Resistance

Section 2.1, 2.2, 2.3, 2.4, 2.5, 2.6

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Courtesy: Rifat Bin Rashid

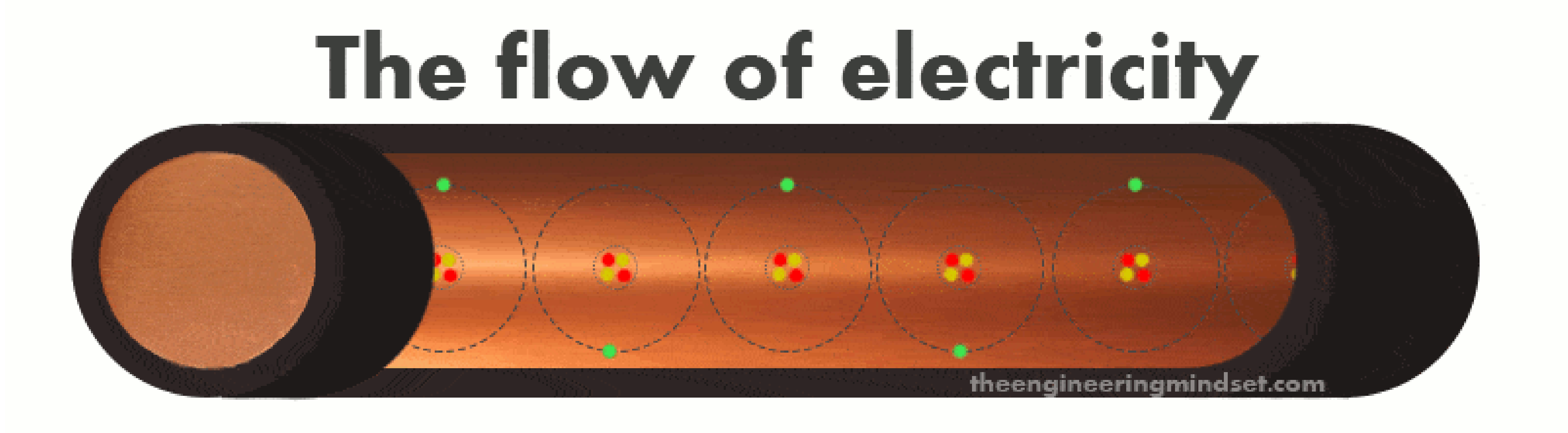
Lecture Outline:

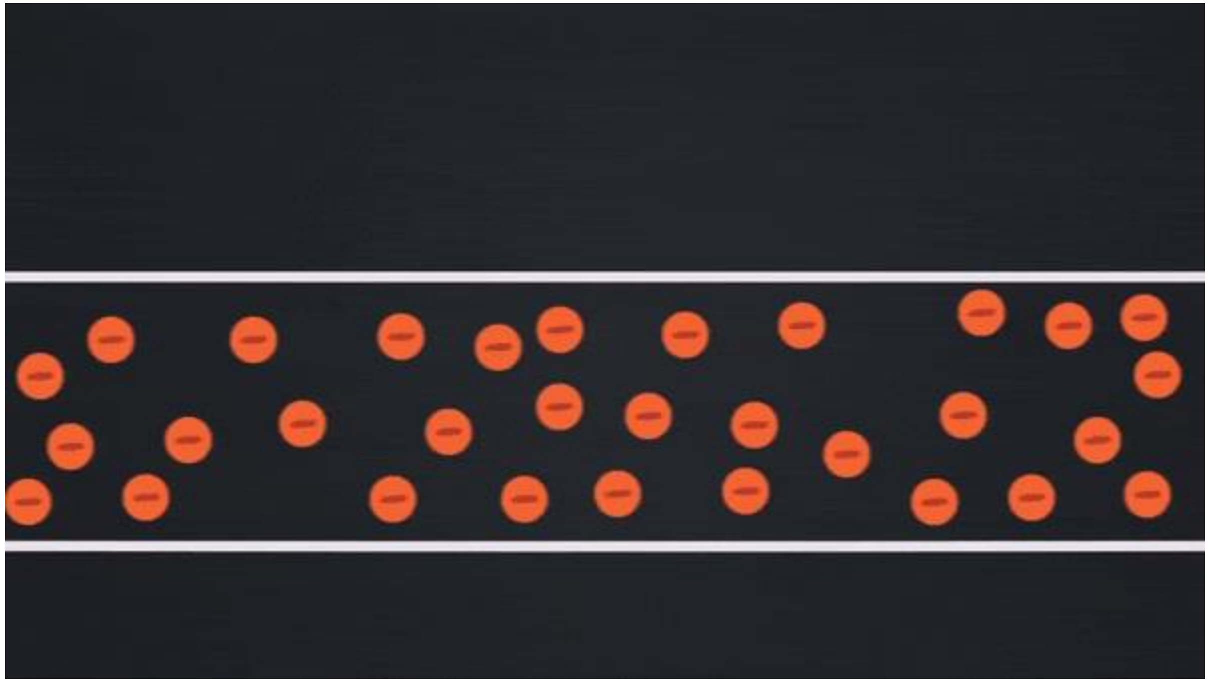
Series and parallel Resistance

Current And voltage division

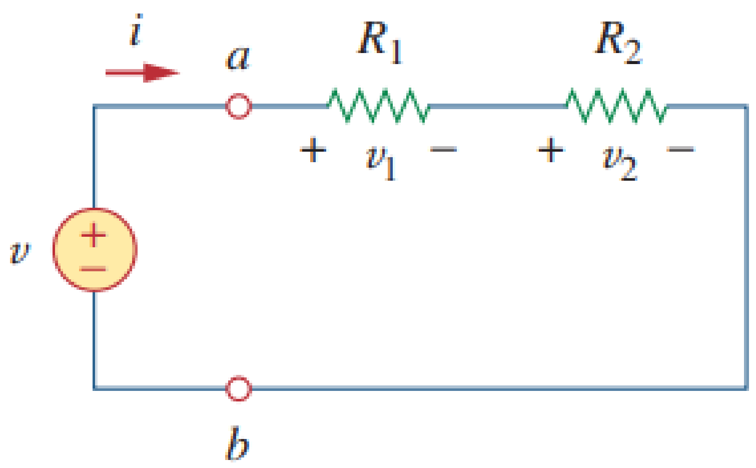
Equivalent resistance

Resistance

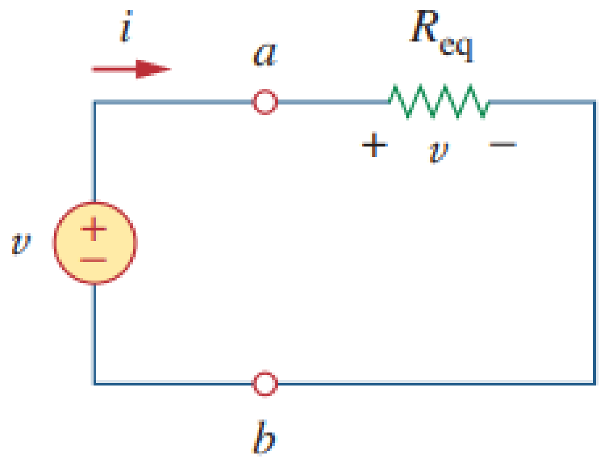




Series Resistors and Voltage Division



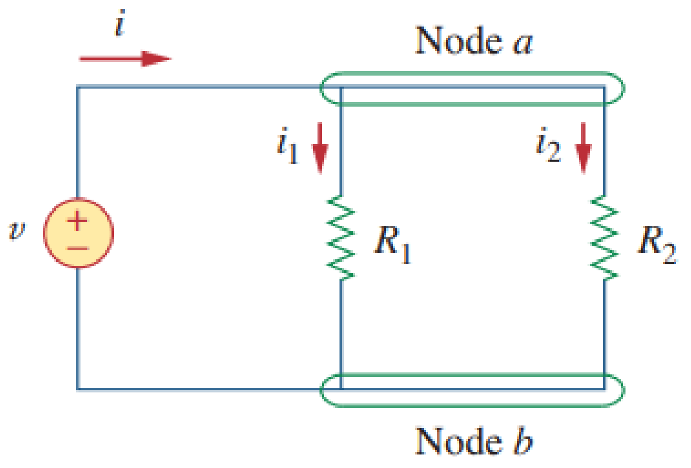
$$R_{eq} = R_1 + R_2$$



The **equivalent resistance** of any number of resistors connected in series is the sum of the individual resistances.

$$v_1 = \frac{R_1}{R_1 + R_2} v, \qquad v_2 = \frac{R_2}{R_1 + R_2} v$$

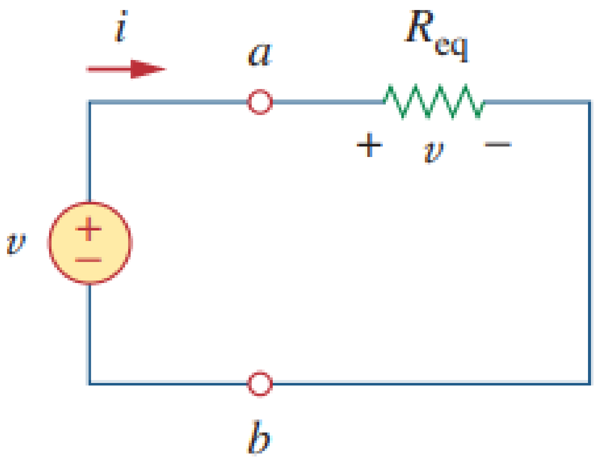
Parallel Resistors and Current Division



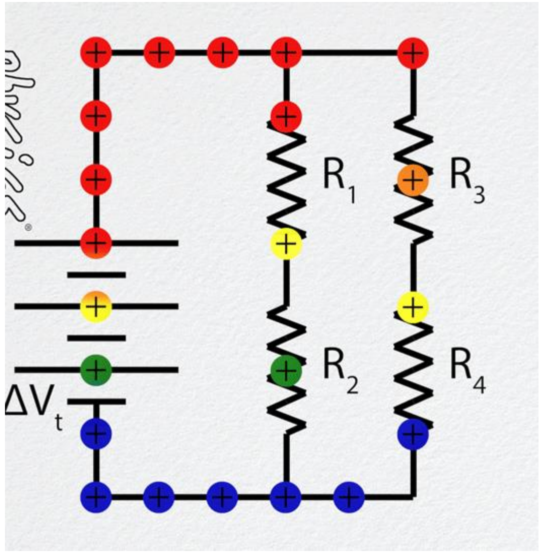
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{R_1 + R_2}{R_1 R_2}$$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$



The **equivalent resistance** of two parallel resistors is equal to the product of their resistances divided by their sum.



$$\boxed{i_1 = \frac{R_2\, i}{R_1 + R_2}, \qquad i_2 = \frac{R_1\, i}{R_1 + R_2}}$$

Math Problem Practice:

By combining the resistors in Fig. 2.36, find R_{eq}.

