Waves and Oscillation

Course- PHY 2105 / PHY 105

Lecture 2 & 3

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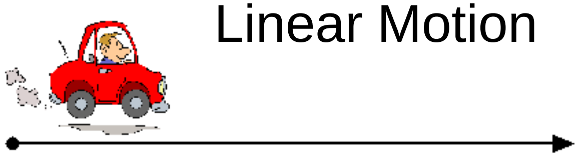
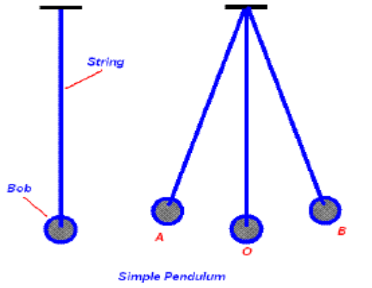
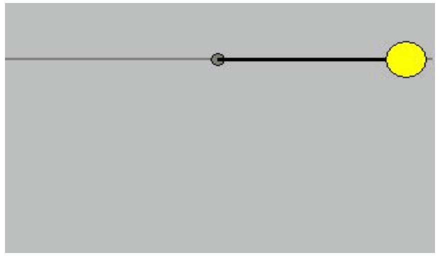
Looking back

Motion
Change of position, with respect to time

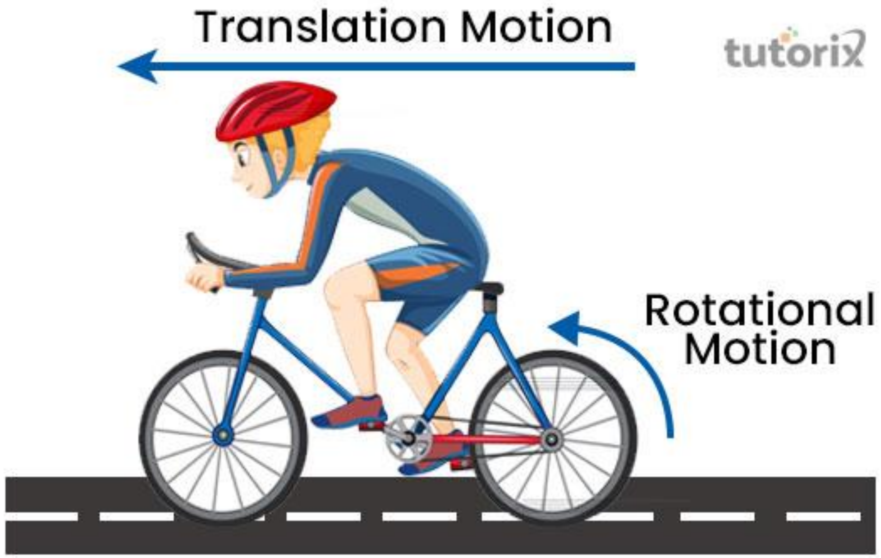


Linear Motion





Oscillatory Motion (Simple Pendulum)



Uniform Circular Motion

Oscillatory Motion (Spring Mass)

Periodic Motion

A motion that repeats itself after an equal interval of time.

Periodicity

Examples:

- the Earth in its orbit
- ceiling fan
- analog clock
- a water wave

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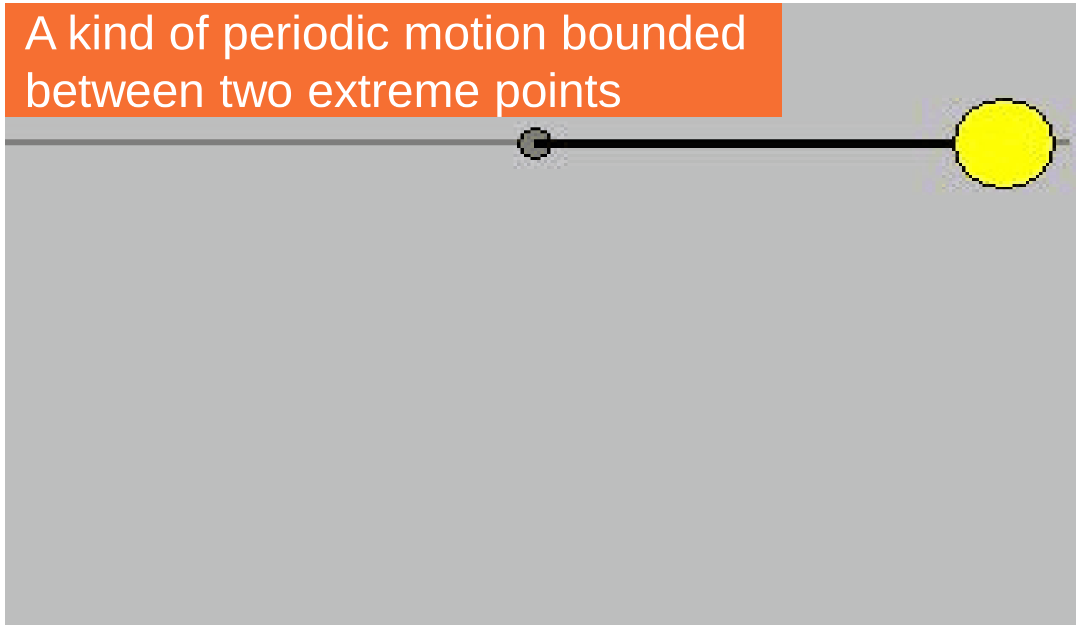
Oscillatory Motion

Periodic motion of an object that moves on either side of the equilibrium (or) mean position is an oscillatory motion.

Examples:

- Power line oscillates when the wind blows past it
- Earthquake oscillations move buildings
- Block attached to a spring
- Motion of a swing
- Motion of a pendulum
- Vibrations on a stringed musical instrument
- Back and forth motion of a piston
- Vibrations of a Quartz crystal



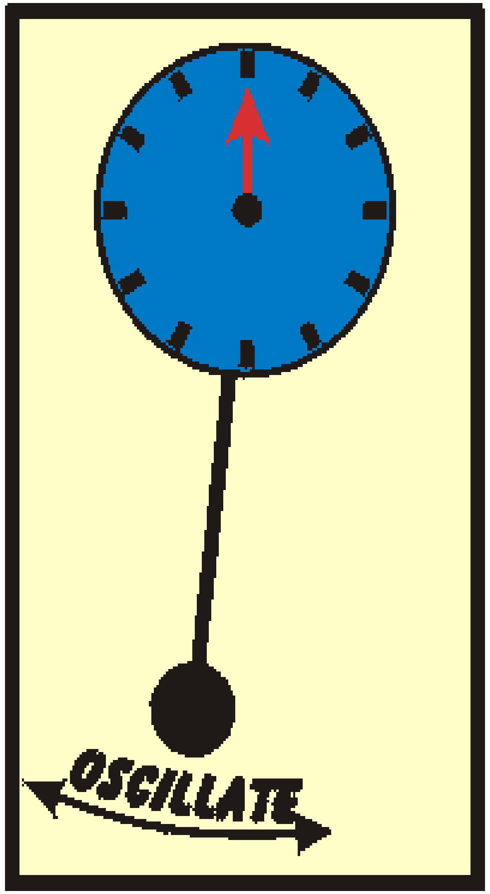
A kind of periodic motion bounded between two extreme points

Differences

Generic Periodic Motion

- There is no equilibrium position.
- There is no restoring force

An oscillatory motion is always periodic. A periodic motion may or may not be oscillatory



