Node Voltage Analysis

Section 3.1, 3.2, 3.4, 3.5

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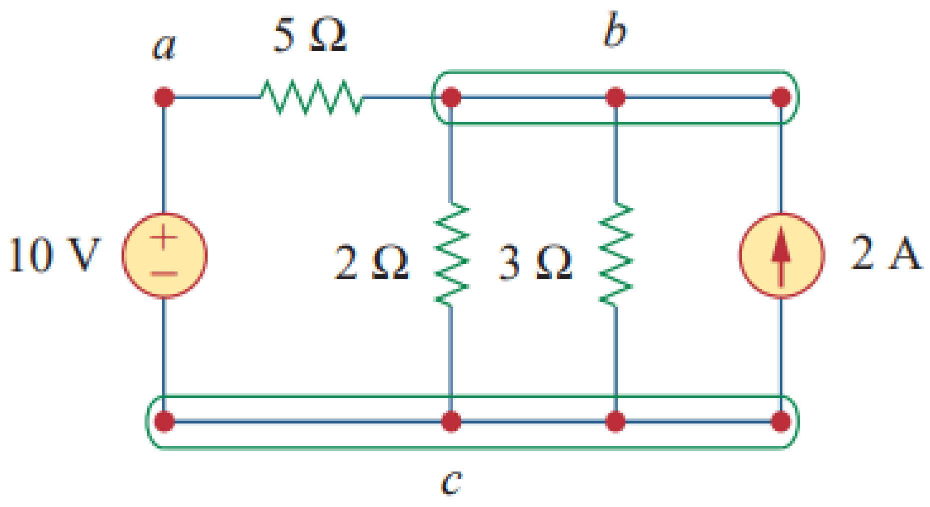
Courtesy: Rifat Bin Rashid

Node , Branch and Loop

Branch: Any two-terminal device

10 V Battery
2A current source
Three Resistances



Node: Junction of different branches

'a' node
'b' node
'c' node

Loop: Any closed path

Nodal Analysis

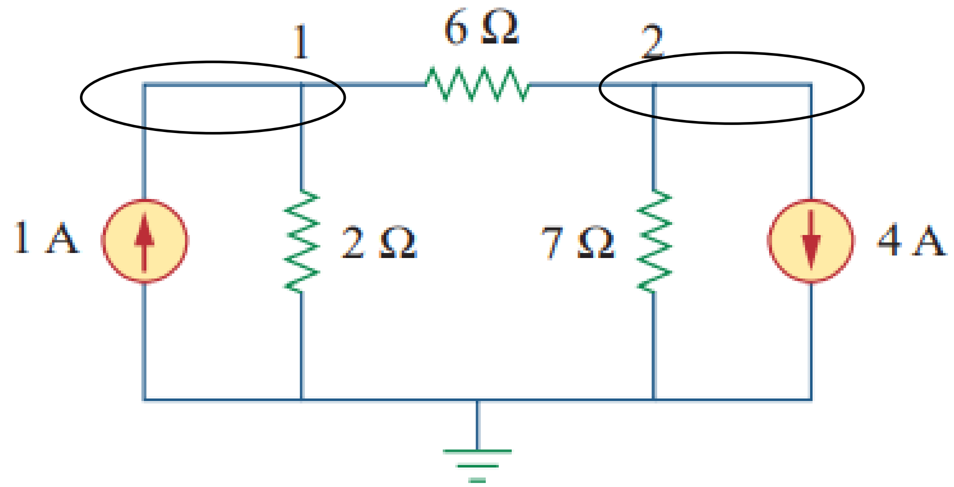
Goal : Finding the node voltages.

Use of Kirchhoff current law **(KCL)** at each node .

$\sum$ Incoming current= $\sum$ Outgoing current

To find V_1 and V_2 , we need two equations.

Less computational complexity !

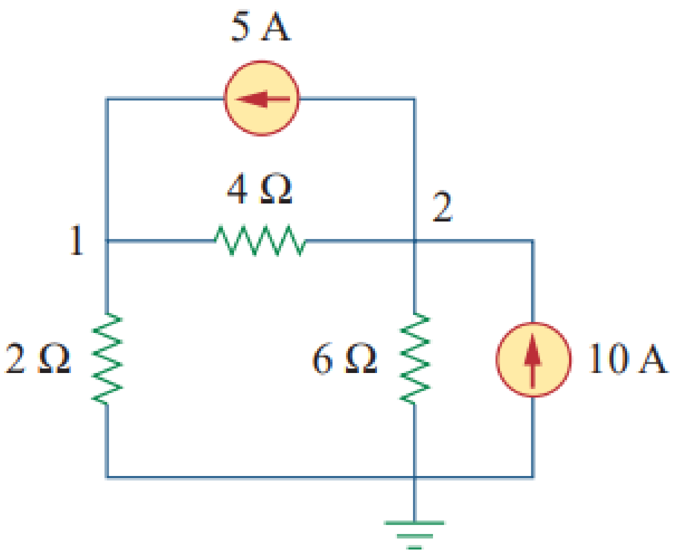


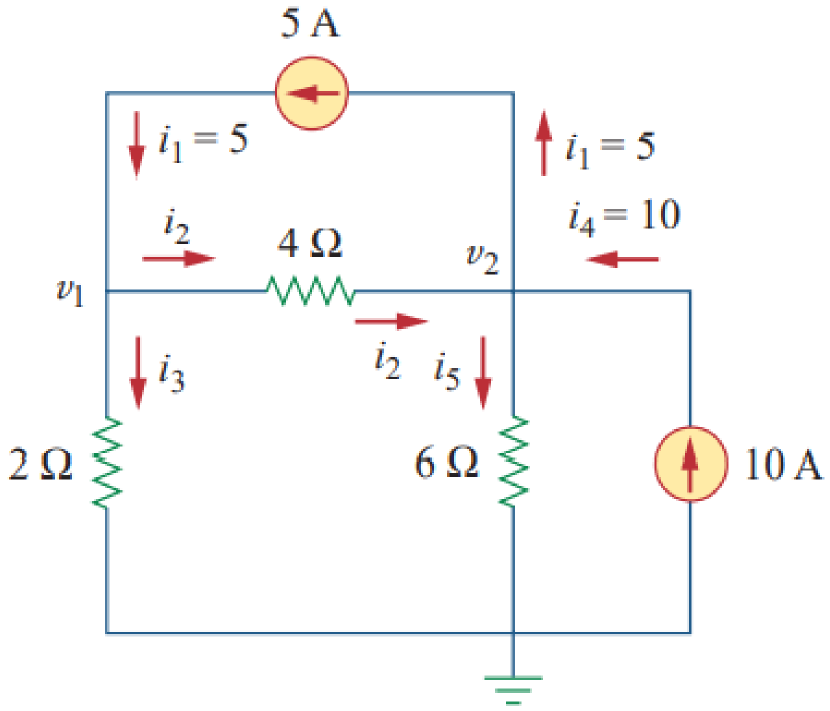
Steps to Determine Node Voltages:

Three Steps

Step 1:

Select a node as the **reference node.** Assign $v_1, v_2, \dots . v_{n-1}$ voltages to the remaining nodes.

