AC POWER ANALYSIS

EFFECTIVE OR RMS VALUE

Section 11.4

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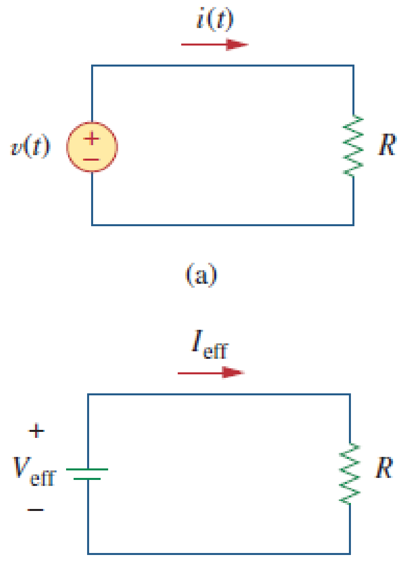
EFFECTIVE OR RMS VALUE

The **effective value** of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.


SOLUTION

CALCULATION



(b)

Figure 11.13
Finding the effective current: (a) ac circuit, (b) dc circuit.

In Fig. 11.13, the circuit in (a) is ac while that of (b) is dc. Our objective is to find I_{eff} that will transfer the same power to resistor R as the sinusoid i. The average power absorbed by the resistor in the ac circuit is

$$P = \frac{1}{T}\int_0^T i^2 R\,dt = \frac{R}{T}\int_0^T i^2\,dt \tag{11.22}$$

while the power absorbed by the resistor in the dc circuit is

$$P = I_{\text{eff}}^2 R \tag{11.23}$$

Equating the expressions in Eqs. (11.22) and (11.23) and solving for I_{eff}, we obtain

$$I_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} \tag{11.24}$$

The effective value of the voltage is found in the same way as current; that is,

$$V_{\text{eff}} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} \tag{11.25}$$

This indicates that the effective value is the (square) *root* of the *mean* (or average) of the *square* of the periodic signal. Thus, the effective value is often known as the *root-mean-square* value, or *rms* value for short; and we write

$$I_{\text{eff}} = I_{\text{rms}}, \qquad V_{\text{eff}} = V_{\text{rms}} \tag{11.26}$$

For any periodic function $x(t)$ in general, the rms value is given by

$$\boxed{X_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T x^2\,dt}} \tag{11.27}$$

The **effective value** of a periodic signal is its root mean square (rms) value.

STEPS TO FIND THE RMS VALUE

- **Step-1:**
  - Find its square
- **Step-2:**
  - Find the mean
- **Step-3:**
  - Find the square root of that mean

CALCULATION OF I(RMS) AND V(RMS)

For the sinusoid $i(t) = I_m \cos \omega t$, the

effective or rms value is

$$
\begin{aligned}
I_{\text{rms}} &= \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2 \omega t \, dt} \\
&= \sqrt{\frac{I_m^2}{T}\int_0^T \frac{1}{2}(1 + \cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}
\end{aligned}
\tag{11.28}
$$

Similarly, for $v(t) = V_m \cos \omega t$,

$$
V_{\text{rms}} = \frac{V_m}{\sqrt{2}} \tag{11.29}
$$

Keep in mind that Eqs. (11.28) and (11.29) are only valid for sinusoidal signals.

AVG POWER IN TERMS OF RMS

The average power in Eq. (11.8) can be written in terms of the rms values.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) \qquad \textbf{(11.30)}$$

Similarly, the average power absorbed by a resistor R in Eq. (11.11) can be written as

$$P = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R} \qquad \textbf{(11.31)}$$

When a sinusoidal voltage or current is specified, it is often in terms of its maximum (or peak) value or its rms value, since its average value is zero. The power industries specify phasor magnitudes in terms of their rms values rather than peak values. For instance, the 110 V available at every household is the rms value of the voltage from the power company. It is convenient in power analysis to express voltage and current in their rms values. Also, analog voltmeters and ammeters are designed to read directly the rms value of voltage and current, respectively.

