AC CIRCUITS

SINUSOIDS AND PHASORS

Section 9.2, 9.5

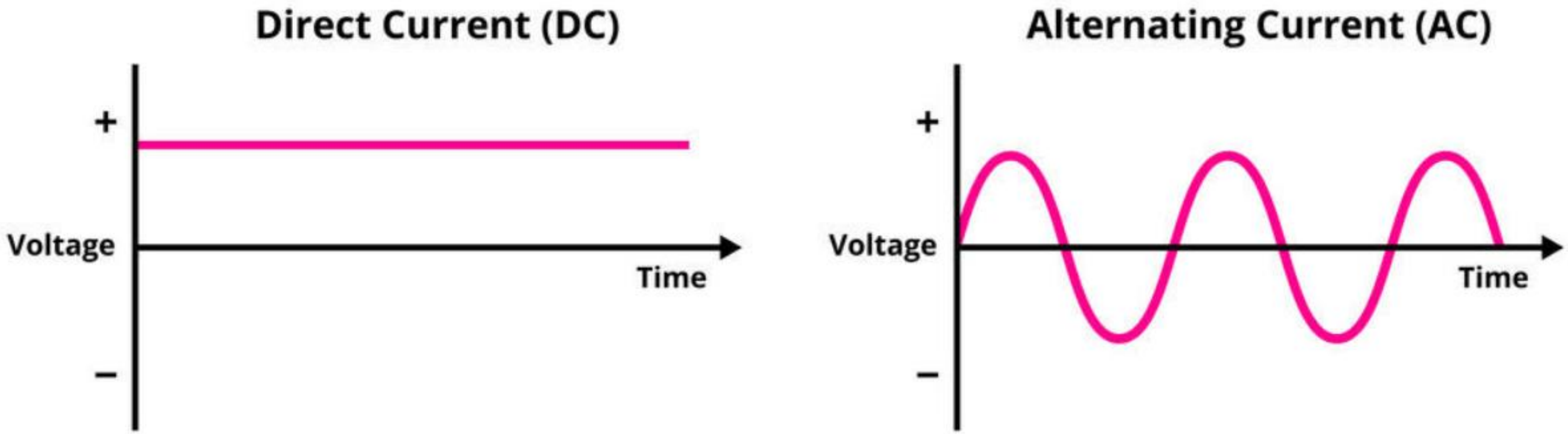
Md. Shafqat Talukder Rakin

Lecturer, Department of CSE,

United International University

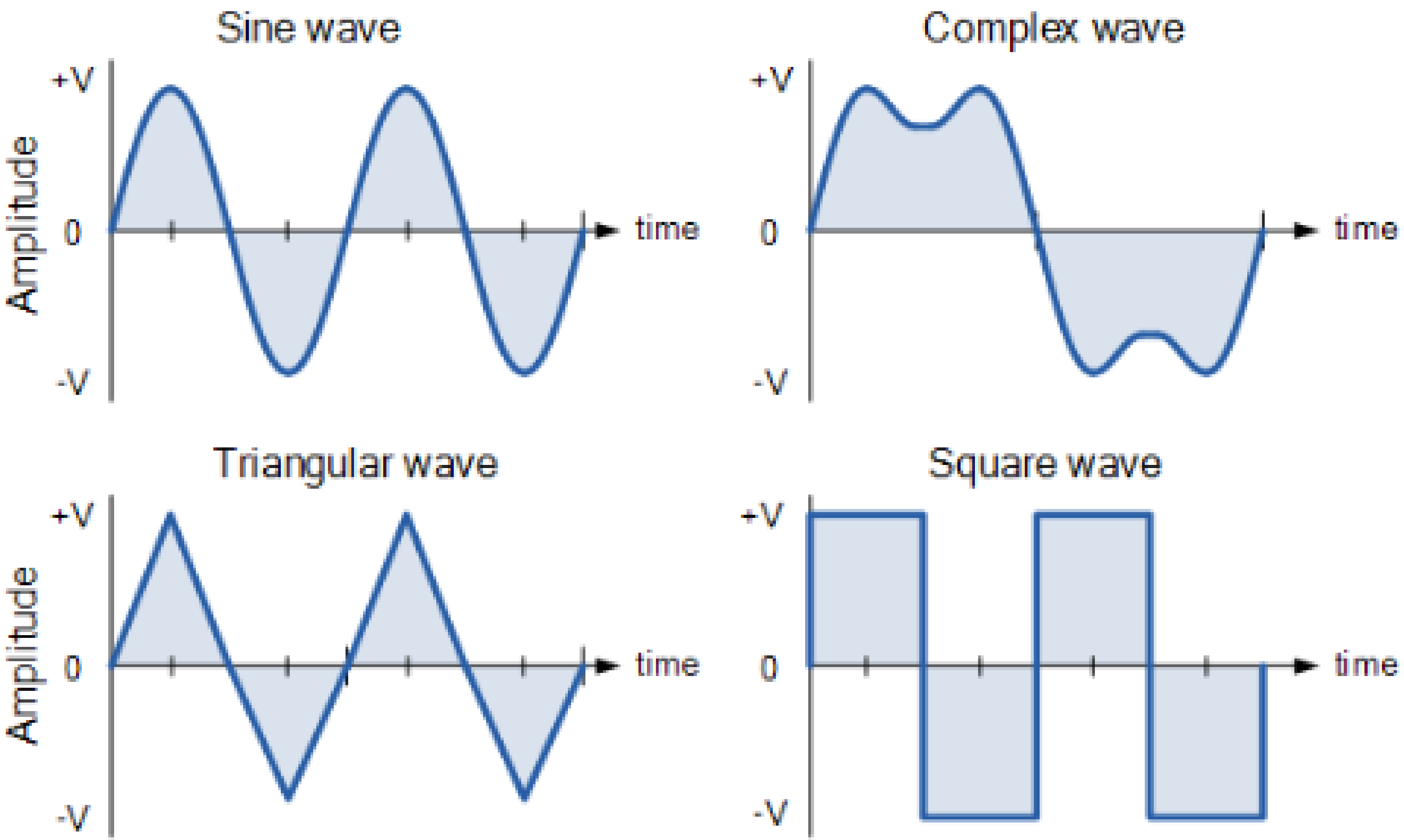
Email id : shafqat@cse.uiu.ac.bd

DC VS AC

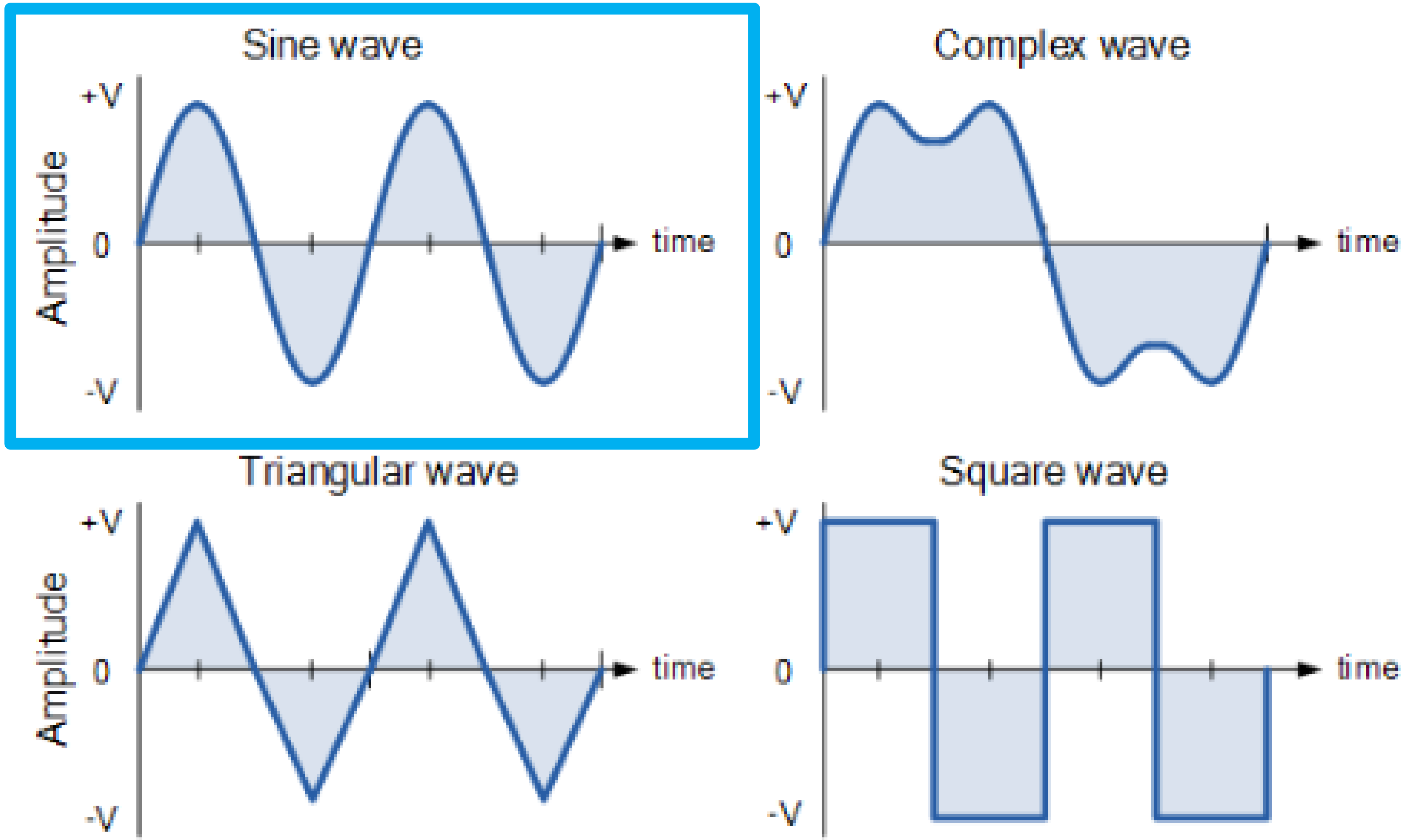


Direct Current (DC)
+
Voltage
Time
−
Alternating Current (AC)
+
Voltage
Time
−

TYPES OF AC WAVEFORMS



OUR FOCUS

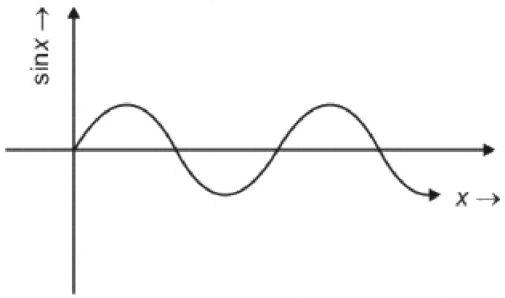


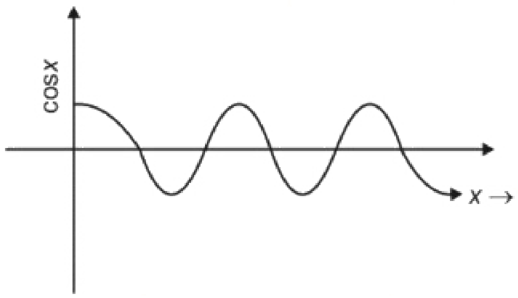
Sine wave
Complex wave
Triangular wave
Square wave
Amplitude
Amplitude
+V
0
-V
time

SINUSOIDS

A **sinusoid** is a signal that has the form of the sine or cosine function.

