



# Assignment (Fall-23)

## United International University

### (Solution)

Name  
(Optional)

ID No.

Section

Invigilator's  
Signature with date

Course Code

Trimester / Semester : Spring / Summer / Fall, 20.....

Name of Exam : Class Test / Mid-term 1 / Mid-term 2 / Final

Date: .....



$L_1 = \{ \text{Strings with equal number of 'a's and b's (in any order)} \}$

$S \rightarrow aSb \mid bSa \mid \epsilon$

$L_2 = \{ \text{Strings not in the form } 0^i 1^j, \text{ where } i=j; i, j \geq 0 \}$

$S \rightarrow A \mid \epsilon$

$A \rightarrow \cancel{0A0} \mid 0A1 \mid 1A0 \mid 10$

$L_3 = \{ a^n b^{2m} \mid n \geq 1, m \geq n \}$

$S \rightarrow aSbb \mid aBbb$

$B \rightarrow bB \mid \epsilon$

$L_4 = \{ a^n b^m \mid n \leq m \leq 2n \}$

$S \rightarrow aSbb \mid aSb \mid \epsilon$

Q2 :

$$\begin{aligned} S &\rightarrow aX/bY/b/ZZc \\ X &\rightarrow Yaa/abZ/\epsilon \\ Y &\rightarrow bXXb/ab/cZ \\ Z &\rightarrow a/b/XZ/\epsilon \end{aligned}$$

Step 1: As Starting Symbol is not on the right side of the production rule, skip this step.

<u>Step 2</u> : Removing $X \rightarrow \epsilon \Rightarrow$	Removing $Z \rightarrow \epsilon \Rightarrow$
$S \rightarrow aX/bY/b/ZZc/a$	$S \rightarrow aX/bY/b/ZZc/a/Zc/c$
$X \rightarrow Yaa/abZ$	$X \rightarrow Yaa/abZ/ab$
$Y \rightarrow bXXb/ab/cZ/bXb/bb$	$Y \rightarrow bXXb/ab/cZ/bXb/bb/c$
$Z \rightarrow a/b/XZ/Z/\epsilon$	$Z \rightarrow a/b/XZ/X$

Step 3: Removing  $Z \rightarrow X \Rightarrow$

$$\begin{aligned} S &\rightarrow aX/bY/b/ZZc/a/Zc/c \\ X &\rightarrow Yaa/abZ/ab \\ Y &\rightarrow bXXb/ab/cZ/bXb/bb/c \\ Z &\rightarrow a/b/XZ/Yaa/abZ/ab \end{aligned}$$

Step 4: ~~Reducing~~ <sup>Reducing</sup> Production rules that has length  $> 2 \Rightarrow$

$$S \rightarrow aX | bY | b | ZA | a | Zc | c$$

$$X \rightarrow Ba | aC | ab$$

$$Y \rightarrow DE | ab | cZ | Db | bb | c$$

$$Z \rightarrow a | b | XZ | Ba | aC | ab$$

$$A \rightarrow Zc$$

$$B \rightarrow Ya$$

$$C \rightarrow bZ$$

$$D \rightarrow bX$$

$$E \rightarrow Xb$$

Step 5: Bring the rules in the CNF format

$$S \rightarrow ZX | ZY | b | ZA | a | ZY | c$$

$$X \rightarrow BZ | ZC | ZZ$$

$$Y \rightarrow DE | ZZ | SZ | DZ | ZZ | c$$

$$Z \rightarrow a | b | XZ | BS | SC | ZZ$$

$$A \rightarrow ZY$$

$$D \rightarrow ZX$$

$$B \rightarrow YZ$$

$$E \rightarrow XZ$$

$$C \rightarrow ZZ$$



ii)

Step 1: Adding a new starting symbol

$$S \rightarrow Q_0$$

$$Q_0 \rightarrow 0Q_0 \mid 1Q_2$$

$$Q_1 \rightarrow 0Q_3 \mid 1Q_0 \mid \epsilon$$

$$Q_2 \rightarrow 0Q_1 \mid 1Q_3 \mid \epsilon$$

$$Q_3 \rightarrow 0Q_4 \mid 1Q_1 \mid \epsilon$$

$$Q_4 \rightarrow 0Q_2 \mid 1Q_4 \mid \epsilon$$

Step 2:

Removing  $Q_1 \rightarrow \epsilon \Rightarrow$

$$S \rightarrow Q_0$$

$$Q_0 \rightarrow 0Q_0 \mid 1Q_2$$

$$Q_1 \rightarrow 0Q_3 \mid 1Q_0$$

$$Q_2 \rightarrow 0Q_1 \mid 1Q_3 \mid 0 \mid \epsilon$$

$$Q_3 \rightarrow 0Q_4 \mid 1Q_1 \mid 1 \mid \epsilon$$

$$Q_4 \rightarrow 0Q_2 \mid 1Q_4 \mid \epsilon$$

Removing  $Q_2 \rightarrow \epsilon \Rightarrow$

$$S \rightarrow Q_0$$

$$Q_0 \rightarrow 0Q_0 \mid 1Q_2 \mid 1$$

$$Q_1 \rightarrow 0Q_3 \mid 1Q_0$$

$$Q_2 \rightarrow 0Q_1 \mid 1Q_3 \mid 0$$

$$Q_3 \rightarrow 0Q_4 \mid 1Q_1 \mid 1 \mid \epsilon$$

$$Q_4 \rightarrow 0Q_2 \mid 1Q_4 \mid 0 \mid \epsilon$$



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Removing  $Q_3 \rightarrow \epsilon \Rightarrow$

$S \rightarrow Q_0$

$Q_0 \rightarrow 0Q_0 | 1Q_2 | 1$

$Q_1 \rightarrow 0Q_3 | 1Q_0 | 0$

$Q_2 \rightarrow 0Q_1 | 1Q_3 | 0 | 1$

$Q_3 \rightarrow 0Q_4 | 1Q_1 | 1$

$Q_4 \rightarrow 0Q_2 | 1Q_4 | 0 | \epsilon$

Removing  $Q_4 \rightarrow \epsilon \Rightarrow$

$S \rightarrow Q_0$

$Q_0 \rightarrow 0Q_0 | 1Q_2 | 1$

$Q_1 \rightarrow 0Q_3 | 1Q_0 | 0$

$Q_2 \rightarrow 0Q_1 | 1Q_3 | 0 | 1$

$Q_3 \rightarrow 0Q_4 | 1Q_1 | 1 | 0$

$Q_4 \rightarrow 0Q_2 | 1Q_4 | 0 | 1$

Step 3: Removing  $S \rightarrow Q_0 \Rightarrow$

$S \rightarrow 0Q_0 | 1Q_2 | 1$

$Q_0 \rightarrow 0Q_0 | 1Q_2 | 1$

$Q_1 \rightarrow 0Q_3 | 1Q_0 | 0$

$Q_2 \rightarrow 0Q_1 | 1Q_3 | 0 | 1$

$Q_3 \rightarrow 0Q_4 | 1Q_1 | 1 | 0$

$Q_4 \rightarrow 0Q_2 | 1Q_4 | 0 | 1$

Step 4: Skip this step as there is no production rules that has length 72.

Step 5:

$S \rightarrow A Q_0 / B Q_2 / 1$

$Q_0 \rightarrow A Q_0 / B Q_2 / 1$

$Q_1 \rightarrow A Q_3 / B Q_0 / 0$

$Q_2 \rightarrow A Q_1 / B Q_3 / 0 / 1$

$Q_3 \rightarrow A Q_4 / B Q_1 / 0 / 1$

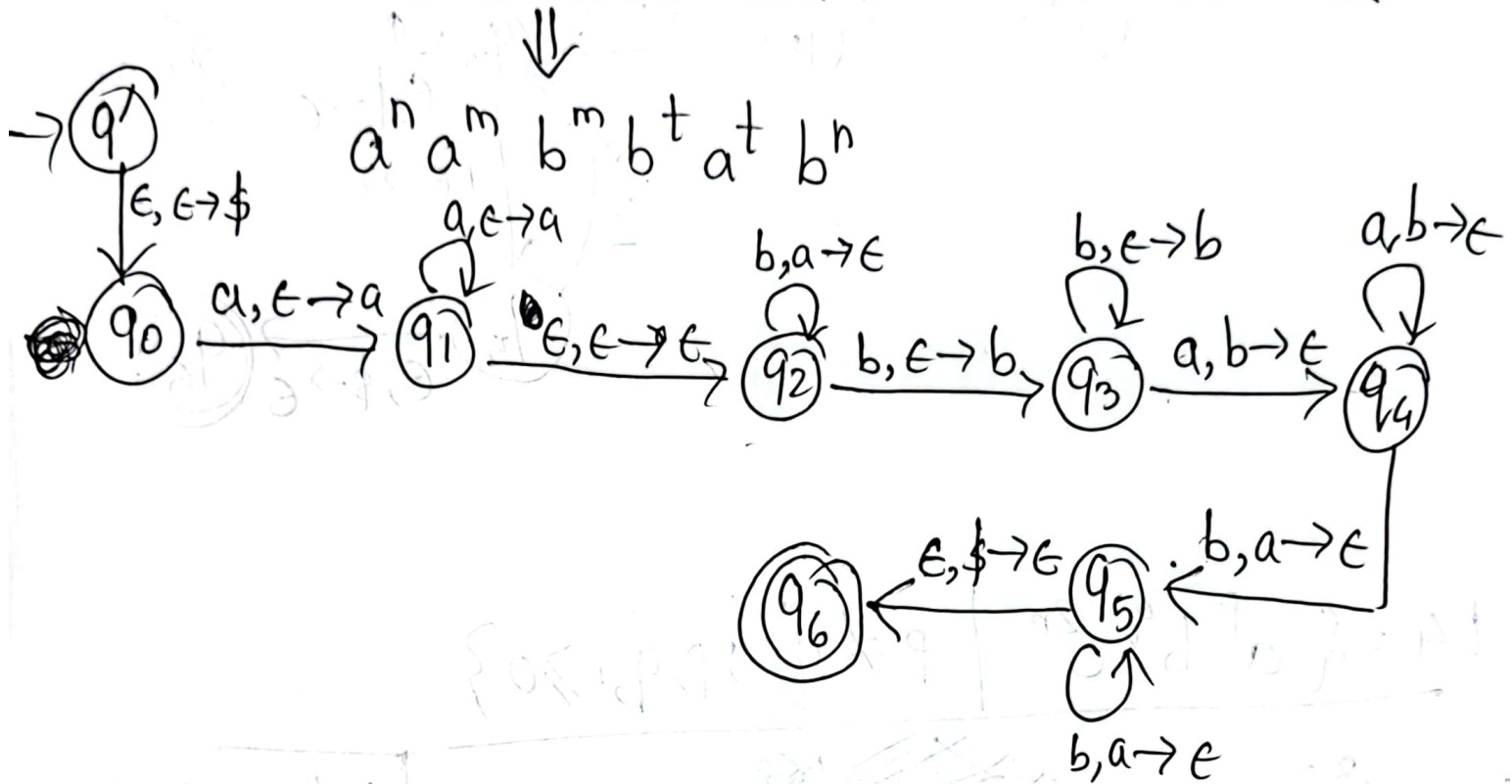
