Electrostatics

Course- PHY 2105 / PHY 105

Lecture 17

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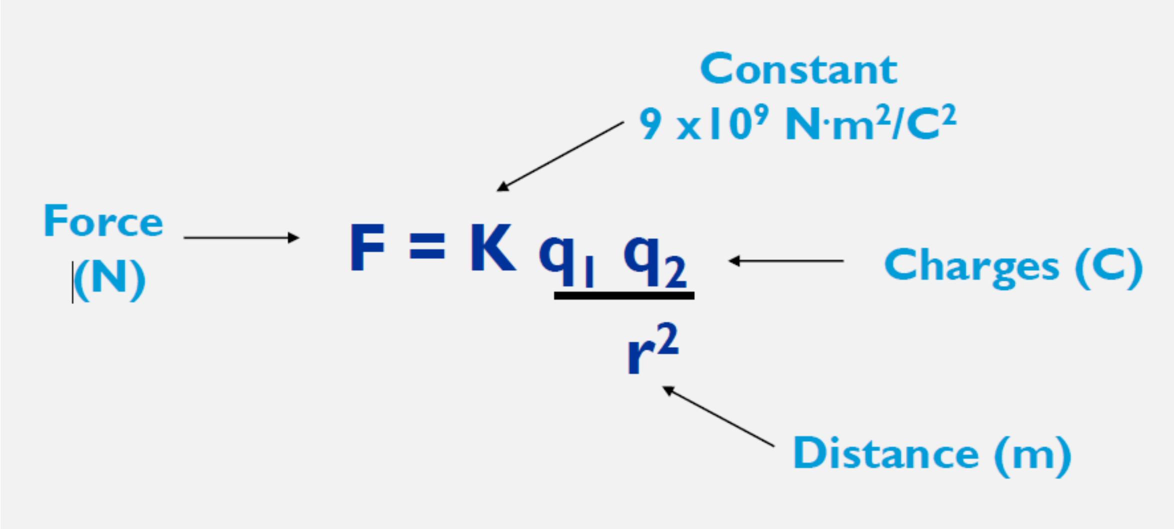
Coulomb's Law

The electrostatic force between two charged object is directly proportional to the product of the amount of charges and inversely proportional to the square of the distance between them



$$k = \frac{1}{4\pi\varepsilon_0}$$

- *Experimental law*
- *Valid for point charges only*
- *Obeys Inverse Square Law*
- *Valid for only charges at rest*

$$Electrostatic\ constant, k = 9 \times 10^9 \ \frac{Nm^2}{C^2}$$

$$Permittivity\ constant, \varepsilon_0 = 8.854 \times 10^{-12} \ \frac{C^2}{Nm^2}$$

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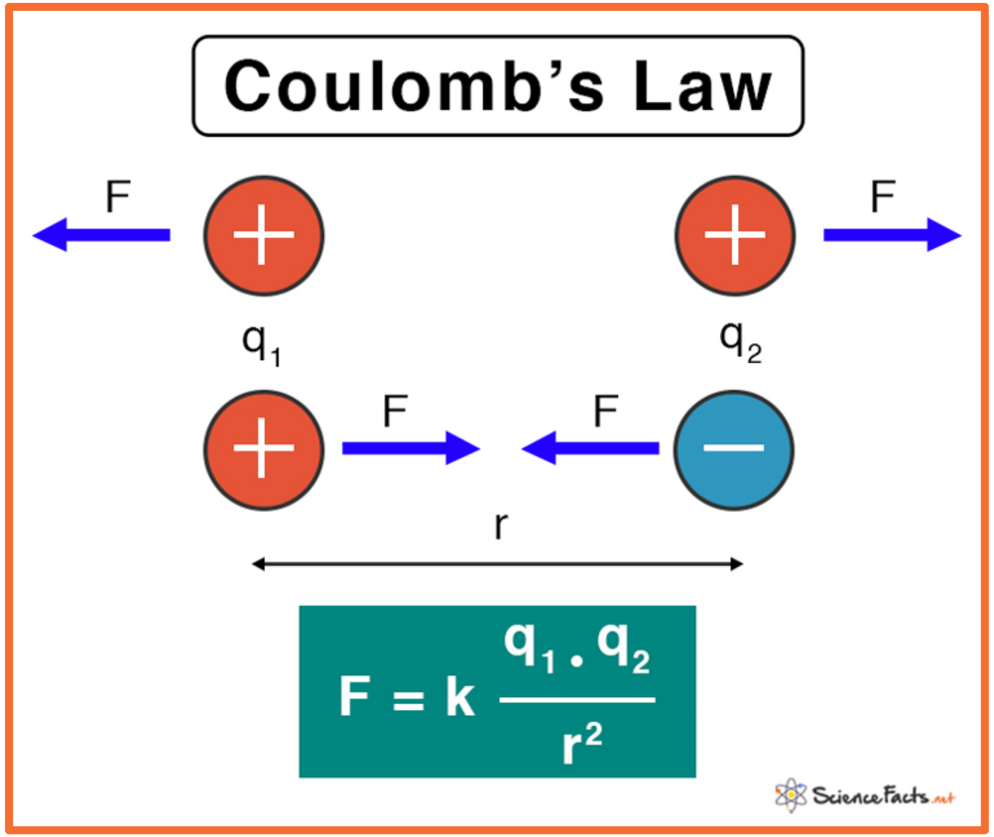
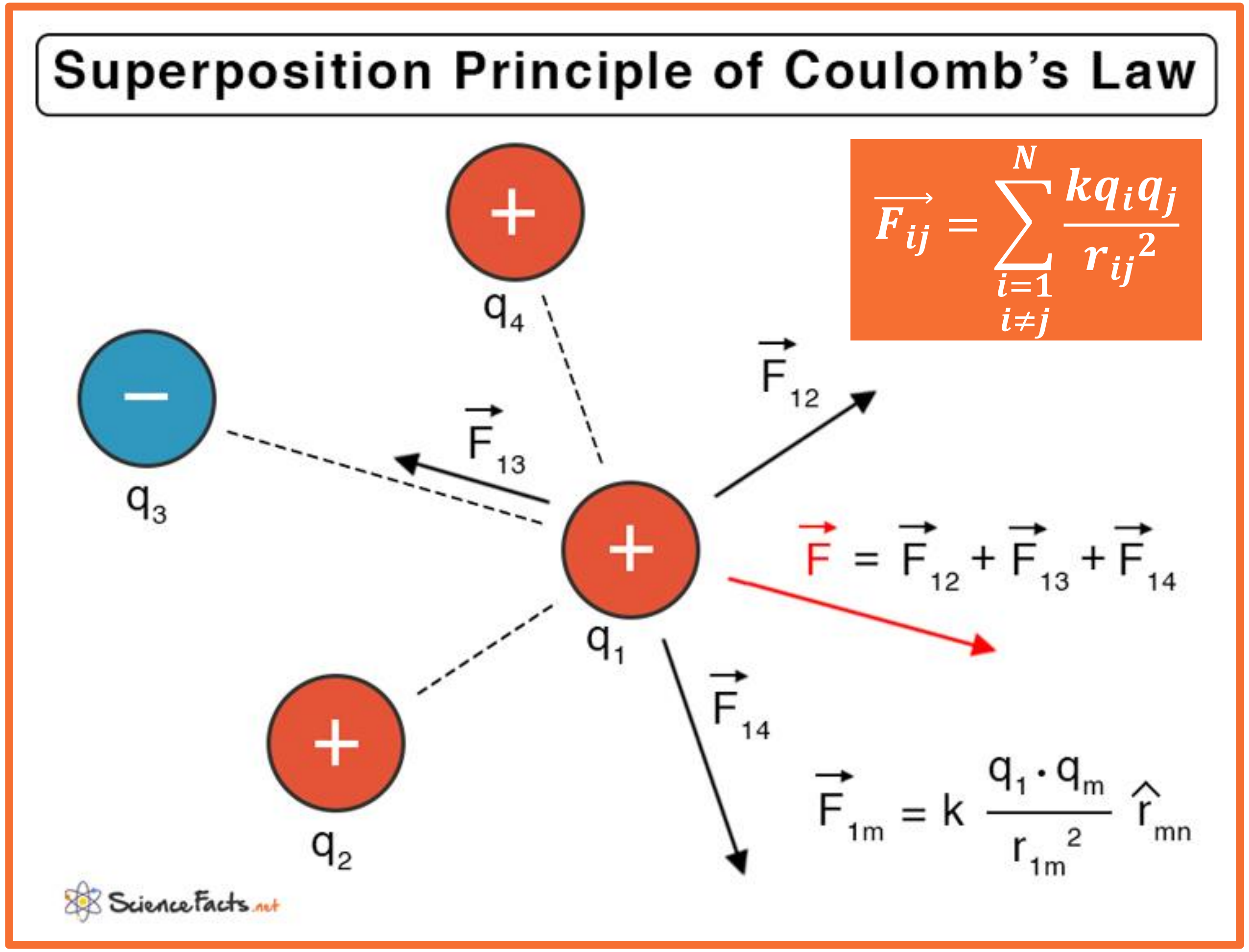
Coulomb’s Law: Superposition

