① **L1**

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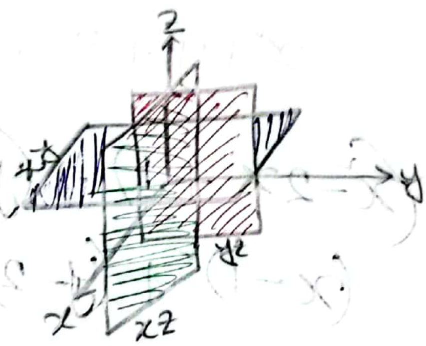
11.1

$xy \to (x, y, 0)$

$yz = (0, y, z)$

$xz = (x, 0, z)$

$x = (x, 0, 0)$



Distance Between 2 point

2D $P_1(x_1, y_1)$, $P_2(x_2, y_2)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

3D (x_1, y_1, z_1) (x_2, y_2, z_2) ~~(x_3, y_3, z_3)~~

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

(2D) Circle equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

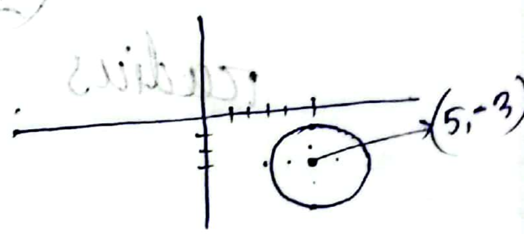
radious

center (x_0, y_0)

$x^2 + y^2 = r^2$

$\hookrightarrow$ center $(0, 0)$

$(x-5)^2 + (y+3)^2 = 2^2$



(3D) Equation of Sphere:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$$

center

$x^2 + (y+1)^2 + (z+4)^2 = 5$

$\hookrightarrow$ center $(0, -1, -4)$ radious $= \sqrt{5}$

$\boxed{x^2+y^2+z^2+Gx+Hy+Iz+J}$

Q: Find center & radious:

$x^2+y^2+z^2-2x-4y+8z+17=0$

$(x^2-2x+1)+(y^2-2\cdot 2y+2^2)+(z^2+2\cdot 4z+4^2)=2^2$

$(x-1)^2+(y-2)^2+(z+4)^2=2^2$

center $(1,2,-4)$

radius 2

$\boxed{\text{Exersise (23-28)}}$

(25) $2x^2+2y^2+2z^2-2x-3y+5z-2=0$

$x^2+y^2+z^2-x-\frac{3}{2}y+\frac{5}{2}z-1=0$

$\Rightarrow \left\{x^2-2\cdot\frac{1}{2}x+\left(\frac{1}{2}\right)^2\right\}+\left\{y^2-2\cdot\frac{3}{4}y+\left(\frac{3}{4}\right)^2\right\}+\left\{z^2+2\cdot\frac{5}{4}z+\left(\frac{5}{4}\right)^2\right\}$

$=$

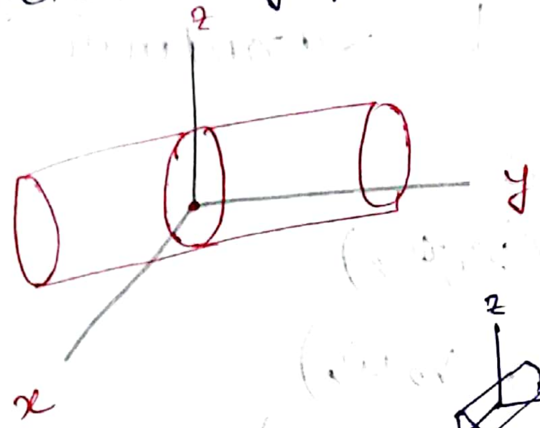
$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{3}{4}\right)^2+\left(z+\frac{5}{4}\right)^2=\frac{27}{8}$

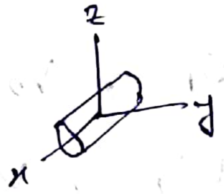
Center $\left(\frac{1}{2},\frac{3}{4},-\frac{5}{4}\right)$

radius : $\left(\sqrt{\frac{27}{8}}\right)$

Center $=\left(\frac{G}{-2},\frac{H}{-2},\frac{I}{-2}\right)$

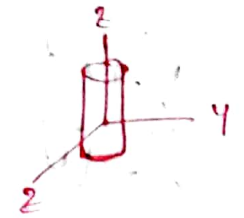
Exm 3 Sketch the graph of $\boxed{x^2+z^2=1}$ in 3 space.





