Electrostatics

Course- PHY 2105 / PHY 105

Lecture 15

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Coulomb's Law

The electrostatic force between two charged object is directly proportional to the product of the amount of charges and inversely proportional to the square of the distance between them

Constant
$9 \times 10^9\ N{\cdot}m^2/C^2$

Force
(N)

$$F = K\frac{q_1\ q_2}{r^2}$$

Charges (C)

Distance (m)

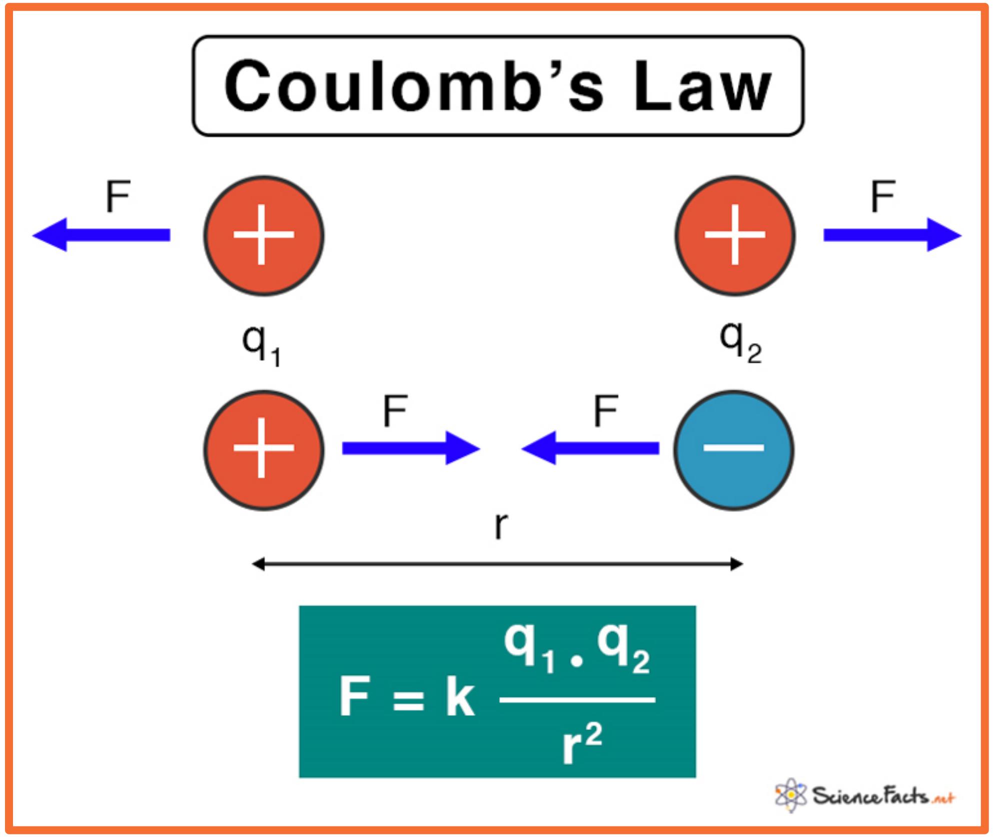
$$k = \frac{1}{4\pi\varepsilon_0}$$

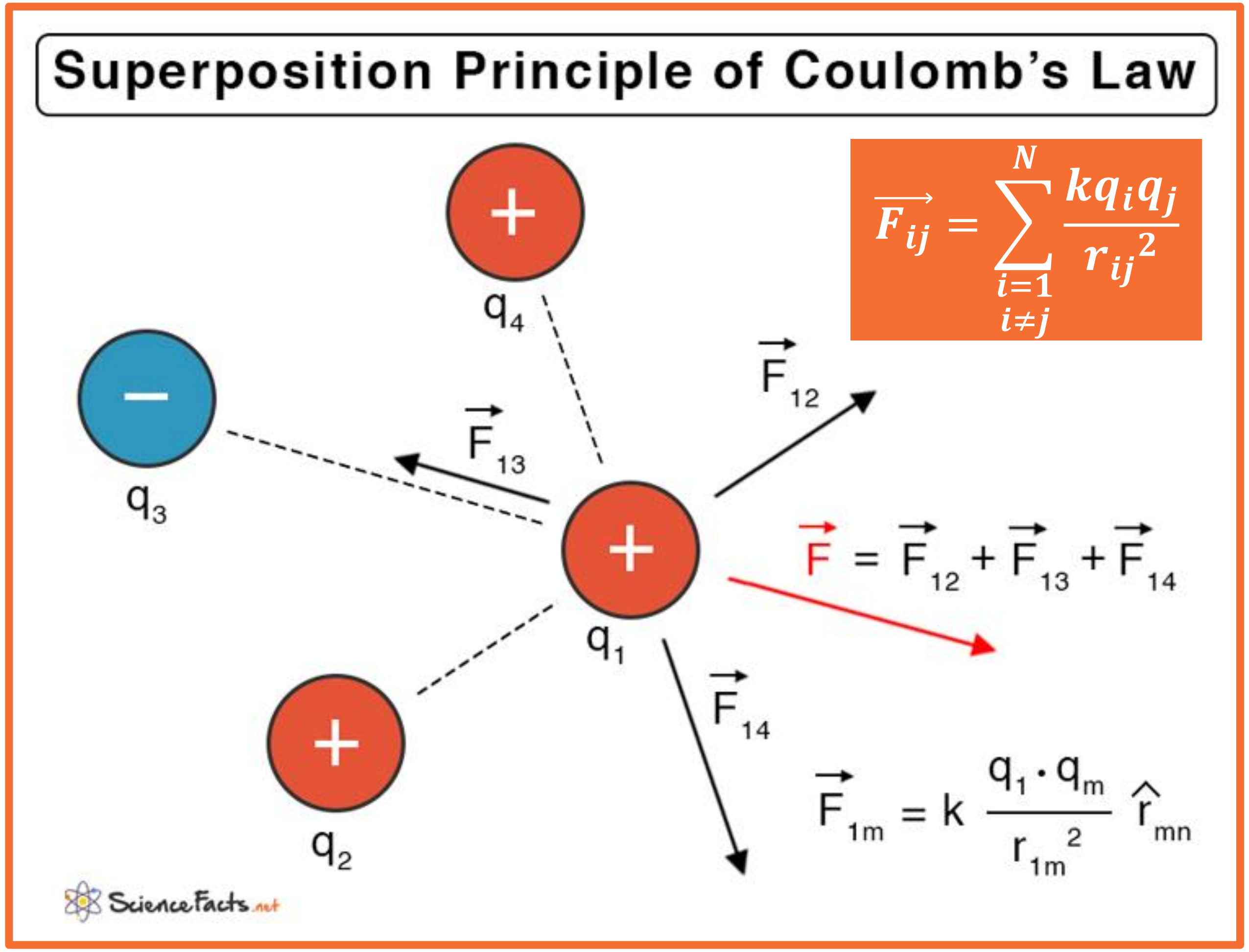
- *Experimental law*
- *Valid for point charges only*
- *Obeys Inverse Square Law*
- *Valid for only charges at rest*

$$Electrostatic\ constant, k = 9 \times 10^9\ \frac{Nm^2}{C^2}$$

$$Permittivity\ constant, \varepsilon_0 = 8.854 \times 10^{-12}\ \frac{C^2}{Nm^2}$$

