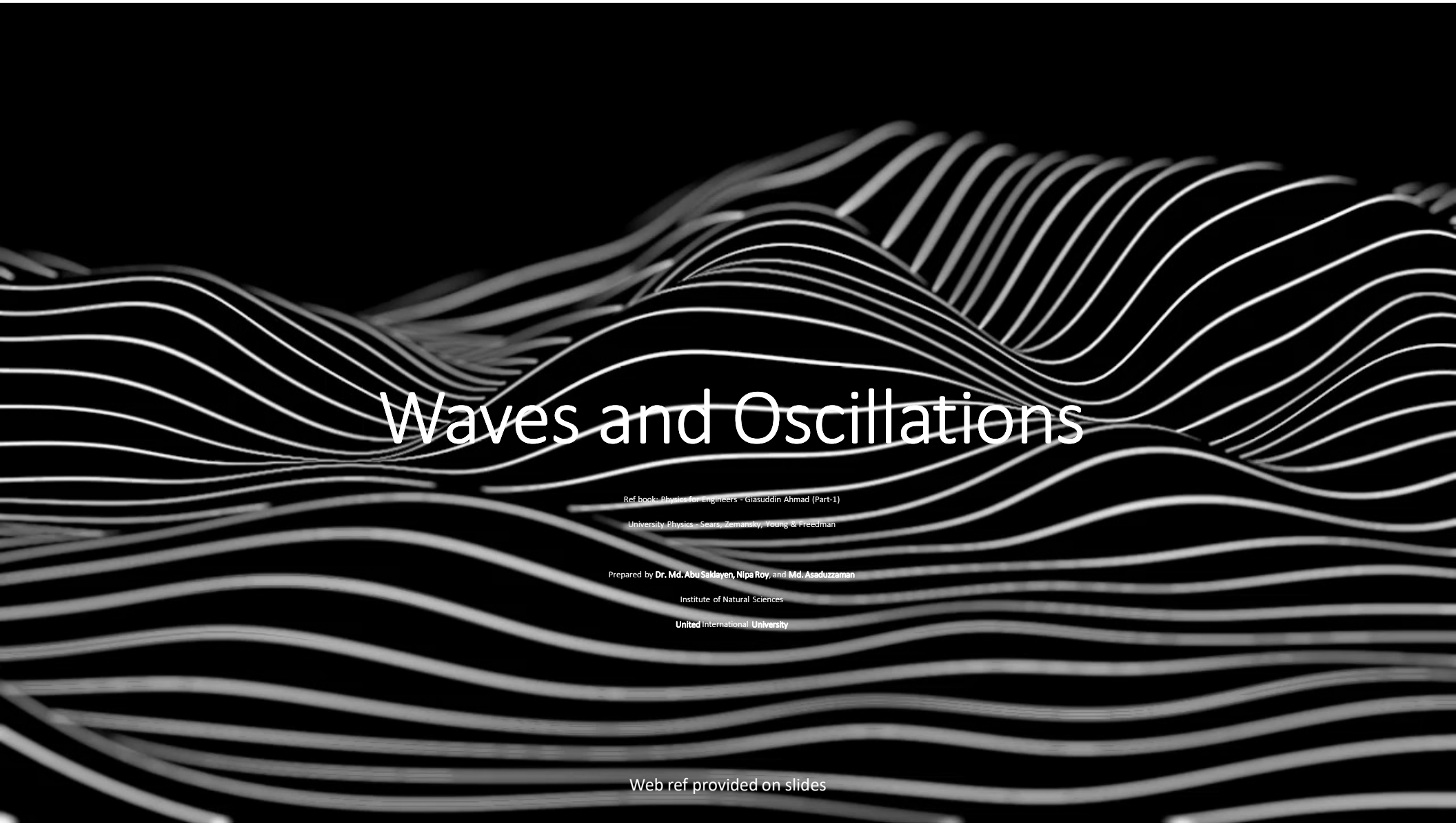


Waves and Oscillations

Ref book: Physics for Engineers - Giasuddin Ahmad (Part-1)

University Physics - Sears, Zemansky, Young & Freedman

Prepared by **Dr. Md. Abu Saklayen, Nipa Roy**, and **Md. Asaduzzaman**

Institute of Natural Sciences

United International **University**

Web ref provided on slides

Simple Harmonic Motion: Sample Problems

A 0.42-kg block is attached to the end of a horizontal ideal spring and rests on a frictionless surface. The block is pulled so that the spring stretches by 2.1 cm relative to its unstrained length. When the block is released, it moves with an acceleration of 9.0 m/s^2. What is the spring constant of the spring?