Answer: 10 Ω.

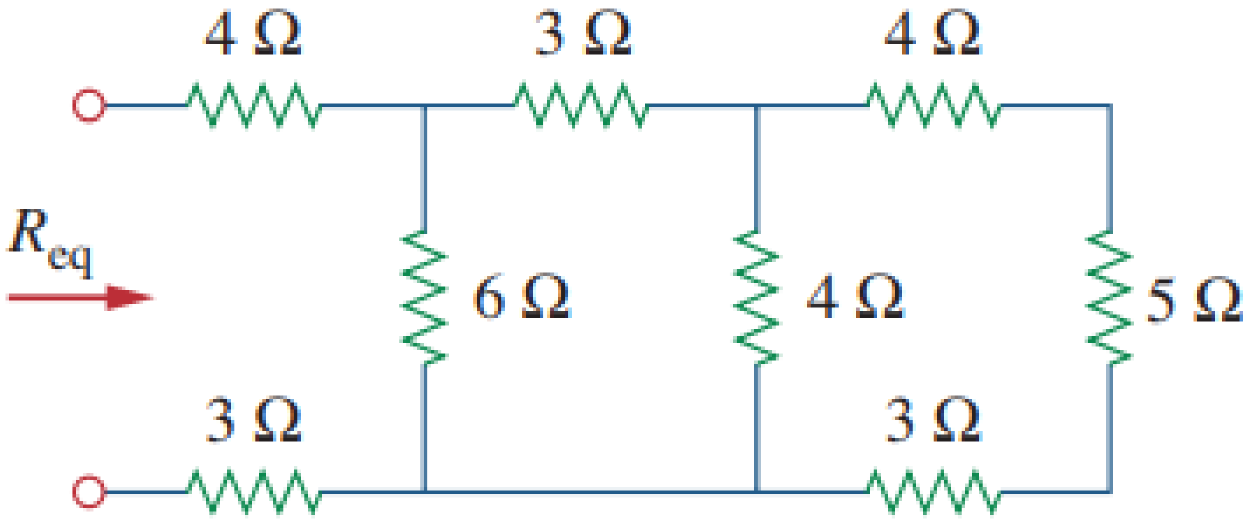
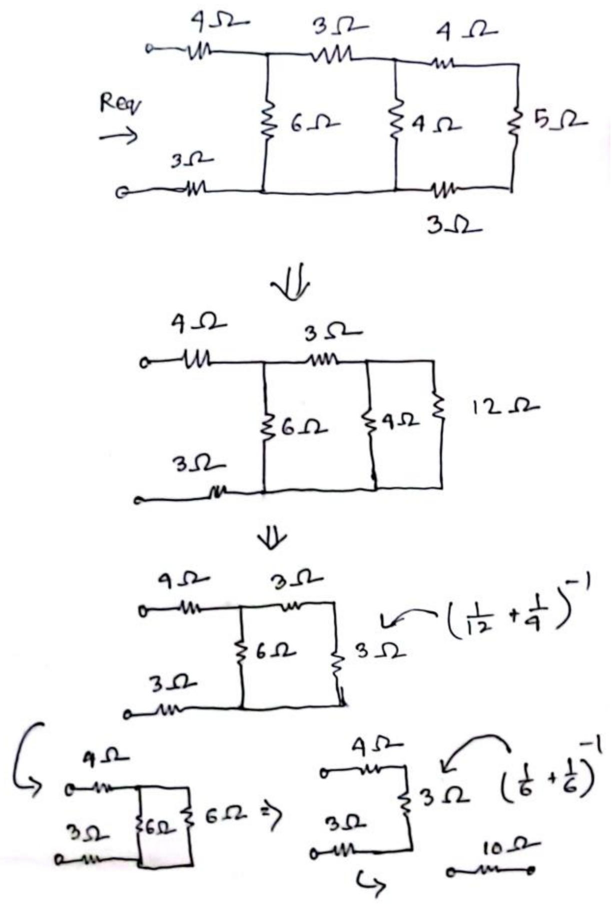


Figure 2.36
For Practice Prob. 2.9.

Ans: R_{eq} = 10 Ω

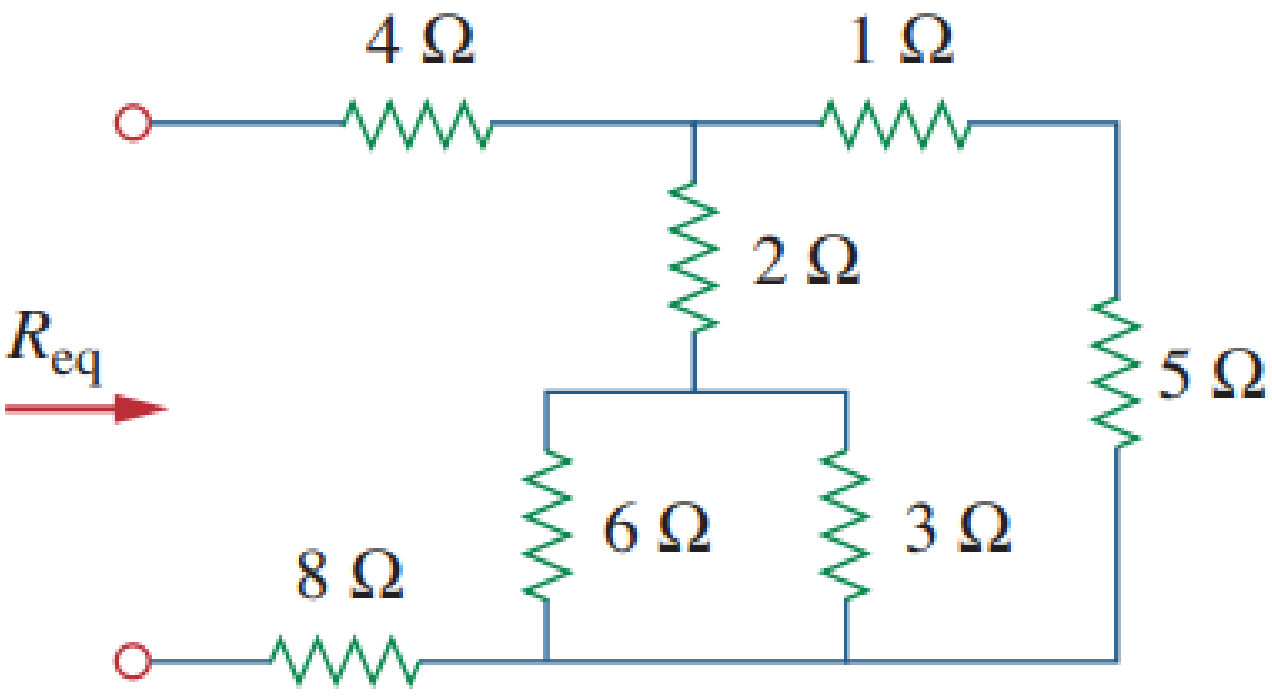
Reference: Sadiku_Practice_Problem_2.9

Solution of Practice problem 2.9



Math Problem Practice:

Find R_{eq} for the circuit shown in Fig.



***Ans*: R_{eq} = 14.4 Ω**

Reference: Sadiku_Example_2.9

Example 2.9

Find R_{eq} for the circuit shown in Fig. 2.34.

Solution:

To get R_{eq}, we combine resistors in series and in parallel. The 6-Ω and 3-Ω resistors are in parallel, so their equivalent resistance is

$$6\ \Omega \parallel 3\Omega = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

(The symbol ∥ is used to indicate a parallel combination.) Also, the 1-Ω and 5-Ω resistors are in series; hence their equivalent resistance is

$$1\ \Omega + 5\ \Omega = 6\ \Omega$$

Thus the circuit in Fig. 2.34 is reduced to that in Fig. 2.35(a). In Fig. 2.35(a), we notice that the two 2-Ω resistors are in series, so the equivalent resistance is

$$2\ \Omega + 2\ \Omega = 4\ \Omega$$

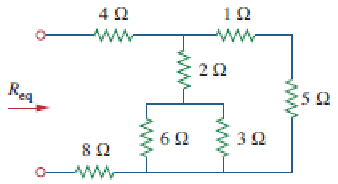


Figure 2.34
For Example 2.9.

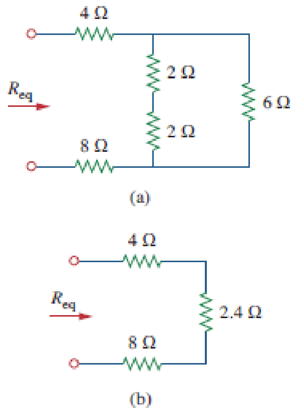


Figure 2.35
Equivalent circuits for Example 2.9.