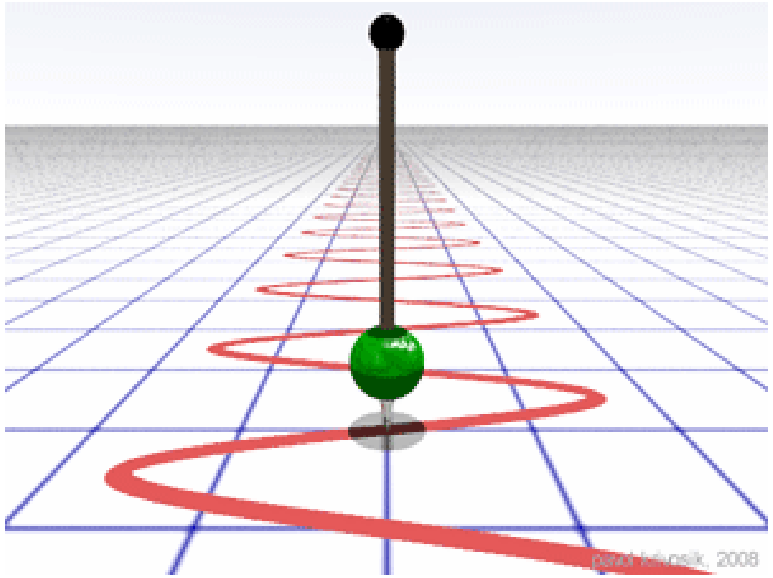
Oscillatory Motion

- There will be a restoring force directed towards the stable equilibrium position (or) mean position

Simple Harmonic Motion

The simplest kind of oscillation occurs when the **restoring** force F_x is directly proportional to the displacement from the equilibrium x, given by equation

$$F_x = -kx$$



This oscillation is called a

Simple Harmonic Motion(SHM).

A system that oscillates with SHM is called a **simple harmonic oscillator**.

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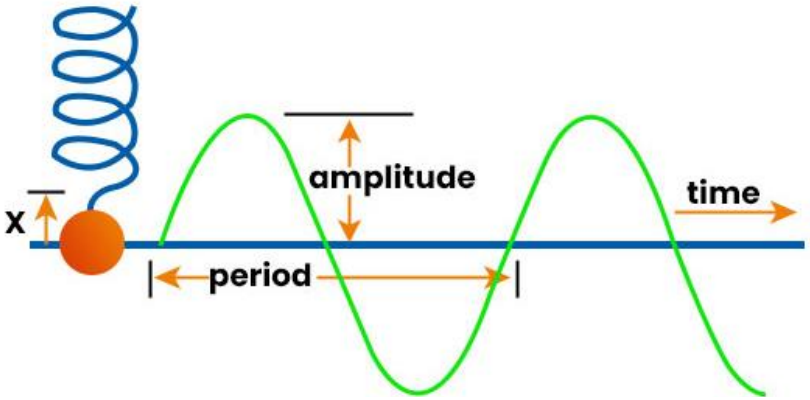
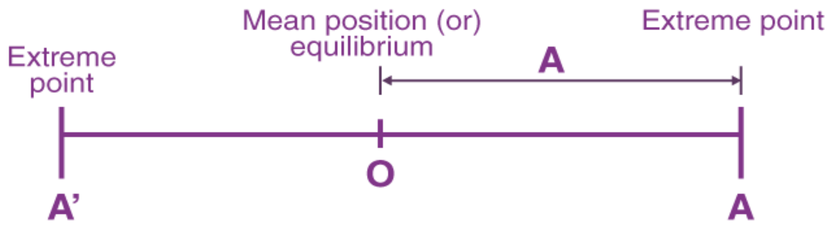
Definitions





A= Amplitude = distance from the mean point to the extreme point

Amplitude, A

The amplitude of the motion, denoted by A, is the maximum magnitude of displacement from the equilibrium position. It is always positive

Period, T

The period T, is the time required for one complete oscillation, or a cycle.

Frequency, f

The frequency, f, is the number of cycles completed in a unit time.

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Formulae

For displacement x, velocity v, acceleration a, frequency f, time t, oscillation period T and angular frequency ω

$$v = \frac{dx}{dt} \qquad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

$$f = \frac{1}{T} \qquad \omega = 2\pi f = \frac{2\pi}{T}$$

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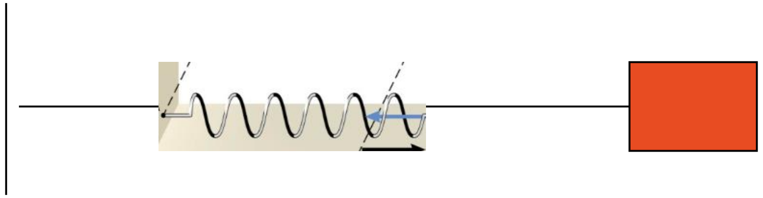
What is the oscillation period of an FM radio station that broadcasts at 100 MHz?

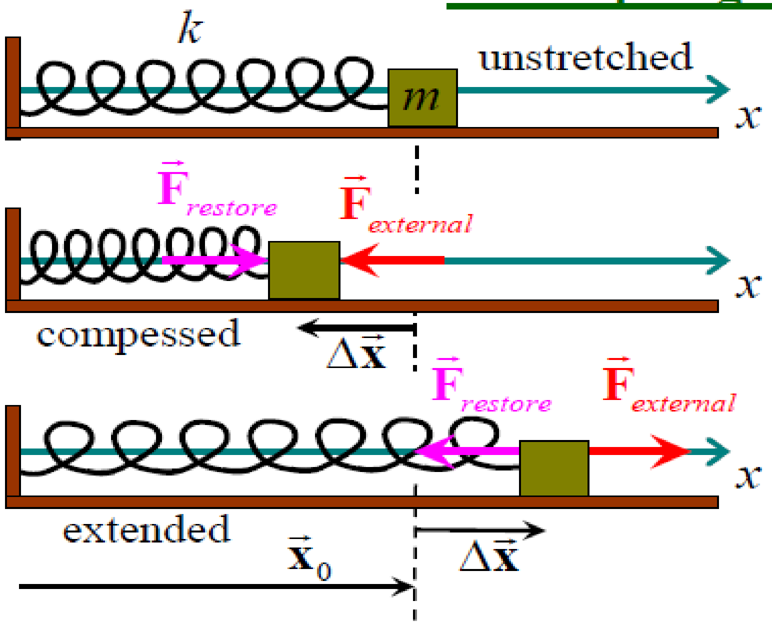
10 ns



$$F = ma = -kx$$

Simple Harmonic Oscillator

Mass-spring oscillator



Hooke's Law:
Restoring force,

$$\vec{\mathbf{F}}_{restore} = -k\Delta\vec{\mathbf{x}}$$

where $\Delta\vec{\mathbf{x}} = \vec{\mathbf{x}} - \vec{\mathbf{x}}_0$

and k is the "spring constant" [N m^{-1}]

Start with the momentum principle: $\dfrac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}_{\text{net}}$

For horizontal forces on the mass: $\dfrac{dp_x}{dt} = -kx$

$$\therefore \frac{d(mv_x)}{dt} = -kx \quad \text{or} \quad \frac{d}{dt}\left(m\frac{dx}{dt}\right) = -kx$$