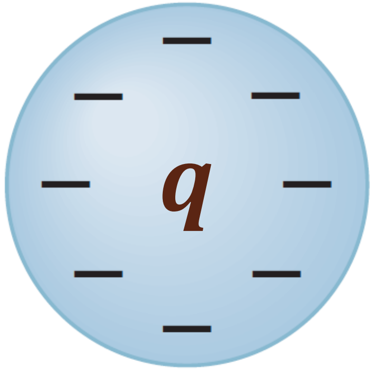




Electric Field

$$\boxed{F = q_0 E}$$

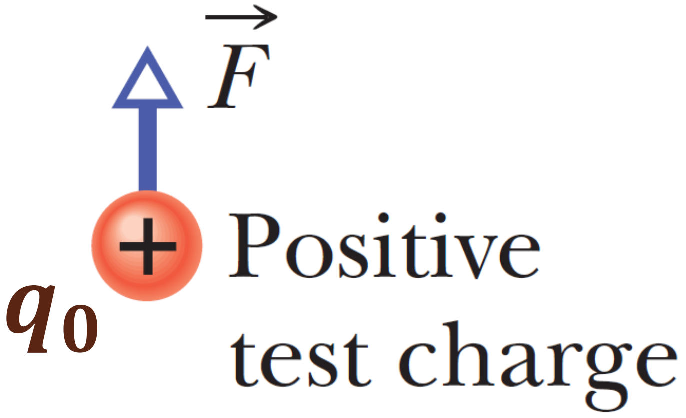
A charge has an effect on its surroundings. The area where it has an effect is generally called an *Electric field.* If any other charge enters that area, it feels an electrostatic Coulomb force.



$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}}$$

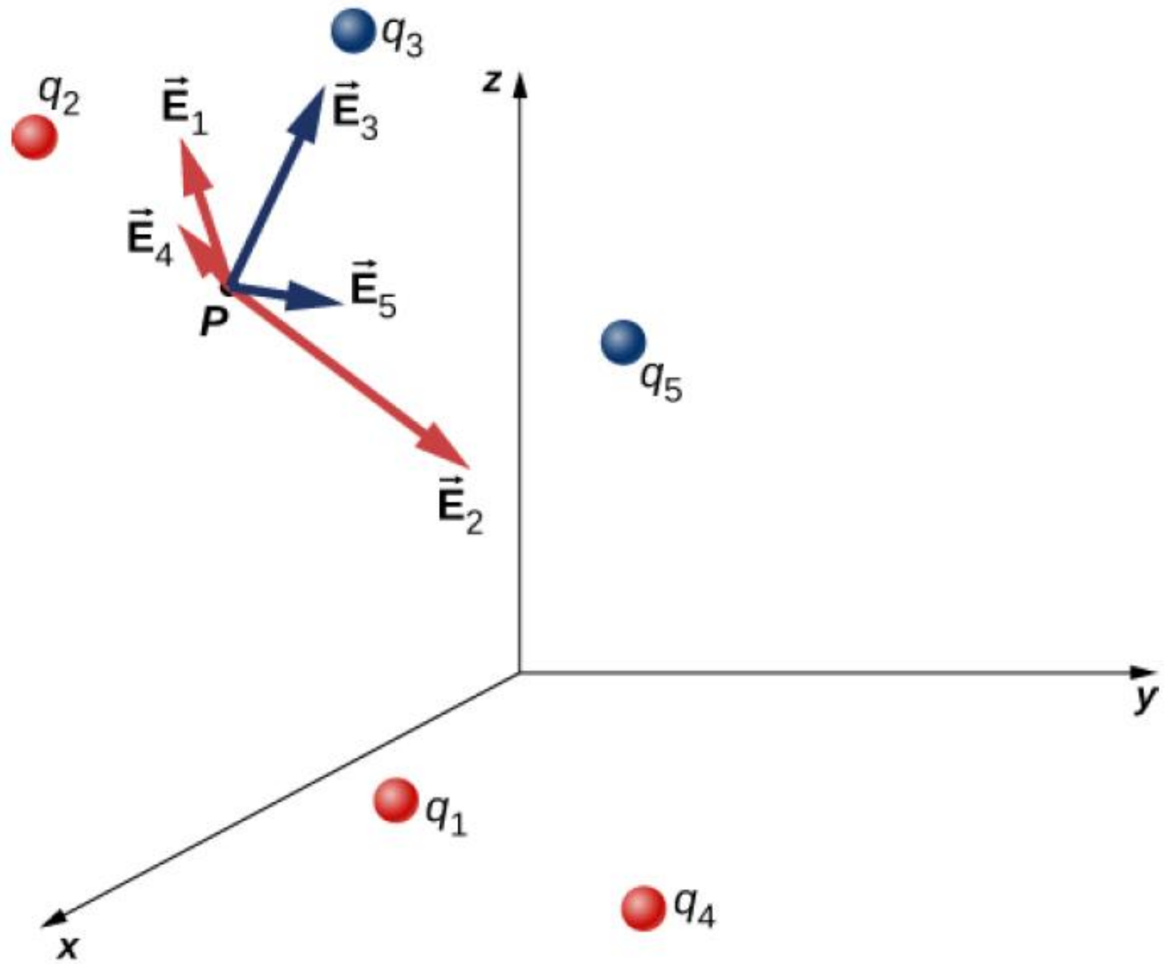
for point test charges only

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.



Superposition of Electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\,\hat{r}$$



- ❑ Treat electric field as a vector quantity
- ❑ q is source charge
- ❑ The test charge is positive

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Electric field due to a dipole

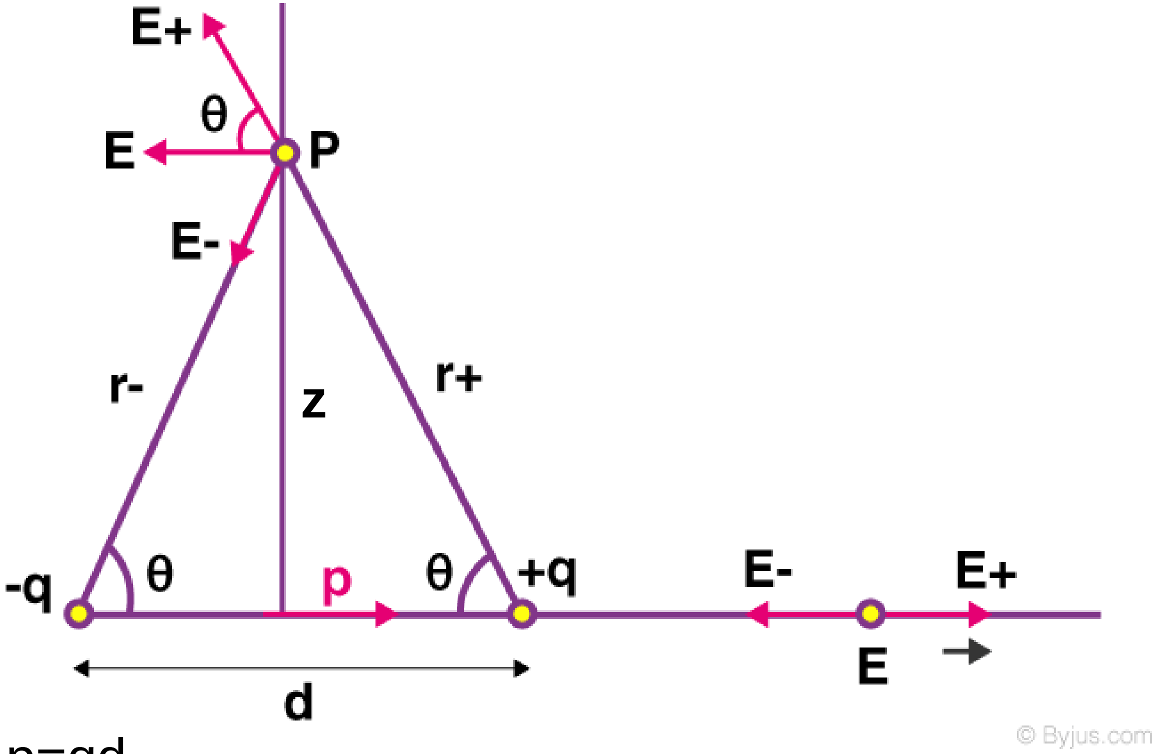
Pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*

