CSE-233 : Section A

Summer 2020

Conversion from/to Regular Expression

Reference:

Book2 Chapter 1.3

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Regular Expression to NFA

Say that R is a ***regular expression*** if R is

1. a for some a in the alphabet Σ,

2. ε,

3. $\emptyset$,

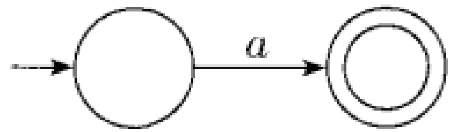
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions,

5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions, or

6. (R_1^*), where R_1 is a regular expression.

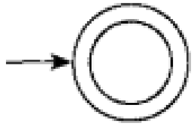
Regular Expression to NFA

1. $R = a$ for some a in Σ. Then $L(R) = \{a\}$, and the following NFA recognizes $L(R)$.



Regular Expression to NFA

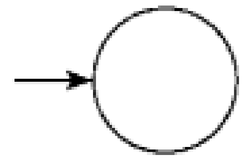
2. $R = \varepsilon$. Then $L(R) = \{\varepsilon\}$, and the following NFA recognizes $L(R)$.



Formally, $N = \big(\{q_1\}, \Sigma, \delta, q_1, \{q_1\}\big)$, where $\delta(r, b) = \emptyset$ for any r and b.

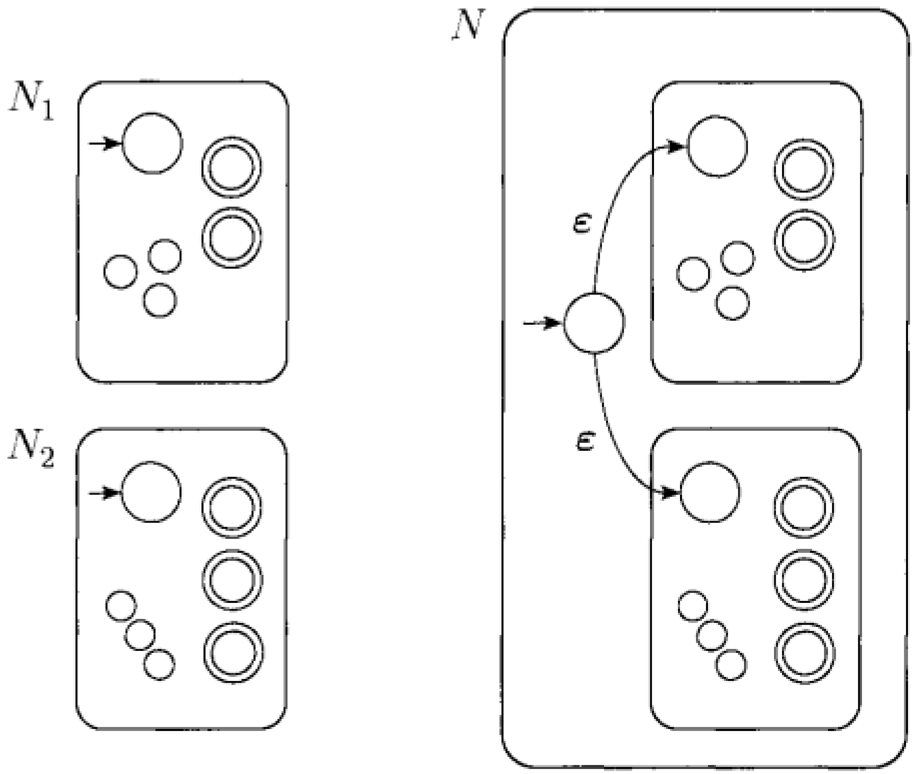
Regular Expression to NFA

3. $R = \emptyset$. Then $L(R) = \emptyset$, and the following NFA recognizes $L(R)$.



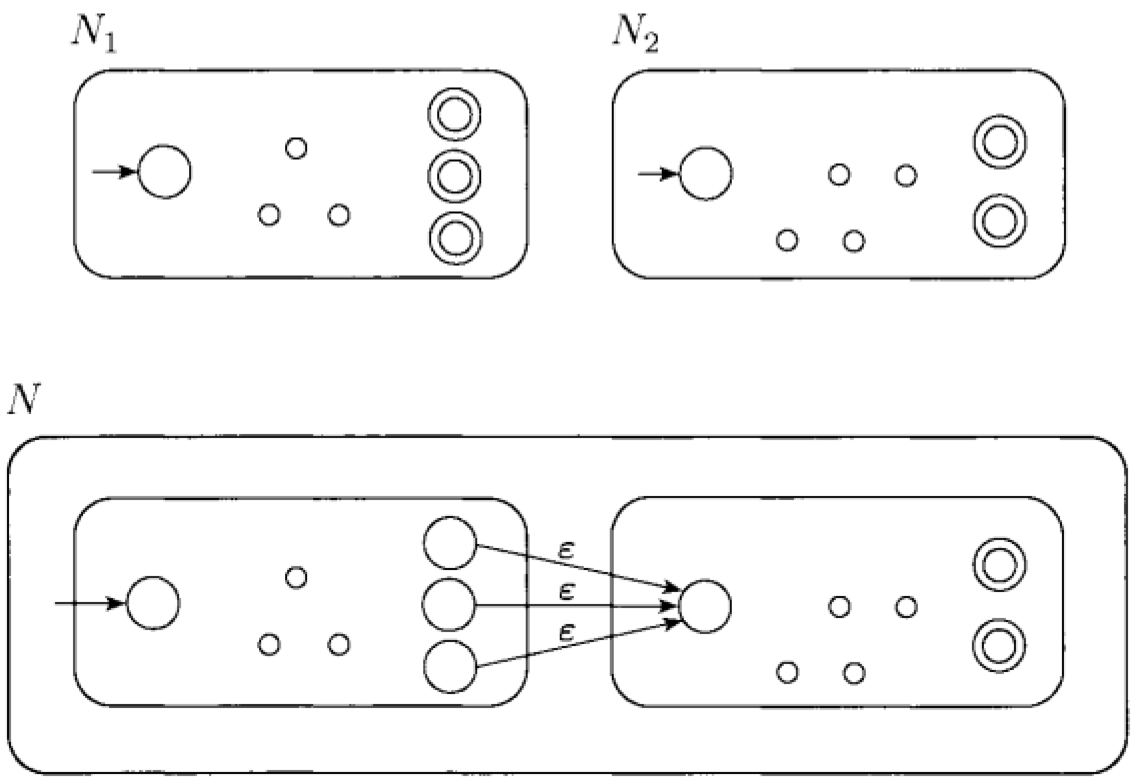
Regular Expression to NFA

4. $R = R_1 \cup R_2$.



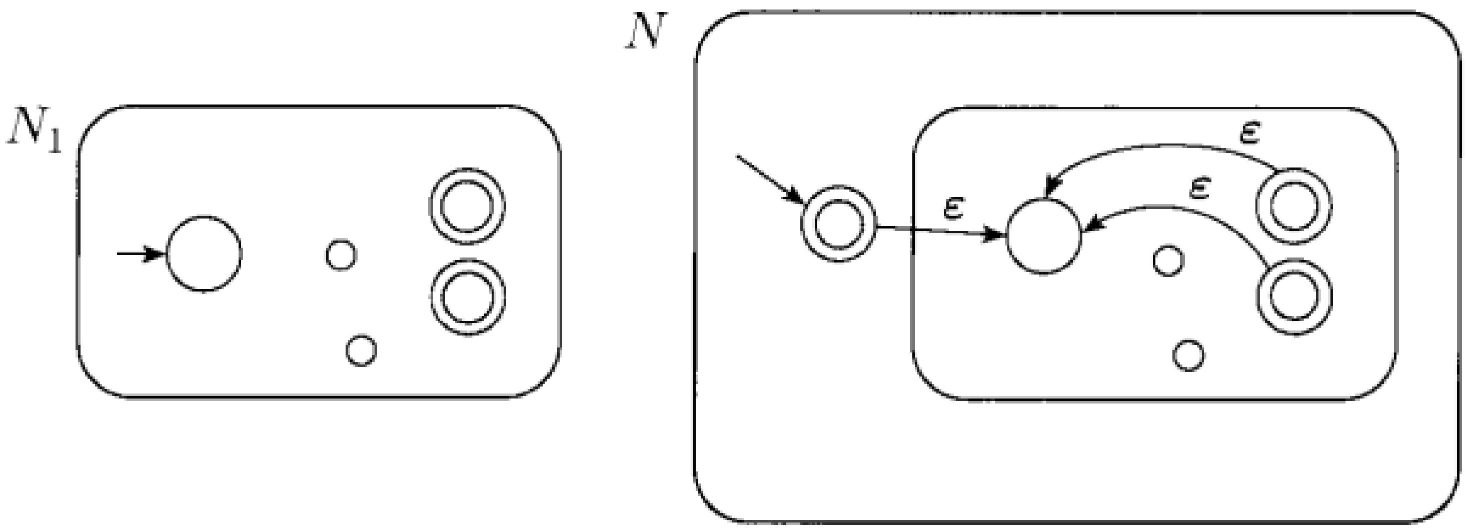
Regular Expression to NFA

5. $R = R_1 \circ R_2$.



