Mesh Analysis

Section 3.1, 3.2, 3.4, 3.5

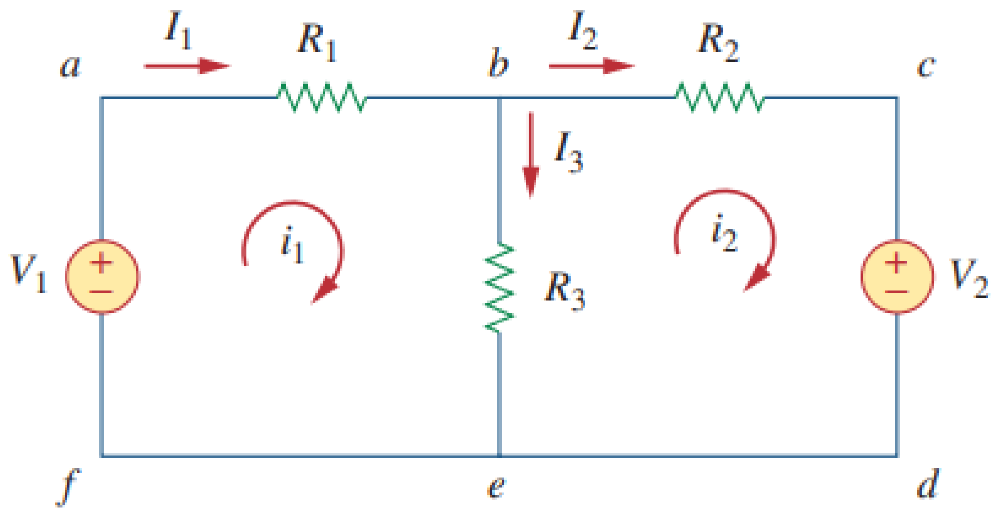
Md. Shafqat Talukder Rakin
Lecturer, Department of CSE,
United International University
Email id : shafqat@cse.uiu.ac.bd

Courtesy: Rifat Bin Rashid

Loop and Mesh

Loop: Any closed path

Mesh : A mesh is a loop which does not contain any other loops within it



Mesh Analysis

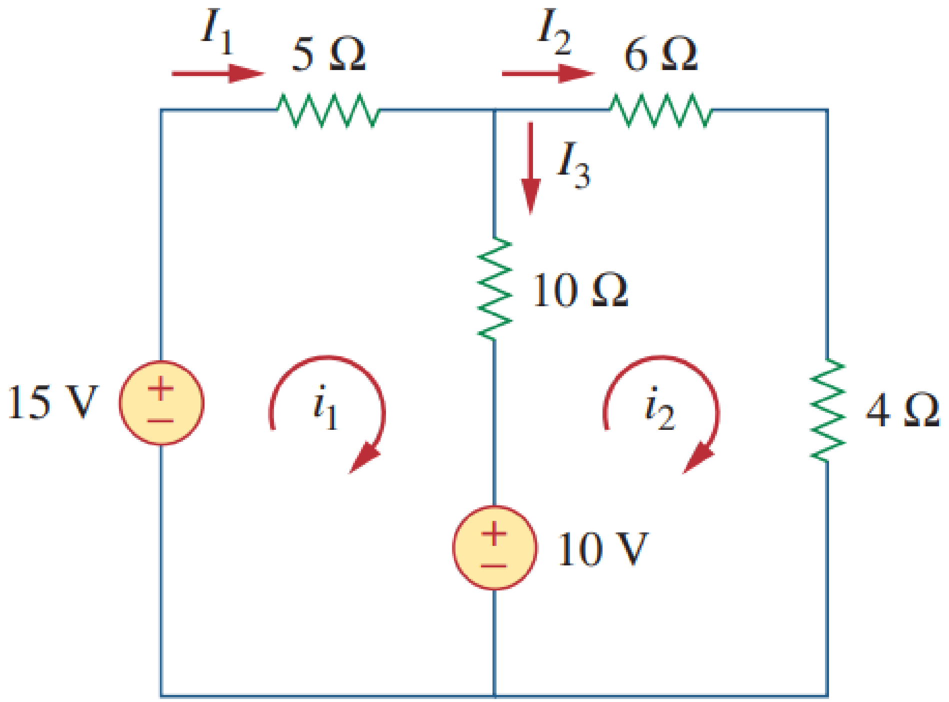
Goal : Finding the mesh Currents.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

Math Problem Practice:

For the circuit in Fig , find the branch currents I_1, I_2, I_3 and using mesh analysis.



Reference: Sadiku_Example_3.5

Math Problem Practice:

Example 3.5

For the circuit in Fig. 3.18, find the branch currents I_1, I_2, and I_3 using mesh analysis.

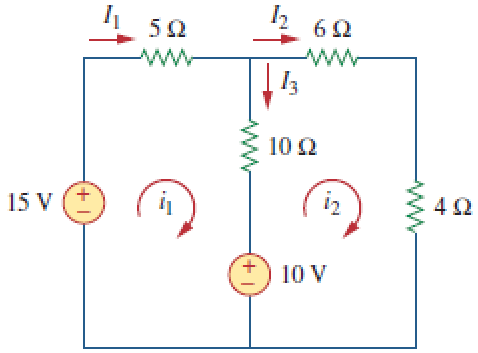


Figure 3.18
For Example 3.5.

Solution:
We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \tag{3.5.1}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \tag{3.5.2}$$

■ **METHOD 1** Using the substitution method, we substitute Eq. (3.5.2) into Eq. (3.5.1), and write