- A sinusoidal current is usually referred to as *alternating current (ac).*
- Such a current reverses at regular time intervals and has *alternately positive and negative values.*
- Circuits driven by sinusoidal current or voltage sources are called *ac circuits*.





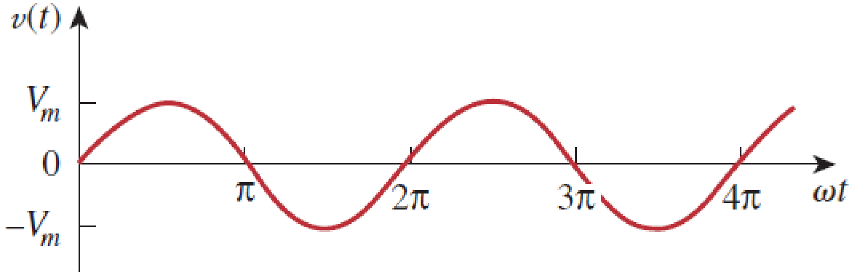
SINUSOIDS EQUATION

Consider the sinusoidal voltage

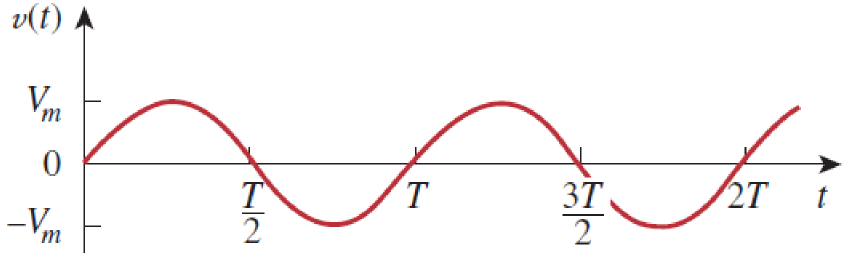
$$v(t) = V_m \sin \omega t \tag{9.1}$$

where

V_m = the *amplitude* of the sinusoid
ω = the *angular frequency* in radians/s
ωt = the *argument* of the sinusoid



(a)



(b)

SOME FORMULAS

- Time period

$$T = \frac{2\pi}{\omega}$$

- Cyclic Frequency

$$f = \frac{1}{T}$$

, f is in hertz (Hz).

- Angular Frequency

$$\omega = 2\pi f$$

ω is in radians per second (rad/s)

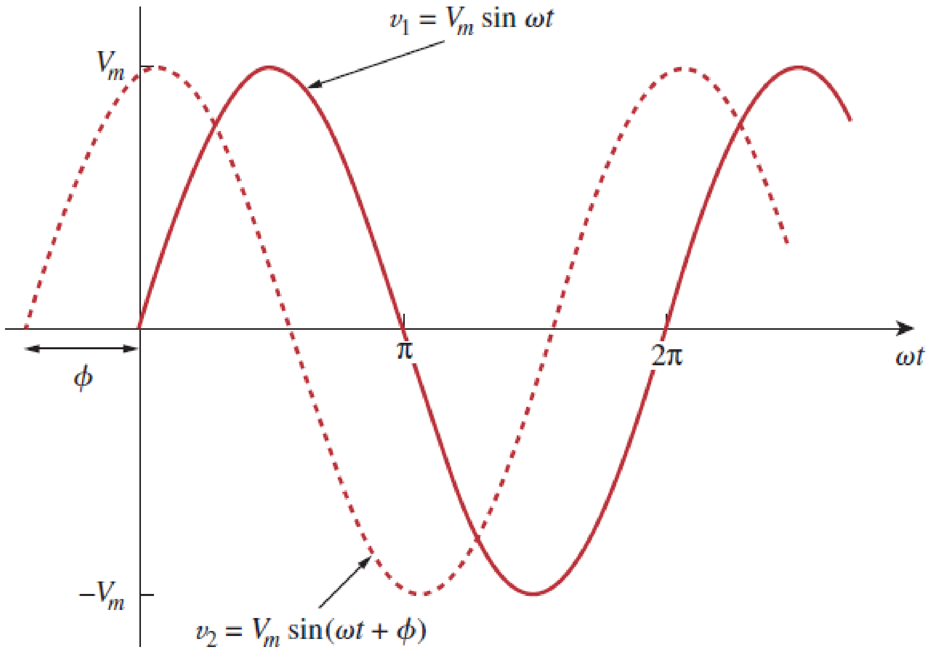
GENERAL EXPRESSION FOR A SINUSOID

$$v(t) = V_m \sin(\omega t + \phi)$$

where $(\omega t + \phi)$ is the argument and ϕ is the *phase*. Both argument and phase can be in radians or degrees.

LEADING LAGGING CONCEPT

$$v_1(t) = V_m \sin \omega t \qquad \text{and} \qquad v_2(t) = V_m \sin(\omega t + \phi)$$



v_2 *leads* v_1 by ϕ or that v_1 *lags* v_2 by ϕ.

NECESSARY FORMULAS FOR COMPARING SINUSOIDS

A sinusoid can be expressed in either sine or cosine form. When comparing two sinusoids, it is expedient to express both as either sine or cosine with positive amplitudes. This is achieved by using the following trigonometric identities:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \end{aligned} \tag{9.9}$$

With these identities, it is easy to show that

$$\begin{aligned} \sin(\omega t \pm 180^\circ) &= -\sin \omega t \\ \cos(\omega t \pm 180^\circ) &= -\cos \omega t \\ \sin(\omega t \pm 90^\circ) &= \pm\cos \omega t \\ \cos(\omega t \pm 90^\circ) &= \mp\sin \omega t \end{aligned} \tag{9.10}$$

Using these relationships, we can transform a sinusoid from sine form to cosine form or vice versa.

MATH PROBLEM PRACTICE:

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

<u>Reference: Sadiku_Example_9.1</u>

MATH PROBLEM PRACTICE:

Find the amplitude, phase, period, and frequency of the sinusoid

$$v(t) = 12\cos(50t + 10^\circ)$$

Solution:

The amplitude is $V_m = 12$ V.
The phase is $\phi = 10^\circ$.
The angular frequency is $\omega = 50$ rad/s.
The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257$ s.
The frequency is $f = \frac{1}{T} = 7.958$ Hz.

Reference: Sadiku_Example_9.1

MATH PROBLEM PRACTICE:

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50^\circ)$ and $v_2 = 12\sin(\omega t - 10^\circ)$. State which sinusoid is leading.

<u>Reference: Sadiku_Example_9.2</u>

MATH PROBLEM PRACTICE:

Calculate the phase angle between $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

Solution:
Let us calculate the phase in three ways. The first two methods use trigonometric identities, while the third method uses the graphical approach.

METHOD 1 In order to compare v_1 and v_2, we must express them in the same form. If we express them in cosine form with positive amplitudes,

$$v_1 = -10\cos(\omega t + 50°) = 10\cos(\omega t + 50° - 180°)$$

$$v_1 = 10\cos(\omega t - 130°) \quad \text{or} \quad v_1 = 10\cos(\omega t + 230°) \qquad \textbf{(9.2.1)}$$

and

$$v_2 = 12\sin(\omega t - 10°) = 12\cos(\omega t - 10° - 90°)$$

$$v_2 = 12\cos(\omega t - 100°) \qquad \textbf{(9.2.2)}$$

It can be deduced from Eqs. (9.2.1) and (9.2.2) that the phase difference between v_1 and v_2 is 30°. We can write v_2 as

$$v_2 = 12\cos(\omega t - 130° + 30°) \quad \text{or} \quad v_2 = 12\cos(\omega t + 260°) \qquad \textbf{(9.2.3)}$$

Comparing Eqs. (9.2.1) and (9.2.3) shows clearly that v_2 leads v_1 by 30°.

METHOD 2 Alternatively, we may express v_1 in sine form:

$$v_1 = -10\cos(\omega t + 50°) = 10\sin(\omega t + 50° - 90°)$$
$$= 10\sin(\omega t - 40°) = 10\sin(\omega t - 10° - 30°)$$

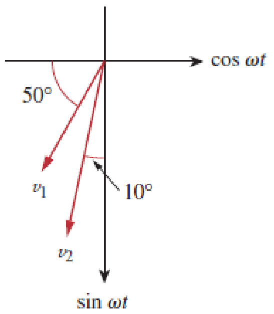


Figure 9.5
For Example 9.2.

But $v_2 = 12\sin(\omega t - 10°)$. Comparing the two shows that v_1 lags v_2 by 30°. This is the same as saying that v_2 leads v_1 by 30°.

METHOD 3 We may regard v_1 as simply $-10\cos\omega t$ with a phase shift of $+50°$. Hence, v_1 is as shown in Fig. 9.5. Similarly, v_2 is $12\sin\omega t$ with a phase shift of $-10°$, as shown in Fig. 9.5. It is easy to see from Fig. 9.5 that v_2 leads v_1 by 30°, that is, $90° - 50° - 10°$.

<u>Reference: Sadiku_Example_9.2</u>

PHASORS

A **phasor** is a complex number that represents the amplitude and phase of a sinusoid.

A complex number z can be written in rectangular form as

$$z = x + jy \tag{9.14a}$$

where $j = \sqrt{-1}$; x is the real part of z; y is the imaginary part of z. In this context, the variables x and y do not represent a location as in two-dimensional vector analysis but rather the real and imaginary parts of z in the complex plane. Nevertheless, we note that there are some resemblances between manipulating complex numbers and manipulating two-dimensional vectors.

