

Regular Expressions : Using regular operations to build up expressions describing languages.

- ① Text search of certain pattern
- ② Token detection in compiler for constant, variable, numerics.
- ③ To generate lexical analyzer in compiler.

Precedence

1. Star (*)
2. Concat (.)
3. Union (|)

To violate precedence
parenthesis required.

Formal Definition

1. a
2. ϵ
3. ϕ
4. $(R_1 \cup R_2)$
5. $(R_1 \circ R_2)$
6. (R_1^*)

$a \in \text{alphabet}$

- Empty

exactly one 1

w contains a single 1 : 0^*10^*

w has at least 1 : $\Sigma^*1\Sigma^*$

string 001 as substring : $\Sigma^*001\Sigma^*$

every 0 in w is followed by at least one 1 .

$1^*(01^+)^*$

110110101

w is a string of even length : $(\Sigma\Sigma)^*$

w is a length is a multiple of 3 : $(\Sigma\Sigma\Sigma)^*$

$(00E)1^* = 01^* \cup 1^*$

all binary strings : $(\Sigma)^*$

Begins with 1 and ends with 1

$1\Sigma^*1$

contain at least three 1s .

$(\Sigma)^*1\Sigma^*1\Sigma^*1\Sigma^*$

contain at least three consecutive 1s

$\Sigma^*111\Sigma^*$

contains substring 110

$\Sigma^*110\Sigma^*$

Doesn't contain

110 101 011 001

$$(0 \cup 10)^* 1^*$$

contain at least two 0s but not consecutive

0s: 101011011

$$(1^* 0 1 1^* (0 + 0 1 1^*))^*$$

has at least 3 char, 3rd one is 0.

$$\Sigma \Sigma 0 \Sigma^*$$

Number of 0s is a multiple of 3.

$$1^* \cup (1^* 0 1^* 0 1^* 0 1^*)^*$$

odd length

even length

$$\Sigma (\Sigma \Sigma)^* \cup (\Sigma \Sigma)^*$$

length is at least 1 and at most 3

$$\Sigma \cup \Sigma \Sigma \cup \Sigma \Sigma \Sigma$$

~# w starts & end with different symbol

$$0 \Sigma^* 1 \cup 1 \Sigma^* 0 \cup 1 \cup 0$$

number of 0's in w is a multiple of 3

$$1^* \cup (1^* 0 1^* 0 1^* 0 1^*)^*$$

w doesn't contain two consecutive 0's

$$\boxed{(01)^* \cup (10)^* \cup 1^*} \cup (01)^* 0 \cup (10)^* 1 \cup 1^* 0$$

} w is of even length if it starts with 0, is
of odd length if it starts with 1

$$0(2Z)^* \cup 1(2Z)^*$$

} w has at least two 1's

$$\Sigma^* 1 \Sigma^* 1 \Sigma^*$$

$$0 \quad 1 \quad \boxed{010} \quad \begin{array}{r} 01001 \\ 001010 \\ \hline 010101 \end{array} \quad \begin{array}{r} 010101 \\ 101010 \\ \hline 10101 \end{array}$$

$$\Sigma = \{0, 1\}$$

i) $w \mid w$ contains at least three 1.

$$\Sigma^* (111)^+ \Sigma^*$$

$$\Sigma^* 111 \Sigma^*$$

$$010101$$

$$\Sigma^* 1 \Sigma^* 1 \Sigma^* 1 \Sigma^*$$

ii) $w \mid w$ has length at least 3 and third symbol is 0

$$\Sigma \cup \Sigma \Sigma \cup \Sigma \Sigma \Sigma$$

$$(\Sigma \Sigma 0)^+$$

$$((011)(011)(011))^+$$

iii) starts with 0 and has odd length or starts with 1 and has even length

$$0(\Sigma \Sigma)^* \cup 1(\Sigma \Sigma \Sigma)^*$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

iv) every odd position of w is 1

$$1(011)^+ 1(011)^+ 1(011)^+$$

$$1(\Sigma 1)^*$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

v) w contains an even number of 0's or contains exactly two 1's.

01001

$$(0^*1^*0^*)^* \cup 0^*1^*0^*1^*0^*$$

$$1^*(00)^*1^* \cup 0^*10^*10^*$$

000 000

vii) w contains even no of a's

$$(b^*ab^*ab^*)^*$$

$$(b^*ab^*ab^*)^*$$

aa b aa

iv) w contains ~~even~~ odd no of a's

$$b^*ab^*(b^*ab^*ab^*)^*$$

a
b a b a a

viii) w contains neither aa nor bb as substring.

$$(\Sigma^*aa\Sigma^* \cup \Sigma^*bb\Sigma^*)'$$

$$a^* \cup b^*$$

$$(ab)^* \cup (ba)^*$$

ababab
bababab

w contain 01 or 10 as substring

$$\Sigma^*01\Sigma^* \cup \Sigma^*10\Sigma^*$$

with or , $0^* \cup 1^*$

iii) starts with a and is a string of w even length.

ab aabq $a(b|a)$

~~aaaa~~

$a \Sigma (\Sigma)^*$

w starts and ends with different symbols.