Regular Expression to NFA

6. $R = R_1^*$.



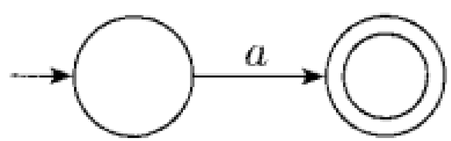
Example: (ab U a)* to NFA

a

Example: (ab U a)* to NFA

a

1. $R = a$ for some a in Σ. Then $L(R) = \{a\}$, and the following NFA recognizes $L(R)$.

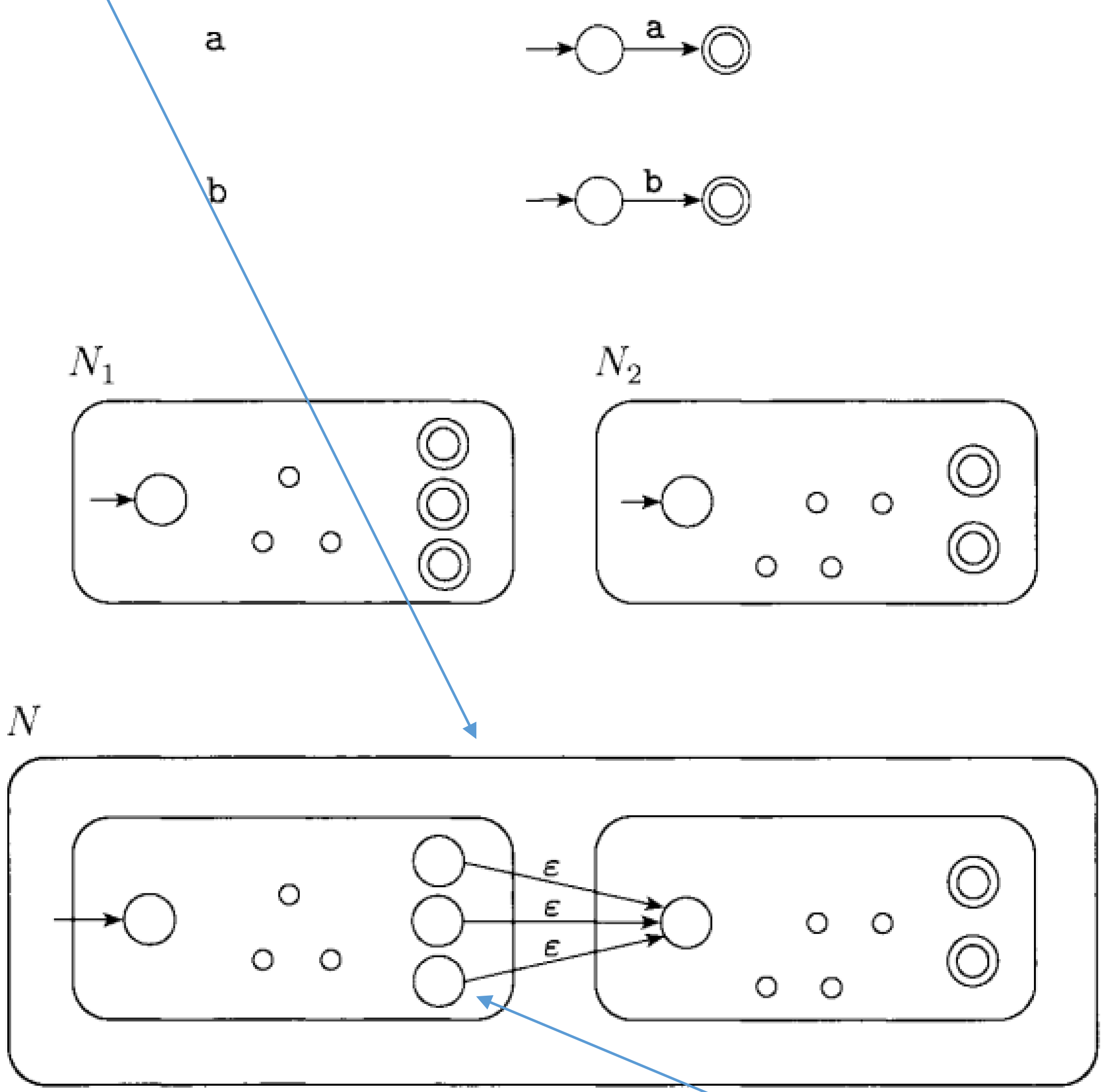


Example: (ab U a)* to NFA

a

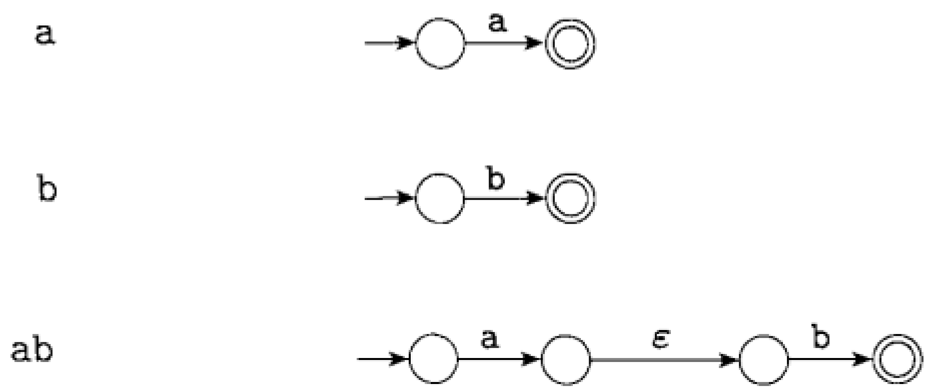
b

Example: (ab U a)* to NFA



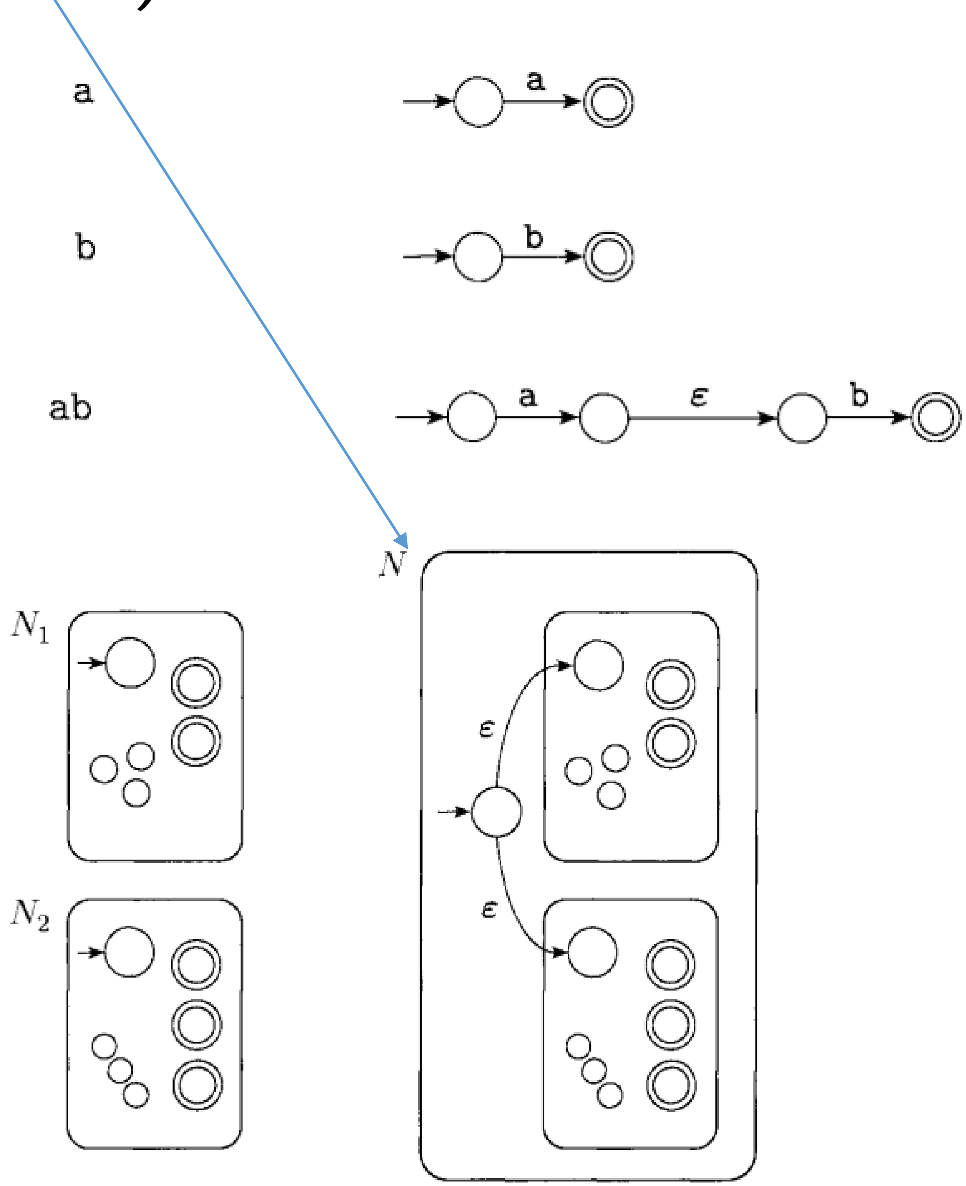
Notice how the final state has changed

Example: (ab U a)* to NFA



Notice how the final state has changed in ab

Example: (ab U a)* to NFA



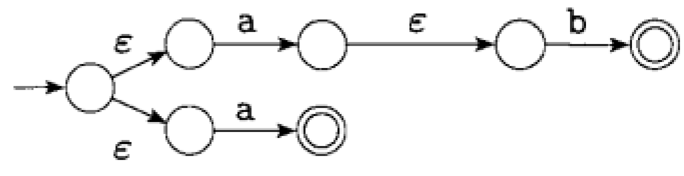
