SHM Formulae

$x = A\cos(\omega t + \varphi)$

$x = A\sin(\omega t + \varphi)$

Here,

x = displacement

ω = angular velocity/frequency

φ = phase angle

t = time

Angular velocity/frequency,

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{m}} = 2\pi f$$

Time period, $T = \frac{1}{f} = \frac{2\pi}{\omega}$

Frequency, $f = \frac{1}{T} = \frac{\omega}{2\pi}$

$velocity, v = \frac{dx}{dt}$

$v = \omega\sqrt{A^2 - x^2}$;

A= Amplitude, x = displacement

$v_{max} = \omega A \ \ (at\ mean\ point)$

$acceleration, a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

$a = -\omega^2 x$

$a_{max} = \omega^2 A \ \ (At\ amplitude)$

1. A SHM is represented by

$$\boldsymbol{y = 10\ cm \sin\left(10t - \frac{\pi}{6}\right)}$$

Calculate (a) Time period & frequency (b) Maximum velocity and acceleration (c) Displacement, velocity and acceleration at t = 1 sec.

Comparing the given equation $\boldsymbol{y = 10\ cm \sin\left(10t - \frac{\pi}{6}\right)}$ with standard SHM equation $\boldsymbol{y = A\sin(\omega t + \varphi)}$, we get, amplitude, **A = 10 cm = 0.1 m**, angular velocity, $\boldsymbol{\omega = 10\ rad/s}$ and $\boldsymbol{\varphi = -\frac{\pi}{6}}$

(a) $T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = 0.628\ s$

$f = \frac{1}{T} = \frac{1}{0.628} = 1.59\ Hz$

(b) $v_{max} = \omega A = 10 \times 0.1 = 1\ m/s$

$a_{max} = \omega^2 A = 10^2 \times 0.1 = 10\ m/s^2$

(c) $At\ t = 1\ s,$

$$y(t = 1) = 0.1\ m \sin\left(10 \times 1 - \frac{\pi}{6}\right)$$
$$= -0.0052\ m$$

$$v(t) = \frac{dy}{dt}$$
$$v(t) = \frac{d}{dt}[0.1 \sin\left(10t - \frac{\pi}{6}\right)]$$
$$v(t) = 0.1 \times 10\ \cos\left(10t - \frac{\pi}{6}\right)$$
$$v(t) = 1\ \cos\left(10t - \frac{\pi}{6}\right)$$
$$\mathrm{v(t = 1)} = 1 \cos\left(10 \times 1 - \frac{\pi}{6}\right)$$
$$= -0.998\ m/s$$

$$a(t) = \frac{dv}{dt}$$
$$a(t) = \frac{d}{dt}[1\ \cos\left(10t - \frac{\pi}{6}\right)]$$
$$a(t) = -1 \times 10 \sin\left(10t - \frac{\pi}{6}\right)$$
$$a(t = 1) = -10 \sin\left(10 \times 1 - \frac{\pi}{6}\right)$$
$$= 0.516\ m/m^2$$

2. A particle executes simple harmonic motion given by equation $x = 3sin(25t - \frac{3\pi}{4})$. Calculate the i) displacement at t = 5 s ii) velocity and acceleration at t = 2.5 s.

$$\text{Displacement, } y = 3\sin\left(25t - \frac{3\pi}{4}\right)$$

$\because$ Displacement at $t = 5\ s$

$$y(5) = 3\sin\left(25 \times 5 - \frac{3\pi}{4}\right)$$
$$= -0.36\ m$$

$$\text{Displacement, } y = 3\sin\left(25t - \frac{3\pi}{4}\right)$$

$$\therefore \text{Velocity, } v = \frac{dy}{dt} = 3 \times 25\cos\left(25t - \frac{3\pi}{4}\right)$$
$$= 75\cos\left(25t - \frac{3\pi}{4}\right)$$

$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = -75 \times 25\sin\left(25t - \frac{3\pi}{4}\right)$$
$$= -1875\sin\left(25t - \frac{3\pi}{4}\right)$$

$\therefore$ Velocity at $t = 2.5\ s$,

$$v(2.5) = 75\cos\left(25 \times 2.5 - \frac{3\pi}{4}\right)$$
$$= -67.42\ m\ s^{-1}$$

$\therefore$ Acceleration at $t = 2.5\ s$,

$$a(2.5) = -1875\sin\left(25 \times 2.5 - \frac{3\pi}{4}\right)$$
$$= 821.54\ m\ s^{-2}$$

3. A block of mass 680 gm is fastened to a spring of spring constant 65 N/m. The block is pulled a distance 11 cm from its equilibrium on a frictionless table and released.