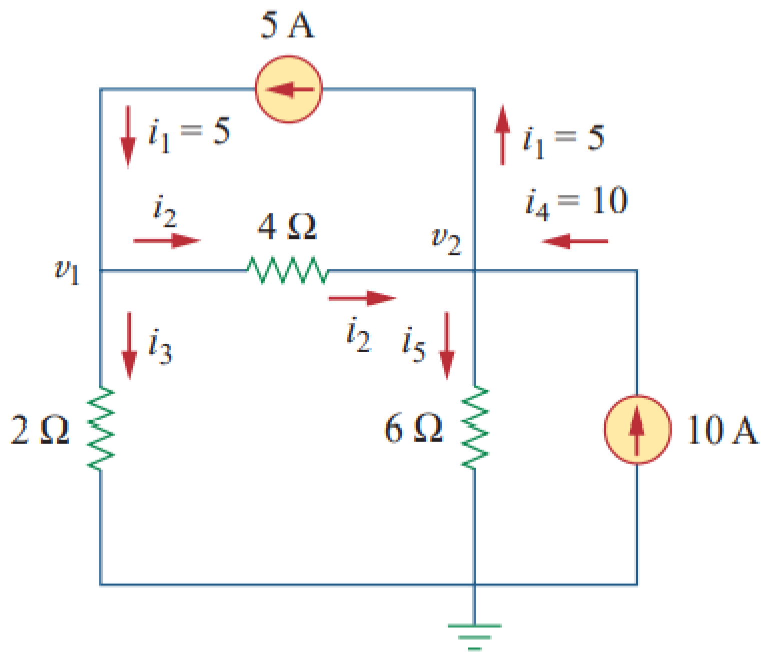




Steps to Determine Node Voltages : *(Continued)*

Step 2:

Apply **KCL** to each of the nonreference nodes.



At node 1,

$$i_1 = i_2 + i_3$$

$$\Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1}{2}$$

$$\Rightarrow \quad 3v_1 - v_2 = 20$$

At node 2,

$$i_2 + i_4 = i_1 + i_5$$

$$\Rightarrow \quad \frac{v_1 - v_2}{4} + 10 = 5 + \frac{v_2}{6}$$

$$\Rightarrow \quad -3v_1 + 5v_2 = 60$$

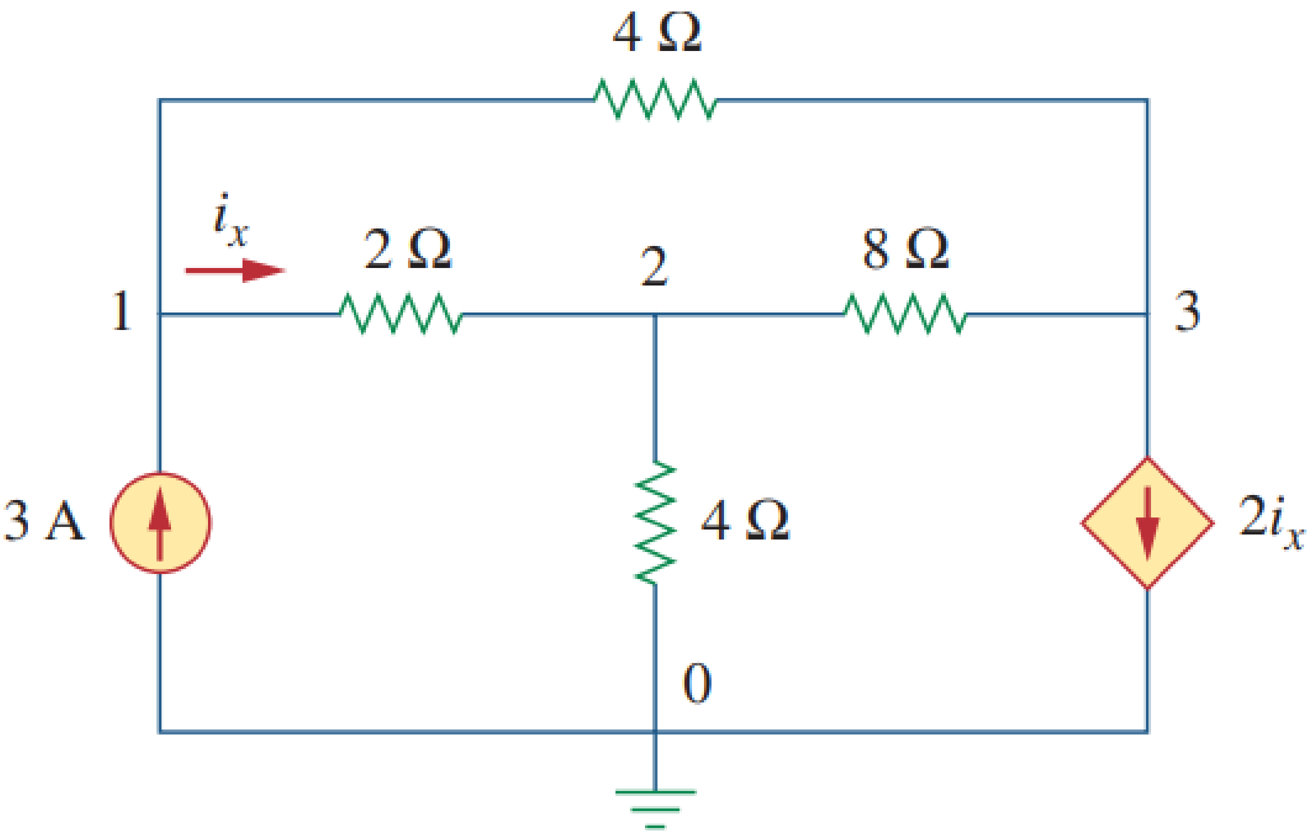
Step 3:

Solve the resulting simultaneous equations.

V_1= 13.333 V
V_2= 20 V

Math Problem Practice:

Determine the voltages at the nodes.



Reference: Sadiku_Example_3.2

Math Problem Practice:

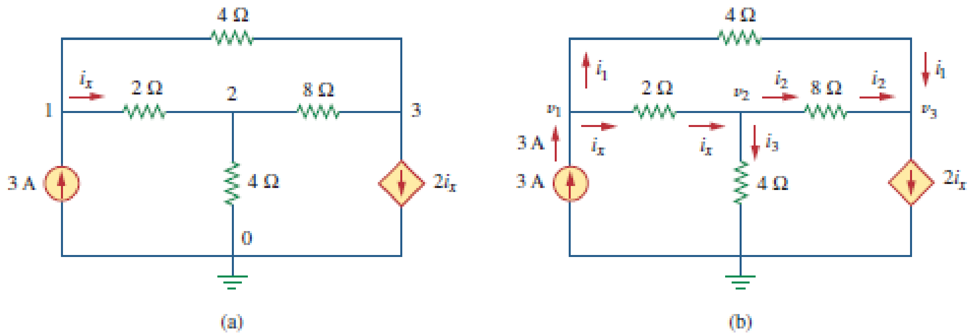


Figure 3.5
For Example 3.2: (a) original circuit, (b) circuit for analysis.