Example: (ab U a)* to NFA

a

b

ab

ab ∪ a

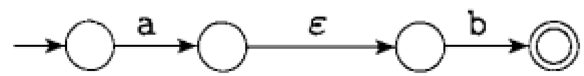


Example: (ab U a)* to NFA

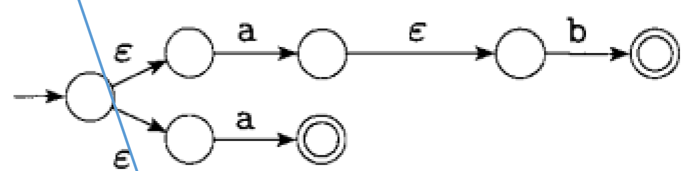
a

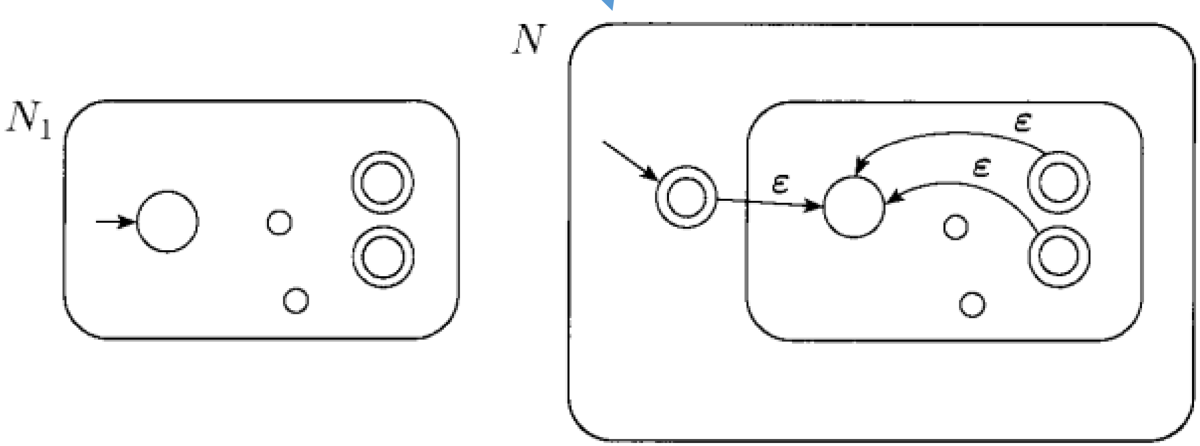
b

ab

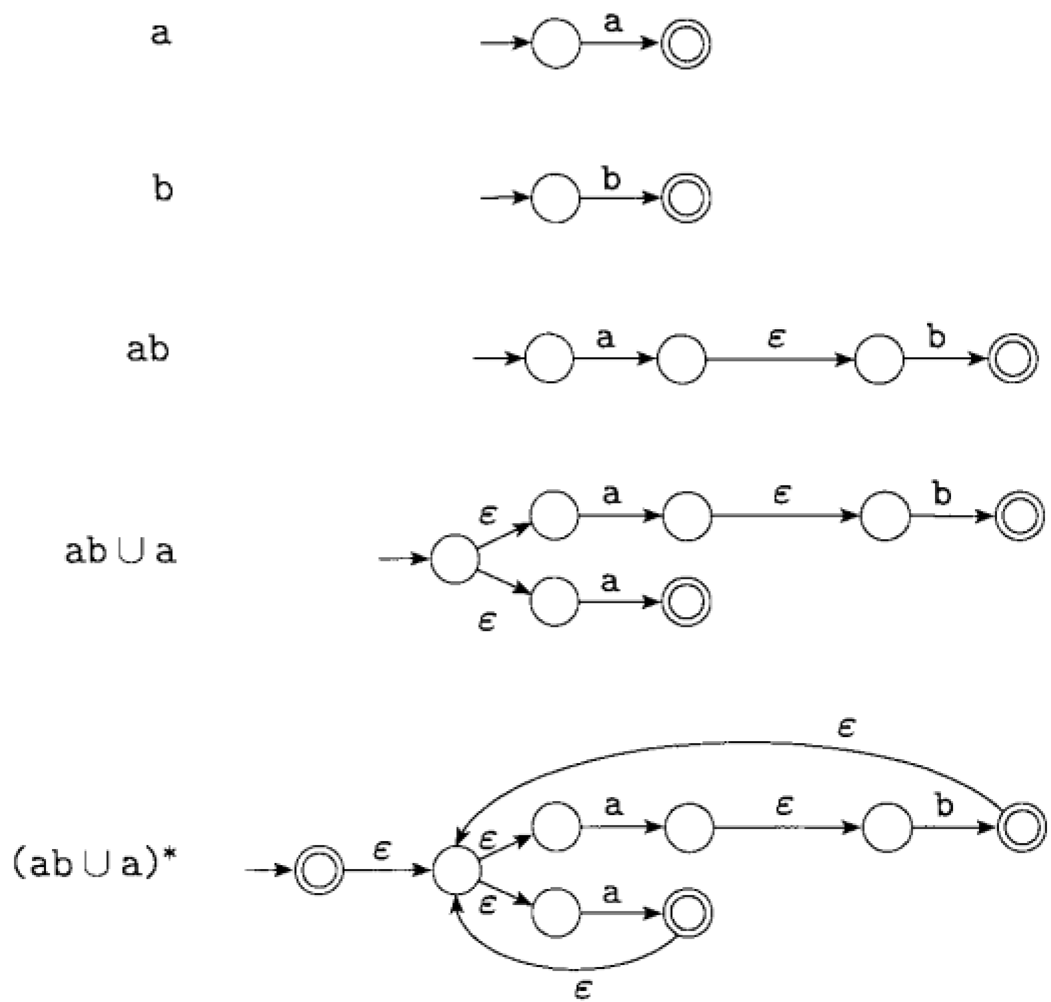


ab ∪ a





Example: (ab U a)* to NFA



Task

Convert the following RE to NFA

$$a+bc^*d$$

Show the following steps-

1. a
2. b
3. c
4. c*
5. bc*d
6. a+bc*d

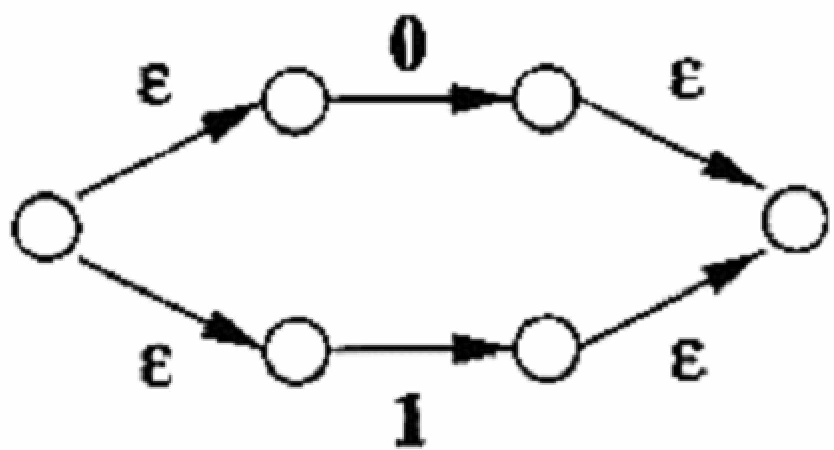
More Examples

$$(0 + 1)^*1(0 + 1)$$

More Examples

$$(0 + 1)^* 1(0 + 1)$$

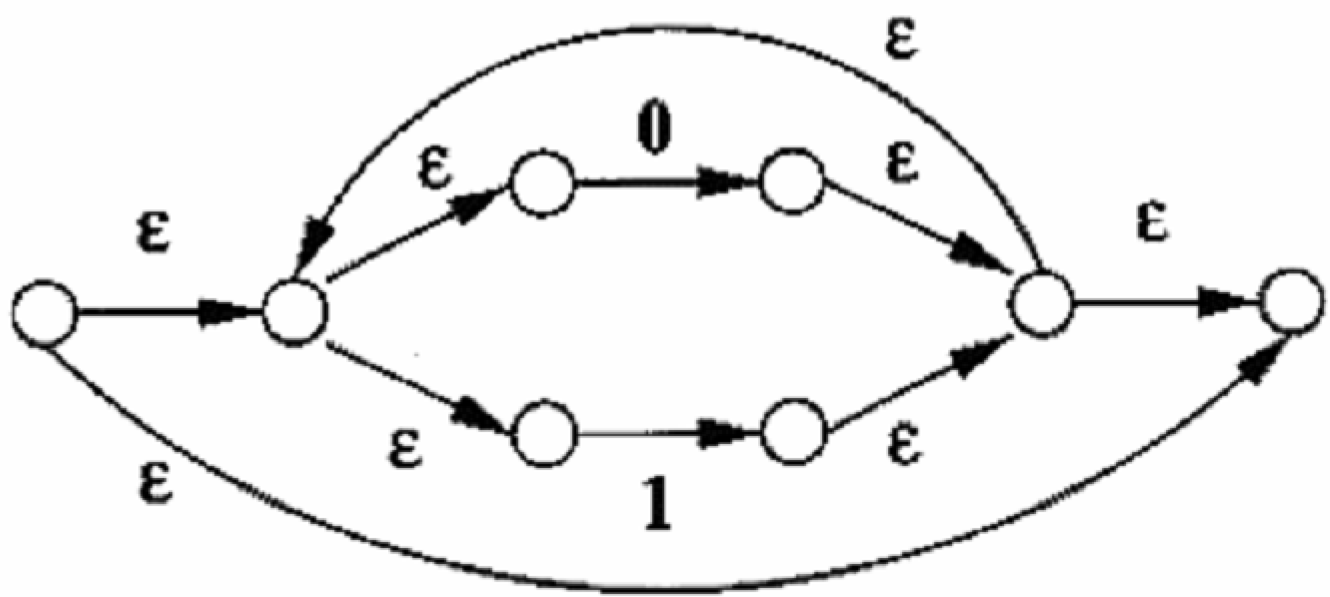
Solution:
Step 1-



