SHM Formulae

Potential energy,

$$E_p = \frac{1}{2}kx^2$$

Here, spring constant, $k = \omega^2 m$

x = displacement

(Potential energy maximum at amplitude)

(Potential energy minimum (0) at equilibrium point)

Kinetic energy,

$$E_k = \frac{1}{2}mv^2$$

$$v = \omega\sqrt{A^2 - x^2}$$

(Kinetic energy maximum at equilibrium point)

(Kinetic energy minimum (0) at amplitude)

Total energy/mechanical energy,

$$E = E_p + E_k = \frac{1}{2}kA^2$$

Total energy/mechanical energy is always constant

1. **A block attached to a spring is suspended vertically. If the block is pushed 7 cm upward from the equilibrium position and released at t = 0. The mass of the block is 5 kg and the spring constant is k = 22 N/m. i) Calculate the potential energy at x = 3 cm. ii) Calculate the kinetic energy at the same position.**

i) We know,
Potential energy,

$$P.E = \frac{1}{2}kx^2$$
$$= \frac{1}{2} \times 22 \times (0.03)^2$$
$$= 0.0099\ J$$

$\therefore$ Potential energy is $0.0099\ J$.

ii) Velocity at x,

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2(A^2 - x^2)$$
$$\text{or, } v^2 = \frac{k}{m}(A^2 - x^2)$$
$$\text{or, } v^2 = \frac{22}{5} \times [(0.07)^2 - (0.03)^2]$$
$$\therefore v^2 = 0.0176\ m\ s^{-1}$$

Here,
$A = 7\ cm = 0.07\ m$
$m = 5\ kg$
$k = 22\ N\ m^{-1}$
$x = 3\ cm = 0.03\ m$
$P.E = ?$
$K.E = ?$

$\therefore$ Kinetic energy at x,

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 5 \times 0.0176$$
$$= 0.044\ J$$

2. **Suppose a spring block-system starts moving from the equilibrium as we apply force on it. The block has mass m = 6.4 kg and is designed to oscillate with angular frequency ω = 56 rads-1 with amplitude 15 cm.**

   Calculate: i) the kinetic energy at x = 14 cm from the equilibrium point, ii) mathematically calculate the position where the kinetic energy is 0.

i) We know,

Kinetic energy,

$$K.E = \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$= \frac{1}{2} \times 6.4 \times (56)^2 \times [(0.15)^2 - (0.14)^2]$$
$$= 28.102\ J$$

Here,
$m = 6.4\ kg$
$\omega = 56\ rad\ s^{-1}$
$A = 15\ cm = 0.15\ m$
$x = 14\ cm = 0.14\ m$
$K.E = ?$

∴ Kinetic energy at $x = 14\ cm$ is $28.102\ J$.

ii) Let, kinetic energy will be 0 at position x.

Now,

$$K.E = 0$$
$$\text{or, } \frac{1}{2}mv^2 = 0$$
$$\text{or, } \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = 0$$
$$\text{or, } A^2 - x^2 = 0$$
$$\text{or, } x^2 = A^2$$
$$\therefore x = A$$

∴ Kinetic energy will be 0 when our displacement will be equal to amplitude.

3. A particle with mass 50 g executes simple harmonic motion given by the equation $y = \sin\left(10t - \frac{\pi}{4}\right)$. Calculate the i) velocity and acceleration at $t = 5\ s$ ii) total energy at $t = 3\ s$.