Coulomb’s Law: Superposition

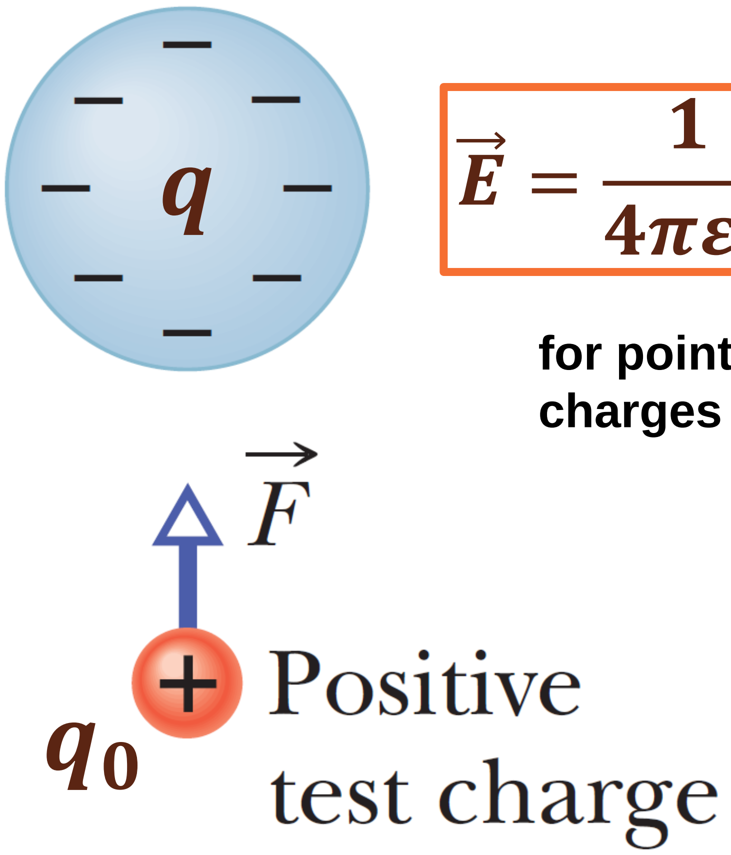




Electric Field

$$\boxed{F = q_0 E}$$

A charge has an effect on its surroundings. The area where it has an effect is generally called an *Electric field.* If any other charge enters that area, it feels an electrostatic Coulomb force.



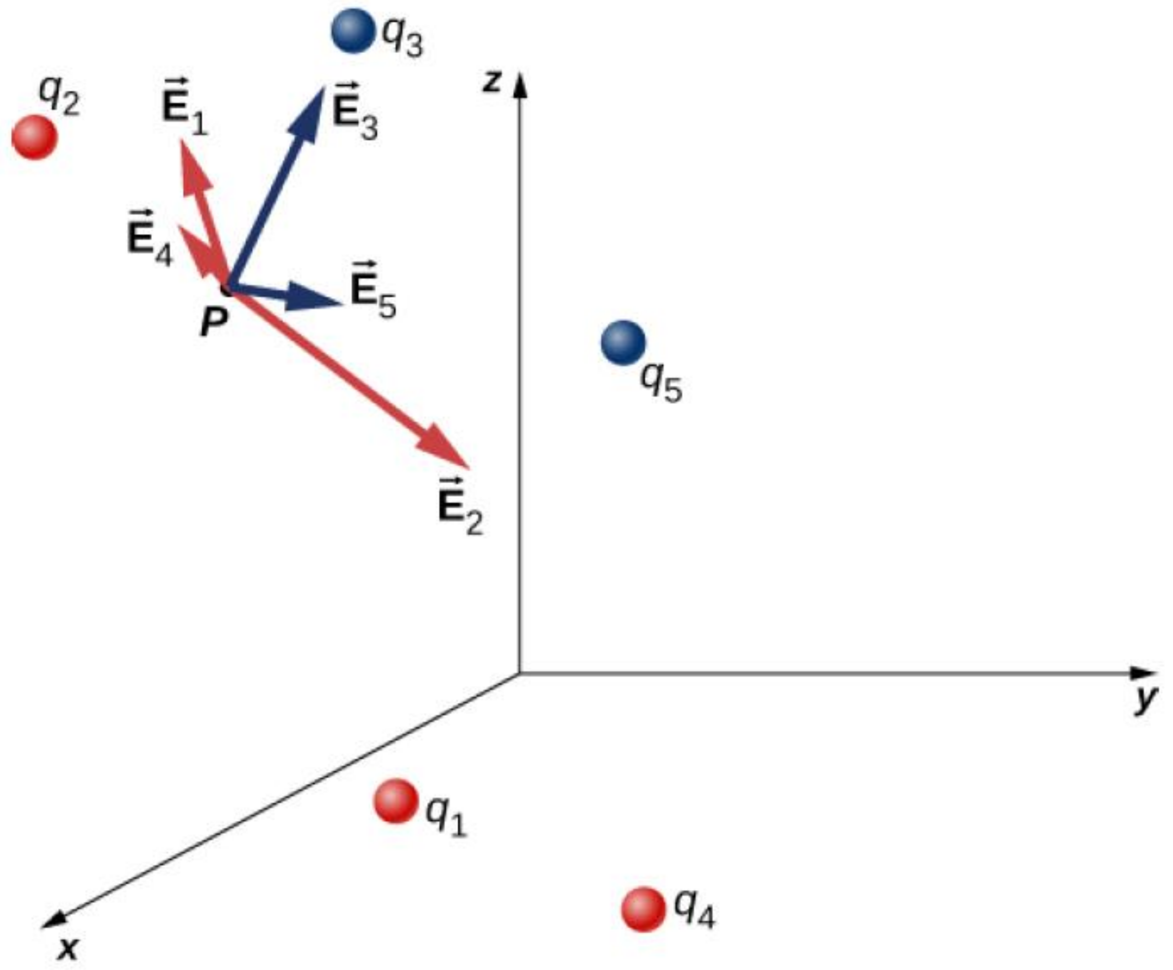
$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{r}}$$

for point test charges only

The electric force on a charged body is exerted by the electric field created by *other* charged bodies.

