STUDENTS' GUIDE TO SIMPLE HARMONIC MOTION

(Review Slide Part: 1.1)

(And YouTube links so you all can pass)

Course: PHY2105

United International University
QUEST FOR EXCELLENCE

Motion: The rate of change of position of an object/particle/body with time.

Periodic Motion: A motion that repeats itself after an equal interval of time.

Periodicity: The property of having the same value / being at the same position after an equal interval of time.

Oscillatory motion / simple harmonic motion / vibration: When a particle/body is subjected to a force (called "the restoring force") which is proportional to the displacement of the particle/body from the equilibrium, it exhibits a specific kind of periodic motion where, it *goes back and forth between two extreme points between set interval of time.*

So the basis of oscillatory motion / simple harmonic motion / vibration is to have a restoring force which is proportional to the displacement of the particle from equilibrium.

Examples:

- Power line oscillates when the wind blows past it
- Earthquake oscillations move buildings
- Block attached to a spring
- Motion of a swing
- Motion of a pendulum
- Vibrations on a stringed musical instrument
- Back and forth motion of a piston
- Vibrations of a Quartz crystal

Difference:

Generic harmonic motion: no restoring force, such us the motion of hands of clock. But Oscillation has a restoring force that causes the body it is active on to go in one direction for half the time, and in the opposite direction for other half of time

Important terms of a simple harmonic motion: Amplitude, Time Period, Frequency, Initial Phase

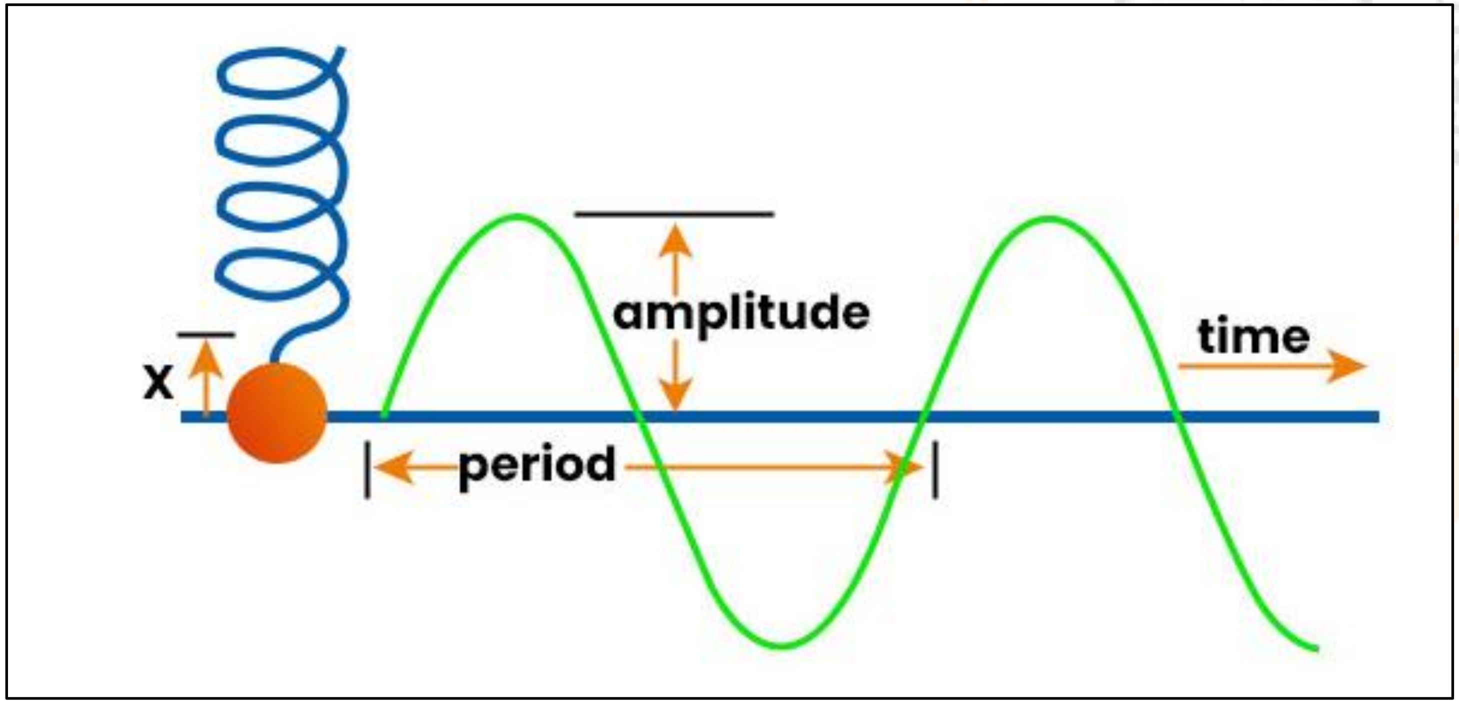
Amplitude, A: The amplitude of the motion, denoted by A, is the maximum magnitude of displacement from the equilibrium position. It is always positive