At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12 \qquad \textbf{(3.2.1)}$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0 \qquad \textbf{(3.2.2)}$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

Multiplying by 8, rearranging terms, and dividing by 3, we get

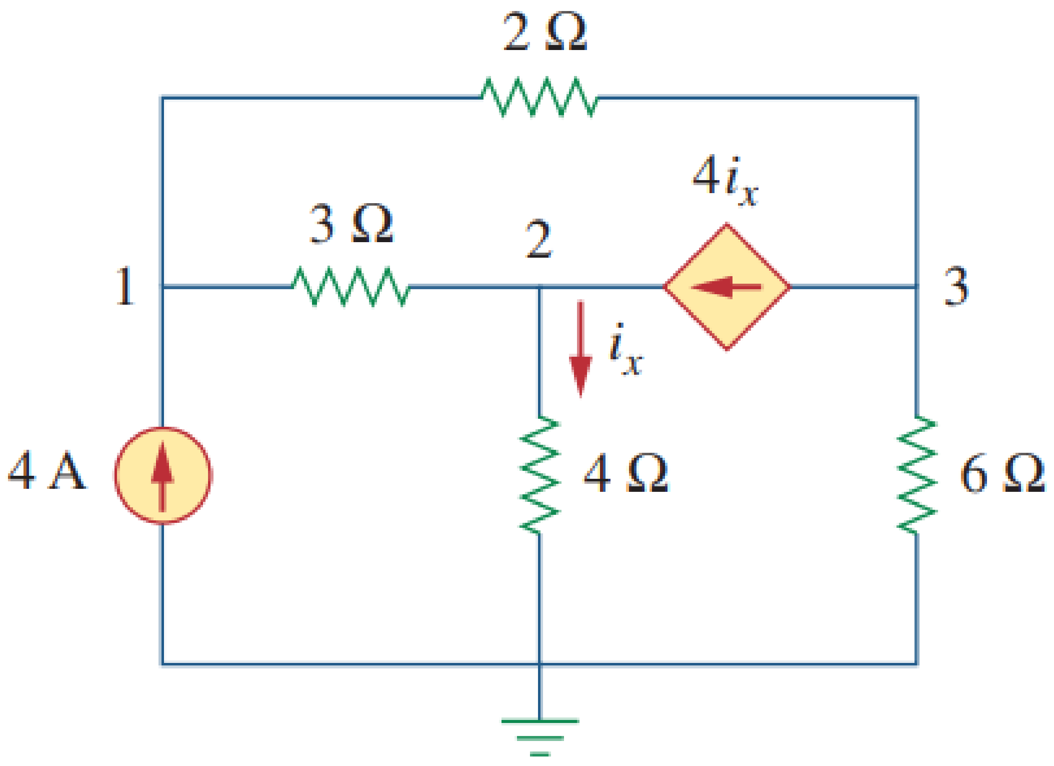
$$2v_1 - 3v_2 + v_3 = 0 \qquad \textbf{(3.2.3)}$$

Reference: Sadiku_Example_3.2

Math Problem Practice:

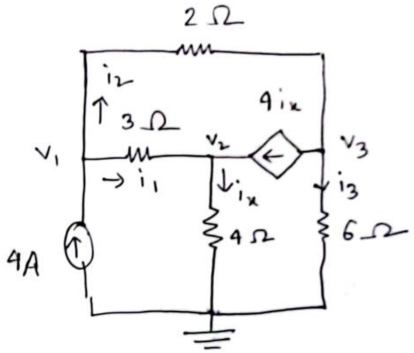
Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

Answer: $v_1 = 32$ V, $v_2 = -25.6$ V, $v_3 = 62.4$ V.



Reference: Sadiku_Practice_Problem_3.2

Sadiku_Practice_Problem_3.2 Solution:



At node-1, $4 = i_1 + i_2$

$\Rightarrow 4 = \dfrac{v_1 - v_2}{3} + \dfrac{v_1 - v_3}{2}$

$\Rightarrow 24 = 2v_1 - 2v_2 + 3v_1 - 3v_3$

$\therefore 5v_1 - 2v_2 - 3v_3 = 24 \cdots (1)$

At node-2, $i_1 + 4i_x = i_x$

$\Rightarrow i_1 + 3i_x = 0$

$\Rightarrow \dfrac{v_1 - v_2}{3} + 3 \times \dfrac{v_2}{4} = 0$

$\Rightarrow 4v_1 - 4v_2 + 9v_2 = 0$

$\Rightarrow 4v_1 + 5v_2 = 0 \cdots (2)$

At node-3, $i_2 = 4i_x + i_3$

$\Rightarrow \dfrac{v_1 - v_3}{2} = 4 \times \dfrac{v_2}{4} + \dfrac{v_3}{6}$

$\Rightarrow 6v_1 - 6v_3 = 12v_2 + 2v_3$

$\Rightarrow 6v_1 - 12v_2 - 8v_3 = 0$

$\Rightarrow 3v_1 - 6v_2 - 4v_3 = 0 \cdots (3)$

$\therefore v_1 = 32\,V$

$v_2 = -25.6\,V$

$v_3 = 62.4\,V$

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Reference: Sadiku_Practice_Problem_3.2

Math Problem Practice:

3.5 Obtain v_o in the circuit of Fig. 3.54.

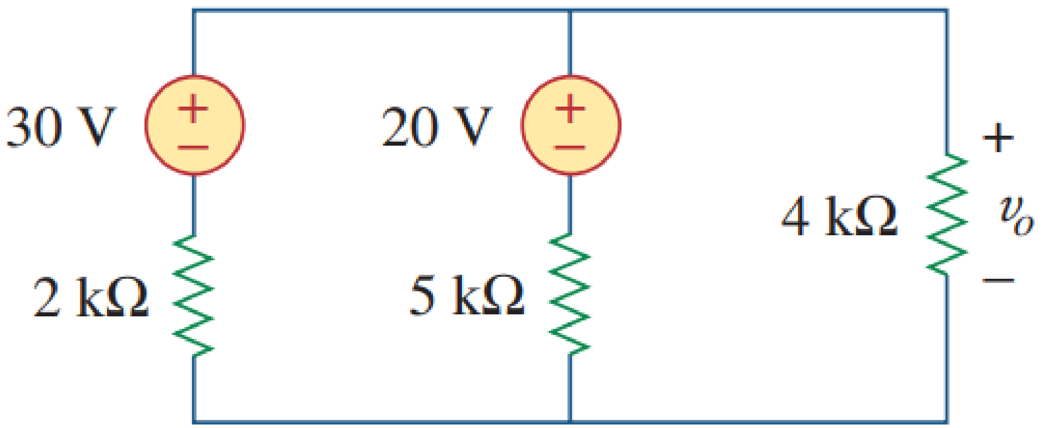


Figure 3.54
For Prob. 3.5.

Reference: Sadiku_Exercise_3.5

Math Problem Practice:

Chapter 3, Problem 5.

Obtain v_0 in the circuit of Fig. 3.54.