The complex number z can also be written in polar or exponential form as

$$z = r\underline{/\phi} = re^{j\phi} \tag{9.14b}$$

where r is the magnitude of z, and ϕ is the phase of z. We notice that z can be represented in three ways:

$$\begin{aligned} z &= x + jy && \text{Rectangular form} \\ z &= r\underline{/\phi} && \text{Polar form} \\ z &= re^{j\phi} && \text{Exponential form} \end{aligned} \tag{9.15}$$

The relationship between the rectangular form and the polar form is shown in Fig. 9.6, where the x axis represents the real part and the y axis represents the imaginary part of a complex number. Given x and y, we can get r and ϕ as

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x} \tag{9.16a}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi \tag{9.16b}$$

Thus, z may be written as

$$\boxed{z = x + jy = r\underline{/\phi} = r(\cos\phi + j\sin\phi)} \tag{9.17}$$

GRAPHICAL REPRESENTATION OF A COMPLEX NUMBER

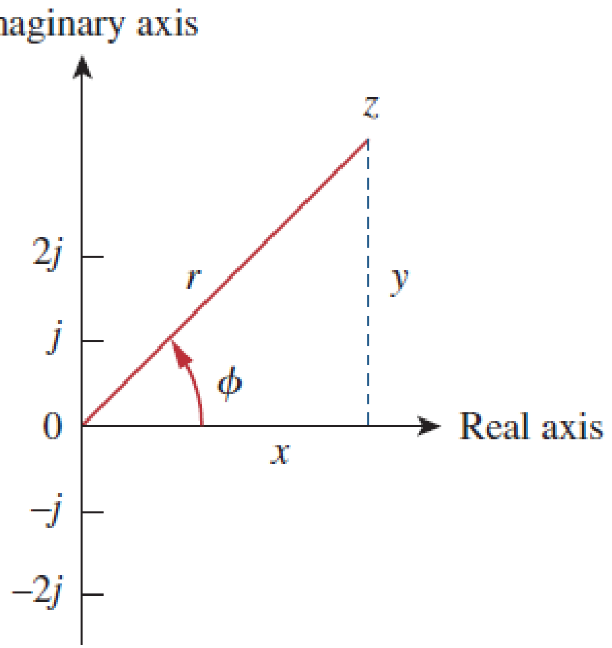


Figure 9.6
Representation of a complex number $z = x + jy = r \angle \phi$.

COMPLEX NUMBERS

Given the complex numbers

$$z = x + jy = r\angle\phi, \qquad z_1 = x_1 + jy_1 = r_1\angle\phi_1$$
$$z_2 = x_2 + jy_2 = r_2\angle\phi_2$$

Addition:

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

Subtraction:

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2) \qquad \textbf{(9.18b)}$$

Multiplication:

$$z_1 z_2 = r_1 r_2 \angle\phi_1 + \phi_2 \qquad \textbf{(9.18c)}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\angle\phi_1 - \phi_2 \qquad \textbf{(9.18d)}$$

Reciprocal:

$$\frac{1}{z} = \frac{1}{r}\angle-\phi \qquad \textbf{(9.18e)}$$

Square Root:

$$\sqrt{z} = \sqrt{r}\,\angle\phi/2 \qquad \textbf{(9.18f)}$$

Complex Conjugate:

$$z^* = x - jy = r\angle-\phi = re^{-j\phi} \qquad \textbf{(9.18g)}$$

Note that from Eq. (9.18e),

$$\frac{1}{j} = -j \qquad \textbf{(9.18h)}$$

TIME DOMAIN -> PHASOR DOMAIN

The idea of phasor representation is based on Euler's identity. In general,

$$\boxed{e^{\pm j\phi} = \cos\phi \pm j\sin\phi} \qquad \textbf{(9.19)}$$

which shows that we may regard $\cos\phi$ and $\sin\phi$ as the real and imaginary parts of $e^{j\phi}$; we may write

$$\cos\phi = \mathrm{Re}(e^{j\phi}) \qquad \textbf{(9.20a)}$$

$$\sin\phi = \mathrm{Im}(e^{j\phi}) \qquad \textbf{(9.20b)}$$

where Re and Im stand for the *real part of* and the *imaginary part of*. Given a sinusoid $v(t) = V_m\cos(\omega t + \phi)$, we use Eq. (9.20a) to express $v(t)$ as

$$v(t) = V_m\cos(\omega t + \phi) = \mathrm{Re}(V_m e^{j(\omega t+\phi)}) \qquad \textbf{(9.21)}$$

or

$$v(t) = \mathrm{Re}(V_m e^{j\phi} e^{j\omega t}) \qquad \textbf{(9.22)}$$

Thus,

$$\boxed{v(t) = \mathrm{Re}(\mathbf{V}e^{j\omega t})} \qquad \textbf{(9.23)}$$

where

$$\mathbf{V} = V_m e^{j\phi} = V_m\underline{/\phi} \qquad \textbf{(9.24)}$$

Equations (9.21) through (9.23) reveal that to get the phasor corresponding to a sinusoid, we first express the sinusoid in the cosine form so that the sinusoid can be written as the real part of a complex number. Then we take out the time factor $e^{j\omega t}$, and whatever is left is the phasor corresponding to the sinusoid. By suppressing the time factor, we transform the sinusoid from the time domain to the phasor domain. This transformation is summarized as follows:

$$\boxed{\underset{\text{(Time-domain representation)}}{v(t) = V_m\cos(\omega t + \phi)} \quad \Leftrightarrow \quad \underset{\text{(Phasor-domain representation)}}{\mathbf{V} = V_m\underline{/\phi}}} \qquad \textbf{(9.25)}$$

PHASOR DIAGRAM

For example, phasors $\mathbf{V} = V_m \underline{/\phi}$ and $\mathbf{I} = I_m \underline{/-\theta}$ are graphically represented in Fig. 9.8. Such a graphical representation of phasors is known as a *phasor diagram*.

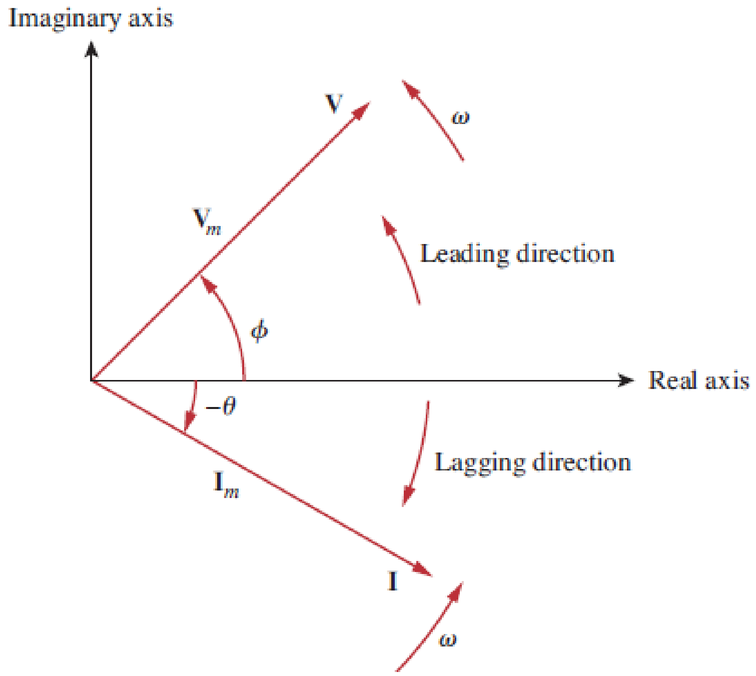


Figure 9.8
A phasor diagram showing $\mathbf{V} = V_m \underline{/\phi}$ and $\mathbf{I} = I_m \underline{/-\theta}$.

FORMULAS FOR PHASOR TRANSFORMATION

TABLE 9.1

Sinusoid-phasor transformation.

Time domain representation	Phasor domain representation
$V_m \cos(\omega t + \phi)$	$V_m \angle \phi$
$V_m \sin(\omega t + \phi)$	$V_m \angle \phi - 90^\circ$
$I_m \cos(\omega t + \theta)$	$I_m \angle \theta$
$I_m \sin(\omega t + \theta)$	$I_m \angle \theta - 90^\circ$

MATH PROBLEM PRACTICE:

Transform these sinusoids to phasors:

(a) $i = 6\cos(50t - 40^\circ)$ A
(b) $v = -4\sin(30t + 50^\circ)$ V

Solution:

(a) $i = 6\cos(50t - 40^\circ)$ has the phasor

$$\mathbf{I} = 6\underline{/-40^\circ} \text{ A}$$

(b) Since $-\sin A = \cos(A + 90^\circ)$,

$$\begin{aligned} v = -4\sin(30t + 50^\circ) &= 4\cos(30t + 50^\circ + 90^\circ) \\ &= 4\cos(30t + 140^\circ) \text{ V} \end{aligned}$$

The phasor form of v is

$$\mathbf{V} = 4\underline{/140^\circ} \text{ V}$$

Reference: Sadiku_Example_9.4

MATH PROBLEM PRACTICE:

Find the sinusoids represented by these phasors:

(a) $\mathbf{I} = -3 + j4$ A
(b) $\mathbf{V} = j8e^{-j20^\circ}$ V

Solution:

(a) $\mathbf{I} = -3 + j4 = 5\underline{/126.87^\circ}$. Transforming this to the time domain gives

$$i(t) = 5\cos(\omega t + 126.87^\circ) \text{ A}$$

(b) Since $j = 1\underline{/90^\circ}$,

$$\begin{aligned}\mathbf{V} = j8\underline{/-20^\circ} &= (1\underline{/90^\circ})(8\underline{/-20^\circ}) \\ &= 8\underline{/90^\circ - 20^\circ} = 8\underline{/70^\circ} \text{ V}\end{aligned}$$

Converting this to the time domain gives

$$v(t) = 8\cos(\omega t + 70^\circ) \text{ V}$$

Reference: Sadiku_Example_9.5

IMPEDANCE

The impedance **Z** of a circuit is the ratio of the phasor voltage **V** to the phasor current **I**, measured in ohms (Ω).

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

PASSIVE ELEMENTS IN AC CIRCUIT(R, L, C)

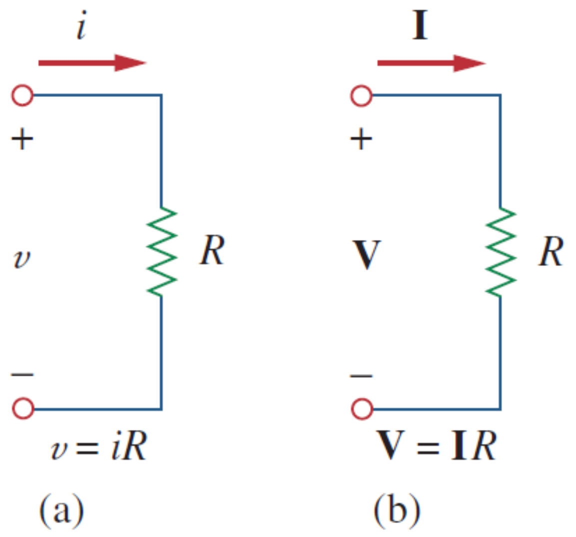


Figure 9.9
Voltage-current relations for a resistor in the: (a) time domain, (b) frequency domain.