More Examples

$$(0 + 1)^*1(0 + 1)$$

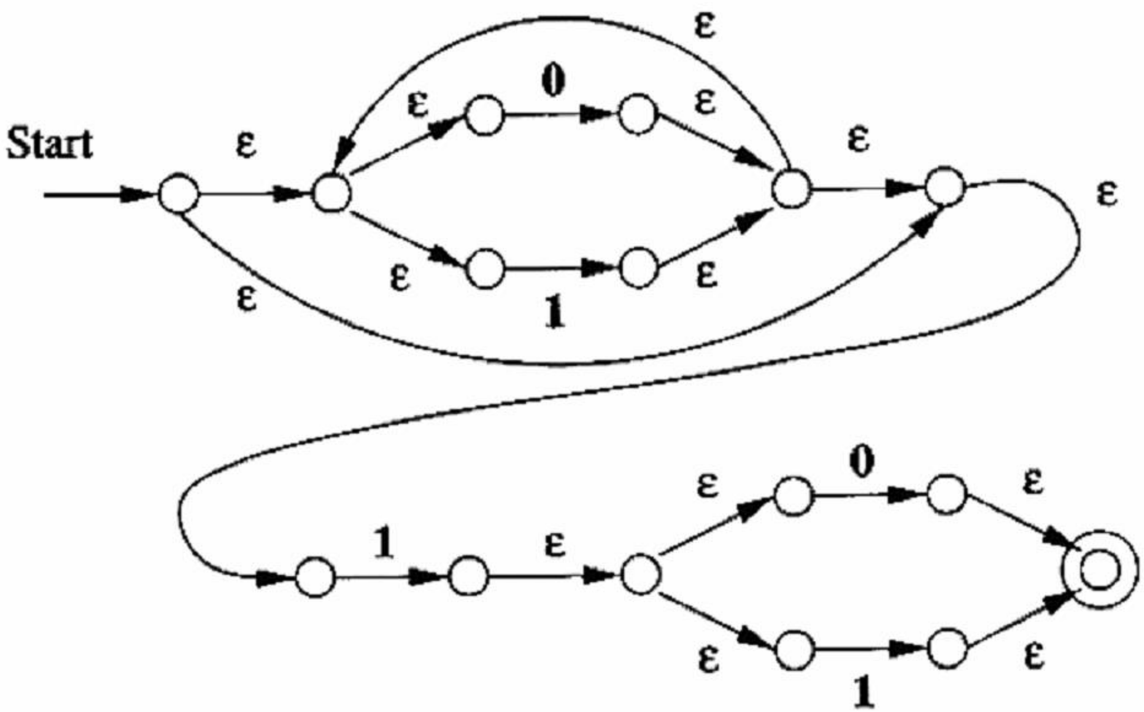
Solution:
Step 2-



More Examples

$$(0+1)^*1(0+1)$$

Solution:
Step 3-



Practice

Exercise 3.2.4: Convert the following regular expressions to NFA's with ϵ-transitions.

* a) $\mathbf{01}^*$.

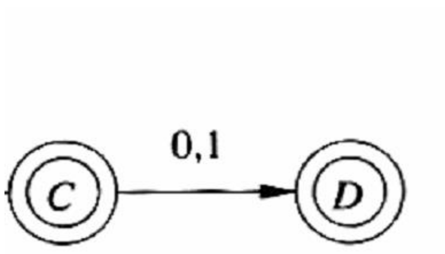
b) $(\mathbf{0} + \mathbf{1})\mathbf{01}$.

c) $\mathbf{00}(\mathbf{0} + \mathbf{1})^*$.

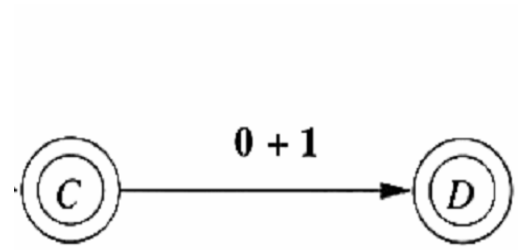
DFA to Regular Expression

1. Convert DFA to GNFA*
2. Convert GNFA to Regular Expression

*GNFA = Generalized NFA (Where transition arrows can have RE)