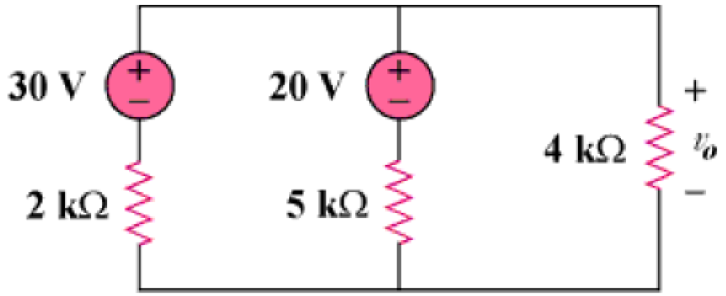


Figure 3.54

Chapter 3, Solution 5

Apply KCL to the top node.

$$\frac{30 - v_0}{2k} + \frac{20 - v_0}{5k} = \frac{v_0}{4k} \longrightarrow v_0 = \underline{\mathbf{20\ V}}$$

Reference: Sadiku_Exercise_3.5

Math Problem Practice:

3.10 Find I_o in the circuit of Fig. 3.59.

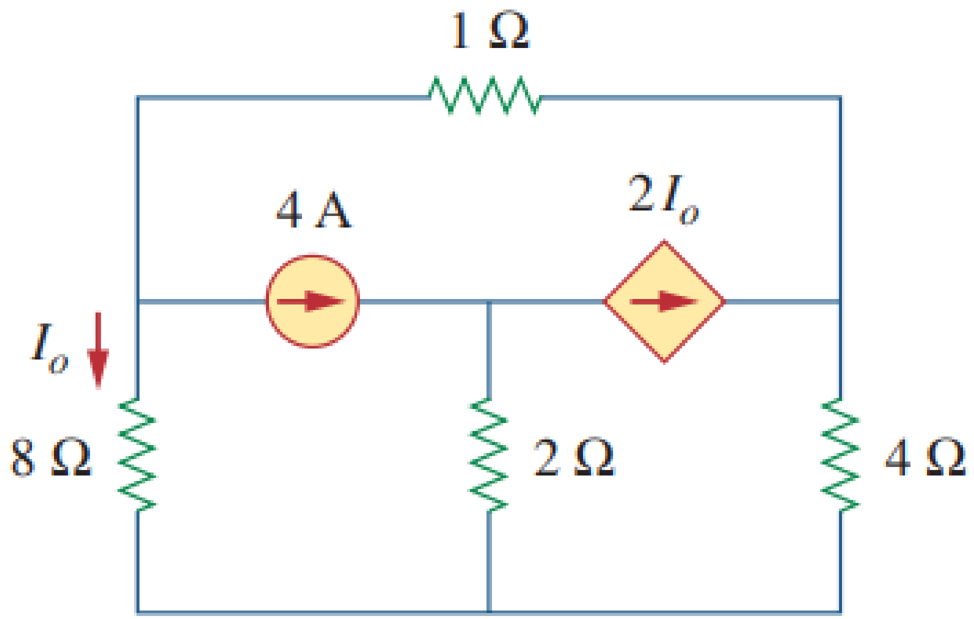


Figure 3.59
For Prob. 3.10.

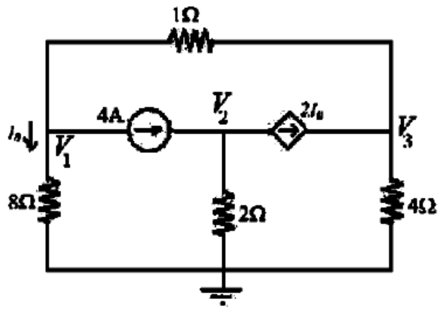
Reference: Sadiku_Exercise_3.10

Math Problem Practice:

Step 1 of 5

3.010P

Given circuit **diagram**:



Step 2 of 5

At node V_1,

$$\frac{V_1-0}{8}+4+\frac{V_1-V_3}{1}=0$$

$$\frac{V_1}{8}+4+V_1=V_3$$

$$\Rightarrow V_3 = \frac{9V_1+32}{8} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Step 3 of 5

At node V_2,

$$-4+\frac{V_2-0}{2}+\frac{2V_1}{8}=0$$

$$\frac{V_2}{2}=4-\frac{2V_1}{8}$$

Or,

$$\Rightarrow V_2=-0.5V_1+8 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Step 4 of 5

At node V_3,

$$\frac{V_3-V_1}{1}-2\left[\frac{V_1}{8}\right]+\frac{V_3-0}{4}=0$$

$$V_3\left[1+\frac{1}{4}\right]-V_1\left[1+\frac{1}{4}\right]=0$$

Or,

$$V_1=V_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

Step 5 of 5

Combining (1) and (3), we get,

$$V_1 = \frac{9V_1+32}{8} \quad \text{(From equation} - (1))$$

$$8V_1=9V_1+32$$

$$V_1=-32\,\text{V}$$

V_1 Value substitute in equation - (2) we get,

$$V_2=24\text{V} \quad \text{And}$$

$$V_3=-32\text{V}$$

$$I_o=\frac{V_1}{8}$$

$$I_0=\frac{-32}{8}$$

$$\boxed{I_0=-4\text{A}}$$

<u>Reference: Sadiku_Exercise_3.10</u>

Math Problem Practice:

3.12 Using nodal analysis, determine V_o in the circuit in Fig. 3.61.

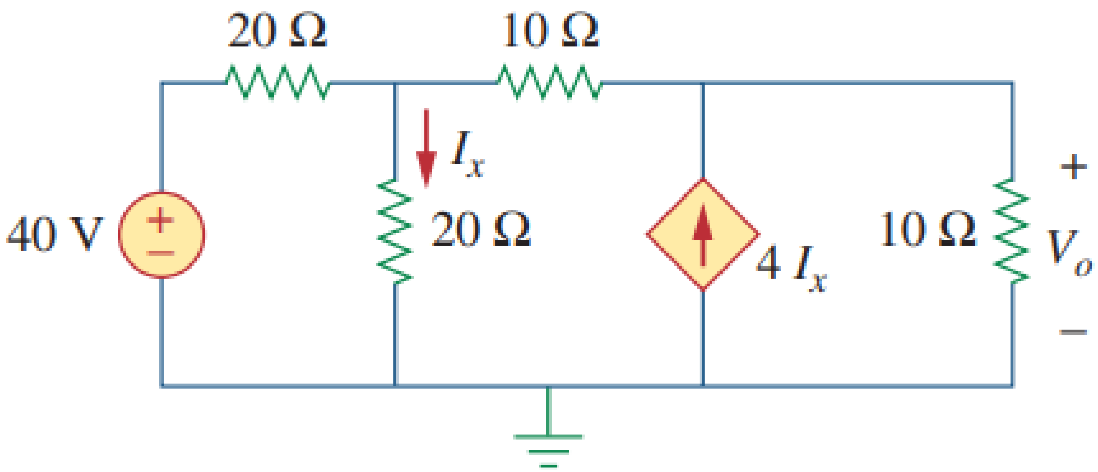


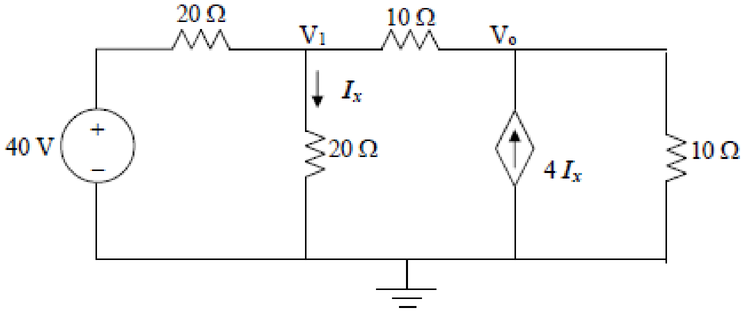
Figure 3.61
For Prob. 3.12.