Equation of SHM: $\therefore \dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$

Motion properties

Shape of a SHM oscillation function : **Sinusoidal**

Functional equation:

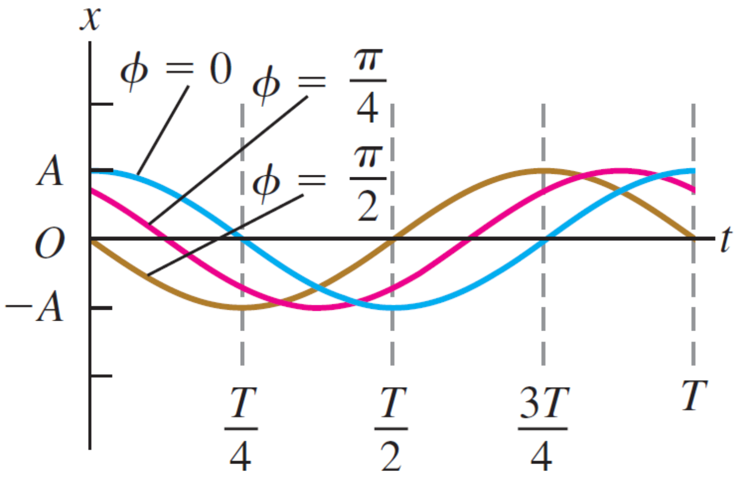
$$x = A\cos(\omega t + \varphi)$$

x = displacement

A = amplitude

ω = angular frequency

Φ = phase angle



These three curves show SHM with the same period T and amplitude A but with different phase angles ϕ.

At $t = 0$, write $x = x_0$ and $v = v_0$.

Then at $t = 0$:

$$\left.\begin{aligned} x_0 &= A\cos(\phi) \\ v_0 &= -\omega_0 A\sin(\phi) \end{aligned}\right\} \quad \tan\phi = -\frac{v_0}{\omega_0 x_0}$$

and $\quad x_0^{\,2} + \left(\frac{v_0}{\omega_0}\right)^2 = A^2\cos^2(\phi) + A^2\sin^2(\phi) = A^2$

$$\therefore A = \sqrt{x_0^{\,2} + \left(\frac{v_0}{\omega_0}\right)^2}$$

Example 2.1

Ex. A block of mass 680gm is fastened to a spring of spring constant 65N/m. The block is pulled a distance 11cm from its equilibrium on a frictionless table and released

(a) What are the angular frequency, the frequency, and the time period of the motion?
(b) What is amplitude of the motion?
(c) What is the maximum speed of the block?

$(a)\ T = 0.643\ s\ and\ f = 1.555\ Hz\ and\ \omega = 9.777\ rad/s$
$(b)\ A = 11\ cm \qquad (c)\ v = 1.075\ m/s$

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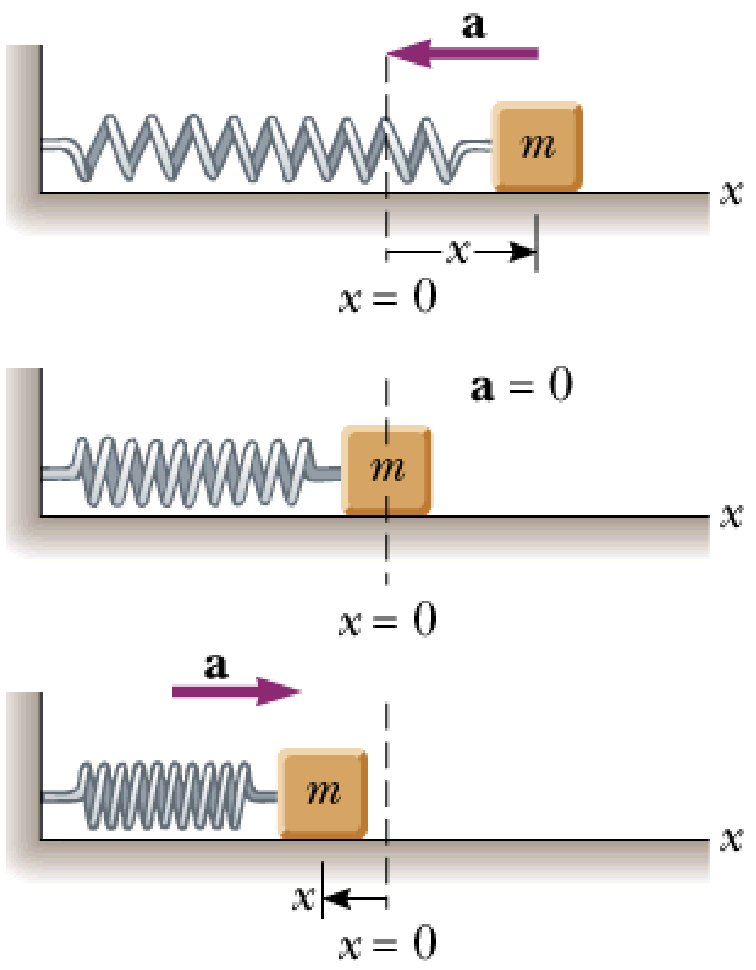
Example 2.2

A spring stretches by 3.90 cm when a 10.0 g mass is hung from it. A 25.0 g mass attached to this spring oscillates in simple harmonic motion.



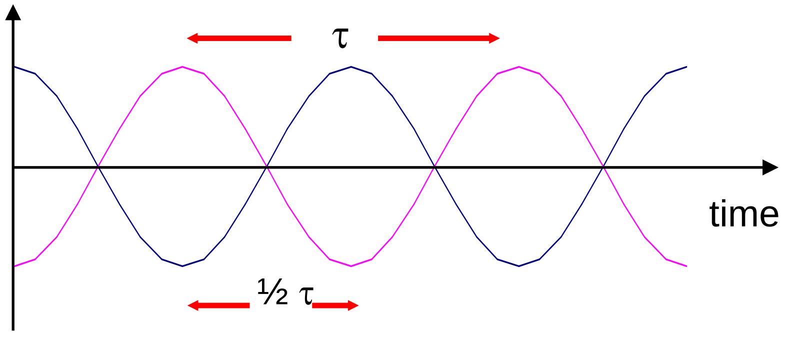
(a) Calculate the period of the motion.

(b) Calculate frequency and the angular velocity of the motion.

