① 1

Vector

Summer 22

Tashin Parvez
011221437

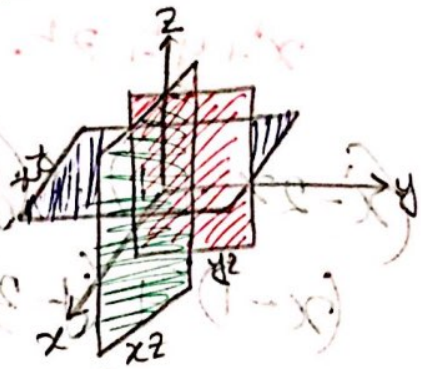
11.1

$xy \rightarrow (x, y, 0)$

$yz = (0, y, z)$

$xz = (x, 0, z)$

$x = (x, 0, 0)$



Distance Between 2 point

2D $P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

3D (x_1, y_1, z_1) (x_2, y_2, z_2) ~~(x_3, y_3, z_3)~~

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(2D) Circle equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

center (x_0, y_0)

radious

$x^2 + y^2 = r^2$

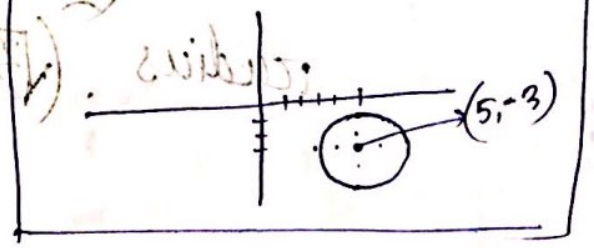
↳ center $(0, 0)$

(3D) Equation of Sphere:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

center

$(x-5)^2 + (y+3)^2 = 2^2$



$x^2 + (y+1)^2 + (z+4)^2 = 5$

↳ center $(0, -1, -4)$ radious $\sqrt{5}$

Scanned with CamScanner

$$\boxed{x^2+y^2+z^2+Gx+Hy+Iz+J=0}$$

Q: Find center & radious:

$$x^2+y^2+z^2-2x-4y+8z+17=0$$

$$(x^2-2x+1)+(y^2-2\cdot 2y+2^2)+(z^2+2\cdot 4z+4^2)=2^2$$

$$(x-1)^2+(y-2)^2+(z+4)^2=2^2$$

center $(1, 2, -4)$

radius 2

Exensise (23–28)

(25) $2x^2+2y^2+2z^2-2x-3y+5z-2=0$

$$x^2+y^2+z^2-x-\tfrac{3}{2}y+\tfrac{5}{2}z-1=0$$

$$\Rightarrow \left\{x^2-2\cdot\tfrac{1}{2}x+\left(\tfrac{1}{2}\right)^2\right\}+\left\{y^2-2\cdot\tfrac{3}{4}y+\left(\tfrac{3}{4}\right)^2\right\}+\left\{z^2+2\cdot\tfrac{5}{4}z+\left(\tfrac{5}{4}\right)^2\right\}=$$

$$\left(x-\tfrac{1}{2}\right)^2+\left(y-\tfrac{3}{4}\right)^2+\left(z+\tfrac{5}{4}\right)^2=\frac{27}{8}$$

Center $\left(\frac{1}{2}, \frac{3}{4}, -\frac{5}{4}\right)$

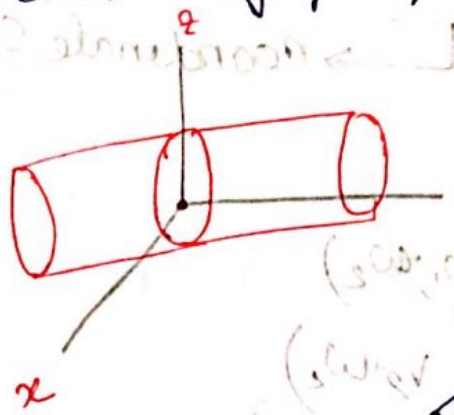
radius $\left(\sqrt{\frac{27}{8}}\right)$

$$\text{center}=\left(\frac{G}{-2}, \frac{H}{-2}, \frac{I}{-2}\right)$$

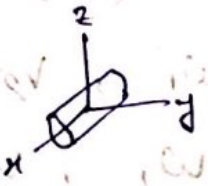
↳ G, H, I, included signs

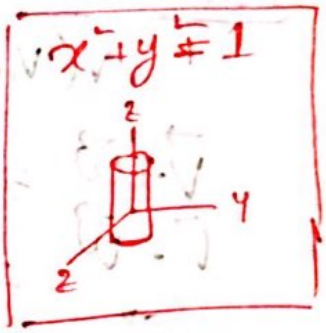
Scanned with CamScanner

Exm 3 Sketch the graph of $\boxed{x^2+z^2=1}$ in 3 space.



*যে Variable missing সেটা parallel হবে.

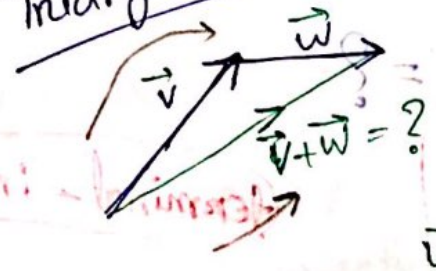




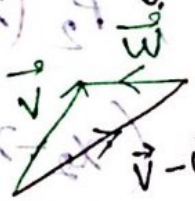
L2

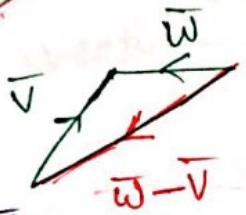
11.2 (Vector)

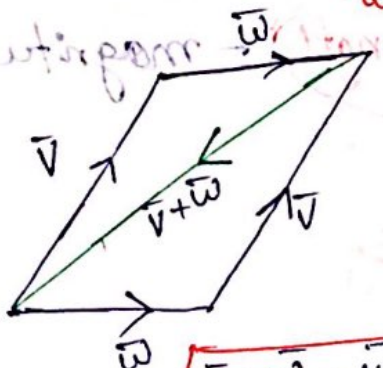
Triangle law



$\boxed{\text{magnitude} = \text{length}}$







$\boxed{\bar{v}+\bar{w}=\bar{w}+\bar{v}}$

* Direction of given 2 vector different the Resultant Vector (লব্ধি ভেক্টর) will be same direction of a vector and vector will be sum of different direction vector - same dim. vector.

Scanned with CamScanner

→ Vector form.

$\vec{V} = 2\hat{i} + 3\hat{j} = (2, 3)$

→ coordinate System

Arithmetic Operation

$V(V_1, V_2)$, $\omega(\omega_1, \omega_2)$

$\vec{V} + \vec{\omega} = (V_1 + \omega_1 , V_2 + \omega_2)$

$\vec{V} - \vec{\omega} = (V_1 - \omega_1 , V_2 - \omega_2)$

$K \cdot \vec{V} = (KV_1 , KV_2)$

↑ constant

$P_1(x_1, y_1)$; $P_2(x_2, y_2)$ $\vec{P_1P_2} = ?$

$$\boxed{\vec{P_1P_2} = (x_2 - x_1 , y_2 - y_1)}$$

terminal - initial

→ P; q two points

Norm Of a Vector

Norm = Length = magnitude

$\vec{V} = (2, 3, 4)$

$\|V\| = \sqrt{2^2 + 3^2 + 4^2}$

$= \sqrt{29}$

Scanned with CamScanner

Norm always Positive.

$$\|kV\| = |k|\,\|V\|$$

$\vec{N} = (-2, 3)$

$\|3V\| = |3|\,\|V\| = 3\|V\|$

$\|-2V\| = |-2|\,\|V\| = 2\|V\|$

$\|-4V\| = ?$

$|-4|\,\|V\| = 4(\sqrt{4+9})$

$= 4\sqrt{13}$

Unit Vector: Length $\boxed{1 = \text{Norm}}$

Norm of i vector $\sqrt{1^2+0+0}$

$= 1$

j vector $(0, 1, 0)$

i vector $(1, 0, 0)$

Normalizing Vector: $\vec{u} = \frac{1}{\|V\|}\vec{V}$

always Unit Vector

Norm

Same direction of $\vec{V}$

Q: Find a $\boxed{\text{unit vector}}$ that has the same direction

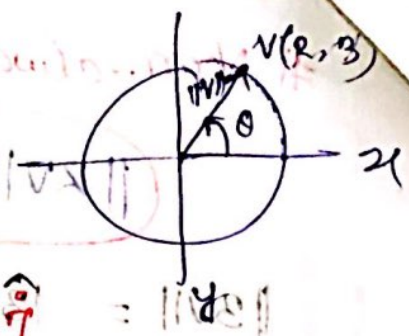
$V = 2i + 3j - k$

$\|V\| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$\vec{u} = \frac{1}{\sqrt{14}}(2i + 3j - k)$

Scanned with CamScanner

V. determined by length & angle



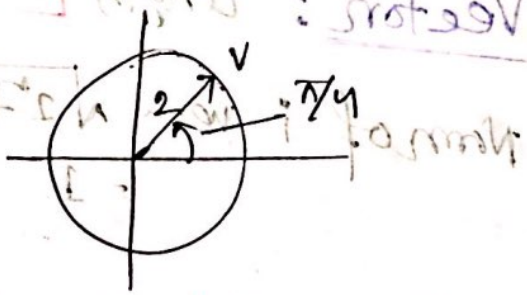
$$V = \|V\| \cdot (\cos\theta, \sin\theta)$$

OR

$$V = \|V\| \cos\theta\,\hat{i} + \|V\| \sin\theta\,\hat{j}$$

θ = angle between vector & x axis

Q: Find a vector of length 2 & makes angle π/4 with x axis.



$$V = 2\cos\frac{\pi}{4}\hat{i} + 2\sin\frac{\pi}{4}\hat{j}$$

$$= \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$$

** **Q:** Find the angle with x axis, $\vec{V}(-\sqrt{3}\hat{i} + \hat{j})$ makes.

$\|V\| = 2$

$$\vec{V} = \|V\| (\cos\theta, \sin\theta)$$

$$\frac{\vec{V}}{\|V\|} = (\cos\theta, \sin\theta)$$

$$\left(\frac{-\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right) = (\cos\theta, \sin\theta)$$

$$\therefore \cos\theta = \frac{-3}{2}$$

$$\theta = \cos^{-1}(-3/2)$$

$$= 150^\circ = 5\pi/6$$

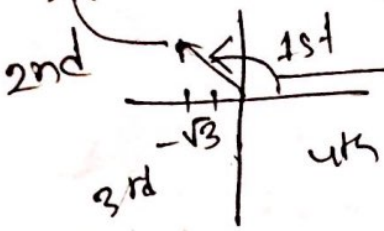
$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}(1/2)$$

$$= 30^\circ$$

Scanned with CamScanner

$V = -\sqrt{3}\,\hat{i} + \hat{j}$



$\rightarrow 150^\circ = \theta > 90$

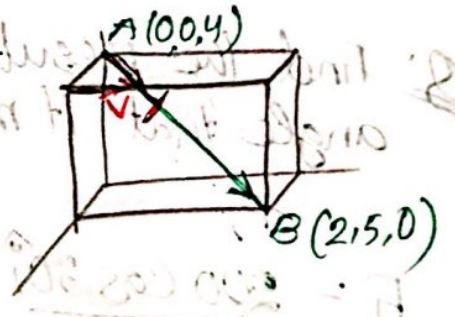
* Vector যে Quadant এ পড়ে Ans সেই Quadrant এ θ.

So ans $5\pi/6$ Ans

L3

$\tan^{-1}$ দিয়ে করলে Ans ভুল হবে।

** Q: $\|V\| = \sqrt{5}$ find vector (V)



$\vec{AB} = (2-0,\ 5-0,\ 0-4)$

$= (2, 5, -4)$

Unit Vector.

$\vec{u} = \frac{1}{\|AB\|}\vec{AB}$

$\|AB\| = \sqrt{2^2+5^2+4^2}$

$= 45 = 3\sqrt{5}$

$= \left(\frac{2}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}\right)$

$V = \|V\| \cdot \vec{u}$

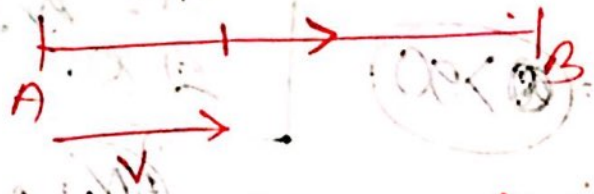
$= \sqrt{5}\left(\frac{2}{3\sqrt{5}}, \frac{5}{3\sqrt{5}}, \frac{-4}{3\sqrt{5}}\right)$

$= \left(\frac{2}{3}, \frac{5}{3}, \frac{-4}{3}\right)$

Usually we use unit vector to get direction

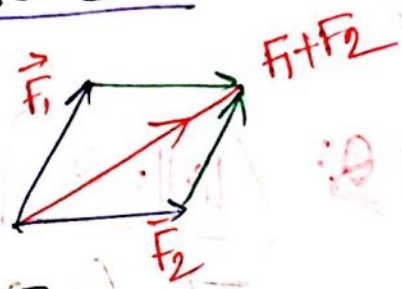
Value = 1

Scanned with CamScanner

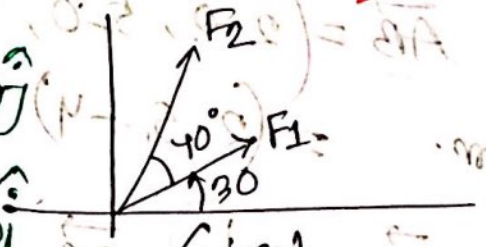


$V = \|V\| \cdot \underbrace{\text{unit vector of } (\vec{AB})}_{\text{Normalizing Vector.}}$

Resultant of two Concurrent Force



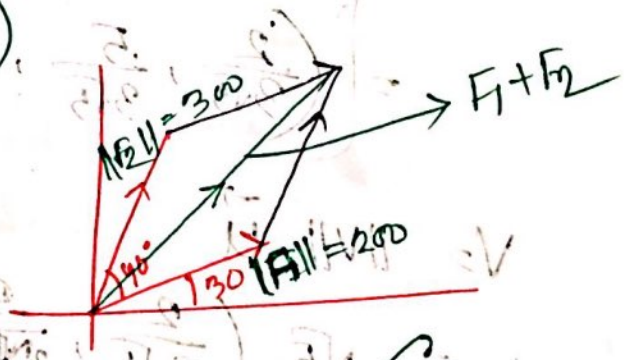
Q: Find the resultant & the angle that it makes with x axis



$F_1 = 200 \cos 30\hat{i} + 200 \sin 30\hat{j}$

$F_2 = 300 \cos 70\hat{i} + 300 \sin 70\hat{j}$ (given)

$F_1 + F_2 = (275.87 \;,\; 381.90)$



$\|F_1 + F_2\| = 471.08$

$\theta = \cos^{-1}\left(\frac{275.87}{471.08}\right)$ or $\theta = \sin^{-1}\left(\frac{381.90}{471.08}\right)$

$= 54.16$ $\qquad = 53.95$

Similar.

Scanned with CamScanner

L3

DOT. Product

11.3

Dot Product (scalar value)

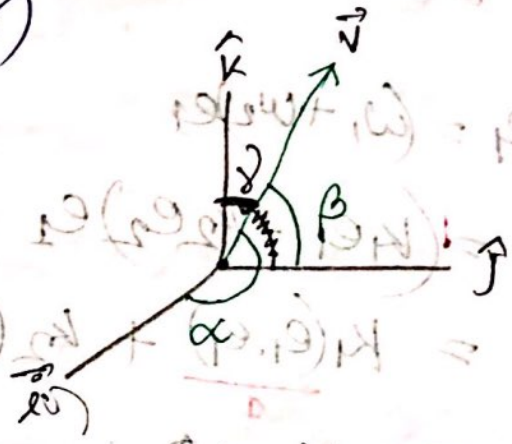
$$u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$$

Angle between Vectors

$$u \cdot v = \|u\| \, \|v\| \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\| \, \|v\|}\right)}$$

Direction Angles (α, β, γ)



$\cos\alpha = \dfrac{\vec{v} \cdot \hat{i}}{\|v\| \, \|i\|} = \dfrac{v_1}{\|v\|}$

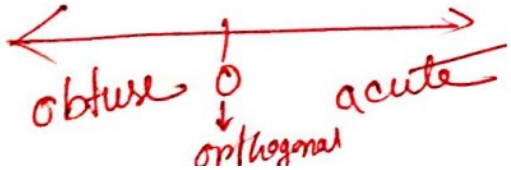
$\cos\beta = \dfrac{\vec{v}\,\hat{j}}{\|v\| \, \|j\|} = \dfrac{v_2}{\|v\|}$

$\cos\gamma = \dfrac{\vec{v}\,\hat{k}}{\|v\| \, \|k\|} = \dfrac{v_3}{\|v\|}$

$$\begin{cases} v \cdot u < 0 = \text{obtuse} \\ u \cdot v > 0 = \text{acute} \\ v \cdot u = 0 = \text{orthogonal} \end{cases}$$

$$\|v\| = \sqrt{v \cdot v}$$

$$v \cdot v = \|v\|^2$$

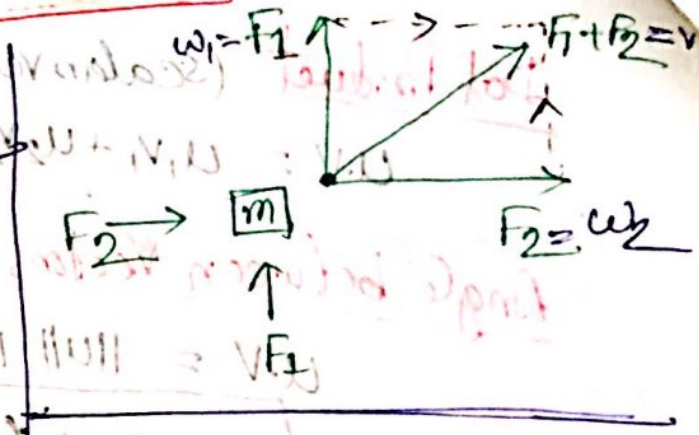


Scanned with CamScanner

Decomposing Vector Into Orthogonal Components

$\rightarrow 90^\circ$

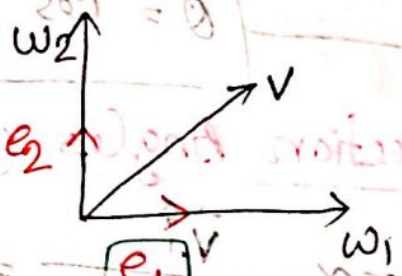
we know V we have to make it into two vectors



$$\boxed{V = \omega_1 + \omega_2}$$

$\omega_1 = k_1 e_1$

$\omega_2 = k_2 e_2$



$\rightarrow$ Unit vector.

