Md. Mishkatul Bary Section: N
01123306669

Assignment -1

1.(a) $x = A\cos(\omega t + \phi)$

$= 0.2 \cos\left(\frac{10\pi}{3} t\right)$

$T = 0.6\,s$

$A = 0.2\,m$

$\omega = \frac{2\pi}{T}$

$= \frac{2\pi}{0.6}$

$= \frac{10\pi}{3}\,rad/s$

(b) $\frac{dx}{dt} = v = -A\omega \sin(\omega t)$

$= -0.2 \times \frac{10\pi}{3} \sin\left(\frac{10\pi}{3} \times 2\right)$ | $t = 2\,sec$

$= -\frac{2\pi}{3} \sin\left(\frac{20\pi}{3}\right)$

$= -1.813\,ms^{-1}$

(c) $P.E = \frac{1}{2} k x^2$

$= \frac{1}{2} \times \frac{25\pi^2}{9} \times (0.2 \cos \omega t)^2$

$= \frac{1}{2} \times \frac{25\pi^2}{9} \times 0.2^2 \cos^2\left(\frac{10\pi}{3} \times 0.25\right)$

$= 0.411\,J$

$m = 0.25\,kg$

$\omega^2 = \frac{k}{m}$

$k = \omega^2 m$

$= \left(\frac{10\pi}{3}\right)^2 \times 0.25$

$= \frac{25\pi^2}{9}\,Nm^{-1}$

2. (a) $Y = 10\cos(2t+\pi)$

$A = 10m$

$\omega = 2\pi ads^{-1}$

$\phi = \pi$

$m = 0.5kg$

(i) $P.E = \frac{1}{2}kx^2$

$= \frac{1}{2} \times 0.5 \times (10\cos(2t+\pi))^2$

$= 1 \times 10^2 \cos^2(2 \times .75 + \pi)$ $\quad t = 0.75s$

$= 0.50\ J$

$k = m\omega^2$

$= .5 \times 2^2$

$= 2Nm^{-1}$

(ii) $K.E = \frac{1}{2}mv^2$

$= \frac{1}{2}m(10 \times 2\sin(2t+\pi))^2$

$= \frac{1}{2} \times m \times 10^2 \times 2^2 \sin^2(2 \times .3 + \pi)$ $\quad t = .3s$

$= \frac{1}{2}m \times 400 \sin^2(0.6+\pi)$

$= \frac{1}{2} \times 0.5 \times 400 \sin^2(0.6+\pi)$

$= 31.88\ J$

(iii) $V_{max} = A\omega$

$= 10 \times 2\pi$

$= 20\ ms^{-1}$

3. P.E $= \frac{1}{2} k x^2$

$= \frac{1}{2} \times 19.74 \times 0$

$= 0$ J

$x = A\cos(\omega t + \phi)$

$= 0.5 \cos(\pi t)$

$x(.25) = 0.5 \cos(2\pi \times .25)$

$= 0$

$T = 1$

$\omega = \frac{2\pi}{T}$

$= 2\pi$

$k = m\omega^2$

$= .5 \times (2\pi)^2$

$= 19.74$ N/m

$m = 0.05$ kg

$k = 20.5$ Nm^{-1}

$A = 0.7$ m

$t = 0.2$ s

$\omega = \sqrt{k/m}$

$= \sqrt{\frac{20.5}{0.05}}$

$= 20.25$ rads^{-1}

4. (i) $x(t) = A\cos(\omega t)$

$= 0.7 \cos(20.25 \times .2)$

$\frac{dx}{dt} = V = -A\omega \sin(\omega t)$

$= 0.7 \times 20.25 \sin(20.25 \times .2)$

$= 11.17$ ms^{-1}

(ii) $\frac{d^2x}{dt^2} = a = -A\omega^2 \cos(\omega t)$ | $t = .25$ s

$= -0.7 \times (20.25)^2 \times \cos(20.25 \times .25)$

$= -98.45$ ms^{-2}

(iii) $K.E = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.05 \times 200.64$

$= 501\ J$

~~$v^2 = \omega^2(A^2 - x^2)$~~

$v = -A\omega \sin(\omega t)$

$= 0.7 \times 20.25 \sin$

$(20.25 \times .75)$

$= -7.04\ ms^{-1}$

$v^2 = 200.64\ ms^{-1}$

(5) (i) $x = A\cos(\omega t)$

$x = 0.005 \cos(480\pi \times .2)$

$= 0.005 \cos(96\pi)$

$= 0.005\ m$

$t = 0.2s$

$A = 0.005\ m$

$f = 240\ Hz$

$\omega = 2\pi f$

$\omega = 480\pi\ rads^{-1}$

(ii) $v_{max} = A\omega$

$= 0.005 \times 480\pi$

$= 7.54\ ms^{-1}$

(iii) $a_{max} = A\omega^2$

$= 0.005 \times (480\pi)^2$

$= 23.68\ ms^{-2}$

6. (i) $f = \frac{\omega}{2\pi}$ | $\omega = \sqrt{\frac{k}{m}} = \frac{460}{\cdot}$