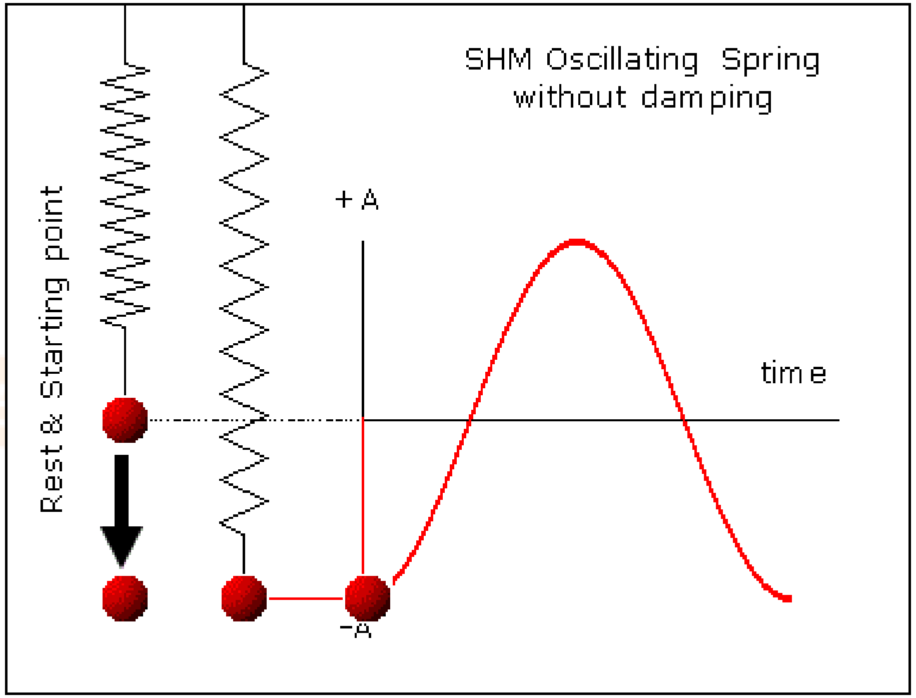
Period, T: The period T, is the time required for one complete oscillation, or a cycle.

Frequency, f: The frequency, f, is the number of cycles completed in a unit time.

Angular velocity, ω: The angle covered per second. A whole cycle is equal to 2π. Therefore, ω=number of cycles per second x $2\pi = 2\pi f$

Initial phase, φ: The initial phase is the angle added to the ωt term in the wave equation, which indicates the angle by which the wave is shifted from perfect sine / cosine graph.





Finding the value of angular velocity of simple harmonic motion (For spring system):

We know, at any point for a particle with simple harmonic motion, the restoring force is proportional to the displacement of the particle from equilibrium. Therefore,

$$F_{restoring} \propto -x$$

$$\text{Or, } F_{restoring} = -kx$$

[Where, k is the spring constant and x is negative because the restoring force has opposite direction of displacement]

$$\text{We know, } F_{restoring} = \text{m. a}$$

Where, m is the mass of the particle, and a is the resulting acceleration due to the restoring force

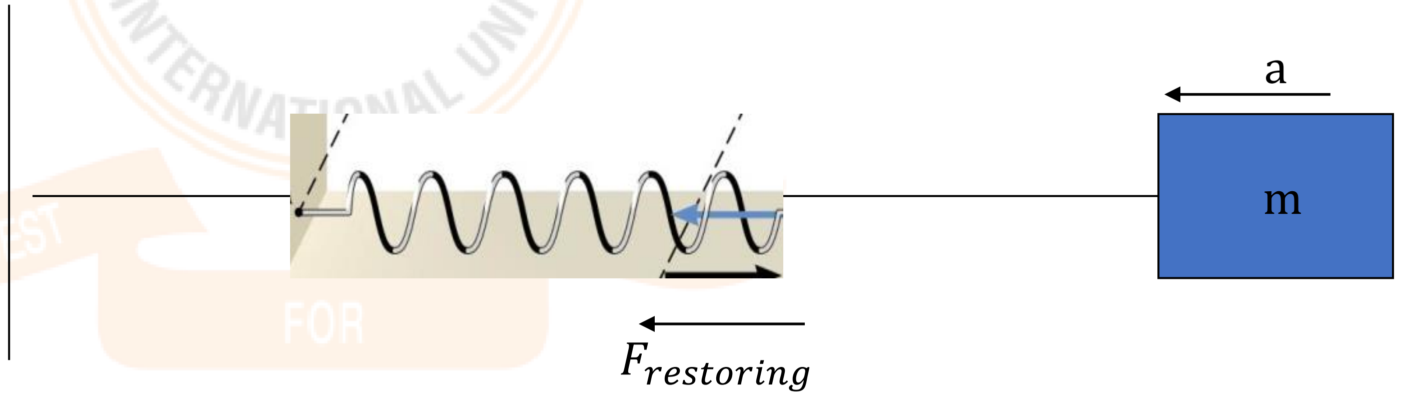
$$\text{Since, a} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The final equation stands as,