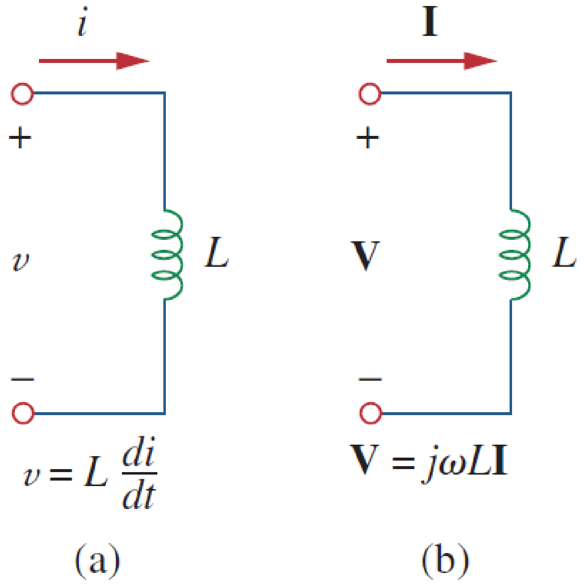


Figure 9.11
Voltage-current relations for an inductor in the: (a) time domain, (b) frequency domain.

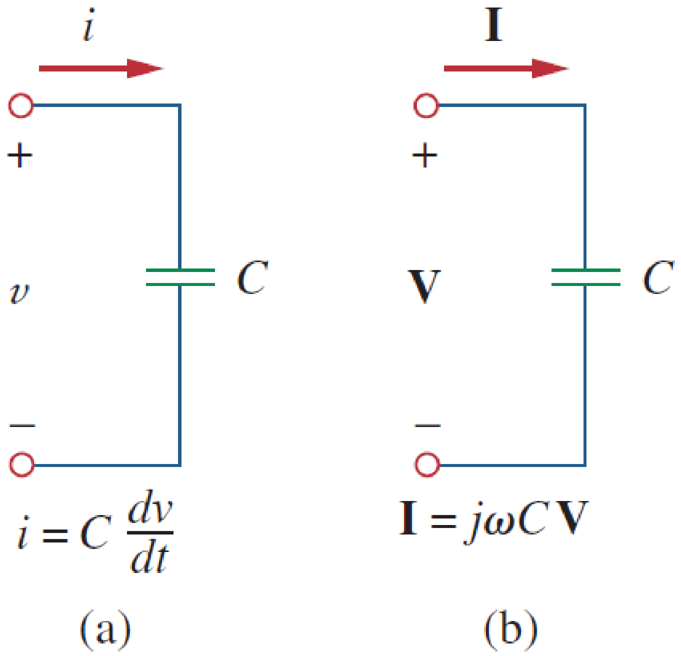


Figure 9.13
Voltage-current relations for a capacitor in the: (a) time domain, (b) frequency domain.

IMPEDANCE(CONTINUED…)

Element	Impedance
R	$\mathbf{Z} = R$
L	$\mathbf{Z} = j\omega L$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$

IMPEDANCE(CONTINUED…)

As a complex quantity, the impedence may be expressed in rectangular form as

$$\mathbf{Z} = R + jX \tag{9.41}$$

where $R = \text{Re}\,\mathbf{Z}$ is the *resistance* and $X = \text{Im}\,\mathbf{Z}$ is the *reactance*. The reactance X may be positive or negative. We say that the impedance is inductive when X is positive or capacitive when X is negative. Thus, impedance $\mathbf{Z} = R + jX$ is said to be *inductive* or lagging since current lags voltage, while impedance $\mathbf{Z} = R - jX$ is capacitive or leading because current leads voltage. The impedance, resistance, and reactance are all measured in ohms. The impedance may also be expressed in polar form as

$$\mathbf{Z} = |\mathbf{Z}| \underline{/\theta} \tag{9.42}$$

Comparing Eqs. (9.41) and (9.42), we infer that

$$\boxed{\mathbf{Z} = R + jX = |\mathbf{Z}| \underline{/\theta}} \tag{9.43}$$

where

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}, \qquad \theta = \tan^{-1}\frac{X}{R} \tag{9.44}$$

and

$$R = |\mathbf{Z}|\cos\theta, \qquad X = |\mathbf{Z}|\sin\theta \tag{9.45}$$

ADMITTANCE

The admittance **Y** is the reciprocal of impedance, measured in siemens (S).

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{\mathbf{I}}{\mathbf{V}}$$

ADMITTANCE(CONTINUED…)

Element	Impedance	Admittance
R	$\mathbf{Z} = R$	$\mathbf{Y} = \dfrac{1}{R}$
L	$\mathbf{Z} = j\omega L$	$\mathbf{Y} = \dfrac{1}{j\omega L}$
C	$\mathbf{Z} = \dfrac{1}{j\omega C}$	$\mathbf{Y} = j\omega C$

ADMITTANCE(CONTINUED...)

As a complex quantity, we may write $\mathbf{Y}$ as

$$\boxed{\mathbf{Y} = G + jB} \tag{9.47}$$

where $G = \text{Re}\,\mathbf{Y}$ is called the *conductance* and $B = \text{Im}\,\mathbf{Y}$ is called the *susceptance*. Admittance, conductance, and susceptance are all expressed in the unit of siemens (or mhos). From Eqs. (9.41) and (9.47),

$$G + jB = \frac{1}{R + jX} \tag{9.48}$$

By rationalization,

$$G + jB = \frac{1}{R + jX} \cdot \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2} \tag{9.49}$$

Equating the real and imaginary parts gives

$$G = \frac{R}{R^2 + X^2}, \qquad B = -\frac{X}{R^2 + X^2} \tag{9.50}$$

showing that $G \neq 1/R$ as it is in resistive circuits. Of course, if $X = 0$, then $G = 1/R$.

MATH PROBLEM PRACTICE:

Example 9.9

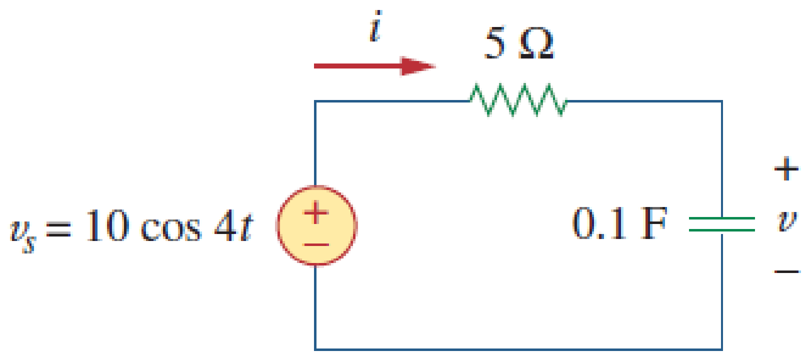


Figure 9.16
For Example 9.9.

Find $v(t)$ and $i(t)$ in the circuit shown in Fig. 9.16.

Reference: Sadiku_Example_9.9

MATH PROBLEM PRACTICE:

Solution:

From the voltage source 10 cos 4*t*, $\omega = 4$,

$$\mathbf{V}_s = 10\angle 0^\circ \text{ V}$$

The impedance is

$$\mathbf{Z} = 5 + \frac{1}{j\omega C} = 5 + \frac{1}{j4 \times 0.1} = 5 - j2.5\ \Omega$$

Hence the current

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ}{5 - j2.5} = \frac{10(5 + j2.5)}{5^2 + 2.5^2} \qquad (9.9.1)$$
$$= 1.6 + j0.8 = 1.789\angle 26.57^\circ \text{ A}$$

The voltage across the capacitor is

$$\mathbf{V} = \mathbf{I}\mathbf{Z}_C = \frac{\mathbf{I}}{j\omega C} = \frac{1.789\angle 26.57^\circ}{j4 \times 0.1}$$
$$= \frac{1.789\angle 26.57^\circ}{0.4\angle 90^\circ} = 4.47\angle -63.43^\circ \text{ V} \qquad (9.9.2)$$

Converting **I** and **V** in Eqs. (9.9.1) and (9.9.2) to the time domain, we get

$$i(t) = 1.789 \cos(4t + 26.57^\circ) \text{ A}$$
$$v(t) = 4.47 \cos(4t - 63.43^\circ) \text{ V}$$

Notice that $i(t)$ leads $v(t)$ by 90° as expected.

Reference: Sadiku_Example_9.9

KIRCHHOFF'S LAWS IN THE FREQUENCY DOMAIN

We cannot do circuit analysis in the frequency domain without Kirchhoff's current and voltage laws. Therefore, we need to express them in the frequency domain.

For KVL, let $v_1, v_2, \ldots, v_n$ be the voltages around a closed loop. Then

$$v_1 + v_2 + \cdots + v_n = 0 \tag{9.51}$$

In the sinusoidal steady state, each voltage may be written in cosine form, so that Eq. (9.51) becomes

$$\begin{aligned} V_{m1}\cos(\omega t + \theta_1) + V_{m2}\cos(\omega t + \theta_2) \\ + \cdots + V_{mn}\cos(\omega t + \theta_n) = 0 \end{aligned} \tag{9.52}$$

This can be written as

$$\mathrm{Re}(V_{m1}e^{j\theta_1}e^{j\omega t}) + \mathrm{Re}(V_{m2}e^{j\theta_2}e^{j\omega t}) + \cdots + \mathrm{Re}(V_{mn}e^{j\theta_n}e^{j\omega t}) = 0$$

or

$$\mathrm{Re}[(V_{m1}e^{j\theta_1} + V_{m2}e^{j\theta_2} + \cdots + V_{mn}e^{j\theta_n})e^{j\omega t}] = 0 \tag{9.53}$$

If we let $\mathbf{V}_k = V_{mk}e^{j\theta_k}$, then

$$\mathrm{Re}[(\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n)e^{j\omega t}] = 0 \tag{9.54}$$

Since $e^{j\omega t} \neq 0$,

$$\mathbf{V}_1 + \mathbf{V}_2 + \cdots + \mathbf{V}_n = 0 \tag{9.55}$$

indicating that Kirchhoff's voltage law holds for phasors.

By following a similar procedure, we can show that Kirchhoff's current law holds for phasors. If we let $i_1, i_2, \ldots, i_n$ be the current leaving or entering a closed surface in a network at time t, then

$$i_1 + i_2 + \cdots + i_n = 0 \tag{9.56}$$

If $\mathbf{I}_1, \mathbf{I}_2, \ldots, \mathbf{I}_n$ are the phasor forms of the sinusoids $i_1, i_2, \ldots, i_n$, then

$$\mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_n = 0 \tag{9.57}$$

which is Kirchhoff's current law in the frequency domain.

Once we have shown that both KVL and KCL hold in the frequency domain, it is easy to do many things, such as impedance combination, nodal and mesh analyses, superposition, and source transformation.