Reference: Sadiku_Exercise_3.12

Math Problem Practice:

Chapter 3, Solution 12

There are two unknown nodes, as shown in the circuit below.



At node 1,

$$\frac{V_1-40}{20}+\frac{V_1-0}{20}+\frac{V_1-V_o}{10}=0 \text{ or}$$

$$(0.05+0.05+.1)V_1 - 0.1V_o = 0.2V_1 - 0.1V_o = 2 \qquad (1)$$

At node o,

$$\frac{V_o-V_1}{10}-4I_x+\frac{V_o-0}{10}=0 \quad \text{and } I_x = V_1/20$$

$$-0.1V_1 - 0.2V_1 + 0.2V_o = -0.3V_1 + 0.2V_o = 0 \text{ or} \qquad (2)$$

$$V_1 = (2/3)V_o \qquad (3)$$

Substituting (3) into (1),

$$0.2(2/3)V_o - 0.1V_o = 0.03333V_o = 2 \text{ or}$$

$$V_o = 60 \text{ V}.$$

<u>Reference: Sadiku_Exercise_3.12</u>

Math Problem Practice:

3.13 Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

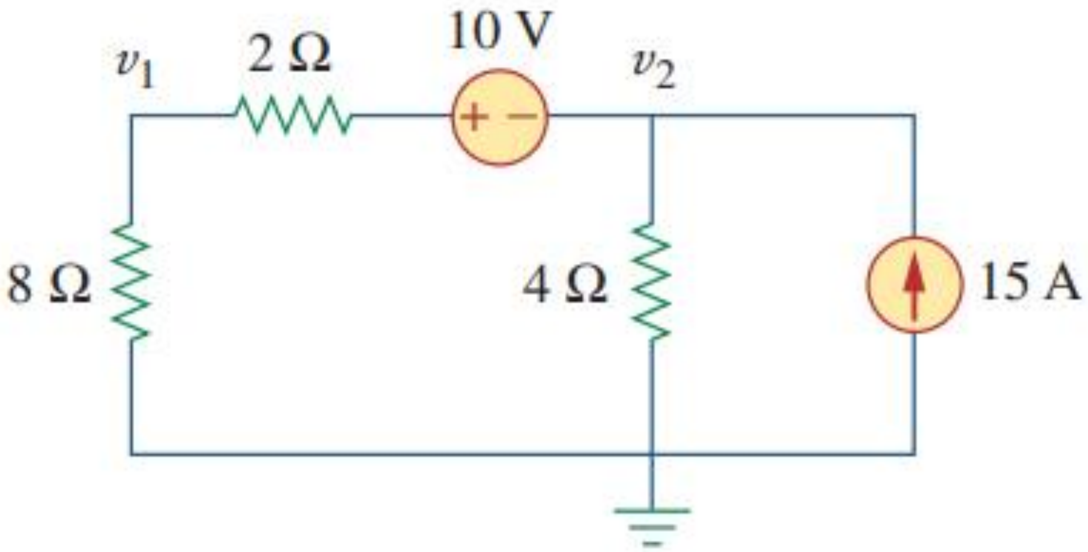


Figure 3.62
For Prob. 3.13.

Reference: Sadiku_Exercise_3.13

Math Problem Practice:

Chapter 3, Solution 13

Calculate v_1 and v_2 in the circuit of Fig. 3.62 using nodal analysis.

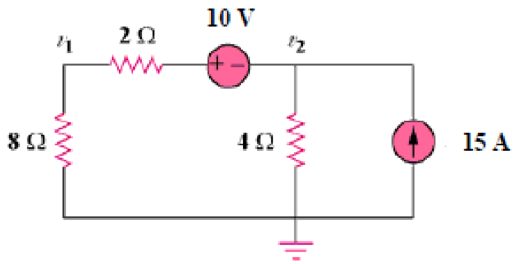


Figure 3.62
For Prob. 3.13.

Solution

At node number 2, $[((v_2 + 10) - 0)/10] + [(v_2-0)/4] - 15 = 0$ or
$(0.1+0.25)v_2 = 0.35v_2 = -1+15 = 14$ or

$$v_2 = \textbf{40 volts}.$$

Next, $I = [(v_2 + 10) - 0]/10 = (40 + 10)/10 = 5$ amps and

$$v_1 = 8x5 = \textbf{40 volts}.$$

Reference: Sadiku_Exercise_3.13

Math Problem Practice:

3.14 Using nodal analysis, find v_o in the circuit of Fig. 3.63.

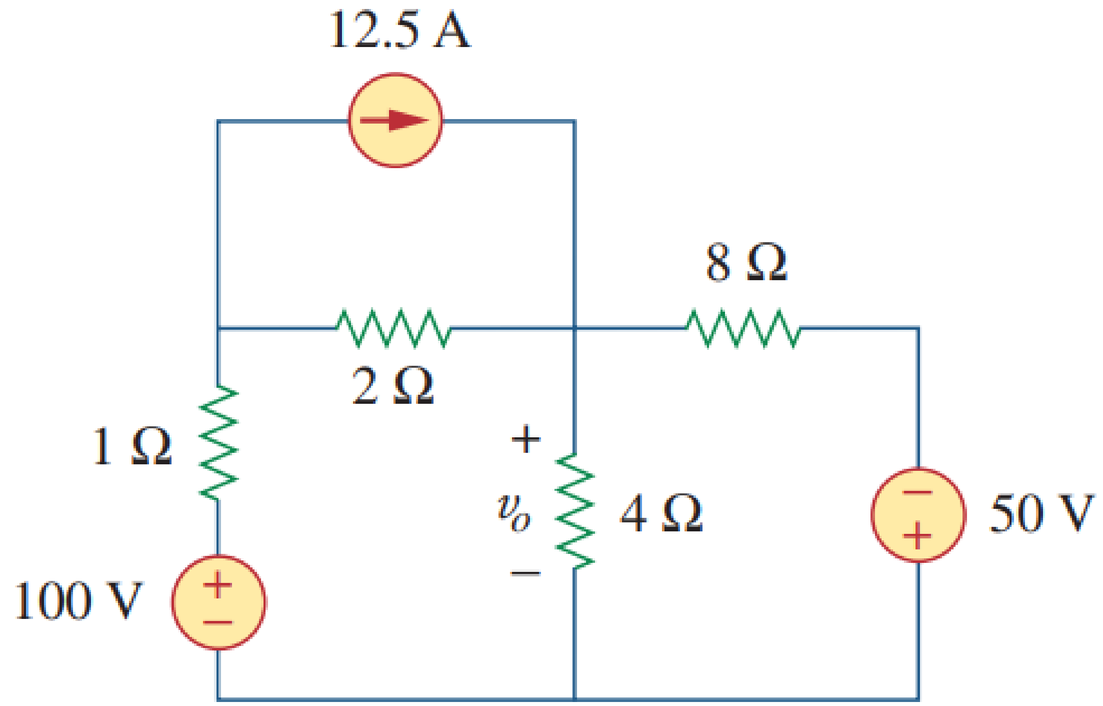


Figure 3.63
For Prob. 3.14.