$$F_{restoring} = m.\frac{d^2x}{dt^2} = -kx$$

$$\text{Or, } \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\text{Or, } \frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$



Since, the equation is: $\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$, a typical form of solving the equation is to "Guess" a value for x. Which you can take either:
a) $x = Acos(\omega t + \varphi)$
Or, b) $x = e^{\lambda t}$

The process for taking $x = e^{\lambda t}$ is more methodical, but a longer process.
You can see the solution process here: https://www.youtube.com/watch?v=k584Zgb_5HM&ab_channel=VirtuallyPassed

However, for exam, you can simply use: $x = Acos(\omega t + \varphi)$
Which returns, $v = \frac{dx}{dt} = -A\omega sin(\omega t + \varphi)$
And, a $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -A\omega^2 cos(\omega t + \varphi)$

Then, solution to the equation;

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

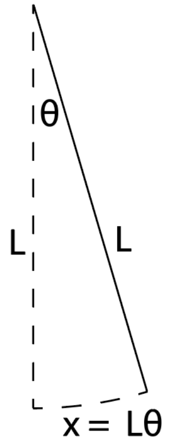
$$-A\omega^2 cos(\omega t + \varphi) + \frac{k}{m}Acos(\omega t + \varphi) = 0$$

$$\omega^2 = \frac{k}{m}$$

Which, returns the value, $\omega = \sqrt{\frac{k}{m}}$

Finding the value of angular velocity of simple harmonic motion (Simple Pendulum):

We know, at any point for a particle with simple harmonic motion, the restoring force is proportional to the displacement of the particle from equilibrium.



From the diagram below, on right, we can find from the free body diagram that, the restoring force is the tangential component of the gravitational force, mg sin θ

Therefore, $F_{restoring} = \text{mg} \sin\theta = \text{mg}\,\theta$

(Since, for small angles, $\sin\theta = \theta$)

We know, We know, $F_{restoring} = \text{m.a}$

Since, $\text{a} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

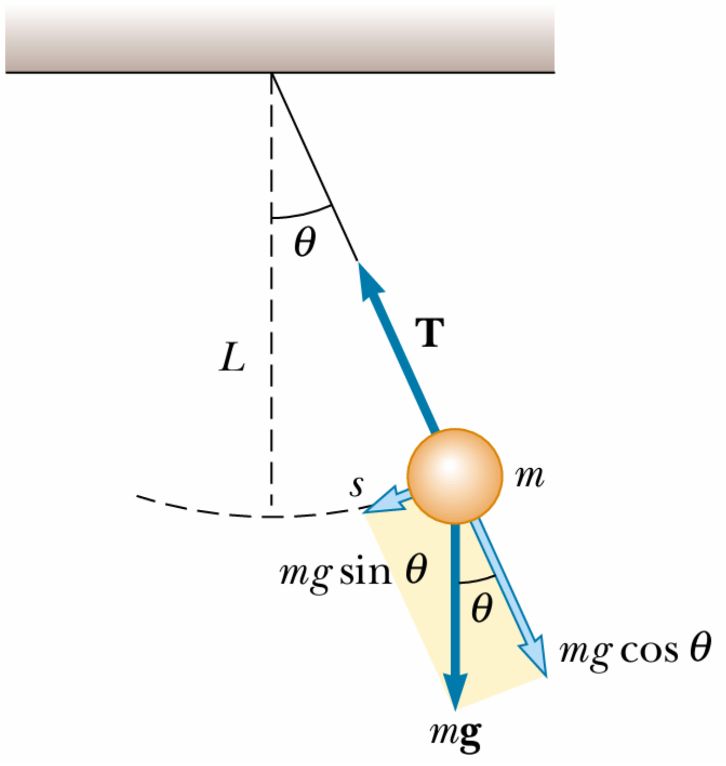
As, $\text{x} = \text{L}\theta$

$$\text{a} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \frac{d^2(L\theta)}{dt^2} = L\frac{d^2(\theta)}{dt^2}$$

