Practice problems (Simple harmonic motions)

Topic 1: Draw graphs from equation:

a) $x = 5\cos(6\pi t)$

b) $x = 8\cos\left(2\pi t + \frac{\pi}{2}\right)$

c) $x = 11\sin\left(5\pi t + \frac{\pi}{2}\right)$

Topic 2: Find equations of displacement, velocity and acceleration from graphs:

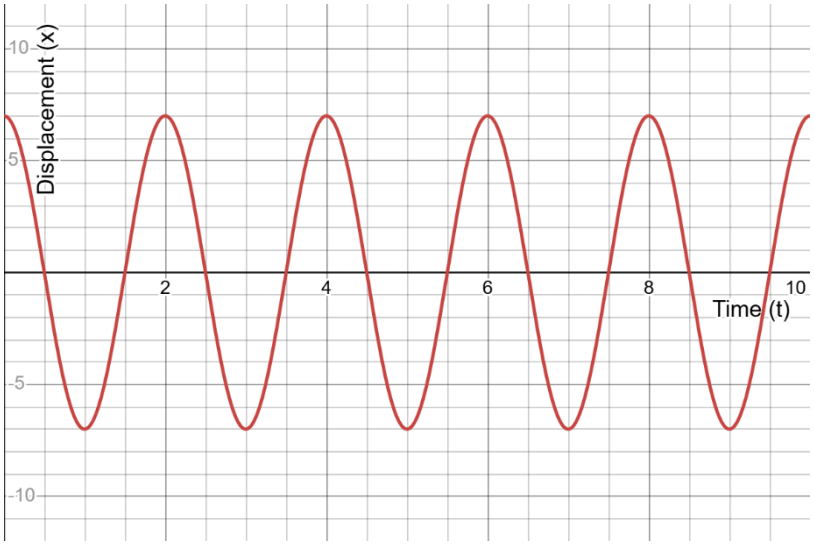

Figure 1: Graph 1

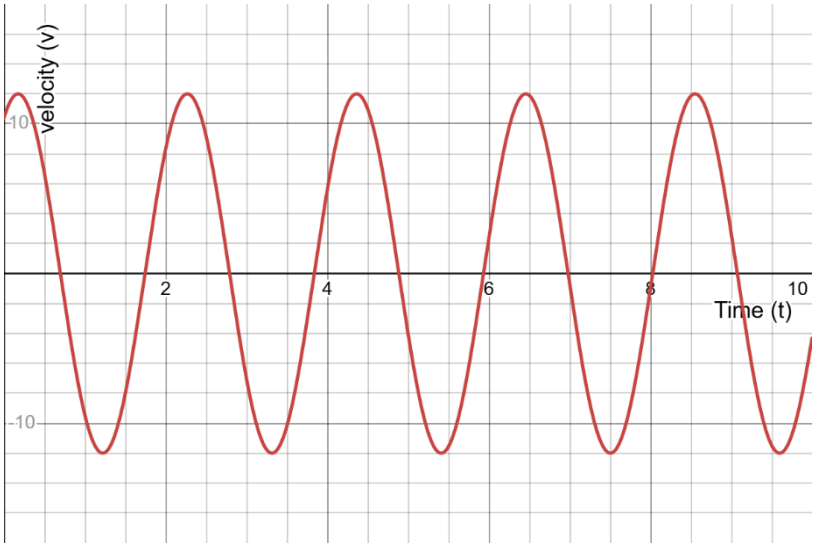

Figure 2: Graph 2

Note that, the first peak at graph 2 occurs at time t = 0.167s.

Topic 3: Find equations of acceleration and velocity from equation of displacement:

a) $x = 14\cos(9.4\pi t)$
b) $x = 11.11\cos(11.11t + 11.11)$
c) $x = 7\sin(3.1416t + 2.71)$

Topic 4: Find equations of displacement and velocity from equation of acceleration:

a) $a = -177\cos(4\pi t - 1.4)$
b) $a = 12\sin\left(t + \frac{\pi}{3}\right)$

Topic 5: Spring

a) A block is suspended from a spring. The mass of the block is equal to last two digit of your ID (take 42 if the last two digits of your id is 00). The spring extends to 7cm to 11cm due to the suspension of the mass. Find the spring constant, and the potential energy stored in the spring length is 11 cm and the maximum kinetic energy if the spring were pulled 2 cm further downwards and released. (Hint: Amplitude 2cm).

b) A block whose mass m is 680 gm is fastened to a spring whose spring constant k is 65 N/m. The block is pulled maximum 11 cm generating cosine form on a frictionless surface and released from rest at t=0. Find out (i) time period, (ii) angular frequency, (iii) phase constant, (iv) v (4 sec), v_{max} , (v) a (4 sec), a_{max}, (vi) displacement at t=0 sec, t=7 sec, and (vii) velocity at a displacement x=0.11 m, x=0.04 m.

c) A 2.30-kg mass on a spring has displacement as a function of time given by

$$x(t) = -(6\ cm)\cos\left(2\pi t + \frac{\pi}{3}\right)$$

Find – (i)the frequency of spring; (ii) the force constant of the spring; (iii) the maximum speed of the mass; (iv) displacement vs time graph and, (v) the position, speed, and acceleration of the mass at t = 3 s.

d) Suppose, A block with mass m = L kg is attached to one end of an ideal spring and moves on a horizontal frictionless surface. Here, L is the last digit of your ID. If last digit of your ID is 0, then take L=5. The other end of the spring is attached to a wall. When the block is at x = +0.5 m, its acceleration is a = -6.0 ms-2 and its velocity is v = +2.5ms-1. Determine- (a) the spring's force constant k; (b) the velocity at x = 0.3m; (c) the maximum speed of the block during its motion; and (d) the maximum magnitude of the block's acceleration during its motion?

e) A body of mass 25gm is attached with a spring of spring constant 400dyns/cm. The body is displaced by 10cm from its equilibrium position and released. Then the body executes simple harmonic motion. Calculate (i) the time period, (ii) frequency, (iii) angular frequency and (iv) maximum velocity.

f) A body of mass 500 gm is suspended from a spring of negligible mass and it stretches the spring by 7 cm. For a displacement of 3 cm, it is given a downward velocity 40 cm/s. Calculate (i) the spring constant, (ii) the angular frequency, (iii) the time period (iv) the initial potential energy, (v) the initial kinetic energy, and (vi) the amplitude of the ensuing motion of the spring.

Topic 6: Simple pendulum

a) The effective length of a simple pendulum is 2.02m, when the angular displacement of the pendulum from equilibrium is 1.54°, find the value of linear displacement, velocity of the bob, acceleration of the bob, kinetic energy and the potential energy.
b) Suppose a simple pendulum has a time period of 5s on Earth. The pendulum was taken to a different planet. The planet has a gravity which is 7 times that of the earth. Also, due to gravity, the effective length of the pendulum increases by 17%. What would be the time period of the pendulum in that planet?

Topic 7: Simple harmonic motion

a) An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (i) period, (ii) frequency, and (iii) amplitude of the motion.
b) Suppose, for a particle moving with a simple harmonic motion, the equation of displacement from equilibrium is x = -11cos(7t+14). Prove that, the basic equation for simple harmonic motion, $\frac{d^2x}{dt^2} + \omega^2 x = 0$ is valid for this particle.