NFA

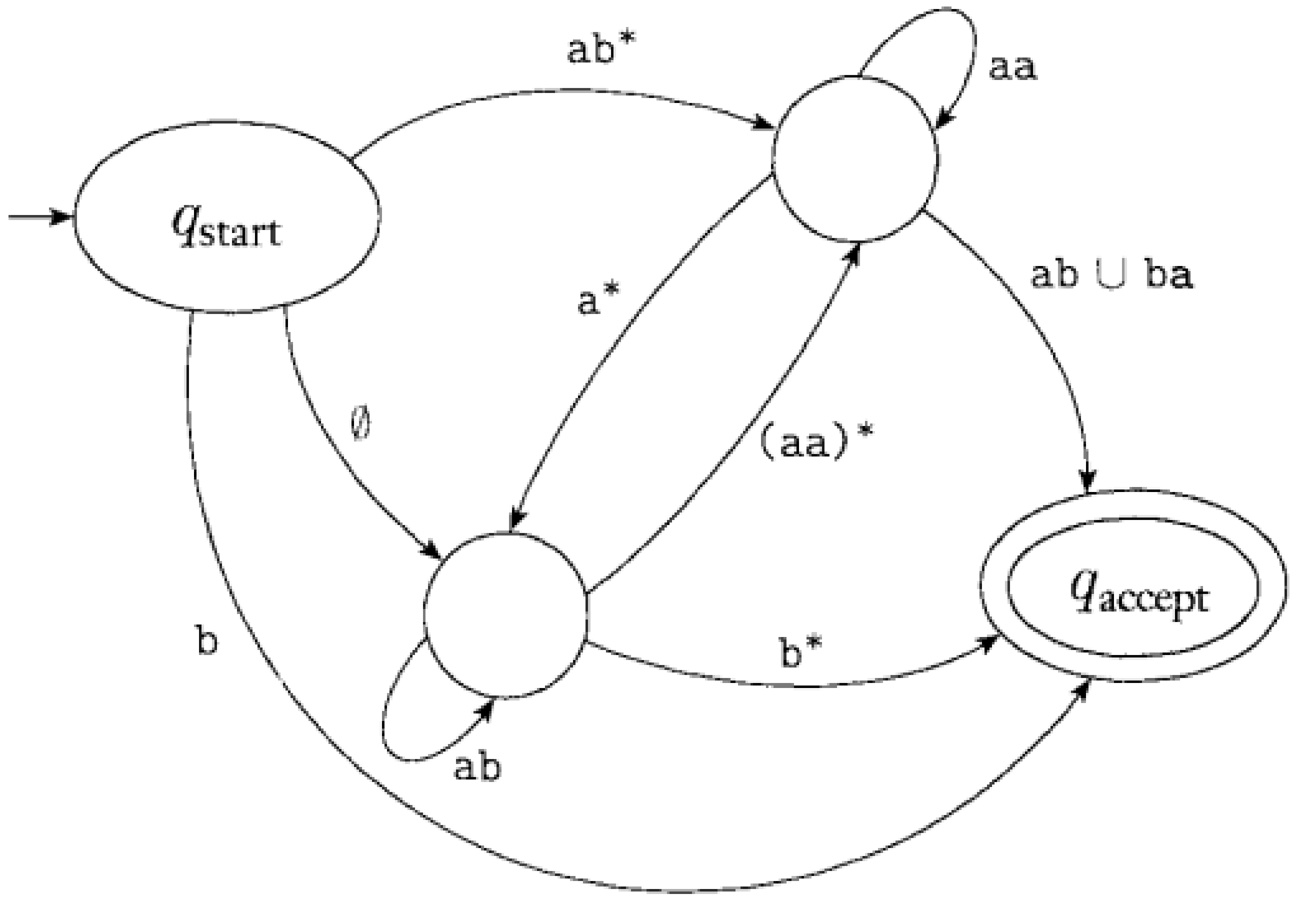


GNFA

Property of GNFA

1. The start state has transition arrows going to every state but no arrows coming in from any other state.
2. There is only a single accept state, and it has arrows coming in from every other state (including itself, ie self-loop) but no arrow going to any other state. This state is not the same as start state.
3. Other than these two, all states have arrows between them.

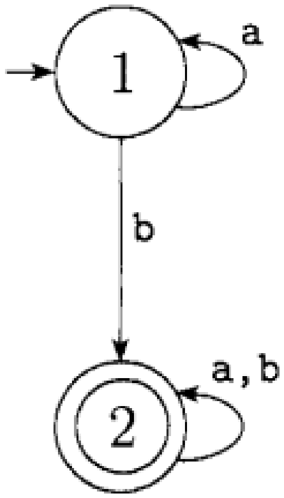
Example of GNFA



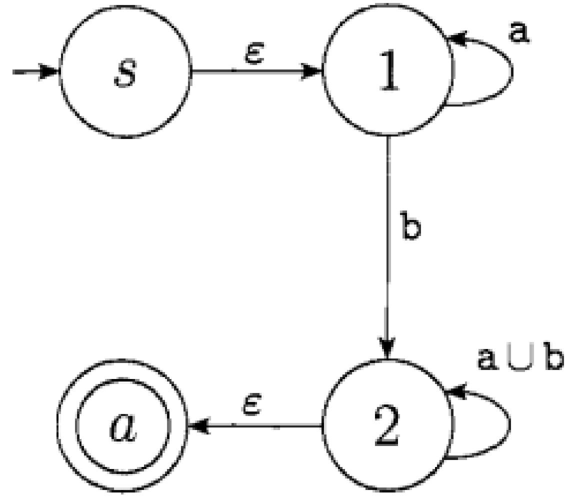
Step 1: DFA to GNFA

1. Add a new start state with epsilon arrow to old start state.
2. Add a new final state with epsilon arrow from old final states.
3. Other than these two, all states having-
   - multiple symbols can be written as union
   - no symbol can be written as ∅

DFA to GNFA Example



(a)



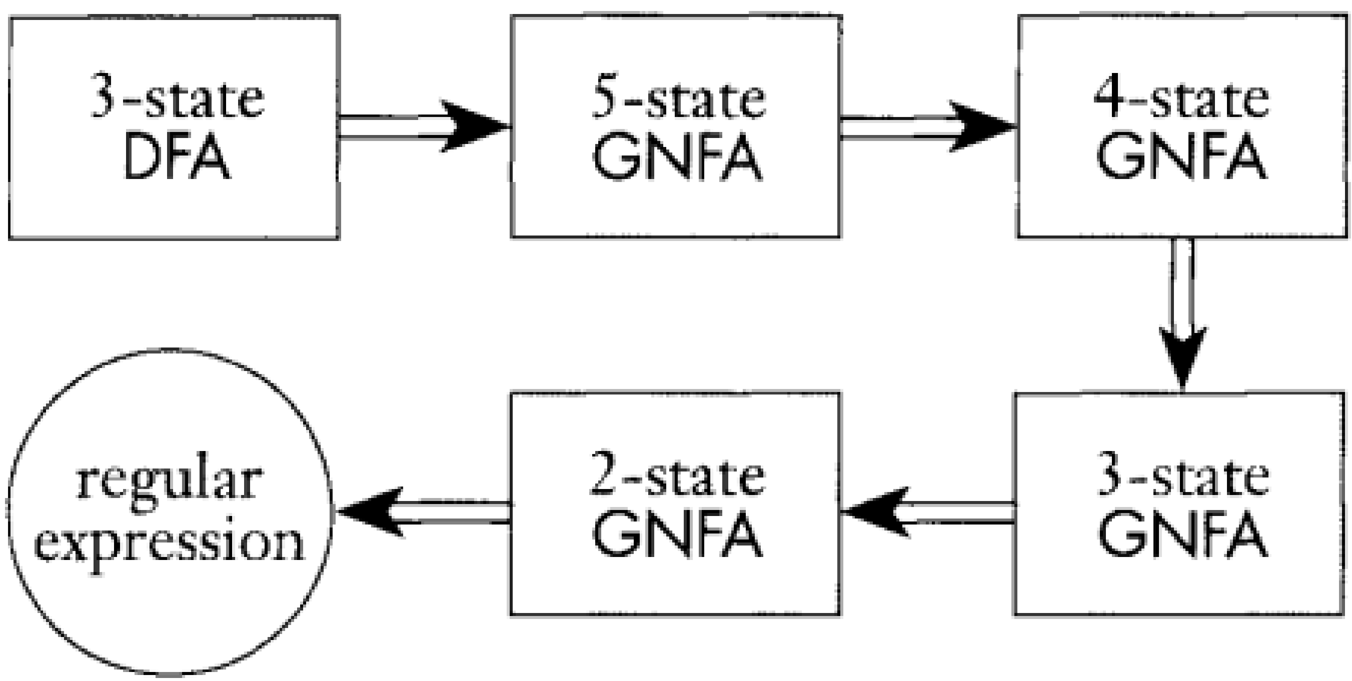
(b)

DFA

GDFA

(Empty arrow from 2 to 1 is ignored)