$$V.e_1 = (\omega_1 + \omega_2)e_1$$

$$= (k_1 e_1 + k_2 e_2)e_1$$

$$= k_1\underbrace{(e_1.e_1)}_{1} + k_2\underbrace{(e_2.e_1)}_{0}$$

$\rightarrow$ Dot product

$$= k_1 + 0$$

$$= k_1$$

similarly, $V.e_2 = k_2$

$$V = \omega_1 + \omega_2$$

$$= k_1 e_1 + k_2 e_2$$

$$V = \underbrace{(V.e_1)}_{\text{scalar}} e_1 + (V.e_2)e_2$$

$\rightarrow$ vector

$\overbrace{V.e_1}^{\text{Dot}}$ = is called scalar Component of v along e_1

$V.e_2$ = " " " " of V " e_2

$w_1 = (V.e_1)e_1$ " " Vector " of v along e_1

$w_2 = \underbrace{(V.e_2)e_2}_{\text{cross}}$ " " " " of V " e_2

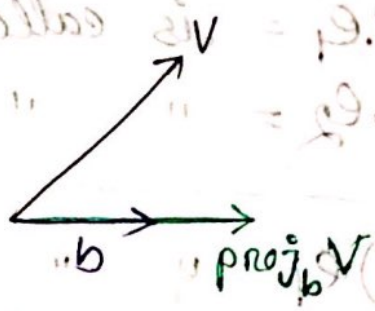
Scanned with CamScanner

Orthogonal Projections

One vector representation of another vector

$$\text{Proj}_{e_1} v = (v \cdot e_1)\, e_1$$

$$\text{Proj}_{e_2} v = (v \cdot e_2)\, e_2$$



$$\boxed{\text{proj}_e v = (v \cdot e)\, e}$$

unit vector

projection of v on b

If e is not a unit vector

$$\text{proj}_e v = \left(v \cdot \frac{\vec{e}}{\|e\|}\right)\left(\frac{\vec{e}}{\|e\|}\right)$$

$$\boxed{\text{proj}_e v = \frac{v \cdot \vec{e}}{\|e\|^2}\, \vec{e}}$$

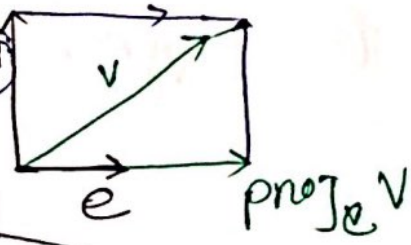
not a unit vector

Orthogonal projection of v on e

parallel to e

Orthogonal projection

$v - \text{proj}_e v$



vector component of v orthogonal to e

orthogonal to e.

Scanned with CamScanner

Q: Find the orthogonal projection of $\boxed{V = i + j + k}$ on $\boxed{b = 2i + 2j}$ and then find the vector component of V orthogonal to b.

$$V \cdot b = (2 \times 1) + (2 \times 1) + 0$$

$$= 4$$

$$\|b\| = \sqrt{4+4}$$

$$= \sqrt{8}$$

O.P $\longrightarrow$ $\text{proj}_b V = 4 \cdot (2i + 2j) \times \frac{1}{\|b\|^2}$

$$= \frac{8i + 8j}{8} = i + j$$

$$\rightarrow \frac{V \cdot b}{\|b\|^2} \cdot b$$

V.C $\longrightarrow$ $\boxed{V - \text{proj}_b V} = (i + j + k) - (i + j)$

$$= k$$

$$= (0, 0, 1)$$

Scanned with CamScanner

Work

W = Force × distance.

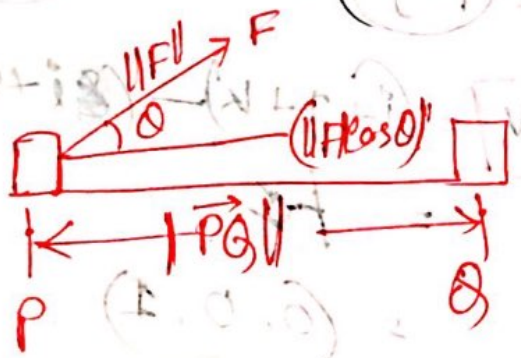
Scalar

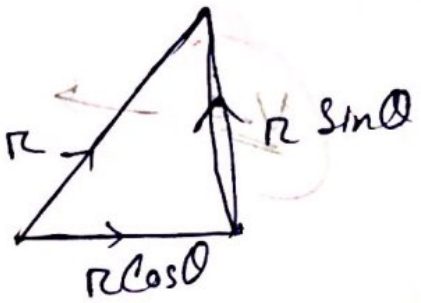
$= Fd$

$= \|F\| d$

$= \|F\| \, \|\overrightarrow{PQ}\|$ ⟶ from P to Q (move)

$= (\|F\| \cos\theta) \cdot \|\overrightarrow{PQ}\|$





Scanned with CamScanner

Cross Product

$u \times v = -(v \times u)$

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$i \times j = k \quad j \times k = i \quad k \times i = j$

$j \times i = -k \quad k \times j = -i \quad i \times k = -j$

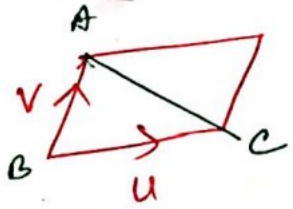
$\boxed{u \times v = -(v \times u)}$

$\boxed{v \times v = 0}$

$\boxed{u \times v = w \quad u \perp w \quad v \perp w}$ → orthogonal.

$$\boxed{\begin{aligned} u \cdot \underbrace{(u \times v)}_{w} &= 0 \\ v \cdot \underbrace{(u \times v)}_{w} &= 0 \end{aligned}}$$

Area of a parallelogram that has u & v as adjacent sides is $A = \|u \times v\|$



$\frac{1}{2}\|A\|$ = Area of triangle

$\|u \times v\| = \|u\| \, \|v\| \sin\theta.$

Scanned with CamScanner

Scalar triple product

$u, v, w \rightarrow u \cdot (v * w)$ | serial maintain

$v, w, u \rightarrow v \cdot (w \times u)$

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

Volume of Parallelepiped that has u, v, w

$$V = |u \cdot (v \times w)|$$

$u \cdot (v \times w) = 0$ means they lie in the same plane.

$0 = (v \times u) \cdot u$

$0 = (v \times u) \cdot v$

$\boxed{\text{Ex - 17 - 27}}$

Scanned with CamScanner

Parametric Eqn

$x = x_0 + t$

$ax + by + cz + d = 0$

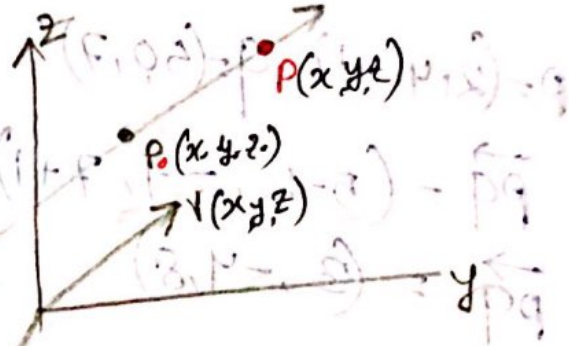
Scanned with CamScanner

L5

Chapter - 11.5

Line determined by a point & a vector:-

P_0 & $\vec{V}$ given → parallel vector.



$$\vec{P_0P} = t\vec{V}$$

$$(x-x_0,\ y-y_0,\ z-z_0) = (ta,\ tb,\ tc)$$

$$x-x_0 = ta \quad y-y_0 = tb \quad z-z_0 = tc$$

$$\boxed{x = ta + x_0,\quad y = tb + y_0,\quad z = tc + z_0}$$

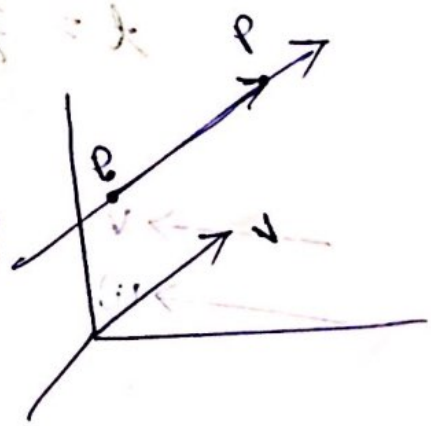
from parallel vector

parametric equation of straight line.

Q: Find parametric equation of a line passing through $(4,2)$ & parallel to $V = (-1,5)$.

x_0 y_0

$P_0 = (4,2) \qquad P = (x,y)$



$$\vec{P_0P} = (x-4,\ y-2)$$

$$\vec{P_0P} = t\vec{V}$$

$$(x-4,\ y-2) = (-1t,\ 5t)$$

$$(x-4 = -1t) \qquad y-2 = 5t$$

$$\Rightarrow x = 4-1t \qquad y = 2+5t$$

Ans.

$$x = x_0 + at$$

Scanned with CamScanner

* **Q:** find p.equ of line pass point $(2,4,-1)$ & ~~parallel~~ $(5,0,7)$ points. & Does it intercept xy plane?

$p=(2,4,-1)$ $q=(5,0,7)$

$\vec{pq} = (5-2, -4, 7+1)$

$\boxed{\vec{pq} = (3,-4,8)}$

* One vector is parallel to it's own vector (itself)

$P_0 = (x, y, z)$

$\vec{PP_0} = (x-2, y-4, z+1)$

$x-2=3t$ $y-4=-4t$ $z+1=8t$

$\boxed{x=3t+2 \quad y=4-4t \quad z=8t-1}$

In intercept ~~pot~~ point in xy plane $z=0$

$z=8t-1$

$0=8t-1$

$t=1/8$

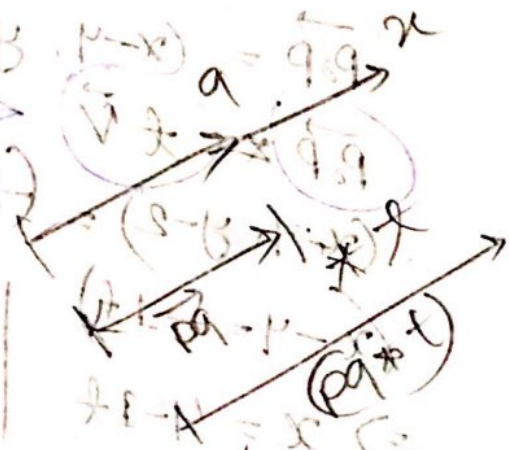
$x=\frac{3}{8}+2=19/8$

$y=4-\frac{4}{8}=7/2$

$\vec{v}$

$\vec{w}$

$\boxed{v=tw}$



xy plane intersect
কী করে কী 2টি t এর মান?

Example 3

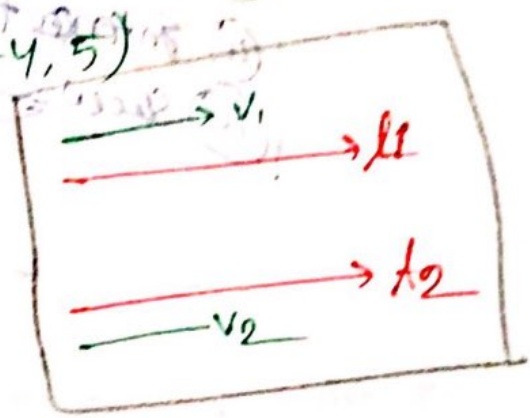
Line 1 $\boxed{x = 1+4t,\ y = 5-4t,\ z = -1+5t}$

line 2 $x = 2+8t,\ y = 4-3t,\ z = 5+t$

are they parallel?

parallel vector 1

parallel vector 2 $(8, -3, 1)$ v_2 $v_1 = (4, -4, 5)$



$v_1 \neq t\, v_2$

scalar value.

So line 1 & 2 are not parallel.

does the line intersect?

Assume intersect point $P(x_0, y_0, z_0)$

$x_0 = 1+4t_1$ $y_0 = 5-4t_1$ $z_0 = -1+5t_1$ $P(x_0, y_0, z_0)$

$x_0 = 2+8t_2$ $y_0 = 4-3t_2$ $z_0 = 5+t_2$

$\therefore$ $1+4t_1 = 2+8t_2$
$5-4t_1 = 4-3t_2$
$-1+5t_1 = 5+t_2$ } solving $(t_1, t_2) = (1/4, 0)$

$-1+5(1/4) = 5+0$ [t_1 & t_2 value used]

$\Rightarrow \frac{1}{4} \neq 5$ so they don't intersect.

Scanned with CamScanner

Skew Lines

In 2D

(i) Parallel
(ii) Intersect

In 3D

(i) parallel
(ii) Intersect
(iii) Skew → Not parallel
→ Not Intersect.

Scanned with CamScanner

Vector equation of lines

$x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$ → parametric equ.

$(x, y, z) = (x_0 + at,\ y_0 + bt \ldots)$

$r = \underbrace{(x_0, y_0, z_0)}_{r_0} + t\underbrace{(a, b, c)}_{V}$ → parallel vec

$\boxed{r = r_0 + tV}$ → vector equation.

$(x, y, z) = (x_0, y_0, z_0) + t(a, b, c)$

Ex: (5-8), (15-16), (19-22), (25-28), (29, 34), (35-38)

$x = x_0 + at$

point

$\langle x, y \rangle = \langle x_0 + at,\ y_0 + bt \rangle$

$\langle x, y \rangle = \langle x_0, y_0 \rangle + \langle a, b \rangle t$

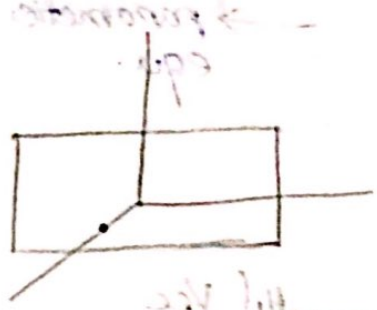
$r = r_0 + vt$

vector

Scanned with CamScanner

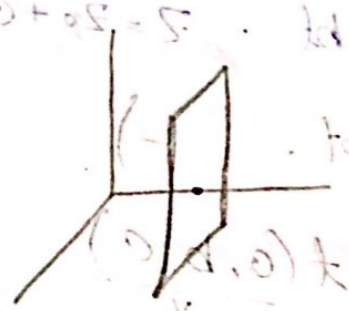
L6

Chapter: 11.6



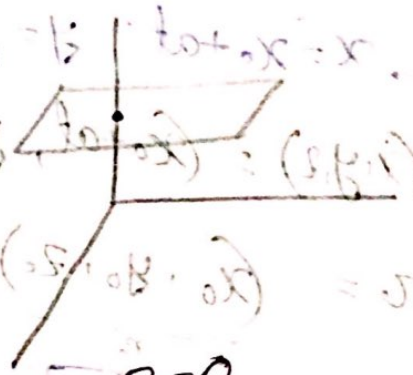
$x = a$
parallel to ~~xy~~ plane
yz

$(a, 0, 0)$



$y = b$
xz plane

$(0, b, 0)$



$z = c$
x,y plane

$(0, 0, c)$

Plane determined by a point & a normal vector

- A point in the plane (P_0)
- A vector perpendicular to the plane (n) → normal vector

One vector is orthogonal to the other vector means product of vector's is 0 (zero).

P another point in the plane.

$(P - P_0)$ is parallel to the plane & orthogonal with n vector.

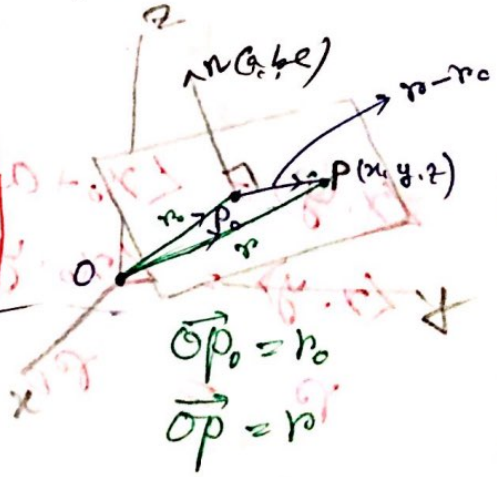
Two vectors is orthogonal if they are perpendicular to each other

$$n \cdot (P - P_0) = 0$$

$$(a, b, c) \cdot (x - x_0, y - y_0, z - z_0) = 0$$

$$\boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

plane equation.



$\vec{OP_0} = r_0$

$\vec{OP} = r$

Scanned with CamScanner

Q: Find an equation of a plane passing through the point $(3,-1,7)$ & perpendicular to the vector $n(4,2,-5)$

$$equ = 4(x-3)+2(y+1)-5(z-7)=0$$

$$\Rightarrow 4x+2y-5z+25=0$$

normal vector $(4,2,-5)$

$$ax+by+cz+d=0$$

normal vector (a,b,c)

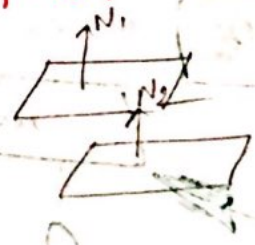
L-7

Q: Determine wheather the $\boxed{\text{planes are parallel}}$

$P_1 = 3x-4y+5z=0$ & $P_2 = -6x+8y-10z-4=0$

Normal of $P_1 = (3,-4,5) = N_1$

$P_2 = (-6,8,-10) = N_2$



N_2 is scaler multiple of N_1 means Normal vectors are parallel.