$$6i_2 - 3 - 2i_2 = 1 \quad \Rightarrow \quad i_2 = 1 \text{ A}$$

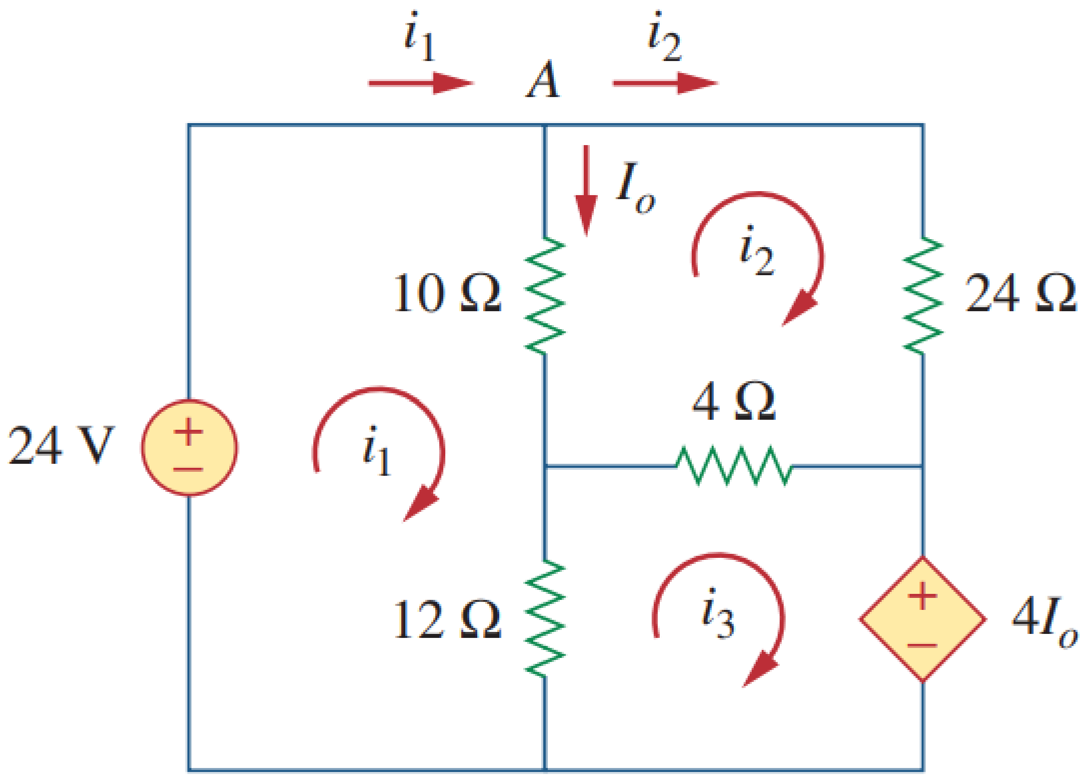
From Eq. (3.5.2), $i_1 = 2i_2 - 1 = 2 - 1 = 1$ A. Thus,

$$I_1 = i_1 = 1 \text{ A}, \qquad I_2 = i_2 = 1 \text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Reference: Sadiku_Example_3.5

Math Problem Practice:

Use **mesh analysis** to find the current $\mathbf{I_o}$ in the circuit of Fig. below



Reference: Sadiku_Example_3.6

Math Problem Practice:

Example 3.6

Use mesh analysis to find the current I_o in the circuit of Fig. 3.20.

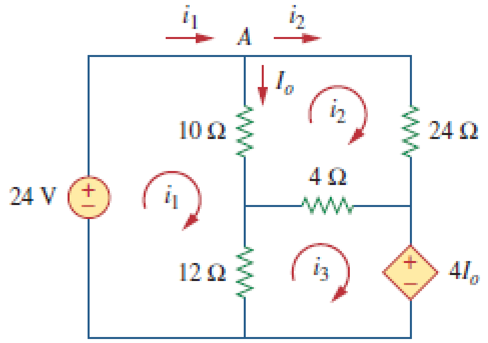


Figure 3.20
For Example 3.6.

Solution:
We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad \textbf{(3.6.1)}$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad \textbf{(3.6.2)}$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

But at node A, $I_o = i_1 - i_2$, so that

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad (3.6.3)$$

In matrix form, Eqs. (3.6.1) to (3.6.3) become

$$\begin{bmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$$

We obtain the determinants as

$$\Delta = \begin{vmatrix} 11 & -5 & -6 \\ -5 & 19 & -2 \\ -1 & -1 & 2 \\ 11 & -5 & -6 \\ -5 & 19 & -2 \end{vmatrix}$$

$$= 418 - 30 - 10 - 114 - 22 - 50 = 192$$

$$\Delta_1 = \begin{vmatrix} 12 & -5 & -6 \\ 0 & 19 & -2 \\ 0 & -1 & 2 \\ 12 & -5 & -6 \\ 0 & 19 & -2 \end{vmatrix} = 456 - 24 = 432$$

$$\Delta_2 = \begin{vmatrix} 11 & 12 & -6 \\ -5 & 0 & -2 \\ -1 & 0 & 2 \\ 11 & 12 & -6 \\ -5 & 0 & -2 \end{vmatrix} = 24 + 120 = 144$$

$$\Delta_3 = \begin{vmatrix} 11 & -5 & 12 \\ -5 & 19 & 0 \\ -1 & -1 & 0 \\ 11 & -5 & 12 \\ -5 & 19 & 0 \end{vmatrix} = 60 + 228 = 288$$

We calculate the mesh currents using Cramer's rule as

$$i_1 = \frac{\Delta_1}{\Delta} = \frac{432}{192} = 2.25\ \text{A}, \qquad i_2 = \frac{\Delta_2}{\Delta} = \frac{144}{192} = 0.75\ \text{A},$$

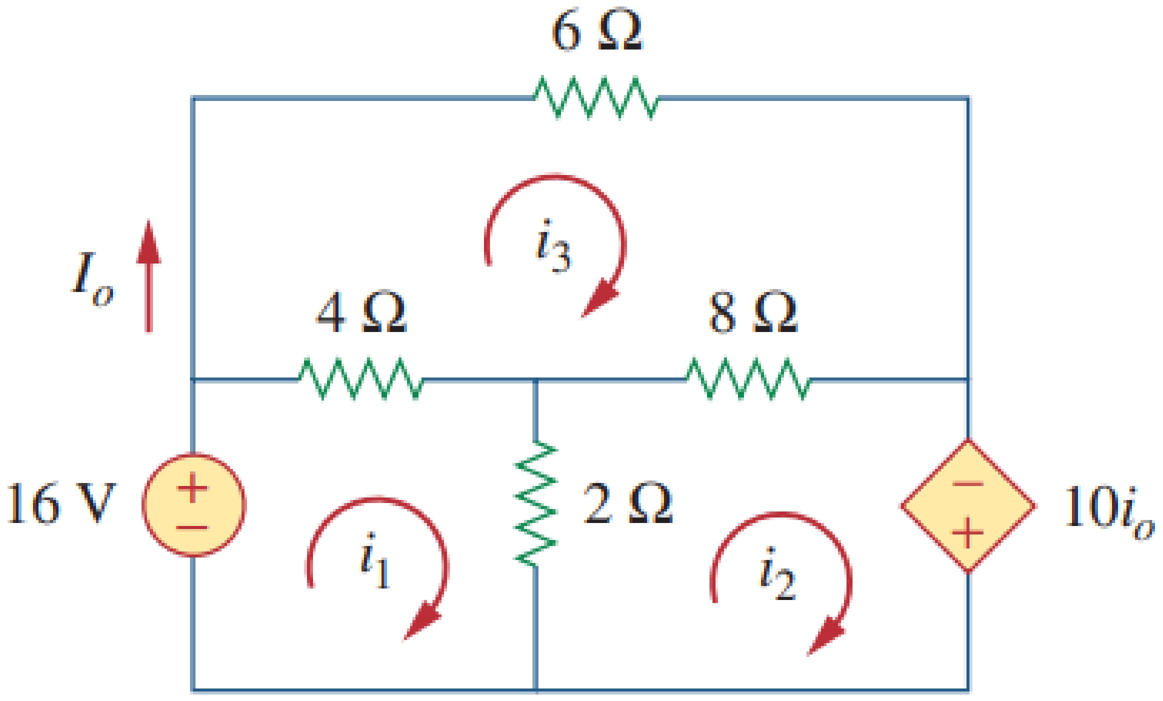
$$i_3 = \frac{\Delta_3}{\Delta} = \frac{288}{192} = 1.5\ \text{A}$$