Superposition of Electric field

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\,\hat{r}$$



- ❑ Treat electric field as a vector quantity
- ❑ q is source charge
- ❑ The test charge is positive

Electric field due to a dipole

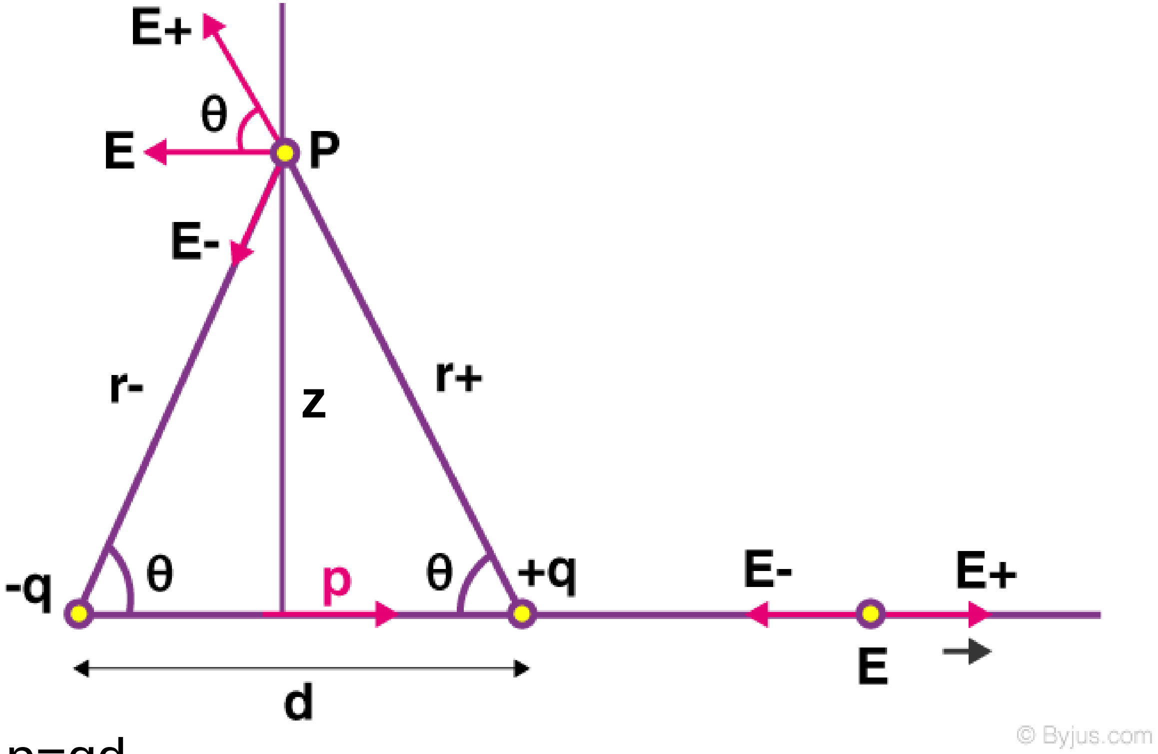
Pairs of point charges with equal magnitude and opposite sign are called *electric dipoles*

At any point

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{p}{z^3}}$$

Along the dipole axis

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{z^3}}$$



Where, dipole moment, p=qd

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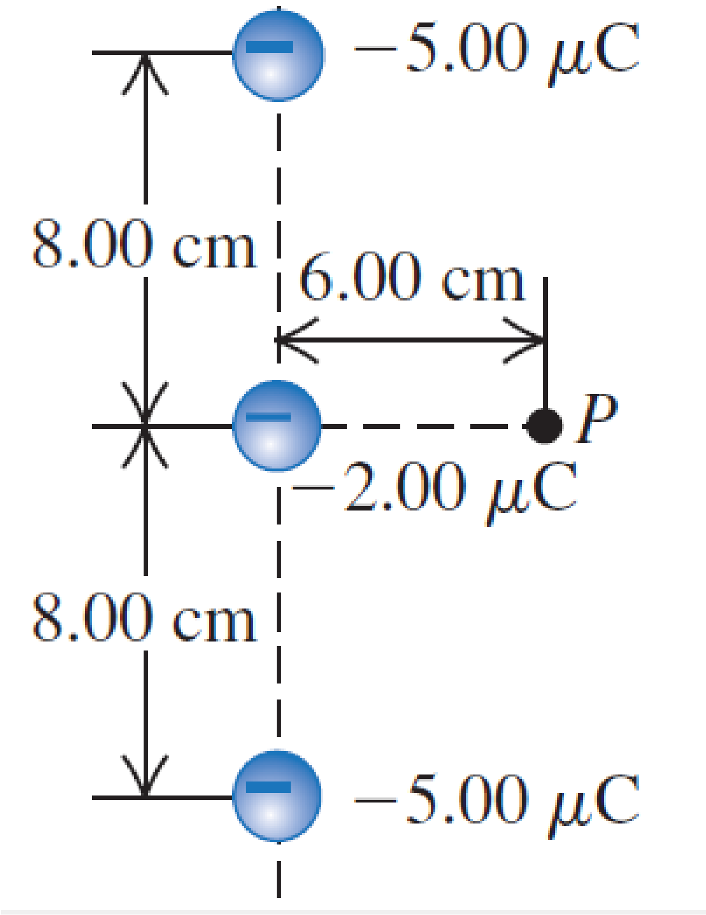
Example 14.4

21.47 • Three negative point charges lie along a line as shown in Fig. E21.47. Find the magnitude and direction of the electric field this combination of charges produces at point P, which lies 6.00 cm from the $-2.00\text{-}\mu\text{C}$ charge measured perpendicular to the line connecting the three charges.