This 4-Ω resistor is now in parallel with the 6-Ω resistor in Fig. 2.35(a); their equivalent resistance is

$$4\ \Omega \parallel 6\ \Omega = \frac{4 \times 6}{4 + 6} = 2.4\ \Omega$$

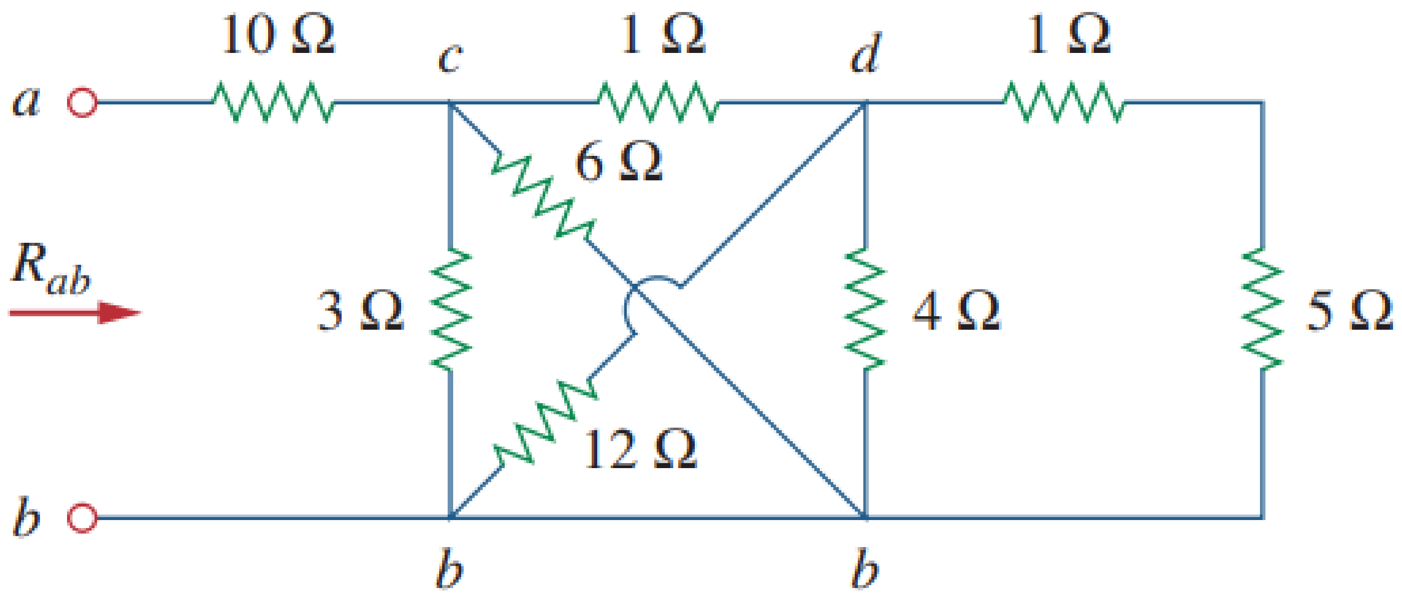
The circuit in Fig. 2.35(a) is now replaced with that in Fig. 2.35(b). In Fig. 2.35(b), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4\ \Omega + 2.4\ \Omega + 8\ \Omega = 14.4\ \Omega$$

Reference: Sadiku_Example_2.9

Math Problem Practice:

Find R_{eq} for the circuit shown in Fig.



Ans: R_{eq} = **11.2 Ω**

Reference: Sadiku_Example_2.10

Calculate the equivalent resistance R_{ab} in the circuit in Fig. 2.37.

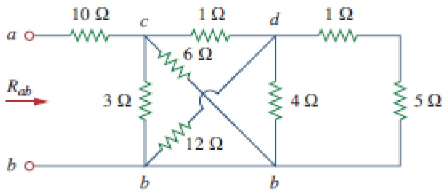


Figure 2.37
For Example 2.10.

Solution:
The 3-Ω and 6-Ω resistors are in parallel because they are connected to the same two nodes c and b. Their combined resistance is

$$3\,\Omega \parallel 6\,\Omega = \frac{3 \times 6}{3 + 6} = 2\,\Omega \qquad \textbf{(2.10.1)}$$

Similarly, the 12-Ω and 4-Ω resistors are in parallel since they are connected to the same two nodes d and b. Hence

$$12\,\Omega \parallel 4\,\Omega = \frac{12 \times 4}{12 + 4} = 3\,\Omega \qquad \textbf{(2.10.2)}$$

Also the 1-Ω and 5-Ω resistors are in series; hence, their equivalent resistance is

$$1\,\Omega + 5\,\Omega = 6\,\Omega \qquad \textbf{(2.10.3)}$$

With these three combinations, we can replace the circuit in Fig. 2.37 with that in Fig. 2.38(a). In Fig. 2.38(a), 3-Ω in parallel with 6-Ω gives 2-Ω, as calculated in Eq. (2.10.1). This 2-Ω equivalent resistance is now in series with the 1-Ω resistance to give a combined resistance of $1\,\Omega + 2\,\Omega = 3\,\Omega$. Thus, we replace the circuit in Fig. 2.38(a) with that in Fig. 2.38(b). In Fig. 2.38(b), we combine the 2-Ω and 3-Ω resistors in parallel to get

$$2\,\Omega \parallel 3\,\Omega = \frac{2 \times 3}{2 + 3} = 1.2\,\Omega$$

This 1.2-Ω resistor is in series with the 10-Ω resistor, so that

$$R_{ab} = 10 + 1.2 = 11.2\,\Omega$$

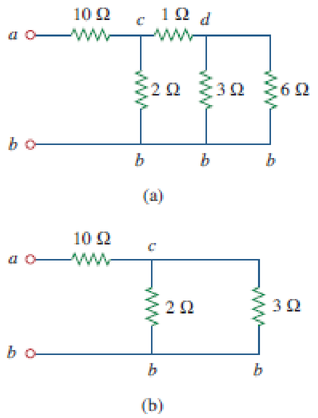
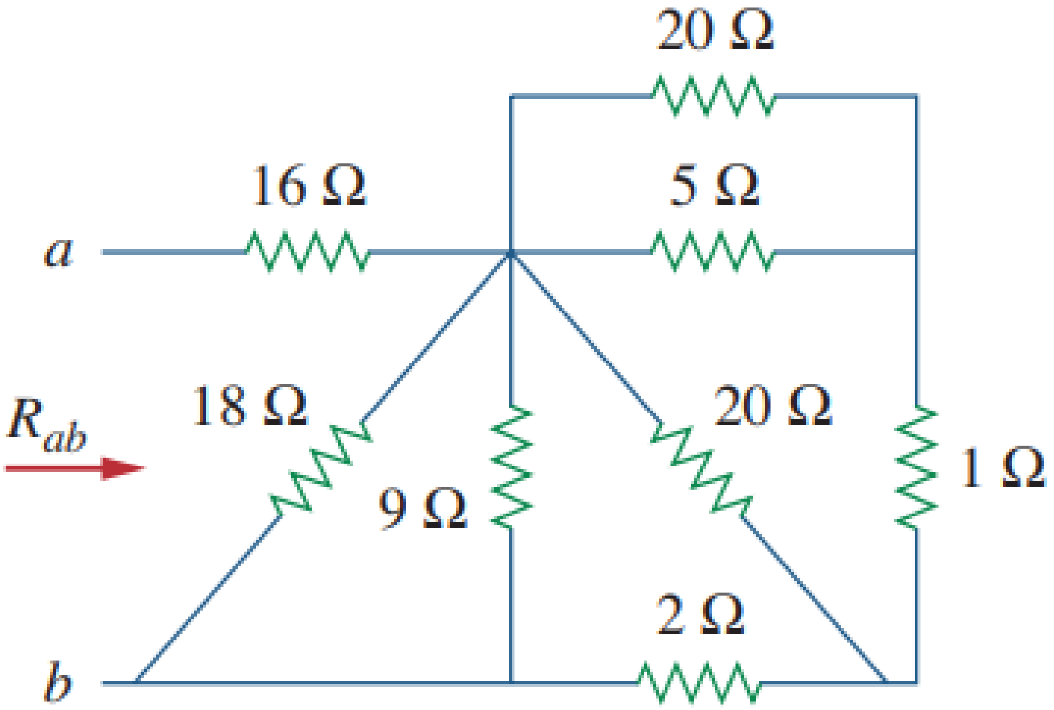


Figure 2.38
Equivalent circuits for Example 2.10.

Math Problem Practice:

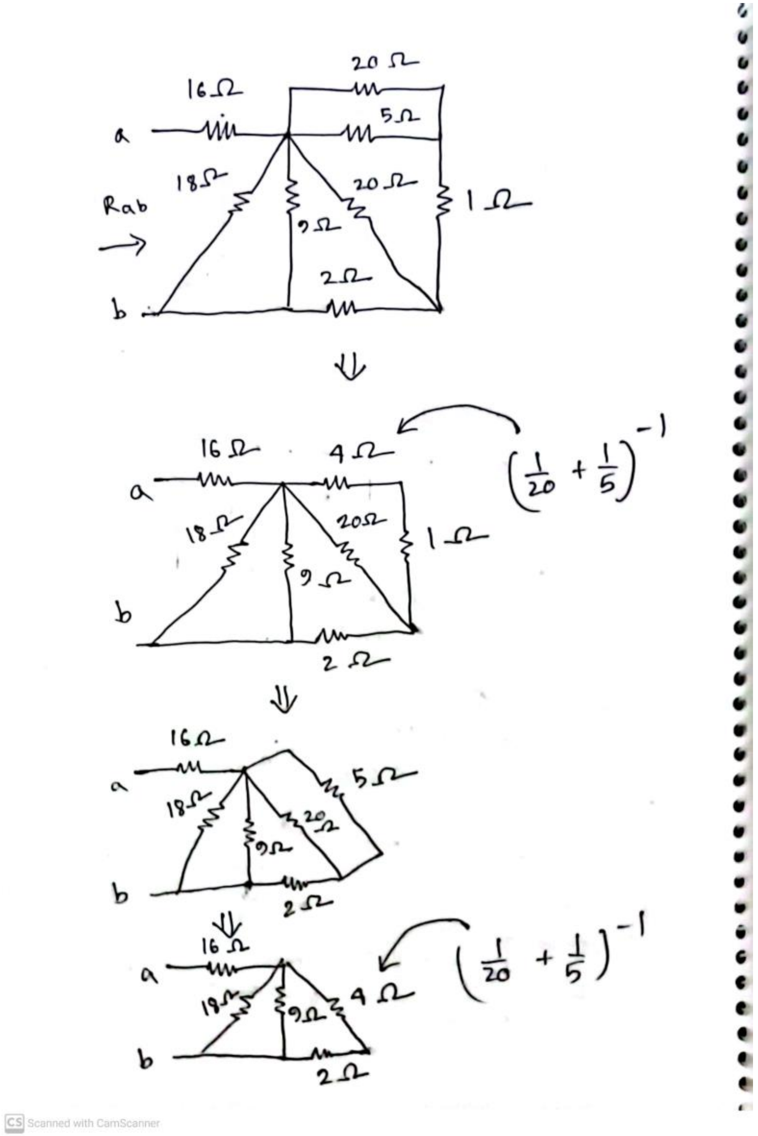
Find R_{eq} for the circuit shown in Fig.