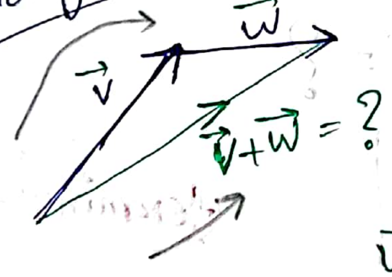
*দুই Variable missing থাকলে parallel হবে।

$\boxed{x^2+y^2=1}$

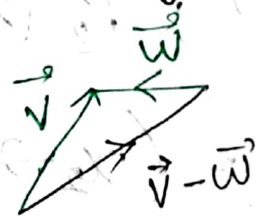


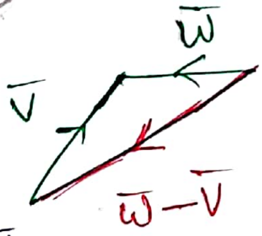
11.2 (Vector)

Triangle law

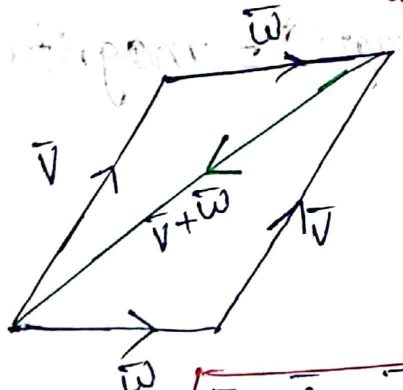


$\boxed{\text{magnitude} = \text{length}}$





* Direction of given 2 vector different the Resultant Vector (লব্ধি (ভেক্টর)) will be same direction of a vector and vector will be sum of different direction vectors - same direction vector.



$\boxed{\bar{v}+\bar{w}=\bar{w}+\bar{v}}$

$$\vec{V} = \underbrace{2\hat{i} + 3\hat{j}}_{\text{Vector form}} = \underbrace{(2, 3)}_{\text{coordinate system}}$$

Arithmetic Operation

$V(V_1, V_2)$, $\omega(\omega_1, \omega_2)$

$\vec{V} + \vec{\omega} = (V_1 + \omega_1 , V_2 + \omega_2)$

$\vec{V} - \vec{\omega} = (V_1 - \omega_1 , V_2 - \omega_2)$

$K \cdot \vec{V} = (KV_1 , KV_2)$

K → constant

$P_1(x_1, y_1)$; $P_2(x_2, y_2)$ $\vec{P_1P_2} = ?$

$$\boxed{\vec{P_1P_2} = (x_2 - x_1 , y_2 - y_1)}$$

terminal - initial

Norm Of a Vector

Norm = Length = magnitude

$\vec{V} = (2, 3, 4)$

$\|V\| = \sqrt{2^2 + 3^2 + 4^2}$

$= \sqrt{29}$

Norm always Positive.

$\|KV\| = |K|\,\|V\|$

$\|3V\| = |3|\,\|V\| = 3\|V\|$

$\|-2V\| = |-2|\,\|V\| = 2\|V\|$

$\vec{N} = (-2, 3)$

$\|-4V\| = ?$

$|-4|\,\|V\| = 4(\sqrt{4+9})$

$= 4\sqrt{13}$

Unit Vector: Length 1 = Norm

Norm of i vec $\sqrt{1^2+0+0}$

$= 1$

j vector $(0, 1, 0)$

i vector $(1, 0, 0)$

Normalizing Vector: $\vec{u} = \frac{1}{\|V\|}\vec{V}$

always Unit Vector

Norm

Same direction of $\vec{v}$

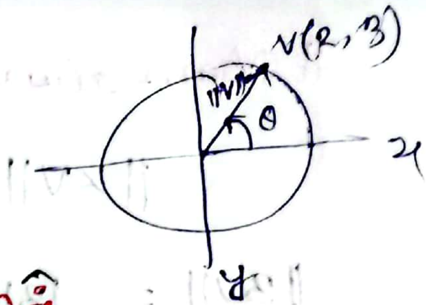
Q: Find a unit vector that has the same direction

$V = 2i + 3j - K$

$\|V\| = \sqrt{2^2+3^2+1^2} = \sqrt{14}$

$\vec{u} = \frac{1}{\sqrt{14}}(2i + 3j - K)$

V. determined by length & angle



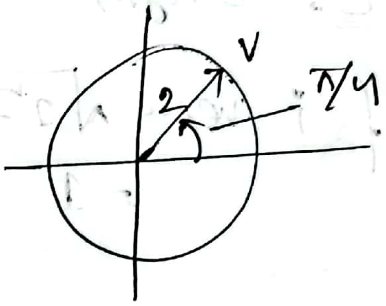
$$V = \|V\| (\cos\theta , \sin\theta)$$

OR

$$V = \|V\| \cos\theta \, \hat{i} + \|V\| \sin\theta \, \hat{j}$$

θ = angle between vector & x axis

Q: Find a vector of length 2 & makes angle $\pi/4$ with x axis.



$$V = 2\cos\frac{\pi}{4}\,\hat{i} + 2\sin\frac{\pi}{4}\,\hat{j}$$

$$= \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$$

* **Q:** Find the angle with x axis, $\vec{V}(-\sqrt{3}\,\hat{i} + \hat{j})$ makes.