Reference: Sadiku_Exercise_3.14

Math Problem Practice:

Chapter 3, Solution 14

Using nodal analysis, find v_o in the circuit of Fig. 3.63.

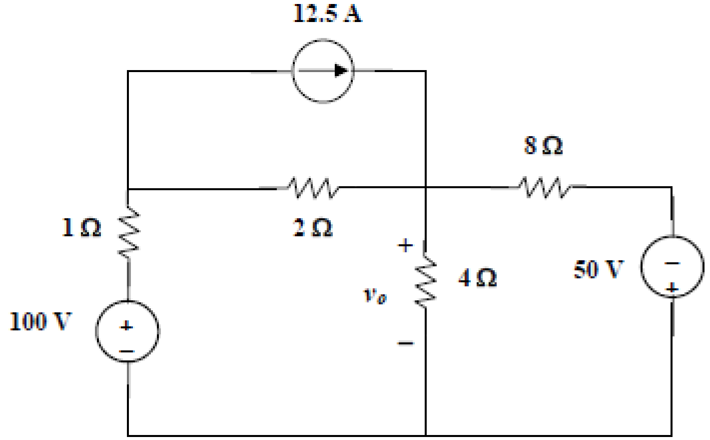
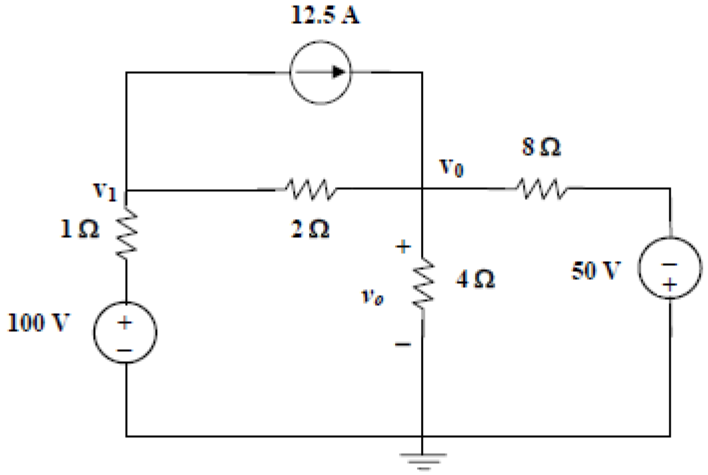


Figure 3.63
For Prob. 3.14.

Solution



At node 1,

$$[(v_1-100)/1] + [(v_1-v_o)/2] + 12.5 = 0 \text{ or } 3v_1 - v_o = 200-25 = 175 \qquad (1)$$

At node o,

$$[(v_o-v_1)/2] - 12.5 + [(v_o-0)/4] + [(v_o+50)/8] = 0 \text{ or } -4v_1 + 7v_o = 50 \qquad (2)$$

Adding 4x(1) to 3x(2) yields,

$$4(1) + 3(2) = -4v_o + 21v_o = 700 + 150 \text{ or } 17v_o = 850 \text{ or}$$

$$v_o = \mathbf{50\ V}.$$

Checking, we get $v_1 = (175+v_o)/3 = 75$ V.

At node 1,

$$[(75-100)/1] + [(75-50)/2] + 12.5 = -25 + 12.5 + 12.5 = 0!$$

At node o,

$$[(50-75)/2] + [(50-0)/4] + [(50+50)/8] - 12.5 = -12.5 + 12.5 + 12.5 - 12.5 = 0!$$

Reference: Sadiku_Exercise_3.14

Math Problem Practice:

3.19 Use nodal analysis to find v_1, v_2, and v_3 in the circuit of Fig. 3.68.

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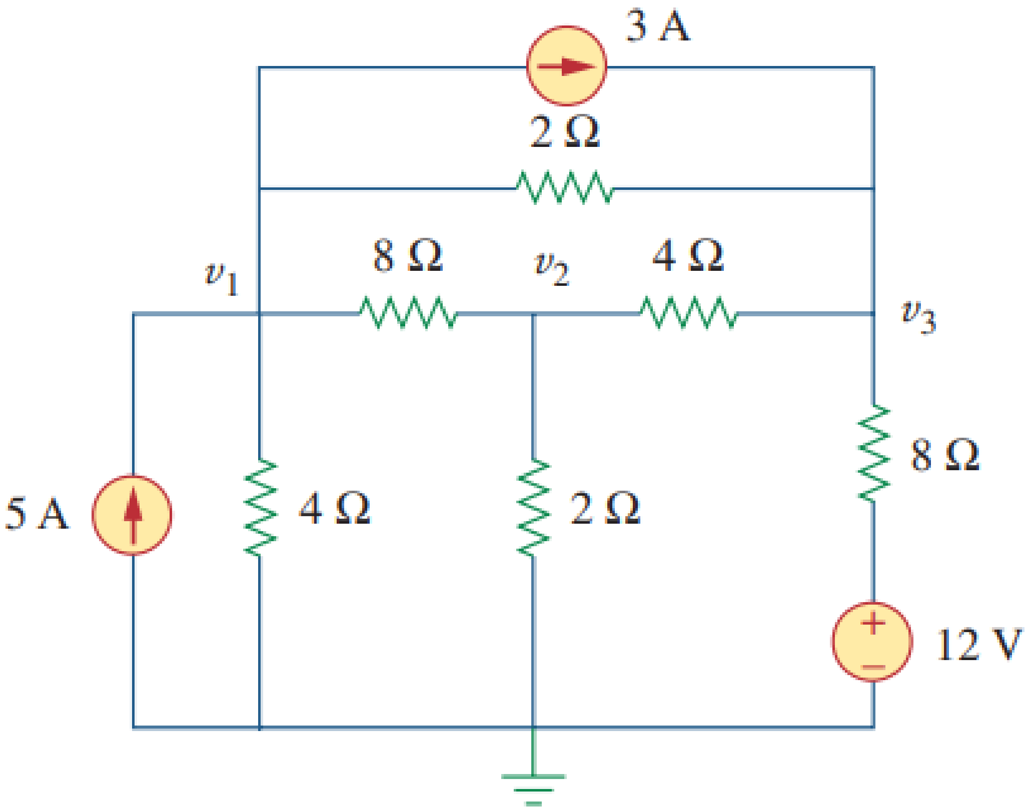

Reference: Sadiku_Exercise_3.19

Math Problem Practice:

Chapter 3, Solution 19

At node 1,

$$5 = 3 + \frac{V_1 - V_3}{2} + \frac{V_1 - V_2}{8} + \frac{V_1}{4} \longrightarrow 16 = 7V_1 - V_2 - 4V_3 \qquad (1)$$

At node 2,

$$\frac{V_1 - V_2}{8} = \frac{V_2}{2} + \frac{V_2 - V_3}{4} \longrightarrow 0 = -V_1 + 7V_2 - 2V_3 \qquad (2)$$