MATH PROBLEM PRACTICE:

Example 11.7

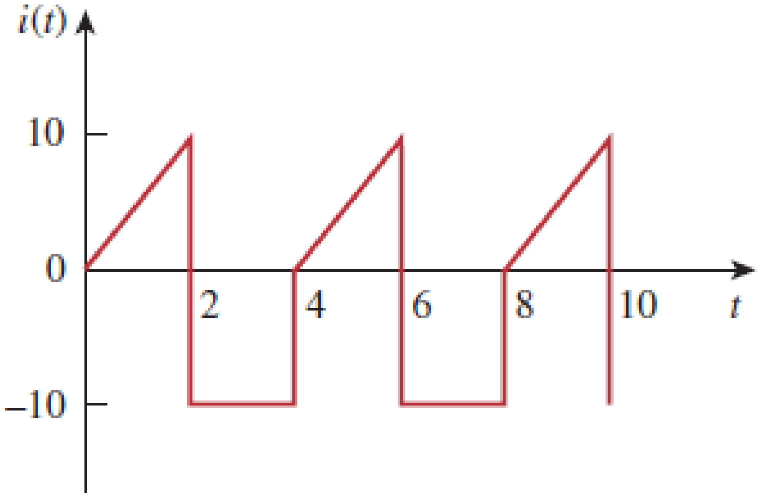


Figure 11.14
For Example 11.7.

Determine the rms value of the current waveform in Fig. 11.14. If the current is passed through a 2-Ω resistor, find the average power absorbed by the resistor.

Reference: Sadiku_Example_11.7

MATH PROBLEM PRACTICE:

Example 11.7

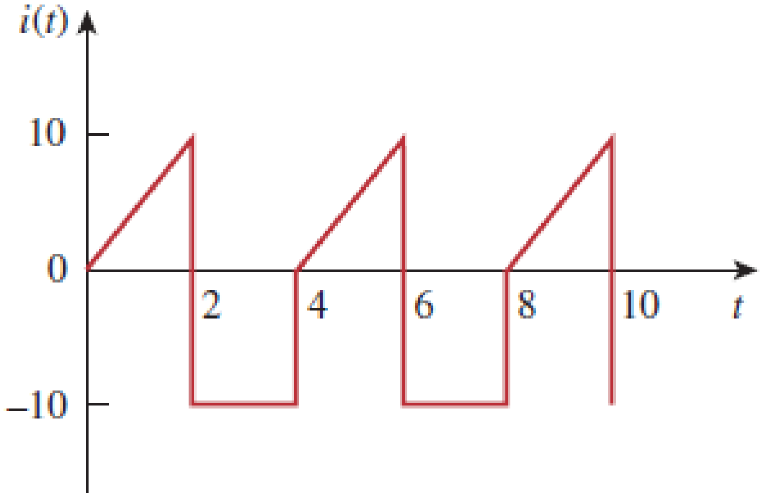


Figure 11.14
For Example 11.7.

Solution:
The period of the waveform is $T = 4$. Over a period, we can write tl
current waveform as

$$i(t) = \begin{cases} 5t, & 0 < t < 2 \\ -10, & 2 < t < 4 \end{cases}$$

The rms value is

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{4}\left[\int_0^2 (5t)^2\,dt + \int_2^4 (-10)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{4}\left[25\frac{t^3}{3}\bigg|_0^2 + 100t\bigg|_2^4\right]} = \sqrt{\frac{1}{4}\left(\frac{200}{3} + 200\right)} = 8.165\text{ A}$$

The power absorbed by a 2-Ω resistor is

$$P = I_{\text{rms}}^2 R = (8.165)^2(2) = 133.3\text{ W}$$

<u>Reference: Sadiku_Example_11.7</u>

MATH PROBLEM PRACTICE:

Practice Problem 11.7

Find the rms value of the current waveform of Fig. 11.15. If the current flows through a 9-Ω resistor, calculate the average power absorbed by the resistor.

Answer: 9.238 A, 768 W.

