CSE-233 : Section A

Summer 2020

# Conversion from NFA to DFA

Reference:

Book2 Chapter 1.2

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# Equivalence of NFA and DFA

- Every language that can be described by some NFA can also be described by some DFA
- DFA in practice has almost the same states as NFA, but it has more transitions
- In worst case the smallest DFA can have $2^n$ transitions (for a smallest NFA with n states)

From NFA to DFA:

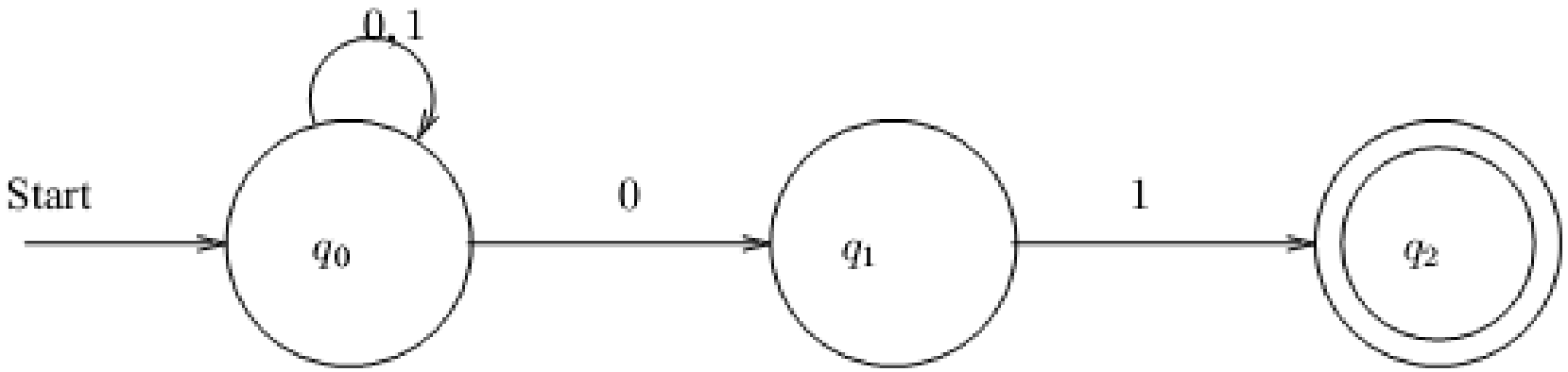
- We have a NFA N
- The goal is to construct a DFA D where L(D) = L(N)

$$N = (Q_N, \Sigma, \delta_N, q_0, F_N) \quad D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$$

# Formal Idea

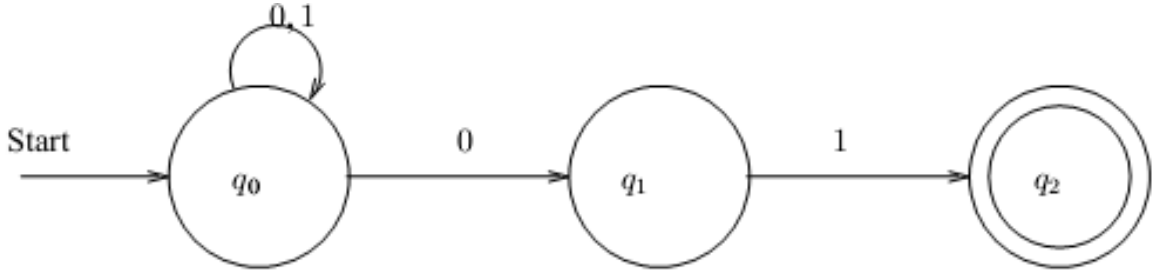
| | NFA | DFA |
|---|---|---|
| states | $q_0, q_1, \ldots, q_n$ | $\{\ \}, \{q_0\}, \{q_1\}, \{q_0,q_1\}, \ldots, \{q_0,\ldots,q_n\}$<br>one for each subset of states in the NFA |
| initial state | $q_0$ | $\{q_0\}$ |
| transitions | d | $d'(\{q_{i1},\ldots,q_{ik}\}, a) =$<br>$d(q_{i1}, a) \cup \ldots \cup d(q_{ik}, a)$ |
| accepting states | $F \subseteq Q$ | $F' = \{S: S$ contains some state in $F\}$ |