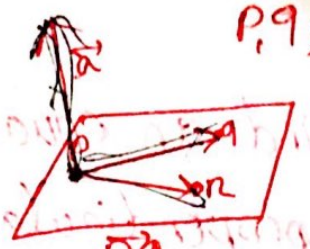
So, P_1 & P_2 are parallel.

Scanned with CamScanner

#

Exam: B

3 points on a plane ~~pass~~ (p, q, r)
equ of plane?



$\vec{pq}$
$\vec{pr}$
$a = \vec{pq} \times \vec{pr}$

Ex: 4

Ex: 5

Intersecting Plane

normal Vectors.

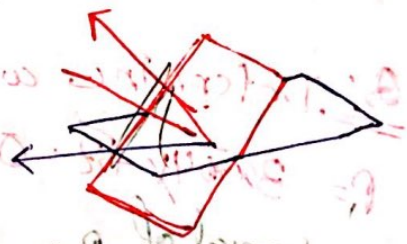
$$\cos\theta = \frac{|n_1 \cdot n_2|}{\|n_1\| \, \|n_2\|}$$

Ex 6

Ex 7: find intersection $\boxed{\text{Line equation}}$ of 2 plane:

$2x - 4y + 4z = 6$ & $6x + 2y - 3z = 4$

$n_1 = (2, -4, 4)$ $n_2 = (6, 2, -3)$



(1) $\boxed{n_1 \times n_2} = (\quad)$

$\boxed{\text{parallel vector of the line}}$

(11) we need a intercept point

(14) Line intercept xy plane so, there $z = 0$

then from 2 equ of plane get (x, y)

you have (point & parallel vector)

$x = (x_0 + at)$

Line
(1) point
(11) parallel vector

Distance between a plane & a point

$(ax+by+cz+d=0)$ (x_0, y_0, z_0)

$$\boxed{D = \frac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2+b^2+c^2}}}$$

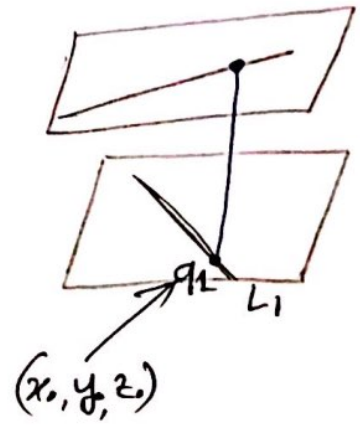
(P)

Ex: 10 Q: Show that given lines are skew. & find distance between them.

L_1: $x = 1+4t$ $y = 5-4t$ $z = -1+5t$

L_2: $x = 2+8t$ $y = 4-3t$ $z = 5+t$

need
(I) point
(II) plane equ.



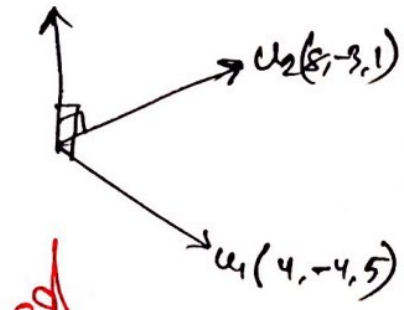
point in L_1

$t=0$

point $(x_0=1, y_0=5, z_0=-1)$

$$n = u_1 \times u_2 = \begin{vmatrix} i & j & k \\ 4 & -4 & 5 \\ 8 & -3 & 1 \end{vmatrix}$$

$$= 11i + 36j + 20k$$



need
plane equ
point

Scanned with CamScanner

L8

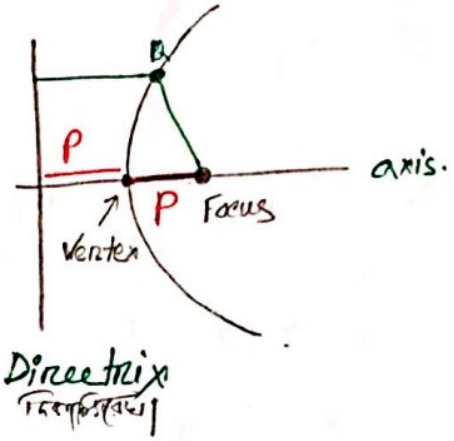
Ch-10.4

Parabola

যার মধ্যে square এর তফাৎ থাকে না.
যার মধ্যে symmetric এর তফাৎ থাকে না

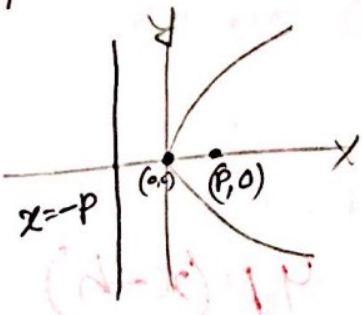
p = distance between vertex & focus / Directrix & vertex distance.

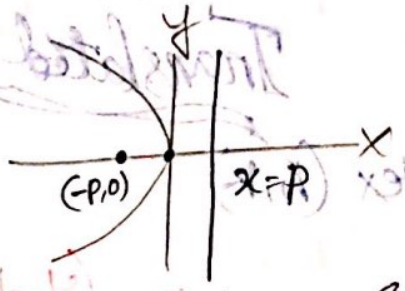
means vertex is the midde point of Directrix & focus.



Equation

$\boxed{y^2 = 4px}$

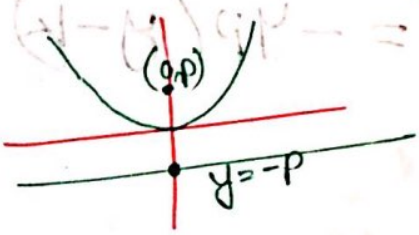


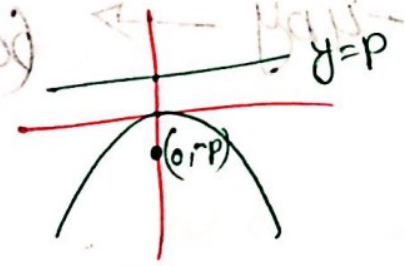


$\boxed{y^2 = -4px}$

* x Axis এর সাপেক্ষে Symmetric. so equ = y^2
* positive portion এ open $+4px$ / $4py$
* Negitive " " $-4px$ / $-4py$

$\boxed{x^2 = 4py}$



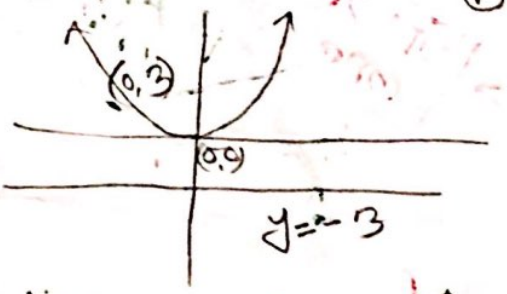


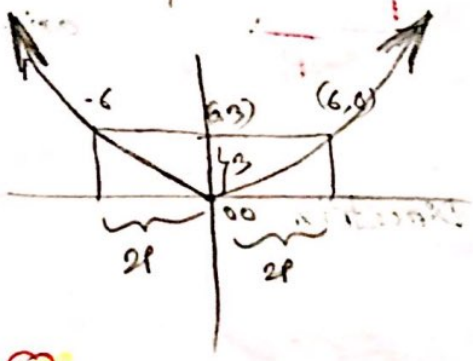
$\boxed{x^2 = -4py}$

* y axis এ open so x^2
* { positive → open $4py$
  Negative → $-4py$

Scanned with CamScanner

Q: $x^2 = 12y = 4.3y$ (P)

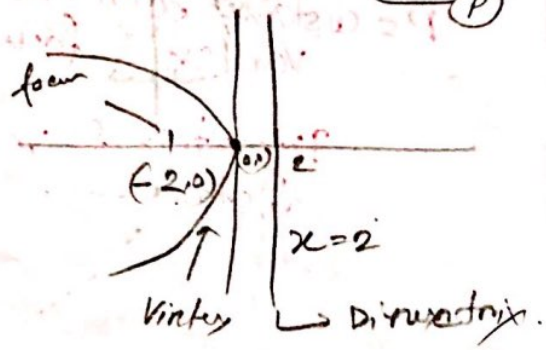




Q: $y^2 + 8x = 0$

$y^2 = -8x$

$= -4.2x$ (P)



online (L)

Translated

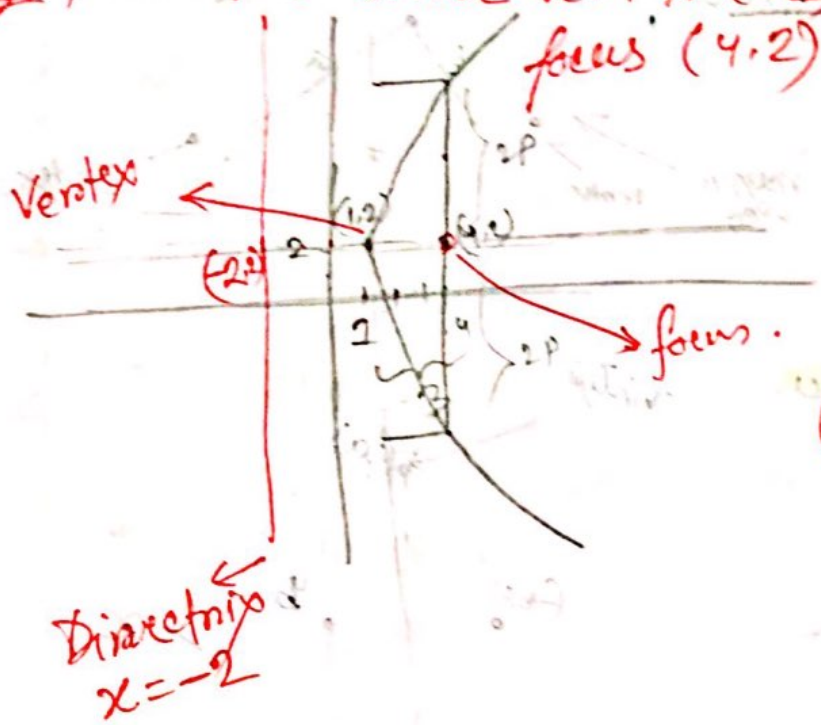
when vertex (h,k)

$$\begin{cases} y^2 = 4px \longrightarrow (y-k)^2 = 4p(x-h) \\ y^2 = -4px \longrightarrow (y-k)^2 = -4p(x-h) \end{cases}$$

$$\begin{cases} x^2 = 4py \longrightarrow (x-h)^2 = 4p(y-k) \\ x^2 = -4py \longrightarrow (x-h)^2 = -4p(y-k) \end{cases}$$

Scanned with CamScanner

Q: parabola with vertex (1,2)
focus (4,2)



$p = 4-1$
$= 3$

$(y-2)^2 = 4\times3(x-1)$

Q $y^2-8x-6y-23=0$ Describe the graph equ

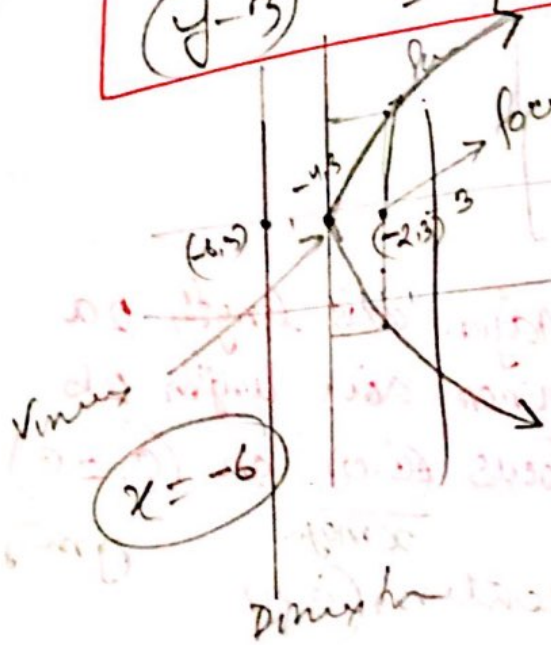
$y^2-2\cdot3y+9-8x-32=0$

$(y-3)^2 = 8x+32$

$= 8(x+4)$

$\boxed{(y-3)^2 = 4\cdot2\cdot(x+4)}$

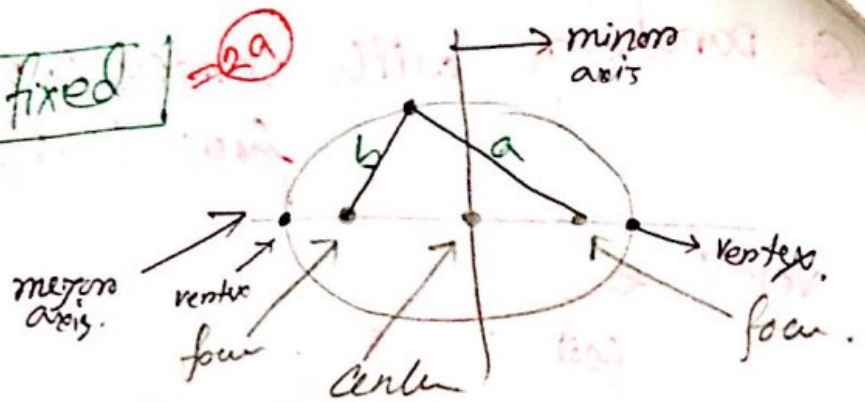
focus $(-2,3)$
vertex $(-4,3)$
$p=2$



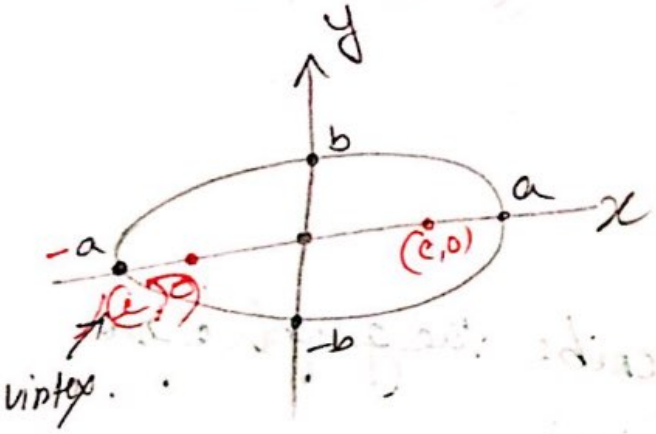
Scanned with CamScanner

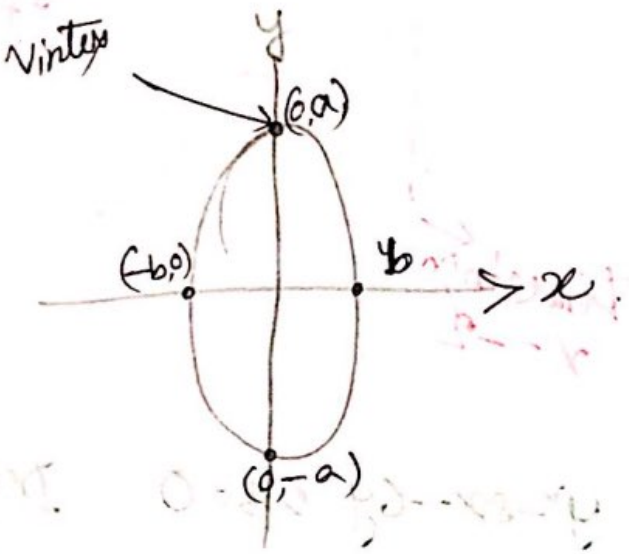
Ellipse

$\boxed{a+b = \text{fixed}}$ = 2a



Standard equ.