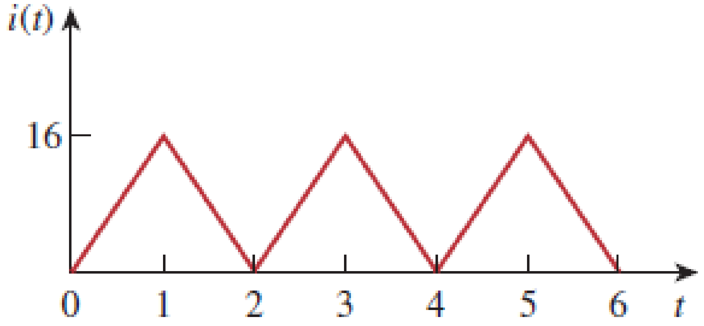


Figure 11.15
For Practice Prob. 11.7.

Reference: Sadiku_Practice_Problem_11.7

MATH PROBLEM PRACTICE:

Practice Problem 11·7

The period, $T = 2$

So,

$$i(t) = \begin{cases} 16t, & 0 < t < 1 \\ -16t + 32, & 1 < t < 2 \end{cases}$$

The rms value is,

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$= \sqrt{\frac{1}{2}\int_0^2 i^2\,dt}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 (16t)^2\,dt + \int_1^2 (-16t+32)^2\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left[\int_0^1 256t^2\,dt + \int_1^2 (256t^2 - 1024t + 1024)\,dt\right]}$$

$$= \sqrt{\frac{1}{2}\left[\left[\frac{256t^3}{3}\right]_0^1 + \left[\frac{256t^3}{3}\right]_1^2 - \left[\frac{1024t^2}{2}\right]_1^2 + \left[1024t\right]_1^2\right]}$$

$$= \sqrt{\frac{1}{2}\left(\frac{256}{3} + \frac{1792}{3} - 1536 + 1024\right)}$$

$$= \sqrt{\frac{1}{2} \times \frac{512}{3}} = \sqrt{\frac{256}{3}} = \frac{16}{\sqrt{3}}$$

$$= 9.238\,A$$

The power absorbed by $9\,\Omega$ resistor,

$$P = I_{rms}^2 R = \left(\frac{16}{\sqrt{3}}\right)^2 \times 9 = 768\,W$$

Reference: Sadiku_Practice_Problem_11.7

MATH PROBLEM PRACTICE:

Example 11.8

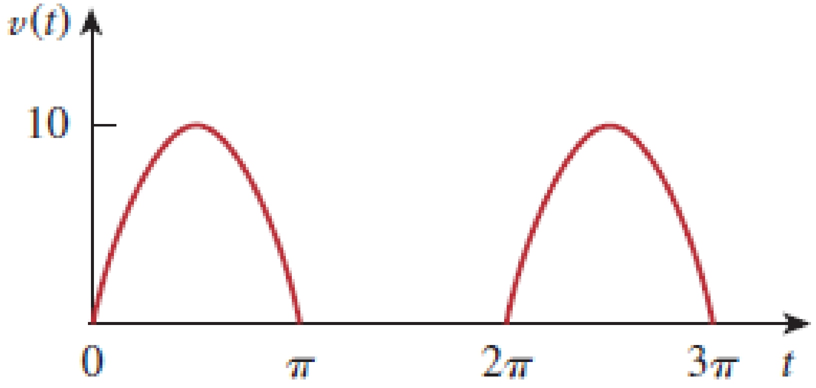


Figure 11.16
For Example 11.8.

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.

Reference: Sadiku_Example_11.8

MATH PROBLEM PRACTICE:

Example 11.8

The waveform shown in Fig. 11.16 is a half-wave rectified sine wave. Find the rms value and the amount of average power dissipated in a 10-Ω resistor.

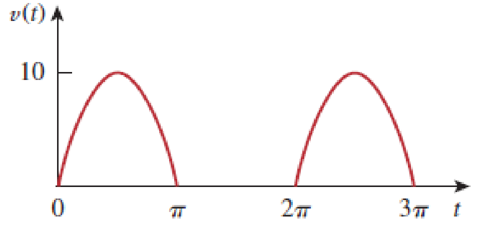


Figure 11.16
For Example 11.8.