$a \Sigma^* b \cup b \Sigma^* a$ ~~$\cup a \cup b$~~

w starts and ends with same symbols.

$a \Sigma^* a \cup b \Sigma^* b$ ~~$\cup a \cup b$~~

#

a's in w always appear in pairs.

baabaa

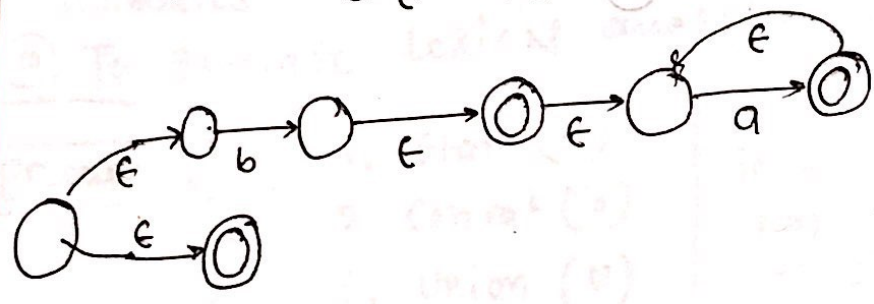
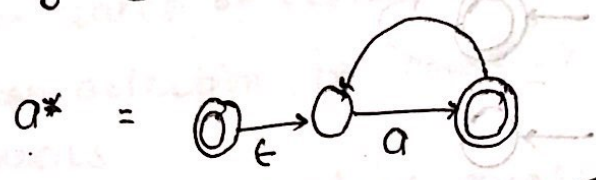
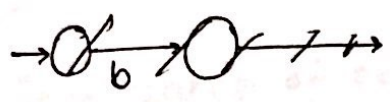
~~$b^*(aa)^*b^*$~~

~~baabaa~~

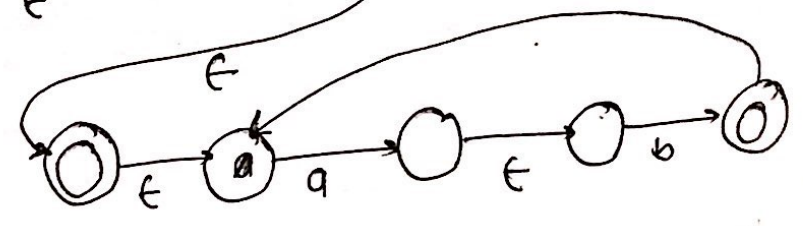
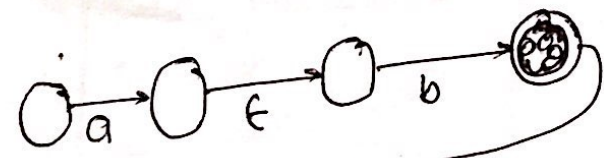
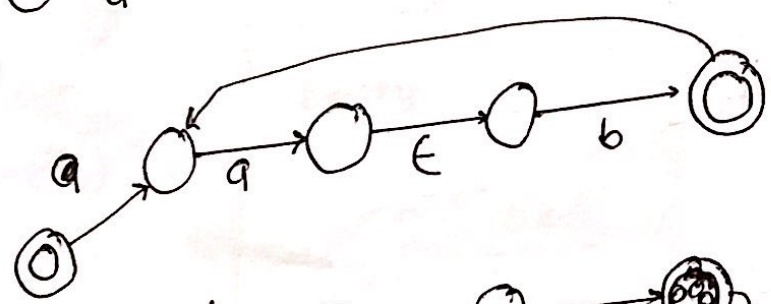
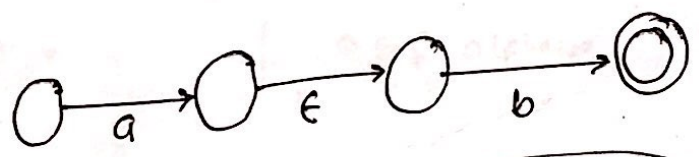
a's in w always in pair ~~$b^*(aa)^*b^*(aa)^*$~~
 ~~$\cup (aa)^*b^*(aa)^*$~~