# Example




We'll need to figure out the equivalent function for DFA

# Equivalent DFA Table



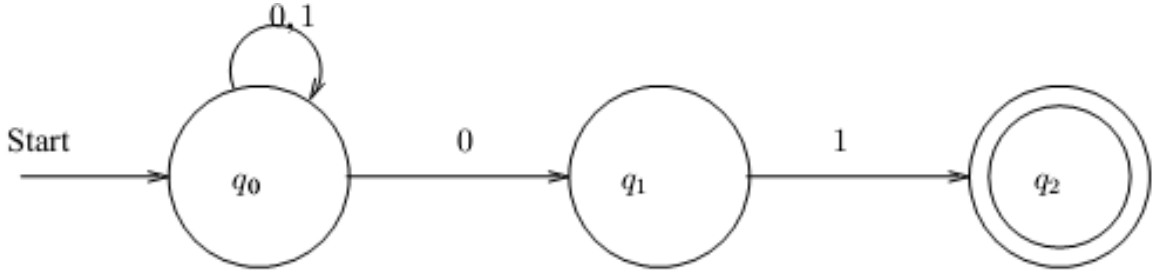

$Q_N = \{q_0, q_1, q_2\}$ then $Q_D = \{\emptyset, \{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_1\} \ldots\}$, *i.e.*, $Q_D$ has $8$ states (each one corresponding to a subset of $Q_N$)

| | | 0 | 1 |
|---|---|---|---|
| | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\rightarrow$ | $\{q_0\}$ | $\{q_0\ q_1\}$ | $\{q_0\}$ |
| | $\{q_1\}$ | $\emptyset$ | $\{q_2\}$ |
| $\star$ | $\{q_2\}$ | $\emptyset$ | $\emptyset$ |
| | $\{q_0, q_1\}$ | $\{q_0\ q_1\}$ | $\{q_0, q_2\}$ |
| $\star$ | $\{q_0, q_2\}$ | $\{q_0\ q_1\}$ | $\{q_0\}$ |
| $\star$ | $\{q_1, q_2\}$ | $\emptyset$ | $\{q_2\}$ |
| $\star$ | $\{q_0, q_1, q_2\}$ | $\{q_0\ q_1\}$ | $\{q_0, q_2\}$ |

Note

1. Arrow means starts state
2. * Denotes the final states

# Equivalent DFA Table




We can rename the states for easier understanding

| | | 0 | 1 |
|---|---|---|---|
| | $\emptyset$ | $\emptyset$ | $\emptyset$ |
| $\rightarrow$ | $\{q_0\}$ | $\{q_0\ q_1\}$ | $\{q_0\}$ |
| | $\{q_1\}$ | $\emptyset$ | $\{q_2\}$ |
| $\star$ | $\{q_2\}$ | $\emptyset$ | $\emptyset$ |
| | $\{q_0, q_1\}$ | $\{q_0\ q_1\}$ | $\{q_0, q_2\}$ |
| $\star$ | $\{q_0, q_2\}$ | $\{q_0\ q_1\}$ | $\{q_0\}$ |
| $\star$ | $\{q_1, q_2\}$ | $\emptyset$ | $\{q_2\}$ |
| $\star$ | $\{q_0, q_1, q_2\}$ | $\{q_0\ q_1\}$ | $\{q_0, q_2\}$ |