IMPEDANCE COMBINATIONS(SERIES)

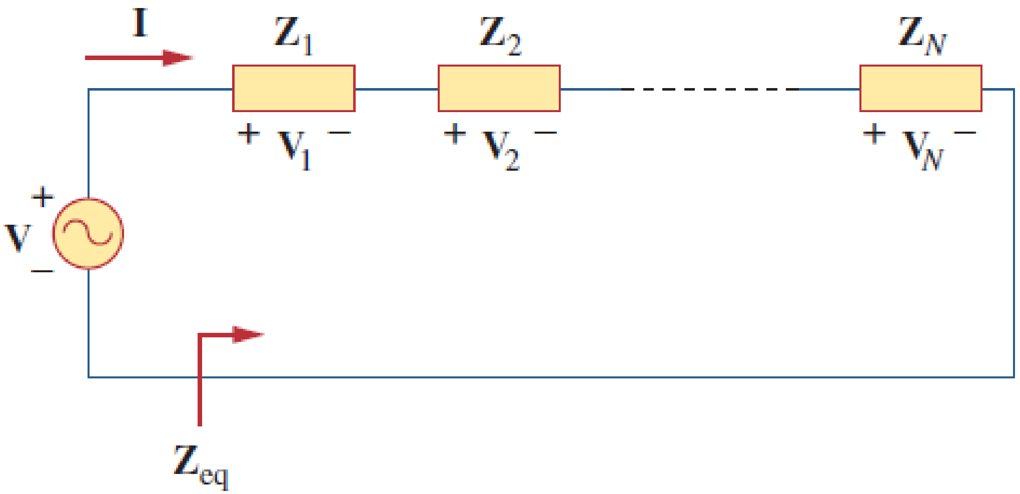


Figure 9.18
N impedances in series.

$$\boxed{Z_{eq} = Z_1 + Z_2 + \cdots + Z_N}$$

VOLTAGE DIVIDER RULE(VDR)

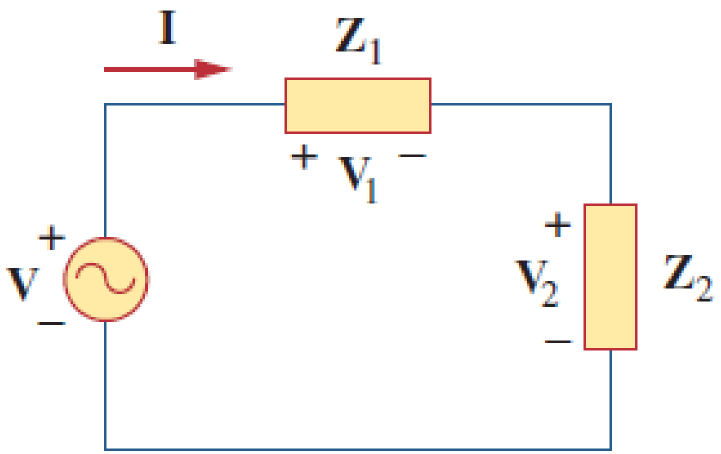


Figure 9.19
Voltage division.

$$\mathbf{V}_1 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}, \qquad \mathbf{V}_2 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}$$

IMPEDANCE COMBINATIONS(PARALLEL)

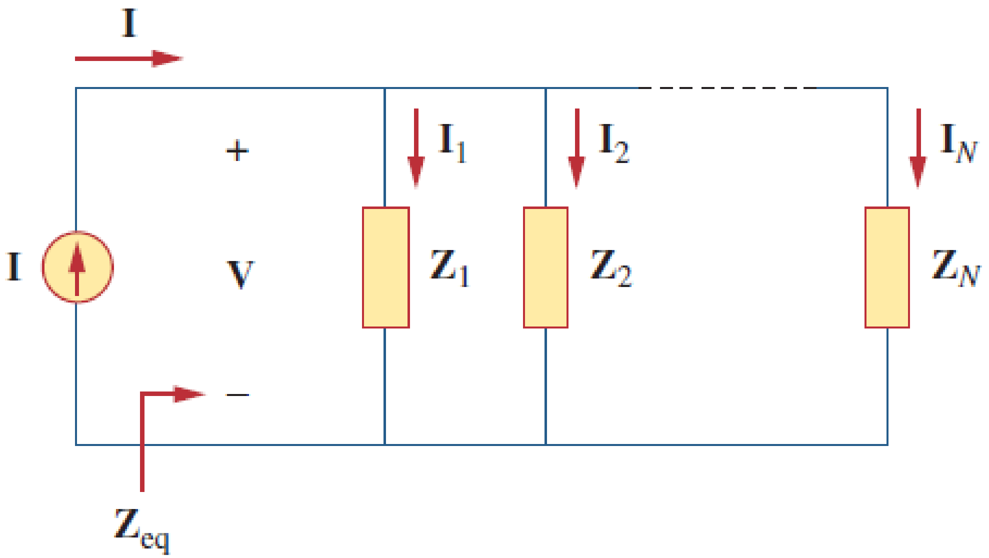


Figure 9.20
N impedances in parallel.

The equivalent impedance is

$$\frac{1}{\mathbf{Z}_{eq}} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots + \frac{1}{\mathbf{Z}_N}$$

and the equivalent admittance is

$$\boxed{\mathbf{Y}_{eq} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots + \mathbf{Y}_N}$$

CURRENT DIVIDER RULE(CDR)

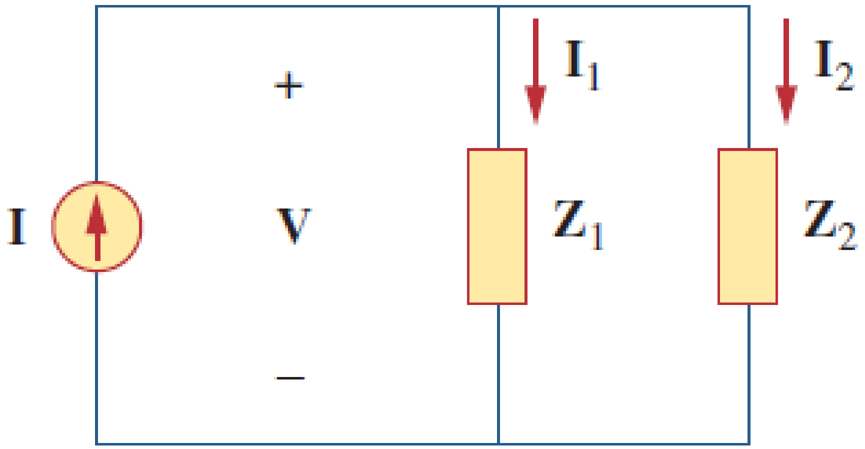


Figure 9.21
Current division.

$$\mathbf{I}_1 = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}, \qquad \mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I}$$

MATH PROBLEM PRACTICE:

Example 9.10

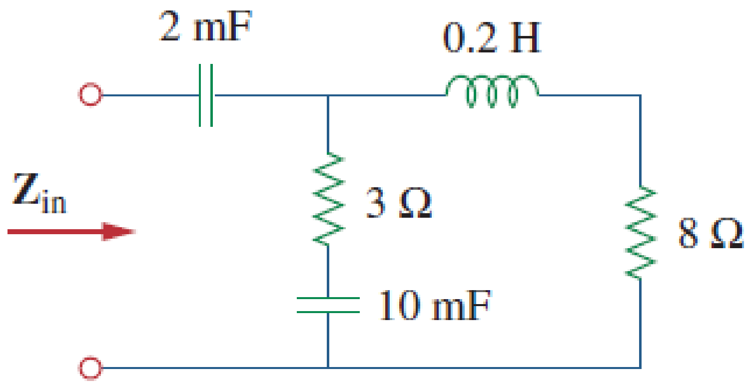


Figure 9.23
For Example 9.10.

Find the input impedance of the circuit in Fig. 9.23. Assume that the circuit operates at $\omega = 50$ rad/s.

Reference: Sadiku_Example_9.10

MATH PROBLEM PRACTICE:

Solution:
Let

$\mathbf{Z}_1$ = Impedance of the 2-mF capacitor
$\mathbf{Z}_2$ = Impedance of the 3-Ω resistor in series with the10-mF capacitor
$\mathbf{Z}_3$ = Impedance of the 0.2-H inductor in series with the 8-Ω resistor

Then

$$\mathbf{Z}_1 = \frac{1}{j\omega C} = \frac{1}{j50 \times 2 \times 10^{-3}} = -j10\ \Omega$$

$$\mathbf{Z}_2 = 3 + \frac{1}{j\omega C} = 3 + \frac{1}{j50 \times 10 \times 10^{-3}} = (3 - j2)\ \Omega$$

$$\mathbf{Z}_3 = 8 + j\omega L = 8 + j50 \times 0.2 = (8 + j10)\ \Omega$$

The input impedance is

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_1 + \mathbf{Z}_2 \| \mathbf{Z}_3 = -j10 + \frac{(3 - j2)(8 + j10)}{11 + j8}$$

$$= -j10 + \frac{(44 + j14)(11 - j8)}{11^2 + 8^2} = -j10 + 3.22 - j1.07\ \Omega$$

Thus,

$$\mathbf{Z}_{\text{in}} = 3.22 - j11.07\ \Omega$$

<u>Reference: Sadiku_Example_9.10</u>

MATH PROBLEM PRACTICE:

Practice Problem 9.10

Determine the input impedance of the circuit in Fig. 9.24 at $\omega = 10$ rad/s.

Answer: $(149.52 - j195)$

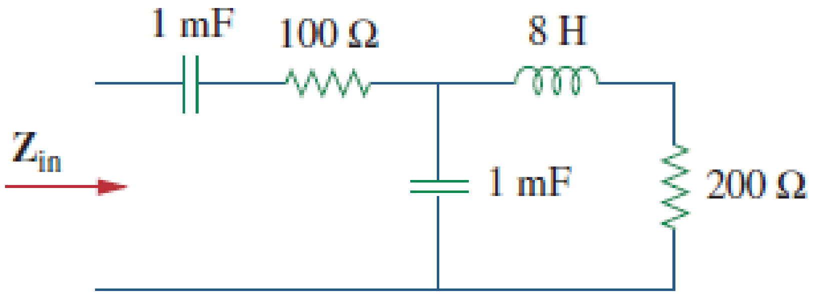


Figure 9.24
For Practice Prob. 9.10.