$Q_4 \rightarrow A Q_2 / B Q_4 / 0 / 1$

$A \rightarrow 0$

$B \rightarrow 1$

3

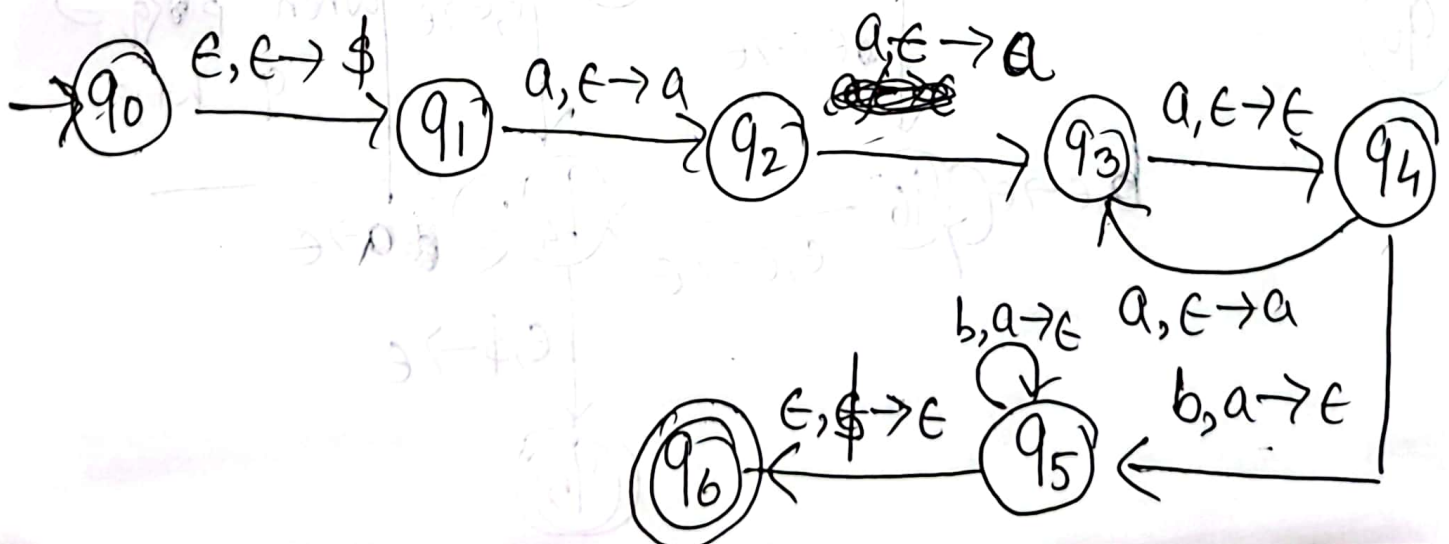
$$L1 = \{ a^{n+m} b^{m+t} a^t b^n \mid n, t > 0 \text{ and } m \geq 0 \}$$