Figure **E21.47**



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Electric field due to any continuous charge distribution

If there is an infinitesimal charge dq contained within an infinitesimal section of a continuous distribution of charge, then the electric field at r distance would be:

$$\boxed{\boldsymbol{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\hat{\boldsymbol{r}}}$$

Some parameters:

Volume charge density

If a charge *Q* is uniformly distributed throughout a volume *V*, the ρ is defined by

$Unit: {}^{C}/_{m^3}$

$$\rho = \frac{dQ}{dV}$$

Surface charge density

If a charge *Q* is uniformly distributed on a surface of area *A*, the σ is defined by

$Unit: {}^{C}/_{m^2}$

$$\sigma = \frac{dQ}{dA}$$

Linear charge density

If a charge *Q* is uniformly distributed along a line of length l, the λ is defined by

$$\lambda = \frac{dQ}{dl}$$

$Unit: {}^{C}/_{m}$

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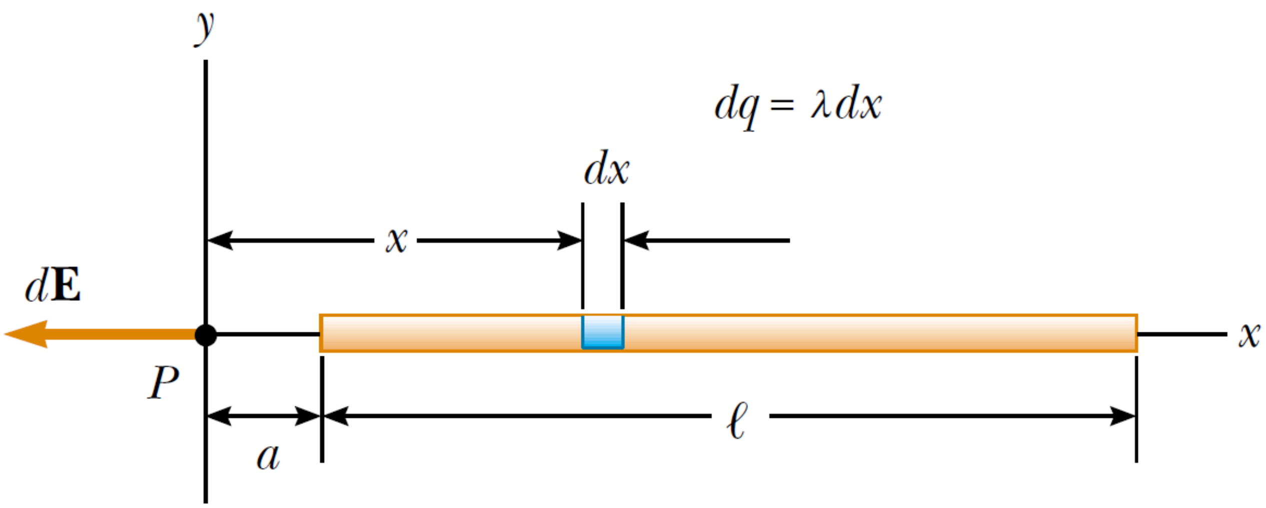
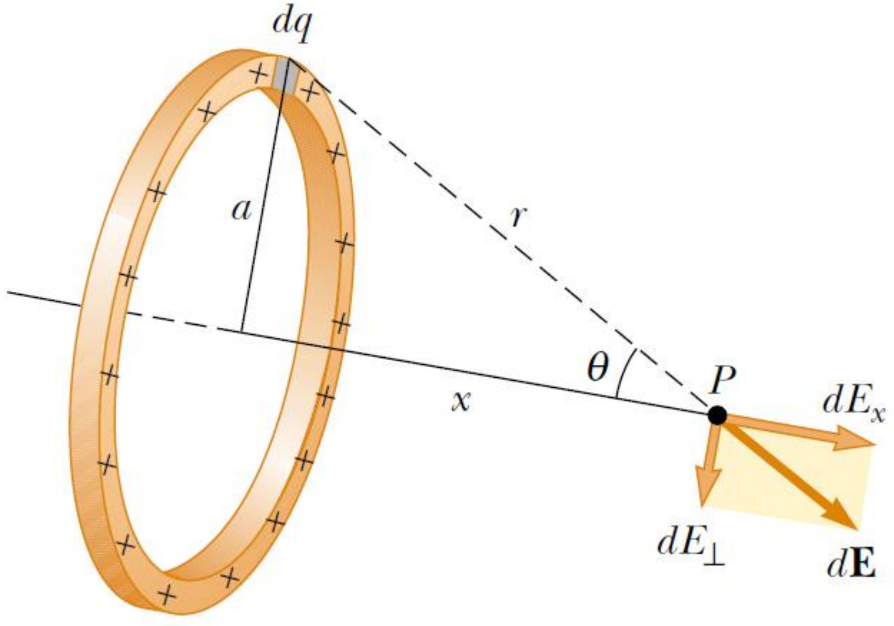
Electric field due to a charged rod



$$E = k_e \lambda \int_a^{\ell+a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell+a}$$

$$= k_e \lambda \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Electric field due to a charged ring



(a)

$$E_x = \int \frac{k_e x}{(x^2 + a^2)^{3/2}}\, dq = \frac{k_e x}{(x^2 + a^2)^{3/2}} \int dq$$

$$= \boxed{\frac{k_e x}{(x^2 + a^2)^{3/2}}\, Q}$$

Que: What is the value of the electric field at the center of the ring?

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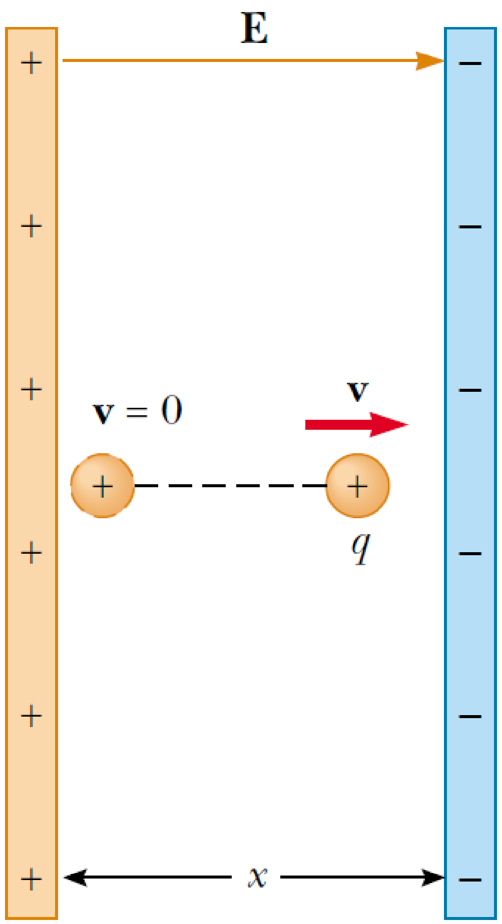
Motion of an accelerating positive charge in an electric field



$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2$$

$$v_{xf} = v_{xi} + a_x t$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

Taking $x_i = 0$ and $v_{xi} = 0$, we have

$$x_f = \tfrac{1}{2}a_x t^2 = \frac{qE}{2m}t^2$$

$$v_{xf} = a_x t = \frac{qE}{m}t$$

$$v_{xf}^2 = 2a_x x_f = \left(\frac{2qE}{m}\right)x_f$$

The kinetic energy of the charge after it has moved a distance $x = x_f - x_i$ is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{2qE}{m}\right)x = qEx$$