(a)What are the angular frequency, the frequency, and the time period of the SHM? (b) What is amplitude of the motion? (c) What is the maximum speed of the block?

Here, mass, m= 680 gm = 0.68 kg, spring constant k = 65 N/m

(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65}{0.68}} = 9.77\ rad/s$

$$f = \frac{\omega}{2\pi} = \frac{9.77}{2\pi} = 1.55\ Hz$$

$$T = \frac{1}{f} = \frac{1}{1.55} = 0.643\ s$$

(b) A = 11 cm = 0.11 m

(c) $v_{max} = \omega A$

$$= 9.77 \times 0.11$$
$$= 1.07\ m/s$$

4. A spring stretches by 3.90 cm when a 10.0 g mass is hung from it. A 25.0 g mass attached to this spring oscillates in simple harmonic motion. (a) Calculate the period of the motion. (b)Calculate frequency

(a)

$$F = kx$$
$$mg = kx$$
$$k = \frac{mg}{x};\ \ here\ m = 10\ g = 0.01kg$$
$$= \frac{0.01 \times 9.8}{0.039} = 2.51\ N/m$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.51}{0.025}}$$
$$= 9.97\ rad/s$$

Here oscillating mass,
m = 25 g = 0.025 kg

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{9.97}$$
$$= 0.63\ s$$

(b)

$$f = \frac{1}{T} = \frac{1}{0.63}$$
$$= 1.58\ Hz$$

5. **A 0.12 kg body undergoes SHM of amplitude 8.5 cm and time period 0.20 s. i) What is the magnitude of the maximum force acting on it? ii) If the oscillations are produced by a spring, what is the spring constant?**

Here,

$m = 0.12\ kg$

$A = 8.5\ cm = 0.085\ m$

$T = 0.20\ s$

$F_{max} = ?$

$k = ?$

i. $F = kx$

$$F_{max} = kA$$
$$= m\omega^2 A$$
$$= m(\frac{2\pi}{T})^2 A$$
$$= 0.12 \times \left(\frac{2\pi}{0.2}\right)^2 \times 0.085$$
$$= 10.067\ N$$

ii. $k = m\omega^2$

$$= m\left(\frac{2\pi}{T}\right)^2$$
$$= 0.12 \times \left(\frac{2\pi}{0.2}\right)^2$$
$$= 118.435\ N/m$$

6. A 3 kg block is attached to a spring and the spring constant is k = 19.6 N/m. The block is held 6 cm from equilibrium the released at t = 0. i) Write an equation for x vs. time. ii) Calculate the velocity at t = 3 and acceleration at t = 0.5 s.

(i)

$$x = A\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{19.6}{3}} = 2.556 \text{ rad s}^{-1}$$

Here,

$m = 3\ kg$

$k = 19.6\ N\ m^{-1}$

$A = 6\ cm = 0.06\ m$

$$x = 0.06\cos(2.556t)$$

ii.

$$v = \frac{dx}{dt}$$
$$= \frac{d}{dt}[A\cos(\omega t)]$$
$$v = -\omega A\sin(\omega t)$$

At t = 3 s

$$v = -2.556 \times 0.06\sin(2.556 \times 3)$$
$$= -0.15\ m/s$$

$$a = \frac{dv}{dt}$$
$$= \frac{d}{dt}[-\omega A\sin(\omega t)]$$
$$= -\omega^2 A\ cos\ (\omega t)$$