Thus, $I_o = i_1 - i_2 = 1.5$ A.

<u>Reference: Sadiku_Example_3.6</u>

Math Problem Practice:

Use **mesh analysis** to find the current $\mathbf{I_o}$ in the circuit of Fig. below

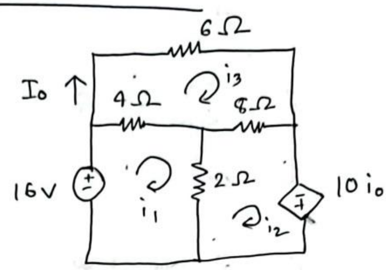


Answer: −4 A.

Reference: Sadiku_Practice_Problem_3.6

Math Problem Practice:

Practic problem 3·6



For mesh-1,

$$4(i_1 - i_3) + 2(i_1 - i_2) - 16 = 0$$
$$\Rightarrow 4i_1 - 4i_3 + 2i_1 - 2i_2 = 16$$
$$\Rightarrow 6i_1 - 2i_2 - 4i_3 = 16 \quad \ldots (1)$$

For mesh-2,

$$2(i_2 - i_1) + 8(i_2 - i_3) - 10I_o = 0$$
$$\Rightarrow 2i_2 - 2i_1 + 8i_2 - 8i_3 - 10i_3 = 0 \quad [\because I_o = i_3]$$
$$\Rightarrow -2i_1 + 10i_2 - 18i_3 = 0 \quad \ldots (2)$$

For mesh-3,

$$6i_3 + 8(i_3 - i_2) + 4(i_3 - i_1) = 0$$
$$\Rightarrow 6i_3 + 8i_3 - 8i_2 + 4i_3 - 4i_1 = 0$$
$$\Rightarrow -4i_1 - 8i_2 + 18i_3 = 0 \quad \ldots (3)$$

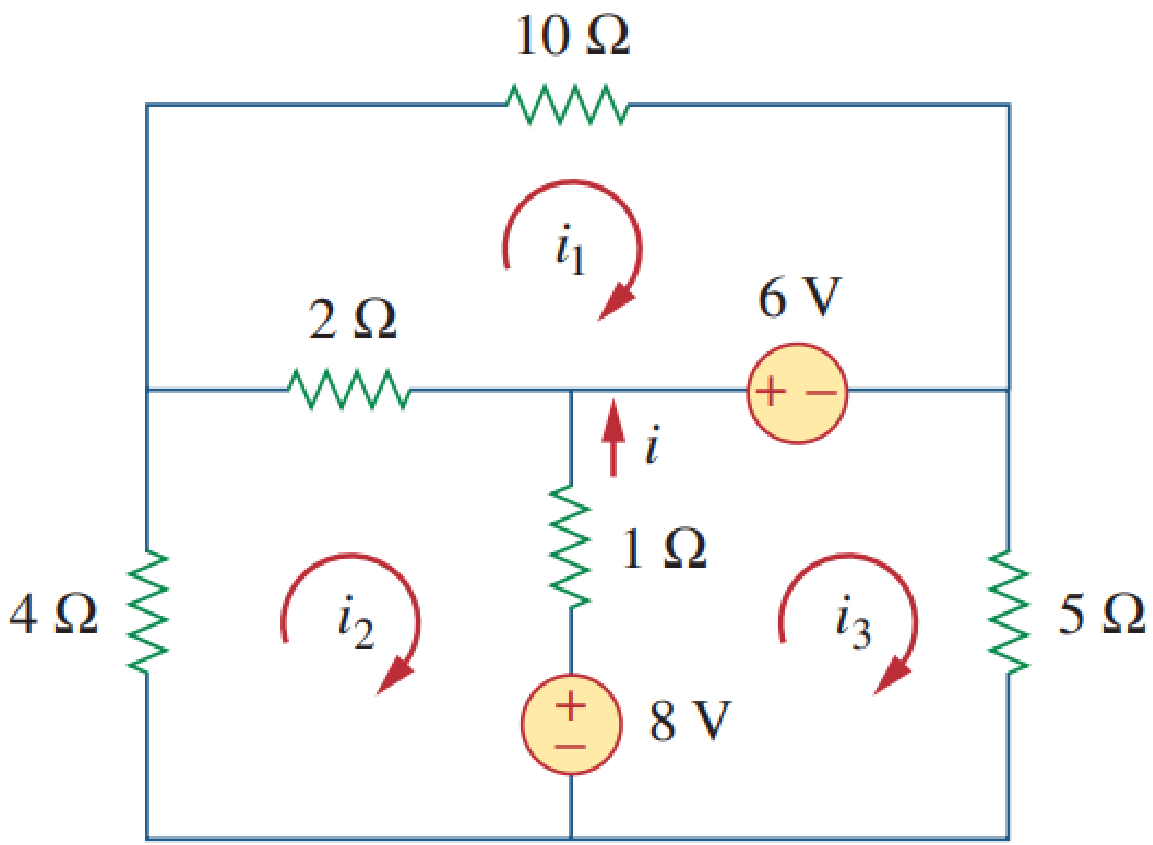
Solving (1), (2), (3) we get,

$$i_1 = -\frac{18}{7}\,A, \quad i_2 = -\frac{54}{7}\,A, \quad i_3 = -4A$$
$$\therefore I_o = -4A$$

Reference: Sadiku_Practice_Problem_3.6

Math Problem Practice:

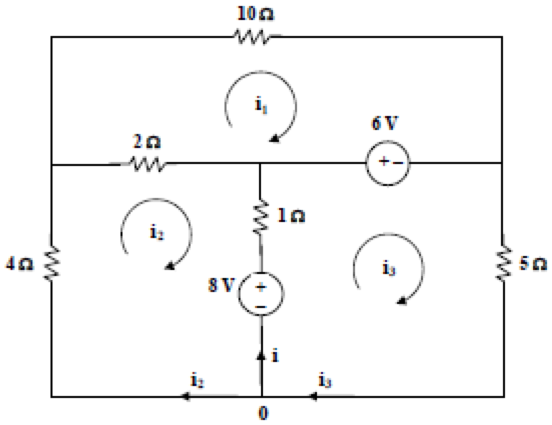
Apply mesh analysis to find i in Fig. 3.87.