At any point

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{z^3}}$$

Along the dipole axis

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{z^3}}$$



Where, dipole moment, p=qd

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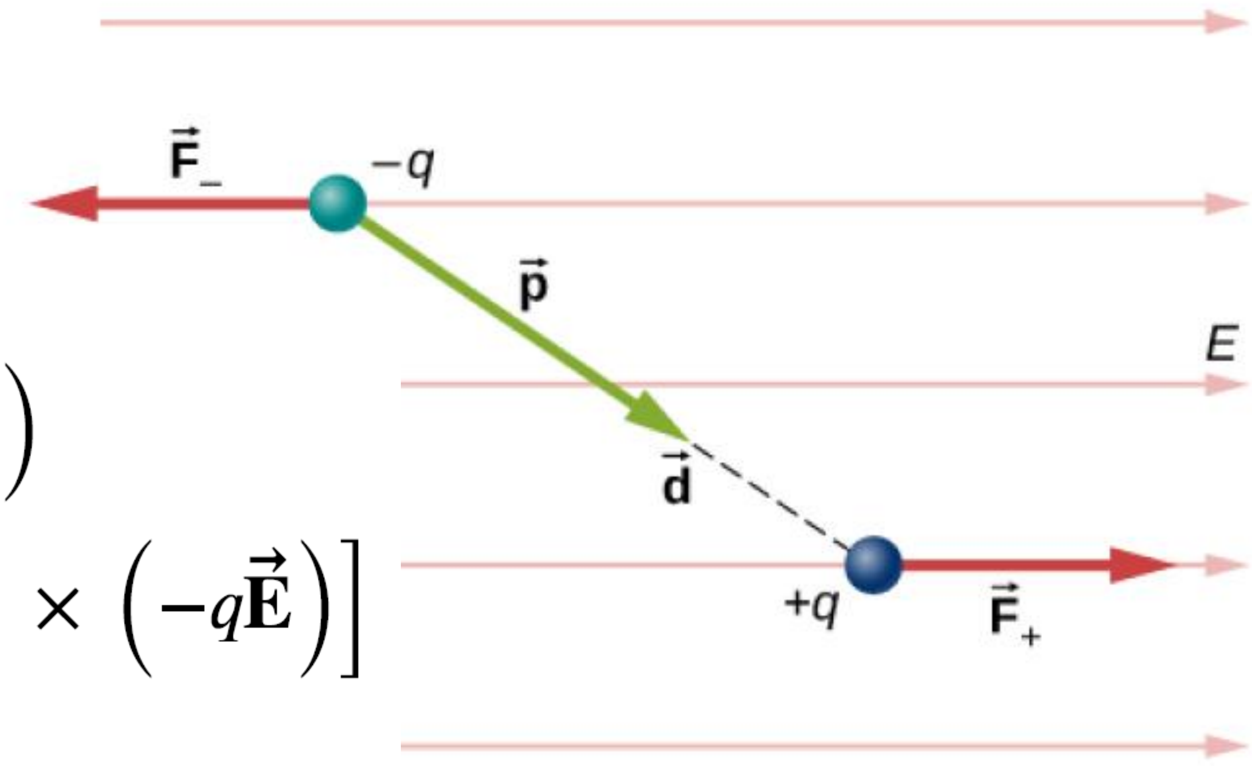
Torque of a dipole

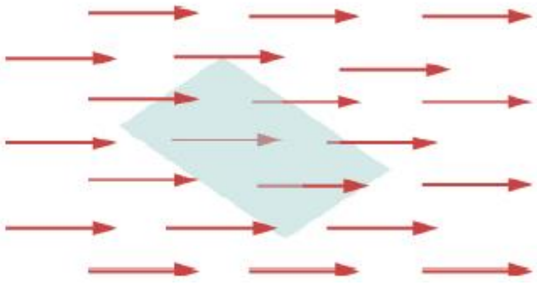
$$\begin{aligned}\vec{\tau} &= \left(\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_+\right) + \left(-\frac{\vec{\mathbf{d}}}{2} \times \vec{\mathbf{F}}_-\right) \\ &= \left[\left(\frac{\vec{\mathbf{d}}}{2}\right) \times \left(+q\vec{\mathbf{E}}\right) + \left(-\frac{\vec{\mathbf{d}}}{2}\right) \times \left(-q\vec{\mathbf{E}}\right)\right] \\ &= q\vec{\mathbf{d}} \times \vec{\mathbf{E}}.\end{aligned}$$

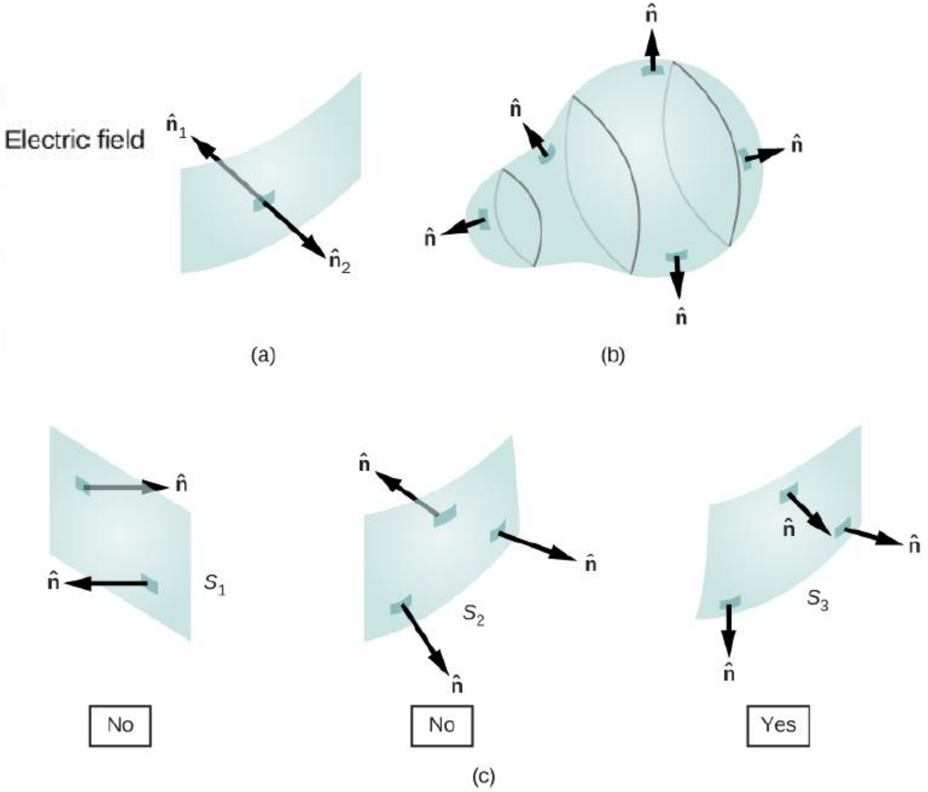


Electric Flux

The concept of **flux** describes how much of something goes through a given area.