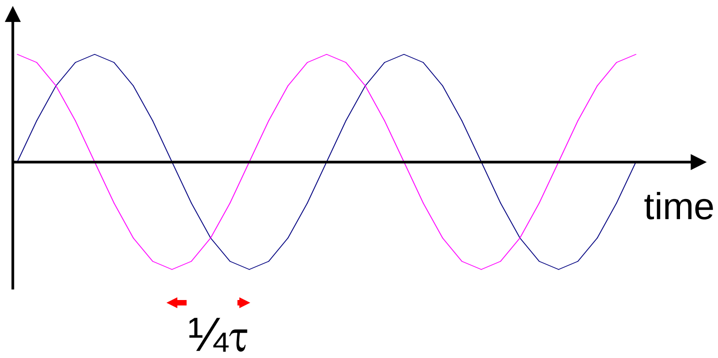
$(a)\ T = 0.63\ s\ (b)\ f = 1.60\ Hz\ and\ \omega = 10\ rad/s$

Phase difference examples

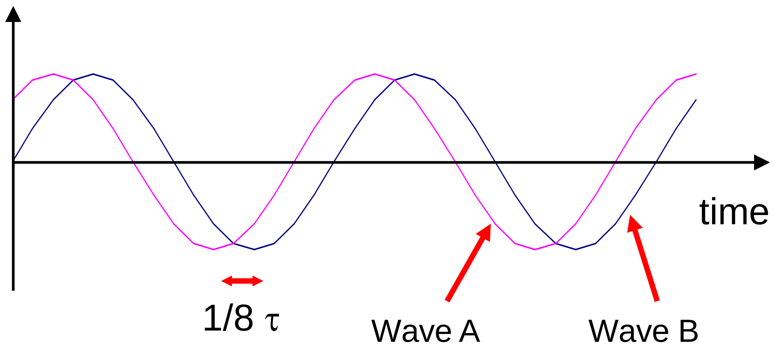
Phase Difference 180°



Phase Difference 90°



Phase Difference 45°



- The phase of periodic wave describes where the wave is in its cycle
- Phase difference is used to describe the phase position of one wave relative to another

$$\frac{d^2x(t)}{dt^2} = \frac{-k}{m}x(t)$$

... a second order differential equation
... we know that if we displace a mass-spring system from its rest position and then release it, it will perform SHM ...

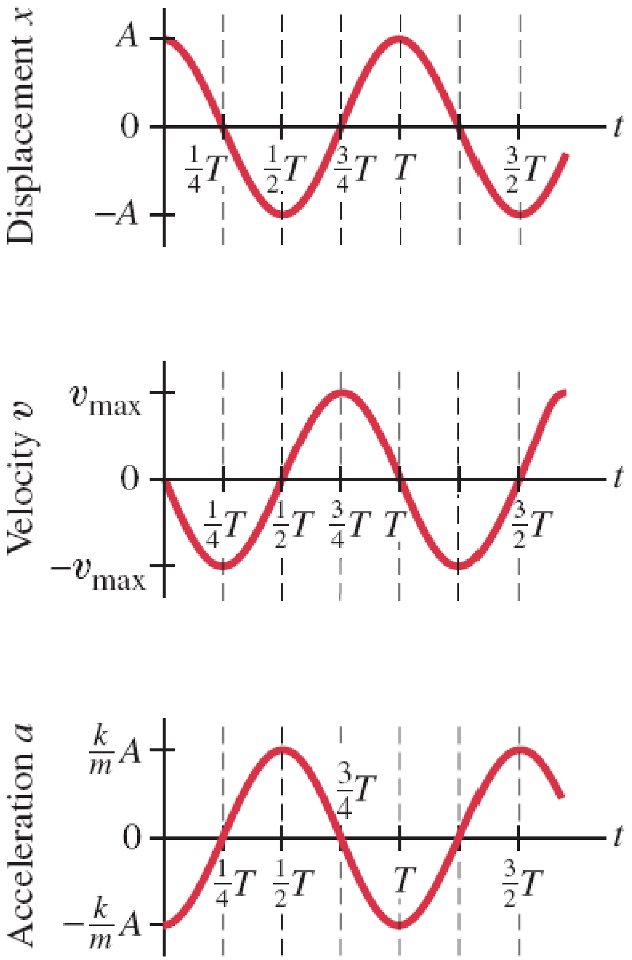
Guess a trial solution: $x(t) = A\cos(\omega t + \phi)$

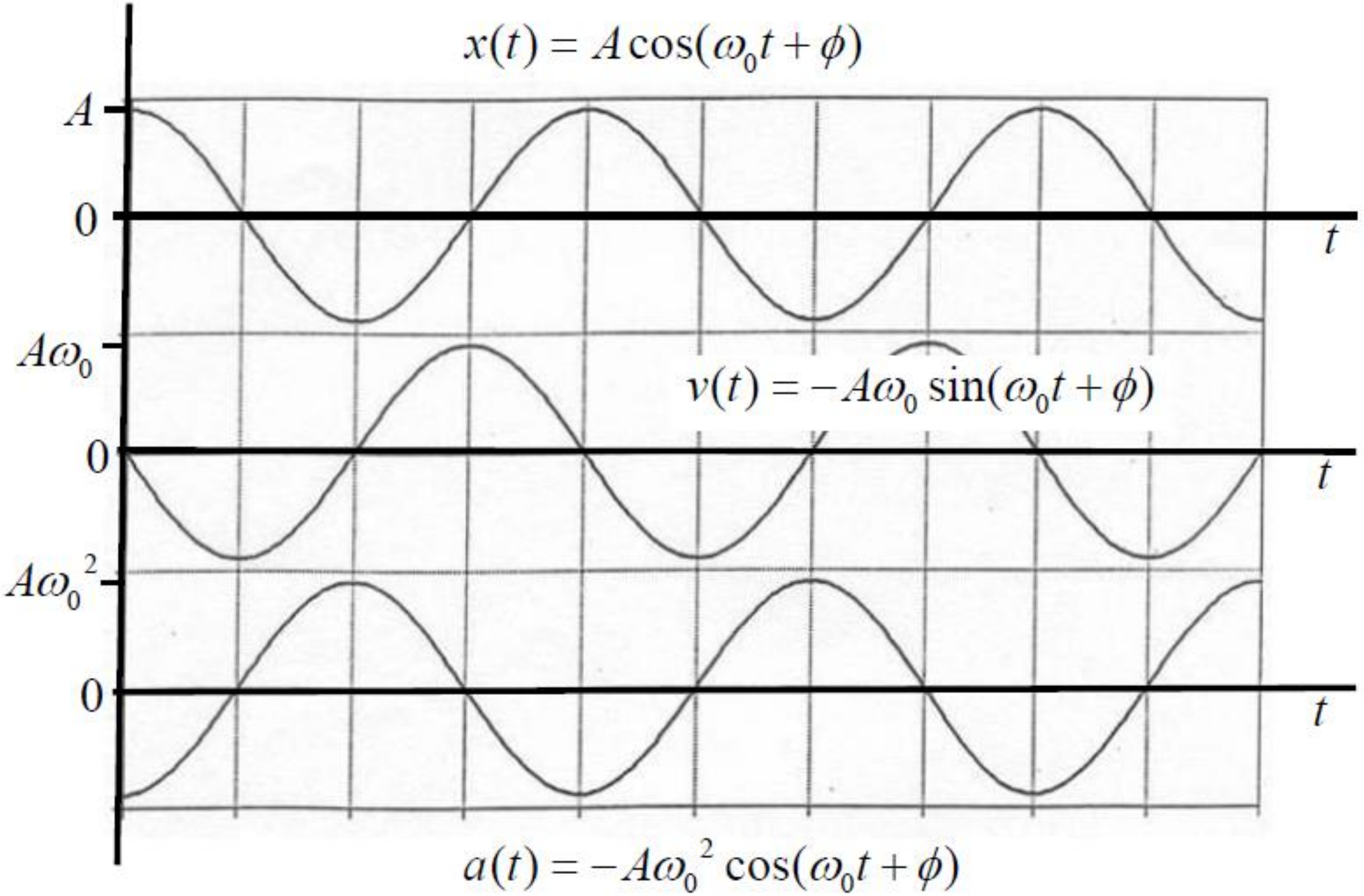
then $\dfrac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$

and substitute into our DE: $-A\omega^2\cos(\omega t + \phi) = -A\dfrac{k}{m}\cos(\omega t + \phi)$

