Areas of Theory of computation

"What are the fundamental capabilities and limitations of computers?"

1. AUTOMATA,
2. COMPUTABILITY, AND
3. COMPLEXITY

Areas of Theory of computation (Complexity)

What makes some problems computationally hard and others easy?

- ➔ *Sorting elements*
- ➔ *Scheduling program (class schedule)*

➔ *Objective is to classify problems as easy ones and hard ones.*

Areas of Theory of computation (Computability)

Can a computer determine whether a mathematical statement is true or false?

➔ *The classification of problems is whether they are solvable or not.*

Areas of Theory of computation (Automata)

- *Deals with the definitions and properties of mathematical models of computation*
  - *Finite automaton,*
  - *Context-free grammar*

Mathematical Notions And Terminology

- ***SETS***
  A set is a group of objects represented as a unit. Sets may contain any type of object, including numbers, symbols, and even other sets. The objects in a set are called its elements or members

$$S = \{7, 21, 57\}$$

Mathematical Notions And Terminology

- ***SEQUENCES AND TUPLES***

  *A **sequence** of objects is a list of these objects in some order. Written as list within parentheses.*

  For example, the sequence 7, 21, 57 would be written as,

  (7, 21, 57)

  *The order doesn't matter in a **set**, but in a **sequence** it does.*

Mathematical Notions And Terminology

➔ ***SEQUENCES AND TUPLES (Continued...)***
Sequences *may be finite or infinite. Finite sequences often are called* ***tuples****. A sequence with k elements is a* ***k-tuple.***

Example: (7, 21, 57) is a ***3-tuple.***

Mathematical Notions And Terminology

- ***FUNCTIONS AND RELATIONS***
  A ***function*** *is an object that sets up an input–output relationship. A function takes an input and produces an output. A function also is called a* ***mapping.***

  $$\boldsymbol{f(a) = b}, \textit{\textbf{ read as f maps a to b}}$$

- *The set of possible inputs to the function is called its* ***domain****. The outputs of a function come from a set called its* ***range****.*

  $f : D \rightarrow R$; **D** *for domain, R for range*

  $$f : \{0, 1, 2, 3, 4\} \rightarrow \{0, 1, 2, 3, 4\}$$

Mathematical Notions And Terminology

➔ ***FUNCTIONS AND RELATIONS (Continued...)***
*A **predicate** or **property** is a function whose range is {TRUE, FALSE}. A **property** whose **domain** is a set of **k-tuples** is called a **relation**.*
*For example: **beats** is a relation.*

beats	**SCISSORS**	**PAPER**	**STONE**
SCISSORS	FALSE	TRUE	FALSE
PAPER	FALSE	FALSE	TRUE
STONE	TRUE	FALSE	FALSE

Mathematical Notions And Terminology

- ***GRAPHS***

  *An **undirected graph**, or simply a **graph**, is a set of points with lines connecting some of the points. The points are called nodes or vertices, and the lines are called edges.*

  *A **directed graph** has arrows instead of lines.*
  - *outdegree*
  - *indegree.*

Mathematical Notions And Terminology

➔ ***STRINGS AND LANGUAGES***

Alphabet *is any nonempty finite set. To designate alphabets* $\boldsymbol{\Sigma}$ *and* $\boldsymbol{\Gamma}$ *are used.*

$\Sigma_1 = \{0,1\}$ *// binary alphabet*

$\Sigma_2 = \{a,b,c,d,e,\ldots\ldots\ldots, z\}$ *// English alphabet*

$\Sigma_3 = \{0,1,2,3,\ldots\ldots\ldots, 9\}$ *// Decimal number alphabet*

Mathematical Notions And Terminology

- ***STRINGS AND LANGUAGES*** *(Continued)*
  *A **string** over an alphabet is a finite sequence of symbols from that **alphabet**, usually written next to one another and not separated by commas.*

  Example of a string of alphabet Σ_1 is **0, 1, 11, 10, 01001, ……**
  Example of a string of alphabet Σ_2 is ***abcd, tree, red, green, xyz, ……***
  Example of a string of alphabet Σ_3 is ***0, 1, 12, 13, 1234, 789 ……***

Mathematical Notions And Terminology

➔ ***STRINGS AND LANGUAGES (Continued)***
Empty string: *string with zero occurrence of symbols. That means length of the string is zero.*

The empty string is denoted by the $\boldsymbol{\varepsilon}$ *(epsilon)*

Length of string '10001' is = |10001| = 5

Mathematical Notions And Terminology

➔ ***STRINGS AND LANGUAGES (Continued)***

*A **set of strings** all of which are chosen from some* Σ^**, where* Σ *is a particular alphabet, called **language**.*

Mathematical Notions And Terminology

➔ ***DEFINITIONS, THEOREMS, AND PROOFS***

Definitions *describe the objects and notions that we use.*

*A **proof** is a convincing logical argument that a statement is true.*

*A **theorem** is a mathematical statement proved true.*

Mathematical Notions And Terminology

➔ ***TYPES OF PROOF***

- ✓ ***proof by construction***
- ✓ ***proof by contradiction (****assume statement as false)*
- ✓ ***proof by induction*** *(basis and induction step)*