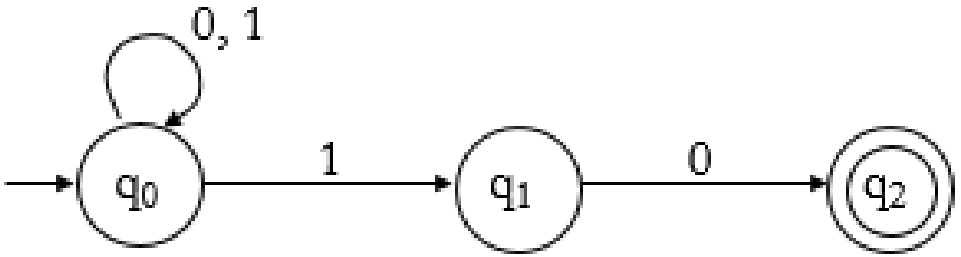
| | | 0 | 1 |
|---|---|---|---|
| | A | A | A |
| $\rightarrow$ | B | E | B |
| | C | A | D |
| $\star$ | D | A | A |
| | E | E | F |
| $\star$ | F | E | B |
| $\star$ | G | A | D |
| $\star$ | H | E | F |

# Shortcut: Subset Construction Method

- No need to write all the subsets
- Start with the initial state, and generate states for all inputs
- For only the generated states, generate next states for all inputs

# Shortcut: Subset Construction Method

NFA:




NFA:

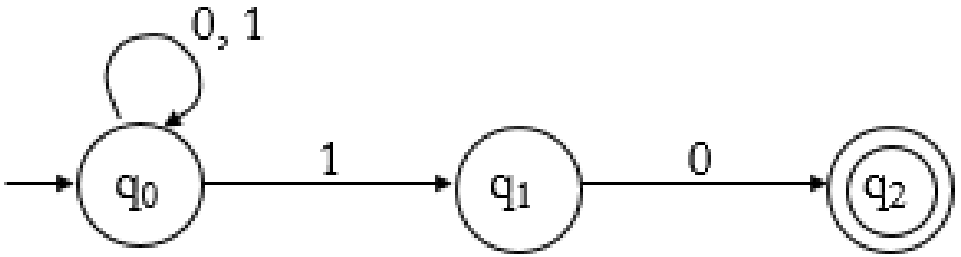
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| $\{q_0\}$ | | |
| | | |
| | | |

# Shortcut: Subset Construction Method

NFA:




NFA:

inputs

| states | 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |
| | | |

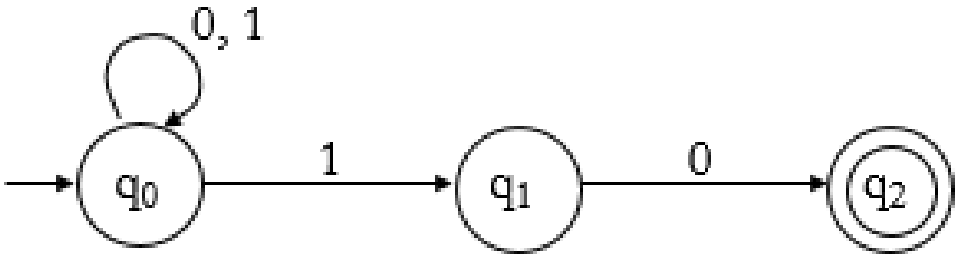
Equivalent DFA:

Inputs

| States | 0 | 1 |
|---|---|---|
| $\{q_0\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| | | |
| | | |

# Shortcut: Subset Construction Method

NFA:




NFA:

| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
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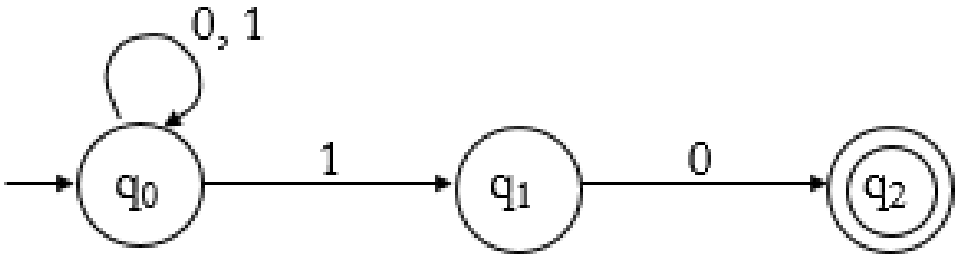
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| $\{q_0\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_2\}$ | $\{q_0, q_1\}$ |
| | | |