$\|V\| = 2$

$$\vec{V} = \|V\| (\cos\theta , \sin\theta)$$

$$\frac{\vec{V}}{\|V\|} = (\cos\theta , \sin\theta)$$

$$\left(\frac{-\sqrt{3}}{2}\,\hat{i} + \frac{1}{2}\,\hat{j}\right) = (\cos\theta ; \sin\theta)$$

$$\therefore \cos\theta = \frac{-3}{2}$$

$$\theta = \cos^{-1}(-3/2)$$

$$= 150^\circ = 5\pi/6$$

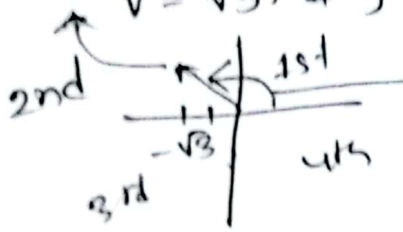
$$\sin\theta = \frac{1}{2}$$

$$\theta = \sin^{-1}(1/2)$$

$$= 30^\circ$$

Ans.

$V = -\sqrt{3}\,\hat{i} + \hat{j}$



$\rightarrow 150^\circ = \theta > 90$

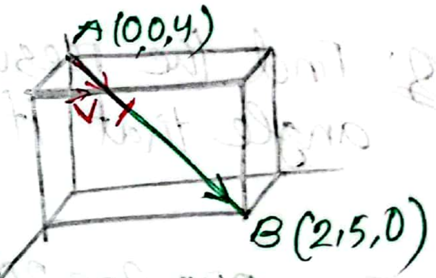
* Vector IV Quadrant এ থাকলে Ans 2nd Quadrant এ θ.

So ans $\frac{5\pi}{6}$ An.

$\tan^{-1}$ দিয়ে করলে Ans ভুল হবে।

L3

** Q: $\|V\| = \sqrt{5}$ find vector (V):



$\vec{AB} = (2-0,\ 5-0,\ 0-4)$

$= (2,\ 5,\ -4)$

Unit Vector.

$\vec{u} = \frac{1}{\|AB\|}\vec{AB}$

$\|AB\| = \sqrt{2^2+5^2+4^2}$

$= 45 = 3\sqrt{5}$

$= \left(\frac{2}{3\sqrt{5}},\ \frac{5}{3\sqrt{5}},\ -\frac{4}{3\sqrt{5}}\right)$

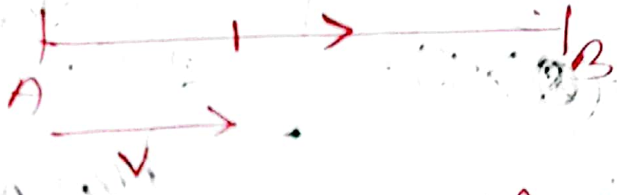
$V = \|V\| \cdot \vec{u}$

$= \sqrt{5}\left(\frac{2}{3\sqrt{5}},\ \frac{5}{3\sqrt{5}},\ \frac{-4}{3\sqrt{5}}\right)$

$= \left(\frac{2}{3},\ \frac{5}{3},\ \frac{-4}{3}\right)$

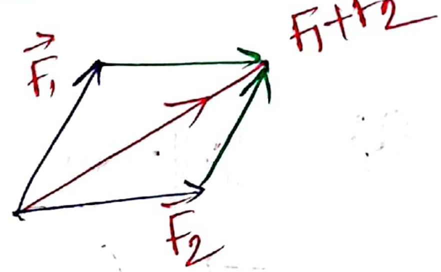
Usually we use unit vector to get direction