180 N/m

Ref: google image

Simple Harmonic Motion: Sample Problems

Energy calculations.

For the simple harmonic oscillation where k = 19.6 N/m, A = 0.100 m, x = -(0.100 m) cos 8.08t, and v = (0.808 m/s) sin 8.08t, determine (a) the total energy, (b) the kinetic and potential energies as a function of time, (c) the velocity when the mass is 0.050 m from equilibrium, (d) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

Ref: google image

Simple Harmonic Motion: Sample Problems

a. $E = \frac{1}{2}kA^2 = \frac{1}{2}\cdot 19.6\text{N/m}\cdot(0.100\text{m})^2 = 9.80\times10^{-2}\,\text{J}.$

b. $U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2\omega t = (9.80\times10^{-2}\,\text{J})\cos^2 8.08t,$

$K = E - U = (9.80\times10^{-2}\,\text{J})\sin^2 8.08t.$

c. $K = E - U, \quad \frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2,$

$$v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \omega\sqrt{A^2 - x^2}$$

$$= 8.08\text{Hz}\cdot\sqrt{(0.100\text{m})^2 - (0.050\text{m})^2} = 0.70\text{m/s}.$$

d. $U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{4}E = 2.5\times10^{-2}\,\text{J},$

$E = K - U = 7.3\times10^{-2}\,\text{J}.$

Ref: google image

Simple Harmonic Motion: Sample Problems

A 500 g block on a spring is pulled a distance of 20 cm and released. The subsequent oscillations are measured to have a period of 0.80 s. At what position (or positions) is the speed of the block 1.0 m/s?

$$T = 0.80 \text{ s} \quad \text{so} \quad \omega = \frac{2\pi}{T} = \frac{2\pi}{(0.80 \text{ s})} = 7.85 \text{ rad/s}$$

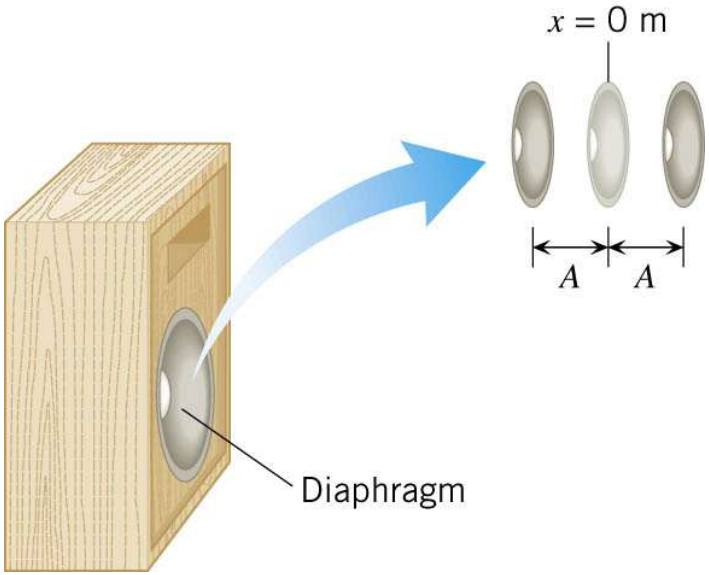
$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \omega\sqrt{A^2 - x^2}$$

$$x = \pm\sqrt{A^2 - \left(\frac{v}{\omega}\right)^2} = \pm\sqrt{(0.20 \text{ m})^2 - \left(\frac{(1.0 \text{ m/s})}{(7.85 \text{ rad/s})}\right)^2} = \pm 0.154 \text{ m} = \pm 15.4 \text{ cm}$$

Ref: google image

Simple Harmonic Motion: Sample Problems

The diaphragm of a loudspeaker moves back and forth in simple harmonic motion to create sound. The frequency of the motion is f = 1.0 kHz and the amplitude is A = 0.20 mm.



(a) What is the maximum speed of the diaphragm?

(b) Where in the motion does this maximum speed occur?

Ref: google image

Simple Harmonic Motion: Sample Problems

(a)

$$V_{max} = A\omega = A(2\pi f) = 1.3 \text{ ms}^{-1}$$

(b) The speed of the diaphragm is zero when the diaphragm momentarily comes to rest at either end of its motion: $x = +A$ and $x = -A$. Its maximum speed occurs midway between these two positions, or at $x = 0$ m.

Ref: google image

Simple Harmonic Motion: Sample Problems

What is the oscillation period of an FM radio station that broadcasts at 100 MHz?

