Answer all 8 questions. Numbers to the right of the questions denote their marks.

1. Consider the language $L = \{a^p b^q c^{3p+2} d^r \mid p, q, r \geq 0\}$ $6 + 2 = 8$

   (a) Design a Pushdown Automaton for the given language.

   (b) Write the seven components of the PDA you designed.

2. Consider the following state diagram of a Pushdown Automata: $2 + 2 = 4$
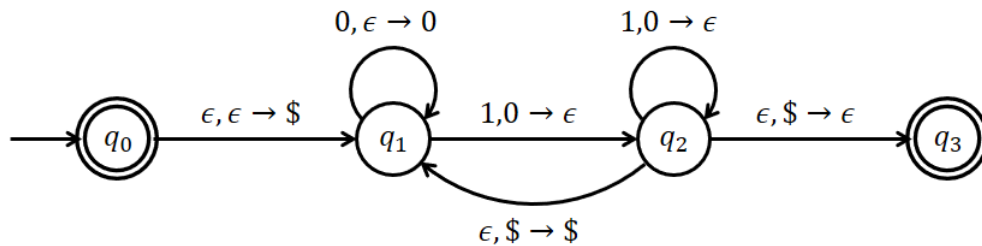
   Show that the following strings are accepted by the given push-down automaton. Show the instantaneous descriptions of the automaton for both strings.
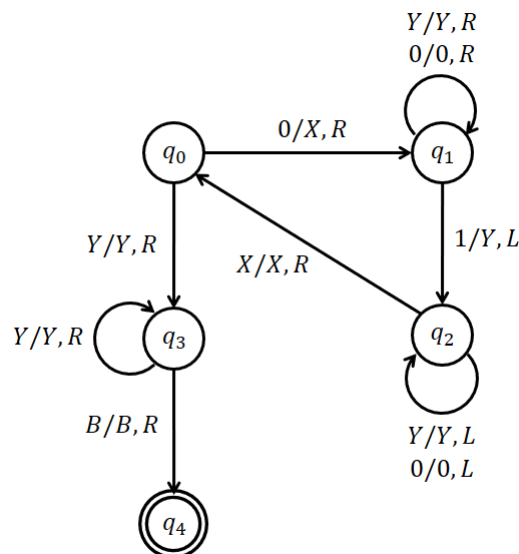
   i. 00011101 ii. 00110101

3. Draw the state diagram of a Turing Machine for the following language: 4

$$L = \{a^{n+2}(cd)^{2n} \mid n \geq 1\}$$

   (Sample accepted strings: *aaacdcd, aaaacdcdcdcd, aaaaacdcdcdcdcdcd*)

4. Consider the following state diagram of a Turing Machine: $2 + 2 = 4$

   Find out whether the following strings are accepted in this Turing Machine. Show description of the input tape for each transition.

   i. 00001111 ii. 0000111

5. Convert the following grammar to Chomsky Normal Form: 4

$$S \rightarrow AS \,|\, BAC$$
$$A \rightarrow A1 \,|\, 0A1 \,|\, 0B1 \,|\, B$$
$$B \rightarrow 0B \,|\, 0 \,|\, \epsilon$$
$$C \rightarrow 1 \,|\, \epsilon$$

6. Find out if the following grammar is ambiguous or not, using the string $x + (0 * 1\%0)$. If the grammar is ambiguous, show two different parse trees for this string. 4

$$E \rightarrow E + E \,|\, E - E \,|\, (E) \,|\, V$$
$$V \rightarrow x \,|\, y \,|\, z \,|\, A$$
$$A \rightarrow A * A \,|\, A\%A \,|\, C$$
$$C \rightarrow 0 \,|\, 1$$

7. Design Context-free Grammars for the following languages: $4 + 4 = 8$

    (a) $L = \{a^i b^j c^k d^l \mid i = 2l \text{ and } 3j = k\}$
    (b) $L = \{0^x 1^y 2^z \mid x < y \text{ or } y < z\}$

8. Consider the following Context-free Grammar: $2 + 2 = 4$

$$E \rightarrow E + E \,|\, E * E \,|\, (E) \,|\, N$$
$$N \rightarrow 0N \,|\, 1N \,|\, 0 \,|\, 1$$

Show a leftmost derivation and a rightmost derivation for the input string $(1 + 10) * 1$.