6 (i) $F = -kx$

$\Rightarrow ma = -kx$

$\therefore m = \frac{-kx}{a}$

$= -\frac{460 \times 0.150}{-125.0}$

$= 0.552$ kg

$k = 460$ N/m
$a = -125$ ms^{-2}
$v = -15$ ms^{-1}
$x = 0.150$ m

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{460}{.552}}}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{460}{.552}}$

$= 0.217$

$f = \frac{1}{T} = 4.59$ Hz

(ii) $F = -kx$

$ma = -kx$

$\therefore m = \frac{-kx}{a} = \frac{460 \times .150}{-125}$

$= 0.552$ kg

(iii) $v^2 = \omega^2(A^2 - x^2)$

$v_2 = -15\ ms^{-1}$

$a = -125\ ms^{-2}$

$\omega^2 = 833.33$

$x = .15$

$\Rightarrow A^2 = \frac{v^2}{\omega^2} + x^2$

$= \frac{(-15)^2}{833.33} + (0.15)^2$

$\therefore A = 0.54\ m$

(7) (i) $a = -\omega^2 x$

$x = 4\ m$

$A = 8\ m$

$a = +80\ ms^{-2}$

$\omega^2 = \frac{a}{x}$

$= \frac{80}{4} = 20$ $rads^{-1}$

$v^2 = \omega^2(A^2 - x^2)$

$= 20(8^2 - 4^2)$

$= 960$

$\therefore v = 8\sqrt{15}\ ms^{-1}$

(ii) $T = \frac{2\pi}{\omega} = \frac{2\pi}{4.47}$

$= 1.41 \text{ sec}$

$\omega = \sqrt{20}$

(iii) $V_{max} = \omega A = 4.47 \times 8$

$= 35.76 \text{ ms}^{-1}$

8) oscillatory checked

$\omega_d^2 > \frac{\gamma^2}{4}$

$\Rightarrow \frac{1}{LC} > \frac{R^2}{4L^2}$

$\Rightarrow \frac{1}{20\times10^{-3}\times0.8\times10^{-9}} > \frac{290^2}{4\times20\times10^{-3}}$

$\Rightarrow R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4\times20\times10^{-3}}{0.8\times10^{-9}}}$

$= 10000$

$\therefore R_c > R$

$C = 0.8 \text{ nF}$

$= 0.8\times10^{-9} \text{ F}$

$L = 20 \text{ mH}$

$= 20\times10^{-3} \text{ H}$

$R = 290 \Omega$

∴ oscillatory damped

$T = \frac{2\pi}{\omega_d}$

$f = \frac{\omega_d}{2\pi} = \frac{2.49\times10^5}{2\pi} = 39.7 \text{ KHz}$

$\omega_d = \sqrt{\omega^2 - \gamma^2/4}$

$= \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

$= 2.49\times10^5$

9. (i) $T = \frac{2\pi}{\omega}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{82}{\cdot 22}}$

$= 19.3 \text{ rad s}^{-1}$

$= \frac{2\pi}{19.3} = 0.32 \text{ s}$

(ii) $A(t) = A_0 e^{-\gamma/2 t}$

$A(t) = \frac{1}{2} A_0$

$\gamma = \frac{b}{m}$

$= \frac{\cdot 063}{\cdot 22}$

$= \cdot 286$

$\Rightarrow A_0\left(\frac{1}{2}\right) = A_0 e^{-\gamma/2 t}$

$\Rightarrow \ln\left(\frac{1}{2}\right) = \ln e^{-\gamma/2 t}$

$\ln\left(\frac{1}{2}\right) = -\gamma/2 t$

$\Rightarrow t = -\frac{2\ln\left(\frac{1}{2}\right)}{\gamma}$

$\Rightarrow t = -\frac{2\ln\left(\frac{1}{2}\right)}{\cdot 286}$

$\therefore t = 4.84 \text{ sec}$

$T = \frac{1}{\gamma} = \frac{m}{b} \times 2$

$= \frac{\cdot 22}{\cdot 063} \times 2$

$= 6.98$

(iii) $\frac{\text{Life time}}{\text{period}} = \frac{T}{T_d}$

$= \frac{6.98}{0.32}$

≈ 21.9

oscillation : ~~≈ 21~~

(iv) Lifetime $T = \frac{1}{\gamma} = \frac{m}{b}$

$= \frac{.22}{.063} \times 2$

$= 3.49 \times 2 \text{sec}$

(v) Damping Energy

$\frac{1}{2} k A^2 (1 - e^{-\gamma t})$

$= \frac{1}{2} \times 82 \times .35^2 \left(1 - e^{-\frac{.063}{.22} \times 6.52}\right)$

$= 4.24 \text{ J}$

$n = \frac{t}{T_d}$

$t = n \times T_d$

$= 20 \times \sqrt{\omega^2 - \frac{\gamma^2}{4}}$

$= 20 \times \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$

$A = 0.35 \text{ m}$

$K = 82 \text{ N/m}$

$\rightarrow t = 6.52$

$\gamma = \frac{b}{m} = \frac{.063}{.22}$

$= 3.49$

(vi) Ratio $= e^{-20 \cdot \frac{bT}{2m}}$

$= e^{-20 \times .0466}$

$= e^{-0.932}$

$= 0.393$

39.3%