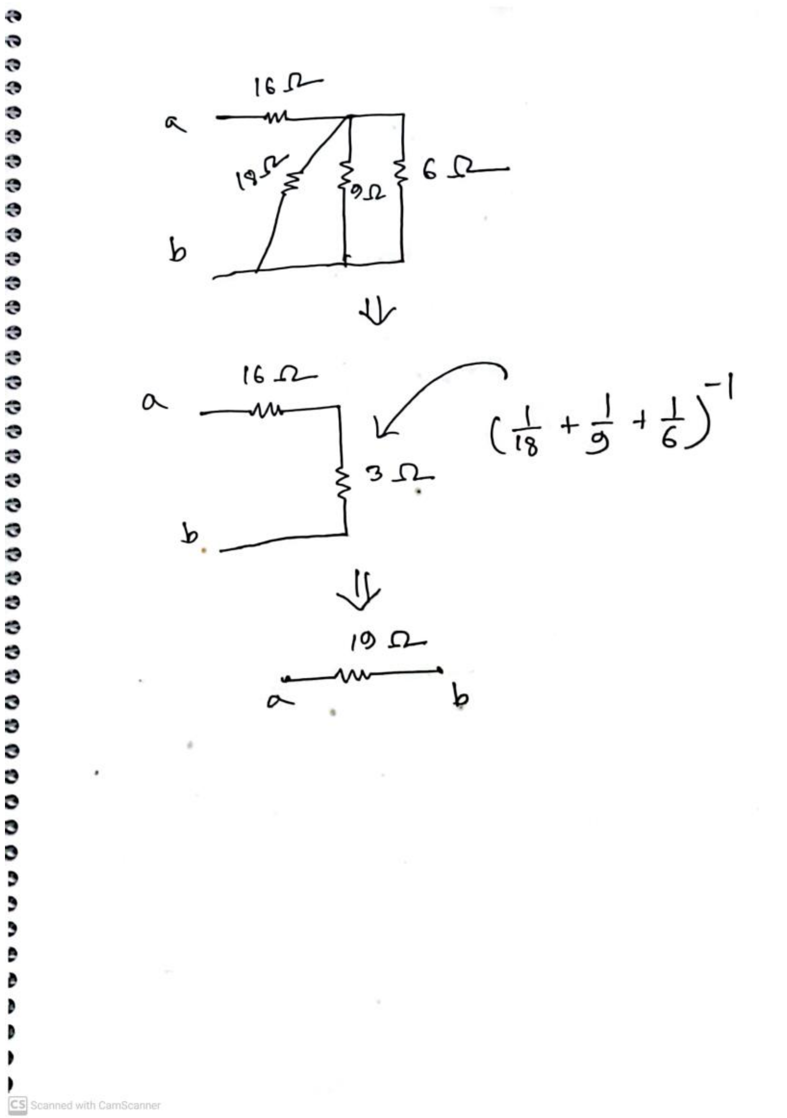


Ans: R_{eq} **= 19 Ω**

Reference: Sadiku_Practice_Problem_2.10

Solution of Practice problem 2.9



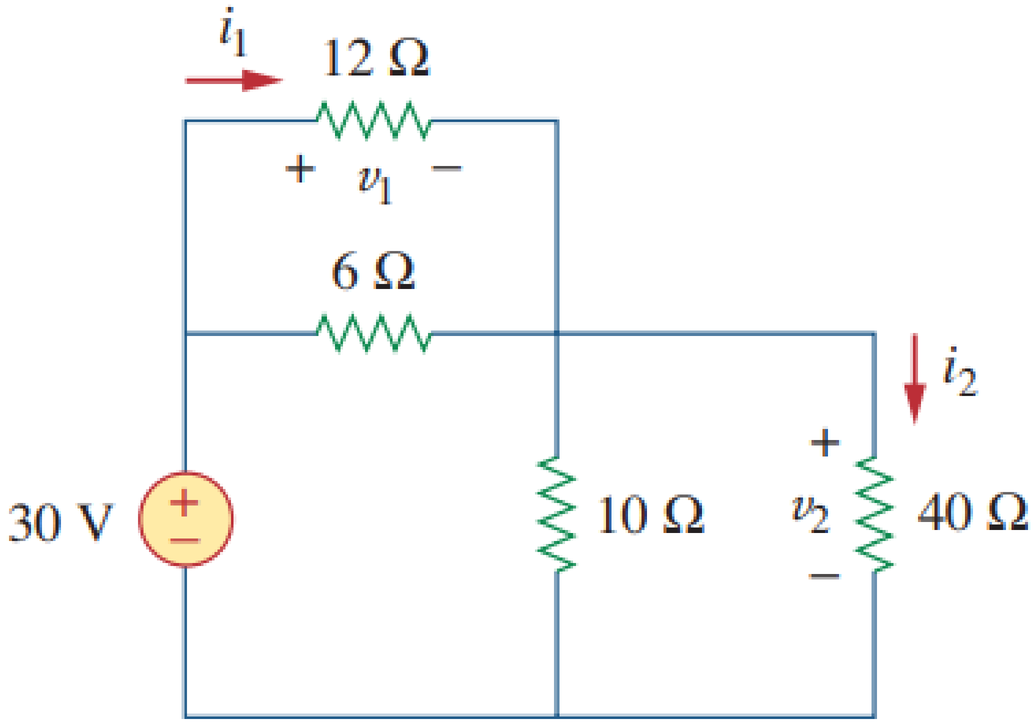


Reference: Sadiku_Practice_Problem_2.10

Math Problem Practice:

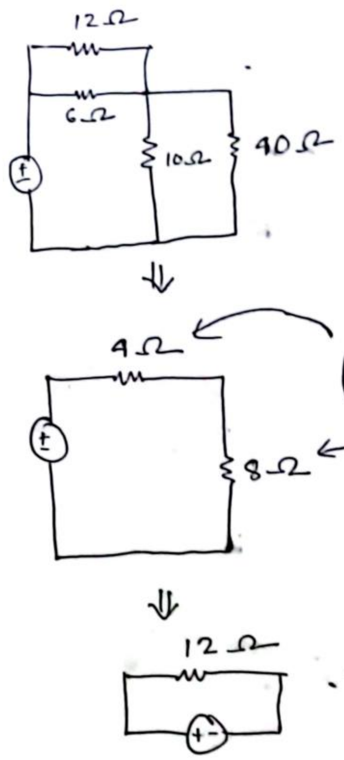
Find v_1 and v_2 in the circuit shown in Fig. 2.43. Also calculate i_1 and i_2 and the power dissipated in the 12-Ω and 40-Ω resistors.

Answer: $v_1 = 10$ V, $i_1 = 833.3$ mA, $p_1 = 8.333$ W, $v_2 = 20$ V, $i_2 = 500$ mA, $p_2 = 10$ W.



Reference: Sadiku_Practice_Problem_2.12

Solution of Practice problem 2.12



$$\left(\frac{1}{12}+\frac{1}{6}\right)^{-1}$$

$$\left(\frac{1}{10}+\frac{1}{40}\right)^{-1}$$

$\therefore R_{eq} = 12\ \Omega$

$$V = IR \Rightarrow I = \frac{V}{R_{eq}} = \frac{30\,V}{12\,\Omega} = 2.5\,A$$

$$i_1 = \frac{6}{6+12} \times 2.5 = \frac{5}{6}\,A = 0.8333\,A$$

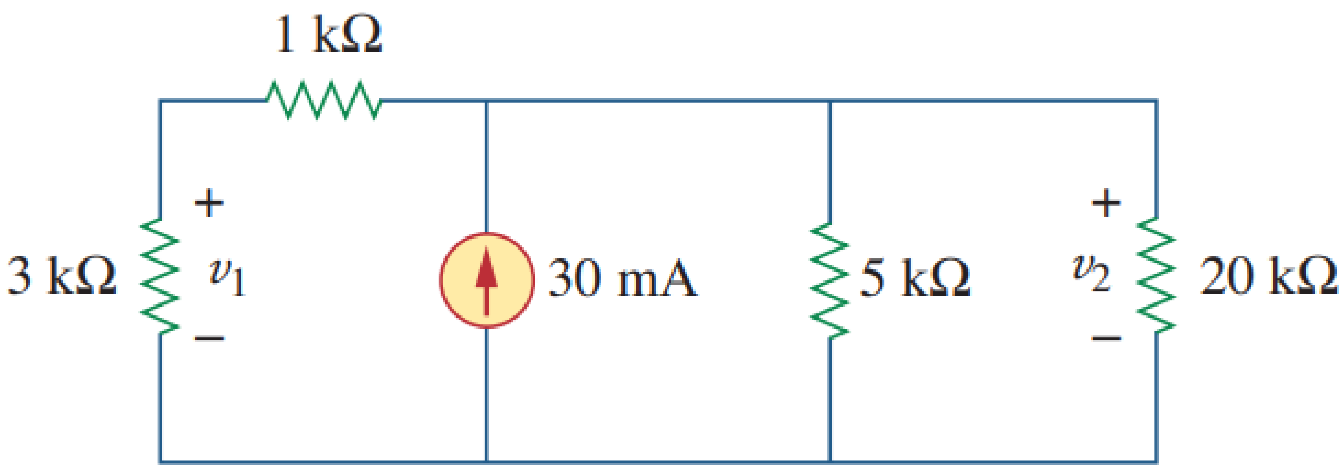
$$i_2 = \frac{10}{10+40} \times 2.5 = 0.5\,A$$

$$v_1 = i_1 R = \frac{5}{6} \times 12 = 10\,V, \quad v_2 = i_2 R = 0.5 \times 40 = 20\,V$$

Reference: Sadiku_Practice_Problem_2.12

Math Problem Practice:

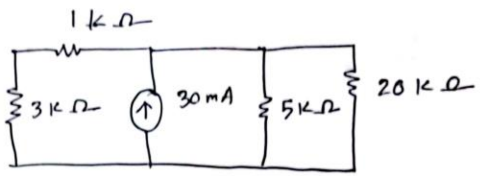
For the circuit shown in Fig. 2.45, find: (a) v_1 and v_2, (b) the power dissipated in the 3-kΩ and 20-kΩ resistors, and (c) the power supplied by the current source.



Answer: (a) 45 V, 60 V, (b) 675 mW, 180 mW, (c) 1.8 W.