Value = 1

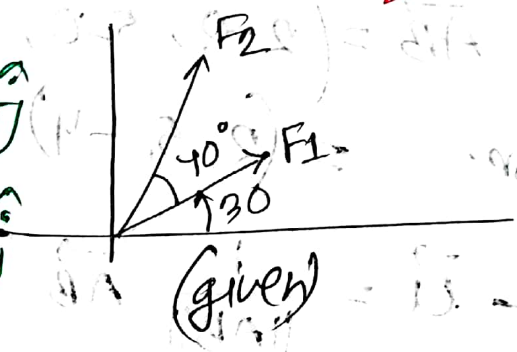


$V = \|V\| \cdot$ unit vector of $(\vec{AB})$

Resultant of two Concurrent Force



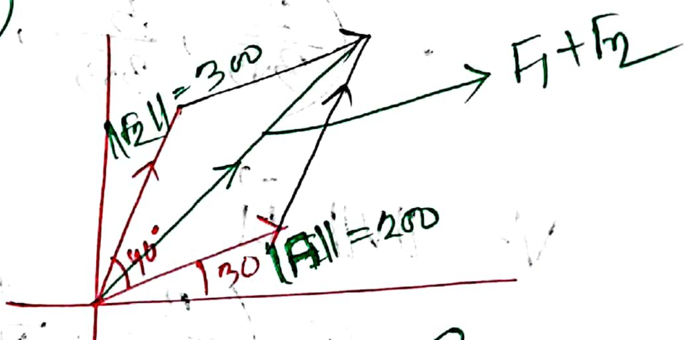
Q: Find the resultant & the angle that it makes with x axis



$F_1 = 200 \cos 30\hat{i} + 200 \sin 30\hat{j}$

$F_2 = 300 \cos 70\hat{i} + 300 \sin 70\hat{j}$

$F_1 + F_2 = (275.81, 381.90)$



$\|F_1 + F_2\| = 471.08$

$\theta = \cos^{-1}\left(\frac{275.81}{471.08}\right)$

$= 54.162$

or $\theta = \sin^{-1}\left(\frac{381.90}{471.08}\right)$

$= 53.95$

similar

L3

11.3

Dot Product (scalar value)

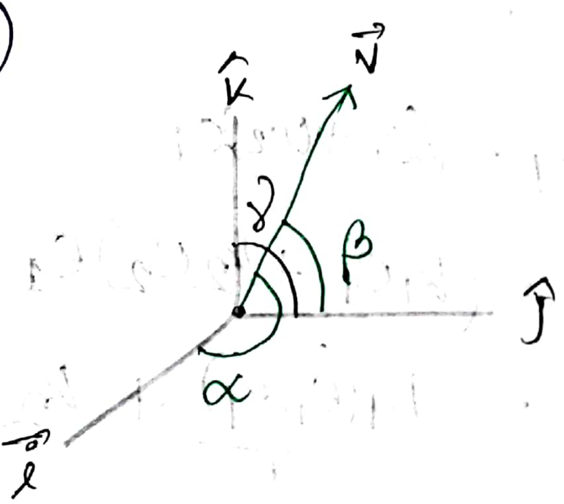
$$u \cdot v = u_1v_1 + u_2v_2 + u_3v_3$$

Angle between Vectors

$$u \cdot v = \|u\| \, \|v\| \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\| \, \|v\|}\right)$$

Direction Angles (α, β, γ)



$$\#\ \cos\alpha = \frac{\vec{v} \cdot \hat{i}}{\|v\| \, \|\hat{i}\|} = \frac{v_1}{\|v\|}$$

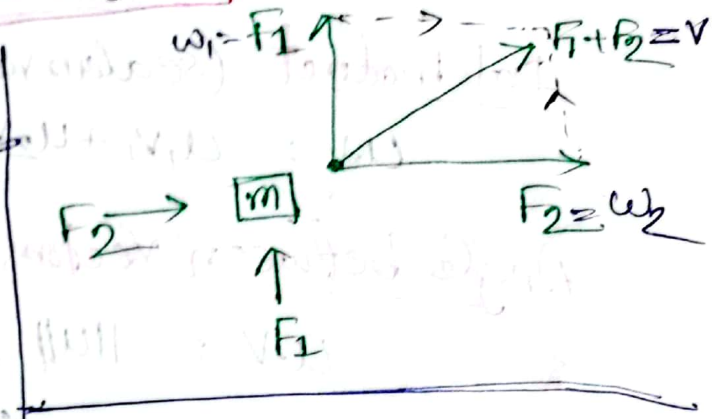
$$\#\ \cos\beta = \frac{\vec{v} \cdot \hat{j}}{\|v\| \, \|\hat{j}\|} = \frac{v_2}{\|v\|}$$

$$\#\ \cos\gamma = \frac{\vec{v} \cdot \hat{k}}{\|v\| \, \|\hat{k}\|} = \frac{v_3}{\|v\|}$$

