United International **University**

School of Science and Engineering

Assignment-2 Trimester: Fall-2024

Course Title: Probability and Statistics

Course Code: Math 2205

Group 1

1. A fair 4 sided die, numbered 1, 2,3, and 5 is rolled twice. The random variable X is the sum of the two outcomes on which the die comes to rest.

(i) Show that $P(x = 8) = \frac{1}{8}$

(ii) Draw up the probability distribution table for X, and find $p(x > 6)$

2.

A bag contains 7 orange balls and 3 blue balls. 4 balls are selected at random from the bag, without replacement. Let X denote the number of blue balls selected.

(i) Show that $P(X = 0) = \frac{1}{6}$ and $P(X = 1) = \frac{1}{2}$.

(ii) Construct a table to show the probability distribution of X.

(iii) Find the mean and variance of X.

3.

The discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	0.3	a	b	0.25

(i) Write down an equation satisfied by a and b.

(ii) Given that $E(X) = 4$, find a and b.

Group 2

1. A driving test is passed by 70% of people at their attempt. Find the probability that
   (i) exactly 5 people out of 10 people will passed the driving test.
   (ii) More than 1 people out of 8 people will passed the driving test.

2.

65% of all watches sold by a shop have a digital display and 35% have an analog display.

(i) Find the probability that, out of the next 12 customers who buy a watch, fewer than 10 choose one with a digital display. [4]

3.

(i) A garden shop sells polyanthus plants in boxes, each box containing the same number of plants. The number of plants per box which produce yellow flowers has a binomial distribution with mean 11 and variance 4.95.

(a) Find the number of plants per box. [4]

(b) Find the probability that a box contains exactly 12 plants which produce yellow flowers. [2]

Group 3

1.

Computer breakdowns occur randomly on average once every 48 hours of use.

(i) Calculate the probability that there will be fewer than 4 breakdowns in 60 hours of use.

(ii) there will be no breakdowns in 24 hours of use.

2.

Between 7 p.m. and 11 p.m., arrivals of patients at the casualty department of a hospital occur at random at an average rate of 6 per hour.

(i) Find the probability that, during any period of one hour between 7 p.m. and 11 p.m., exactly 5 people will arrive. [2]

(ii) A patient arrives at exactly 10.15 p.m. Find the probability that at least one more patient arrives before 10.35 p.m. [3]

3.

The number of radioactive particles emitted per second by a certain metal is random and has mean 1.7. The radioactive metal is placed next to an object which independently emits particles at random such that the mean number of particles emitted per second is 0.6. Find the probability that the total number of particles emitted in the next 3 seconds is 6, 7 or 8. [4]

Group 4

1.

The distance in metres that a ball can be thrown by pupils at a particular school follows a normal distribution with mean 35.0 m and standard deviation 11.6 m.

(i) Find the probability that a randomly chosen pupil can throw a ball between 30 and 40 m. [3]

(ii) The school gives a certificate to the 10% of pupils who throw further than a certain distance. Find the least distance that must be thrown to qualify for a certificate. [3]

2.

(i) In a normal distribution with mean μ and standard deviation σ, $P(X > 3.6) = 0.5$ and $P(X > 2.8) = 0.6554$. Write down the value of μ, and calculate the value of σ. [4]

(ii) If four observations are taken at random from this distribution, find the probability that at least two observations are greater than 2.8. [4]

3.

The waiting time in a doctor's surgery is normally distributed with mean 15 minutes and standard deviation 4.2 minutes.

(i) Find the probability that a patient has to wait less than 10 minutes to see the doctor. [3]

(ii) 10% of people wait longer than T minutes. Find T. [3]

(iii) In a given week, 200 people attend the surgery. Estimate the number of these who wait more than 20 minutes. [3]

4.

In a certain country the time taken for a common infection to clear up is normally distributed with mean μ days and standard deviation 2.6 days. 25% of these infections clear up in less than 7 days.

(i) Find the value of μ. [4]

In another country the standard deviation of the time taken for the infection to clear up is the same as in part **(i)**, but the mean is 6.5 days. The time taken is normally distributed.

(ii) Find the probability that, in a randomly chosen case from this country, the infection takes longer than 6.2 days to clear up. [3]