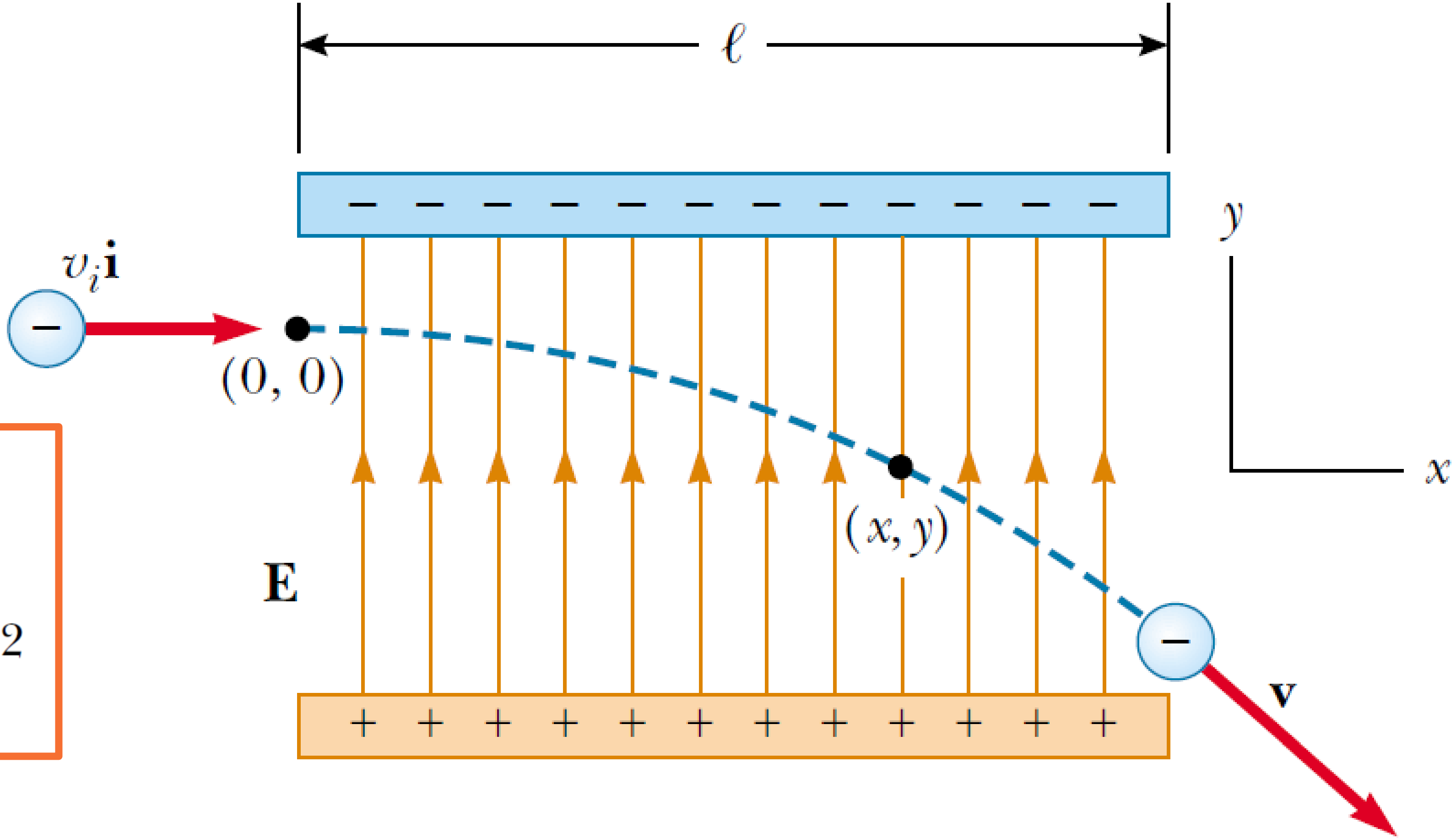
Motion of an accelerating electron in an electric field

$$v_x = v_i = \text{constant}$$

$$v_y = a_y t = -\frac{eE}{m} t$$

$$x = v_i t$$

$$y = \frac{1}{2} a_y t^2 = -\frac{1}{2} \frac{eE}{m} t^2$$



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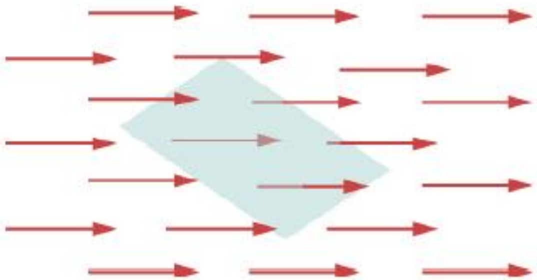
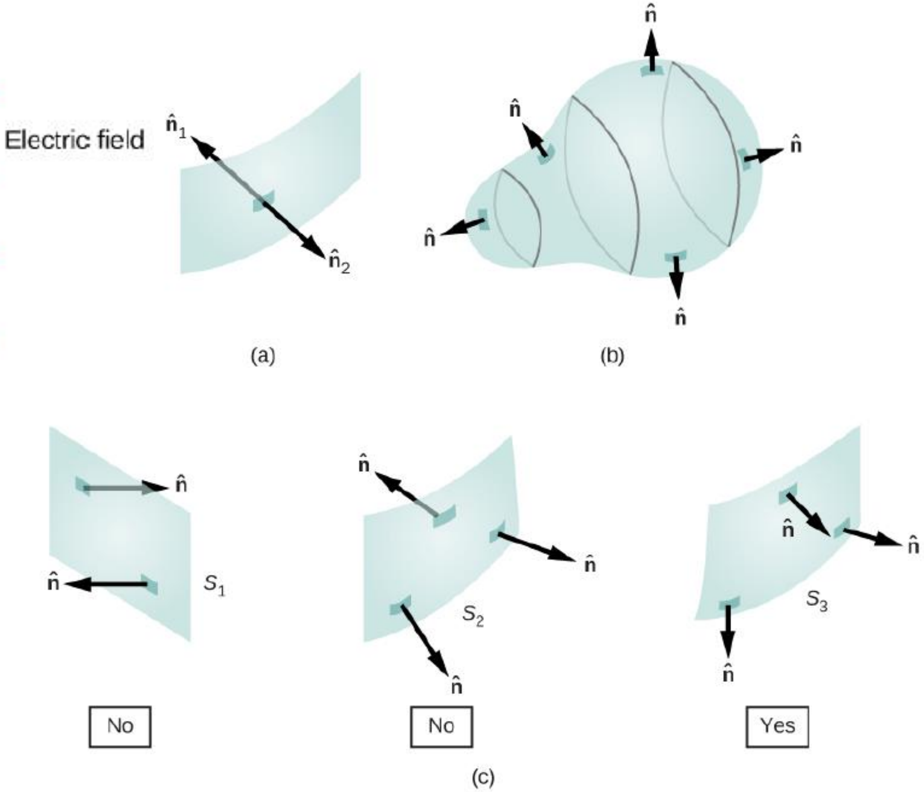
Electric Flux

The concept of **flux** describes how much of something goes through a given area.