Reference: Sadiku_Practice_Problem_2.13

Solution of Practice problem 2.13



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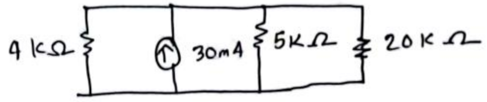

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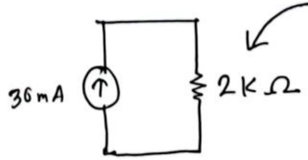


$\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{20}\right)^{-1}$

$V = IR \Rightarrow V = 30\times10^{-3} \times 2\times10^{3}$

$\therefore V = 60\,V$

(a) $V_1 = \dfrac{3\times10^3}{3\times10^3 + 1\times10^3} \times 60 = 45\,V$

~~V_2 =~~ $V_2 = V = 60\,V$

(b) $P_{3k\Omega} = \dfrac{V^2}{R} = \dfrac{45^2}{3\times10^3} = 0.675\,W$

$P_{20k\Omega} = \dfrac{V^2}{R} = \dfrac{60^2}{20\times10^3}$

$= 0.18\,W$

(c) $P = VI$

$= 60 \times 30\times10^{-3}$

$\therefore P = 1.8\,W$

Reference: Sadiku_Practice_Problem_2.13

Math Problem Practice:

2.26 For the circuit in Fig. 2.90, $i_o = 3$ A. Calculate i_x and the total power absorbed by the entire circuit.

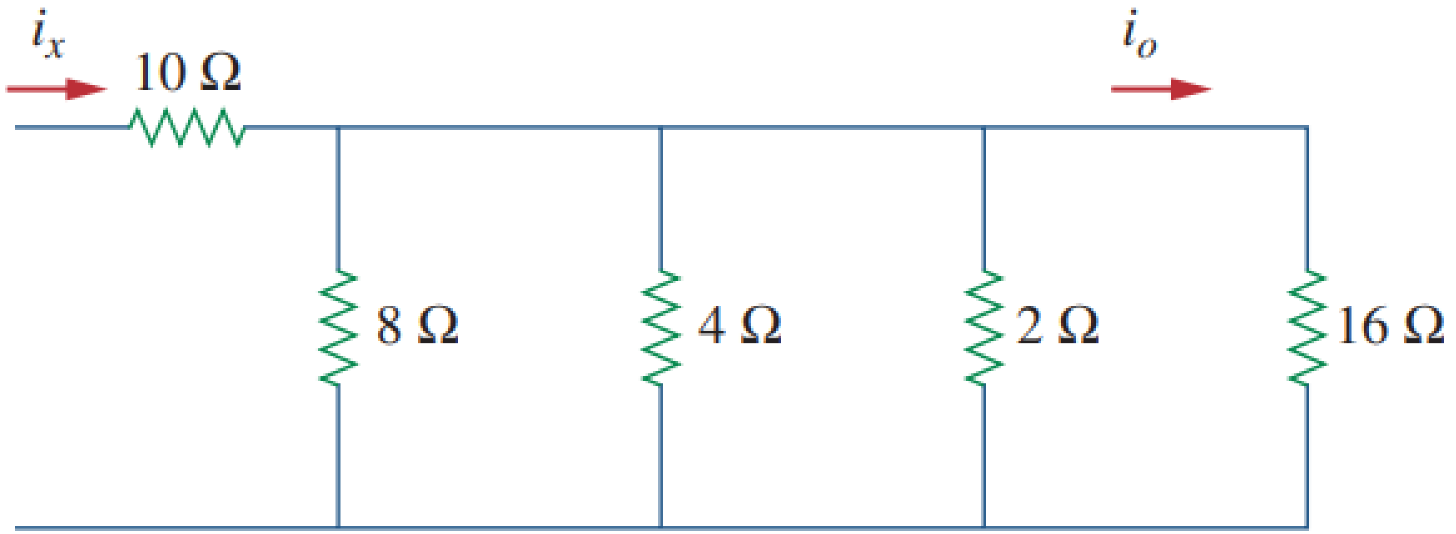


Figure 2.90
For Prob. 2.26.

Reference: Sadiku_Exercise_Problem_2.26

Solution of 2.26:

Step 1 of 5

2.026P

Consider the following circuit diagram:

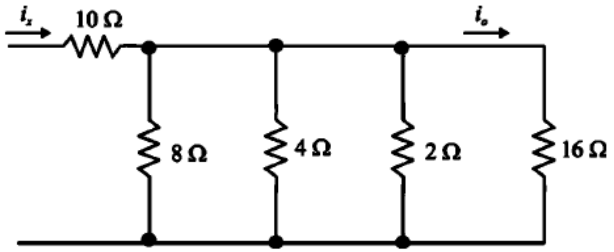


Figure 1

Step 2 of 5

Consider the following data:

The current $i_o = 3\,\text{A}$

By using Ohm's law, the voltage across the 16 ? resistor is,

$$\begin{aligned} v_{16\Omega} &= i_o R \\ &= (3)(16) \\ &= 48\,\text{V} \end{aligned}$$

Since the 8 ?, 4 ? and 2 ? resistors are connected in parallel; the voltage across them is same as the 16 ? resistor (48 V).

By using the Ohm's law, the current flowing through the 8 ? resistor is,

$$\begin{aligned} i_{8\Omega} &= \frac{v}{R} \\ &= \frac{48}{8} \\ &= 6\,\text{A} \end{aligned}$$

By using the Ohm's law, the current flowing through the 4 ? resistor is,

$$\begin{aligned} i_{4\Omega} &= \frac{v}{R} \\ &= \frac{48}{4} \\ &= 12\,\text{A} \end{aligned}$$

By using the Ohm's law, the current flowing through the 2 ? resistor is,

$$\begin{aligned} i_{2\Omega} &= \frac{v}{R} \\ &= \frac{48}{2} \\ &= 24\,\text{A} \end{aligned}$$

The total current i_x is the sum of the branch currents.

The total current i_x is the sum of the branch currents.

$$\begin{aligned} i_x &= i_{8\Omega} + i_{4\Omega} + i_{2\Omega} + i_o \\ &= 6+12+24+3 \\ &= 45\,\text{A} \end{aligned}$$

Therefore, the current i_x is $\boxed{45\,\text{A}}$.

Step 3 of 5

The 8 ? and 2 ? resistors are in parallel. Their combined resistance is,

$$\begin{aligned} 8\Omega \| 2\Omega &= \frac{8\times 2}{8+2} \\ &= 1.6\Omega \end{aligned}$$

The 4 ? and 16 ? resistors are in parallel. Their combined resistance is,

$$\begin{aligned} 4\Omega \| 16\Omega &= \frac{4\times 16}{4+16} \\ &= 3.2\Omega \end{aligned}$$

Step 4 of 5

Then the circuit in Figure 1 is reduced to the circuit in Figure 2.

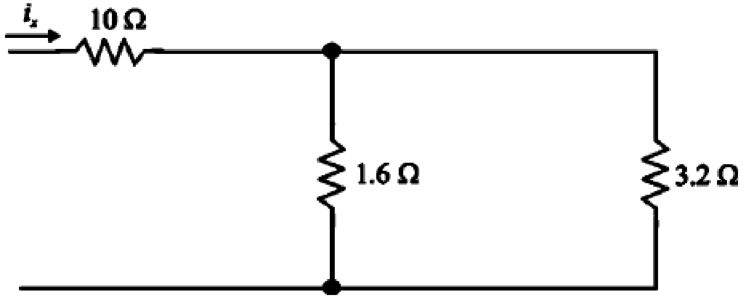


Figure 2

Step 5 of 5

These 1.6? and 3.2? resistors are in parallel. Their combined resistance is,

$$\begin{aligned} 1.6\Omega \| 3.2\Omega &= \frac{1.6\times 3.2}{1.6+3.2} \\ &= 1\Omega \end{aligned}$$

This 1? and 10? resistors are in series. Their combined resistance is,

$$1\Omega + 10\Omega = 11\Omega$$

The power absorbed by the entire circuit can be calculated as follows.

$$\begin{aligned} p &= i_x^2 R \\ &= (45)^2 (11) \\ &= 22.275\,\text{kW} \end{aligned}$$

Therefore, the power absorbed by the entire circuit is $\boxed{22.275\,\text{kW}}$.

Reference: Sadiku_Exercise_Problem_2.26

Math Problem Practice:

2.30 Find R_{eq} for the circuit in Fig. 2.94.

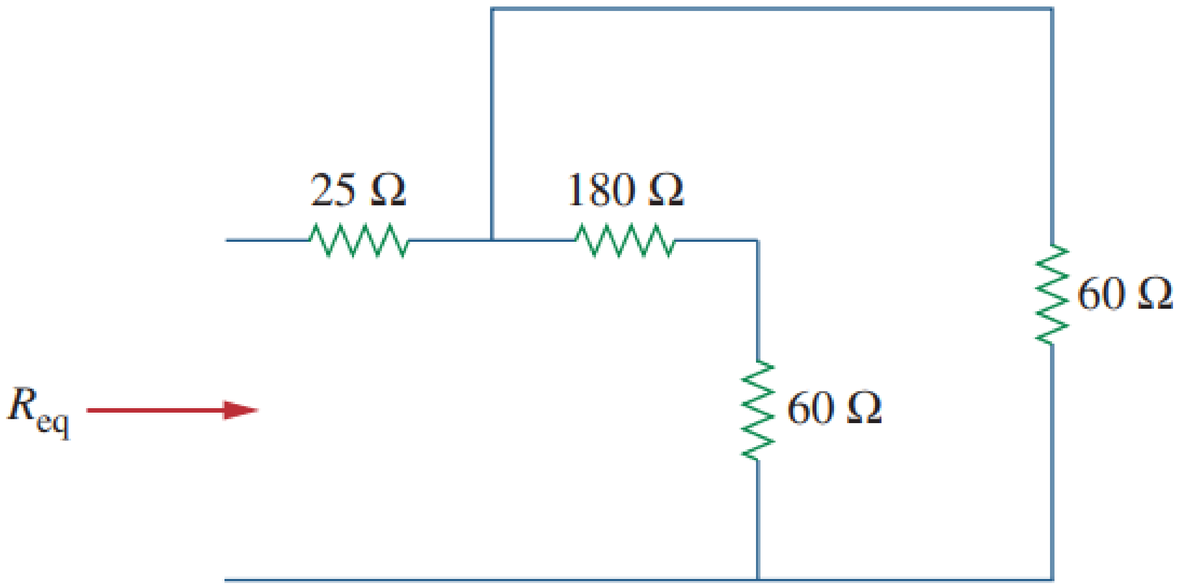


Figure 2.94
For Prob. 2.30.

Reference: Sadiku_Exercise_Problem_2.30

Solution of 2.30:

Step 1 of 6 2.030P

Consider the following circuit diagram:

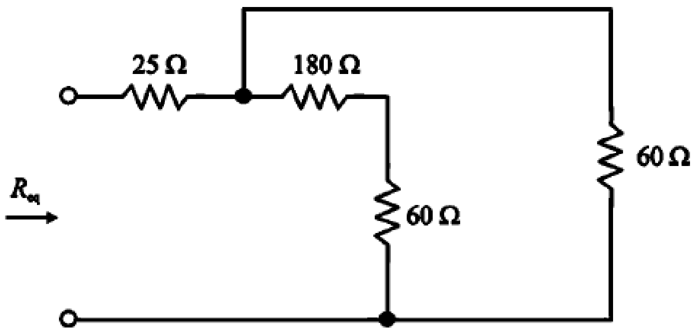


Figure 1

Step 2 of 6

To get R_{eq}, we combine the resistors in series and in parallel.