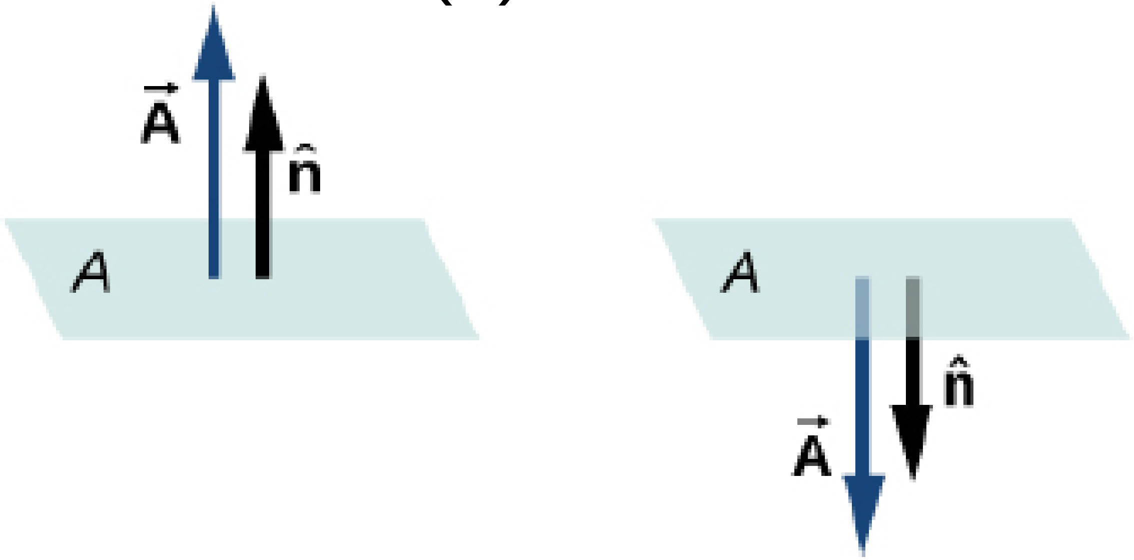
The flux of an electric field as a measure of the number of electric field lines passing through an area





The **area vector** of a flat surface of area A has the following magnitude and direction:

- ❑ Magnitude is equal to area (A)
- ❑ Direction is along the normal to the surface ($\hat{n}$); that is, **perpendicular** to the surface



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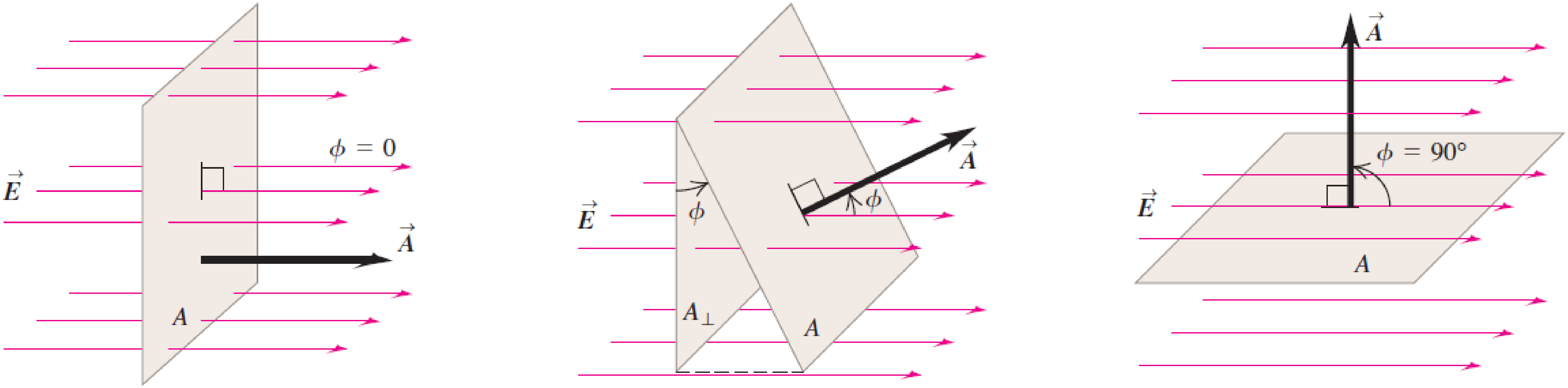
(a) Surface is face-on to electric field:
- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.

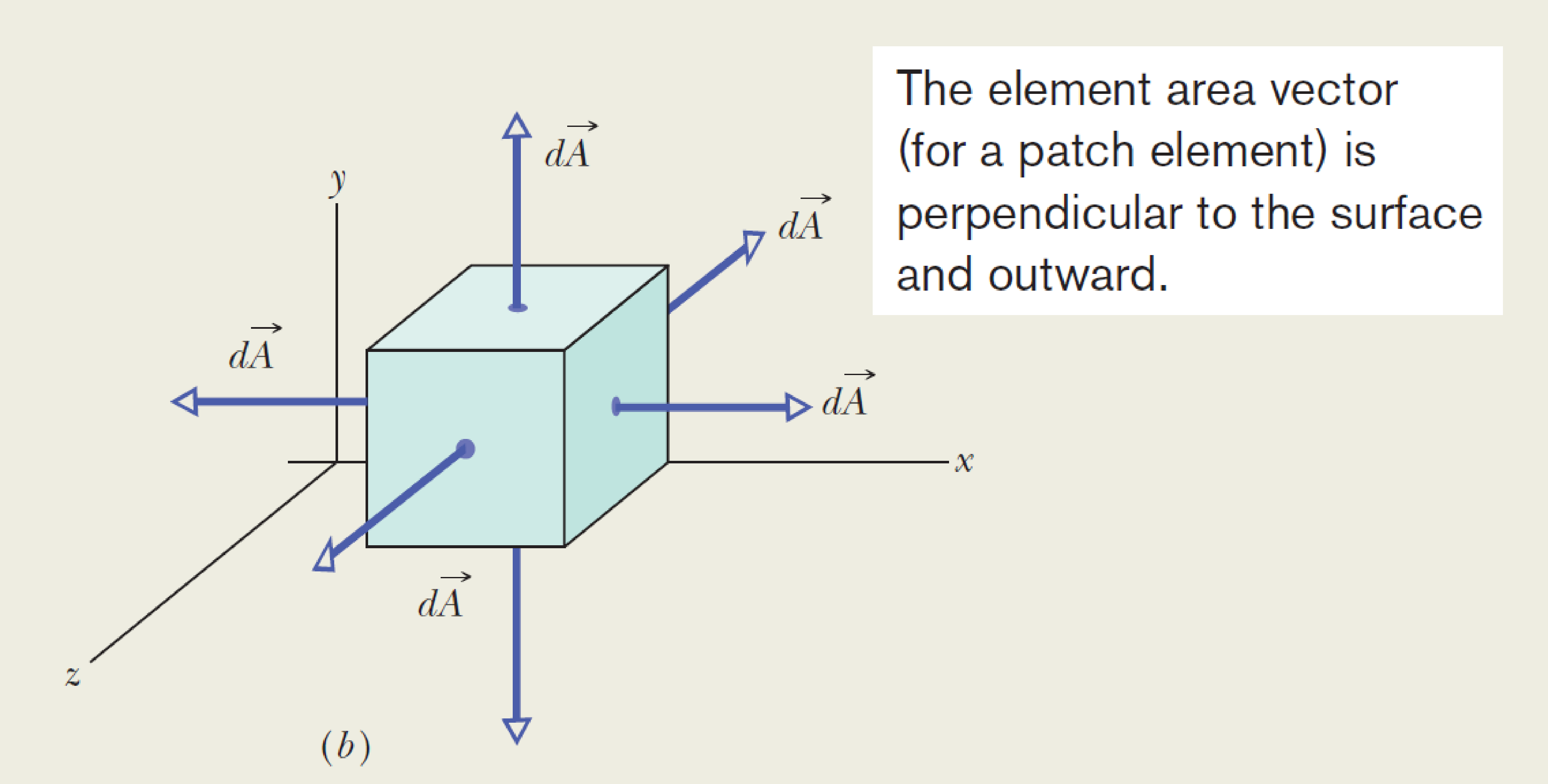
(b) Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.