... which is true provided $\omega^2 = \dfrac{k}{m}$

Therefore our solution is $x(t) = A\cos(\omega t + \phi)$ where $\omega = \sqrt{\dfrac{k}{m}}$





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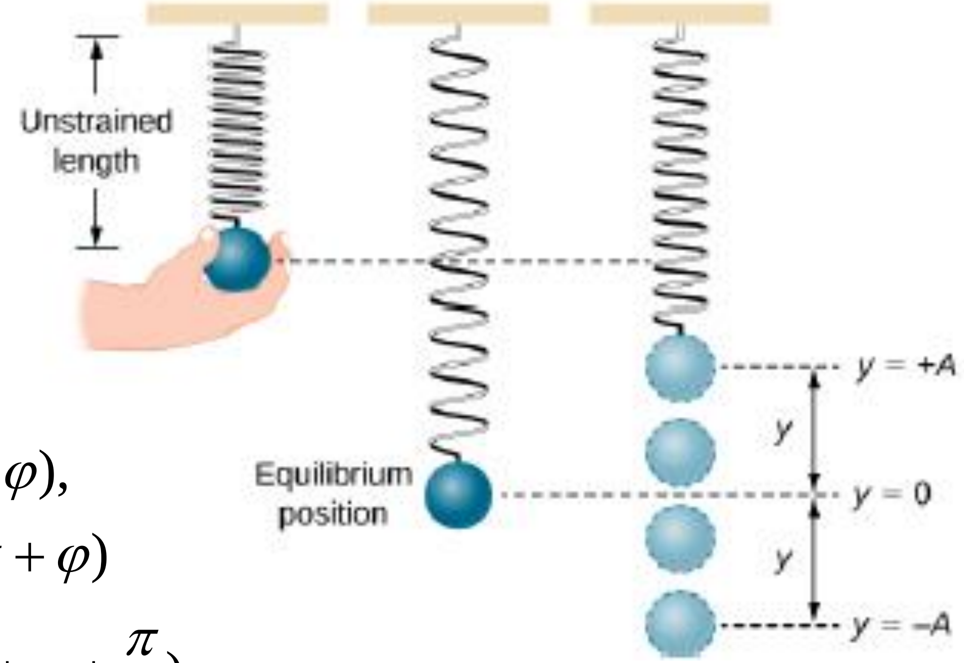
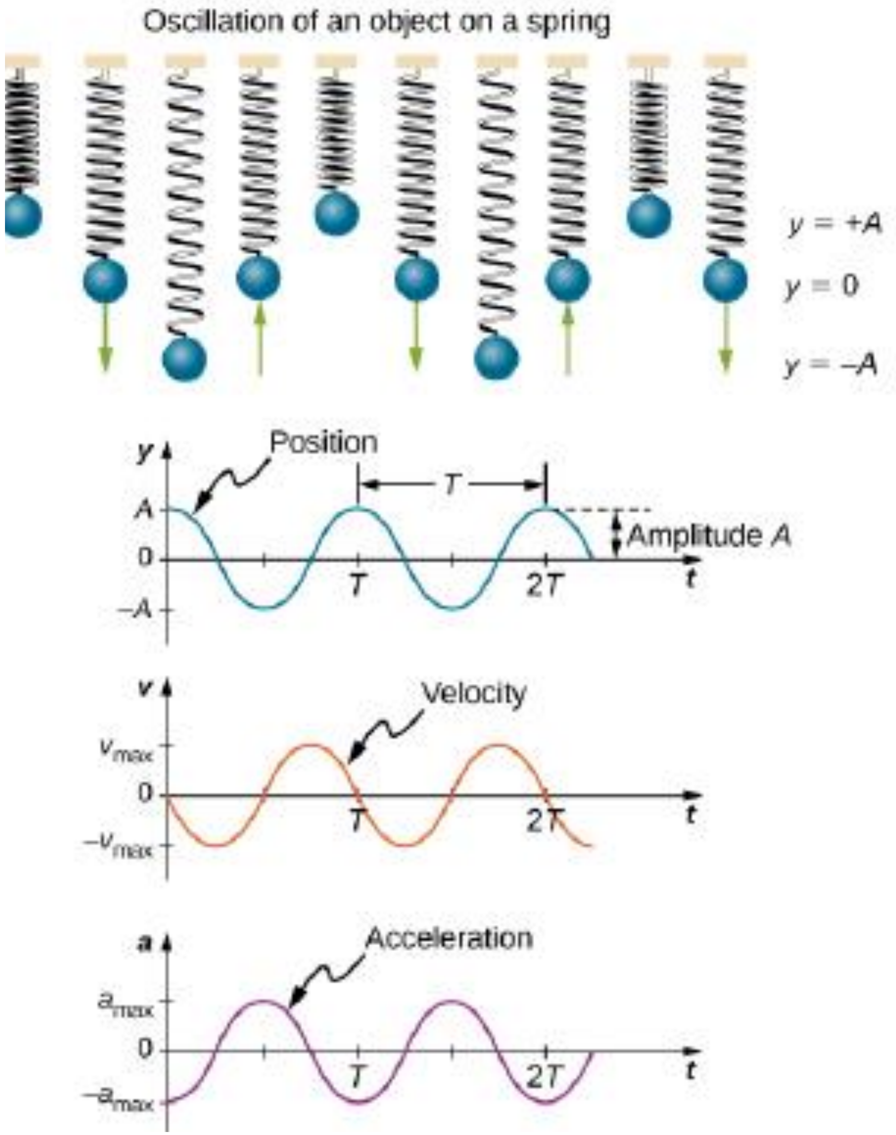