The 180 ? and 60 ? resistors are in series. Their combined resistance is,

$$180\Omega + 60\Omega = 240\Omega$$

Step 3 of 6

Then the reduced circuit is shown in Figure 2.

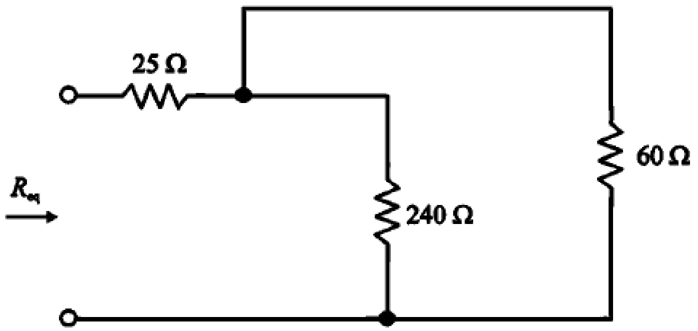


Figure 2

Step 4 of 6

The 240 ? and 60 ? resistors are in parallel. Their combined resistance is,

$$240\Omega \| 60\Omega = \frac{240 \times 60}{240 + 60}$$
$$= 48\Omega$$

Step 5 of 6

Then the reduced circuit is shown in Figure 3.

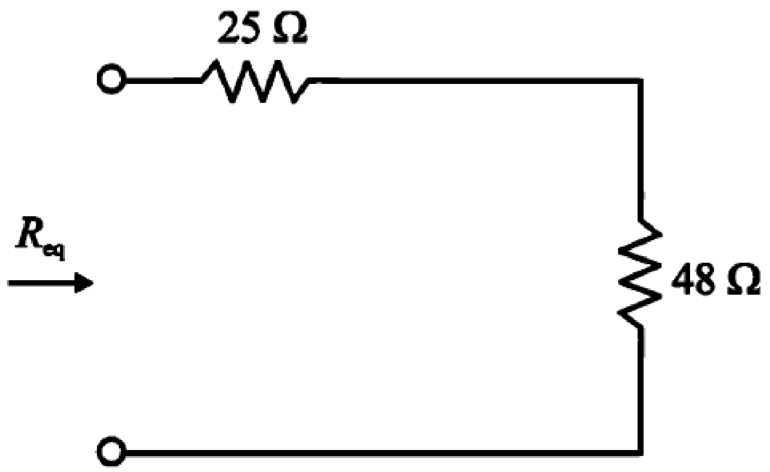


Figure 3

Step 6 of 6

The 25 ? and 48 ? resistors are in series. Their combined resistance is,

$$R_{eq} = 25\Omega + 48\Omega$$
$$= 73\Omega$$

Therefore, the resistance R_{eq} is $\boxed{73\Omega}$.

Reference: Sadiku_Exercise_Problem_2.30

Math Problem Practice:

2.31 For the circuit in Fig. 2.95, determine i_1 to i_5.

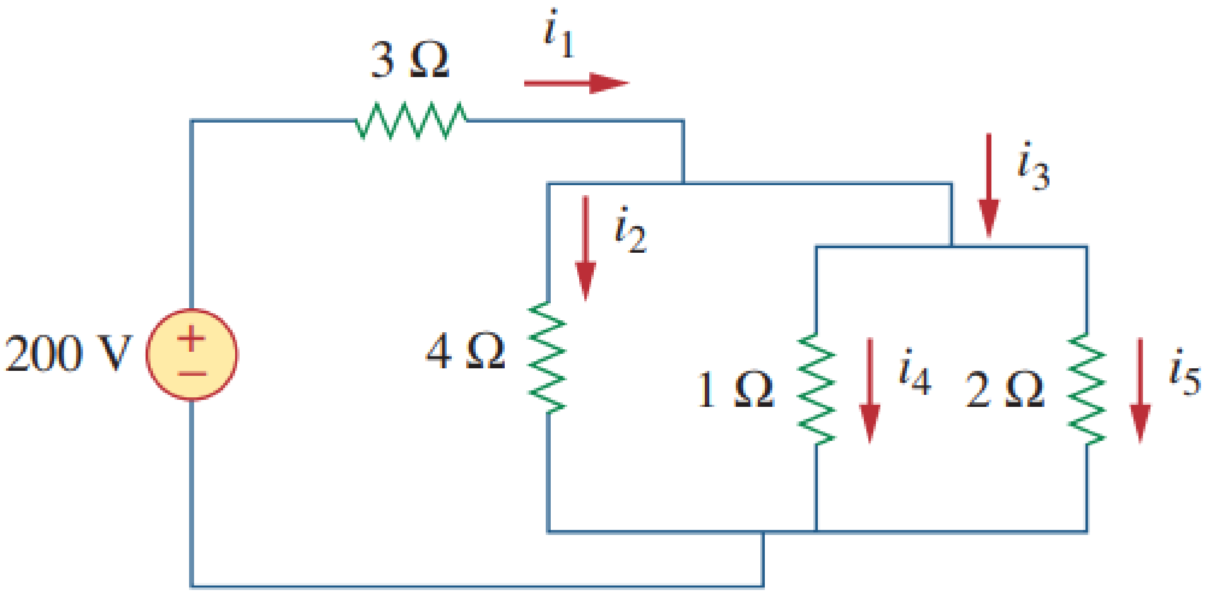


Figure 2.95
For Prob. 2.31.

Reference: Sadiku_Exercise_Problem_2.31

Solution of 2.31:

Step 1 of 9 2.031P

Consider the following circuit diagram:

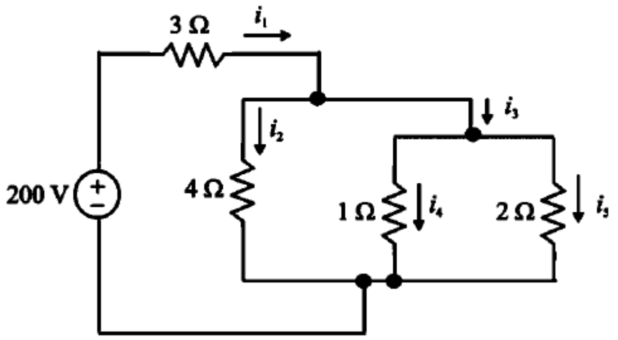


Figure 1

Step 2 of 9

To get R_{eq}, we combine the resistors in series and in parallel.

The 1 ? and 2 ? resistors are in parallel. Their combined resistance is,

$$1\Omega \| 2\Omega = \frac{1\times 2}{1+2} = 0.66\Omega$$

Step 3 of 9

Then the reduced circuit is shown in Figure 2.

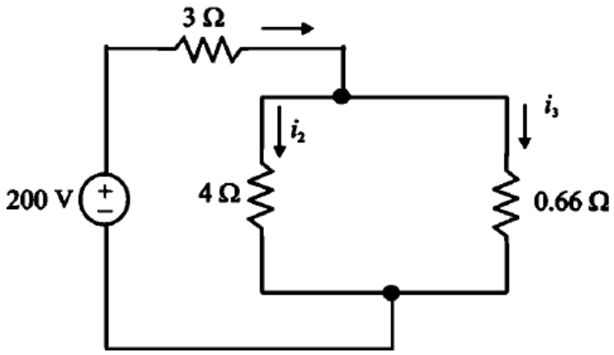


Figure 2

Step 4 of 9

The 0.66 ? and 4 ? resistors are in parallel. Their combined resistance is,

$$0.66\Omega \| 4\Omega = \frac{0.66\times 4}{0.66+4} = 0.57\Omega$$

Step 5 of 9

Then the reduced circuit is shown in Figure 3.

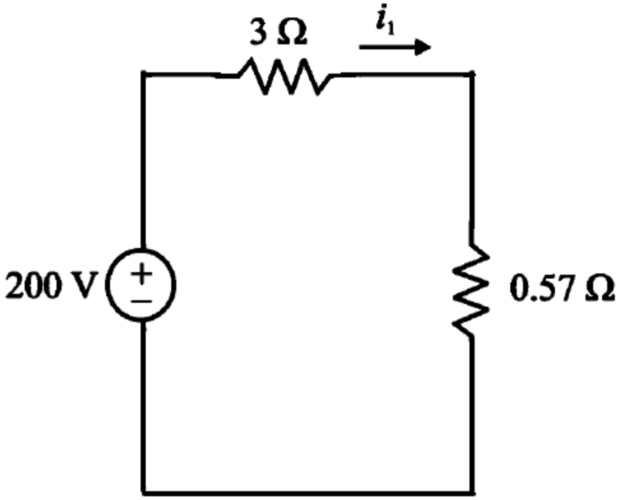


Figure 3

Step 6 of 9

The 0.57 ? and 3 ? resistors are in series. Their combined resistance is,

$$0.57\Omega + 3\Omega = 3.57\Omega$$

By using Ohms law, the current i_1 can be calculated as follows.

$$i_1 = \frac{v}{R} = \frac{200}{3.57} = 56\,\text{A}$$

Therefore, the current i_1 is $\boxed{56\,\text{A}}$.

Step 7 of 9

By using current division, the current i_2 can be calculated as follows.

$$i_2 = \frac{0.66}{0.66+4} i_1 = \frac{0.66}{0.66+4}(56) = 8\,\text{A}$$

Therefore, the current i_2 is $\boxed{8\,\text{A}}$.

Applying Kirchhoff's current law, the sum of currents entering any node is equal to the sum of currents leaving from that node. The current i_3 can be calculated as follows.

$$i_1 = i_2 + i_3$$
$$56 = 8 + i_3$$
$$i_3 = 48\,\text{A}$$

Therefore, the current i_3 is $\boxed{48\,\text{A}}$.

Step 8 of 9

By using current division, the current i_4 can be calculated as follows.

$$i_4 = \frac{2}{2+1}(i_3) = \frac{2}{2+1}(48) = 32\,\text{A}$$

Therefore, the current i_4 is $\boxed{32\,\text{A}}$.