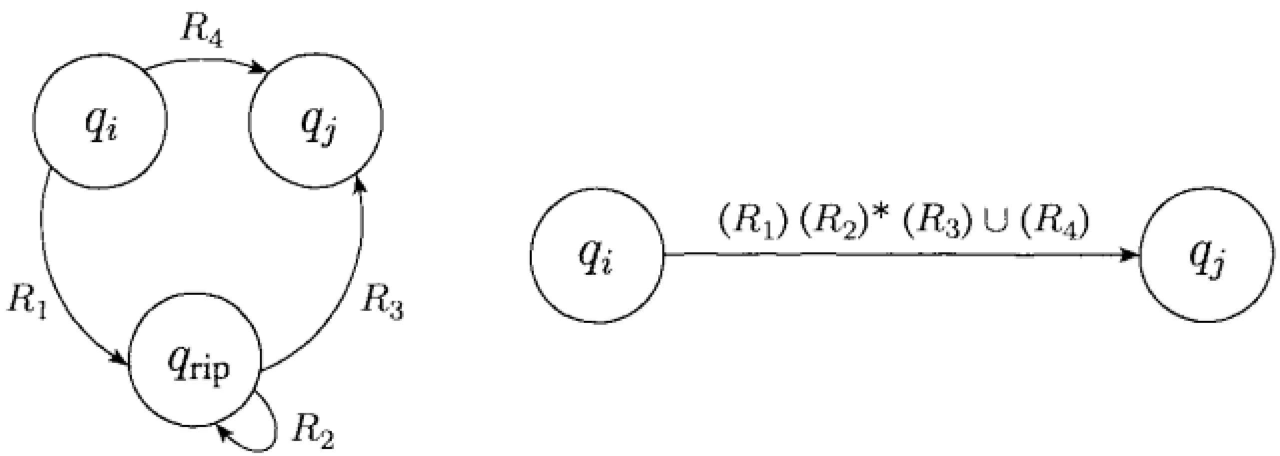
Step 2: GNFA to RE



Step 2: GNFA to RE

Take a middle state, remove it and repair the rest of the states

This is called **State Elimination Technique**

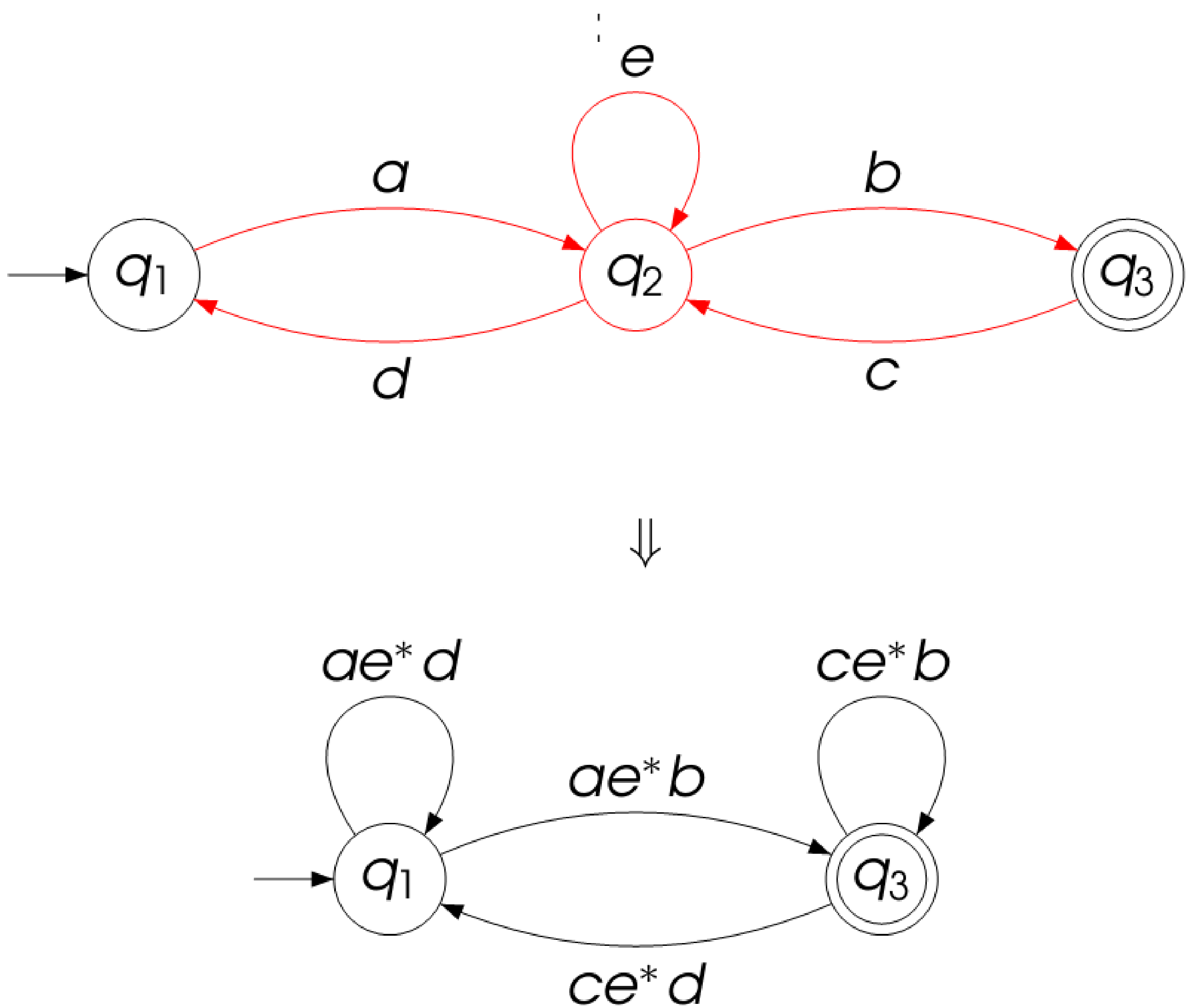


before

after

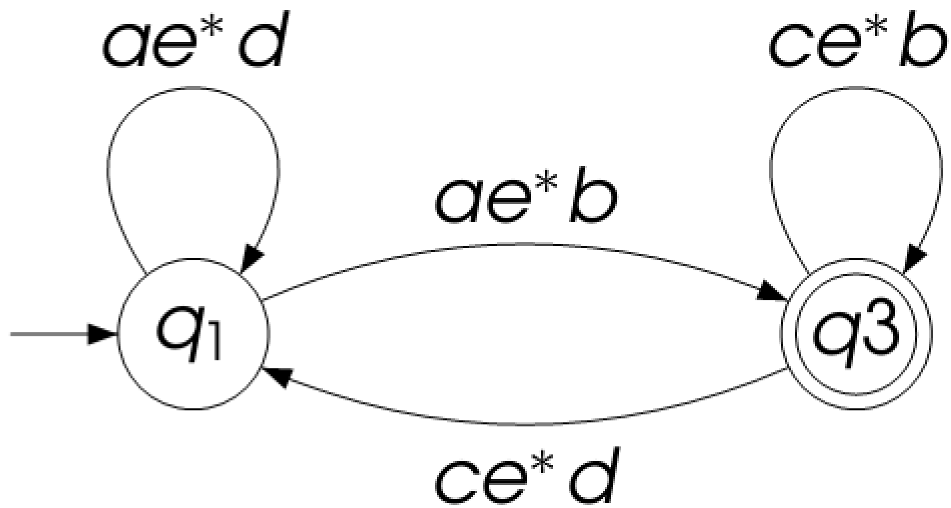
Step 2: GNFA to RE

Another Example



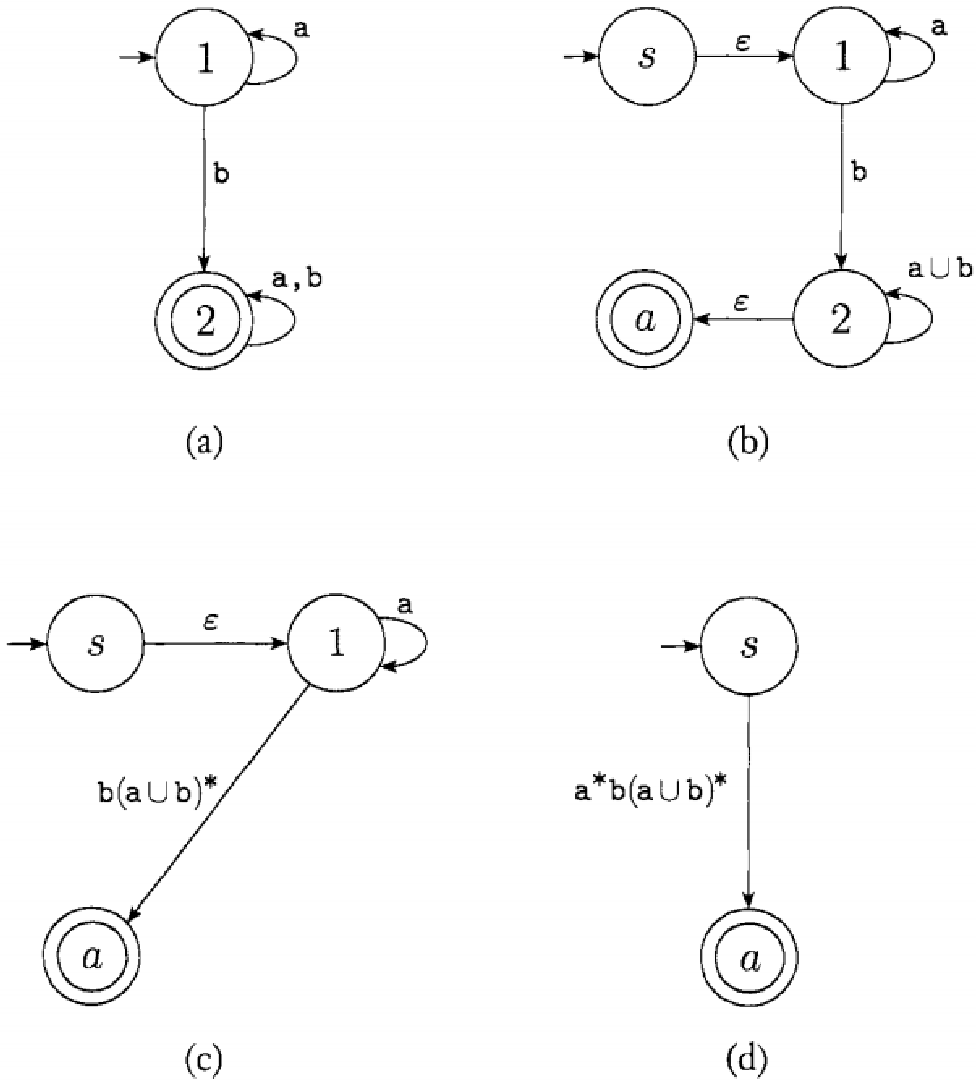
Step 2: GNFA to RE

Final Regular Expression

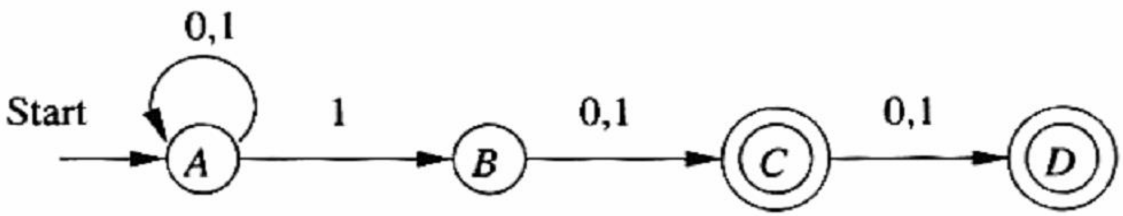


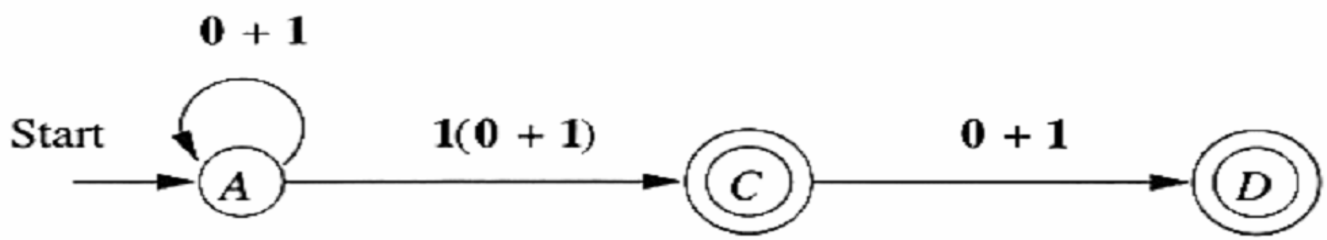
$$r = (ae^* d)^* ae^* b(ce^* b + ce^* d(ae^* d)^* ae^* b)^* .$$

