PUSHDOWN AUTOMATA (PDA)

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PUSHDOWN AUTOMATA

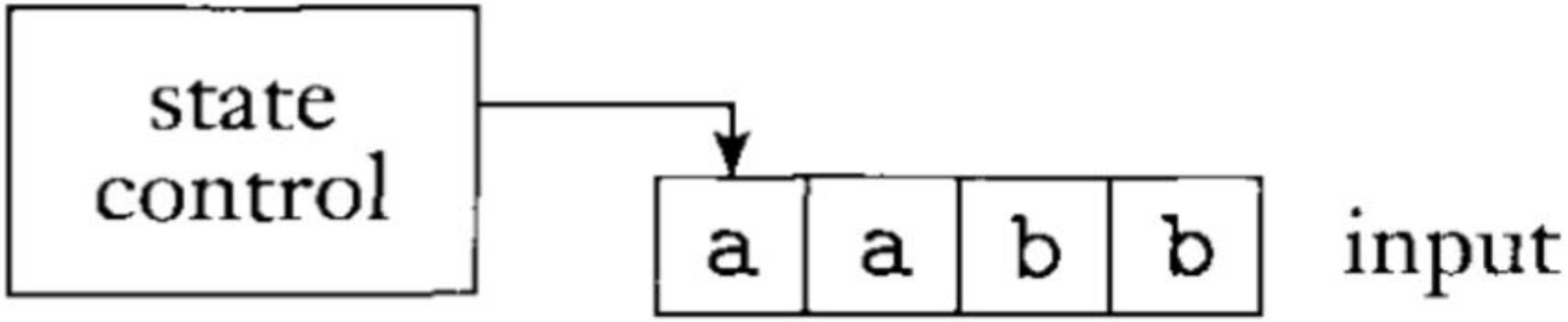


FIGURE 2.11
Schematic of a finite automaton

Pushdown Automata (Continuation…)

