Teacher's Signature:____________
Date: ______________________

United International **University**

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Section: A Batch: ____________ Date: ______________________

Experiment No. 04

Name of the Experiment: Determination of the spring constant and effective mass of a given spiral spring.

Theory:

If a spring is clamped vertically at the end P, and loaded with a mass m_0 at the other end A, then the period of vibration of the spring along a vertical line is given by,

$$T = 2\pi\sqrt{\frac{m' + m_0}{k}} = 2\pi\sqrt{\frac{M}{k}} \quad \dots\dots\dots (1)$$

Where, m' is a constant called the effective mass of the spring and k, the spring constant i.e. the ratio between the added force and the corresponding stretch of the spring.

The contribution of the mass of the spring to the effective mass of the vibrating system can be shown as follows: Consider the kinetic energy of a spring and its load undergoing simple harmonic motion. At the instant under consideration, let the load m_0 be moving with velocity v_0 as shown in the figure.

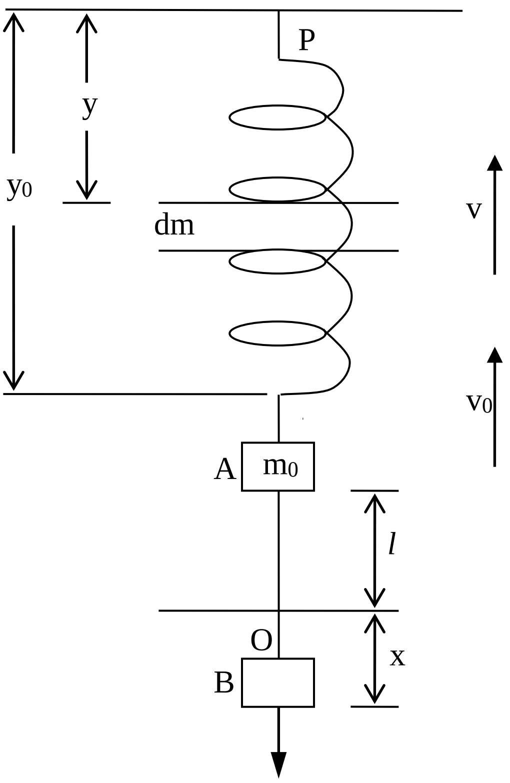


Fig. 01

At the same instant, an element dm of the mass m of the spring will also be moving up but with a velocity v which is smaller than v_0. It is evident that the ratio between v and v_0 is just the ratio between y and y_0. Hence, $\frac{v}{y} = \frac{v_0}{y_0}$ i.e. $v = \frac{v_0}{y_0} y$. The kinetic energy of the spring alone will be $\int \frac{v^2}{2} dm$. But dm maybe written as $\frac{m}{y_0} dy$, where, m is the mass of the spring.

Thus the integral equals to $\frac{1}{2}\left(\frac{m}{3}\right) v_0^2$. The total kinetic energy of the system will then be $\frac{1}{2}\left(m_0 + \frac{m}{3}\right) v_0^2$ and the total mass of the system is therefore, $M = \left(m_0 + \frac{m}{3}\right)$

Hence, effective mass, $m' = \frac{m}{3}$. The applied force $m_0 g$ is proportional to the extension l within the elastic limit.

$$\text{Therefore, } mg = kl \text{ or, } l = \frac{g}{k} m$$

$$k = \frac{g}{l/m} = \frac{g}{\text{slope of l vs m graph}} =$$ (980/0.2) = 4900 dyne/cm = 49 N/m

Apparatus:

- A spiral spring
- Convenient masses with hanging arrangement
- Clamp or a hook attached to a rigid framework of heavy metal rods
- Weighing balance
- Stopwatch and scale

Experimental Data:

(A) Initial length of the Spring, L_0 = 48 cm

(B) Table for determining extensions and time periods:

No. of Obs.	Added Loads, m_0 (gm.)	Length of the Spring, L (cm)	Extension, $l = L_0 - L$ (cm.)	No. of vibrations	Total time (sec.)	Period, T (sec.)	T^2
1	50	56	8	20	11.51	0.58	0.33
2	100	65	17	20	16.22	0.81	0.66
3	150	72	24	20	19.92	0.99	0.99
4	200	81	33	20	22.91	1.145	1.31
5	250	89	41	20	25.59	1.28	1.64

Calculation:

(A) Effective mass, $m^{/}$ (from graph) = 30 g

(B) Spring Constant $k = \frac{g}{\text{slope of l vs m graph}} =$ (981/0.16) = 6131.25 dyne/cm

(C) Difference = [(Experimental Result ~ Theoretical Result)/Theoretical Result] x100%
= ((30 - 16.67) / 16.67) * 100 = 79.96% =

(D) Accuracy= 100% - % Difference = (100 - 79.96) = 20.04%

Results:

(A) Value of the Effective Mass, *m´* = 30

(B) Value of the Spring Constant, *k* = 6131.25 dyne/cm

Discussions:

Q: What do you understand by the term *Spring Constant*?