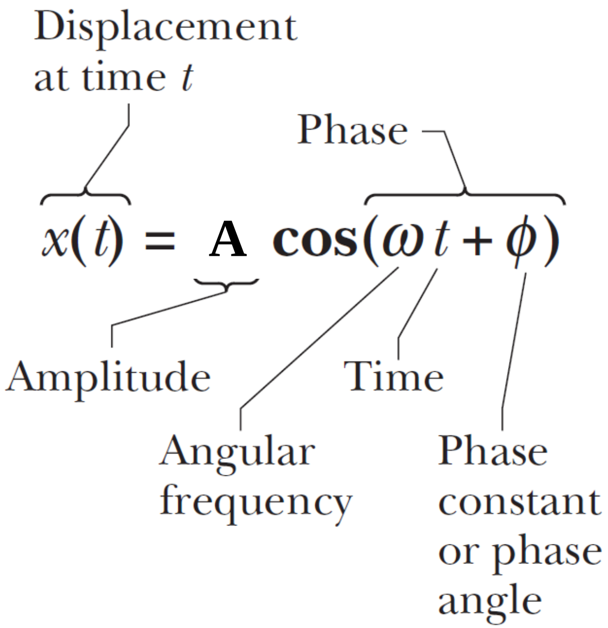


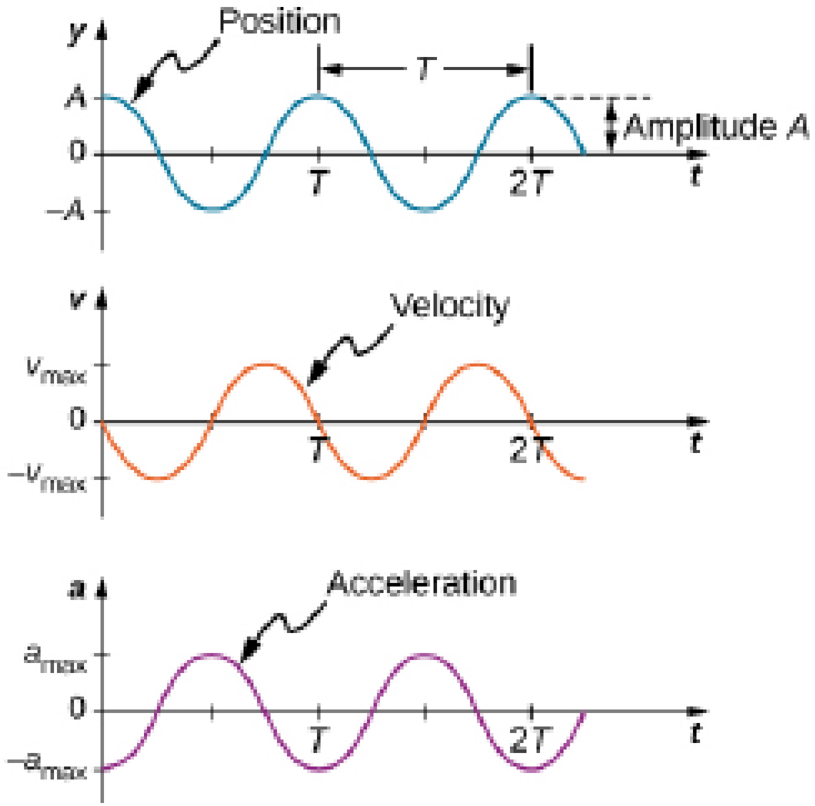
$$x = A\cos(\omega t + \varphi),$$
$$v = -\omega A\sin(\omega t + \varphi)$$
$$= \omega A\cos(\omega t + \varphi + \frac{\pi}{2}),$$
$$a = -\omega^2 A\cos(\omega t + \varphi)$$
$$= -\omega^2 A\sin(\omega t + \varphi + \frac{\pi}{2}).$$

Equations

Equation of SHM: $\frac{d^2x}{dt^2} = -\frac{k}{m}x$

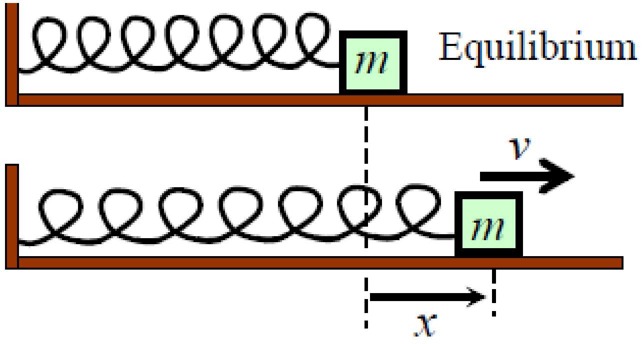
$$x(t) = A\cos(\omega t + \phi)$$
$$v(t) = -v_{\max}\sin(\omega t + \phi)$$
$$a(t) = -a_{\max}\cos(\omega t + \phi)$$
$$x_{\max} = A$$
$$v_{\max} = A\omega$$
$$a_{\max} = A\omega^2.$$





Mass-spring oscillator: an energy approach

For our mass-spring system: $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$



Suppose that the mass has a speed v when it has displacement x

Kinetic energy of mass = $\frac{1}{2}mv^2$

Potential energy of spring = $\int_0^x F dx' = \int_0^x kx' dx' = \frac{1}{2}kx^2$

There are no dissipative mechanisms in our model (no friction).
… the total energy of the mass-spring system is conserved.

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = 0$$

$$\therefore mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0$$

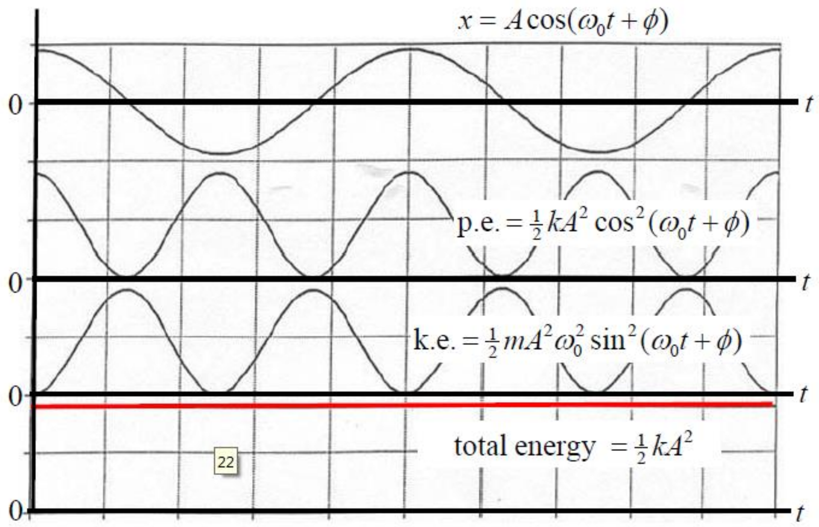
$$\therefore mv\frac{dv}{dt} + kxv = 0$$

$$\therefore m\frac{dv}{dt} + kx = 0$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

Energy

Energy of the mass-spring simple harmonic oscillator



For the mass-spring system: $x = A\cos(\omega_0 t + \phi)$

Potential energy = $\frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi)$

k.e. = $\frac{1}{2}mv^2 = \frac{1}{2}m[-A\omega_0\sin(\omega_0 t + \phi)]^2 = \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$

Total energy = p.e. + k.e

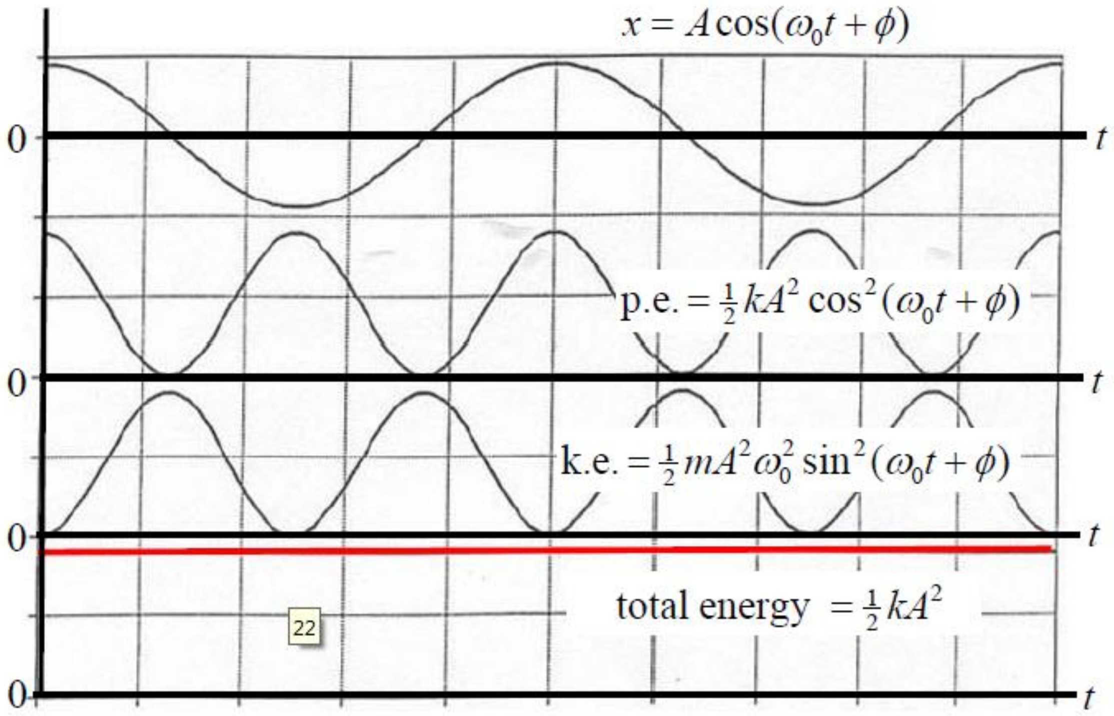
$$= \frac{1}{2}kA^2\cos^2(\omega_0 t + \phi) + \frac{1}{2}mA^2\omega_0^2\sin^2(\omega_0 t + \phi)$$