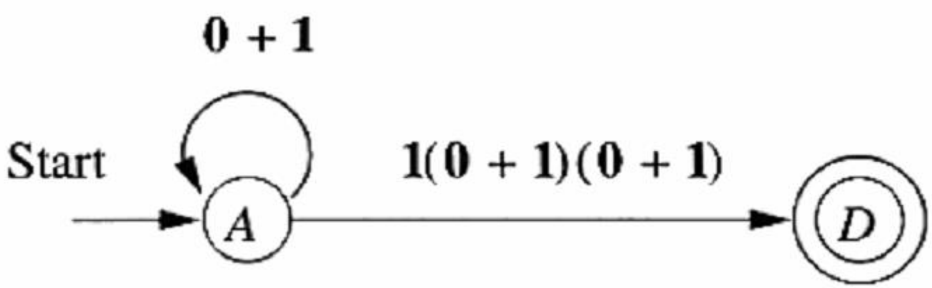
Complete Example: DFA to RE

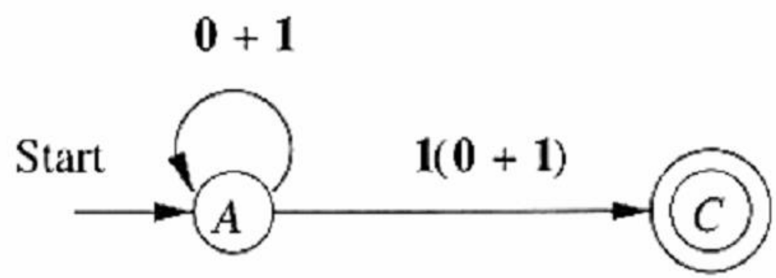


DFA to RE : Example 2





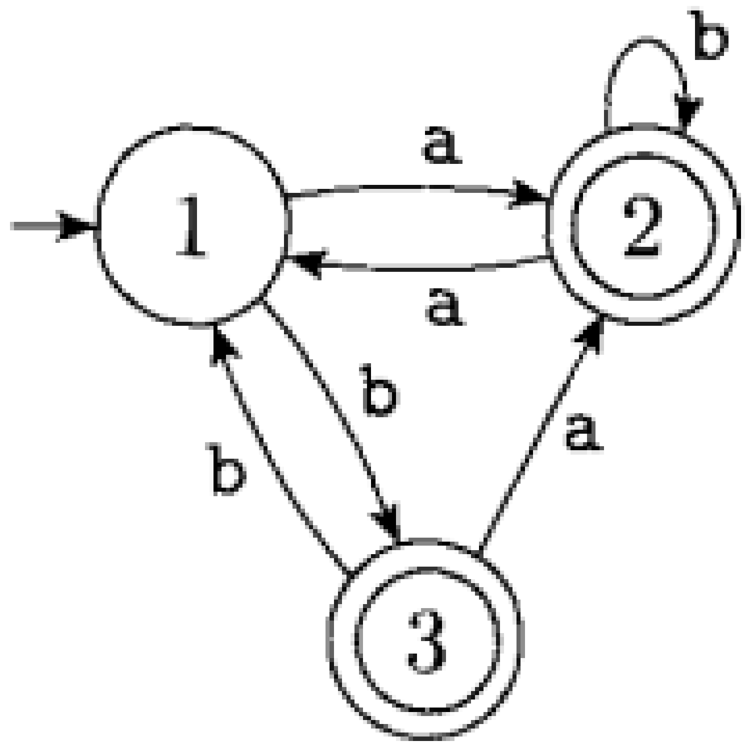




$$(\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1}) + (\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1})(\mathbf{0}+\mathbf{1})$$