At t = 0.5 s,

$$a = -2.556^2 \times 0.06\ cos\ (2.556 \times 0.5)$$
$$= -0.113\ m/s^2$$

7. An oscillator consists of a block attached to a spring (k=400 N/m). At some time t, the position, velocity, and acceleration of the block are x = 0.100 m, v = −13.6 m/s, and a = −123 m/s^2. Calculate a) the mass of the block and b) the amplitude of the motion.

$$F = -kx$$
$$\text{or, } ma = -kx$$
$$\text{or, } m = -\frac{kx}{a}$$
$$\text{or, } m = -\frac{400 \times 0.1}{-123}$$
$$\therefore m = 0.325\ kg$$

$$k = 400\ N\ m^{-1}$$
$$x = 0.1\ m$$
$$v = -13.6\ m\ s^{-1}$$
$$a = -123\ m\ s^{-2}$$
$$m = ?$$
$$A = ?$$

$$v = \pm\omega\sqrt{A^2 - x^2}$$
$$\text{or, } v^2 = \omega^2\left(A^2 - x^2\right)$$
$$\text{or, } \frac{v^2}{\omega^2} = A^2 - x^2$$
$$\text{or, } \frac{v^2}{\frac{k}{m}} + x^2 = A^2$$
$$\text{or, } A = \sqrt{v^2 \times \frac{m}{k} + x^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\text{or, } A = \sqrt{(-13.6)^2 \times \frac{0.325}{400} + (0.1)^2}$$
$$\therefore A = 0.400\ m$$

8. **Consider a simple harmonic motion (SHM) where the velocity is 7 cm/s when the displacement is 2 cm, and 5 cm/s when the displacement is 3 cm. Determine the amplitude of the oscillation.**

We know that,

$$V = \omega\sqrt{A^2 - x^2}$$

Given:

$$7 = \omega\sqrt{A^2 - 4} \quad (1), \quad 5 = \omega\sqrt{A^2 - 9} \quad (2)$$

Dividing (1) by (2):

$$\frac{7}{5} = \frac{\sqrt{A^2-4}}{\sqrt{A^2-9}}$$

$$\frac{49}{25} = \frac{A^2-4}{A^2-9}$$

$$49A^2 - 441 = 25A^2 - 100$$

$$24A^2 = 341$$

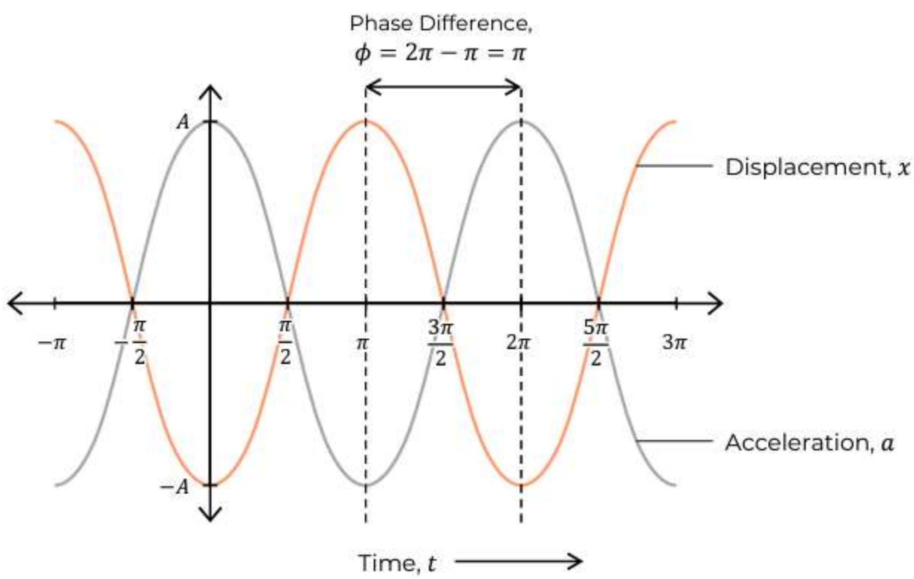
$$A = \pm\sqrt{\frac{341}{24}} \quad \text{cm}$$

9. The equation of displacement of a simple harmonic oscillator is
$x = A\cos(\omega t + \pi)$. Plot displacement vs. time and acceleration vs. time graphically. What is phase difference between displacement and acceleration?