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

↳ x axis is major so a is with x.

$$\frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$$

↳ major y axis.

$$\boxed{a = \text{vertex}}$$

$$\boxed{c = \sqrt{a^2 - b^2}}$$

(focus)

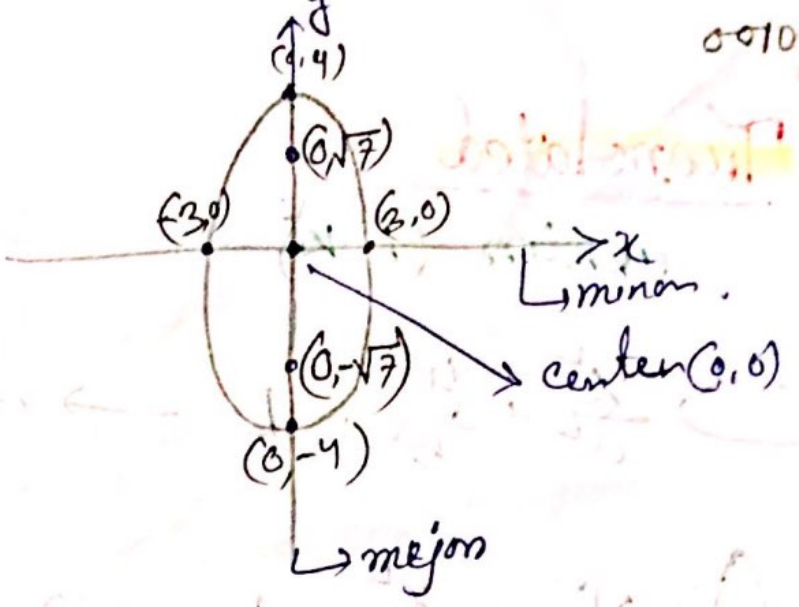
Major axis length 2a
minor axis length 2b
focus $(\pm c, 0)$ or $(0, \pm c)$
x major y major.
center $(0,0)$

Scanned with CamScanner

Q: $\frac{x^2}{9} + \frac{y^2}{16} = 1$

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1$$

b, a



$$c = \sqrt{a^2 - b^2}$$
$$= \sqrt{16-9}$$
$$= \sqrt{7}$$

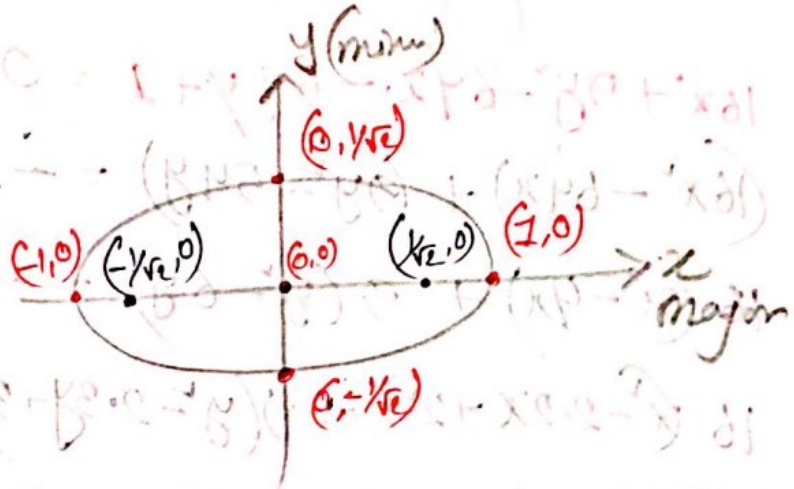
Major axis $= 2 * 4 = 8$

minor axis $= 2 * 3 = 6$

Q: $x^2 + 2y^2 = 1$

$$\frac{x^2}{1^2} + \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1$$

a, b

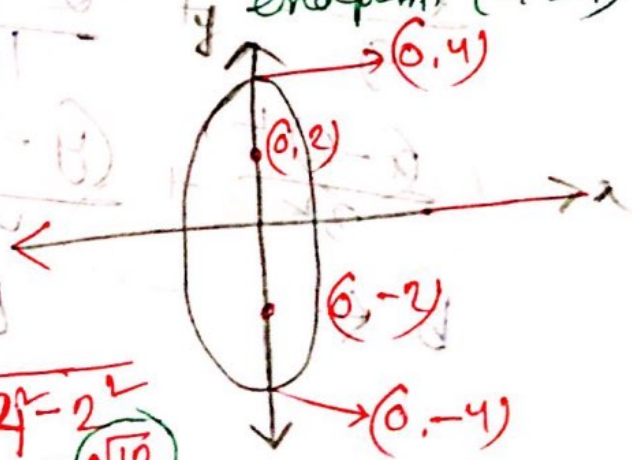


$$c = \sqrt{a^2 - b^2}$$
$$= 1/\sqrt{2}$$

$$\begin{cases} \text{major Axis} = 2 \times 1 = 2 \\ \text{Minor Axis} = 2 \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times 2 \\ \qquad = \sqrt{2} \end{cases}$$

Q: Find ellips equ focus $(0, \pm 2)$, major axis endpoint $(0, \pm 4)$

$$\frac{y^2}{4^2} + \frac{x^2}{(\sqrt{12})^2} = 1$$



$$c = \sqrt{a^2 - b^2}$$
$$2^2 = 4^2 - b^2 \Rightarrow b = \sqrt{4^2 - 2^2}$$
$$= \sqrt{12}$$

Scanned with CamScanner

(28)

Translated

center (h,k)

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \longrightarrow \frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1 \longrightarrow \frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$$

Q: $16x^2+9y^2-64x-54y+1=0$

$\hookrightarrow (16x^2-64x)+(9y^2-54y)=-1$

$\hookrightarrow 16(x^2-4x)+9(y^2-6y)=-1$

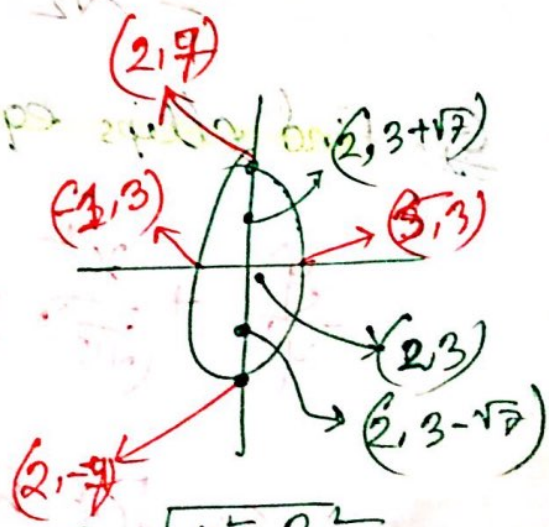
$16(x^2-2\cdot 2x+2^2)+9(y^2-2\cdot 3y+3^2)=-1+\underbrace{64}_{(2^2\times 16)}+\underbrace{81}_{3^2\times 9}$

$$\frac{(x-2)^2}{\left(\frac{1}{16}\right)}+\frac{(y-3)^2}{\left(\frac{1}{9}\right)}=144$$

$$\frac{(x-2)^2}{9}+\frac{(y-3)^2}{16}=1$$

$$\frac{(x-2)^2}{3^2}+\frac{(y-3)^2}{4^2}=1$$

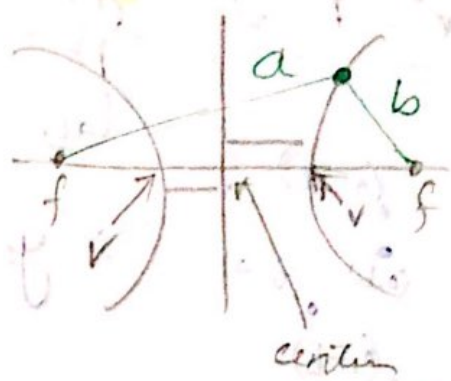
$b \leftarrow$ $\quad$ $\hookrightarrow a$



$c=\sqrt{4^2-3^2}$
$=\sqrt{7}$

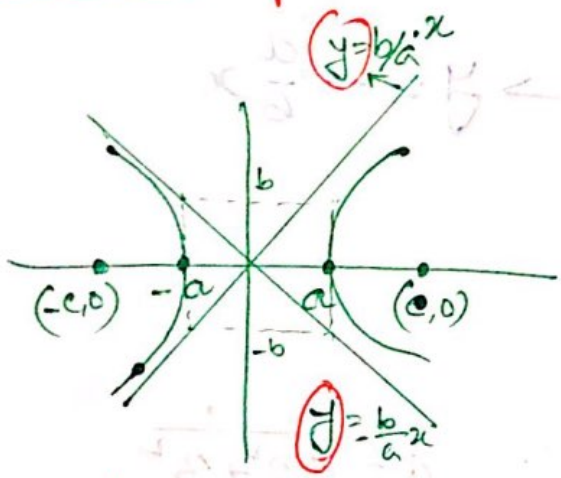
Scanned with CamScanner

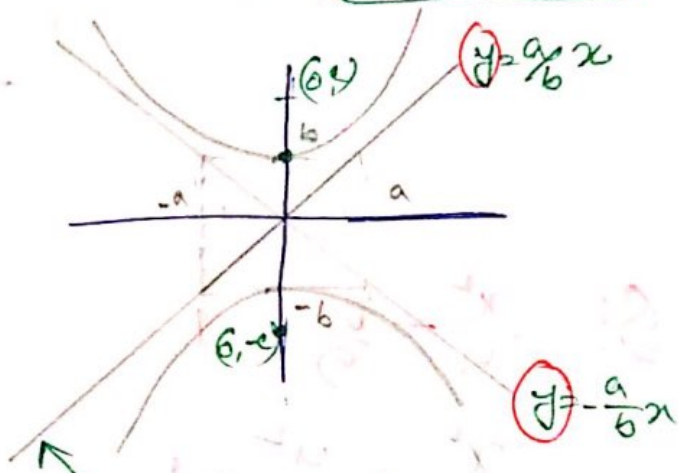
Hypenbola



Standard equation

$a - b = \text{fixed}$





$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

যার: আগে Negative এবং touch করবে না.

$\boxed{a, b \text{ এর বড় সমস্যা নাই}}$

$c = \sqrt{a^2 + b^2}$

x এর নিচে a fixed
y এর নিচে b

then asymtotic $= y = \pm \frac{b}{a} x$

Scanned with CamScanner

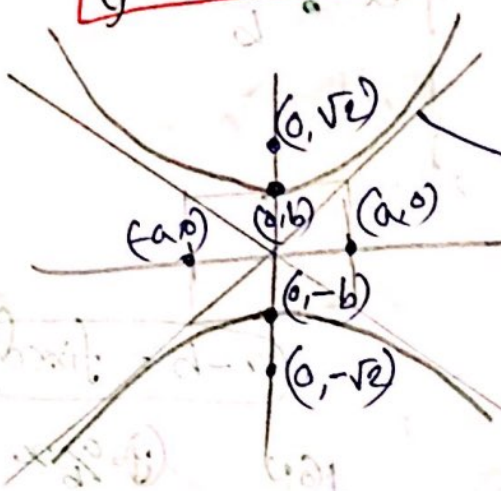
Exm 5

$\boxed{y^2 - x^2 = 1} \Rightarrow \dfrac{y^2}{1^2} - \dfrac{x^2}{1^2} = 1$

$b \quad a$

$c = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$



$\longrightarrow y = \dfrac{a}{b}x$

$\longrightarrow y = -\dfrac{a}{b}x$

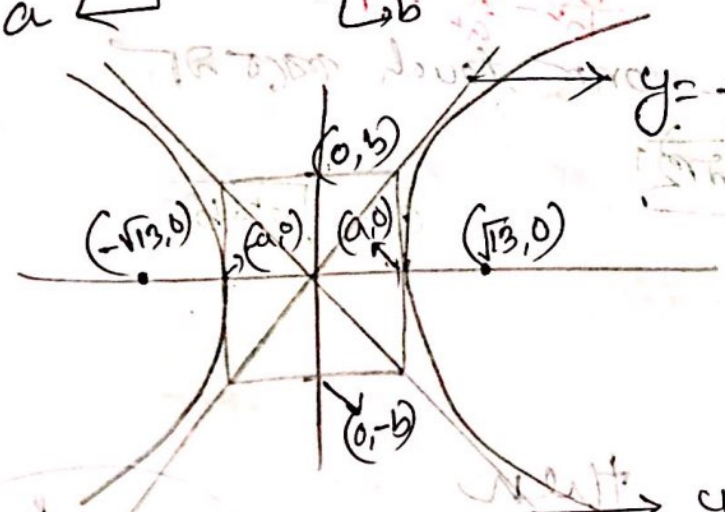
Q: $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$

$\dfrac{x^2}{2^2} - \dfrac{y^2}{3^2} = 1$

$a \quad b$

$c = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

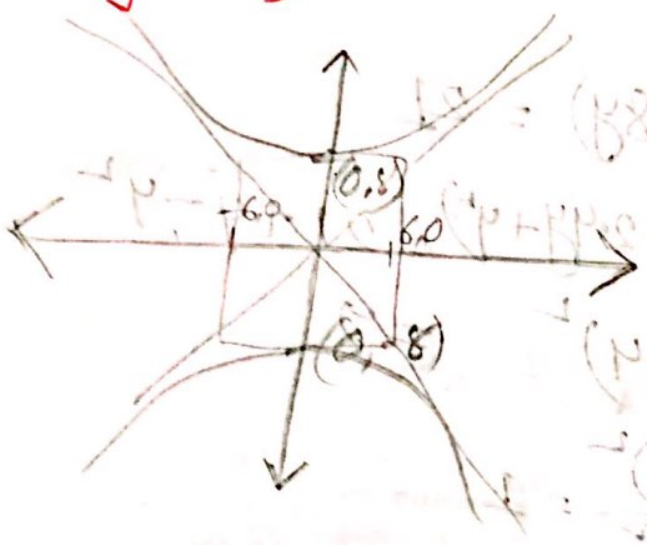


$\longrightarrow y = \dfrac{b}{a}x$

$\longrightarrow y = -\dfrac{b}{a}x$

Scanned with CamScanner

Q: Find Hyperbola equ vertices $(0, \pm 8)$ & asymptotes $y = \pm \frac{4}{3}x$.



$a = 8$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$y = \pm \frac{a}{b}x$$

$$= \pm \frac{4}{3}x$$

$$= \pm \frac{8}{6}x$$

$b = 6$

Translated (center) $= (h, k)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Asymptotes $(y-k) = \pm \frac{b}{a}(x-h)$

Scanned with CamScanner

Q: Find the graph equation

$$x^2 - y^2 - 4x + 8y - 21 = 0$$

↳ hyperbola

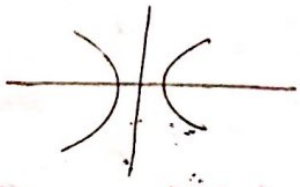
$$(x^2 - 4x) - (y^2 - 8y) = 21$$

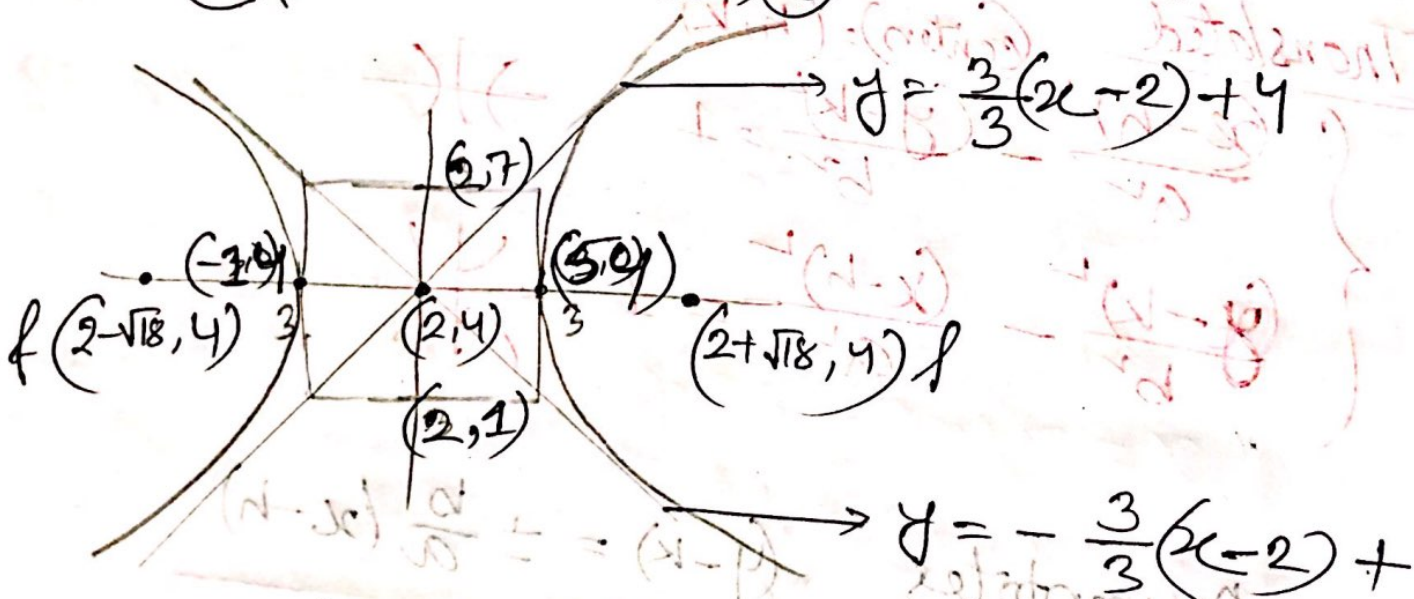
$$(x^2 - 2\cdot 2x + 2^2) - (y^2 - 2\cdot 4y + 4^2) = 21 + 4 - 4^2$$

$$(x-2)^2 - (y-4)^2 = 9$$

$$\frac{(x-2)^2}{3^2} - \frac{(y-4)^2}{3^2} = 1$$

a ↲ ↳ b





centre (2,4)

$$c = \sqrt{a^2 + b^2}$$

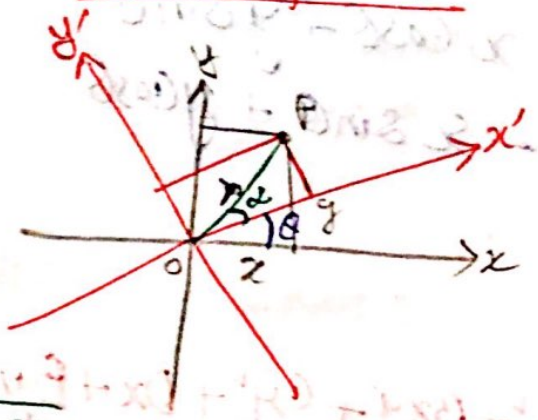
$$= \sqrt{9+9}$$

$$= \sqrt{18}$$

$$=$$

Scanned with CamScanner

10.5 Rotation of Axis



$$\boxed{r\cos(\theta+\alpha) = x} \longleftarrow \cos(\theta+\alpha) = \frac{x}{r}$$

$$\boxed{r(\sin(\theta+\alpha) = y} \longleftarrow \sin(\theta+\alpha) = \frac{y}{r}$$

$$\cos\alpha = \frac{x'}{r} \longrightarrow x' = r\cos\alpha$$

$$\sin\alpha = \frac{y'}{r} \longrightarrow y' = r\sin\alpha$$

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

সর্ব বের করার জন্য
Cos ও Sin এর আগে
(+)

* $\sin(A+B) = \sin A\cos B + \cos A\sin B$
* $\cos(A+B) = \cos A\cos B - \sin A\sin B$.