Decomposing Vector Into Orthogonal Components

$\rightarrow 90^\circ$

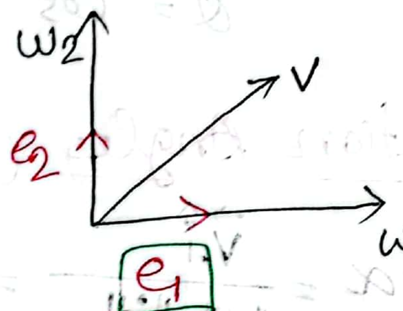
→ we know V we have to make it into two vectors



$$\boxed{V = w_1 + w_2}$$

$w_1 = k_1 e_1$

$w_2 = k_2 e_2$



→ Unit vectors.

$$V.e_1 = (w_1 + w_2)e_1$$
$$= (k_1 e_1 + k_2 e_2)e_1$$
$$= k_1 \underbrace{(e_1.e_1)}_{1} + k_2 \underbrace{(e_2.e_1)}_{0}$$

→ Dot product.

$$= k_1 + 0$$
$$= k_1$$

similarly, $V.e_2 = k_2$

$$V = w_1 + w_2$$
$$= k_1 e_1 + k_2 e_2$$
$$V = \underbrace{(V.e_1)}_{\text{scalar}} e_1 + (V.e_2)e_2$$

→ ~~scalar~~.

scalar

vector

L4

Dot

$\widehat{V.e_1}$ = is called scalar Component of v along e_1

$V.e_2$ = " " " of V " e_2

$(V.e_1)e_1$ " " Vector " of V along e_1

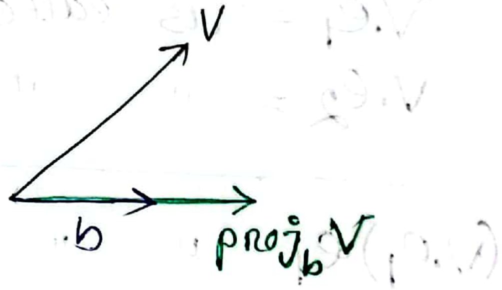
$\underbrace{(V.e_2)e_2}_{\text{cross}}$ " " " of V " e_2

Orthogonal Projections

→ One vector representation on another vector

$$\text{Proj}_{e_1} V = (V \cdot e_1)\, e_1$$

$$\text{Proj}_{e_2} V = (V \cdot e_2)\, e_2$$



projection of V on b

$$\boxed{\text{proj}_e V = (V \cdot e)\, e}$$

→ unit vector

If e is not a unit vector

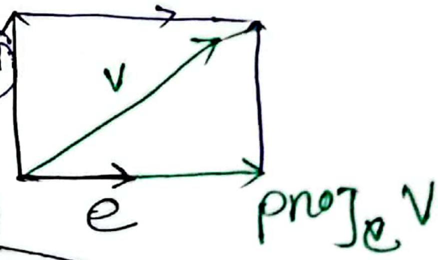
$$\text{proj}_e V = \left(V \cdot \frac{\vec{e}}{\|e\|}\right)\left(\frac{\vec{e}}{\|e\|}\right)$$

$$\boxed{\text{proj}_e V = \frac{V \cdot \vec{e}}{\|e\|^2}\, \vec{e}}$$

→ not a unit vector

Orthogonal projection

$V - \text{proj}_e V$



vector component of v orthogonal to e

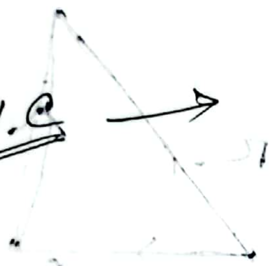
Q: Find the orthogonal projection of $\boxed{V = i + j + K}$ on $\boxed{b = 2i + 2j}$ and then find the vector component of V orthogonal to b.

$$V \cdot b = (2 * 1) + (2 * 1) + 0$$
$$= 4$$

$$\|b\| = \sqrt{4+4}$$
$$= \sqrt{8}$$

O.P $\longrightarrow$ $\text{proj}_b V = 4 \cdot (2i + 2j) \times \frac{1}{\|b\|^2}$ $\rightarrow$ $\frac{V \cdot b}{\|b\|^2} \cdot b$

$$= \frac{8i + 8j}{8} = i + j$$



V.C $\longrightarrow$ $V - \text{proj}_b V = (i + j + k) - (\cancel{8}i + \cancel{8}j)$

$$= k$$

$$= (0, 0, 1)$$