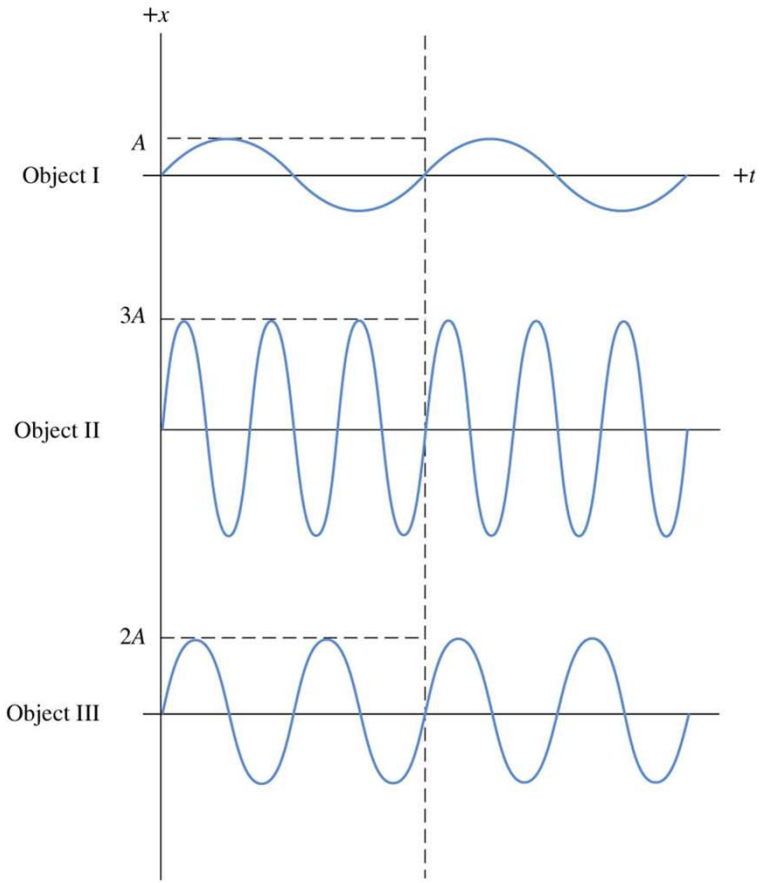
$$f = 100\ \text{MHz} = 1.0 \times 10^{8}\ \text{Hz}$$

$$T = 1/f = \frac{1}{1.0 \times 10^{8}\ \text{Hz}} = 1.0 \times 10^{-8}\ \text{s} = 10\ \text{ns}$$

Note that 1/Hz = s

Ref: google image

Simple Harmonic Motion: Sample Problems



The drawing shows plots of the displacement *x* versus the time *t* for three objects undergoing simple harmonic motion. Which object, I, II, or III, has the greatest maximum velocity?

II

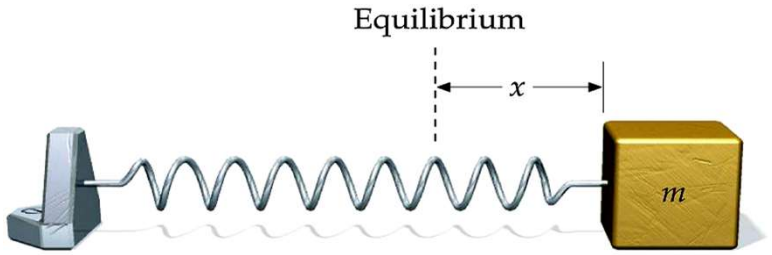
Ref: google image

Simple Harmonic Motion: Sample Problems

A 2.00 kg block is attached to a spring as shown. The force constant of the spring is $k = 196$ N/m. The block is held a distance of 5.00 cm from equilibrium and released at $t = 0$.

(a) Find the angular frequency ω, the frequency f, and the period T.

(b) Write an equation for x vs. time.



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Simple Harmonic Motion: Sample Problems

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{(196\ \text{N/m})}{(2.00\ \text{kg})}} = 9.90\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{(9.90\ \text{rad/s})}{2\pi} = 1.58\ \text{Hz}$$

$$T = 1/f = 0.635\ \text{s} \qquad A = 5.00\ \text{cm and } \delta = 0$$

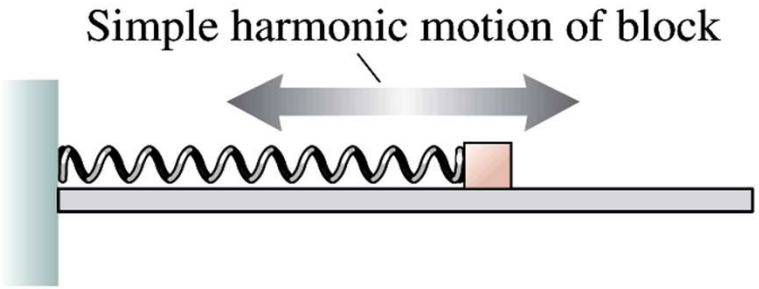
$$\boxed{x = (5.00\ \text{cm})\cos\left[(9.90\ \text{rad/s})t\right]}$$