The flux of an electric field as a measure of the number of electric field lines passing through an area





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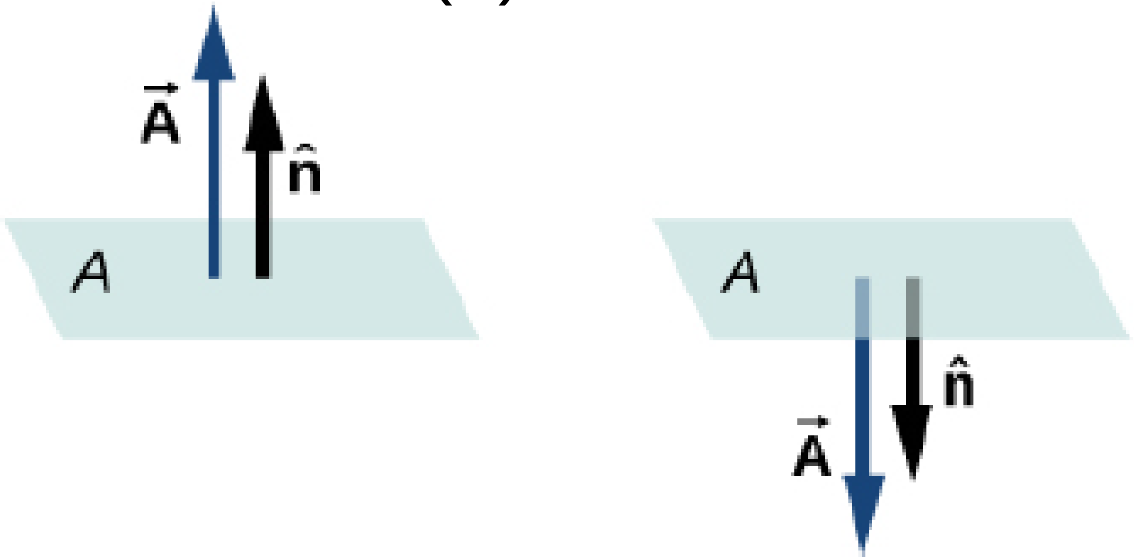
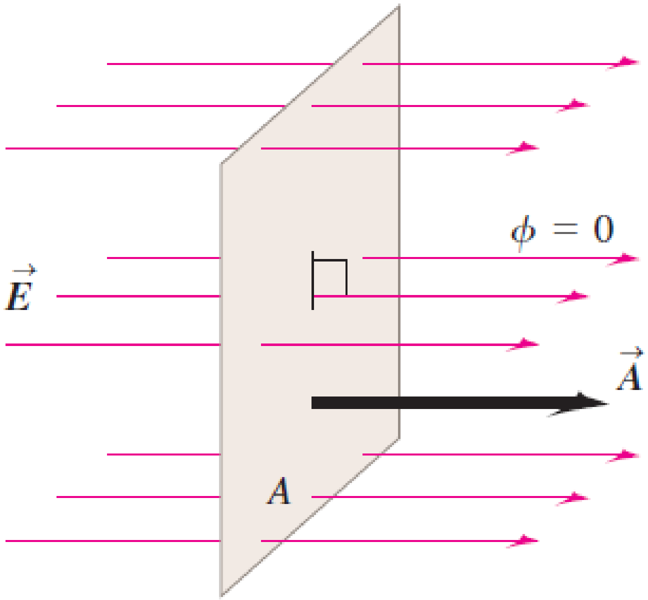
The **area vector** of a flat surface of area A has the following magnitude and direction:

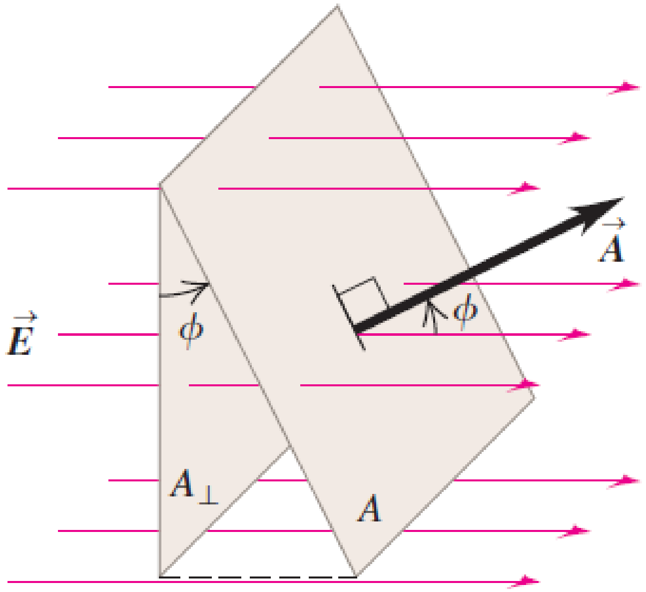
- ❑ Magnitude is equal to area (A)
- ❑ Direction is along the normal to the surface ($\hat{n}$); that is, **perpendicular** to the surface



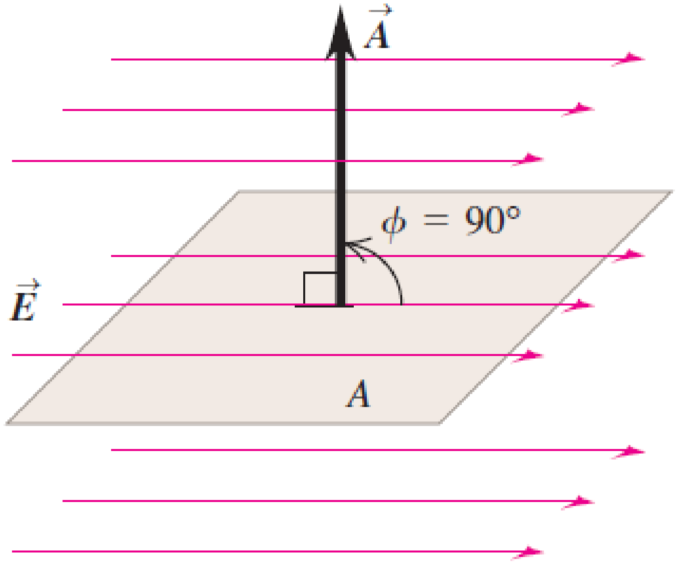
(a) Surface is face-on to electric field:
- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.