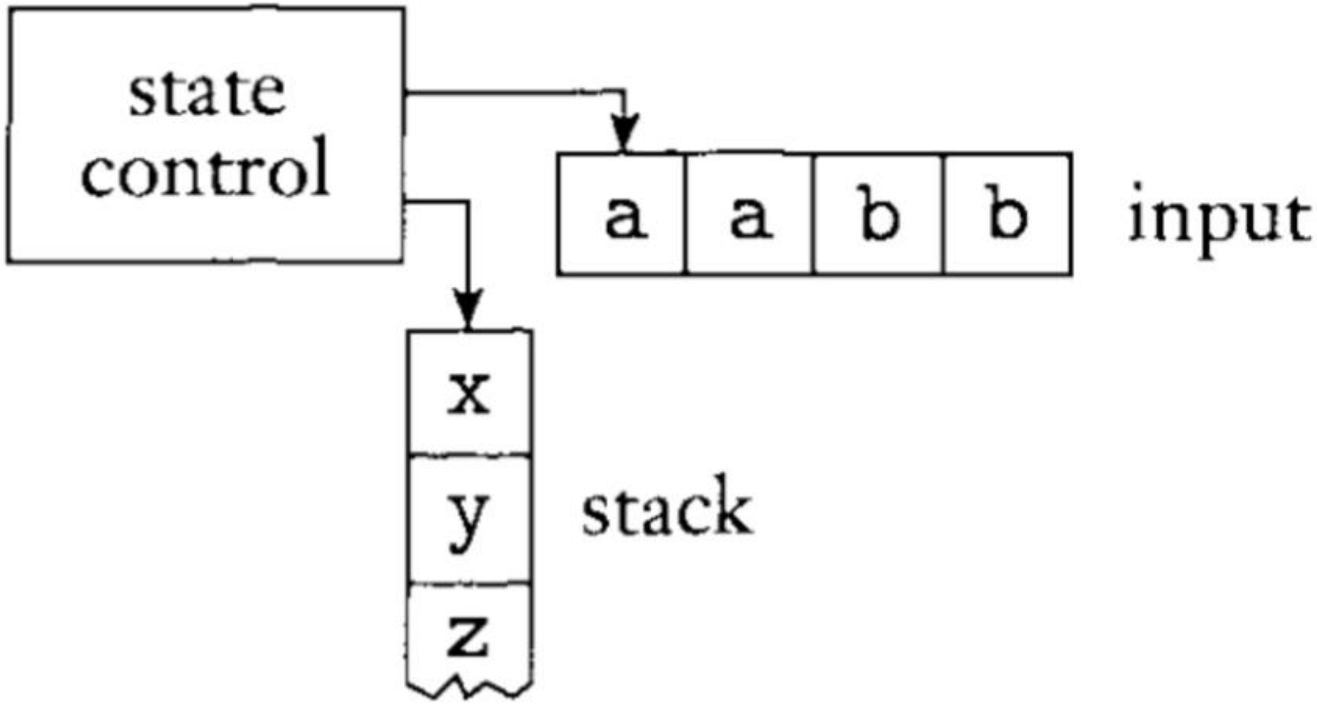


FIGURE 2.12
Schematic of a pushdown automaton

Formal Definition of a Pushdown Automaton

DEFINITION 2.13

A ***pushdown automaton*** is a 6-tuple $(Q, \Sigma, \Gamma, \delta, q_0, F)$, where Q, Σ, Γ, and F are all finite sets, and

1. Q is the set of states,
2. Σ is the input alphabet,
3. Γ is the stack alphabet,
4. $\delta\colon Q \times \Sigma_\varepsilon \times \Gamma_\varepsilon \longrightarrow \mathcal{P}(Q \times \Gamma_\varepsilon)$ is the transition function,
5. $q_0 \in Q$ is the start state, and
6. $F \subseteq Q$ is the set of accept states.

Examples of Pushdown Automaton

EXAMPLE 2.14 ....

The following is the formal description of the PDA (page 110) that recognizes the language $\{0^n1^n \mid n \geq 0\}$. Let M_1 be $(Q, \Sigma, \Gamma, \delta, q_1, F)$, where