(c) Surface is edge-on to electric field:
- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.



$$\boxed{\varphi = \vec{E}.\hat{n}A = EA\,cos\varphi}$$

Electric Flux through a Cube



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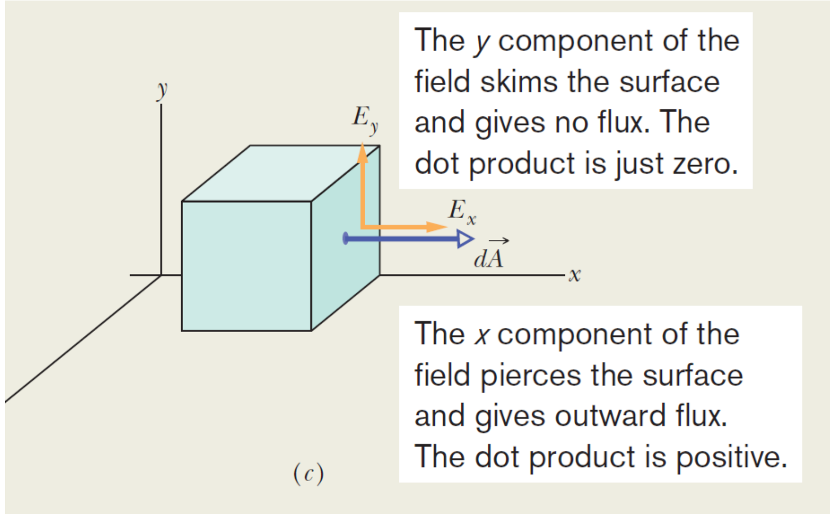
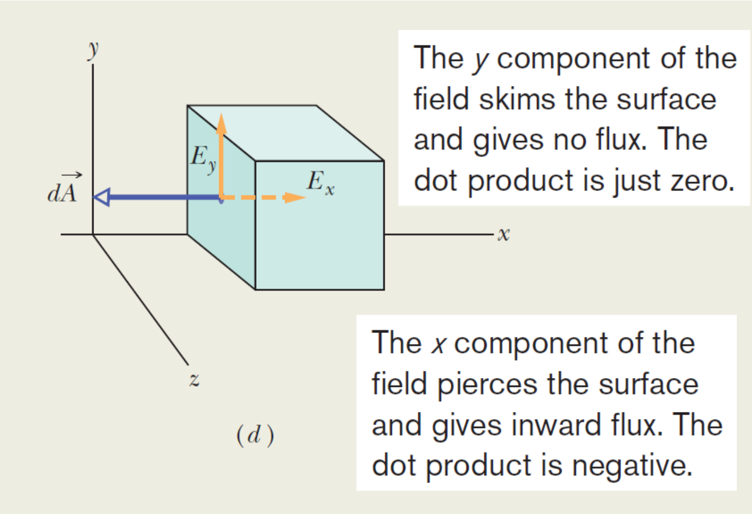
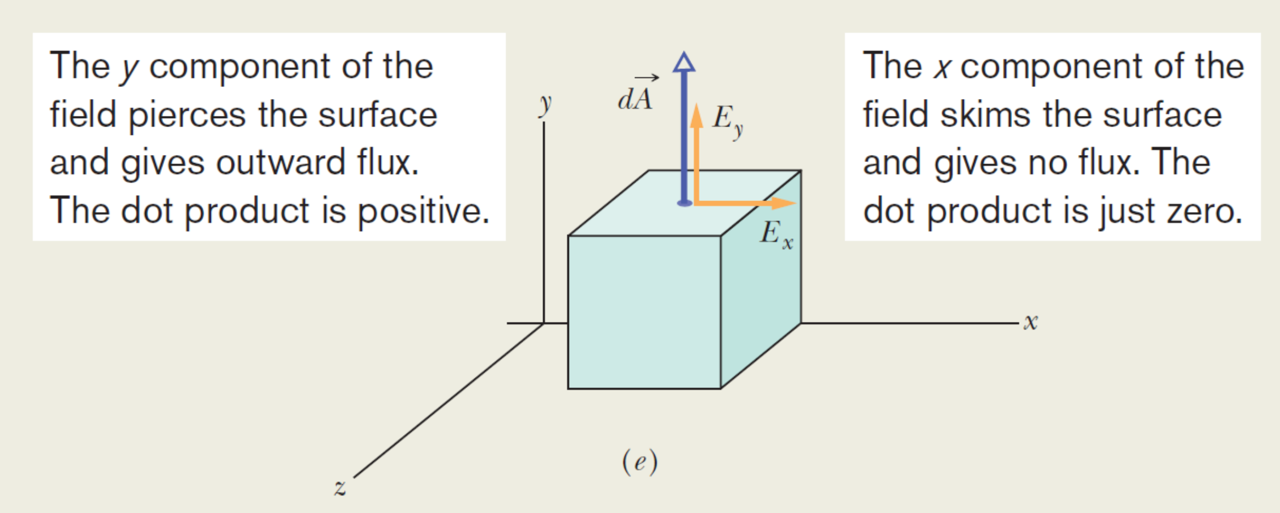
Electric Flux through a Cube







Flux in a closed surface

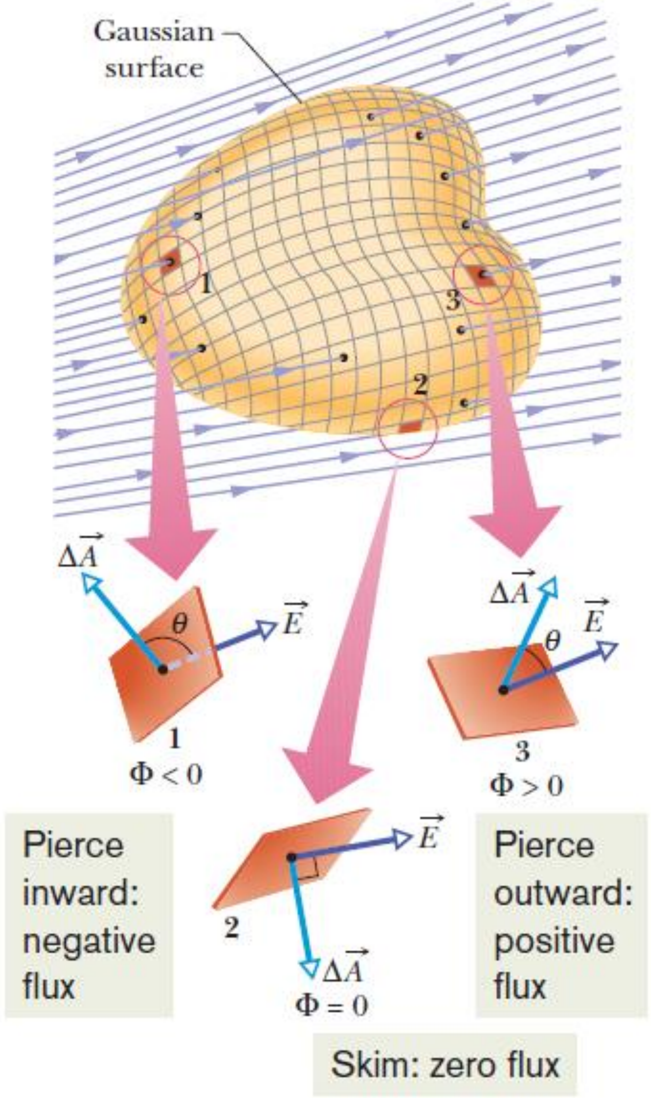
- An inward piercing field is negative flux.
- An outward piercing field is positive flux.
- A skimming field is zero flux.

$$\boxed{\boldsymbol{\Phi} = \sum \vec{E}.\Delta\vec{A} = \int \vec{E}.d\vec{A}}$$