Reference: Sadiku_Exercise_Problem_3.41

Math Problem Practice:

Chapter 3, Solution 41



For loop 1,

$$6 = 12i_1 - 2i_2 \longrightarrow 3 = 6i_1 - i_2 \qquad (1)$$

For loop 2,

$$-8 = -2i_1 + 7i_2 - i_3 \qquad (2)$$

For loop 3,

$$-8 + 6 + 6i_3 - i_2 = 0 \longrightarrow 2 = -i_2 + 6i_3 \qquad (3)$$

We put (1), (2), and (3) in matrix form,

$$\begin{bmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 8 \\ 2 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 6 & -1 & 0 \\ 2 & -7 & 1 \\ 0 & -1 & 6 \end{vmatrix} = -234, \quad \Delta_2 = \begin{vmatrix} 6 & 3 & 0 \\ 2 & 8 & 1 \\ 0 & 2 & 6 \end{vmatrix} = 240$$

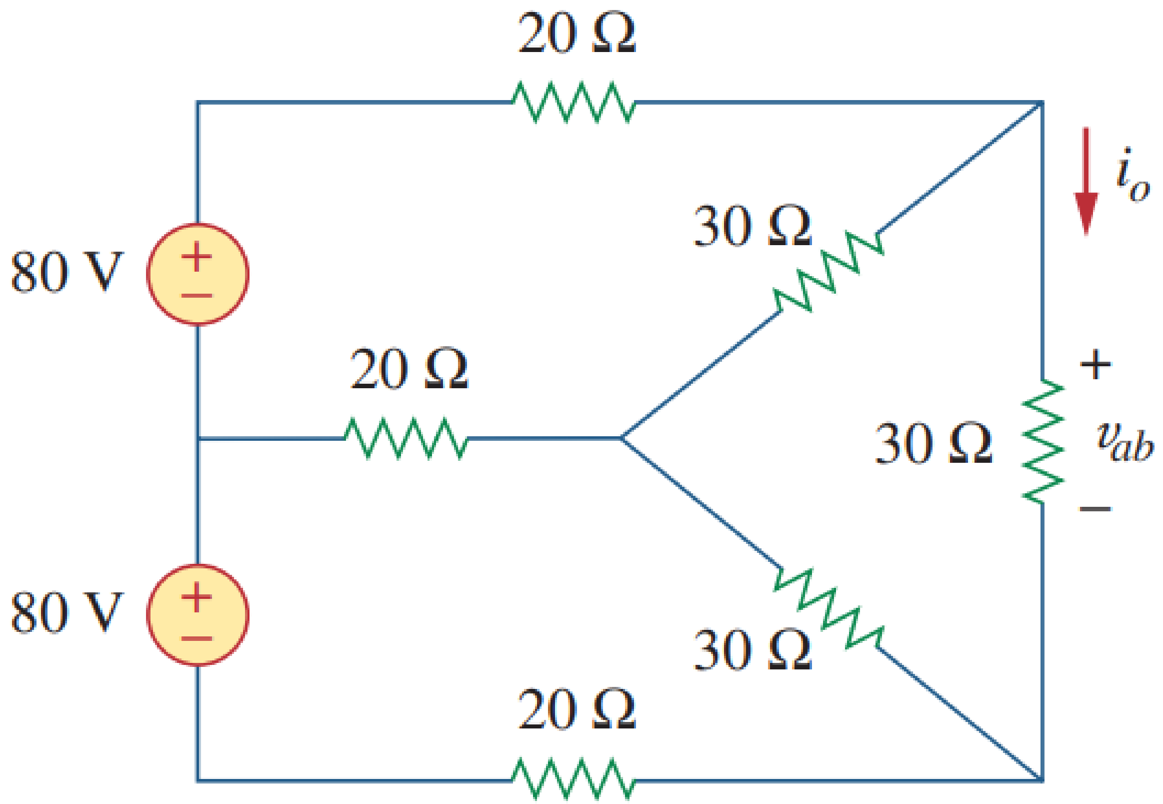
$$\Delta_3 = \begin{vmatrix} 6 & -1 & 3 \\ 2 & -7 & 8 \\ 0 & -1 & 2 \end{vmatrix} = -38$$

At node 0, $i + i_2 = i_3$ or $i = i_3 - i_2 = \frac{\Delta_3 - \Delta_2}{\Delta} = \frac{-38 - 240}{-234} = \mathbf{1.188\ A}$

<u>Reference: Sadiku_Exercise_Problem_3.41</u>

Math Problem Practice:

Use mesh analysis to find v_{ab} and i_o in the circuit of Fig. 3.89.



Reference: Sadiku_Exercise_Problem_3.43

Math Problem Practice:

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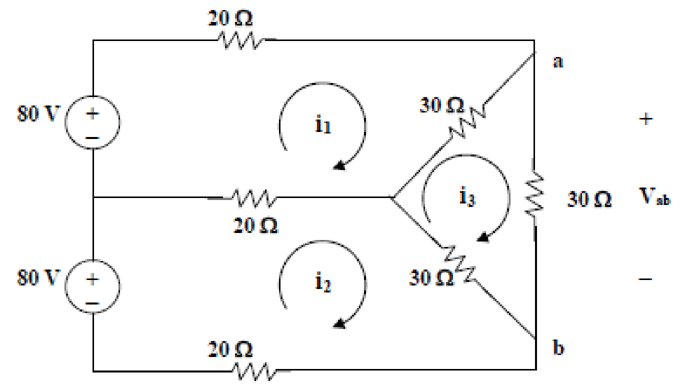


For loop 1,

$80 = 70i_1 - 20i_2 - 30i_3 \longrightarrow 8 = 7i_1 - 2i_2 - 3i_3$ (1)