# Shortcut: Subset Construction Method

NFA:




NFA:

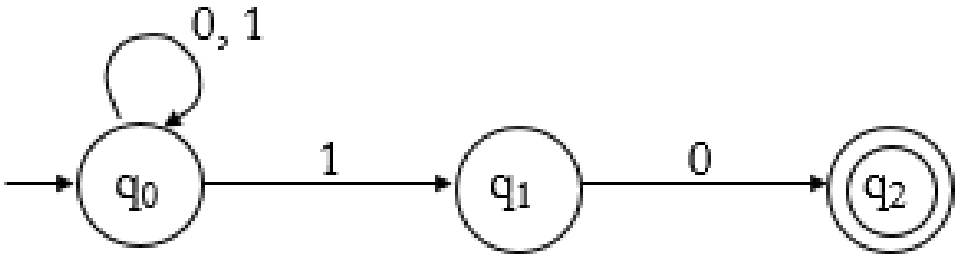
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| $\{q_0\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_2\}$ | $\{q_0, q_1\}$ |
| $\{q_0, q_2\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |

# Shortcut: Subset Construction Method

NFA:




NFA:

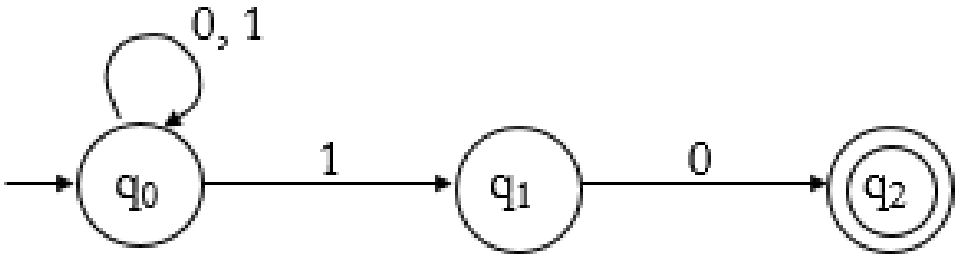
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| → $\{q_0\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_2\}$ | $\{q_0, q_1\}$ |
| * $\{q_0, q_2\}$ | $\{q_0\}$ | $\{q_0, q_1\}$ |

# Shortcut: Subset Construction Method

NFA:




NFA:

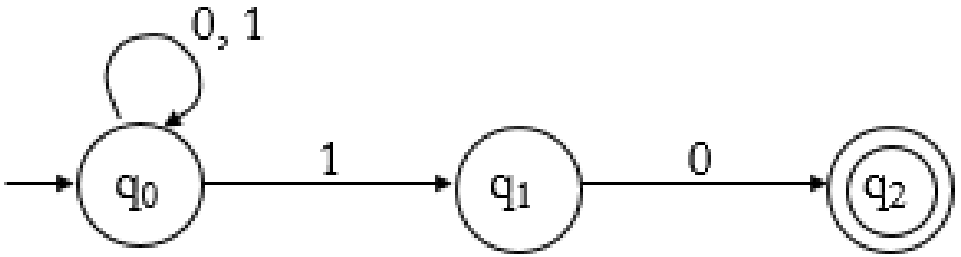
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| → A | A | $\{q_0, q_1\}$ |
| $\{q_0, q_1\}$ | $\{q_0, q_2\}$ | $\{q_0, q_1\}$ |
| * $\{q_0, q_2\}$ | A | $\{q_0, q_1\}$ |

# Shortcut: Subset Construction Method

NFA:




NFA:

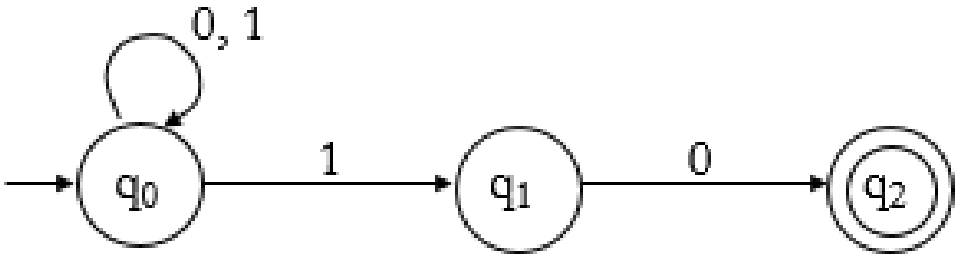
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| → A | A | B |
| B | $\{q_0, q_2\}$ | B |
| * $\{q_0, q_2\}$ | A | B |

# Shortcut: Subset Construction Method

NFA:




NFA:

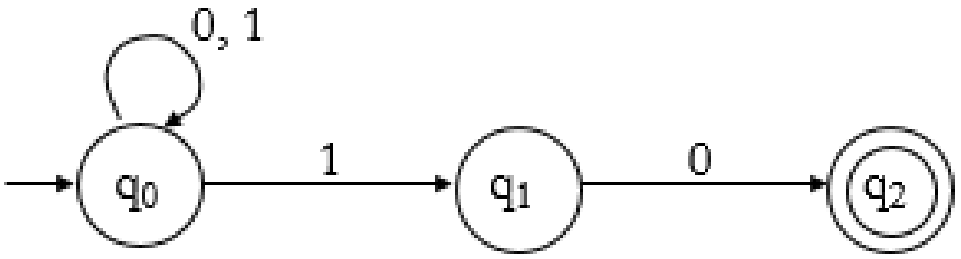
| states | inputs 0 | 1 |
|---|---|---|
| $q_0$ | $\{q_0\}$ | $\{q_0, q_1\}$ |
| $q_1$ | $\{q_2\}$ | $\varnothing$ |
| $q_2$ | $\varnothing$ | $\varnothing$ |

Equivalent DFA:

| States | Inputs 0 | 1 |
|---|---|---|
| → A | A | B |
| B | C | B |
| * C | A | B |

# Shortcut: Subset Construction Method

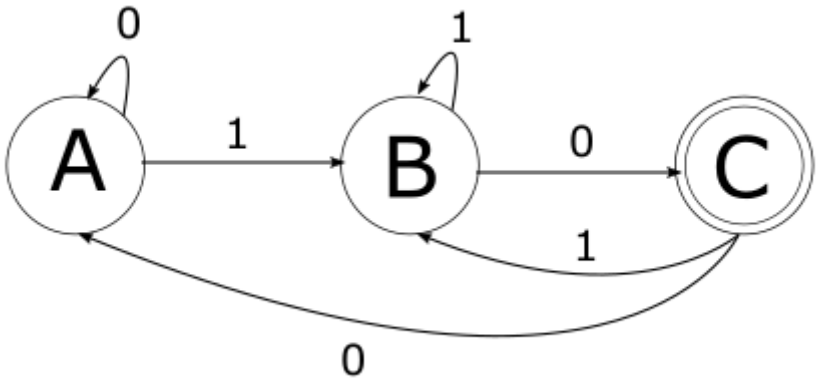
NFA:




**Equivalent DFA:**

| States | Inputs: 0 | Inputs: 1 |
|---|---|---|
| → A | A | B |
| B | C | B |
| * C | A | B |

**DFA:**



# Example: Subset Construction

| | | r | b |
|---|---|---|---|
| → | 1 | 2,4 | 5 |
| | 2 | 4,6 | 1,3,5 |
| | 3 | 2,6 | 5 |
| | 4 | 2,8 | 1,5,7 |
| | 5 | 2,4,6,8 | 1,3,7,9 |
| | 6 | 2,8 | 3,5,9 |
| | 7 | 4,8 | 5 |
| | 8 | 4,6 | 5,7,9 |
| * | 9 | 6,8 | 5 |

| | r | b |
|---|---|---|
| → {1} | {2,4} | {5} |
| {2,4} | | |
| {5} | | |

Alert: What we're doing here is the *lazy* form of DFA construction, where we only construct a state if we are forced to.