Scanned with CamScanner

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

(L9) $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$\cot 2\theta = \frac{A-C}{B}$ → If angle isn't given.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

prime 2nd just sign of comp. for change

$$\boxed{\cot 2\theta = \frac{A-C}{B}} \rightarrow 2\theta = \tan^{-1}\left(\frac{B}{A-C}\right)$$

$$A' = A\cos^2\theta + B\cos\theta.\sin\theta + C\sin^2\theta.$$

$$B' = B(\cos^2\theta - \sin^2\theta) + 2(C-A)\sin\theta\cos\theta.$$

$$C' = A\sin^2\theta - B\sin\theta\cos\theta + C\cos^2\theta.$$

$$D' = D\cos\theta + E\sin\theta.$$
$$E' = -D\sin\theta + E\cos\theta.$$

$\rightarrow$ Same as x', y'

$$F' = F$$

$$2\cos^2\theta = 1 + \cos 2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$

Scanned with CamScanner

Ac (17)

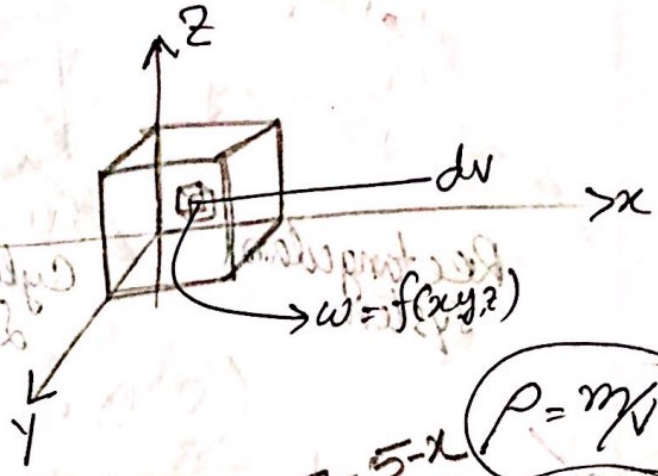
8.09.23

Ch-14.5

$w = f(x, y, z)$ | $dv = dx \cdot dy\, dz$

mass = ρ

$\iiint_R f(x,y,z)\, dv$

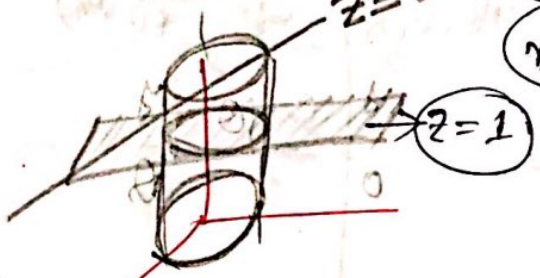


$\rho = m/v$

$m = \rho v$

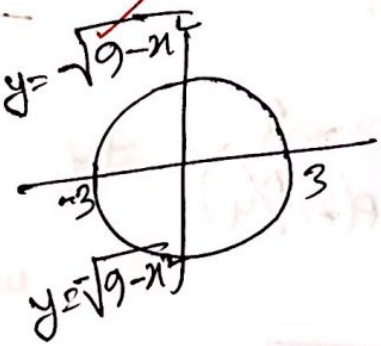
Exm 3

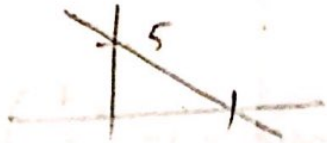
$x^2 + y^2 = 3^2$



$\iiint_R 1\, dv$

$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{1}^{5-x} 1\, dz\, dy\, dx$





$y = 5 - x$

0	5
1	4
5	0
9	-4

$= \iint_D (5 - x - 1)\, dy\, dx$

$= \int_{-3}^{3} \left[4y - xy \right]_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} dx$

$= \int_{-3}^{3} \left(4\sqrt{9-x^2} - x\sqrt{9-x^2} + 4\sqrt{9-x^2} - x\sqrt{9-x^2} \right)$

$= 8\int_{-3}^{3} \sqrt{9-x^2}\, dx - 2\int_{-3}^{3} x\sqrt{9-x^2}\, dx.$

$= 4\left(8 \times \frac{9}{2}\pi\right) - (2 \times 0)$

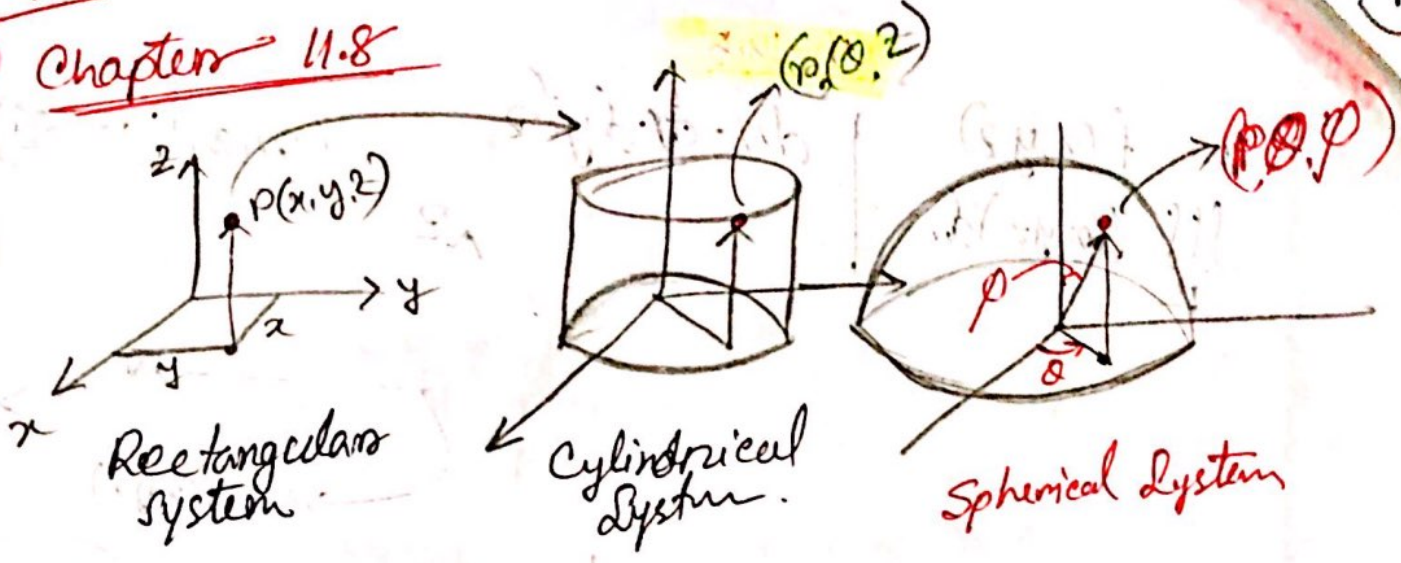
$\Rightarrow 36\pi - 0$

$\Rightarrow 36\pi$ Ans.

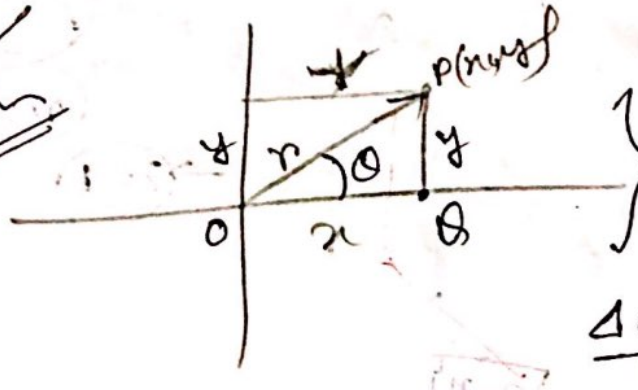
Scanned with CamScanner

Ad-18

Chapter 11.8



polar system

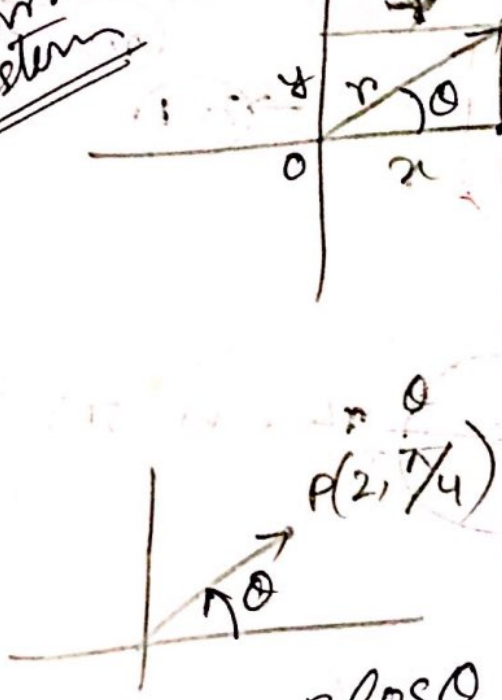


$$r = \sqrt{x^2+y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

ΔOPQ

$\cos\theta = \frac{x}{r}$ | $\sin\theta = \frac{y}{r}$

$\theta = \cos^{-1}\left(\frac{x}{r}\right)$ | $\theta = \sin^{-1}\left(\frac{y}{r}\right)$

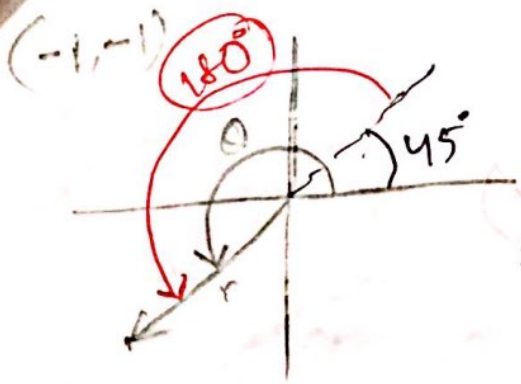


$x = r\cos\theta$
$= 2\cos\pi/4$
$= \sqrt{2}$

$y = r\sin\theta$
$= 2\sin\pi/4$
$= \sqrt{2}$

→ যার সাথে কোন ফিক্সড কর্ড সেটি $\cos\theta$.

Scanned with CamScanner



$r = \sqrt{1+1}$

$= \sqrt{2}$

$\theta = \tan^{-1}\left(\frac{-1}{-1}\right)$

$= 45°$

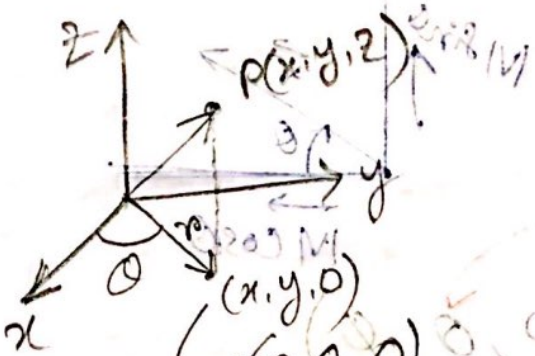
$\therefore \theta = 180° + 45°$

$= 225°$

$(x, y, z) \longrightarrow (r, \theta, z)$

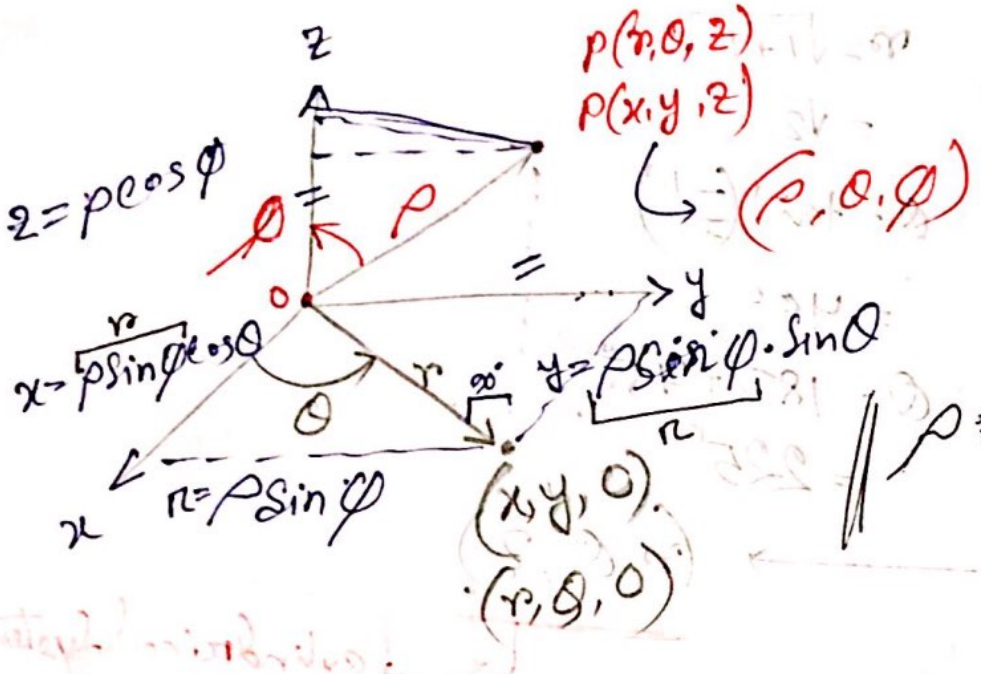
Rectangular System

cylindrical System

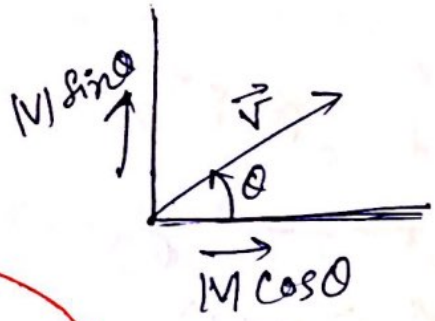


r comes from xy plane

Spherical System



ρ = from center to p cordinate



$$(r, \theta, z) \longrightarrow (\rho, \theta, \phi)$$

with x with z

ϕ z অক্ষের সাথে গঠিত কোণ হবে সে. $\cos\theta$.

$$\rho^2 = r^2 + z^2$$

$$\rho = \sqrt{r^2 + z^2}$$

$$z = \rho \cos\phi$$

$$\phi = \cos^{-1}(z/\rho)$$

Scanned with CamScanner

Ad-19

14.6

$$V = \iiint_G 1 \, dv$$

Hemisphere (অর্ধগোলক)

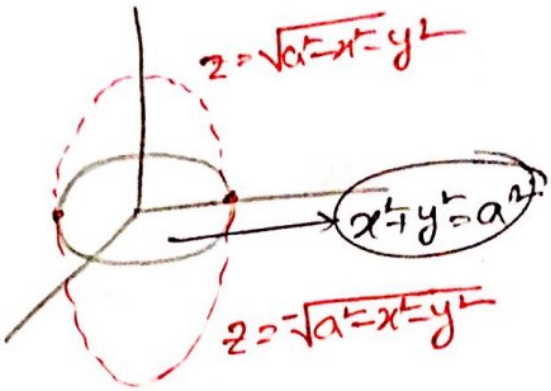
sphere $x^2+y^2+z^2=a^2$

$z=0$

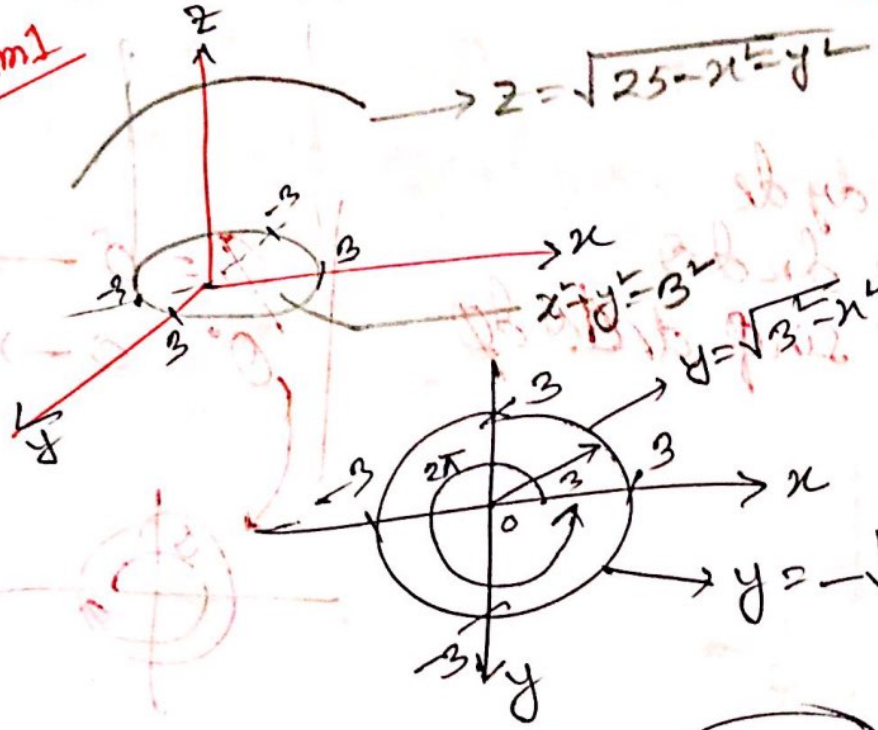
circle $x^2+y^2=a^2$

$$z^2 = a^2-x^2-y^2$$

$$z = \pm\sqrt{a^2-x^2-y^2}$$



Exm1



$0+y^2=3^2$

$y=\pm 3$

$x^2+0=3^2$

$x=\pm 3$

(r, θ, z)

$\theta = 0 \to 2\pi$

$r \to 0 \to 3$

Scanned with CamScanner

#

$$V = \int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{25-x^2-y^2}} dz\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{3}\int_0^{\sqrt{25-r^2}} dz\, dr\, d\theta$$