Step 8 of 9

Applying Kirchhoff's current law, the sum of currents entering any node is equal to the sum of currents leaving from that node. The current i_5 can be calculated as follows.

$$i_3 = i_4 + i_5$$
$$48 = 32 + i_5$$
$$i_5 = 16\,\text{A}$$

Therefore, the current i_5 is $\boxed{16\,\text{A}}$.

<u>Reference: Sadiku_Exercise_Problem_2.31</u>

Math Problem Practice:

2.35 Calculate V_o and I_o in the circuit of Fig. 2.99.

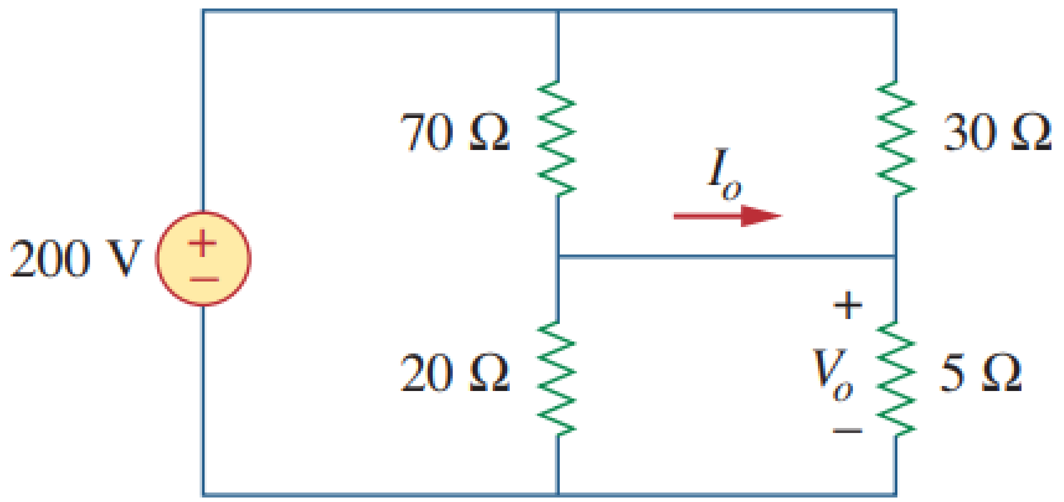


Figure 2.99
For Prob. 2.35.

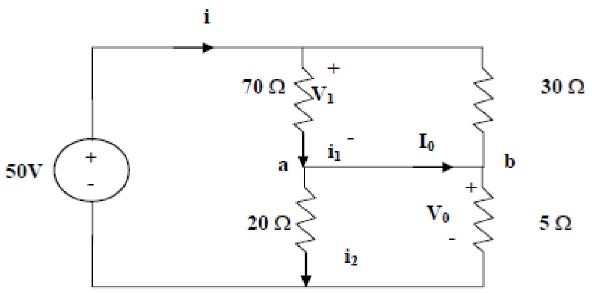
Reference: Sadiku_Exercise_Problem_2.35

Solution of 2.35:

Chapter 2, Problem 35.

Calculate V_o and I_o in the circuit of Fig. 2.99.

Chapter 2, Solution 35



Combining the versions in parallel,

$$70\|30 = \frac{70x30}{100} = 21\Omega \,, \qquad 20\|5 = \frac{20x5}{25} = 4\,\Omega$$

$$i = \frac{50}{21+4} = 2\text{ A}$$

$$v_i = 21i = 42\text{ V},\ v_0 = 4i = 8\text{ V}$$

$$i_1 = \frac{v_1}{70} = 0.6\text{ A},\ i_2 = \frac{v_2}{20} = 0.4\text{ A}$$

At node a, KCL must be satisfied

$$i_1 = i_2 + I_0 \longrightarrow 0.6 = 0.4 + I_0 \longrightarrow I_0 = 0.2\text{ A}$$

Hence $v_0 = \underline{8\text{ V}}$ and $I_0 = \underline{\mathbf{0.2A}}$

Reference: Sadiku_Exercise_Problem_2.35

Math Problem Practice:

2.36 Find i and V_o in the circuit of Fig. 2.100.

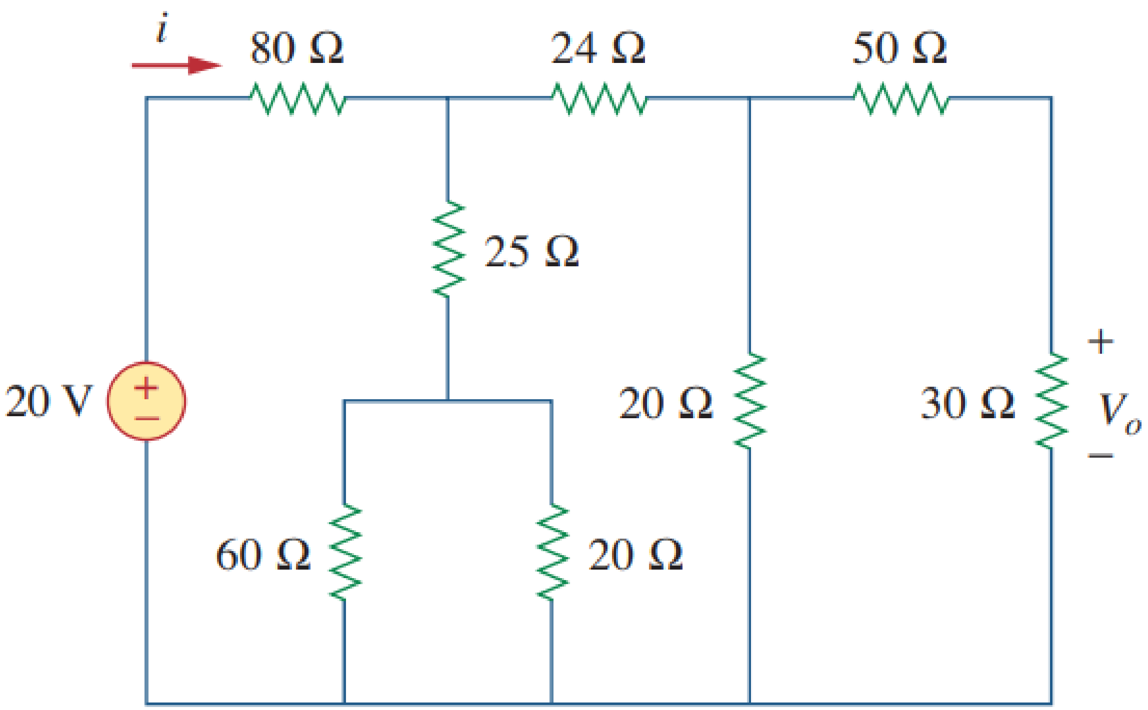


Figure 2.100
For Prob. 2.36.

<u>Reference: Sadiku_Exercise_Problem_2.36</u>

Solution of 2.36:

Step 1 of 6 2.036P

Consider the following circuit diagram:

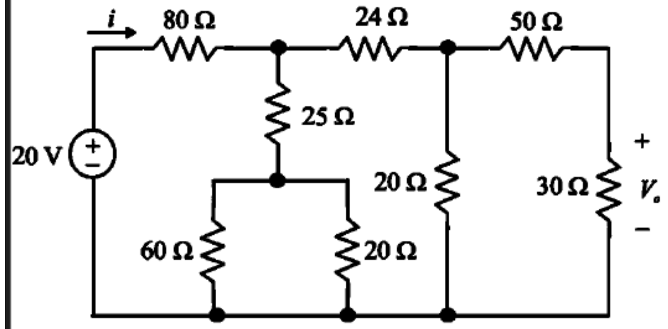


Figure 1

Step 2 of 6

The 50 ? and 30 ? resistors are in series. Their combined resistance is,

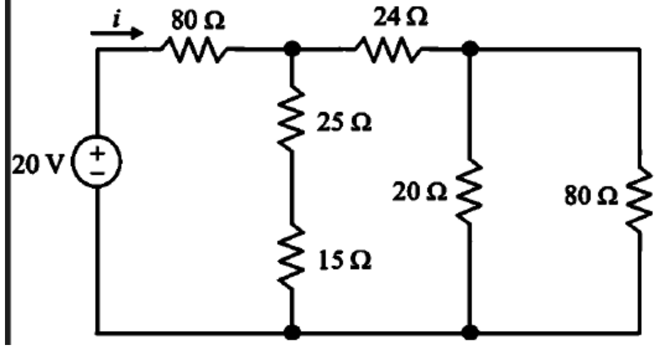
$$50\Omega + 30\Omega = 80\Omega$$

The 20? and 60? resistors are in parallel. Their combined resistance is,

$$20\Omega \| 60\Omega = \frac{20\times 60}{20+60} = 15\Omega$$

Step 3 of 6

Then the reduced circuit is shown in Figure 2.



Step 4 of 6

The 15 ? and 25 ? resistors are in series. Their combined resistance is,

$$15\Omega + 25\Omega = 40\Omega$$

The 20 ? and 80 ? resistors are in parallel. Their combined resistance is,

$$20\Omega \| 80\Omega = \frac{20\times 80}{20+80} = 16\Omega$$

Step 5 of 6

Then the reduced circuit is shown in Figure 3.

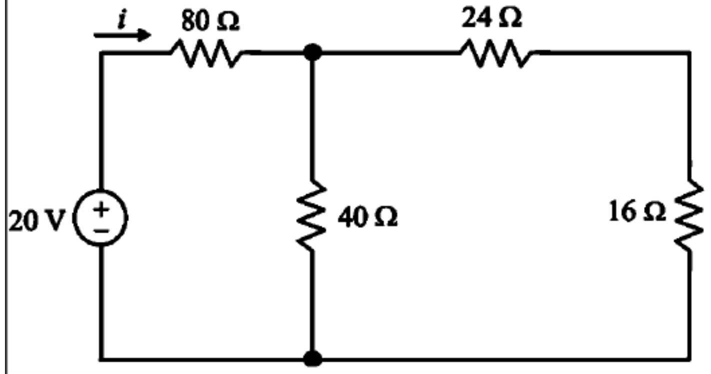


Figure 3

Step 6 of 6

The 16 ? and 24 ? resistors are in series. Their combined resistance is,

$$16\Omega + 24\Omega = 40\Omega$$

This 40 ? and 40 ? resistors are in parallel. Their combined resistance is,

$$40\Omega \| 40\Omega = \frac{40\times 40}{40+40} = 20\Omega$$

This 20 ? and 80 ? resistors are in series. Their combined resistance is,

$$20\Omega + 80\Omega = 100\Omega$$

By using Ohms law, the current i can be calculated as follows.

$$i = \frac{v}{R} = \frac{20}{100} = 200\text{mA}$$

Therefore, the value of current i is $\boxed{200\,\text{mA}}$.