For loop 2,

$80 = 70i_2 - 20i_1 - 30i_3 \longrightarrow 8 = -2i_1 + 7i_2 - 3i_3$ (2)

For loop 3,

$0 = -30i_1 - 30i_2 + 90i_3 \longrightarrow 0 = i_1 + i_2 - 3i_3$

(3)

Solving (1) to (3), we obtain $i_3 = 16/9$

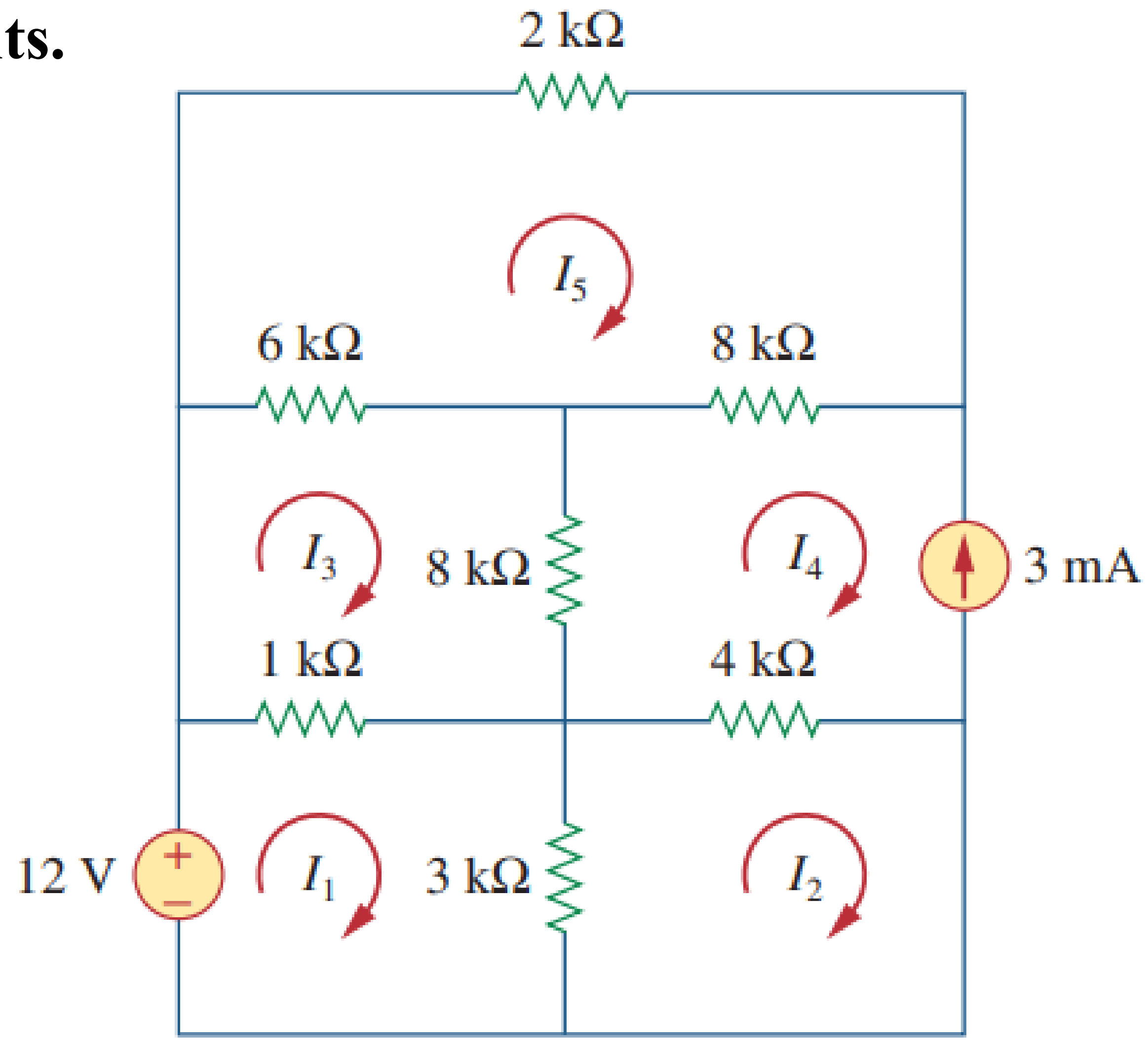
$I_o = i_3 = 16/9 =$ **1.7778 A**

$V_{ab} = 30i_3 =$ **53.33 V**.

Reference: Sadiku_Exercise_Problem_3.43

Math Problem Practice:

Find the mesh currents.



Reference: Sadiku_Exercise_Problem_3.53

Math Problem Practice:

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Applying mesh analysis leads to;