(i)

$$\text{Displacement, } y = \sin\left(10t - \frac{\pi}{4}\right)$$

$$\therefore \text{Velocity, } v = \frac{dy}{dt} = \frac{d}{dt}\left[\sin\left(10t - \frac{\pi}{4}\right)\right]$$

$$= 10\cos\left(10t - \frac{\pi}{4}\right)$$

$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}\left[10\cos\left(10t - \frac{\pi}{4}\right)\right]$$

$$= -10 \times 10\sin\left(10t - \frac{\pi}{4}\right)$$

$$= -100\sin\left(10t - \frac{\pi}{4}\right)$$

Here,

$m = 50\ g = 0.05\ kg$

$\omega = 10\ rad\ s^{-1}$

$v(5) = ?$

$a(5) = ?$

$E(3) = ?$

$\therefore$ Velocity at $t = 5\ s$,

$$v(5) = 10\cos\left(10 \times 5 - \frac{\pi}{4}\right)$$

$$= 4.968\ m\ s^{-1}$$

$\therefore$ Acceleration at $t = 5\ s$,

$$a(5) = -300\sin\left(10 \times 5 - \frac{\pi}{4}\right)$$

$$= 86.786\ m\ s^{-2}$$

(ii) $E = \frac{1}{2}kA^2 = \frac{1}{2}\omega^2 mA = \frac{1}{2} \times 10^2 \times 0.05 \times 1 = 2.5$ J

4. **Suppose a spring block-system move between top and bottom point of a tall building as a moving mass. The block has mass m = 5.7×10³ kg and designed to oscillate at frequency f = 50 Hz with amplitude A = 15 cm.**
Calculate: i) the potential energy at the equilibrium point, ii) the block speed as it passes through the equilibrium point, iii) the maximum acceleration of the spring block-system.

Here,
$m = 5.7 \times 10^3\ kg$
$f = 50\ Hz$
$A = x_{max} = 15\ cm = 0.15\ m$
$P.E = ?$
$v_{max} = ?$
$a_{max} = ?$

i) We know,
At equilibrium position, $x = 0$

∴ Potential energy,

$$P.E = \frac{1}{2} kx^2 = \frac{1}{2} k(0)^2 = 0$$

ii) We know,
At equilibrium position, SHM oscillation have its maximum speed.

∴ Maximum velocity,

$$\begin{aligned} v_{max} &= A\omega \\ &= A2\pi f \quad [\because \omega = 2\pi f] \\ &= 0.15 \times 2\pi \times 50 \\ &= 47.12\ \mathrm{m\ s^{-1}} \end{aligned}$$

iii) We know,
Maximum acceleration,

$$\begin{aligned} a &= -A\omega^2 \\ &= -A(2\pi f)^2 \quad [\because \omega = 2\pi f] \\ &= -0.15 \times (2\pi \times 50)^2 \\ &= 14804.48\ \mathrm{m\ s^{-2}} \end{aligned}$$

5. A body of mass 300 gm is attached with a spring of spring constant 5000 dynes/cm. The body is displaced by 7 cm from its equilibrium position and released. Then the body executes SHM. Calculate the i) frequency, ii) angular frequency, iii) total energy of the mass spring system.

Here,
$m = 300\ g$
$k = 5000\ dynes\ cm^{-1}$
$A = 7\ cm$
$f = ?$
$\omega = ?$
$E = ?$

i) We know,
Frequency,

$$
\begin{aligned}
f &= \frac{\omega}{2\pi} \\
&= \frac{1}{2\pi} \times \sqrt{\frac{k}{m}} \\
&= \frac{1}{2\pi} \times \sqrt{\frac{5000}{300}} \\
&= 0.65\ Hz
\end{aligned}
$$

ii) We know,
Angular frequency,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5000}{300}} = 4.082\ rad\ s^{-1}$$

iii) We know,
Total energy,

$$
\begin{aligned}
E &= \frac{1}{2}kA^2 \\
&= \frac{1}{2} \times 5000 \times (7)^2 \\
&= -0.15 \times (2\pi \times 50)^2 \\
&= 122500\ erg
\end{aligned}
$$

6. **Suppose the block has m=2.75×105 kg and is designed to oscillate at frequency f=10.0 Hz and amplitude A=20.0 cm. i) What is the total energy E of the spring-block system? ii) What is the KE and PE at x=10 cm iii) At what position KE=PE?**