$(x, y, z) \longrightarrow (r, \theta, z)$

$dA = dx\,dy$

$dA = r\,dr\,d\theta$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

$-r^2\cos^2\theta - r^2\sin^2\theta -$

$-(r^2\{\cos^2\theta + \sin^2\theta\})$

1

$-r^2$

(ρ, θ, ϕ)

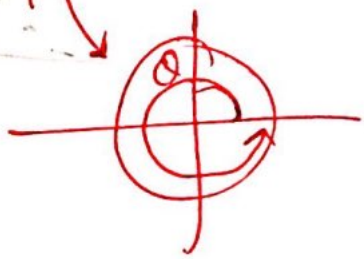
$dV = dx\,dy\,dz$

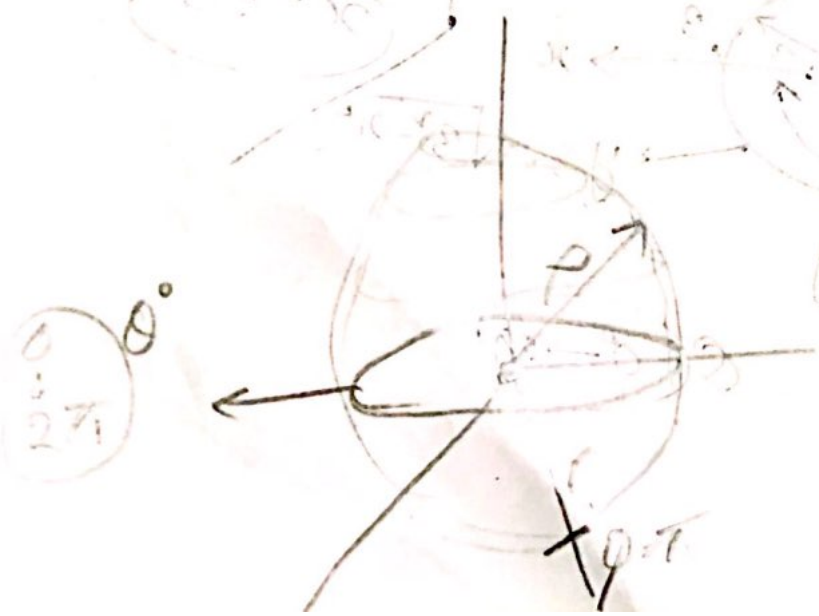
$dV = r\,dz\,dr\,d\theta$

$dV = \rho^2 \sin\phi\, d\rho\, d\theta\, d\phi$

$\phi = 0^\circ \longrightarrow \pi$

$\theta^\circ = 0 \longrightarrow 2\pi$

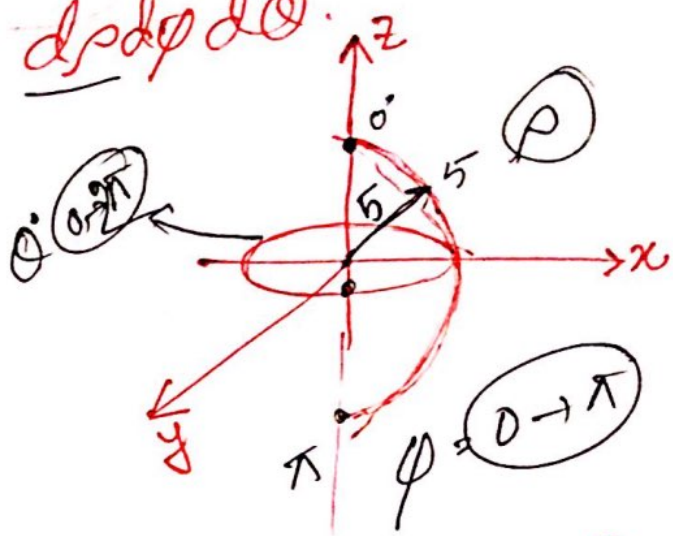




Scanned with CamScanner

$x^2+y^2+z^2=5^2$

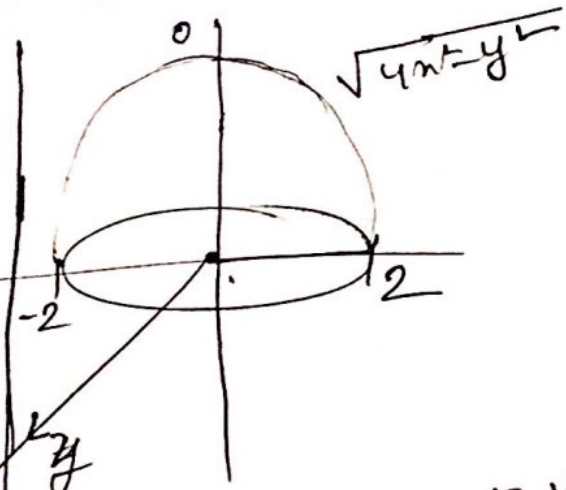
$$V=\int_0^{2\pi}\int_0^{\pi}\int_0^{5} \rho^2 \sin\phi \; d\rho\, d\phi\, d\theta$$



Spherical Coor

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_0^{\sqrt{4x^2-y^2}} z^2\sqrt{x^2+y^2+z^2}\; dz\, dy\, dx.$$

$$\int_0^{2\pi}\int_0^{\pi/2}\int_0^{2} (\rho\cos\phi)^2\, \rho.\; d\rho\, d\phi\, d\theta$$



$$\underbrace{\rho^2\cos^2\theta}_{z^2}.\underbrace{\rho}_{\rho}.\underbrace{\rho\sin}_{\text{for formula}}$$

$z^2 = 4x^2-y^2$

$$\boxed{\begin{aligned}\rho &= \sqrt{x^2+y^2+z^2}\\ z &= \rho\cos\phi\end{aligned}}$$

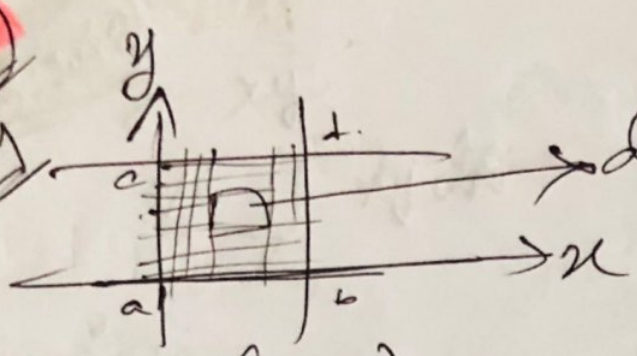
Scanned with CamScanner

cone $\longrightarrow$ $\theta = \pi/4 - \phi$

Scanned with CamScanner

14.1

Ad-14



$dA = dx\,dy$

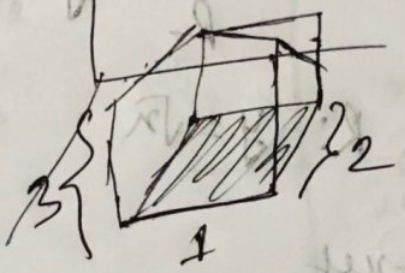
$z = f(x,y)$

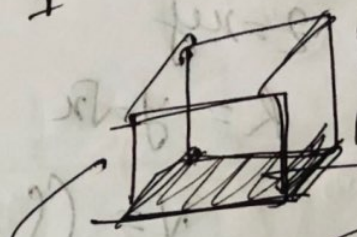
$dV = z \cdot dA$

$V = \iint_R dV$

$= \iint_R z\,dA$

$= \iint_R z\,dx\,dy$





$1 \times 2 = 2$

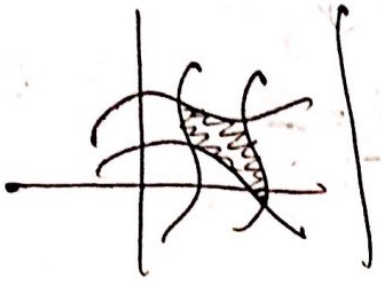
$2 \times 3 = 6$

$V =$ Area × height

$z = f(x,y)$

Ad-15

14.2



① Evaluate $\iint_R xy \, dA$

$R: \; y = \sqrt{x} \qquad y = \frac{1}{2}x$

(a) $z = xy$

$R = y = \sqrt{x} \quad y = \frac{1}{2}x$

$V = \iint_R z \, dA \Rightarrow \iint_R xy \, dA$

(b) Area $z = 1$

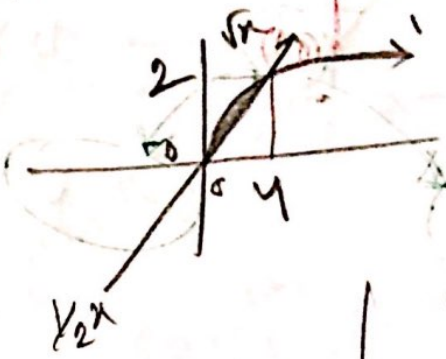
$R = y = \sqrt{x} \quad y = \frac{1}{2}x$

$\iint_R 1 \, dA$

Scanned with CamScanner

Q. 1.42 $z = xy$

$$V = \iint_R xy \, dA \qquad R = y = \tfrac{1}{2}x \quad y = \sqrt{x}$$

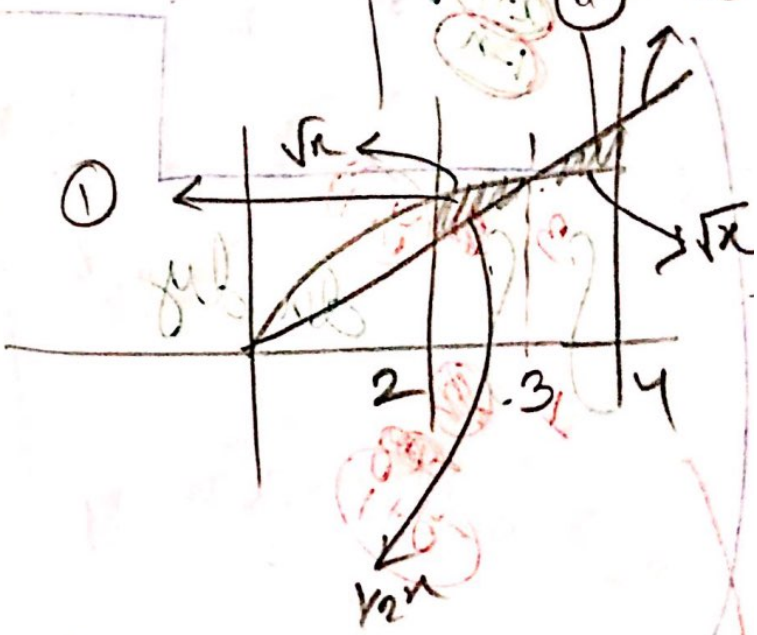


$$V = \int_0^4 \int_{\frac{1}{2}x}^{\sqrt{x}} xy \, dy \, dx$$

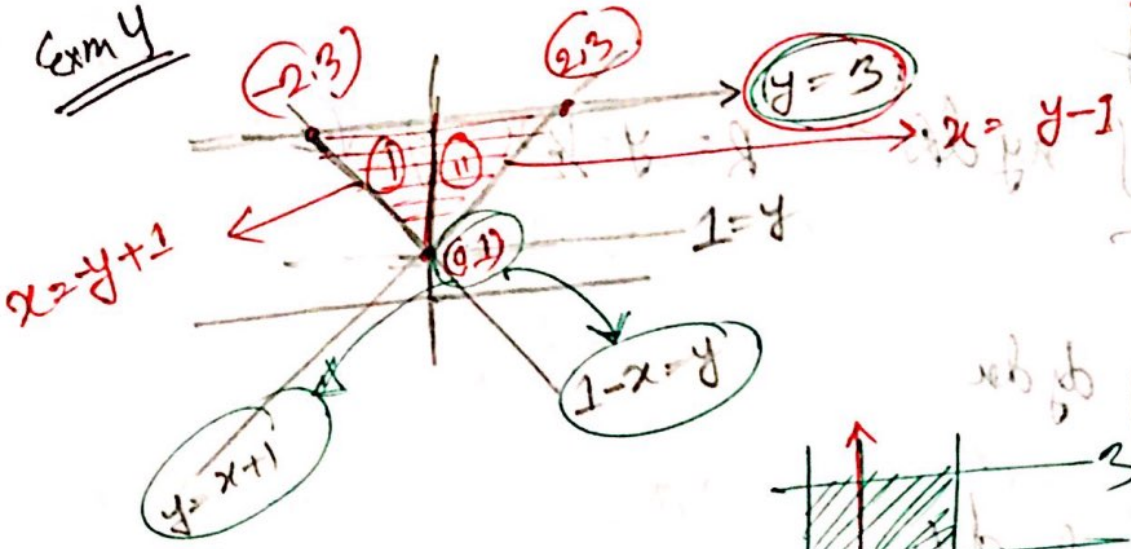
$$A = \int_0^4 \int_{\frac{1}{2}x}^{\sqrt{x}} 1 \, dy \, dx$$

$\sqrt{x} = \frac{1}{2}x$

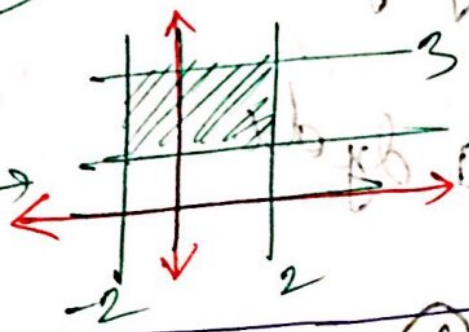
$\Rightarrow x = \frac{1}{4}x^2$



Exm 4



$$\int_{-2}^{2}\int_{x+1}^{3} dy\, dx$$



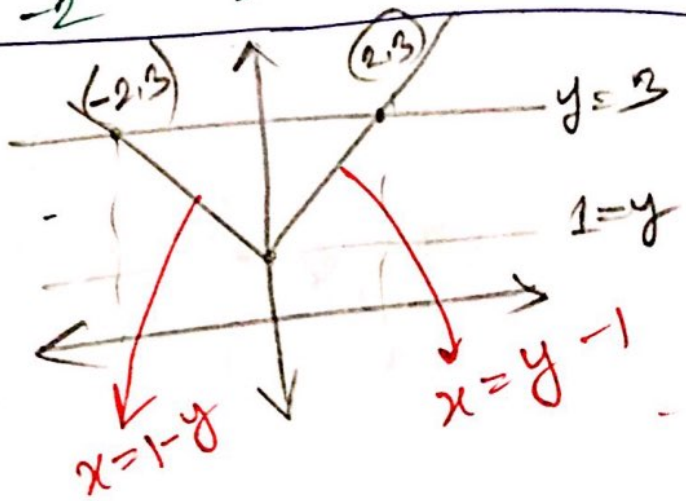
for Part 1

$$\int_{-2}^{0}\int_{x+1}^{3} 2x-y^2\, dy\, dx$$

Part 2

$$\int_{0}^{2}\int_{x+1}^{3} 2x-y^2\, dy$$

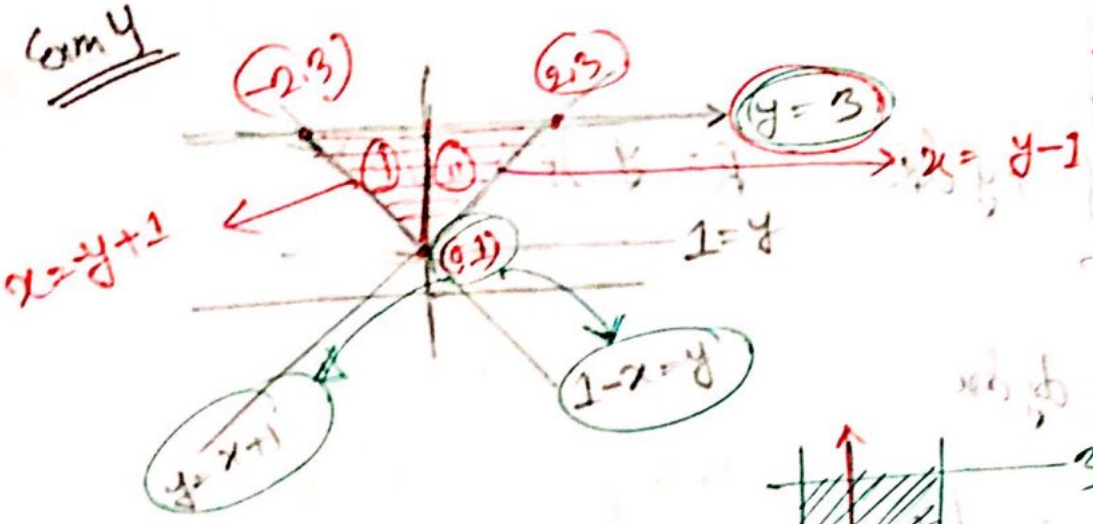
$$\int_{1}^{3}\int_{-y+1}^{y-1} dx\, dy$$



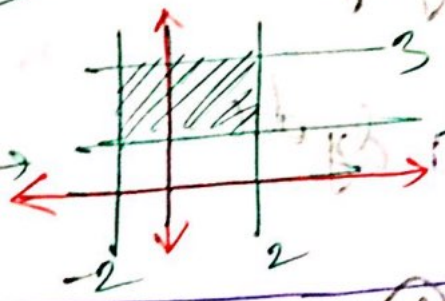
→ Here if we fixed x values then in y value upper limit one it's 3 But lower limit becomes 2 functions.