# Example: Subset Construction

| | | r | b |
|---|---|---|---|
| → | 1 | 2,4 | 5 |
| | 2 | 4,6 | 1,3,5 |
| | 3 | 2,6 | 5 |
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| | 5 | 2,4,6,8 | 1,3,7,9 |
| | 6 | 2,8 | 3,5,9 |
| | 7 | 4,8 | 5 |
| | 8 | 4,6 | 5,7,9 |
| * | 9 | 6,8 | 5 |

| | r | b |
|---|---|---|
| → {1} | {2,4} | {5} |
| {2,4} | {2,4,6,8} | {1,3,5,7} |
| {5} | | |
| {2,4,6,8} | | |
| {1,3,5,7} | | |

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| * | 9 | 6,8 | 5 |

| | | r | b |
|---|---|---|---|
| → | {1} | {2,4} | {5} |
| | {2,4} | {2,4,6,8} | {1,3,5,7} |
| | {5} | {2,4,6,8} | {1,3,7,9} |
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| | | r | b |
|---|---|---|---|
| → | {1} | {2,4} | {5} |
| | {2,4} | {2,4,6,8} | {1,3,5,7} |
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| | {2,4,6,8} | {2,4,6,8} | {1,3,5,7,9} |
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| | | r | b |
|---|---|---|---|
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| | {2,4} | {2,4,6,8} | {1,3,5,7} |
| | {5} | {2,4,6,8} | {1,3,7,9} |
| | {2,4,6,8} | {2,4,6,8} | {1,3,5,7,9} |
| | {1,3,5,7} | {2,4,6,8} | {1,3,5,7,9} |
| * | {1,3,7,9} | | |
| * | {1,3,5,7,9} | | |

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|---|---|---|---|
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| | 5 | 2,4,6,8 | 1,3,7,9 |
| | 6 | 2,8 | 3,5,9 |
| | 7 | 4,8 | 5 |
| | 8 | 4,6 | 5,7,9 |
| * | 9 | 6,8 | 5 |

| | | r | b |
|---|---|---|---|
| → | {1} | {2,4} | {5} |
| | {2,4} | {2,4,6,8} | {1,3,5,7} |
| | {5} | {2,4,6,8} | {1,3,7,9} |
| | {2,4,6,8} | {2,4,6,8} | {1,3,5,7,9} |
| | {1,3,5,7} | {2,4,6,8} | {1,3,5,7,9} |
| * | {1,3,7,9} | {2,4,6,8} | {5} |
| * | {1,3,5,7,9} | | |

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| | | r | b |
|---|---|---|---|
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| | 2 | 4,6 | 1,3,5 |
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| | 4 | 2,8 | 1,5,7 |
| | 5 | 2,4,6,8 | 1,3,7,9 |
| | 6 | 2,8 | 3,5,9 |
| | 7 | 4,8 | 5 |
| | 8 | 4,6 | 5,7,9 |
| * | 9 | 6,8 | 5 |

| | | r | b |
|---|---|---|---|
| → | {1} | {2,4} | {5} |
| | {2,4} | {2,4,6,8} | {1,3,5,7} |
| | {5} | {2,4,6,8} | {1,3,7,9} |
| | {2,4,6,8} | {2,4,6,8} | {1,3,5,7,9} |
| | {1,3,5,7} | {2,4,6,8} | {1,3,5,7,9} |
| * | {1,3,7,9} | {2,4,6,8} | {5} |
| * | {1,3,5,7,9} | {2,4,6,8} | {1,3,5,7,9} |

# Practice

Convert to a DFA the following NFA:

| | 0 | 1 |
|---|---|---|
| $\rightarrow p$ | $\{p, q\}$ | $\{p\}$ |
| $q$ | $\{r\}$ | $\{r\}$ |
| $r$ | $\{s\}$ | $\emptyset$ |
| $*s$ | $\{s\}$ | $\{s\}$ |