$Q = \{q_1, q_2, q_3, q_4\}$,

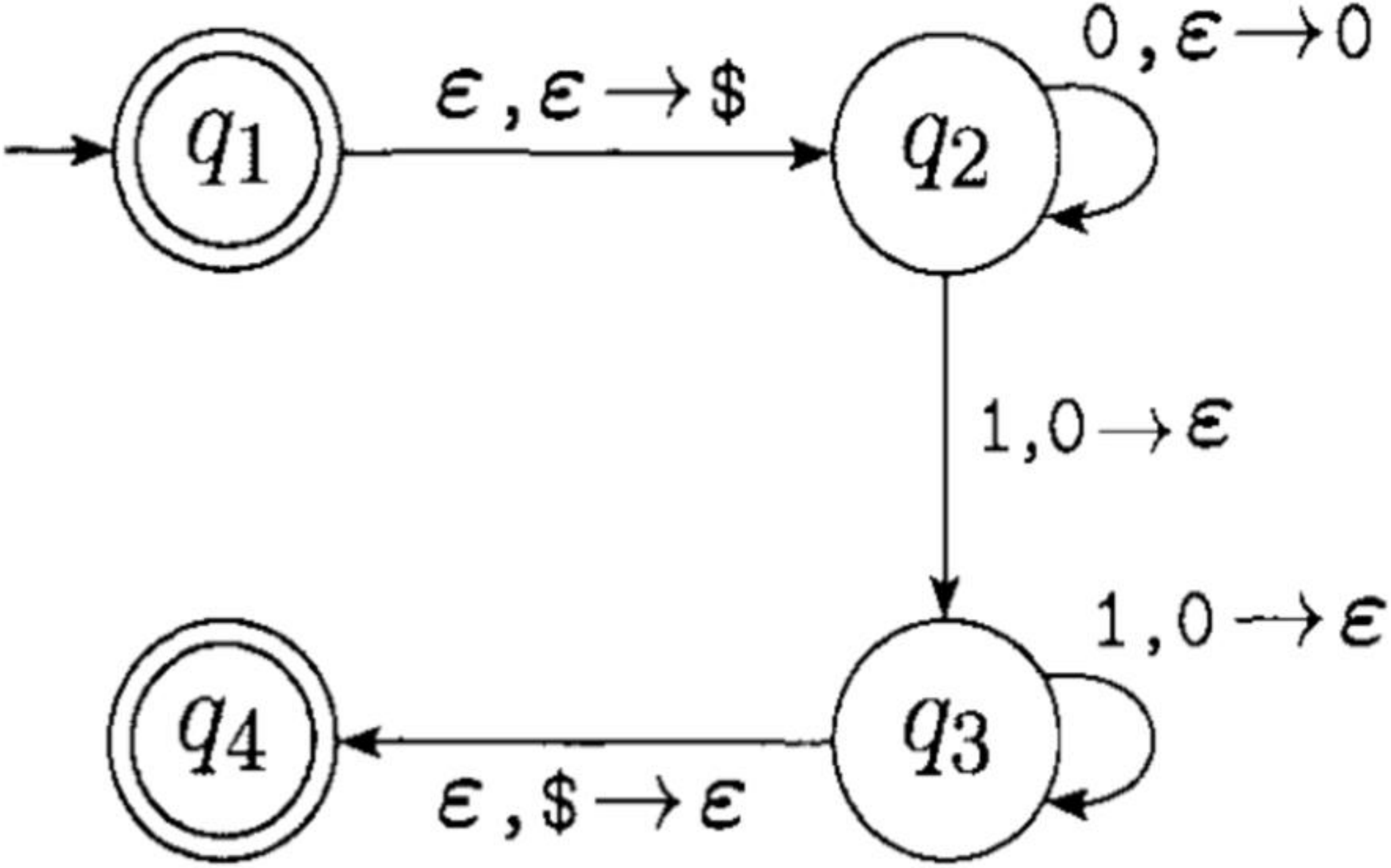
$\Sigma = \{0,1\}$,

$\Gamma = \{0, \$\}$,

$F = \{q_1, q_4\}$, and

δ is given by the following table, wherein blank entries signify $\emptyset$.

Input:	0			1			ε		
Stack:	0	\$	ε	0	\$	ε	0	\$	ε
q_1									$\{(q_2, \$)\}$
q_2			$\{(q_2, 0)\}$	$\{(q_3, \varepsilon)\}$					
q_3				$\{(q_3, \varepsilon)\}$				$\{(q_4, \varepsilon)\}$	
q_4									