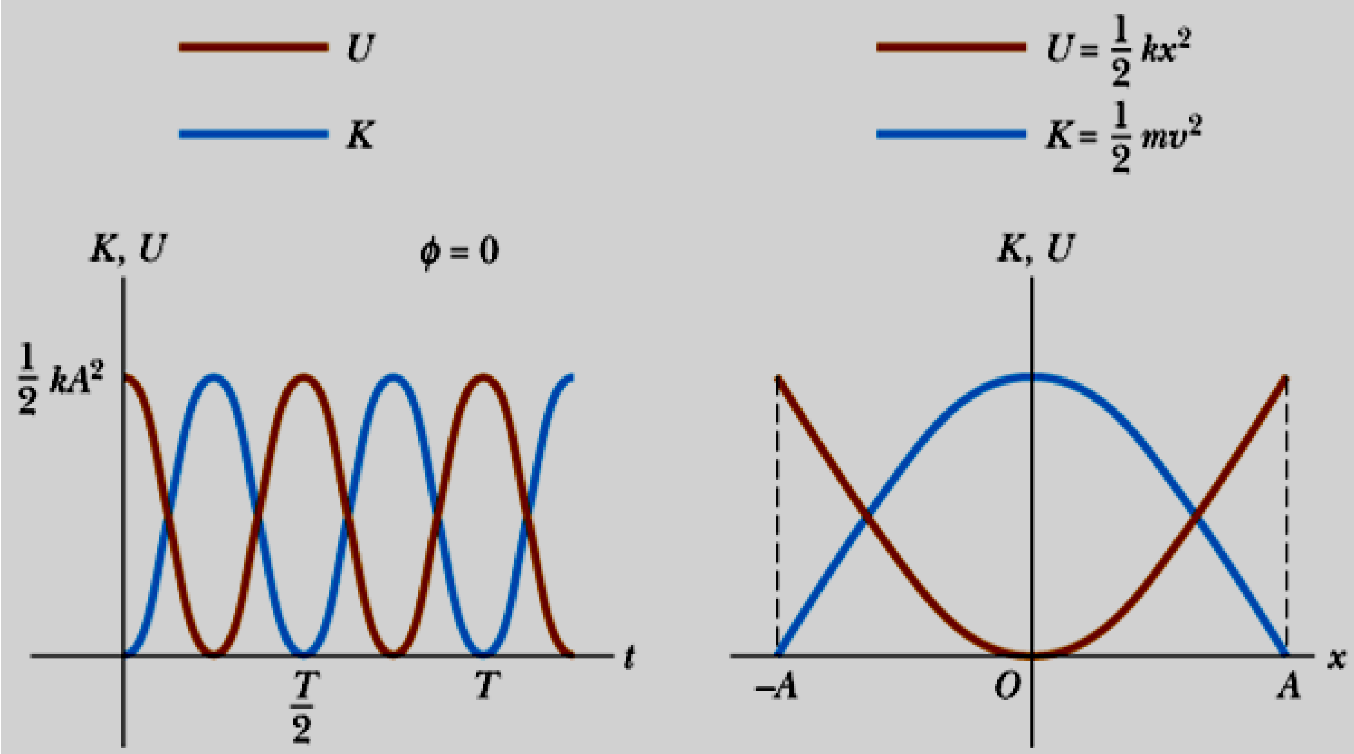
$$= \frac{1}{2}kA^2 \quad \left(= \frac{1}{2}m\omega_0^2 A^2\right) \qquad (\therefore E \propto A^2)$$

We can now write: $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$

$$\therefore \boxed{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} \quad \text{or} \quad v(x) = \pm\omega_0\sqrt{A^2 - x^2}}$$

Energy of the mass-spring simple harmonic oscillator





Example 2.3

For the simple harmonic oscillation where k = 19.6 N/m, A = 0.100 m, x = -(0.100 m) cos 8.08t, and v = (0.808 m/s) sin 8.08t, determine:

(a) the total energy

(b) the kinetic and potential energies as a function of time

(c) the velocity when the mass is 0.050 m from equilibrium

(d) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

$(a)\ E_{total} = 9.8 \times 10^{-2} J$ (b) $\mathrm{U} = 9.8 \times 10^{-2} J\ \cos^2(8.08t)$ and $\mathrm{K} = 9.8 \times 10^{-2} J\ \sin^2(8.08t)$

$(c)\ v = 0.7\frac{m}{s}$ $(d)\ \mathrm{U} = 2.45 \times 10^{-2} J$ and $\mathrm{K} = 7.35 \times 10^{-2} J$

Example 2.4

A 500 g block on a spring is pulled a distance of 20 cm and released. The subsequent oscillations are measured to have a period of 0.80 s. At what position (or positions) is the speed of the block 1.0 m/s?

$$T = 0.80\ \text{s} \quad \text{so} \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{(0.80\ \text{s})} = 7.85\ \text{rad/s}$$

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \omega\sqrt{A^2 - x^2}$$

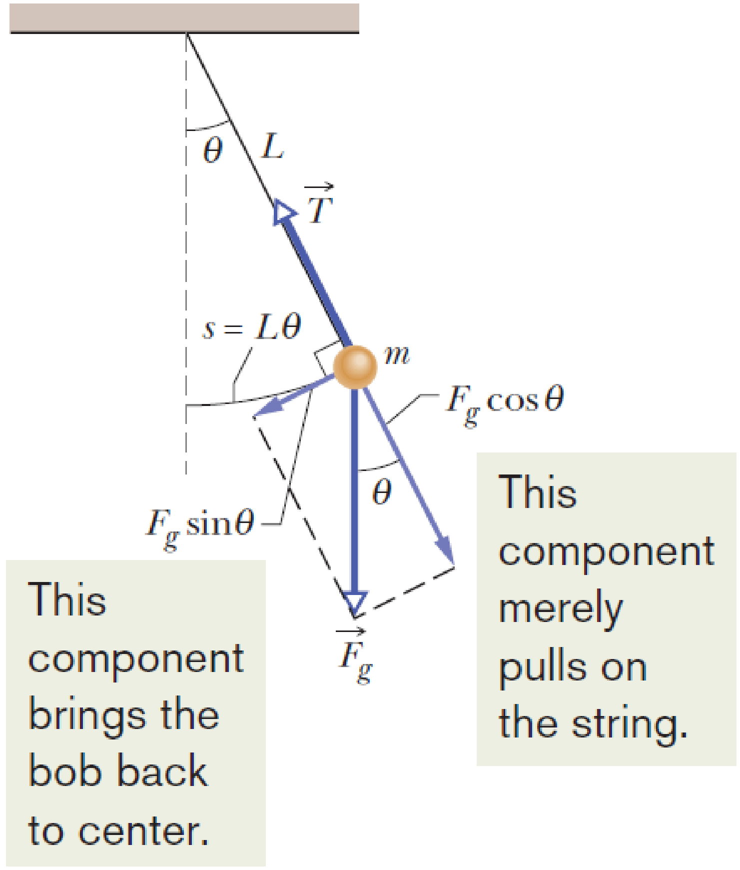
$$x = \pm\sqrt{A^2 - \left(\frac{v}{\omega}\right)^2} = \pm\sqrt{(0.20\ \text{m})^2 - \left(\frac{(1.0\ \text{m/s})}{(7.85\ \text{rad/s})}\right)^2} = \pm 0.154\ \text{m} = \pm 15.4\ \text{cm}$$