Ref: google image

Simple Harmonic Motion: Sample Problems

An air-track glider is attached to a spring, pulled 20 cm to the right, and released at t-=0. It makes 15 complete oscillations in 10 s.

a. What is the period of oscillation?
b. What is the object's maximum speed?
c. What is its position and velocity at t=0.80 s?



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Simple Harmonic Motion: Sample Problems

$$f = \frac{15 \text{ oscillations}}{10 \text{ s}}$$

$$= 1.5 \text{ oscillations/s} = 1.5 \text{ Hz}$$

$$T = 1/f = 0.667 \text{ s} \qquad v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.20 \text{ m})}{(0.667 \text{ s})} = 1.88 \text{ m/s}$$

$$x = A\cos\frac{2\pi t}{T} = (0.20 \text{ m})\cos\frac{2\pi(0.80 \text{ s})}{(0.667 \text{ s})} = 0.062 \text{ m} = 6.2 \text{ cm}$$

$$v = -v_{\max}\sin\frac{2\pi t}{T} = -(1.88 \text{ m/s})\sin\frac{2\pi(0.80 \text{ s})}{(0.667 \text{ s})} = -1.79 \text{ m/s}$$

Ref: google image

Simple Harmonic Motion: Sample Problems

A mass, oscillating in simple harmonic motion, starts at $x = A$ and has period T.

At what time, as a fraction of T, does the mass first pass through $x = ½A$?

$$x = \tfrac{1}{2} A = A\cos\frac{2\pi t}{T}$$

$$t = \frac{T}{2\pi}\cos^{-1}\left(\tfrac{1}{2}\right) = \frac{T}{2\pi}\frac{\pi}{3} = \tfrac{1}{6}T$$

Ref: google image

Simple Harmonic Motion: Sample Problems

A particle execute s simple harmonic motion given by the equation

$$y = 12 \sin\left(\frac{2\pi t}{10} + \frac{\pi}{4}\right)$$

Calculate (i) amplitude, (ii) frequency, (iii) displacement at t= 1.25s, (iv) velocity at t= 2.5s (v) acceleration at t= 5s.

Ref: google image

Simple Harmonic Motion: Sample Problems

A particle execute s simple harmonic motion given by the equation

$$y = 10 \sin(10 t - \frac{\pi}{6})$$

Calculate (i) frequency, (ii) time period (iii) the maximum displacement (iv)the maximum velocity (v) the maximum acceleration acceleration.

Ref: google image

Sample Problems

Problem#1:

A block of mass m=680 g is fatened to a spring whose spring constant k is 65 N/m. The block is pulled a distance of x=11 cm from its equilibrium position at x=0 on a frictionless surface and released from rest at t=0.

(a) What are the angular frequency, the frequency and the period of the resulting motion?

(b) What is the amplitude of the motion?

(c) What is the maximum speed of the block and the where the block when it has this speed?

(d) What is the magnitude of a_{max} of the oscillation?

(e) What is the phase constant?

(f) What is the displacement function $x(t)$ for the spring-block system at t=1.5 s?

Solution:
(a) Angular frequency

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65\,N/m}{0.68\,kg}} = 9.78\,rad/s \approx 9.8\,rad/s$$

The frequency

$$\nu = \frac{\omega}{2\pi} = \frac{9.78\,rad/s}{2\pi\,rad} = 1.56\,Hz$$

The time period

$$T = \frac{1}{\nu} = \frac{1}{1.56\,HZ} = 0.64\,s = 640\,ms$$

(b) The amplitude is the maximum displacement. So,

$$A = 11\,cm = 0.11\,m$$

(c) The maximum speed is

$$v_m = \omega\, A = (9.78\, rad/s)(0.11\, m) = 1.1\, m/s$$

The maximum speed occurs whwn the block crosses the equilibrium position i.e. $x = 0$.
(d) The maximum acceleration

$$a_m = \omega^2\, A = (9.78\, rad/s)(0.11\, m) = 11\, m/s^2$$

(e) The phase constant ϕ can be evaluted in the following way

$$\begin{aligned} & x(t) = A\, \cos(\omega t - \phi) \\ & \text{At} \quad t = 0,\ x = A, \quad \text{So} \\ & 1 = \cos\phi \quad \therefore \quad \phi = \cos^1 1 = 0\, rad \end{aligned}$$