Total energy,

$$E = \frac{1}{2}\mathrm{kA}^2$$
$$= \frac{1}{2} \times 1.086 \times 10^9 \times (0.2)^2$$
$$= 2.172 \times 10^7\ J$$

ii) Velocity when $x = 10\ cm = 0.1\ m$,

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2(A^2 - x^2)$$
$$\text{or, } v^2 = (62.83)^2\left[(0.2)^2 - (0.1)^2\right]$$
$$\therefore v^2 = 118.43\ m\ s^{-1}$$

Kinetic energy at $x = 10\ cm$,

$$KE = \frac{1}{2}\ mv^2$$
$$= \frac{1}{2} \times 2.75 \times 10^5 \times 118.43$$
$$= 1.628 \times 10^7\ J$$

$$\omega = 2\pi f$$
$$= 2\pi \times 10$$
$$= 62.83\ rad\ s^{-1}$$

$$k = m\omega^2$$
$$= 2.75 \times 10^5 \times (62.83)^2$$
$$= 1.086 \times 10^9\ N\ m^{-1}$$

Potential energy at $x = 10\ cm$,

$$PE = \frac{1}{2}\ kx^2$$
$$= \frac{1}{2} \times 1.086 \times 10^9 \times (0.1)^2$$
$$= 5.43 \times 10^6\ J$$

iii) Let, KE will equal to PE at displacement x.

Now,

$$KE = PE$$
$$\text{or, } \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$
$$\text{or, } \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}kx^2$$
$$[\because \mathrm{v}^2 = \omega^2(A^2 - x^2)]$$
$$\text{or, } \frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}kx^2 \quad [\because k = m\omega^2]$$
$$\text{or, } A^2 - x^2 = x^2$$
$$\text{or, } A^2 = 2x^2$$
$$\text{or, } x = \sqrt{\frac{A^2}{2}}$$
$$\text{or, } x = \sqrt{\frac{(0.2)^2}{2}}$$
$$\therefore x = 0.14\ m$$

7. An oscillating block has kinetic energy equal to potential energy of 25 J (KE=PE= 25 J) when the block is at x=+0.50 m. i) What is the amplitude of oscillation? ii) What is the kinetic energy when the block is x=0?

i) Given, Potential energy at $x = 0.5\ m$, $PE = \frac{1}{2}kx^2$ or, $25 = \frac{1}{2}k(0.5)^2$ or, $25 \times 2 = k \times \frac{1}{4}$ $\therefore k = 200\ N\ m^{-1}$	Total energy, $E = PE + KE$ or, $\frac{1}{2}kA^2 = 25 + 25$ or, $\frac{1}{2} \times 200 \times A^2 = 50$ or, $100 \times A^2 = 50$ or, $A = \sqrt{\frac{50}{100}}$ $\therefore A = 0.707\ m$

ii. We know, At x=0, potential energy is 0 and kinetic energy is equal to total energy = 25 J + 25 J = 50 J

8. **An oscillator consists of a block attached to a spring (k=400 N/m). At some time t, the position, velocity, and acceleration of the block are $x = 0.100\ m,\ v = -13.6\ m/s,\ and\ a = -123\ m/s^2$. Calculate a) the mass of the block and b) the amplitude of the motion.**

a) We know,

$$F = -kx$$
$$\text{or, } ma = -kx$$
$$\text{or, } m = -\frac{kx}{a}$$
$$\text{or, } m = -\frac{400 \times 0.1}{-123}$$
$$\therefore m = 0.325\ kg$$

b) We know,

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2\left(A^2 - x^2\right)$$
$$\text{or, } \frac{v^2}{\omega^2} = A^2 - x^2$$
$$\text{or, } \frac{v^2}{\frac{k}{m}} + x^2 = A^2$$
$$\text{or, } A = \sqrt{v^2 \times \frac{m}{k} + x^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\therefore A = 0.400\ m$$