Since the two branch resistances are equal 40 ? and 40 ?, the current is divided equally. Hence the current through the 24 ? resistor is 100mA.

Apply current division rule, the current passing through 30? resistor is,

$$i_{30\Omega} = \frac{20}{20+80} i_{24\Omega} = \frac{20}{20+80}(100\text{mA}) = 20\,\text{mA}$$

By using Ohms law, the current V_o can be calculated as follows.

$$V_o = i_{30\Omega} R = (20\text{mA})(30) = 0.6\,\text{V}$$

Therefore, the value of voltage, V_o is $\boxed{0.6\,\text{V}}$.

Reference: Sadiku_Exercise_Problem_2.36

Math Problem Practice:

2.38 Find R_{eq} and i_o in the circuit of Fig. 2.102.

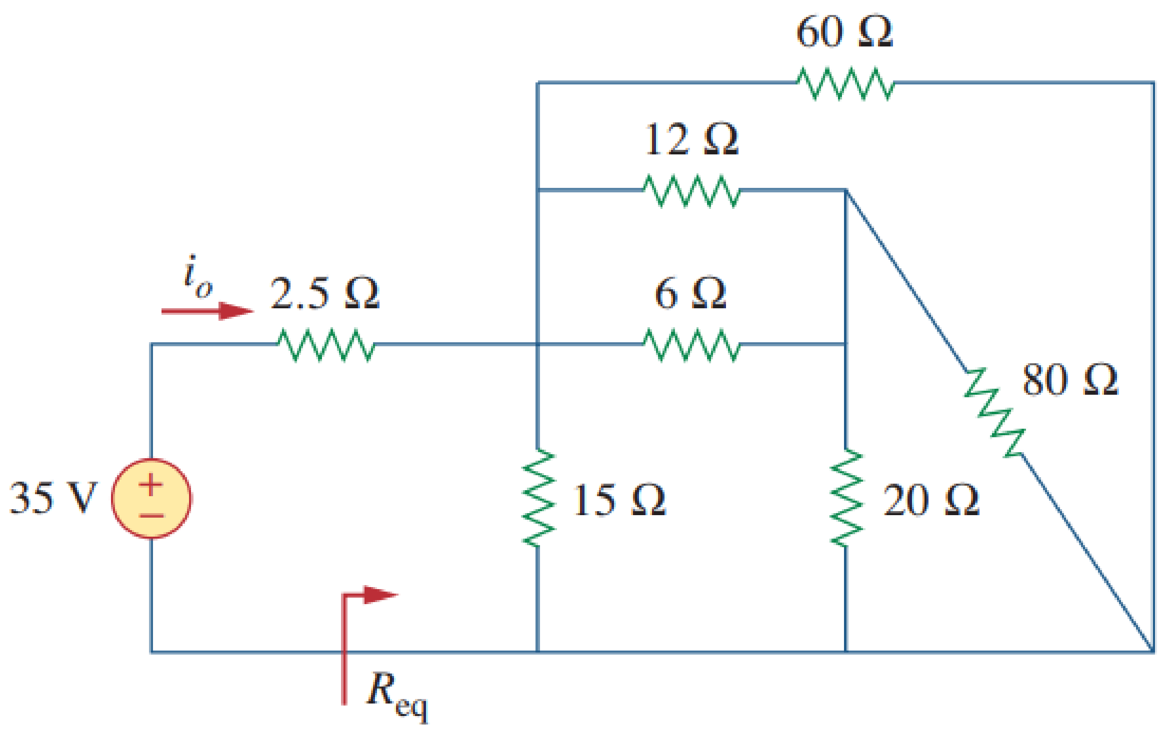


Figure 2.102
For Prob. 2.38.

Reference: Sadiku_Exercise_Problem_2.38

Solution of 2.38:

2.038P

Step 1 of 7

Consider the following circuit diagram:

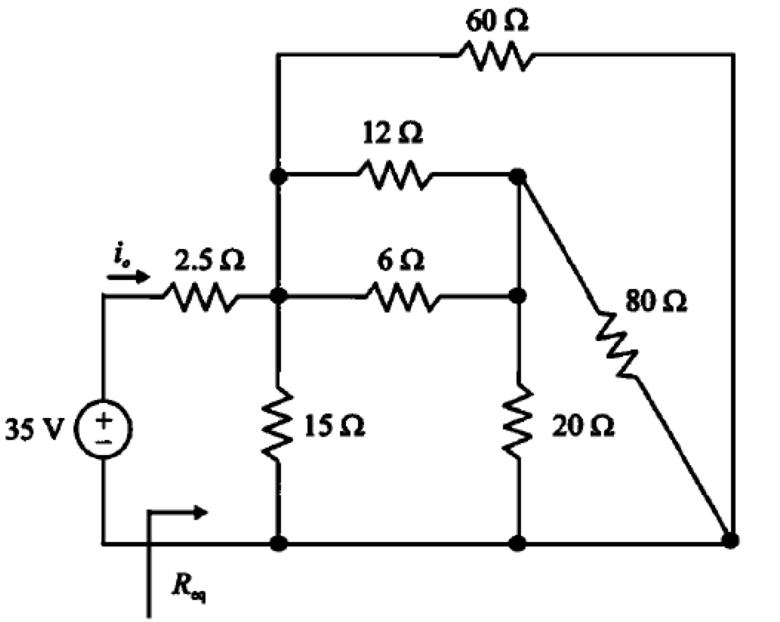


Figure 1

Step 2 of 7

To get the R_{eq}, we combine the resistors in series and in parallel.

The 12 ? and 6 ? resistors are in parallel. Their combined resistance is,

$$12\Omega \| 6\Omega = \frac{12\times 6}{12+6} = 4\Omega$$

The 20 ? and 80 ? resistors are in parallel. Their combined resistance is,

$$20\Omega \| 80\Omega = \frac{20\times 80}{20+80} = 16\Omega$$

Step 3 of 7

Then the reduced circuit is shown in Figure 2.

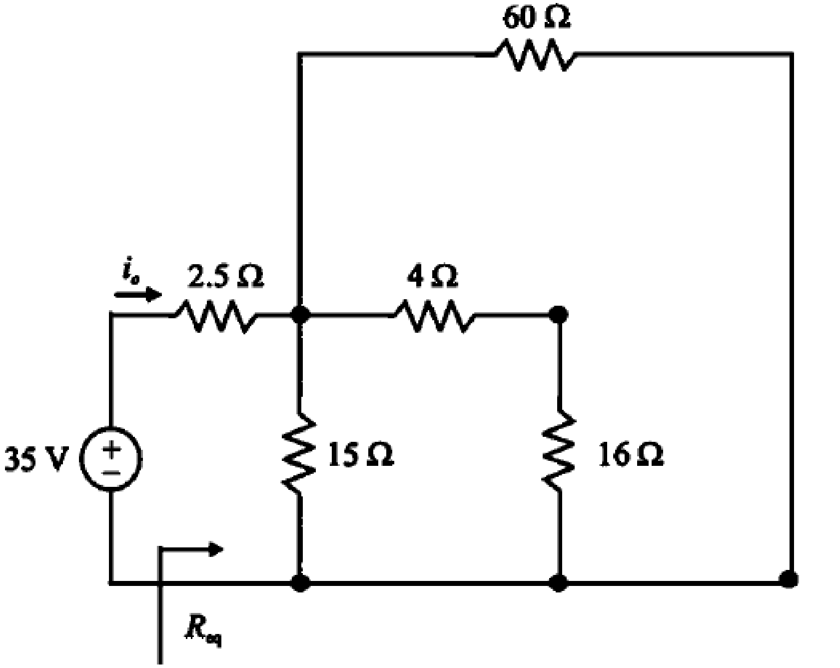


Figure 2

Step 4 of 7

The 4 ? and 16 ? resistors are in series. Their combined resistance is,

$$4\Omega + 16\Omega = 20\Omega$$

This 20 ? and 60 ? resistors are in parallel. Their combined resistance is,

$$20\Omega \| 60\Omega = \frac{20\times 60}{20+60} = 15\Omega$$

Step 5 of 7

Then the reduced circuit is shown in Figure 3.

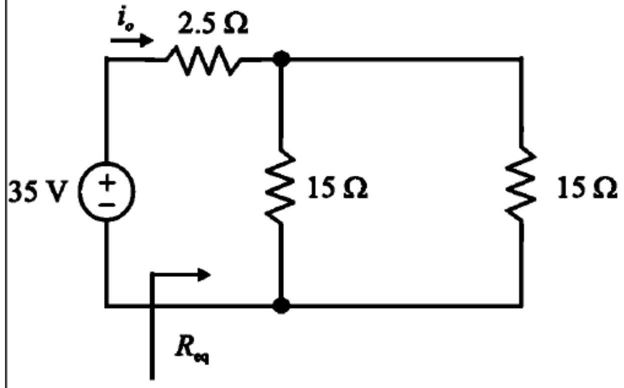


Figure 3

Step 6 of 7

The 15 ? and 15 ? resistors are in parallel. Their combined resistance is,

$$15\Omega \| 15\Omega = \frac{15\times 15}{15+15} = 7.5\Omega$$

This 7.5 ? and 2.5 ? resistors are in series. Their combined resistance is,

$$R_{eq} = 7.5\Omega + 2.5\Omega = 10\Omega$$

Therefore the equivalent resistance R_{eq} seen by the source is $\boxed{10\Omega}$.

Step 7 of 7

By using Ohms law, the current i_o can be calculated as follows.

$$i_o = \frac{v}{R} = \frac{35}{10} = 3.5\text{A}$$

Therefore, the value of current i_o is $\boxed{3.5\text{A}}$.

Reference: Sadiku_Exercise_Problem_2.38

Math to Practice from the Book for Exam

- **Chapter 2**
  - **Example:**
    - 2.3, 2.5, 2.6, 2.7, 2.9, 2.10, 2.12, 2.16
  - **Practice Problem:**
    - 2.3, 2.5, 2.6, 2.7, 2.9, 2.10, 2.12
  - **Problem:**
    - 2.15, 2.16, 2.18, 2.22, 2.23, 2.26, 2.30, 2.31, 2.32, 2.35, 2.36, 2.37, 2.38, 2.39, 2.40, 2.41(**), 2.42, 2.43, 2.44(**), 2.45, 2.46, 2.47(**)
- **N:B: Please note that the Wye-Delta concept is not in the present syllabus!**

Thank You