$$-12 + 4kI_1 - 3kI_2 - 1kI_3 = 0 \quad (1)$$

$$-3kI_1 + 7kI_2 - 4kI_4 = 0$$

$$-3kI_1 + 7kI_2 = -12 \quad (2)$$

$$-1kI_1 + 15kI_3 - 8kI_4 - 6kI_5 = 0$$

$$-1kI_1 + 15kI_3 - 6k = -24 \quad (3)$$

$$I_4 = -3mA \quad (4)$$

$$-6kI_3 - 8kI_4 + 16kI_5 = 0$$

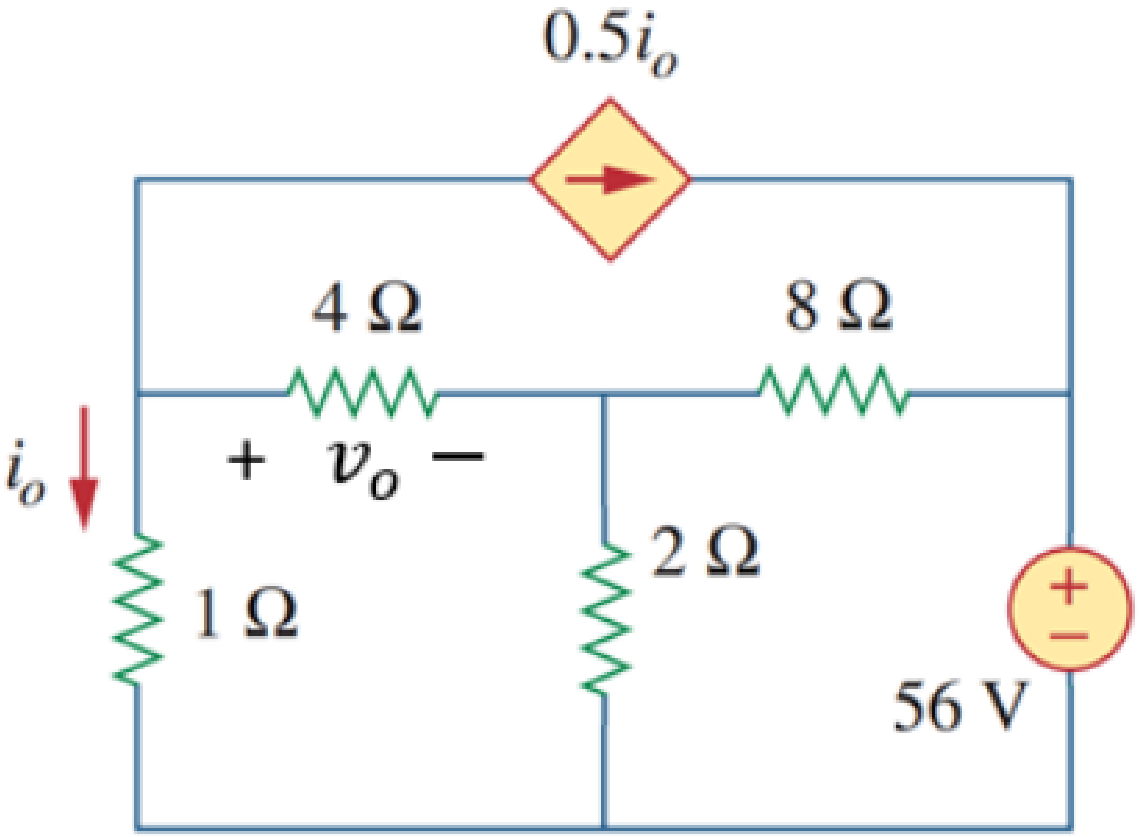
$$-6kI_3 + 16kI_5 = -24 \quad (5)$$

<u>Reference: Sadiku_Exercise_Problem_3.53</u>

Math Problem Practice:

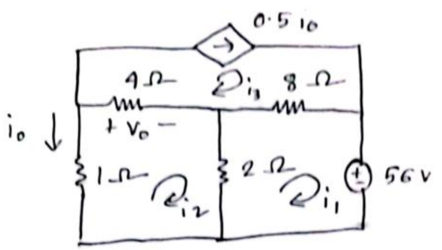
Summer 2022

Calculate the currents through all the resistors using mesh analysis method and find vo.



Math Problem Practice:

Summer 2022



For loop-1,

$2(i_1 - i_2) + 8(i_1 - i_3) + 56 = 0$

$\Rightarrow 10i_1 - 2i_2 - 8i_3 = -56 \quad \cdots (1)$

For loop-2,

$1 \times i_2 + 4(i_2 - i_3) + 2(i_2 - i_1) = 0$

$\Rightarrow -2i_1 + 7i_2 - 4i_3 = 0 \quad \cdot (2)$

and $i_2 = -i_0 \quad \cdots (3)$

From loop-3,

$i_3 = 0.5\, i_0$

$\Rightarrow i_3 = 0.5(-i_2)$ (from (3))

$\Rightarrow 0.5\, i_2 + i_3 = 0 \quad \cdots (4)$

Solving (1), (2) and (4) we get,

$i_1 = -5.362\,A$

$i_2 = -1.191\,A$

$i_3 = 0.596\,A$

$4\Omega \rightarrow i = i_2 - i_3 = -1.787\,A$

$8\Omega \rightarrow i = i_1 - i_3 = -5.958\,A$

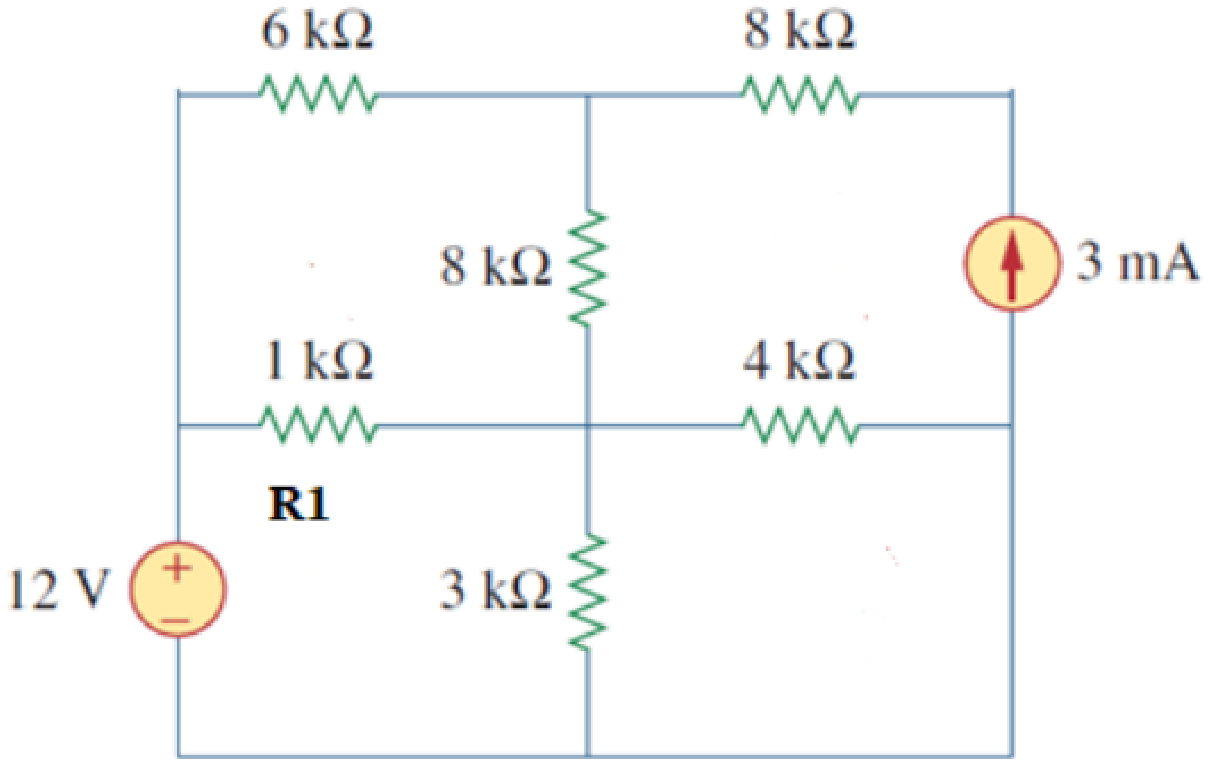
$1\Omega \rightarrow i = i_2 = -1.191\,A$