$$L2 = \{ a^{2n+1} b^n \mid n > 0 \}$$

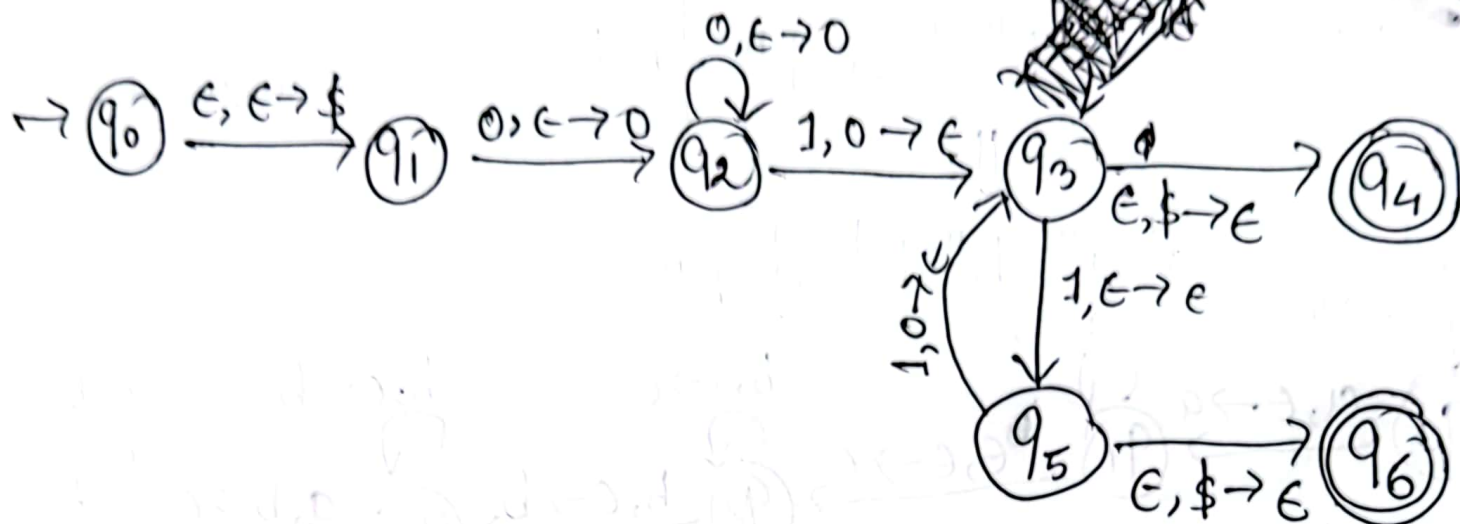
$\Downarrow$

$a \cdot a^{2n} \cdot b^n$

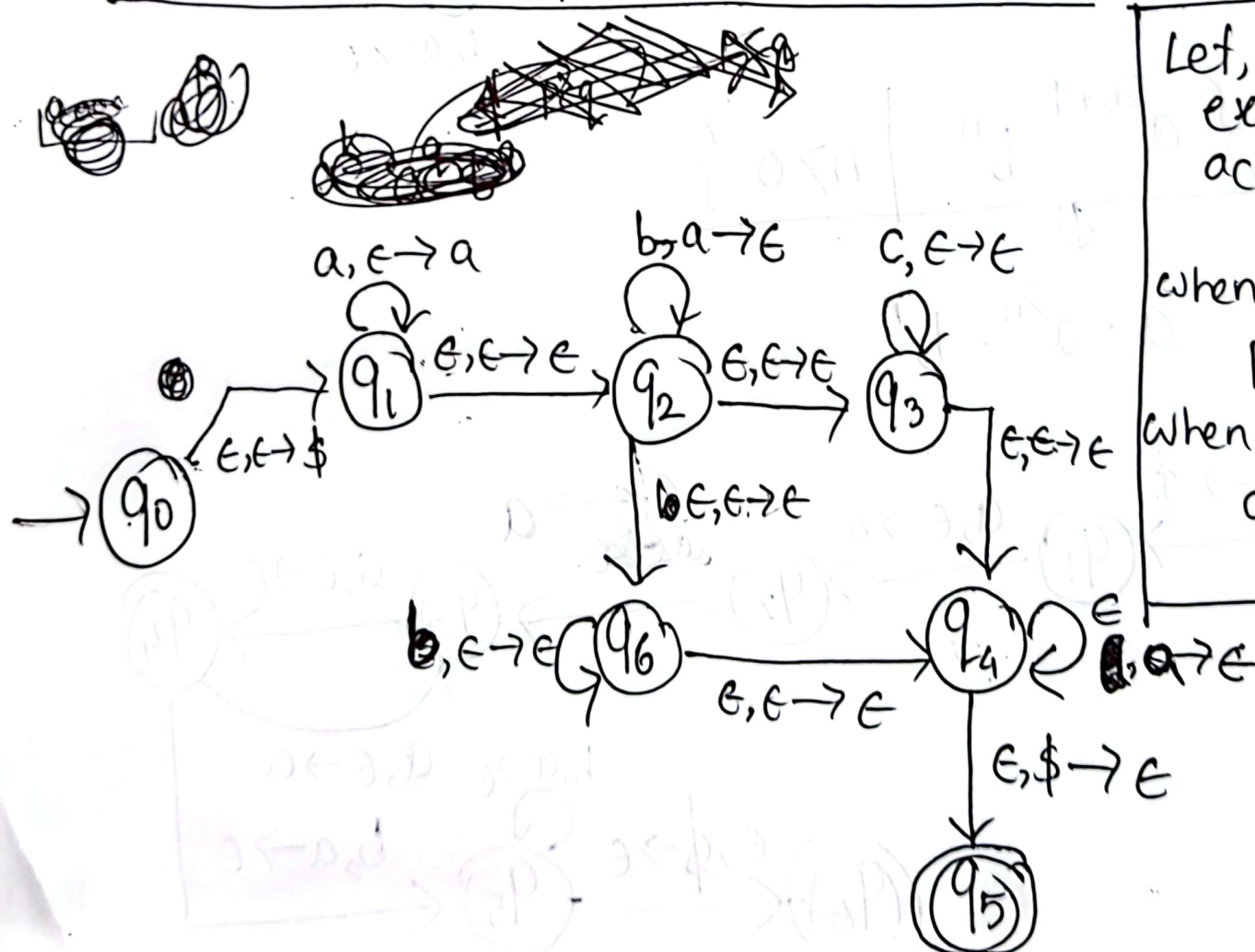




$$L3 = \{ 0^i 1^j \mid i \leq j \leq 2i \}$$



$$L4 = \{ a^p b^q c^{2r} \mid p \neq q, p, q, r \geq 0 \}$$



Let,  $k$  is the extra symbol according to condition when  $p > q = p = k + q$  when  $p < q = q = k + 1$