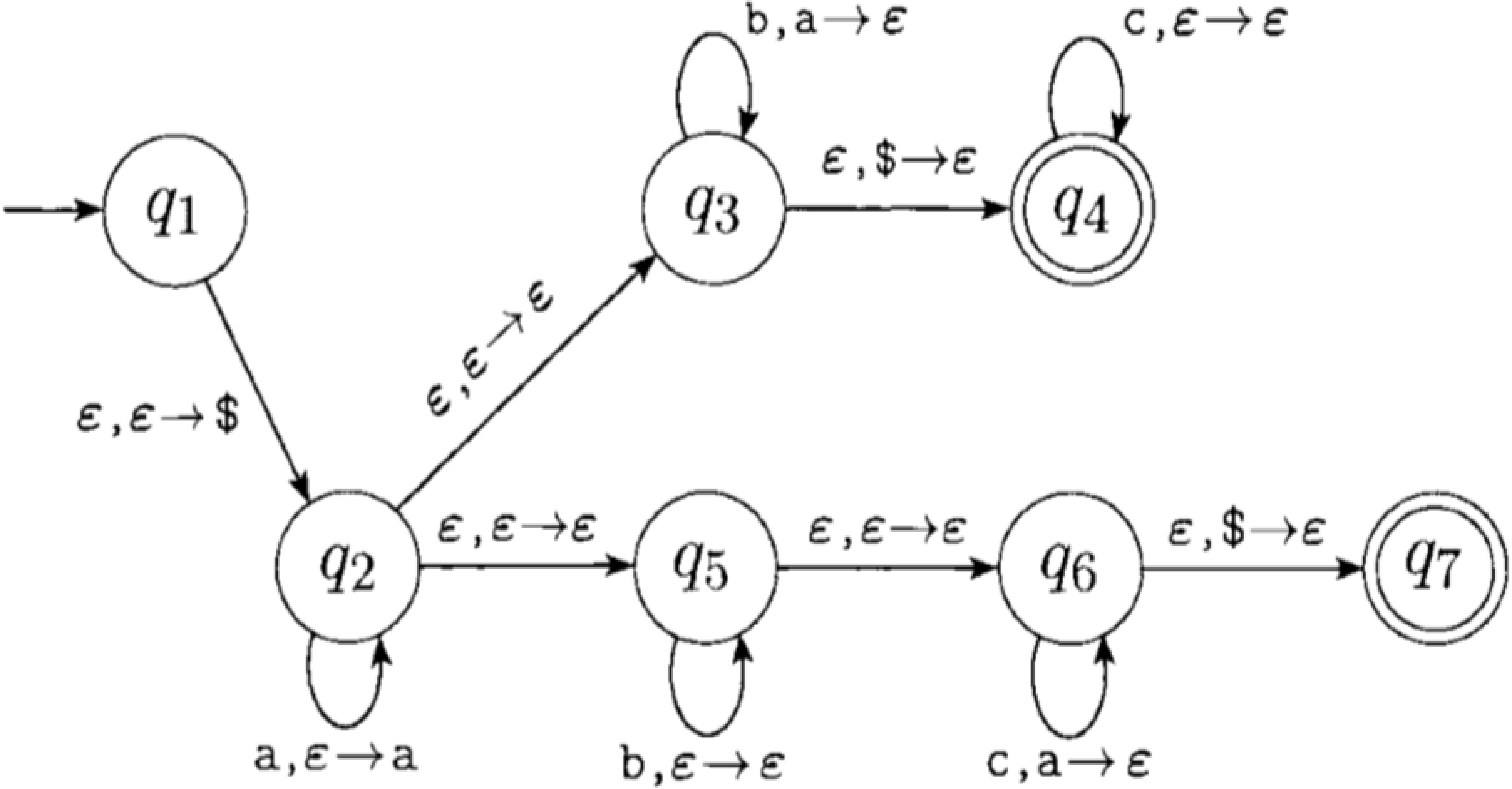

q_1
$\varepsilon, \varepsilon \rightarrow \$$
q_2
$0, \varepsilon \rightarrow 0$
$1, 0 \rightarrow \varepsilon$
q_3
$1, 0 \rightarrow \varepsilon$
q_4
$\varepsilon, \$ \rightarrow \varepsilon$

Examples of Pushdown Automaton : 2

EXAMPLE 2.16 ....

This example illustrates a pushdown automaton that recognizes the language

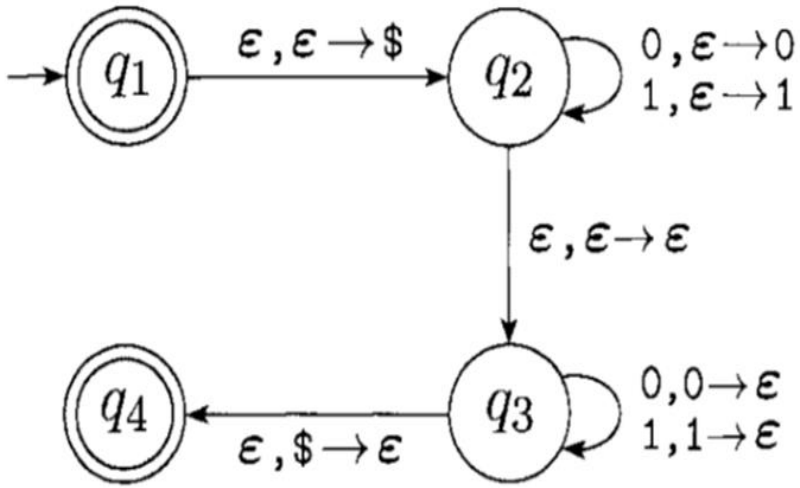
$$\{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}.$$


b,a→ε
c,ε→ε
ε,$→ε
q1
q3
q4
ε,ε→ε
ε,ε→$
ε,ε→ε
ε,ε→ε
ε,$→ε
q2
q5
q6
q7
a,ε→a
b,ε→ε
c,a→ε

Examples of Pushdown Automaton: 3

EXAMPLE 2.18

In this example we give a PDA M_3 recognizing the language $\{ww^{\mathcal{R}} | w \in \{0,1\}^*\}$.


q_1
$\varepsilon, \varepsilon \rightarrow \$$
q_2
$0, \varepsilon \rightarrow 0$
$1, \varepsilon \rightarrow 1$
$\varepsilon, \varepsilon \rightarrow \varepsilon$
q_3
$0, 0 \rightarrow \varepsilon$
$1, 1 \rightarrow \varepsilon$
q_4
$\varepsilon, \$ \rightarrow \varepsilon$

Practice Problems of Pushdown Automaton

- Design a NPDA for the following languages:
  - $\{0^n1^{2n} \mid \text{where } n>=1\}$
  - $\{0^{2n}1^n \mid \text{where } n>=1\}$
  - $\{a^nb^mc^r \mid \text{where } m,n,r>=0 \text{ and } r=n+m\}$
  - $\{a^nb^mc^r \mid \text{where } m,n,r>=0 \text{ and } r=n-m\}$