Solution:

The period of the voltage waveform is $T = 2\pi$, and

$$v(t) = \begin{cases} 10 \sin t, & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{cases}$$

The rms value is obtained as

$$V_{\text{rms}}^2 = \frac{1}{T}\int_0^T v^2(t)\, dt = \frac{1}{2\pi}\left[\int_0^\pi (10 \sin t)^2\, dt + \int_\pi^{2\pi} 0^2\, dt\right]$$

But $\sin^2 t = \frac{1}{2}(1 - \cos 2t)$. Hence,

$$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^\pi \frac{100}{2}(1 - \cos 2t)\, dt = \frac{50}{2\pi}\left(t - \frac{\sin 2t}{2}\right)\Bigg|_0^\pi$$

$$= \frac{50}{2\pi}\left(\pi - \frac{1}{2}\sin 2\pi - 0\right) = 25, \qquad V_{\text{rms}} = 5\ \text{V}$$

The average power absorbed is

$$P = \frac{V_{\text{rms}}^2}{R} = \frac{5^2}{10} = 2.5\ \text{W}$$

Reference: Sadiku_Example_11.8

MATH PROBLEM PRACTICE:

Practice Problem 11.8

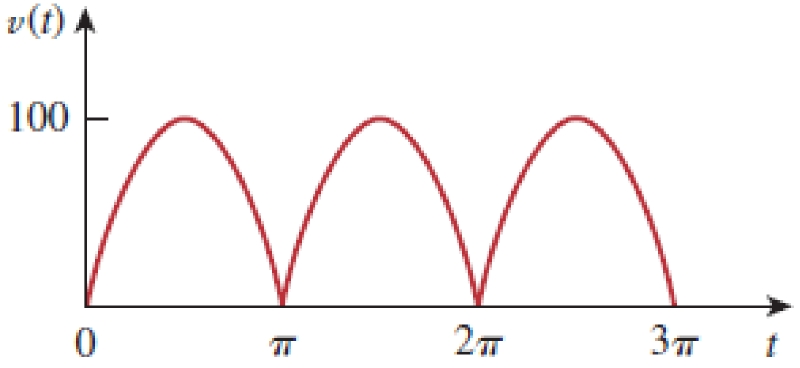


Figure 11.17
For Practice Prob. 11.8.

Find the rms value of the full-wave rectified sine wave in Fig. 11.17. Calculate the average power dissipated in a 6-Ω resistor.

Answer: 70.71 V, 833.3 W.

Reference: Sadiku_Practice_Problem_11.8

MATH PROBLEM PRACTICE:

Practice Problem 11.8

The period, $T = \pi$, $\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{\pi} = 2$

So,

$v(t) = 100\sin(\omega t) = 100\sin 2t$

The rms value is,

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2 dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^\pi (100\sin 2t)^2 dt}$$

$$= \sqrt{\frac{1}{\pi}\int_0^\pi 10000\sin^2 2t\, dt}$$

$$= \sqrt{\frac{10^4}{\pi}\int_0^\pi \frac{1}{2}(2\sin^2 2t)dt}$$

$$= \sqrt{\frac{10^4}{\pi}\int_0^\pi \frac{1}{2}(1-\cos 2t)dt}$$

$$= \sqrt{\frac{10^4}{\pi} \times \frac{1}{2}\left[[t]_0^\pi - \left[\frac{1}{4}\sin 2t\right]_0^\pi\right]}$$

$$= \sqrt{\frac{10^4}{\pi} \times \frac{1}{2}(\pi - 0)}$$

$$= \sqrt{\frac{10^4}{\pi} \times \frac{\pi}{2}} = \sqrt{5000}$$

$$= 50\sqrt{2} = 70.71V$$

The avg. power absorbed by 6Ω resistor,

$$P = \frac{V_{rms}^2}{R} = \frac{(50\sqrt{2})^2}{6} = 833.33\ W$$

<u>Reference: Sadiku_Practice_Problem_11.8</u>

MATH PROBLEM PRACTICE:

11.25 Find the rms value of the signal shown in Fig. 11.56.

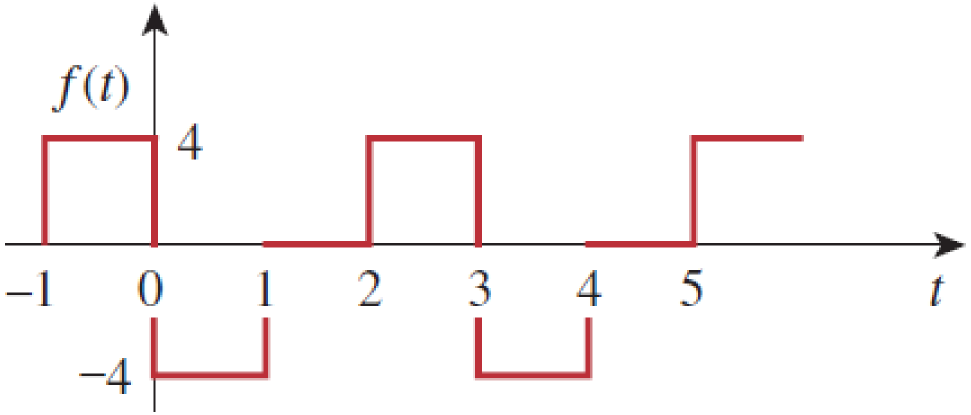


Figure 11.56
For Prob. 11.25.

Reference: Sadiku_Exercise_11.25

MATH PROBLEM PRACTICE:

Chapter 11, Solution 25.

$$f_{rms}^2 = \frac{1}{T}\int_0^T f^2(t)dt = \frac{1}{3}\left[\int_0^1 (-4)^2 dt + \int_1^2 0dt + \int_2^3 4^2 dt\right]$$

$$= \frac{1}{3}[16 + 0 + 16] = \frac{32}{3}$$

$$f_{rms} = \sqrt{\frac{32}{3}} = \underline{3.266}$$

$f_{rms} = \mathbf{3.266}$

Reference: Sadiku_Exercise_11.25

MATH PROBLEM PRACTICE:

11.26 Find the effective value of the voltage waveform in Fig. 11.57.

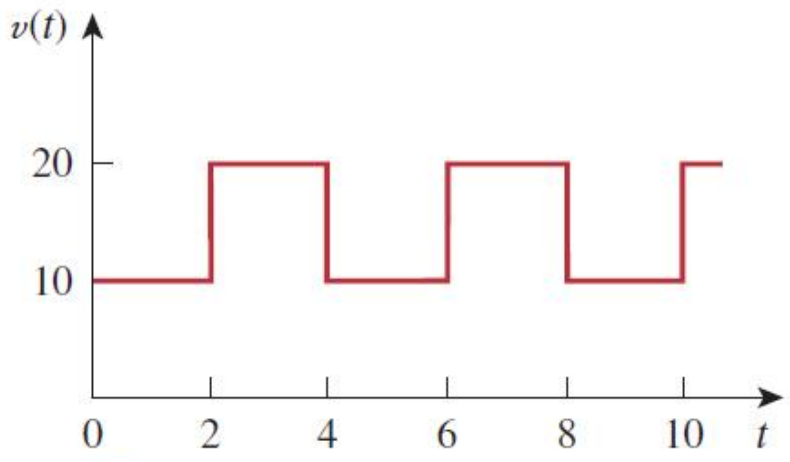


Figure 11.57
For Prob. 11.26.

Reference: Sadiku_Exercise_11.26

MATH PROBLEM PRACTICE:

Chapter 11, Solution 26.

$$T = 4, \qquad v(t) = \begin{cases} 5 & 0 < t < 2 \\ 20 & 2 < t < 4 \end{cases}$$

$$V_{rms}^2 = \frac{1}{4}\left[\int_0^2 10^2\, dt + \int_2^4 (20)^2\, dt \right] = \frac{1}{4}[200 + 800] = 250$$

$$V_{rms} = \mathbf{15.811\ V}.$$

Reference: Sadiku_Exercise_11.26

Fall 2023

For the circuit shown in **Figure 4a**, determine I_m if the rms value of such current is 5A. Now, determine i_o and average real power absorbed by a 3-ohm resistor using **CDR** in the circuit shown in **Figure 4b** if the angular frequency is 100 rad/s in the circuit.