Example 2.5

A mass, oscillating in simple harmonic motion, starts at $x = A$ and has period T. At what time, as a fraction of T, does the mass first pass through $x = \frac{1}{2}A$?

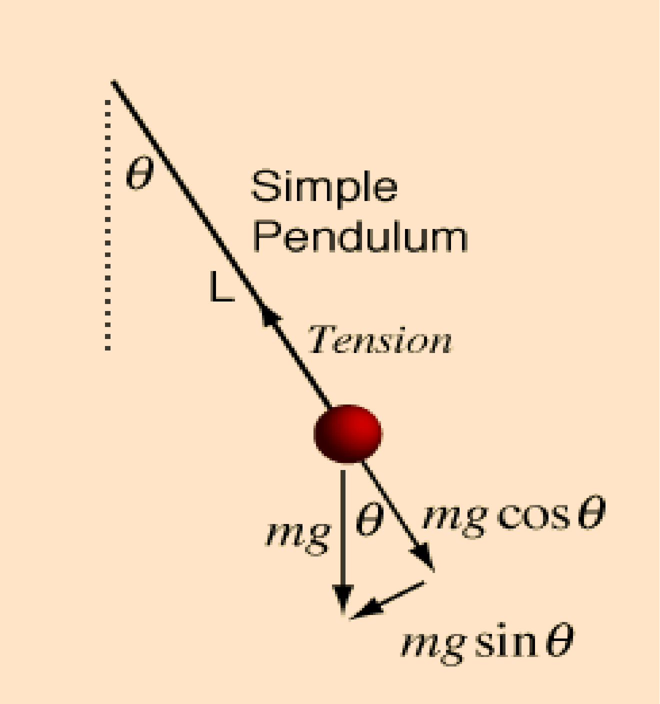
$$t = \frac{T}{2\pi}\cos^{-1}\left(\frac{1}{2}\right) = \frac{T}{6}$$

Simple Pendulum



A simple pendulum consists of a particle of mass *m* (called the *bob* of the pendulum) suspended from one end of an unstretchable, massless string of length *L* that is fixed at the other end.

The only forces acting on the bob are the force of gravity (i.e., the weight of the bob) and tension from the string.



From the above figure restoring force

$$F = -mg\sin\theta$$

If the angle θ is very small sinθ is very nearly equal to θ,
The displacement along the arc is-

$$x = L\theta$$

$$F = -mg\theta$$

Acceleration $\dfrac{d^2x}{dt^2} = L\dfrac{d^2\theta}{dt^2}$ $\quad$ $Force = mL\dfrac{d^2\theta}{dt^2}$

$$mL\frac{d^2\theta}{dt^2} = -mg\theta$$

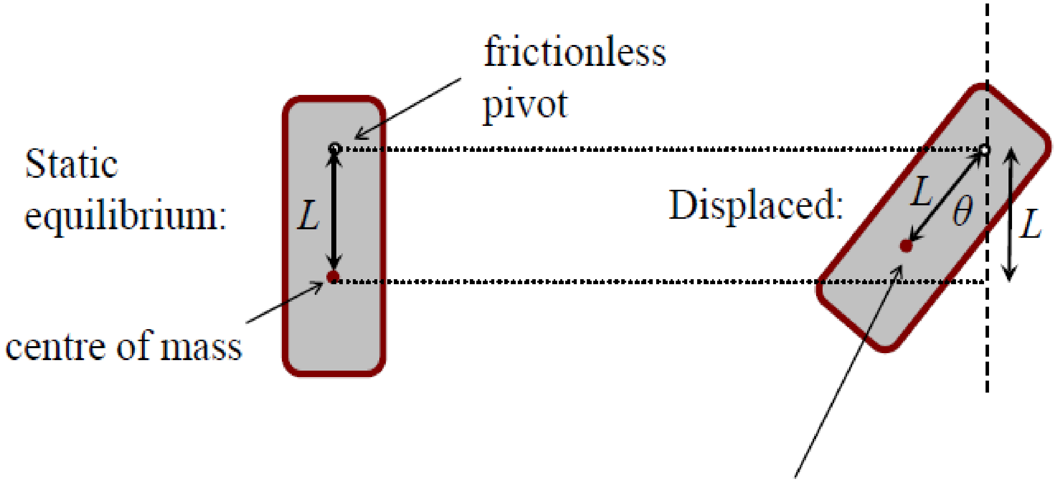
$$\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Therefore, $\boxed{\omega^2 = \dfrac{g}{L} \quad \text{And} \quad T = 2\pi\sqrt{\dfrac{L}{g}}}$

Physical Pendulum

A **physical pendulum** is any object whose oscillations are similar to those of the simple pendulum, but cannot be modeled as a point mass on a string, and the mass distribution must be included into the equation of motion.



In displaced position, centre of mass is $L - L\cos\theta$ above the equilibrium position.

Recall $\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$ For small angles, $\cos\theta \approx 1 - \frac{\theta^2}{2}$

Gravitational potential energy = $mgL(1-\cos\theta) = mgL\frac{\theta^2}{2}$

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Gravitational potential energy = $\frac{1}{2} mgL\theta^2$

Kinetic energy = $\frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2$

Total energy = $\frac{1}{2} I \left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2} mgL\theta^2 = \text{constant}$

$$\therefore I \frac{d\theta}{dt}\frac{d^2\theta}{dt^2} + mgL\theta\frac{d\theta}{dt} = 0 \qquad \ldots \text{ true for all } \frac{d\theta}{dt}$$

$$\therefore \frac{d^2\theta}{dt^2} = -\frac{mgL}{I}\theta = -\omega_0^2\theta \qquad \text{where } \omega_0 = \sqrt{\frac{mgL}{I}}$$

Equation of SHM

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The moment of inertia of the pendulum about an passing through the point of suspension is

$$= mK^2 + mL^2$$

K= radius of gyration
L= distance between suspension and oscillation points
= distance of suspension point and Center of gravity

Therefore, $\omega_0 = \sqrt{\dfrac{gL}{K^2 + L^2}}$

Time Period

$$T = 2\pi\sqrt{\frac{K^2 + L^2}{Lg}}$$

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Example 2.6

Determine the length of a simple pendulum that will swing back and forth in simple harmonic motion with a period of 1.00 s.

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \text{s})^2 (9.80\ \text{m/s}^2)}{4\pi^2} = \boxed{0.248\ \text{m}}$$

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