A characteristic of a spring which is defined as the ratio of the force affecting the spring to the displacement caused by it. Usually denoted with the letter "k" in formulae, as in the formula F = kx, where "F" is the force applied and "x" is the displacement.

Q: What is the *Effective Mass* of a spring?

The effective mass of the spring in a spring-mass system when using an ideal spring of uniform linear density is 1/3 of the mass of the spring and is independent of the direction of the spring mass system.

Q: Infer the extension of your spring when a load of 300 gm is used from the load ~ extension curve.

(102-48) cm = 54 cm

Q: Does the time period of oscillation depends on the displacement from the equilibrium position? Explain

time period does not depends on the displacement from equilibrium position. Spring will move faster but time wont change. From equation of period $T=2\pi \sqrt{(m/k)}$

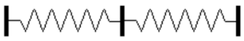
Q: What happens to the time period if you keep increasing the load?

If the load is increased then time period will increase. From equation, $T=2\pi \sqrt{}(m/k)$
Here k and π constants. So , $T2 \propto m$.
So , if we increase mass then time period will increase.

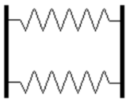
Q: What type of motion does the spring-block system have? Write the differential equation for such motion. Write the solution of the differential equation.

Simple harmonic motion . Differential equation for simple harmonic motion is $d2x/dt2 + \omega 2x = 0$

Q: Suppose two spring with the same spring constant *k*. Draw the diagram, when these two springs are in (i)series and (ii) parallel combination.



Springs in Series



Springs in Parallel

Show that the equivalent spring constant

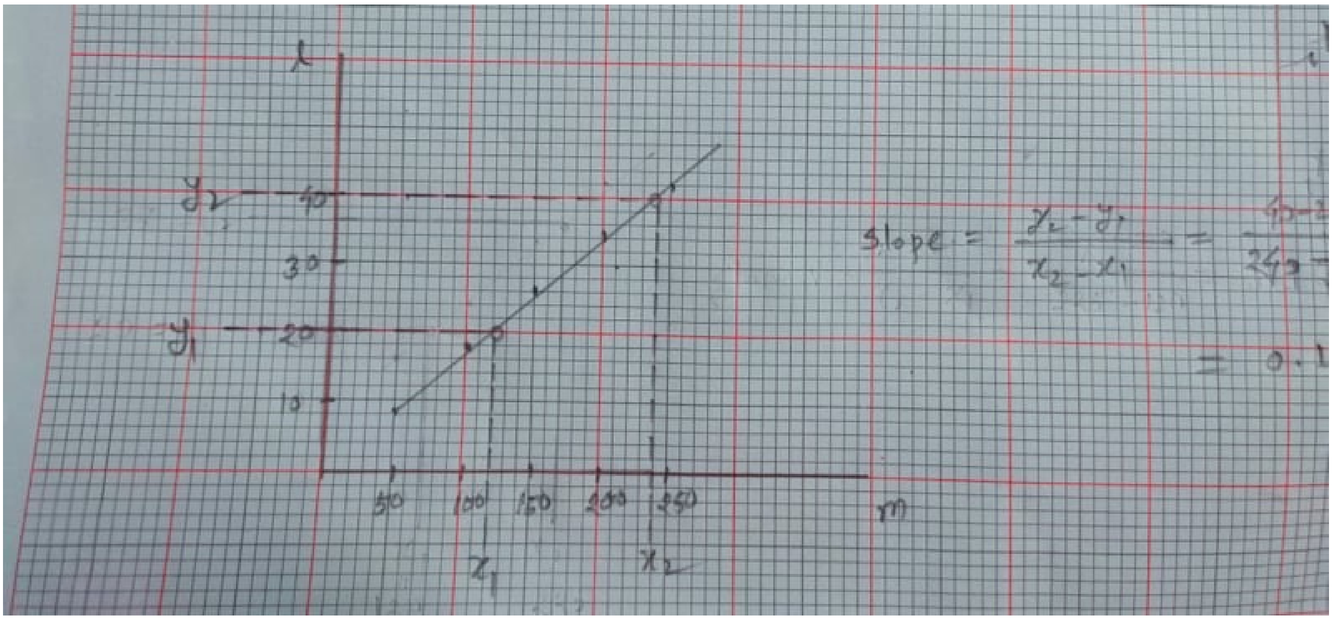
(i) $k_{series} = \frac{k}{2}$

When the spring joined in series the total extension in spring is,
$y=y1+y2=-F/k1-F/k2$
$\Rightarrow y=y1+y2=-F/k1-F/k2$
$\Rightarrow y=F[k1+k2] \Rightarrow$