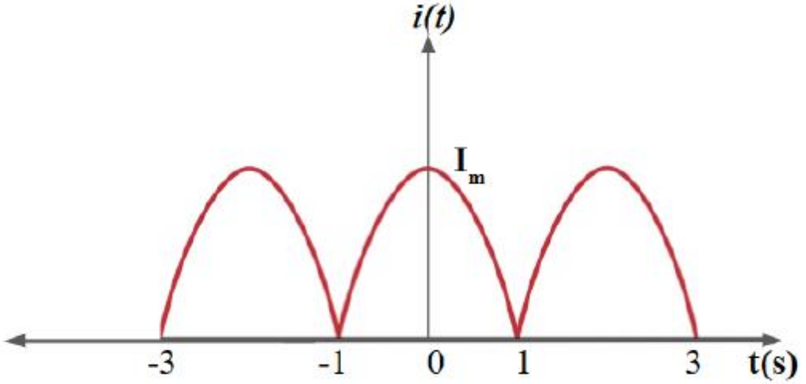


Figure 4a.

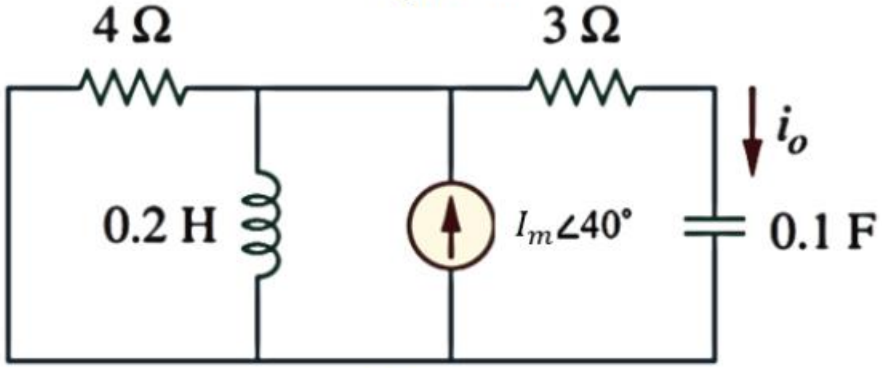


Figure 4b.

Fall 2023

Q4: waveform

$-1 \leq t \leq 1, \quad I_m \cos(\pi t)$

$T = 2$

$f = 0.5$

$\omega = 2\pi f = \pi$

$RMS = \sqrt{\frac{1}{T}\int i(t)^2 \, dt}$

$5 = \sqrt{\frac{1}{2}\int_{-1}^{1} I_m^2 \cos^2(\pi t)\, dt}$

$50 = I_m^2 \times 1$

$\boxed{I_m = 5\sqrt{2}}$

$\frac{1}{2}\int_{-1}^{1} 1 + \cos 2\pi t$

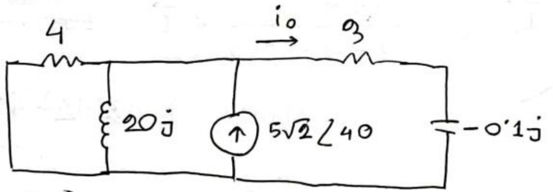
$\frac{1}{2}[1-(-1)]$

$+ \left[\frac{\sin 2\pi t}{2\pi}\right]_{-1}$

$= 1$

AC ckt

$\omega = 100$



$j\omega L$

$\frac{1}{j\omega C}$

$(4 \parallel 20j) = 3.84 + 0.77j$

$i_o = \frac{3.84 + 0.77j}{3.84 + 0.77j + 3 - 0.1j} \times 5\sqrt{2}\angle 40^\circ$

$\boxed{i_o = 4.03\angle 45.744}$

Considering Im as ~~rms~~ peak value

$\boxed{\text{Real } P_{avg} = \frac{1}{2}|I|^2 R = \frac{1}{2}\times(4.03)^2 \times 3 = 24.36\,W}$

Summer 2023

For the waveform shown in **Figure 4a**, determine the rms value of the current, i_{rms}. Also, determine the power absorbed by 5Ω resistance for the circuit shown in **Figure 4b**.

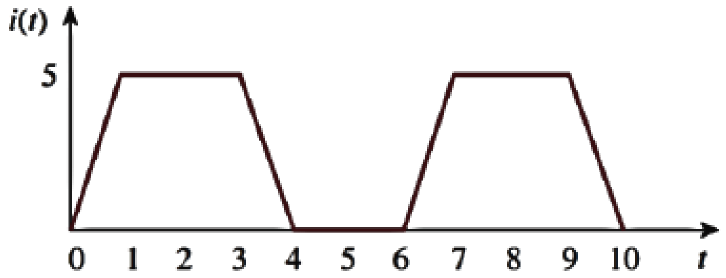


Figure 4a.

