United International University

School of Science and Engineering

Department of Computer Science and Engineering

Final Examination Trimester- Summer – 2024

Course: Math -2205

Total marks - 50; Duration – 2 hours

[Note that the number of marks is given in brackets [ ] at the end of each question or part question. You are requested to answer all the questions in order.]

Q1

(a) Two ordinary fair dices are rolled. Using a possibility diagram or otherwise, find the probability of obtaining

(i) at least one 6

(ii) two numbers whose product is 6.

(b) A coin is biased such that the probability that three successive tosses all result in heads in $\frac{125}{512}$. Find the probability of tail to appear when tossed.

(c) In a survey, 50% of the participants own a desktop (D), 60% own a laptop (L) and 15% own both. By using Venn diagram or otherwise, find what percentage of participants owns neither a desktop nor a laptop?

(d) The following table gives information about all the animals on a farm. Find the probability that a randomly selected animal is:

(i) male or a goat

(ii) a sheep or female

	Male	Female
Goats	5	25
Sheep	3	22

[4+2+2+4 = 12]

Q2

(a) A factory produces half-liters tins of oil. The volume of oil in a tin is normally distributed with a mean 506 ml and standard deviation 2.9 ml. A tin is randomly selected what is the probability that it will be (i) less than half a liter (500 ml) of oil.

(ii) Within one standard deviation

(b) A footballer has a 95% chance of scoring each penalty kick that she takes. Find the probability that she: (i) scores from all of her next 10 penalty kicks

(ii) fails to score exactly 2 of her next 10 penalty kicks.

(c) The probability distribution for random variables Y is given in the following table

y	0	1	2	3	4
$P(Y=y)$	0.03	$2p$	0.32	p	0.05

(i) Find the value of p.

(ii) Find the standard deviation of Y

[6+4+4 = 14]

Q3

(a) The weekly petrol consumption, in hundreds of liters of a sales representative, may be modeled by the random variable X with probability density function (*pdf*)

$$f(x) = \{a\,x(4-3x)\} \quad for\ 0 \le x \le \tfrac{4}{3}.$$

(i) Find the value of a.

(ii) Find the mean of the weekly consumption

(iii) Find the probability that the weekly consumption will be less than 1 hundred liters.

(iv) Find the mode of the weekly consumption.

(v) Construct the corresponding *cdf* for the above *pdf*

[2+3+3+3+3 = 14]

Q4

(a) A machine is supposed to produce metal rods which are 5.7 cm long. A random sample of 100 rods produced by the machine are measured with mean 5.71 cm and standard deviation 0.048 cm. Calculate a 95% confidence interval for the mean length of a rod produced by the machine.

(b) A logistics company claims that the average delivery time for packages is 5 days. A customer service team wants to test if the actual average delivery time is longer than this claim. The delivery times follow normal distribution with a known standard deviation of $\sigma = 1.5$ days. A random sample of 40 deliveries shows an average delivery time of 5.4 days. Perform a one-tailed test at a 5% level of significance to determine if the true average delivery time exceeds 5 days, [use σ instead of $\sigma/\sqrt{n}$.]

(c) State possibly what kind of error may occur in this situation.

[3+6+1 = 10]

Important Formulae

Distribution	pmf/pdf
Binomial	$f(x) = {}_nC_x p^x (1-p)^{n-x}\ ;\ x = 0, 1, 2, \ldots . n$
Poisson	$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}\ ;\ x = 0, 1, 2, \ldots ..$
Uniform	$f(x) = \frac{1}{b-a}\ ;\ a \le x \le b$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\ ;\ -\infty < x < \infty$

For continuous random variables

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$\text{Var}(x) = \int_{-\infty}^{\infty} x^2 f(x) dx - (E(x))^2$$

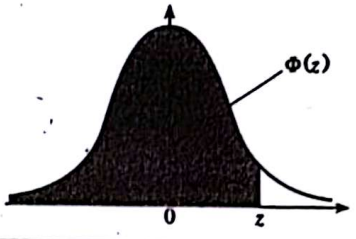
$$CI = \bar{x} \pm z\frac{s}{\sqrt{n}}$$

The normal distribution function

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leq z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
											ADD		
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4 8 12	16 20 24	28 32 36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4 8 12	16 20 24	28 32 36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4 8 12	15 19 23	27 31 35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4 7 11	14 18 22	25 29 32
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4 7 11	14 18 22	25 29 32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3 7 10	14 17 20	24 27 31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3 7 10	13 16 19	23 26 29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3 6 9	12 15 18	21 24 27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3 5 8	11 14 16	19 22 25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3 5 8	10 13 15	18 20 23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2 5 7	9 12 14	16 19 21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2 4 6	8 10 12	14 16 18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2 4 6	7 9 11	13 15 17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2 3 5	6 8 10	11 13 14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1 3 4	6 7 8	10 11 13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1 2 4	5 6 7	8 10 11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1 2 3	4 5 6	7 8 9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1 2 3	4 4 5	6 7 8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1 1 2	3 4 4	5 6 6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1 1 2	2 3 4	4 5 5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0 1 1	2 2 3	3 4 4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0 1 1	2 2 2	3 3 4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0 1 1	1 2 2	2 3 3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0 1 1	1 1 2	2 2 2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0 0 1	1 1 1	1 2 2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0 0 0	1 1 1	1 1 1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0 0 0	0 1 1	1 1 1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0 0 0	0 0 1	1 1 1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0 0 0	0 0 0	0 1 1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0 0 0	0 0 0	0 0 0