(b) Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.

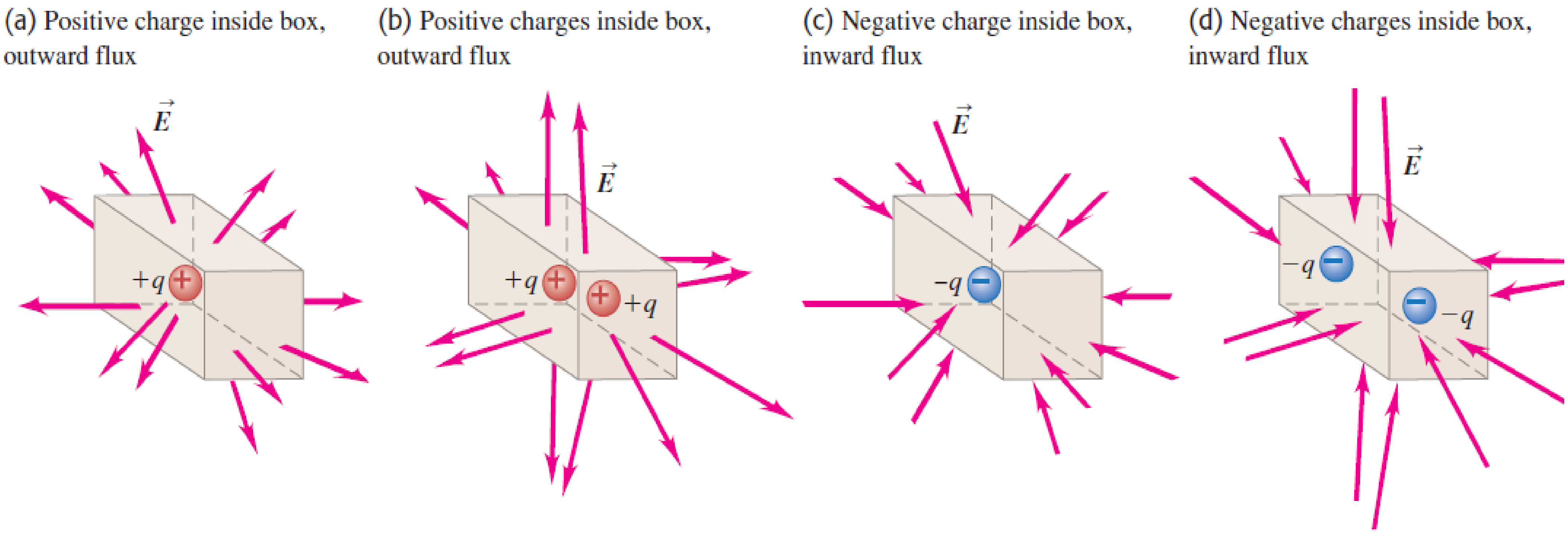


(c) Surface is edge-on to electric field:
- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.

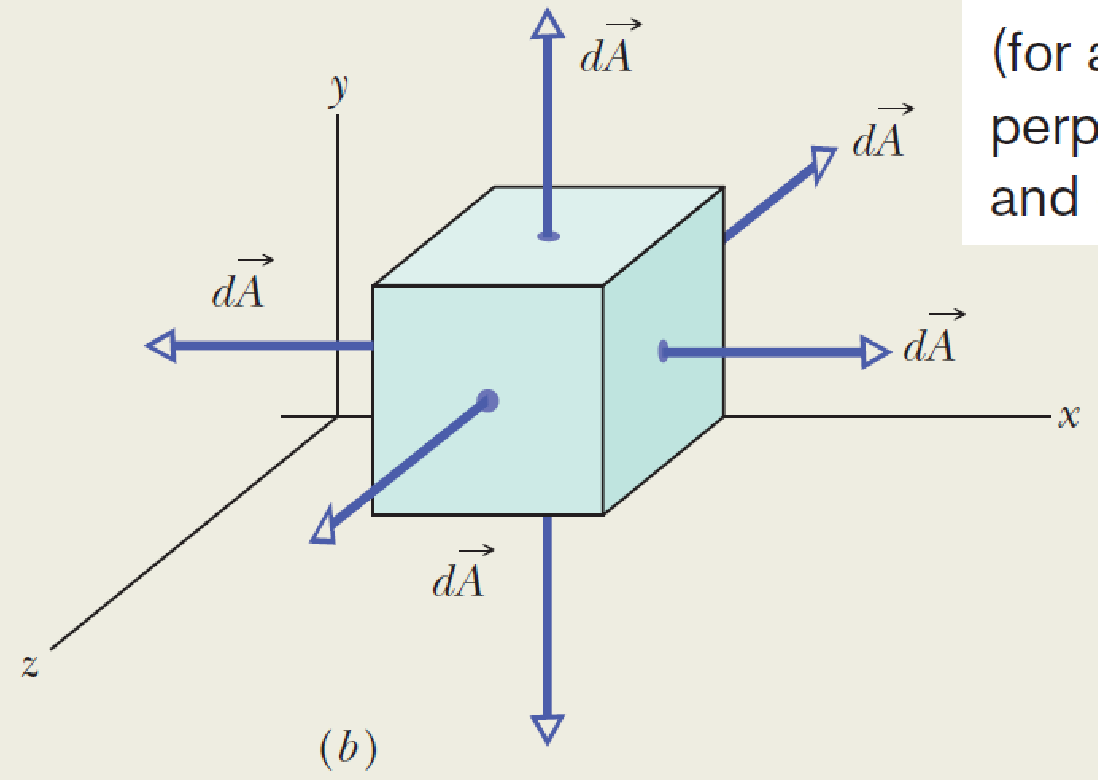


$$\boxed{\varphi = \vec{E}.\hat{n}A = EA\cos\varphi}$$

Flux in an enclosed space



Electric Flux through a Cube



The element area vector (for a patch element) is perpendicular to the surface and outward.