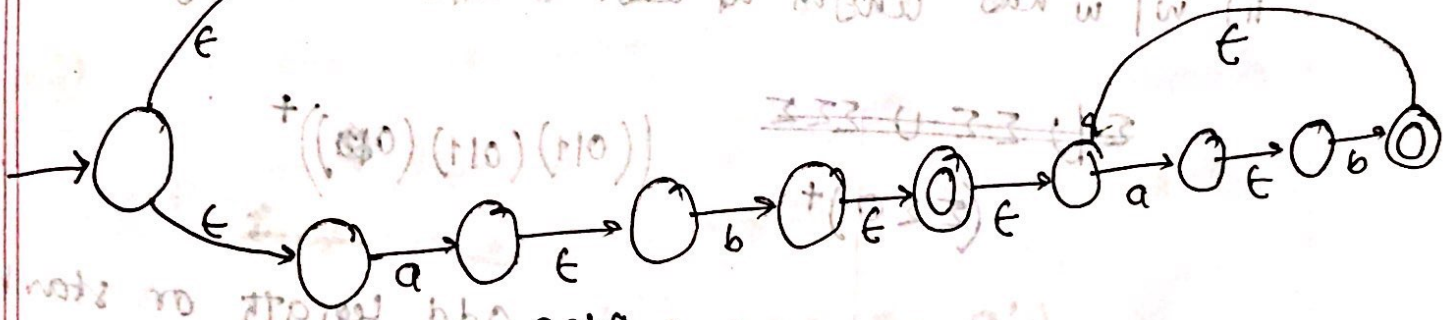
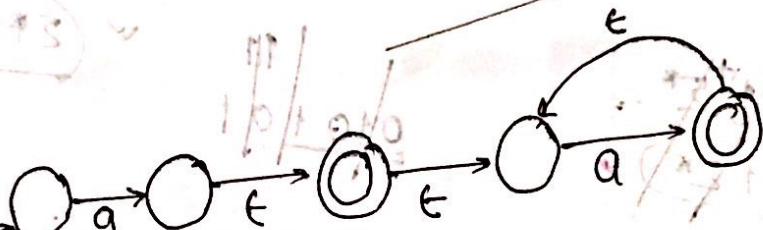
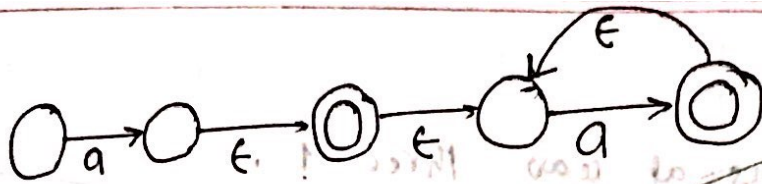
$(b^*(aa)^*)^*b^* \cup ((aa)^*b^*)(aa)^*$

$b a^* + \epsilon$



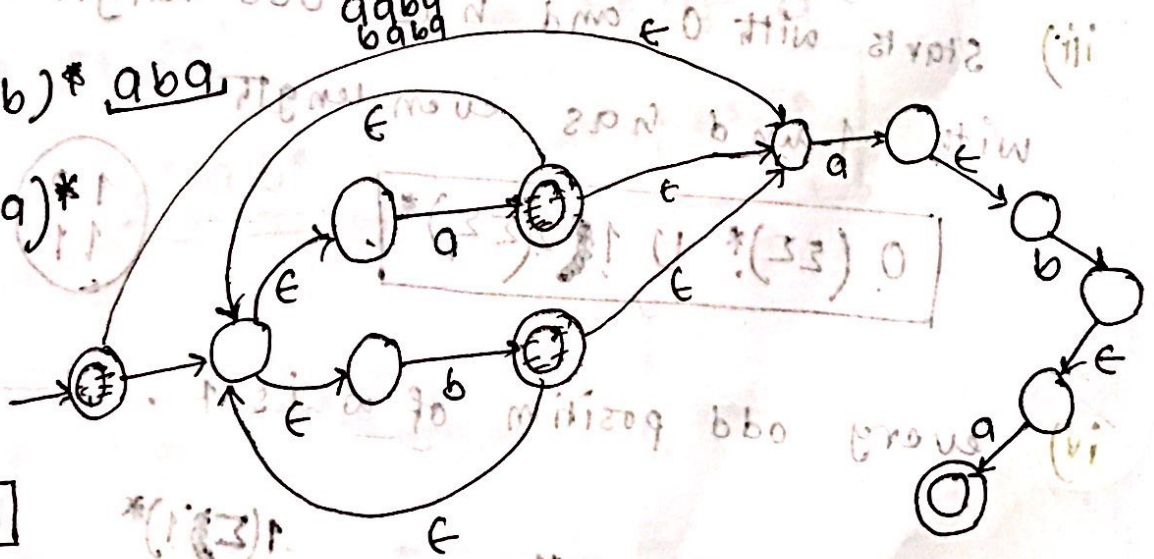
$a^+ \cup (ab)^+$ = $a a^* \cup (ab)^+ (ab)^*$





$(a \cup b)^* abq$

$(ab \cup a)^*$



$q abq$