 of 4

Apply loops to the circuit as shown in Figure 2.

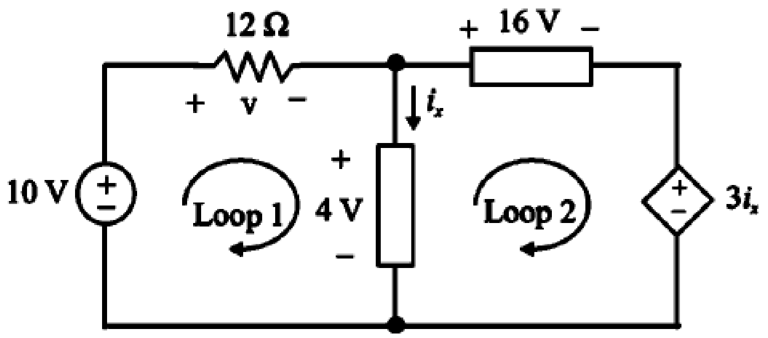


Figure 2

Step 3 of 4

Apply Kirchhoff's voltage law; the algebraic sum of all voltages around a closed path (or loop) is zero. Calculate the voltage v.

$$-10+v+4=0$$
$$v=6\text{V}$$

Therefore, the voltage v is $\boxed{6\text{V}}$.

Step 4 of 4

Apply Kirchhoff's voltage law, calculate the current i_x.

$$-4+16+3i_x=0$$
$$i_x=-4\text{A}$$

Therefore, the current i_x is $\boxed{-4\text{A}}$.

Reference: Sadiku_Exercise_Problem_2.15

Thank You