→ If we fixed y values for x values we have 2 functions for upper & lower limit so No problem

Com 4



$$\int_{-2}^{2}\int_{1}^{3} dy\,dx$$



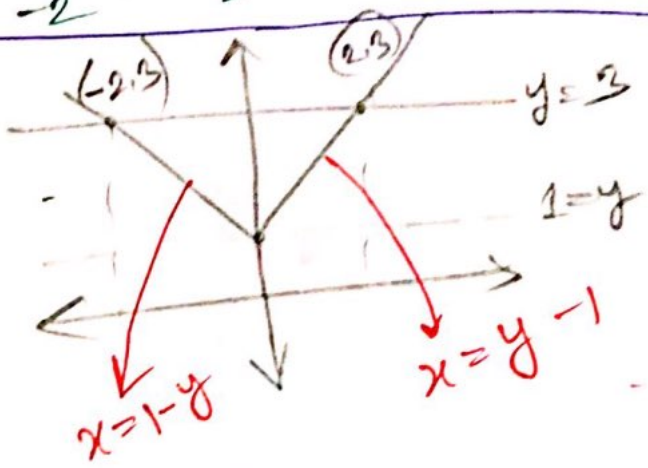
for Part 1

$$\int_{-2}^{0}\int_{x+1}^{3} 2x+y^2\,dy\,dx$$

part 2

$$\int_{0}^{2}\int_{x+1}^{3} 2x-y^2\,dy\,dx$$

$$\int_{1}^{3}\int_{-y+1}^{y-1} dx\,dy$$



→ Here if we fixed x values then in y value upper limit one it's 3 But lower limit becomes 2 functions.

→ If we fixed y values for x values we have 2 functions for upper & lower limit so No problem.

Scanned with CamScanner

5-9-23
L-18

CH-14.1

Double Integrals → Volume

Q: $\int_1^3 \int_2^4 (40-2xy)\,dy\,dx$

$= \int_1^3 \left[40y - \frac{\cancel{2}xy^2}{\cancel{2}}\right]_2^4 dx$

$= \int_1^3 \left[(40\times4 - x4^2) - (40\times2 - x2^2)\right] dx$

$= \int_1^3 (80 - 16x + 4x)\,dx$

$= \int_1^3 (80 - 12x)\,dx$

$= \left(80x - \frac{\overset{6}{\cancel{12}}x^2}{\cancel{2}}\right)_1^3$

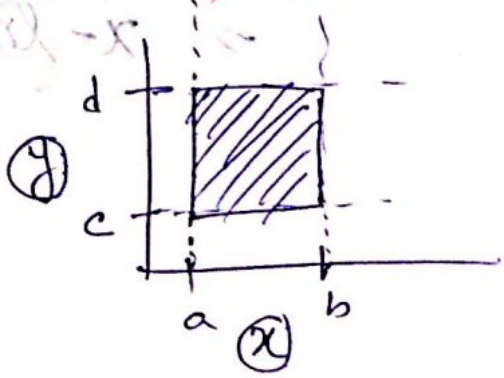
$= (80\times3 - 6\times3^2) - (80\times3 - 6\times3^2)$

$= 112$

Fubini's Theorem

$a \le x \le b \mid c \le y \le d$

$$\int_c^d \int_a^b f(x,y)\,dx\,dy = \int_a^b \int_c^d f(x,y)\,dy\,dx.$$



Scanned with CamScanner

Exercise

Q1 $\int_0^1 \int_0^1 \frac{x}{(xy+1)^2}\, dy\, dx$

d/dy

Now

$\int_0^1 \frac{x}{(xy+1)^2}\, dy$

$xy+1 = u$

$\frac{du}{dy} = x+0$

$\Rightarrow du = x\,dy$

$y=0 \longrightarrow u=1$

$y=1 \longrightarrow u=x+1$

$= \int_1^{x+1} \frac{1}{u^2}\, du$

$= \left[\frac{u^{-1}}{-1}\right]_1^{x+1}$

$= \left(-\frac{1}{x+1} + \frac{1}{1}\right)$

$= 1 - \frac{1}{x+1}$

now, $\int_0^1 1 - \frac{1}{x+1}\, dx$

$= [0x]_0^1 - \left[\ln(x+1)\right]_0^1$

$= (x-0) - \{\ln(2) - \ln(1)\}$

$= x - \ln(2)$ Answer

Scanned with CamScanner

(14) $\iint_R \frac{xy}{\sqrt{x^2+y^2+1}}\, dA$; $R = \{(x,y) : 0 \le x \le 1,\ 0 \le y \le 1\}$

$$\int_0^1\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\, dy\, dx$$

Now $\int_0^1 \frac{xy}{\sqrt{x^2+y^2+1}}\, dy.$

$$= x\frac{1}{2}\int_{x^2+1}^{x^2+2} \frac{1}{\sqrt{u}}\, du.$$

$$= \frac{x}{2}\left(\frac{u^{-1/2+1}}{1/2}\right)_{x^2+1}^{x^2+2}$$

$$= \frac{x}{2}\cdot\frac{2}{1}\left((x^2+2)^{1/2} - (x^2+1)^{1/2}\right)$$

$$= x\left\{(x^2+2)^{1/2} - (x^2+1)^{1/2}\right\}$$

let $x^2+y^2+1 = u$ $\rightarrow d/dy$

$0 + 2y + 0 = \frac{du}{dy}$

$\cancel{2}\cdot y\cdot dy = \frac{1}{2}du$

$y = 1 \rightarrow u = x^2+2$

$y = 0 \rightarrow u = x^2+1$

Now $\int_0^1 x\left\{(x^2+2)^{1/2} - (x^2+1)^{1/2}\right\} dx.$

~~0.789 − 1.147~~

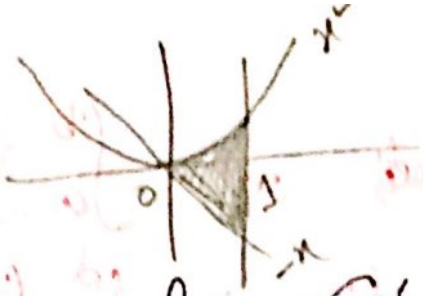
$= \text{Ans}$ 0.179

Scanned with CamScanner

Ch-14.2

Exm:1

$\int_0^1 \int_{-x}^{x^2} y^2 x \, dy \, dx$



$= \int_0^1 x\left[\frac{y^3}{3}\right]_{-x}^{x^2} dx \Rightarrow \int_0^1 \left(\frac{x \cdot x^6}{3} + \frac{x \cdot x^3}{3}\right) dx$

$\Rightarrow \int_0^1 \left(\frac{x^7}{3} + \frac{x^4}{3}\right) dx$

$\Rightarrow \frac{1}{3}\left(\frac{x^8}{8} + \frac{x^5}{5}\right]_0^1 \Rightarrow \frac{1}{3}\left(\frac{1}{8} + \frac{1}{5} - (0+0)\right)$

$\Rightarrow \frac{13}{120}$

2 types of Non Rectangular Regions

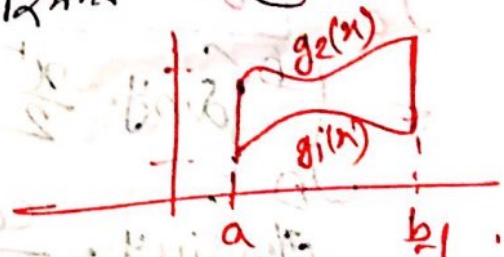
* Type-1:

x এর value Numerical ভাবে দেয়া থাকলে — (fixed)

& y এর " function হিসাবে থাকলে।

$x_1 = a$ $x_2 = b$

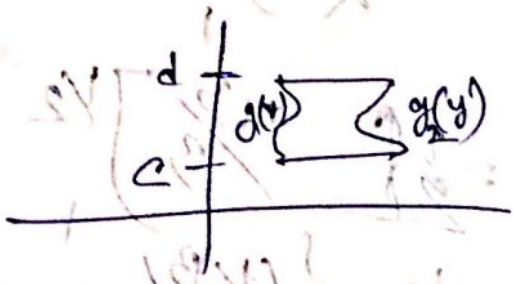
$y_1 = g_1(x)$ $y_2 = g_2(x)$



x (fixed) হলে y এর সকলকে নিয়ে Integration.

then x " ".

Type 2 y fixed

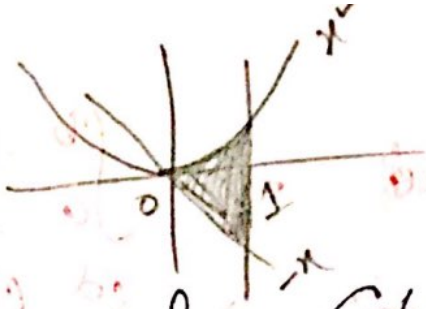


Scanned with CamScanner

Ch-14.2

Exam!

$\int_0^1 \int_{-x}^{x^2} \overbrace{y^2}^{z} \overbrace{x}^{A}\, dy\, dx$:



$= \int_0^1 x\left[\frac{y^3}{3}\right]_{-x}^{x^2} dx \Rightarrow \int_0^1 \left(\frac{x \cdot x^6}{3} + \frac{x \cdot x^3}{3}\right) dx$

$\Rightarrow \int_0^1 \left(\frac{x^7}{3} + \frac{x^4}{3}\right) dx$

$\Rightarrow \frac{1}{3}\left(\frac{x^8}{8} + \frac{x^5}{5}\right]_0^1 \Rightarrow \frac{1}{3}\left(\frac{1}{8} + \frac{1}{5} - (0+0)\right)$

$\Rightarrow \frac{13}{120}$

2 types of Non Rectangular Regions

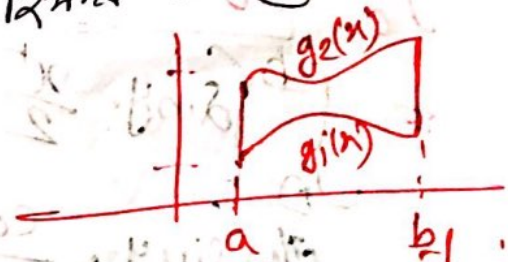
* Type 1:

x এর value Numerical ভাবে দেয়া থাকলে — (fixed)

& y এর " function হিসেবে থাকলে।

$x_1 = a$ $x_2 = b$

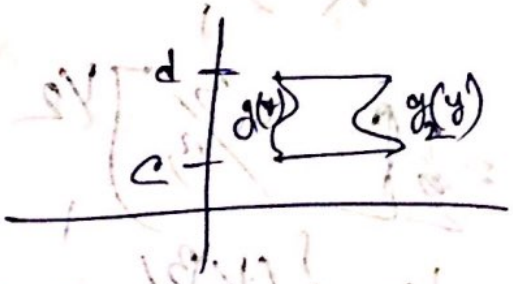
$y_1 = g_1(x)$ $y_2 = g_2(x)$



x (fixed) হলে y এর সাপেক্ষে আগে Integration.

then x " ".

Type 2 y fixed



Scanned with CamScanner

x (fixed) $\Rightarrow \int_a^b \int_{\square}^{\square} f(x,y)\, dy\, dx$

y (fixed) $\rightarrow \int_c^d \int_{\square}^{\square} f(x,y)\, dx\, dy.$

$\uparrow$ y increase so down function $\int_{\square}^{\square}$ — upper fn.

$\rightarrow$ x from so $\int_{\square}^{\square}$ — Right fun.

left limit/fun

$\int_0^{\pi/3} \boxed{\int_0^{\cos y} x \sin y\, dx}\, dy.$

$= \int_0^{\pi/3} \left[\sin y \cdot \frac{x^2}{2}\right]_0^{\cos y} dy.$

$= \int_0^{\pi/3} \left[\sin y \cdot \frac{\cos^2 y}{2}\right] dy$

$= \frac{1}{2} \int_1^{1/2} -u^2\, du$

$= \frac{1}{2}\left[-\frac{u^3}{3}\right]_1^{1/2}$

$= 1/2 \left\{ \frac{(1/2)^3}{3} - \frac{1}{3} \right\} = +\frac{7}{48}$

d/dy.

let $\cos y = u$

$-\sin y = \frac{du}{dy}.$

$\sin y\, dy = -du.$

$y = \pi/3 \rightarrow u = 1/2$

$y = 0 \rightarrow u = 1$

Scanned with CamScanner

Exm 4

$$\int_1^3\int_{-2}^{0}(2x-y^3)\,dx\,dy + \int_1^3\int$$

$y = 4x \longrightarrow$ x এর উপর পড়ে

Similarly

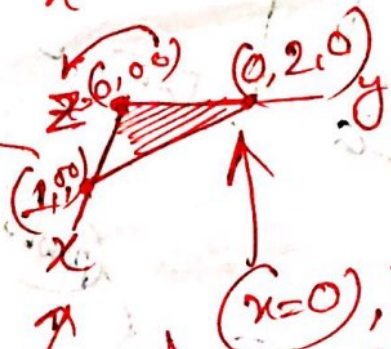
$z = 2x + 9y$ x & y " " "

Exm 5

$z = 4 - 4x - 2y$

$2y + 0 = 4 - 4x$

$y = 2 - 2x$



$(x=0), (z=0)$

$0 = 4 - 0 - 2y$

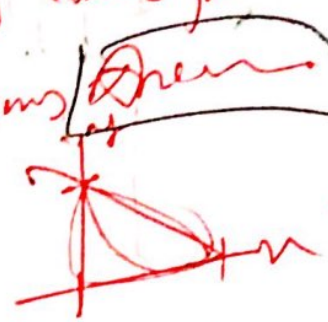
$y = 2$

$(y=0, z=0)$

$0 = 4 - 4x - 0$

$x = 1$

x, y limits comes from the Ground means Area



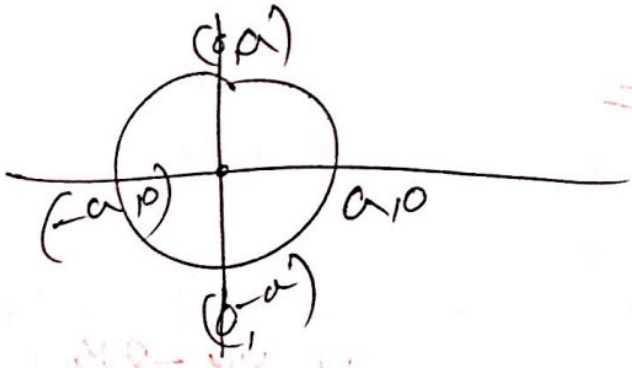
$z = 1$

Area

$$\iint_R dA$$

$x^2 + y^2 = a^2$

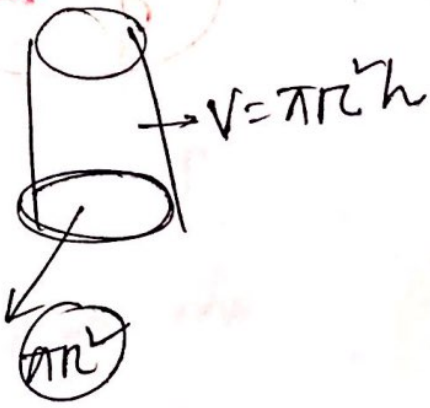
$$y = \pm\sqrt{a^2 - x^2}$$



$$\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}}$$

Imp

Exm: 5 6 7 problem



A116 Done

Scanned with CamScanner

Ad-22
8-9-23

15.4

Green's Theorem

① Must be not conservative

$$\int_C f(x,y)\,dx + g(x,y)\,dy = \iint_R \left(\frac{\partial}{\partial x} g - \frac{\partial}{\partial y} f\right) dA$$

Exm 1

$$\int_C \underbrace{xy\,dx}_{f(x,y)} + \underbrace{x\,dy}_{g(x,y)}$$

$$= \iint_R \left[\frac{\partial}{\partial x}\underbrace{x}_{g(x,y)} - \frac{\partial}{\partial y}\underbrace{(x^2 y)}_{f(x,y)}\right] dA$$

$$= \iint_R (1 - x^2)\, dA$$

$$= \int_0^1 \int_0^{2x} (1 - xy)\, dy\, dx$$

$$= \int_0^1 \left[y - x^2 y\right]_0^{2x} dx$$

$$= \int_0^1 (2x - x^2 \cdot 2x)\, dx \Rightarrow (2x - 2x^3)$$

$$= \left[x^2 - \frac{2}{4}x^4\right]_0^1$$

$$\Rightarrow \left(1 - \frac{1}{2}\right) - (0 - 0)$$

$$= 1/2$$

here limits come from graph

Scanned with CamScanner

15.5

Surface Integrals

a surface equation $z = g(x,y)$.

$$\iint_{\sigma} f(x,y,z)\,dS = \iint_{R} f(x,y,g(x,y)) \cdot \sqrt{\left(\frac{\delta z}{\delta x}\right)^2 + \left(\frac{\delta z}{\delta y}\right)^2 + 1}\; dA$$

density

$$\rho = \frac{M}{V}$$

$$dM = f\,ds$$

$$M = \iint_{\sigma} f\,ds$$

mass

Exm2 $\iint_{\sigma} xz\,dS$

$\sigma \longrightarrow x+y+z = 1$

$\Rightarrow z = 1-x-y$

$$\iint_{\sigma} xz\,dS = \iint_{R} x(1-x-y)\sqrt{\left\{\frac{\delta}{\delta x}(1-x-y)\right\}^2 + \left\{\frac{\delta}{\delta y}(1-x-y)\right\}^2 + 1}\; dA$$

$$= \iint_{R} x(1-x-y)\sqrt{(-1)^2 + (-1)^2 + 1}\; dA$$

$$= \int_0^1 \int_0^{1-x} (x - x^2 - xy)\sqrt{3}\; dy\, dx$$

$$= \int_0^1 \left(\sqrt{3}\left(xy - x^2y - \frac{xy^2}{2}\right)\right)_0^{1-x} dx$$