The final equation stands as,

$$F_{restoring} = mL.\frac{d^2(\theta)}{dt^2} = \text{mg}\,\theta$$

$$\frac{d^2(\theta)}{dt^2} + \left(\frac{\text{g}}{\text{L}}\right)\theta = 0$$



Assuming, $x = Acos(\omega t + \varphi)$

Returns, $v = \frac{dx}{dt} = -A\omega sin(\omega t + \varphi)$

And, a $= \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = -A\omega^2 cos(\omega t + \varphi)$

Then, solution to the equation;

$$\frac{d^2x}{dt^2} + \frac{g}{L}x = 0$$

$$-A\omega^2 cos(\omega t + \varphi) + \frac{g}{L}Acos(\omega t + \varphi) = 0$$

$$\omega^2 = \frac{g}{L}$$

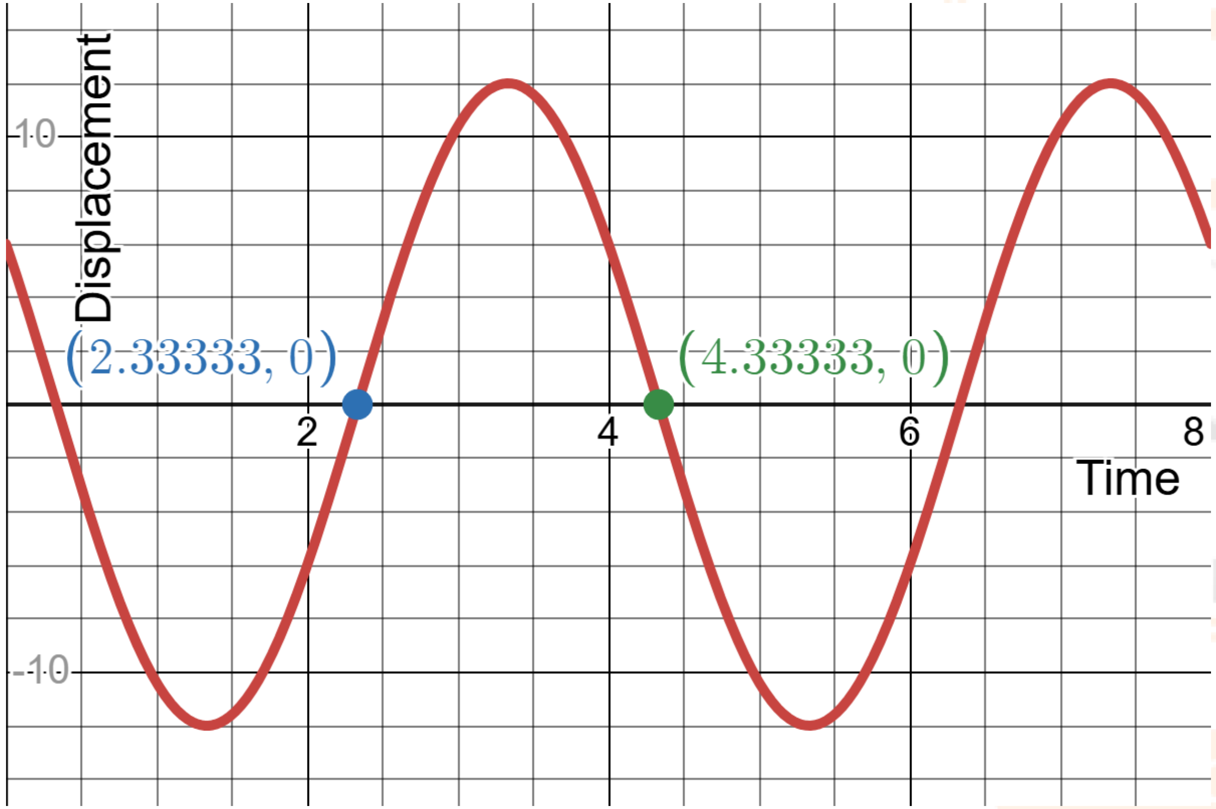
Which, returns the value, $\omega = \sqrt{\frac{g}{L}}$

Read Halliday and Resnick Ch 13 for details on this approach.