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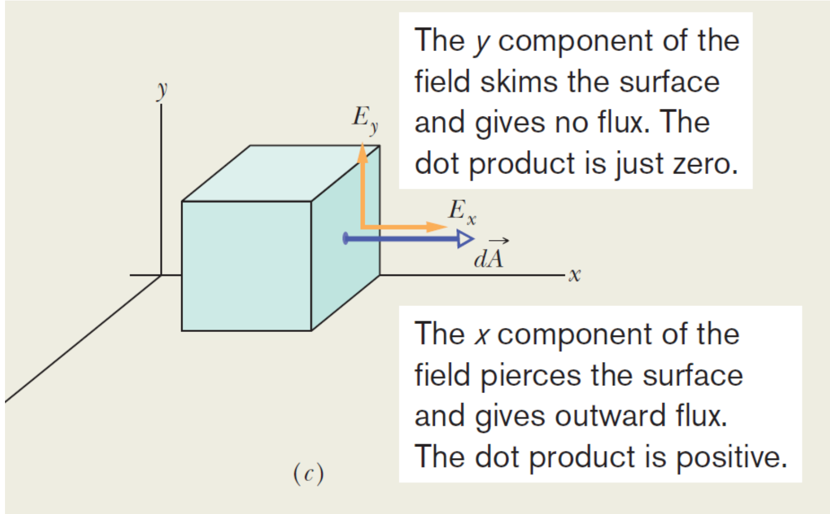
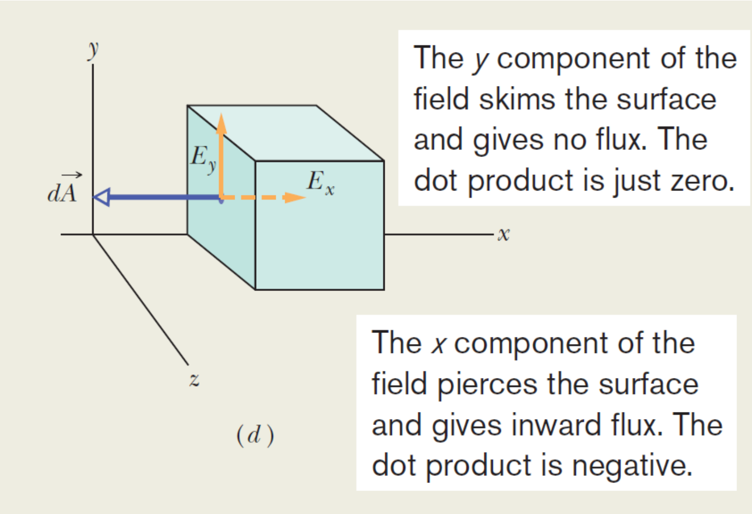
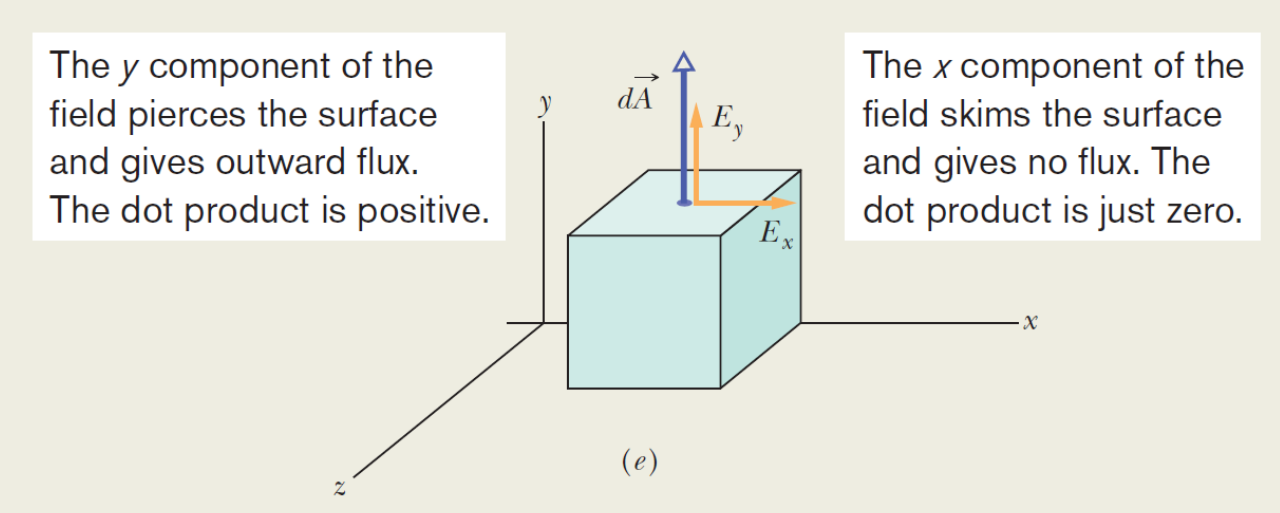
Electric Flux through a Cube

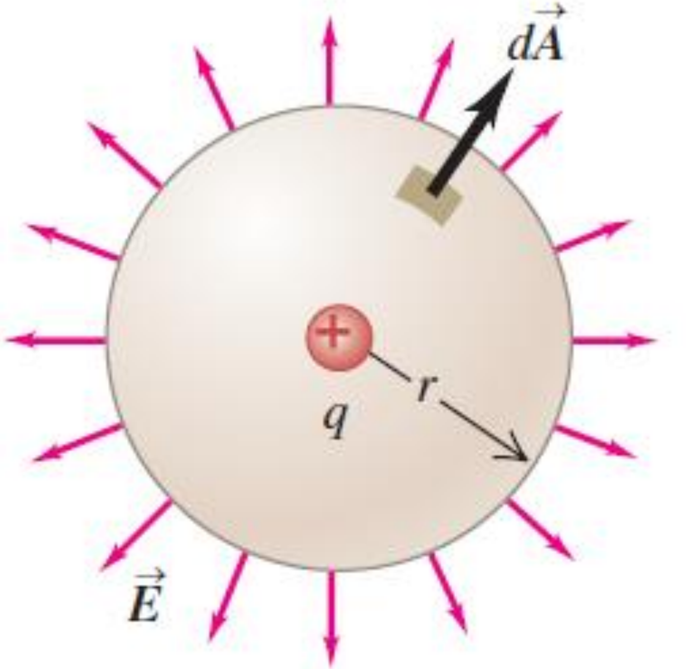






Example 14.5

A point charge q= +3μC is surrounded by an imaginary sphere of radius r= 0.2m centered on the charge. Find the resulting electric flux through the sphere.



$$\Phi_E = EA = \frac{q}{4\pi\epsilon_0 r^2} 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$= \frac{3.0 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 3.4 \times 10^5\ \text{N}\cdot\text{m}^2/\text{C}$$

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