Scanned with CamScanner

$$= \sqrt{3}\int_0^1 \left\{x(1-x) - x^2(1-x) - \frac{x}{2}(1-x)^2\right\} dx$$

$$= \sqrt{3}\int_0^1 \left\{x - x^2 - x^2 + x^3 - x/2\,(1-2x+x^2)\right\} dx$$

$$= \sqrt{3}\int_0^1 \left(x - 2x^2 + x^3 - x/2 + x^2 - \frac{1}{2}x^3\right) dx$$

$$= \sqrt{3}\left[\frac{x^2}{2} - \frac{2x^3}{3} - \frac{x^4}{4} - \frac{1}{4}x^2 + \frac{x^3}{3} - \frac{1}{8}x^4\right]_0^1$$

$$= \sqrt{3}\left(\frac{1}{2} - \frac{2}{3} - \frac{1}{4} - \frac{1}{4} + \frac{1}{3} - \frac{1}{8}\right)$$

$$=$$

Scanned with CamScanner

Exm 3

$$\iint_{\sigma} y^2 z^2 \, dS$$

$$z = \sqrt{x^2+y^2}$$

$$\iint_{\sigma} y^2 z^2 \, dS = \iint_{R} y^2 \left(\sqrt{x^2+y^2}\right)^2 \sqrt{\left(\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1\right)}$$

$$= \sqrt{\frac{\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}} + 1}$$

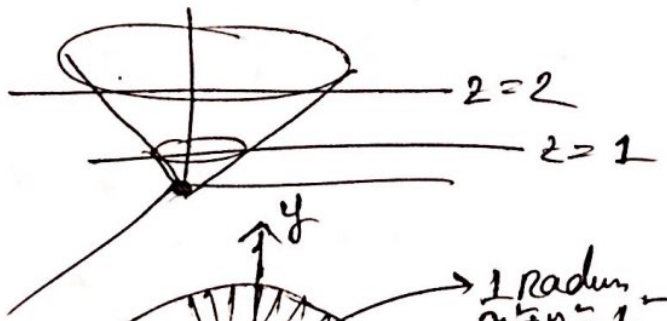
$$= \iint_{R} y^2 \left(\sqrt{x^2+y^2}\right)^2 \cdot \sqrt{2} \, dA$$

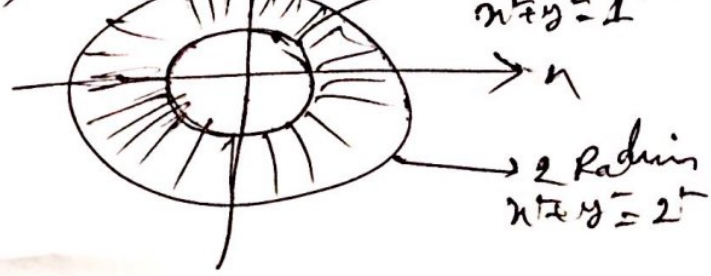
polar form →

$$= \int_0^{2\pi}\int_1^2 (r\sin\theta)^2 \, \cancel{\sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}} \; r^2 \sqrt{2} \, r \, dr \, d\theta$$

$$\frac{\partial}{\partial x}\left(\sqrt{x^2+y^2}\right) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} \cdot (\cancel{2}x)$$

$$\frac{\partial}{\partial y}\left(\sqrt{x^2+y^2}\right) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}} (\cancel{2}y)$$





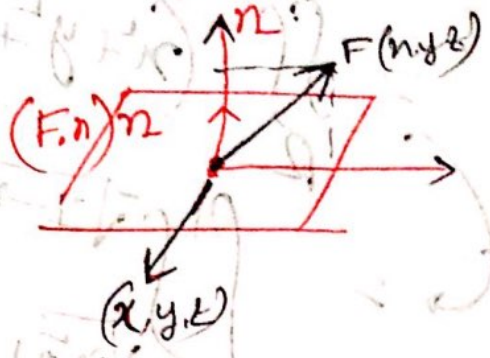
$$0 \le \theta \le 2\pi$$

$$1 \le r \le 2$$

15.6

FLUX (flow)

$$\phi = \iint_{\sigma} F(x,y,z) \cdot n(x,y,z)\, dS$$



or,

$$= \iint_{\sigma} \vec{F}\vec{n}\, dS$$

$$= \iint_{R} \vec{F}\, \vec{\nabla} G\, dA$$

Oriented up
z positive

xm2

Given $\boxed{z = 1 - x^2 - y^2}$

$$x^2 + y^2 + z - 1 = 0$$

$$\boxed{G(x,y,z)} = 0$$

$$\therefore \vec{n} = \vec{\nabla} \cdot G$$

$$= \frac{\partial}{\partial x} G i + \frac{\partial}{\partial y} G j + \frac{\partial}{\partial z} G k$$

$$= 2xi + 2yj + 1k$$

$$\therefore \iint_{R} (xi + yj + 2k) \cdot (2xi + 2yj + k)\, dA$$

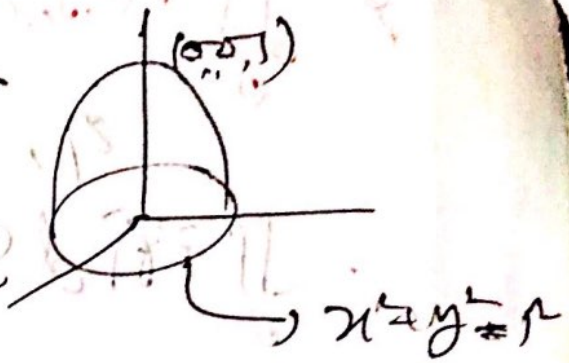
$$\Rightarrow \iint_{R} (2x^2 + 2y^2 + 2)\, dA$$

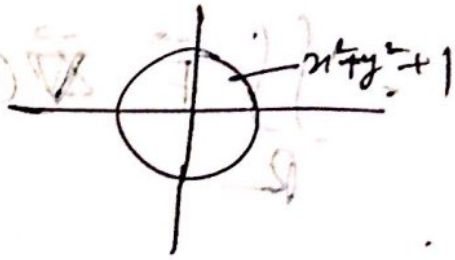
Scanned with CamScanner

$$= \iint_R 2x^2+2y^2+(1-x^2-y^2)\,dA$$

$$= \iint_R (x^2+y^2+1)\,dA$$

$$= \int_0^1\int_{\sqrt{x^2+1}}^{\sqrt{1-x^2}} (x^2+y^2+1)\,dy\,dx$$





OR

$$= \int_0^{2\pi}\int_0^1 (r^2+1)\,r\,dr\,d\theta$$

$$=$$

15.7

$$\phi = \iint_{\sigma} \vec{F} \cdot \vec{n} \, dS$$

$$= \iiint_{G} \operatorname{div} F \, dV.$$

* Exm.4

Scanned with CamScanner

Ad-20 7-9-23

Ch-15.1

vector field

Gradient

(def) $\vec{\nabla} = i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z}$ Gradient

Exm $\varphi(x,y,z) = x^2y + yz^2 + z$

$\vec{\nabla}\varphi = \frac{\delta}{\delta x}(x^2y + yz^2 + z)\,i + \ldots$

$= (y2x + 0 + 0)\,i + (x^2 + z^2 + 0)\,j + (0 + 2yz + 1)\,k$

$\vec{\nabla}.\varphi$ = Divergence

$\vec{\nabla}\times\varphi$ = Curl

$$\vec{\nabla}\times\varphi = \begin{vmatrix} i & j & k \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ f & g & h \end{vmatrix}$$

$\varphi(x,y,z) = \phi(x,y,z \ldots)$

Divergence

$$\text{div } F = \frac{\delta f}{\delta x} + \frac{\delta g}{\delta y} + \frac{\delta h}{\delta z}$$

$F(x,y,z) = f(x,y,z)\,i + g(x,y,z)\,j + h(x,y,z)$

Scanned with CamScanner

(Exm) Q: (4) Find divergence & curl

$$f(x,y,z) = \boxed{x^2y\,i} + \boxed{2y^3z\,j} + \boxed{3z\,K}$$

$$\text{div } F = \frac{\delta}{\delta x} f(x,y,z) + \frac{\delta}{\delta y} g(x,y,z) + \frac{\delta}{\delta z} h(x,y,z)$$

$$= \frac{\delta}{\delta x}(\quad) + \quad$$

$$= \cancel{(2xy+6+0)}$$

$$= 2xy + 6y^2z + 3 \quad \text{Ans}$$

$$\text{Curl } F = \begin{vmatrix} i & j & K \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ x^2y & 2y^3z & 3z \end{vmatrix}$$

$$= i\left(\frac{\delta}{\delta y}(3z) - \frac{\delta}{\delta z}(2y^3z)\right) - j\left(\frac{\delta}{\delta x}(3z) - \frac{\delta}{\delta z}(x^2y)\right) + K\left(\frac{\delta}{\delta x}(2y^3z) - \frac{\delta}{\delta y}(x^2y)\right)$$

$$= (0-2y^3)i - (0-0)j + (0-x^2)K$$

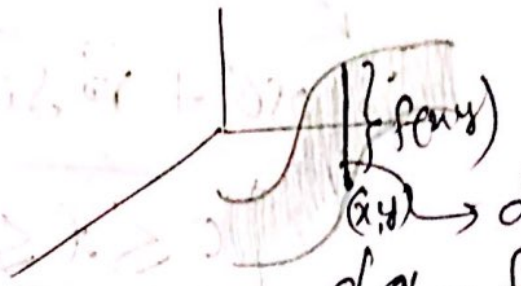
$$= \boxed{-2y^3 i - x^2 K}$$

Ans.

Scanned with CamScanner

Ch-15.2

Line Integrals



$dm = f\,ds$

$M = \int_C dm$

$M = \int_C f\,ds$

$\rho = m/v$

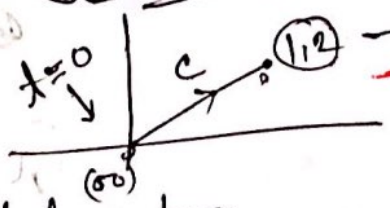
$dA = f\,ds$

$A = \int_C dA$

$= \int_C f\,ds$

Exm 5

(a) $\int_C 3xy\,dy$



straight line

parallel vector

$\vec{V} = (1-0,\ 2-0)$

$\boxed{(0,0)}$ P.V $\vec{V}(1,2)$ parametric equation.

$x = x_0 + at \longrightarrow x = 1t$

$y = y_0 + at \longrightarrow y = 2t$

orientation
Starting point

$y = 2t$

$dy = 2dt$

$\therefore \int_C 3xy\,\boxed{dy}$

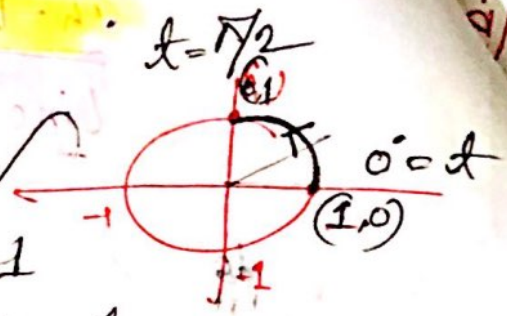
$\Rightarrow \int_0^1 3 \cdot t \cdot 2t\,2dt$

$\Rightarrow \frac{12}{3}(t^3)\Big|_0^1$

$\Rightarrow 4(1-0) \longrightarrow = 4$

Scanned with CamScanner

* (27) $C: x^2+y^2=1$



$r\cos 0^\circ + r\sin \pi/2 = 1$

$\boxed{0 \leq t \leq \pi/2}$

$x = r\cos t$
$y = r\sin t$

$\int_0^{\pi/2} (r^2(\cos 0^\circ)^2 + r^2(\sin \pi/2)^2)$

$r(-\sin t)dt$ — $r\cos t$

$r\cos t\, dt$

$dx = r(-\sin t)dt$

$dy = r\cos t\, dt$

$= \int_0^{\pi/2} (r^2 + r^2) r$

$\vec{r} = r$ Ans

---

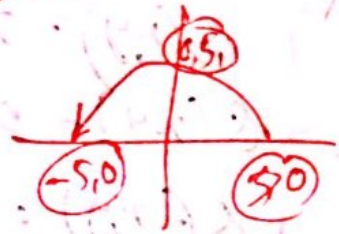
problem Ex (6) (9) | P.P. (13) (25)
(13)

(imp) Exm (18)

D-21

33

34 6



$x = 5\cos t$

$y = 5\sin t$

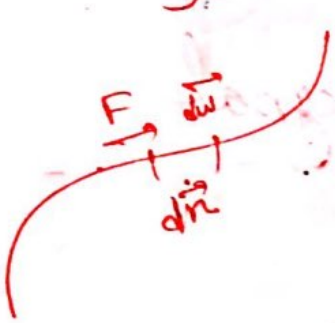
$t = 0$

$t =$

D21

Ch-15.3

$\int_c F\,dr$ $\quad \vec{w} = \int_c d\vec{w}$ $\quad d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$



$= \int_c \vec{F}\,d\vec{r}$

$= \int_c f(x,y,z)\,dx + g(x,y,z)\,dy + h(x,y,z)\,dz$

Fundamental theorem of Calculus

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Scanned with CamScanner

Potential force

$\vec{F} = \vec{\nabla}\varphi$ → conservative field

φ → potential function.

Scalar function

$\varphi(x,y) = x^2 + y^2$

$\varphi(x,y) = 2x^2 y$

$\varphi(x,y) = \sqrt{x}\, y$

→ 2 point's energy difference always constant.

→ all work done same.

Fundamental theorem of line Integral

① conservative field.

$\vec{F} = \vec{\nabla}\varphi$

$\varphi(x_0, y_0)$ → $\varphi(x,y)$

(x_0, y_0) → (x,y)

$$W = \int_C \vec{F}\, d\vec{r}$$ → work Done

$$= \int_C \vec{\nabla}\varphi \cdot d\vec{r}$$

$$= \varphi(x, y) - \varphi(x_0, y_0)$$

Scanned with CamScanner

Conservative proof

$\vec{F}(x,y) = y\hat{i} + x\hat{j}$

$\phi(x,y) = xy$

$\vec{F} = \vec{\nabla}\phi$

$\vec{\nabla}\phi = \hat{i}\frac{\partial(xy)}{\partial x} + \hat{j}\frac{\partial}{\partial y}(xy)$

$= y\hat{i} + x\hat{j}$

$= \vec{F}$

(So conservative field)

15.3.3 Conservative field test

$\vec{F}(x,y) = f(x,y)\hat{i} + g(x,y)\hat{j}$

if $\frac{d}{dy}(f(x,y)) = \frac{d}{dx}(g(x,y))$

then it is conservative field

potential fone check

if $\vec{F}(x,y,z) = \vec{\nabla}\phi$ → checking this fone

Scanned with CamScanner

Finding Potential fun

$\nabla \varphi$

$$F(x,y) = \underbrace{\boxed{2xy^3 i}}_{f(x,y)} + \underbrace{(1+3x^2y^2)}_{g(x)} j$$

$$\frac{d(f)}{dy} = \frac{f(g)}{g(x)} \quad \text{(conservative)}$$

$$\varphi = \int f(x)\,dx \longrightarrow \boxed{x^2y^3} + g(y)$$

$\partial/\partial y$

$$\frac{\partial \varphi}{\partial y} = 3x^2y^2 + g'(y) = (1+3x^2y^2)$$

$$g'(y) = 1$$

$$g(y) = y + C$$

$$\varphi$$

$$K'(y) = 0$$

$$\frac{d\,K(y)}{dy} = 0$$

$$\int d\,K(y) = \int 0\,dy$$

$$K(y) = C$$

Scanned with CamScanner

(39) Proved $\operatorname{div}(\operatorname{curl} \vec{F}) = 0$

$$\operatorname{curl} \vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} = \hat{i}\left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right) - \hat{j}\left(\frac{\partial h}{\partial x} - \frac{\partial f}{\partial z}\right) + \hat{k}\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$$\operatorname{div}(\operatorname{curl} \vec{F}) = \frac{\partial}{\partial x}\left(\frac{\partial h}{\partial y} - \frac{\partial g}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial h}{\partial x} - \frac{\partial f}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$$= \cancel{\frac{\partial^2 h}{\partial x \partial y}} - \cancel{\frac{\partial^2 g}{\partial x \partial z}} - \cancel{\frac{\partial^2 h}{\partial y \partial x}} + \cancel{\frac{\partial^2 f}{\partial y \partial z}} + \cancel{\frac{\partial^2 g}{\partial z \partial x}} - \cancel{\frac{\partial^2 f}{\partial z \partial y}}$$

$= 0$ proved

15.2

(i) take all in one unknown (t)

(ii) change dy or dx to new (dt) → $\frac{d}{dt}$ (f)

(iii) change limit

parametrized the unknowns.

$x = x_0 + at$

$y = y_0 + bt$

$(a, b) = (x_2 - x_1, y_2 - y_1)$

Exm 1 (b) $r(t) = (1-t)\hat{i} + (2-2t)\hat{j}$ $(0 \le t \le 1)$

$\int_C (1 + xy^2)\, ds$

$r(t) = \underbrace{(1-t)}_{x}\hat{i} + \underbrace{(2-2t)}_{y}\hat{j}$

$r'(t) = (0-1)\hat{i} + (0-2)\hat{j}$

$= -\hat{i} - 2\hat{j}$

$ds = \|r'(t)\|\, dt$

$= \sqrt{(-1)^2 + (-2)^2}\, dt$

$= \sqrt{5}\, dt$

$\int_0^1 \left[1 + (1-t)(2-2t)^2\right]\sqrt{5}\, dt$

$\Rightarrow \int_0^1 \sqrt{5}\left[1 + (1-t)(4 - 8t + 4t^2)\right] dt$

$\int_0^1 \left(1 + (4 - 8t + 4t^2 - 4t + 8t^2 - 4t^3)\right) dt$

$\int_0^1 \left(1 + 4 - 4t - 4t^2 + 4t^3 - 4t + 8t^2 - 4t^3\right) dt$

$\sqrt{5}$ $\sqrt{5}$

$\Rightarrow \sqrt{5}\,(x+y+z+y+z-x)$

$\Rightarrow$

$\sqrt{\frac{|a|^2}{2|a|}}$ (4)

Scanned with CamScanner