$2\Omega \rightarrow i = i_1 - i_2 = -4.171\,A$

$V_0 = 4(i_2 - i_3) = -7.148\,V$

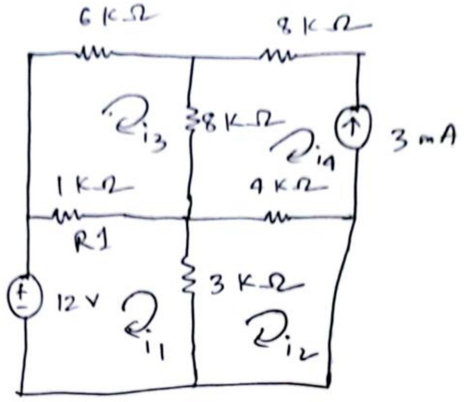
Math Problem Practice: **_Fall 2022_**

(a) Determine the currents in all resistors using Mesh Analysis.
(b) Determine the power absorbed the in R1 resistor.



Math Problem Practice:

Fall 2022



For loop-1,

$$-12 + 1\times(i_1 - i_3) + 3(i_1 - i_2) = 0$$

$$\Rightarrow 4i_1 - 3i_2 - i_3 = 12 \quad \cdots (1)$$

For loop-2,

$$3(i_2 - i_1) + 4(i_2 - i_4) = 0$$

$$\Rightarrow -3i_1 + 7i_2 - 4i_4 = 0 \quad \cdots (2)$$

For loop-3,

$$6i_3 + 8(i_3 - i_4) + 1(i_3 - i_1) = 0$$

$$\Rightarrow -i_1 + 15i_3 - 8i_4 = 0 \quad \cdots (3)$$

For loop-4,

$$i_4 = -3\text{ mA}$$

$$(2) \Rightarrow -3i_1 + 7i_2 - 4\times(-3) = 0$$

$$\Rightarrow -3i_1 + 7i_2 = -12 \quad \cdots (4)$$

$$(3) \Rightarrow -i_1 + 15i_3 - 8\times(-3) = 0$$

$$\Rightarrow -i_1 + 15i_3 = -24 \quad \cdots (5)$$

Solving (1), (4), (5), we get,

$$i_1 = 1.986\text{ mA}$$

$$i_2 = -0.863\text{ mA}$$

$$i_3 = -1.468\text{ mA}$$

$6\text{K}\Omega \rightarrow i = i_3 = -1.468\text{ mA}$

$8\text{K}\Omega \rightarrow i = i_3 - i_4 = 1.532\text{ mA}$

$1\text{K}\Omega \rightarrow i = i_3 - i_1 = -3.454\text{ mA}$

$3\text{K}\Omega \rightarrow i = i_1 - i_2 = 2.899\text{ mA}$

$4\text{K}\Omega \rightarrow i = i_2 - i_4 = 2.137\text{ mA}$

(b) For R1, $P = I^2R = (-3.459\times10^{-3})^2 \times 1\times10^3$

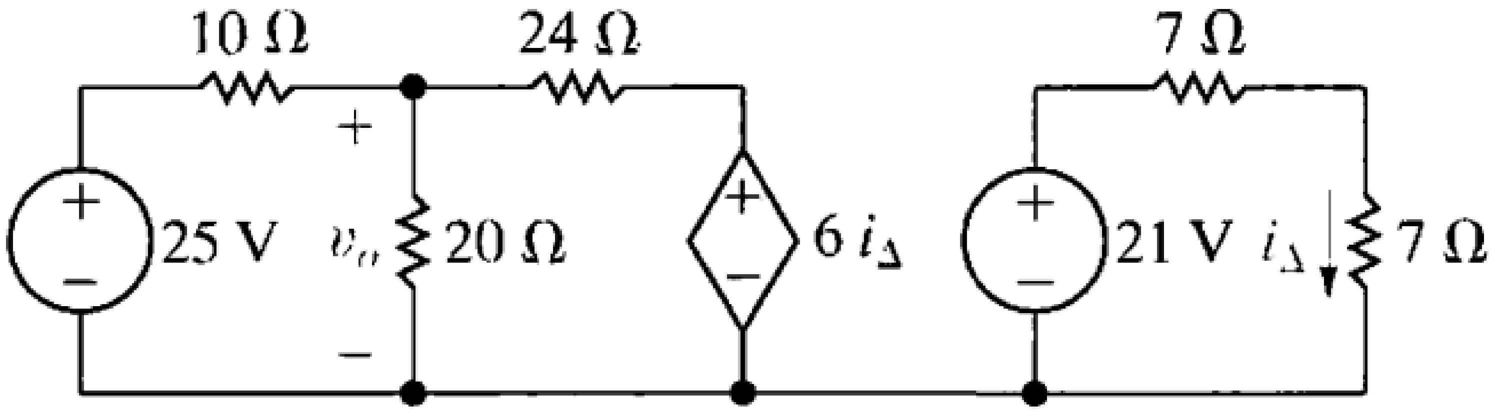
$= 11.93\text{ mW}$

Math Problem Practice:

a) Use the mesh-current method to find v_o in the circuit in Fig. P4.37.

b) Find the power delivered by the dependent source.

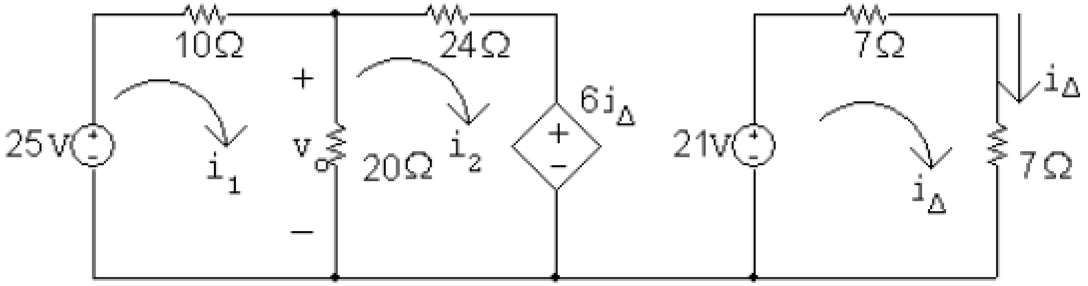
Figure P4.37



Reference: Nilsson_Exercise_Problem_4.37

Math Problem Practice:

P 4.37 [a]



$$25 = 30i_1 - 20i_2 + 0i_\Delta$$

$$0 = -20i_1 + 44i_2 + 6i_\Delta$$

$$21 = 0i_1 + 0i_2 + 14i_\Delta$$

Solving, $i_1 = 1$ A; $\quad i_2 = 0.25$ A; $\quad i_\Delta = 1.5$ A

$$v_o = 20(i_1 - i_2) = 20(0.75) = 15 \text{ V}$$

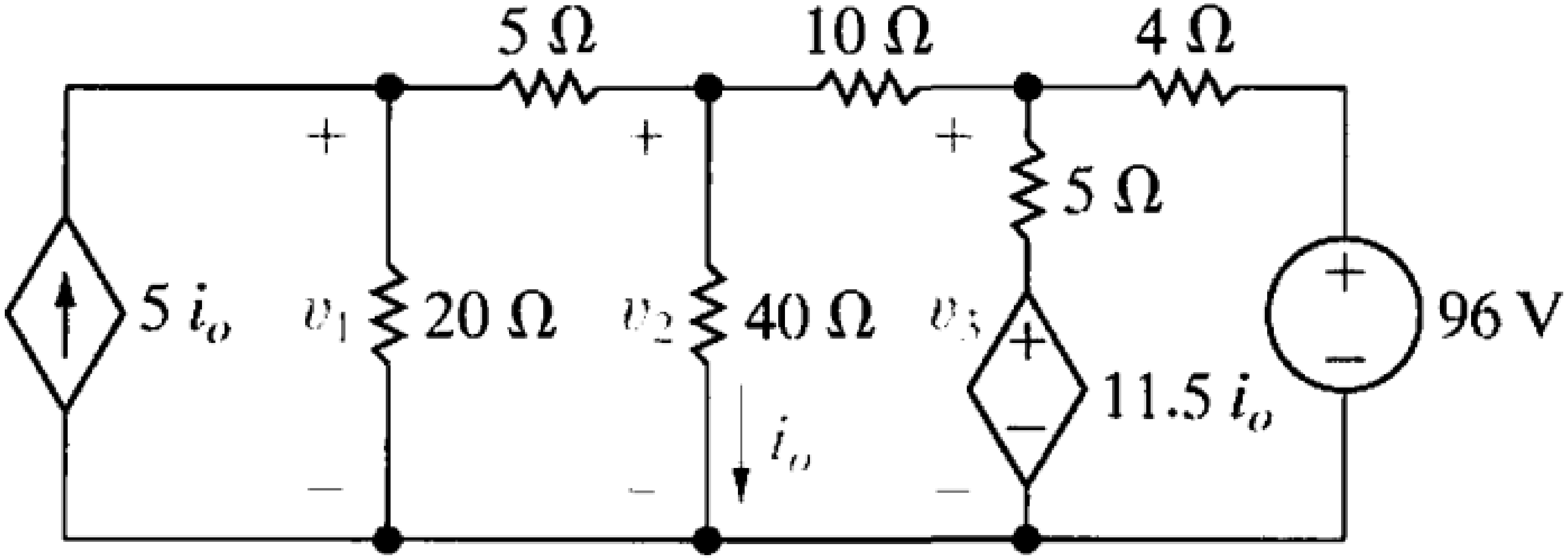
[b] $p_{6i_\Delta} = 6i_\Delta i_2 = (6)(1.5)(0.25) = 2.25$ W (abs)

$\therefore \quad p_{6i_\Delta}$ (deliver) $= -2.25$ W

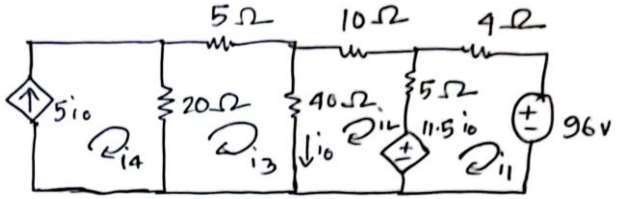
Reference: Nilsson_Exercise_Problem_4.37

Math Problem Practice:

Find the mesh currents.



Math Problem Practice:



For loop-1,

$$4i_1 + 96 - 11.5i_0 + 5(i_1 - i_2) = 0$$

$$\Rightarrow 4i_1 + 96 - 11.5(i_3 - i_2) + 5(i_1 - i_2) = 0$$

$$\Rightarrow 4i_1 - 11.5i_3 + 11.5i_2 + 5i_1 - 5i_2 = -96$$

$$\therefore 9i_1 + 6.5i_2 - 11.5i_3 = -96 \ldots (1)$$

For loop-2,

$$5(i_2 - i_1) + 11.5i_0 + 40(i_2 - i_3) + 10i_2 = 0$$

$$\Rightarrow 5(i_2 - i_1) + 11.5(i_3 - i_2) + 40(i_2 - i_3) + 10i_2 = 0$$

$$\Rightarrow 5i_2 - 5i_1 + 11.5i_3 - 11.5i_2 + 40i_2 - 40i_3 + 10i_2 = 0$$

$$\therefore -5i_1 + 43.5i_2 - 28.5i_3 = 0 \ldots (2)$$

For loop-3,

$$5i_3 + 40(i_3 - i_2) + 20(i_3 - i_4) = 0$$

$$\therefore -40i_2 + 65i_3 - 20i_4 = 0 \ldots (3)$$

For loop-4,

$$i_4 = 5i_0$$

$$\Rightarrow i_4 = 5(i_3 - i_2)$$

$$\Rightarrow i_4 = 5i_3 - 5i_2$$

$$\Rightarrow 5i_2 - 5i_3 + i_4 = 0 \ldots (4)$$

Solving (1), (2), (3) and (4) we get,

$$i_1 = -4.5A$$

$$i_2 = 4.2A$$

$$i_3 = 7.2A$$

$$i_4 = 15A$$

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Math to Practice from the Book for Exam

- **<u>Chapter 3</u>**
  - **<u>Example:</u>**
    - 3.1, 3.2, 3.5, 3.6, 3.7
  - **<u>Practice Problem:</u>**
    - 3.1, 3.2, 3.5, 3.6, 3.7
  - **<u>Problem:</u>**
    - 3.2 to 3.9, 3.12, 3.14, 3.19, 3.38, 3.40, 3.41, 3.42, 3.43, 3.44, 3.45, 3.46, 3.48, 3.50, 3.51, 3.52, 3.55, 3.58, 3.63

  - **N:B: Please note that the supernode concept is not in the present syllabus!**

Thank You