MATH PROBLEM PRACTICE:

Practice Problem 9.10

let,

Z_1 = Impedance of the 8H inductor in series with the 200Ω resistor

Z_2 = ~~Capacitance~~ Impedance of the 1mF capacitor

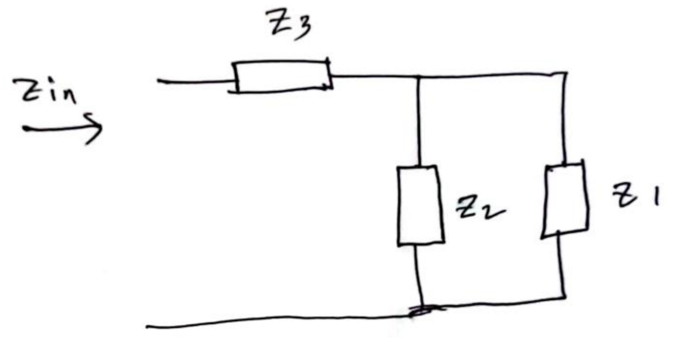
Z_3 = ~~Inda~~ Impedance of the 1mF capacitor in series with the 100Ω resistor

Then

$$Z_1 = 200 + j\omega L$$
$$= 200 + (10\times8)\,j$$
$$= 200 + 80j$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j\times10\times1\times10^{-3}} = -100j$$

$$Z_3 = 100 + \frac{1}{j\omega C}$$
$$= 100 + \frac{1}{j\times10\times1\times10^{-3}} = 100 - 100j$$



$$Z_{in} = Z_{eq} = Z_3 + (Z_1 \,||\, Z_2)$$
$$= 149.505 - 195.0995\,j$$

Reference: Sadiku_Practice_Problem_9.10

MATH PROBLEM PRACTICE:

Example 9.11

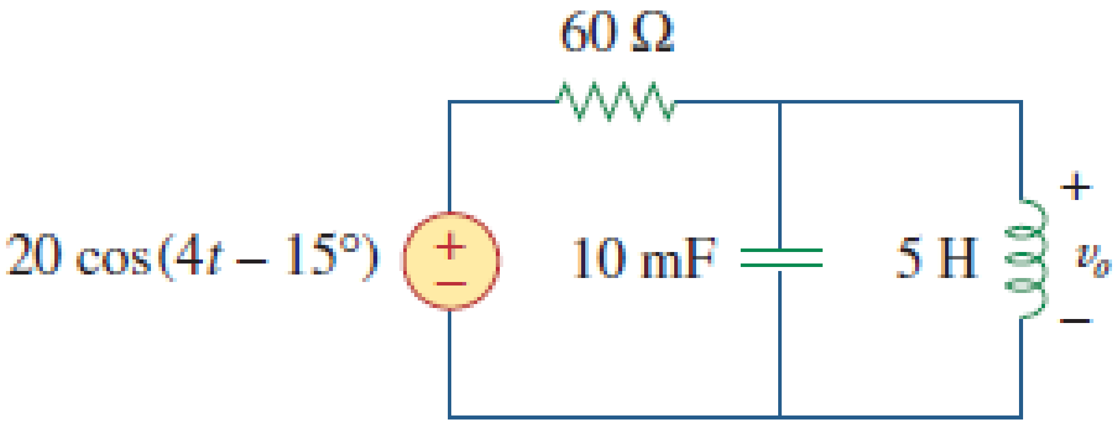


Figure 9.25
For Example 9.11.

Determine $v_o(t)$ in the circuit of Fig. 9.25.

Reference: Sadiku_Example_9.11

MATH PROBLEM PRACTICE:

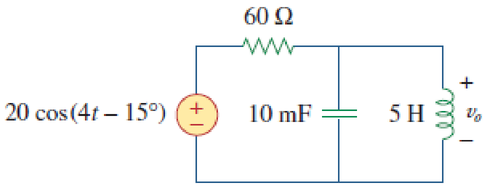


Figure 9.25
For Example 9.11.

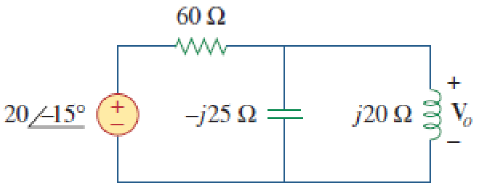


Figure 9.26
The frequency domain equivalent of the circuit in Fig. 9.25.

Solution:

To do the analysis in the frequency domain, we must first transform the time domain circuit in Fig. 9.25 to the phasor domain equivalent in Fig. 9.26. The transformation produces

$$v_s = 20\cos(4t - 15°) \quad \Rightarrow \quad \mathbf{V}_s = 20\angle -15°\ \text{V}, \qquad \omega = 4$$

$$10\text{ mF} \quad \Rightarrow \quad \frac{1}{j\omega C} = \frac{1}{j4 \times 10 \times 10^{-3}} = -j25\ \Omega$$

$$5\text{ H} \quad \Rightarrow \quad j\omega L = j4 \times 5 = j20\ \Omega$$

Let

$\mathbf{Z}_1$ = Impedance of the 60-Ω resistor

$\mathbf{Z}_2$ = Impedance of the parallel combination of the 10-mF capacitor and the 5-H inductor

Then $\mathbf{Z}_1 = 60\ \Omega$ and

$$\mathbf{Z}_2 = -j25 \parallel j20 = \frac{-j25 \times j20}{-j25 + j20} = j100\ \Omega$$

By the voltage-division principle,

$$\mathbf{V}_o = \frac{\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{V}_s = \frac{j100}{60 + j100}(20\angle -15°)$$
$$= (0.8575\angle 30.96°)(20\angle -15°) = 17.15\angle 15.96°\ \text{V}$$

We convert this to the time domain and obtain

$$v_o(t) = 17.15\cos(4t + 15.96°)\ \text{V}$$

<u>Reference: Sadiku_Example_9.11</u>

MATH PROBLEM PRACTICE:

Practice Problem 9.11 Calculate v_o in the circuit of Fig. 9.27.

Answer: $v_o(t) = 35.36 \cos(10t - 105°)$ V.

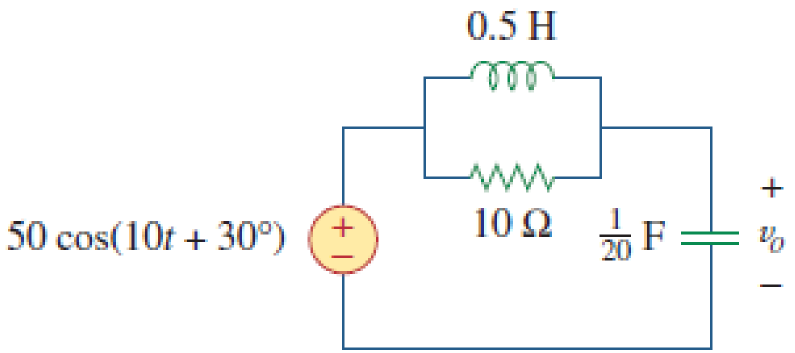


Figure 9.27
For Practice Prob. 9.11.

Reference: Sadiku_Practice_Problem_9.11

MATH PROBLEM PRACTICE:

Practice Problem 9.11

let,

Z_1 = Impedance of the 0.5 H inductor

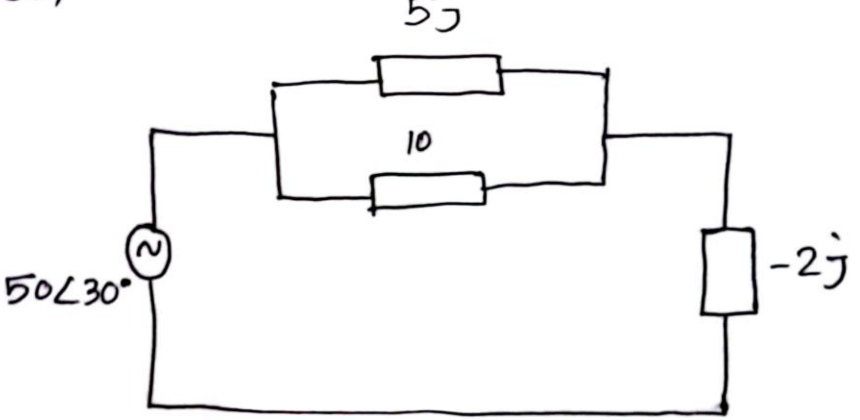
Z_2 = Impedance of the $\frac{1}{20}$ F capacitor

Here, $V_s = 50\cos(10t + 30^\circ)$, So, $\omega = 10$

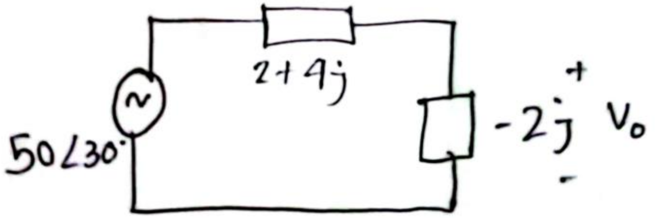
$$Z_1 = j\omega L = j \times 10 \times 0.5 = 5j$$

$$Z_2 = \frac{1}{j\omega C} = \frac{1}{j \times 10 \times \frac{1}{20}} = -2j$$

So,



$\Downarrow$ $(5j \parallel 10) = 2 + 4j$



$$\therefore V_o = \frac{-2j}{2+4j-2j} \times 50\angle 30^\circ$$

$$= -9.1506 - 34.1506j$$

$$= 35.355\angle -105$$

$$\therefore v_o(t) = 35.36\cos(10t - 105^\circ)$$

Reference: Sadiku_Practice_Problem_9.11

MATH PROBLEM PRACTICE:

9.39 For the circuit shown in Fig. 9.46, find $\mathbf{Z}_{eq}$ and use that to find current **I**. Let $\omega = 10$ rad/s.

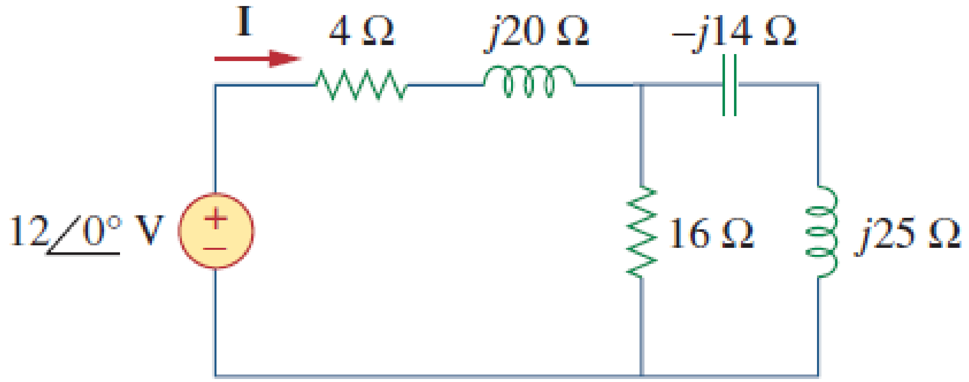


Figure 9.46
For Prob. 9.39.