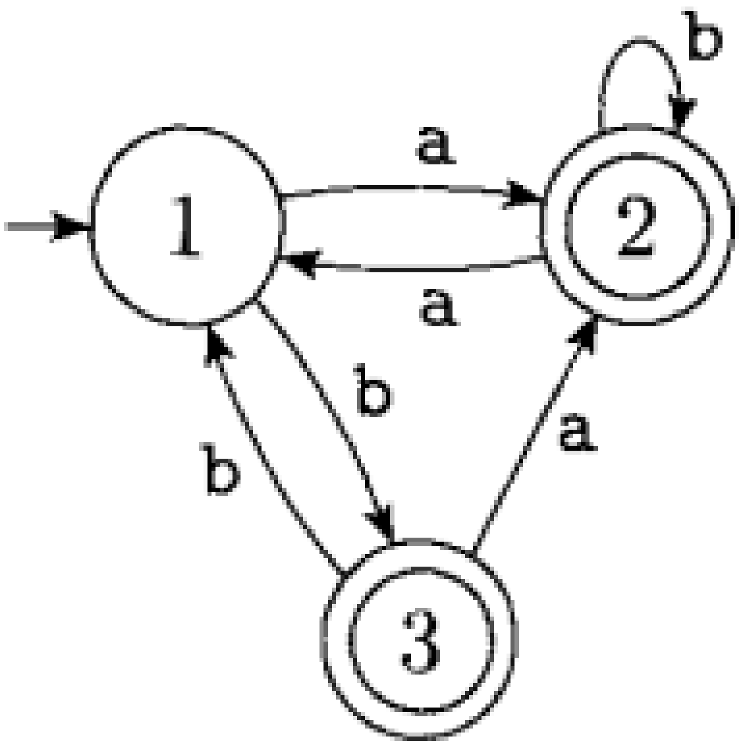
Task

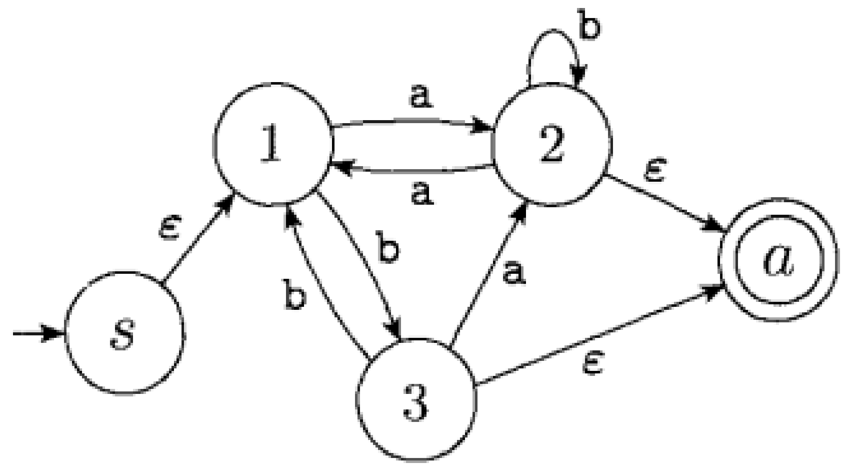
Convert the following DFA to RE



Solution

Conversion from the following DFA to RE

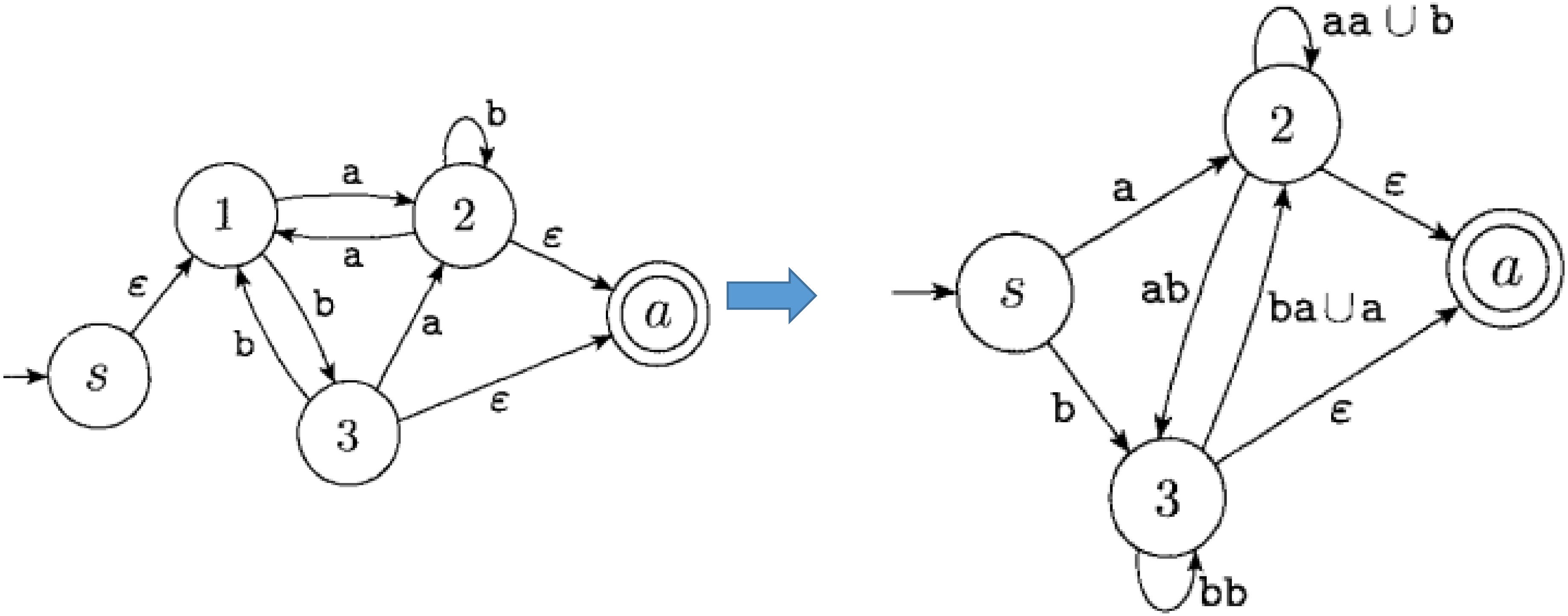




Adding the extra start and final state

Solution

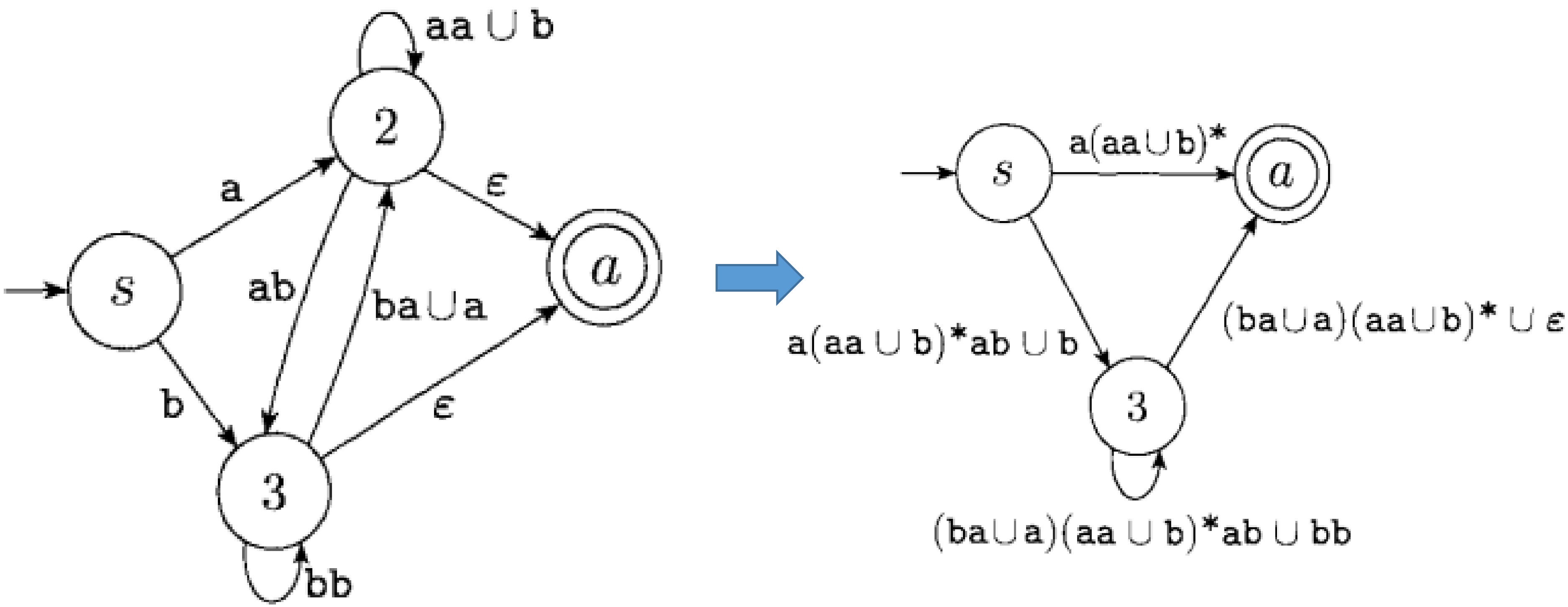
Conversion from the following DFA to RE



Eliminate state 1

Solution

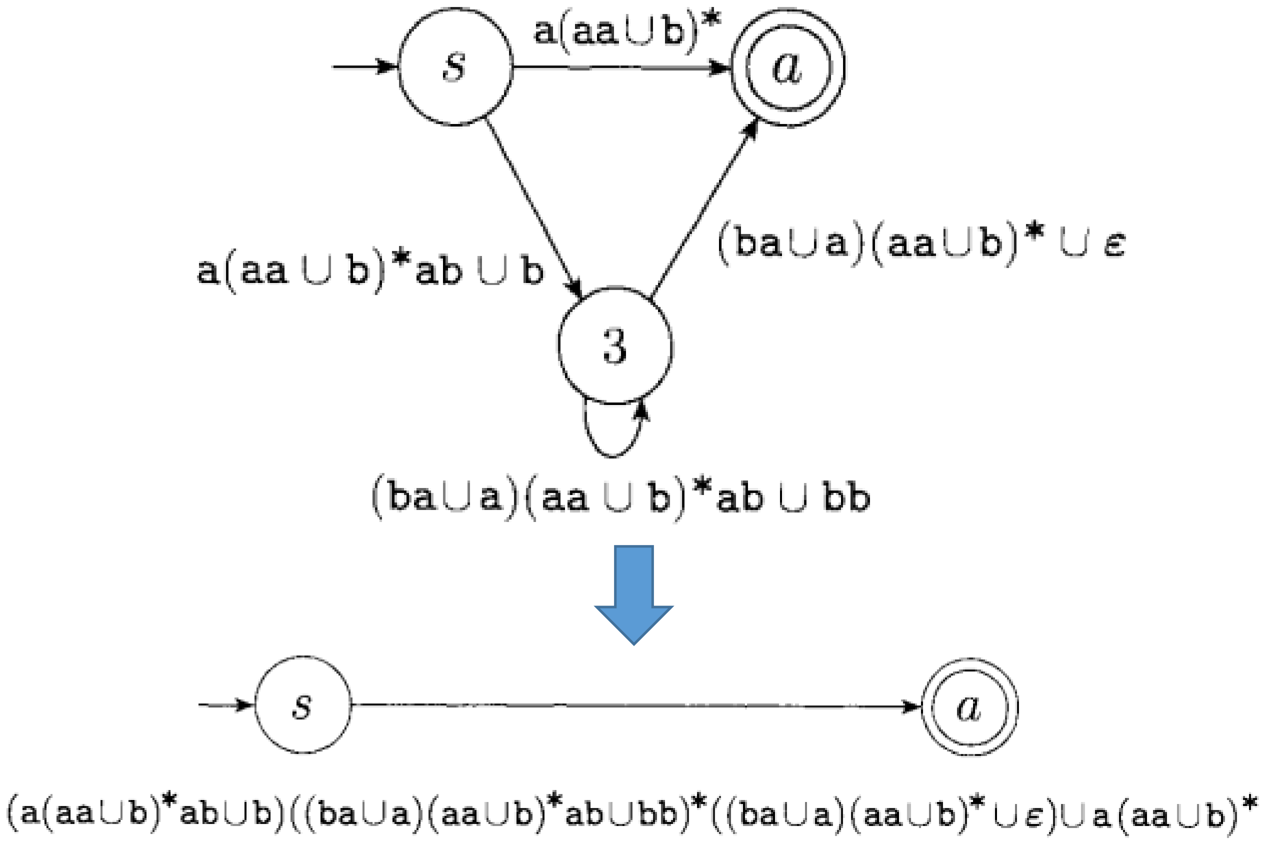
Conversion from the following DFA to RE



Eliminate state 2

Solution

Conversion from the following DFA to RE



Eliminate state 3

Practice

Exercise 3.2.3: Convert the following DFA to a regular expression, using the state-elimination technique of Section 3.2.2.

	0	1
$\rightarrow *p$	s	p
q	p	s
r	r	q
s	q	r