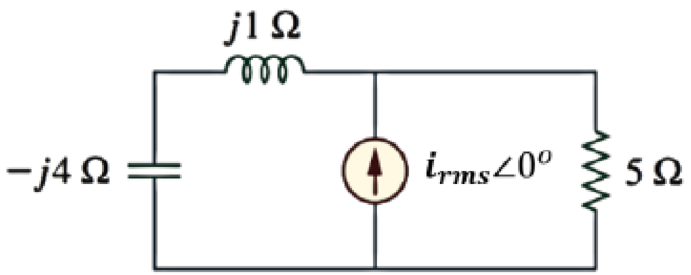


Figure 4b.

Summer 2023

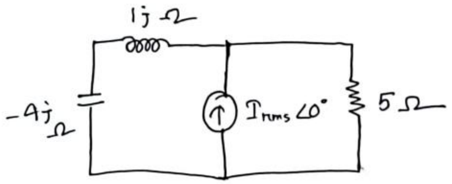
$$I_{rms}^2 = \frac{1}{T}\int_0^T i^2(t)dt = \frac{1}{6}\left[\int_0^1 25t^2 dt + \int_1^3 25dt + \int_3^4 (-5t+20)^2 dt\right]$$

$$I_{rms}^2 = \frac{1}{6}\left[25\frac{t^3}{3}\bigg|_0^1 + 25(3-1) + (25\frac{t^3}{3} - 100t^2 + 400t)\bigg|_3^4\right] = 11.1056$$

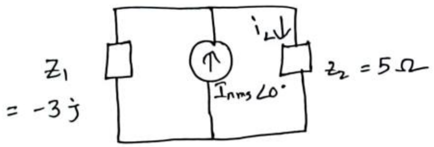
$$I_{\mathrm{rms}} = \mathbf{3.332\ A}$$

Summer 2023

$I_{rms} = 3.332\,A$



$\Downarrow$



As,
1j inductor
and -4j
capacitor are
in series

So, $Z_1 = 1j - 4j$
$= -3j$

$$i_2 = \frac{Z_1}{Z_1 + Z_2} \times I$$

$$= \frac{-3j}{-3j + 5} \times 3.332\angle 0^\circ$$

$$\therefore i_2 = 1.7143 \angle -59.036^\circ$$

$$P_{5\Omega} = \frac{1}{2} I_m^2 R = \frac{1}{2} \times (1.7143)^2 \times 5$$

$$= 7.347 \text{ watt}$$

REFERENCES FOR BETTER UNDERSTANDING

- **Links to go through in Power Calculation:**
  - RMS and average value concept:
    - What is RMS value? (& why should we care?) | Alternating currents | Physics | Khan Academy (youtube.com)
  - RMS value and power calculation math:
    - RMS (Effective) Value || Average Power || Example 11.7, 11.8 & Practice 11.7, 11.8 || (Bangla) (youtube.com)
    - RMS Value (Solved Problem) (youtube.com)
    - Complex Power (No more Confusion) || Example 11.11 || Practice Problem 11.11 || ENA 11.6 (B) (youtube.com)

REFERENCES FOR CALCULATOR USE FOR COMPLEX NUMBERS

- Complex Number calculations in calculator:
  - Complex Number Operations in Calculator (Bangla Tutorial) #ComplexNumberOperationsBanglaTutorial (youtube.com)
  - FX 991ES + Scientific Calculator Conversion of Rectangular to Polar & Polar to Rectangular (youtube.com)
  - Rectangular to Polar form in fx-991MS calculator (youtube.com)
  - Casio FX-991EX Classwiz Complex Numbers: Rectangular to Polar Form Conversions (and Vice Versa) (youtube.com)

MATH TO PRACTICE FROM THE BOOK FOR EXAM

- **Chapter 11**
  - **Example:**
    - 11.8
  - **Practice Problem:**
    - 11.8
  - **Problem:**
    - 11.25, 11.26

THANK YOU