Reference: Sadiku_Exercise_9.39

MATH PROBLEM PRACTICE:

Chapter 9, Solution 39.

$$Z_{eq} = 4 + j20 + 10//(-j14 + j25) = \underline{9.135 + j27.47 \ \ \Omega}$$

$$= \mathbf{(9.135+j27.47)\ \Omega}$$

$$I = \frac{V}{Z_{eq}} = \frac{12}{9.135 + j27.47} = 0.4145 < -71.605^o$$

$$i(t) = \mathbf{414.5cos(10t–71.6^{\circ})\ mA}$$

<u>Reference: Sadiku_Exercise_9.39</u>

MATH PROBLEM PRACTICE:

9.45 Find current $\mathbf{I}_o$ in the network of Fig. 9.52.

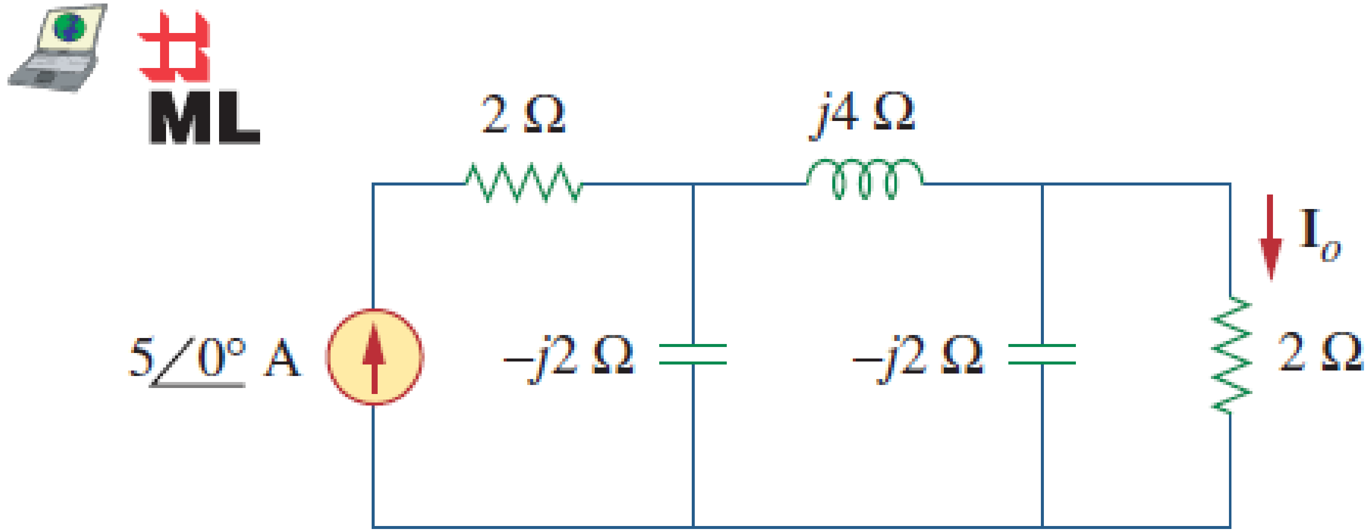


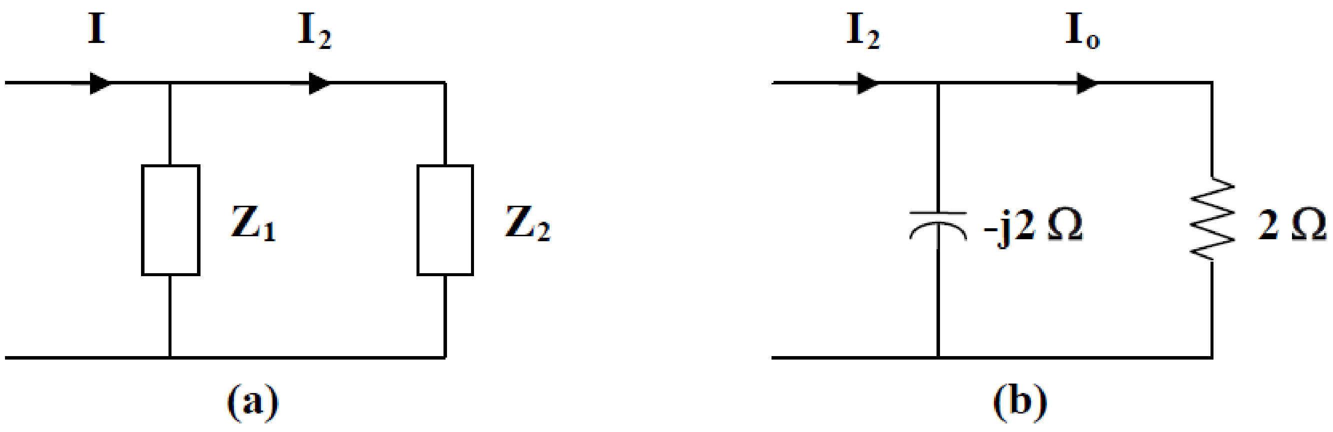
Figure 9.52
For Prob. 9.45.

Reference: Sadiku_Exercise_9.45

MATH PROBLEM PRACTICE:

Chapter 9, Solution 45.

We obtain I_o by applying the principle of current division twice.



$$\mathbf{Z}_1 = \text{-j2}\,, \qquad \mathbf{Z}_2 = \text{j4} + (\text{-j2}) \,||\, 2 = \text{j4} + \frac{\text{- j4}}{2 \text{ - j2}} = 1 + \text{j3}$$

$$\mathbf{I}_2 = \frac{\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2}\mathbf{I} = \frac{-j2}{-j2 + 1 + j3}(5\angle 0°) = \frac{\text{- j10}}{1 + \text{j}}$$

$$\mathbf{I}_o = \frac{\text{- j2}}{2 \text{ - j2}}\mathbf{I}_2 = \left(\frac{\text{- j}}{1 \text{ - j}}\right)\left(\frac{\text{- j10}}{1 + \text{j}}\right) = \frac{-10}{1+1} = \mathbf{-5\ A}$$

Reference: Sadiku_Exercise_9.45

MATH PROBLEM PRACTICE:

9.54 In the circuit of Fig. 9.61, find $\mathbf{V}_s$ if $\mathbf{I}_o = 2\angle 0^\circ$ A.

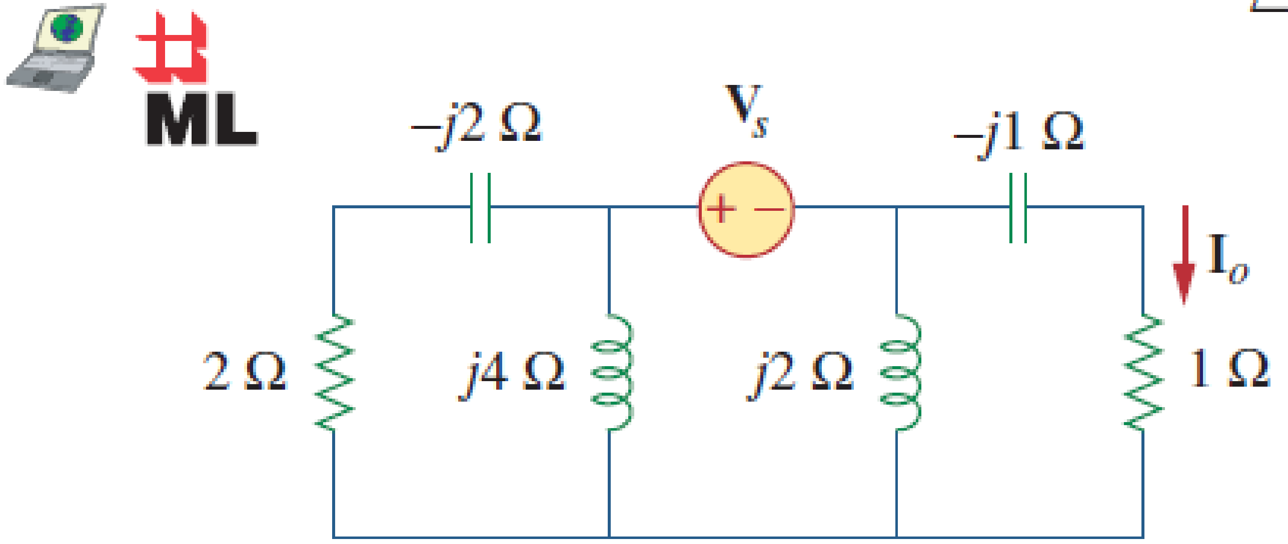


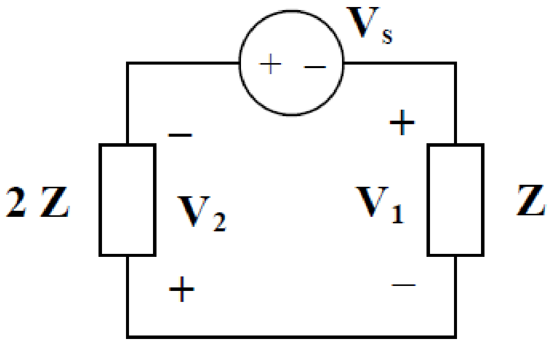
Figure 9.61
For Prob. 9.54.

Reference: Sadiku_Exercise_9.54

MATH PROBLEM PRACTICE:

Chapter 9, Solution 54.

Since the left portion of the circuit is twice as large as the right portion, the equivalent circuit is shown below.



$$\mathbf{V}_1 = \mathbf{I}_o(1-j) = 2(1-j)$$

$$\mathbf{V}_2 = 2\mathbf{V}_1 = 4(1-j)$$

$$\mathbf{V}_2 + \mathbf{V}_s + \mathbf{V}_1 = 0 \text{ or}$$

$$\mathbf{V}_s = -\mathbf{V}_1 - \mathbf{V}_2 = -6(1-j) = (6\angle 180°)(1.4142\angle -45°)$$

$$\mathbf{V}_s = \mathbf{8.485\angle 135°\ V}$$

<u>Reference: Sadiku_Exercise_9.54</u>

Fall 2023

Answer the following questions for the circuit shown in **Figure 3**:
i)Determine Z_T. ii) Current, I. iii) Find the currents through 4 Ω and 3 Ω resistors.
iv) Is the source voltage or the current, I leading in this circuit?