Total flux

$$\boxed{\boldsymbol{\Phi} = \oint \vec{E}.d\vec{A}}$$

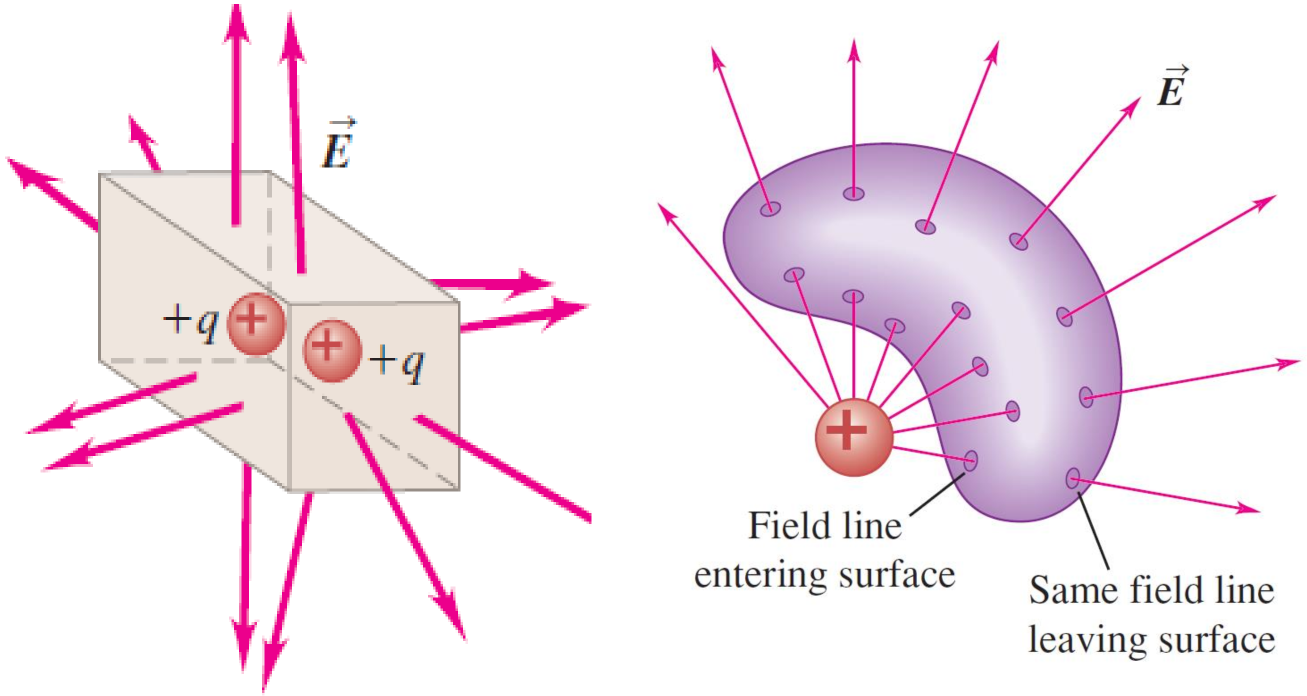
Net flux in a closed surface



Gauss's Law

The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ϵ_0.

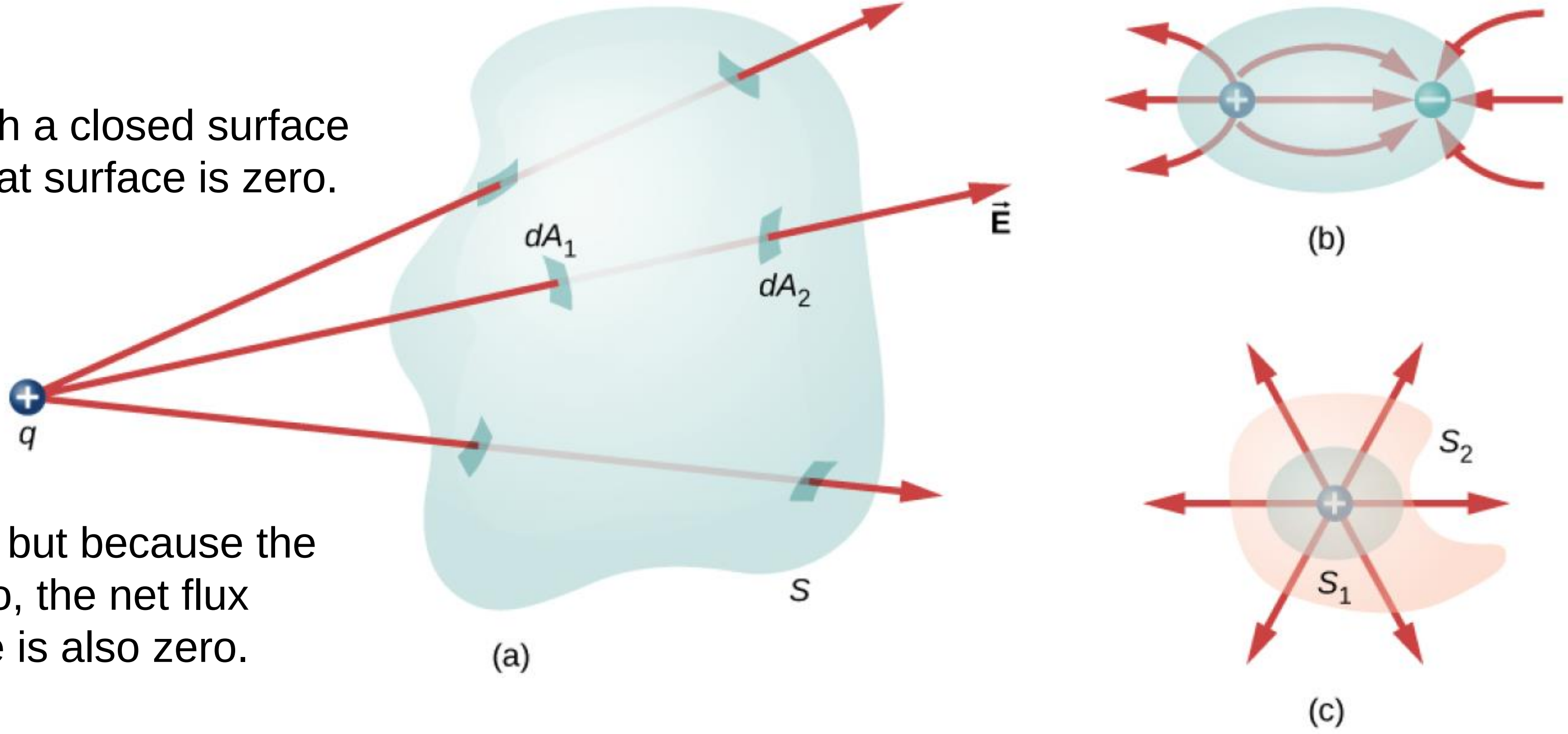
$$\boxed{\boldsymbol{\Phi} = \oint \vec{E}.d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}}$$



Flux and Field Lines

(a) The electric flux through a closed surface due to a charge outside that surface is zero.

(b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero.



(c) The shape and size of the surfaces that enclose a charge does not matter because all surfaces enclosing the same charge have the same flux.

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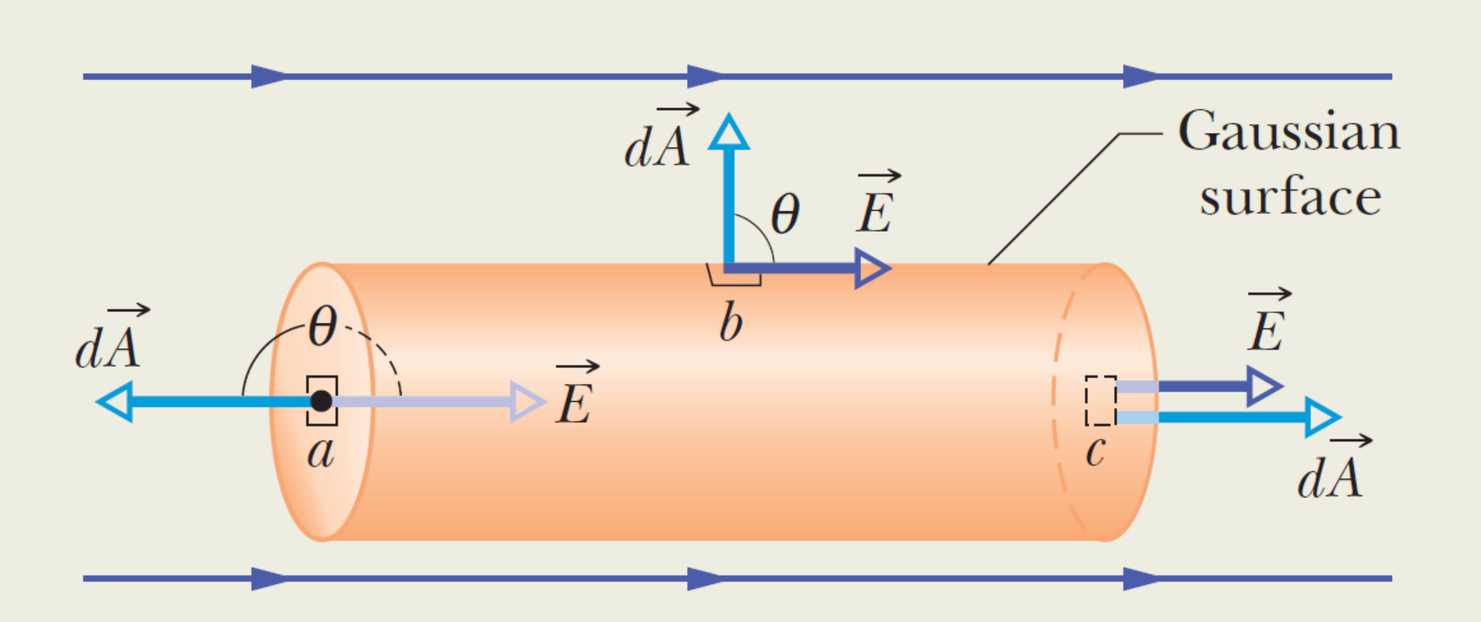
Flux through a cylinder



$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}.$$

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180^\circ)\, dA = -E \int dA = -EA,$$

where $\int dA$ gives the cap's area A $(= \pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90^\circ)\, dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

The net flux is zero because the field lines that represent the electric field all pass entirely through the Gaussian surface, from the left to the right.