$$\text{Displacement, } x = A\cos(\omega t + \pi)$$

$$\therefore \text{Velocity, } v = \frac{dx}{dt} = -A\omega\sin(\omega t + \pi)$$

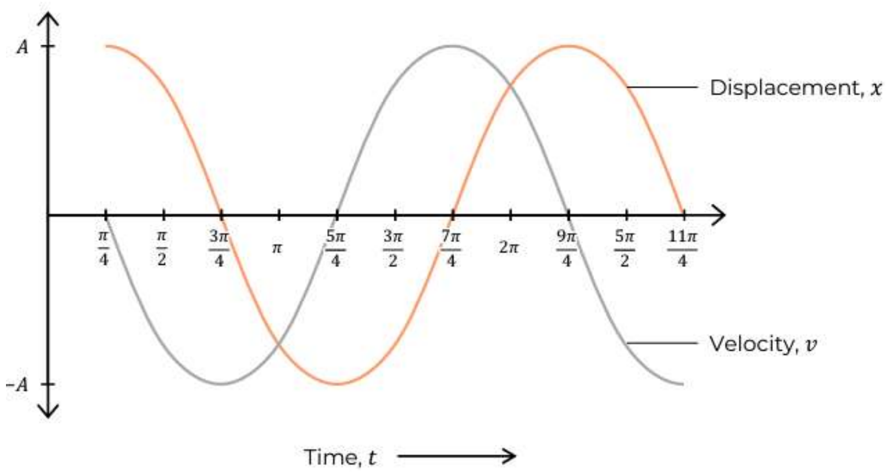
$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \pi)$$



10. The equation of displacement of a simple harmonic oscillator is x = A cos (ωt – π/4). Graphically represent the displacement and velocity of the oscillator.

$$\text{Displacement, } x = A\cos\left(\omega t - \frac{\pi}{4}\right)$$

$$\therefore \text{Velocity, } v = \frac{dx}{dt} = -A\omega\sin\left(\omega t - \frac{\pi}{4}\right)$$