At node 3,

$$3 + \frac{12 - V_3}{8} + \frac{V_1 - V_3}{2} + \frac{V_2 - V_3}{4} = 0 \longrightarrow -36 = 4V_1 + 2V_2 - 7V_3 \quad (3)$$

From (1) to (3),

$$\begin{pmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ 4 & 2 & -7 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 16 \\ 0 \\ -36 \end{pmatrix} \longrightarrow AV = B$$

Using MATLAB,

$$V = A^{-1}B = \begin{bmatrix} 10 \\ 4.933 \\ 12.267 \end{bmatrix} \longrightarrow \underline{V_1 = 10\text{ V},\ V_2 = 4.933\text{ V},\ V_3 = 12.267\text{ V}}$$

<u>Reference: Sadiku_Exercise_3.19</u>

Math Problem Practice:

3.26 Calculate the node voltages v_1, v_2, and v_3 in the circuit of Fig. 3.75.

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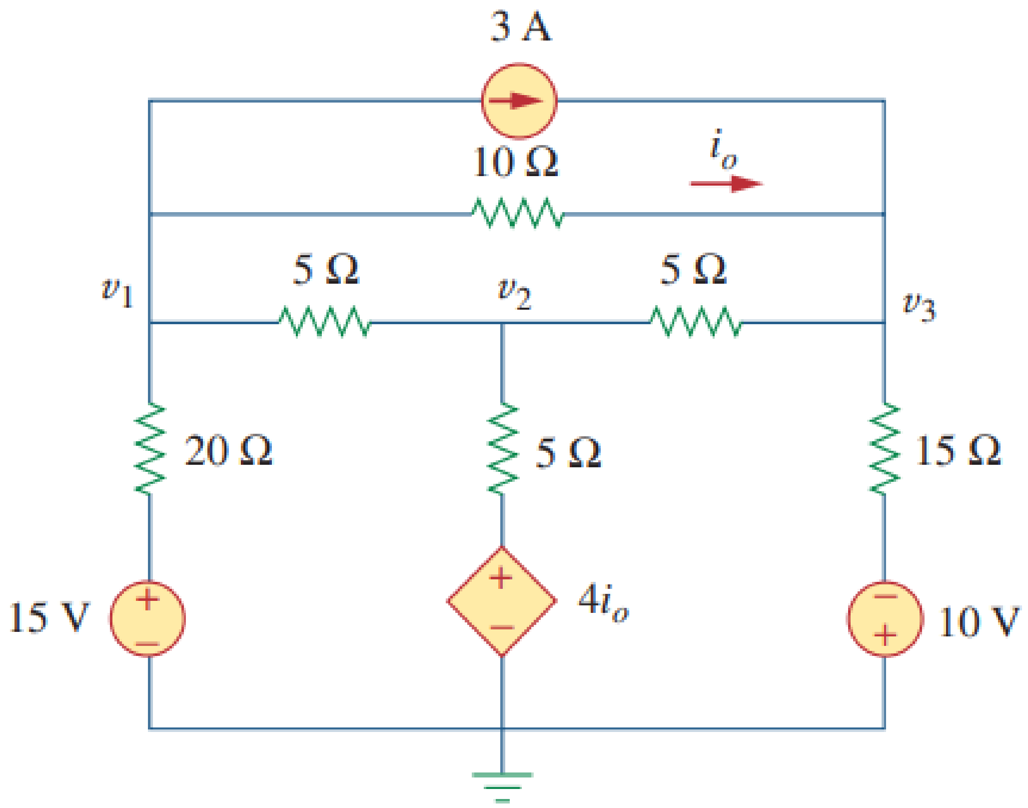

Reference: Sadiku_Exercise_3.26

Math Problem Practice:

Chapter 3, Solution 26

At node 1,

$$\frac{15-V_1}{20}=3+\frac{V_1-V_3}{10}+\frac{V_1-V_2}{5} \longrightarrow -45=7V_1-4V_2-2V_3 \qquad (1)$$

At node 2,

$$\frac{V_1-V_2}{5}+\frac{4I_o-V_2}{5}=\frac{V_2-V_3}{5} \qquad (2)$$

But $I_o=\frac{V_1-V_3}{10}$. Hence, (2) becomes

$$0=7V_1-15V_2+3V_3 \qquad (3)$$

At node 3,

$$3+\frac{V_1-V_3}{10}+\frac{-10-V_3}{15}+\frac{V_2-V_3}{5}=0 \longrightarrow 70=-3V_1-6V_2+11V_3 \qquad (4)$$

Putting (1), (3), and (4) in matrix form produces

$$\begin{pmatrix} 7 & -4 & -2 \\ 7 & -15 & 3 \\ -3 & -6 & 11 \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}=\begin{pmatrix} -45 \\ 0 \\ 70 \end{pmatrix} \longrightarrow AV=B$$

Using MATLAB leads to

$$V=A^{-1}B=\begin{pmatrix} -7.19 \\ -2.78 \\ 2.89 \end{pmatrix}$$

Thus,

$$\mathbf{V_1=-7.19V;\ V_2=-2.78V;\ V_3=2.89V.}$$

<u>Reference: Sadiku_Exercise_3.26</u>

Math Problem Practice:

***3.27** Use nodal analysis to determine voltages v_1, v_2, and v_3 in the circuit of Fig. 3.76.

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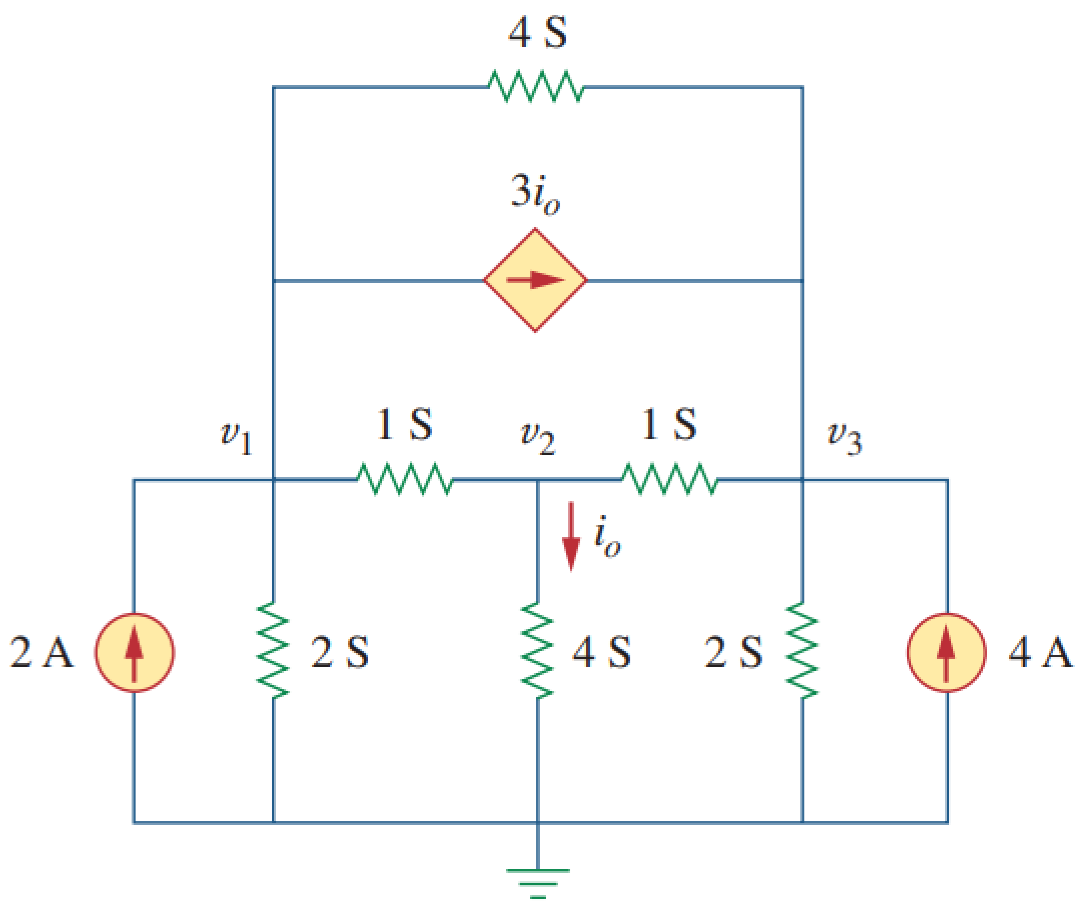