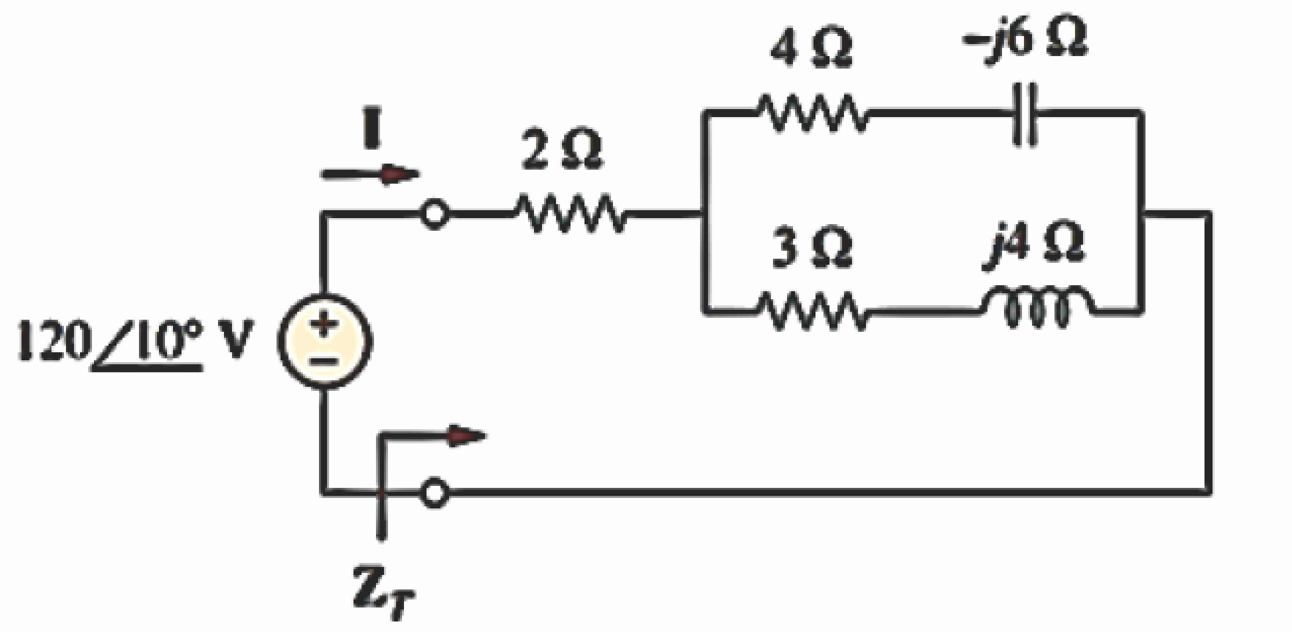
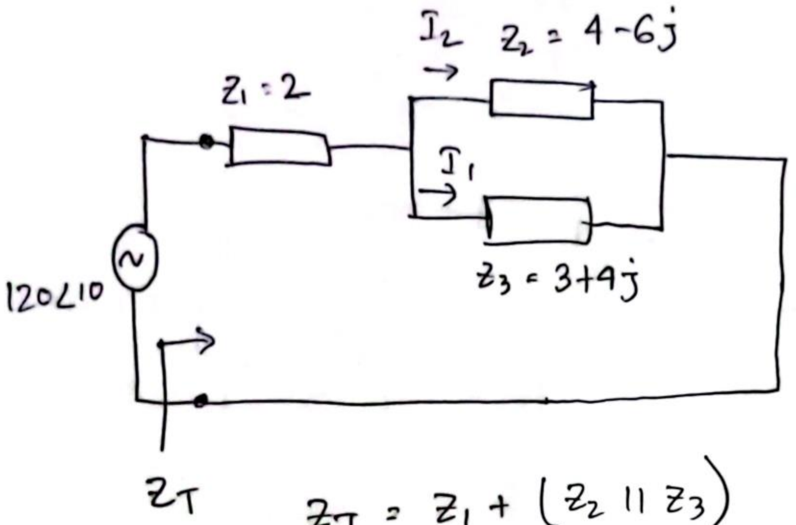


Figure 3.

Fall 2023



$$Z_T = Z_1 + (Z_2 \parallel Z_3)$$
$$= 6.8302 + 1.0943j$$

i) $\therefore Z_T = 6.917\angle 9.103$

ii) $I = \dfrac{V}{Z_T} = \dfrac{120\angle 10}{6.917\angle 9.103} = 17.3978\angle 0.8974$

iii) $I_1 = \dfrac{Z_2}{Z_2+Z_3} \times I = 17.183\angle -39.467°$

$I_2 = \dfrac{Z_3}{Z_2+Z_3} \times I = 11.9145\angle 69.9729$

Current through 4Ω is I_2
" " 3Ω " I_1.

iv) $V = 120\angle 10° = 120\cos(\omega t + 10°)$

$I = 17.3478\angle 0.8974$

$= 17.3978\cos(\omega t + 0.8974)$

So, V is leading.

Summer 2023

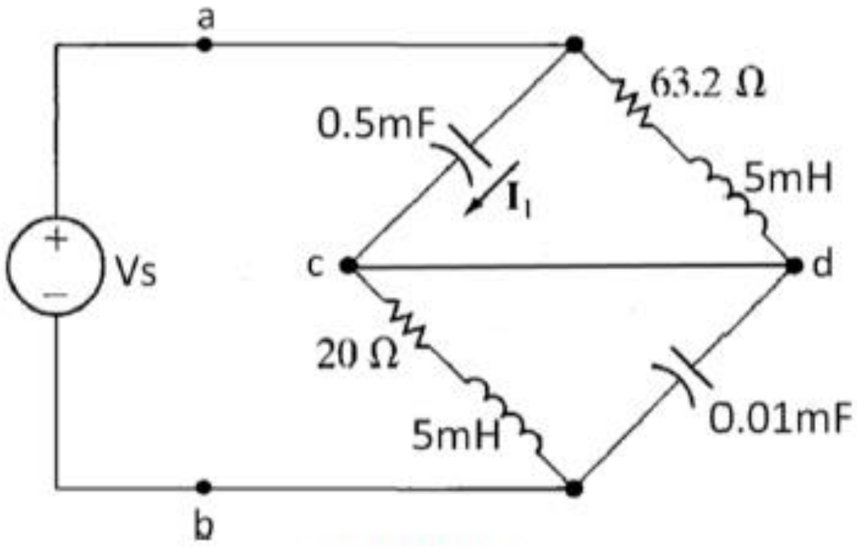


Figure 3.

For the circuit shown in **Figure 3**, $V_s(t) = 15\cos(100t + 30^\circ)$. Now, determine the following questions:

(a) Find equivalent impedance at terminals a – b.

(b) Find $I_1(t)$, $V_c(t)$, $V_d(t)$ and $V_{cd}(t)$.

Summer 2023

let,

z_1 = Impedance of the 0·5 mF capacitor

z_2 = Impedance of the 5 mH inductor in series with the 63·2 Ω resistance

z_3 = Impedance of the 5mH inductor in series with the 20 Ω resistance

z_4 = Impedance of the 0·01 mF capacitor

Here, $V_s(t) = 15\cos(100t + 30^\circ)$ So, $\omega = 100$

$$z_1 = \frac{1}{j\omega C} = \frac{1}{j \times 100 \times 0{\cdot}5 \times 10^{-3}}$$

$$= -20j$$

$$z_2 = 63{\cdot}2 + j\omega L = 63{\cdot}2 + j \times 100 \times 5 \times 10^{-3}$$

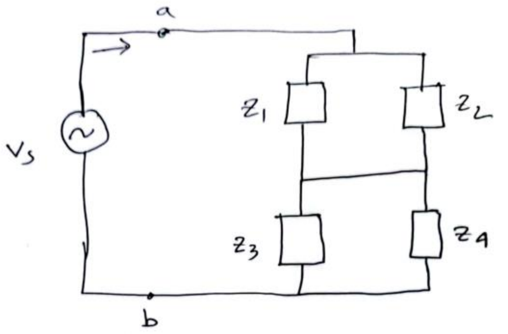
$$= 63{\cdot}2 + 0{\cdot}5j$$

$$z_3 = 20 + j\omega L = 20 + j \times 100 \times 5 \times 10^{-3}$$

$$= 20 + 0{\cdot}5j$$

$$z_4 = \frac{1}{j\omega C} = \frac{1}{j \times 100 \times 0{\cdot}01 \times 10^{-3}} = -1000j$$

So,



(a) $Z_{ab} = (z_1 \parallel z_2) + (z_3 \parallel z_4)$

$$= 25{\cdot}196 - 21{\cdot}617j$$

$$= 33{\cdot}199 \angle -40{\cdot}629^\circ$$

(b) $$I = \frac{V}{Z_{ab}} = \frac{15\angle 30^\circ}{33{\cdot}199\angle -40{\cdot}629}$$

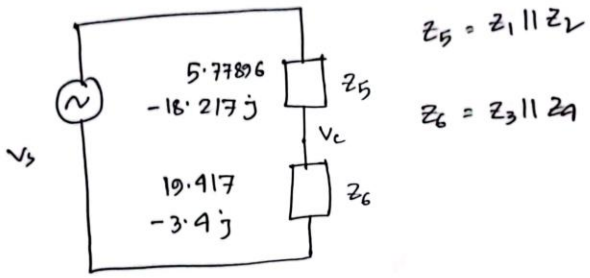
$$= 0{\cdot}4518 \angle 70{\cdot}629^\circ$$

$$I_1 = \frac{z_2}{z_1 + z_2} \times I$$

$$= 0{\cdot}4318 \angle 88{\cdot}229^\circ$$

$\therefore I_1(t) = 0{\cdot}4318\cos(100t + 88{\cdot}229^\circ)$

$V_{cd}(t) = 0$ because $V_c(t) = V_d(t)$



$z_5 = z_1 \parallel z_2$

$z_6 = z_3 \parallel z_4$

$$V_c = \frac{z_6}{z_5 + z_6} \times V_s$$

$$= 15{\cdot}472 \angle 92{\cdot}466^\circ$$

$\therefore V_c(t) = V_d(t) = 15{\cdot}472\cos(100t + 92{\cdot}466^\circ)$

REFERENCES FOR BETTER UNDERSTANDING

- Links to go through in AC Circuit:
  - Basics of sinusoids and lead-lag concept:
    - What is Sinusoid | Lead Lag Sine wave (in Bangla) #Sinusoid (youtube.com)
  - Basics of Impedance and Reactance:
    - What is reactance and impedance | Difference between reactance and impedance [in bangla] (youtube.com)
  - Equivalent Impedance Math:
    - Example 1 Equivalent Impedance (youtube.com)
    - Example 2 Equivalent Impedance (youtube.com)
    - Example : Impedance seen from a terminal (youtube.com)
    - 4. AC Circuit Voltage Divide Rule Problems || AC circuit bangla tutorial (youtube.com)

REFERENCES FOR CALCULATOR USE FOR COMPLEX NUMBERS

- Complex Number calculations in calculator:
  - Complex Number Operations in Calculator (Bangla Tutorial) #ComplexNumberOperationsBanglaTutorial (youtube.com)
  - FX 991ES + Scientific Calculator Conversion of Rectangular to Polar & Polar to Rectangular (youtube.com)
  - Rectangular to Polar form in fx-991MS calculator (youtube.com)
  - Casio FX-991EX Classwiz Complex Numbers: Rectangular to Polar Form Conversions (and Vice Versa) (youtube.com)

MATH TO PRACTICE FROM THE BOOK FOR EXAM

- **Chapter 9**
  - **Example:**
    - 9.1, 9.2, 9.10
  - **Practice Problem:**
    - 9.10
  - **Problem:**
    - 9.1, 9.2, 9.6, 9.39, 9.40, 9.44, 9.45, 9.51, 9.54, 9.55

THANK YOU