Gauss' Law and Coulomb's Law

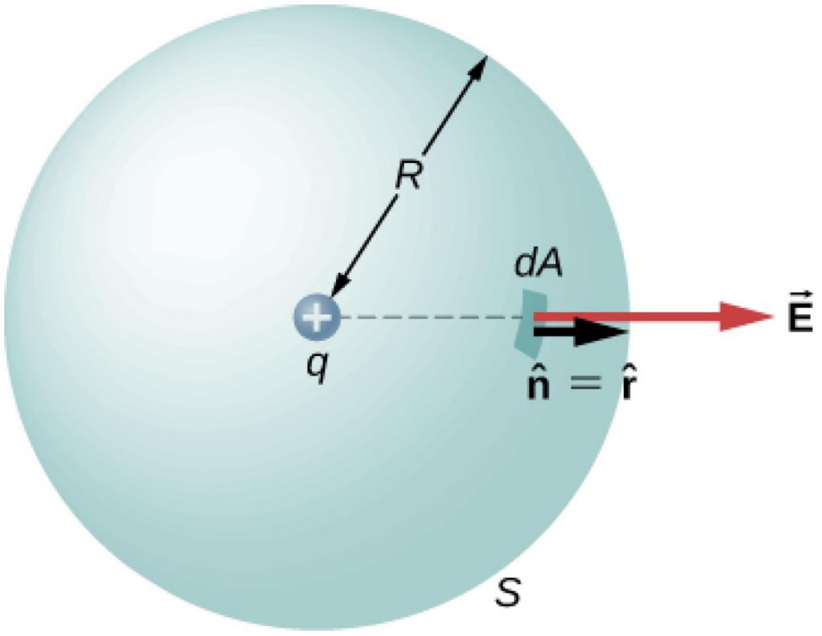


Figure 23-9 A spherical Gaussian surface centered on a particle with charge q.

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{enc}. \tag{23-8}$$

Here $q_{enc} = q$. Because the field magnitude E is the same at every patch element, E can be pulled outside the integral:

$$\varepsilon_0 E \oint dA = q. \tag{23-9}$$

The remaining integral is just an instruction to sum all the areas of the patch elements on the sphere, but we already know that the total area is $4\pi r^2$. Substituting this, we have

$$\varepsilon_0 E(4\pi r^2) = q$$

or

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}. \tag{23-10}$$

This is exactly Eq. 22-3, which we found using Coulomb's law.