Reference: Sadiku_Exercise_3.27

Math Problem Practice:

Chapter 3, Solution 27

At node 1,

$$2 = 2v_1 + v_1 - v_2 + (v_1 - v_3)4 + 3i_0, \quad i_0 = \ \ 4v_2. \text{ Hence,}$$

$$2 = 7v_1 + 11v_2 - 4v_3 \qquad (1)$$

At node 2,

$$v_1 - v_2 = 4v_2 + v_2 - v_3 \longrightarrow \qquad 0 = -\ v_1 + 6v_2 - v_3 \qquad (2)$$

At node 3,

$$2v_3 = 4 + v_2 - v_3 + 12v_2 + 4(v_1 - v_3)$$

or

$$-4 = 4v_1 + 13v_2 - 7v_3 \qquad (3)$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

In matrix form,

$$\begin{bmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & 11 & -4 \\ 1 & -6 & 1 \\ 4 & 13 & -7 \end{vmatrix} = 176, \quad \Delta_1 = \begin{vmatrix} 2 & 11 & -4 \\ 0 & -6 & 1 \\ -4 & 13 & -7 \end{vmatrix} = 110$$

$$\Delta_2 = \begin{vmatrix} 7 & 2 & -4 \\ 1 & 0 & 1 \\ 4 & -4 & -7 \end{vmatrix} = 66, \quad \Delta_3 = \begin{vmatrix} 7 & 11 & 2 \\ 1 & -6 & 0 \\ 4 & 13 & -4 \end{vmatrix} = 286$$

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{110}{176} = 0.625\text{V}, \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{66}{176} = 0.375\text{V}$$

$$v_3 = \frac{\Delta_3}{\Delta} = \frac{286}{176} = 1.625\text{V}.$$

$$v_1 = \mathbf{625\ mV}, \ v_2 = \mathbf{375\ mV}, \ v_3 = \mathbf{1.625\ V}.$$

Reference: Sadiku_Exercise_3.27

Math Problem Practice:

4.21 Use the node-voltage method to find i_o in the circuit in Fig. P4.21.

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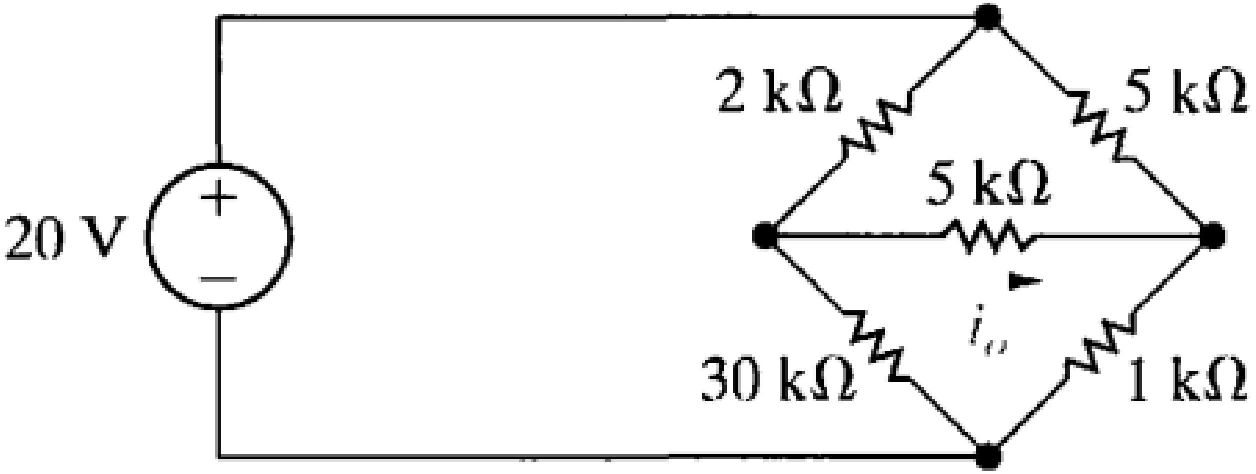
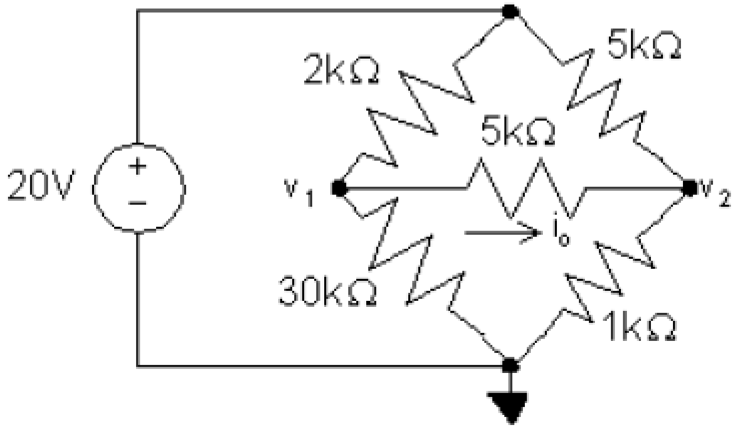
Figure P4.21



Reference: Nilsson_Exercise_4.21

Math Problem Practice:

P 4.21



$$\frac{v_1}{30{,}000} + \frac{v_1 - v_2}{5000} + \frac{v_1 - 20}{2000} = 0 \qquad \text{so} \qquad 22v_1 - 6v_2 = 300$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{5000} + \frac{v_2 - 20}{5000} = 0 \qquad \text{so} \qquad -v_1 + 7v_2 = 20$$

Solving, $v_1 = 15$ V; $\qquad v_2 = 5$ V

Thus, $i_o = \dfrac{v_1 - v_2}{5000} = 2$ mA

Reference: Nilsson_Exercise_4.21

Thank You for patient Hearing