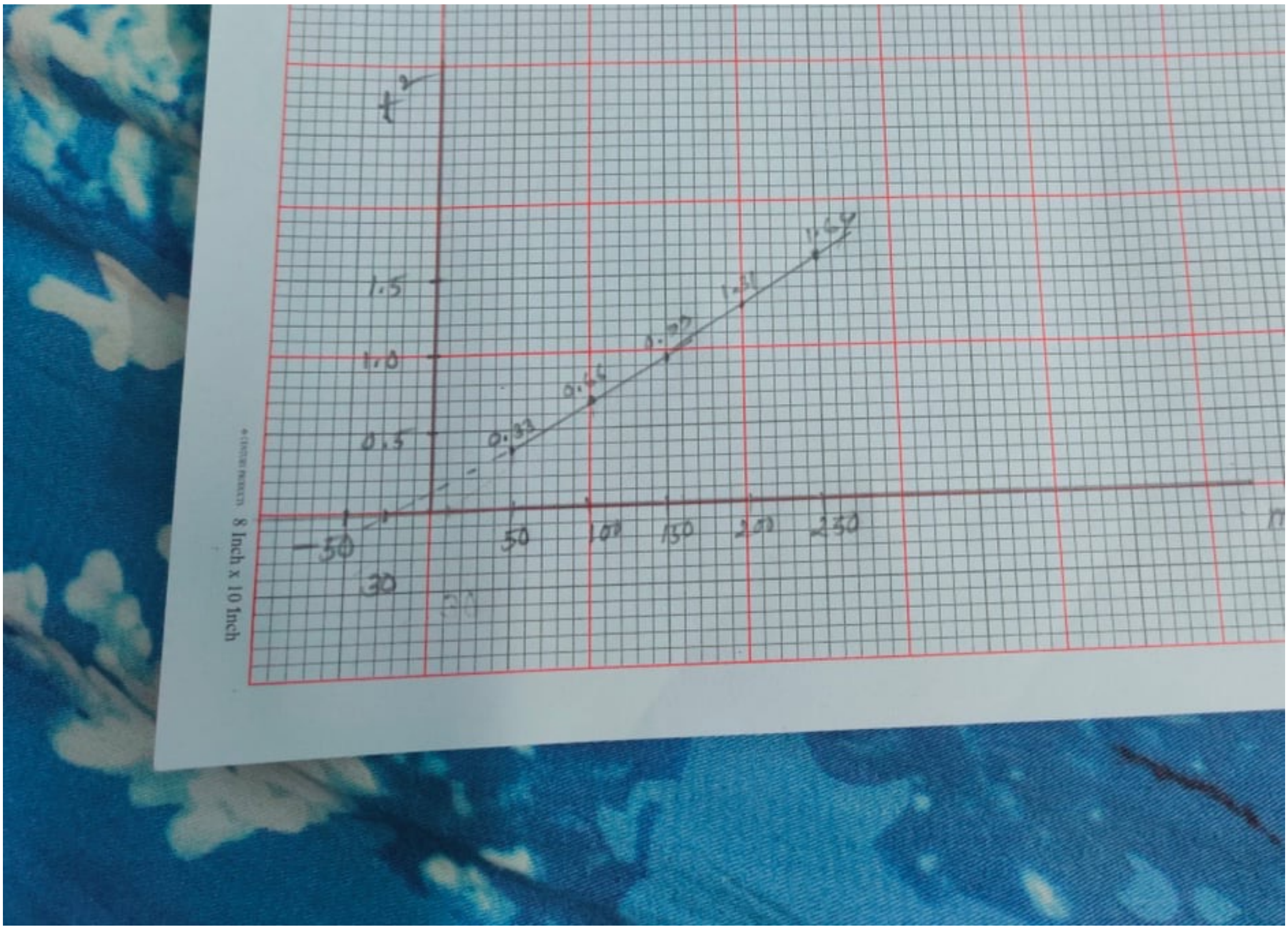
Thus spring constant in this case becomes,
$\Rightarrow k=k1k2/(k1+k2)$
As $k1=k2 = k'$,
$K = k'2/2k'$ $K=k'/2$

(ii) $k_{parallel} = 2k$

When the spring joined in parallel the total extension in spring is,
$F=F1+F2$
$kx = k1x+k2x$ $k= k1+k2$
as $k1=k2=k'$ $k=2k'$



l
y2
40
30
y1
20
10
50
100
150
200
250
m
x1
x2
slope = (y2 - y1)/(x2 - x1) =



t2
1.5
1.0
0.5
0.33
1.64
50
100
150
200
250
-50
30
8 Inch x 10 Inch