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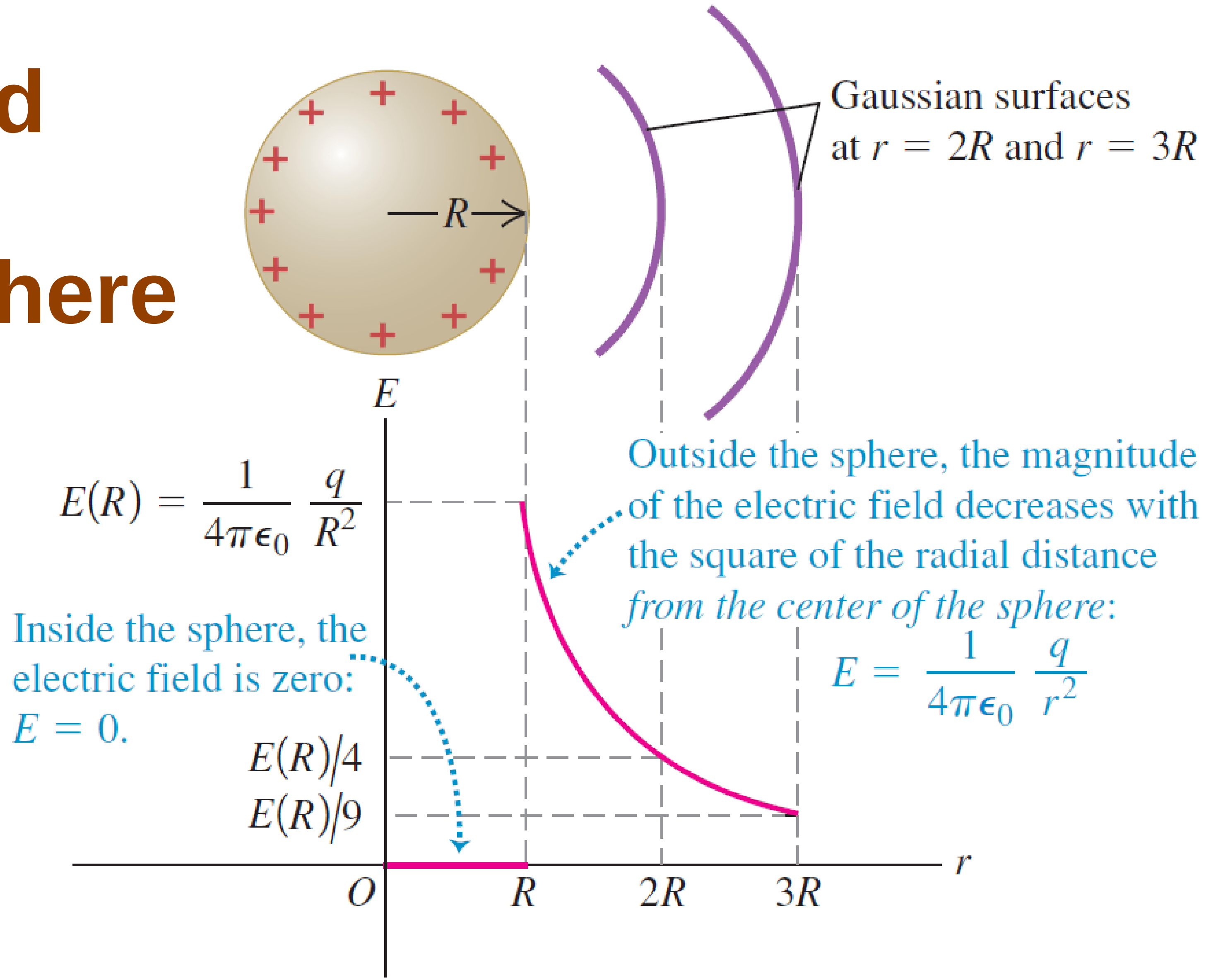
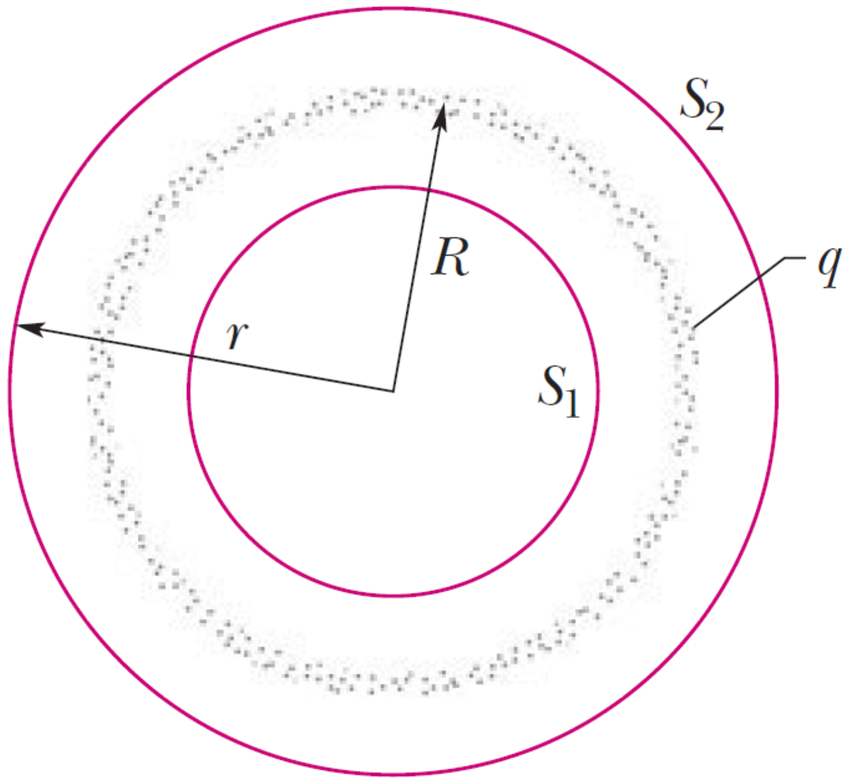
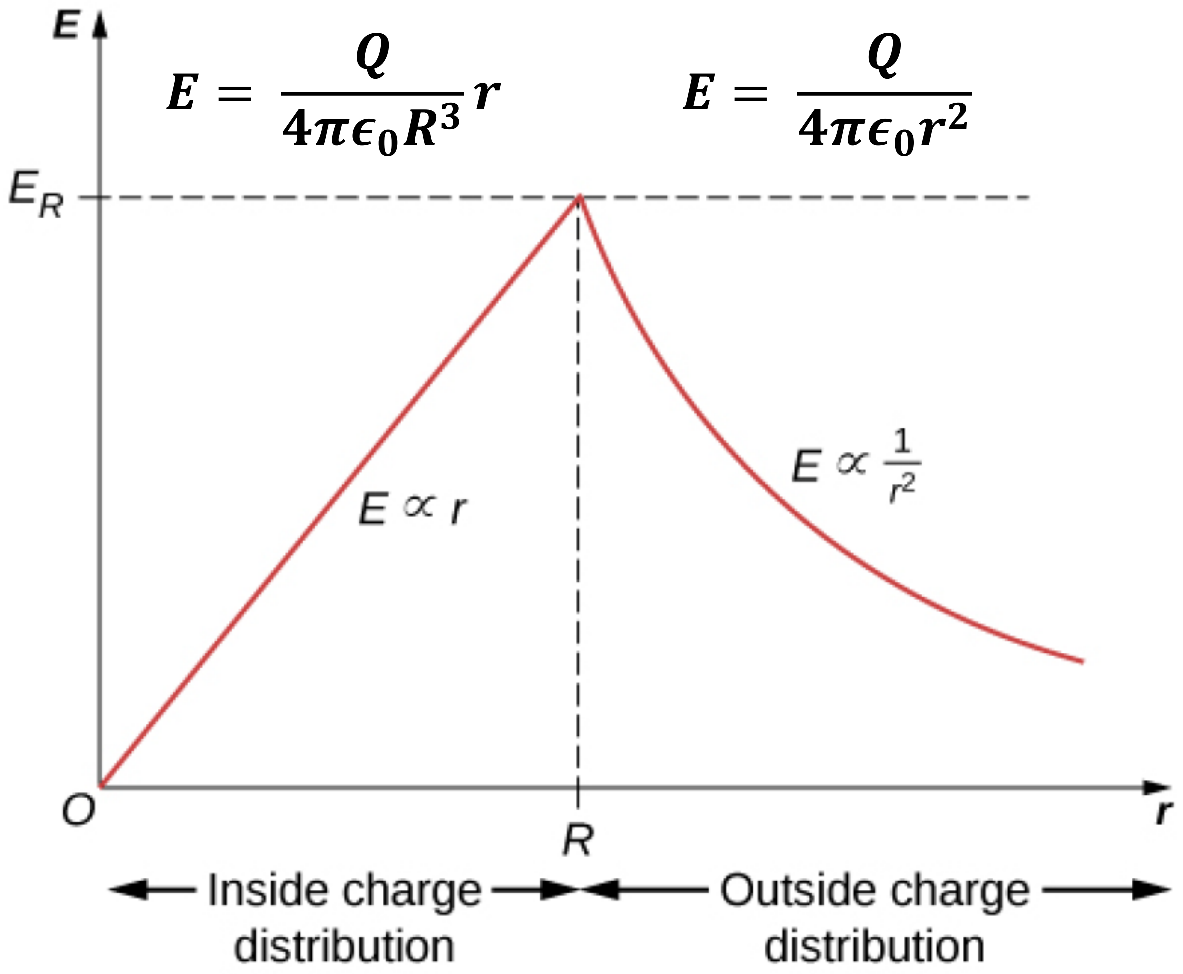
Electric field around any charged sphere



E for a uniformly charged sphere



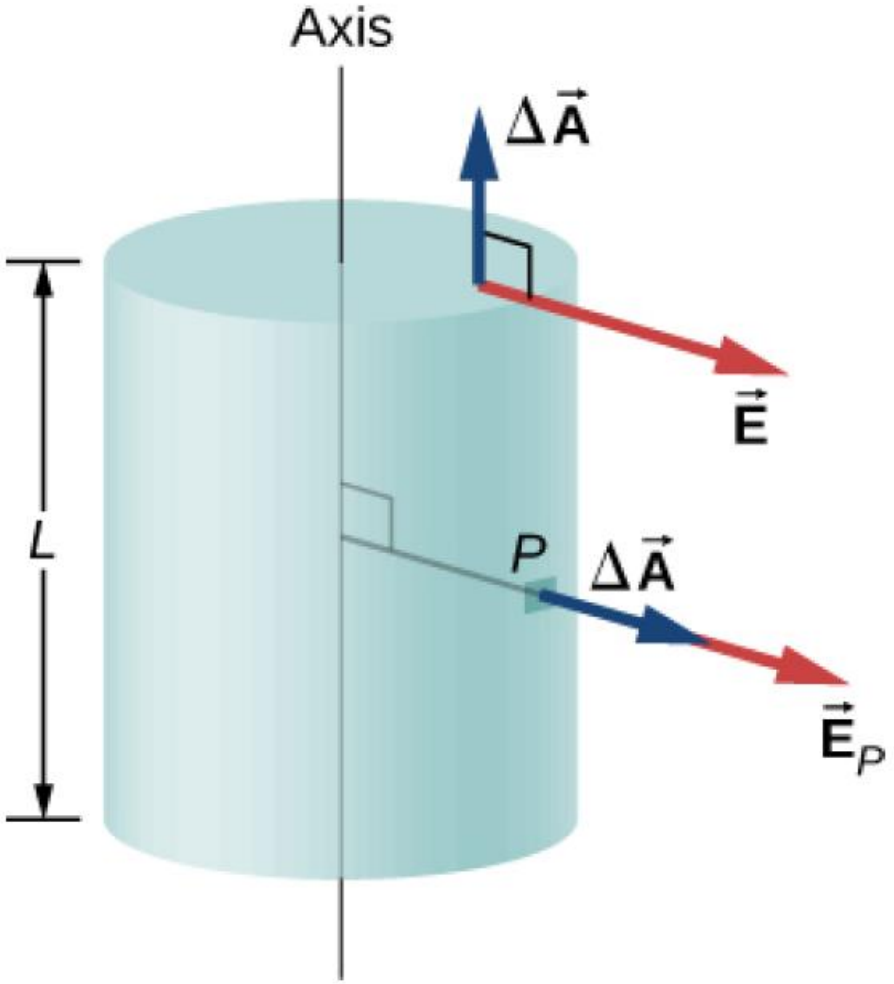


Electric field for a cylinder

$$\lambda = \frac{q_{enc}}{L}$$

$$\boxed{\text{Magnitude: } E(r) = \frac{\lambda_{\text{enc}}}{2\pi\varepsilon_0}\frac{1}{r}.}$$

$$\therefore \boldsymbol{E} \propto \frac{\mathbf{1}}{\boldsymbol{r}}$$



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