10. The displacement of a Simple Harmonic Motion (SHM) is

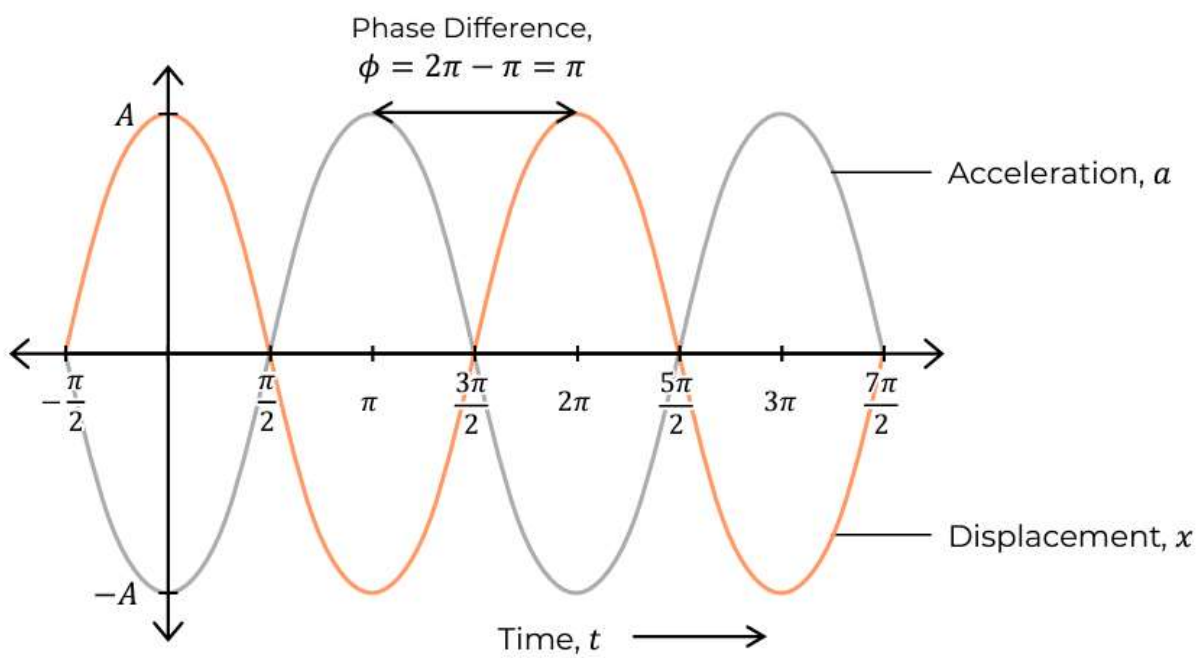
$$y = Asin(\omega t + \frac{\pi}{2}).$$

Graphically show that the displacement and acceleration are out of phase to each other.

$$\text{Displacement, } x = A\sin\left(\omega t + \frac{\pi}{2}\right)$$

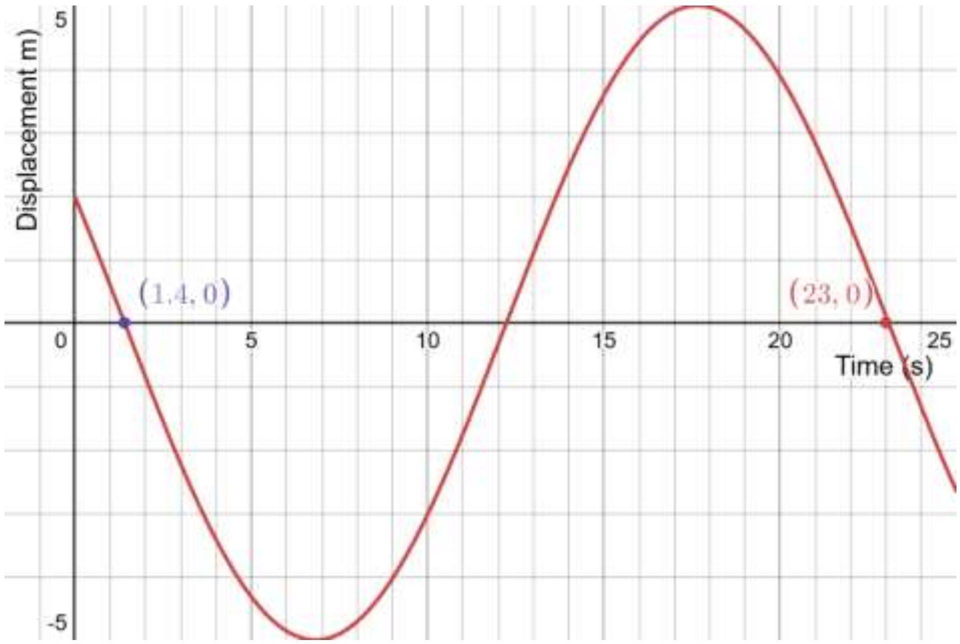
$$\therefore \text{Velocity, } v = \frac{dx}{dt} = A\omega\cos\left(\omega t + \frac{\pi}{2}\right)$$

$$\therefore \text{Acceleration, } a = \frac{dv}{dt} = -A\omega^2\sin\left(\omega t + \frac{\pi}{2}\right)$$



We know, when two sinusoidal waves have phase difference of π, then we say that these two sinusoidal waves are out phase to each other. Since, phase difference between in the acceleration and displacement is π, therefore displacement and acceleration are out phase to each other.

11.



(a) What is the equation of SHM of this graph

$$x = A\cos(\omega t + \varphi)$$

$$From\ graph,$$

$$A = 5\ m$$

$$T = 23 - 1.4 = 21.6\ s$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{21.6} = 0.29\ rad/s$$

$$At\ t\ =\ 0, x\ =\ 2\ m$$

$$x = A\cos(\omega t + \varphi)$$

$$2 = 5\cos(0.29 \times 0 + \varphi)$$

$$\cos\varphi = \frac{2}{5}$$

$$\varphi = \cos^{-1}\frac{2}{5} = 1.159$$

$$\therefore\ x = 5\cos(0.29t + 1.159)$$