Equations related to a SHM:

Parameters	**Spring System Representation**	**Simple Pendulum Representation**
Amplitude, A	Amplitude, A	Amplitude, A
Angular velocity, ω	Angular velocity, $\omega = \sqrt{\frac{k}{m}}$ Where, m = suspended mass k = Spring constant of the spring	Angular velocity, $\omega = \sqrt{\frac{g}{L}}$ Where, L = Effective Length g = Gravitational acceleration
Frequency, f = $\frac{\omega}{2\pi}$	Frequency, f = $\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$	Frequency, f = $\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$
Time period, T = $\frac{1}{f}$	Time period, T = $\frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$	Time period, T = $\frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$
Total Energy, E=E_T	Total Energy, E=$E_T = \frac{1}{2}KA^2$	Total Energy, $E_T = mgL(1 - cos\theta_{max})$
Potential energy, E_P	Potential energy, $E_P = \int_0^x kx.dx = \frac{1}{2}kx^2$ $= \frac{1}{2}kx^2 = \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \varphi) = \frac{1}{2}m\omega^2x^2$	Potential energy, E_P $= E_T = mgL(1 - cos\theta)$
Kinetic Energy, E_k	Kinetic Energy, $E_k = \frac{1}{2}mv^2$ $= \frac{1}{2}mA^2\omega^2\sin^2(\omega t + \varphi)$ $= \frac{1}{2}kA^2\sin^2(\omega t + \varphi)$ $= \frac{1}{2}k\sqrt{A^2 - x^2}$	Kinetic Energy, E_k E_k = Total energy – Potential Energy $= E_T - E_p$ $= mgL(cos\theta - cos\theta_{max})$

Problem: How to find the equation of a SHM from graph:



Step 1: Find the Amplitude
From the graph, the highest displacement is 12 m
Therefore, A = 12 m

Step 2: Find the time period
From the graph, time for a half wave = (4.33333-2.33333) = 2 s
Therefore, Time for a full wave = 4s = Time period

Step 3: Find Angular velocity, ω
We know, $\omega = 2\pi f = 2\pi/T = 2\pi/4 = \pi/2$ rad/sec

Step 4: Find the initial phase,

As time = 0, x = 6 (from the graph)

Assuming the equation to be cosine, $x = A\cos(\omega t+\varphi)$
Therefore, $6 = 12\cos(\pi/2 * 0 + \varphi)$
Or, $1/2 = \cos(\varphi)$
Or, $\cos(\pi/3) = \cos(\varphi)$
Or, $\varphi = \pi/3$

Therefore, the equation = $12\cos\left(\frac{\pi}{2}t+\frac{\pi}{3}\right)$

Other helpful equations:

1. Angular velocity ω = 2πf = 2π/T
2. Time period T = 1/f
3. Velocity at any point v =$-\omega\sqrt{A^2 - x^2} = -\omega A \sin(\omega t + \varphi)$
4. Maximum velocity is at x = 0, maximum velocity $v_{max} = -\omega A$
5. Acceleration at any point, a = $-\omega^2 x = -\omega^2 A \cos(\omega t + \varphi)$
6. Maximum acceleration is at x = A, and maximum acceleration $a_{max} = -\omega^2 A$
7. If the velocity, and acceleration is given at any point as v_i and a_i

Then, Amplitude, A can be found using = $\sqrt{\frac{v_i^2}{\omega^2} + \frac{a_i^2}{\omega^4}}$

Useful links:

On basics of simple harmonic motion:

Book: Halliday and Resnick, Chapter 13, Page 390 to 404
University Physics, Chapter 14, Up to Physical Pendulum

YouTube Lectures:

i. Basic of SHM Math, Spring-Mass System etc.:
https://www.youtube.com/watch?v=iubb3eFBQ9U&t=740s&ab_channel=TheOrganicChemistryTutor
ii. Simple Pendulum: https://www.youtube.com/watch?v=1Q15fgz-lUk&t=1395s&ab_channel=TheOrganicChemistryTutor
iii. Others: https://www.youtube.com/watch?v=QvnmfBqdVIQ&ab_channel=TheOrganicChemistryTutor
https://www.youtube.com/watch?v=Hs_GeBx15Ws&ab_channel=PhysicswithProfessorMattAnderson (Recommended)
https://www.youtube.com/watch?v=zJtCmpH--70&ab_channel=KhanAcademy (Graph of energy, very recommended)

Do go over the practice problems which were provided before class test 1

[N.B: Physical Pendulum is not discussed. Will provide another updated slide with derivation for physical pendulum and other topics]

(In case you need something, mail me at Nayeem.zahid12@gmail.com . Please do not call me – I do not receive calls. Drop a text in WhatsApp if it is an emergency. Else, mailing is fine)