Here,
$k = 400\ N\ m^{-1}$
$x = 0.1\ m$
$v = -13.6\ m\ s^{-1}$
$a = -123\ m\ s^{-2}$
$m = ?$
$A = ?$

9. Show that for a particle executing SHM, the instantaneous velocity is $v = \omega\sqrt{A^2 - x^2}$ and the maximum velocity is $\sqrt{\frac{2E}{m}}$, where symbols have their usual meanings.

$$v = \frac{dx}{dt}$$
$$v = \frac{d}{dt}[A\cos(\omega t + \varphi)]$$
$$= -A\sin(\omega t + \varphi)\frac{d}{dt}(\omega t + \varphi)$$
$$= -\omega A\sin(\omega t + \varphi)$$
$$= -\omega\sqrt{A^2 \sin^2(\omega t + \varphi)}$$
$$= -\omega\sqrt{A^2[1 - \cos^2(\omega t + \varphi)]}$$
$$= -\omega\sqrt{A^2 - A^2\cos^2(\omega t + \varphi)}$$
$$v = \pm\omega\sqrt{A^2 - x^2}$$

We know,
We got maximum velocity, v_{max} at $x = 0$.
And, potential energy, PE is 0 at $x = 0$.

Now,

$$E = PE + KE$$
$$\text{or, } E = 0 + \frac{1}{2}mv_{max}^2 \quad [\because PE = 0]$$
$$\text{or, } 2E = mv_{max}^2$$
$$\text{or, } \frac{2E}{m} = v_{max}^2$$
$$\therefore v_{max} = \sqrt{\frac{2E}{m}} \quad \text{(Showed)}$$

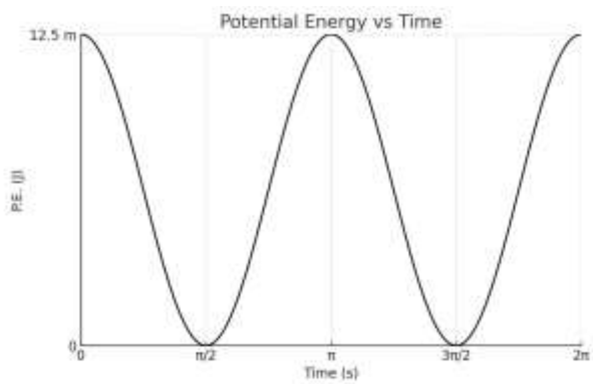
10. Show that, for a mass spring system with the equation of displacement $x = 5cost$, the potential and kinetic energy depends on time while total energy is time independent. [Use equations and graphical figures to justify your answer]

Cthe given equation with $x = Acos\,\omega t$ implies, $\omega = 1\, rad/s$

Potential energy, $E_p = \frac{1}{2}kx^2 = \frac{1}{2}k(\,5\text{cost})^2 = \frac{1}{2}25kcos^2t$

$$= \frac{1}{2}25\omega^2 mcos^2t = \frac{1}{2}25 \times 1^2 \times mcos^2t$$

$$= \frac{1}{2}25mcos^2t$$

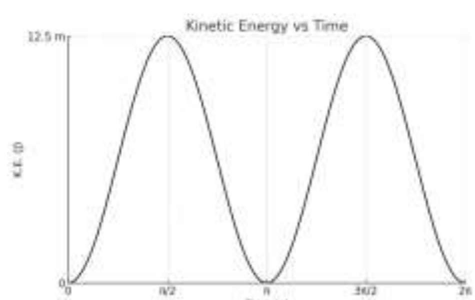


$$v = \frac{dy}{dx} = -5sint$$

Kinetic energy, $E_k = \frac{1}{2}mv^2$

$$= \frac{1}{2}25msin^2t$$

This indicates K.E. depends on time.



Total energy, E = P.E. + K.E. $=\frac{1}{2}25mcos^2t + \frac{1}{2}25msin^2t = \frac{1}{2}25m(cos^2t + sin^2t) = \frac{1}{2}25m$

This shows that total energy time-independent.