(f)

$$x(t) = A\, \cos(\omega t - \phi) = (0.11\, m)(\cos(9.8 \times 1.5 - 0) = -0.059\, m$$

Problem#2:

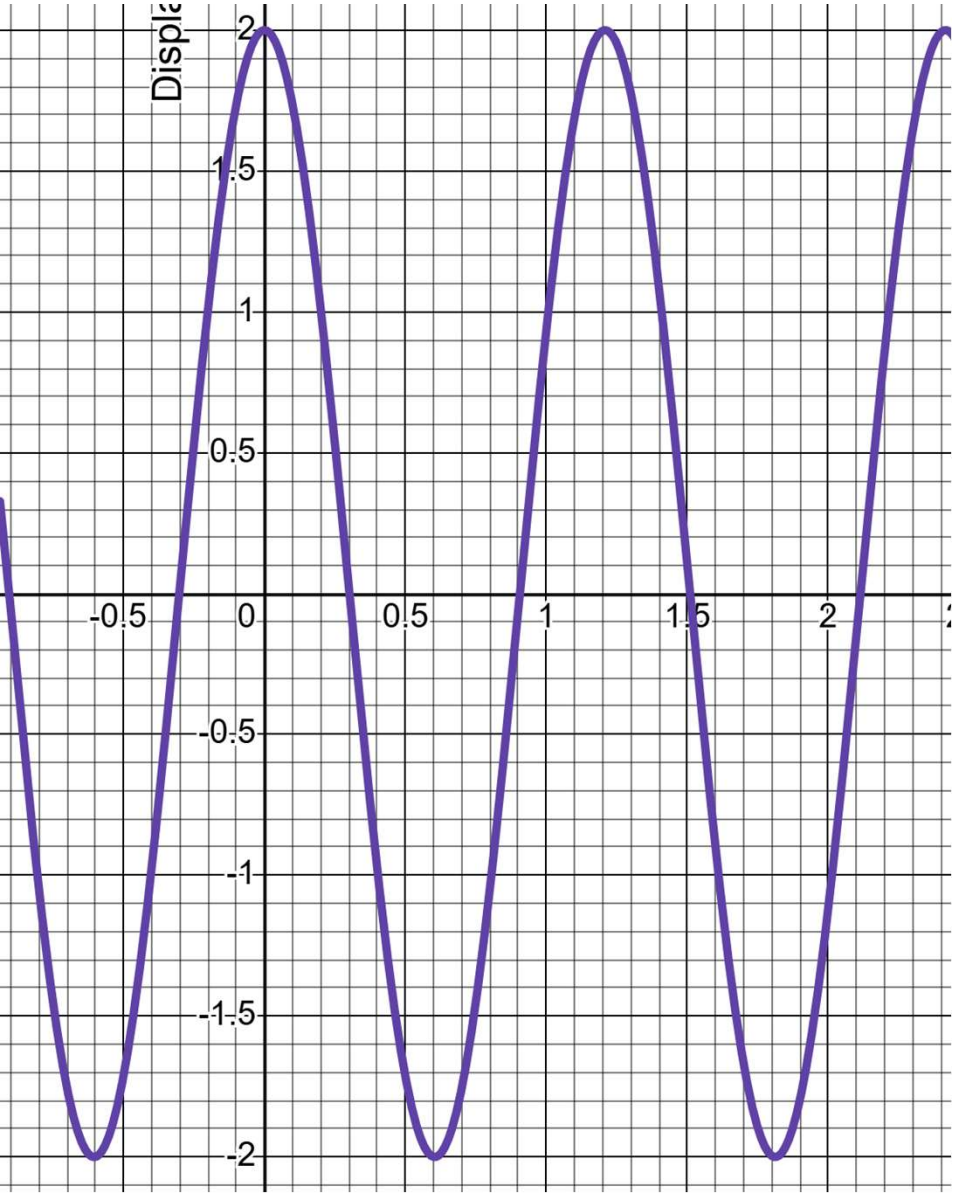
An object undergoing simple harmonic motion takes 0.25 s to travel from one point of zero velocity to the next such point. The distance between those points is 36 cm. Calculate the (a) period, (b) frequency and (c) amplitude of the motion.

Problem#3:

What is the maximum acceleration of a platform that oscillates at amplitudes 2.20 cm and frequency 6.60 Hz.

Problem#4:

Given $x(t) = 6.0\ \cos(2.5t - \frac{\pi}{3})$. Find the velocity of the body and acceleration at $t = 1.4$s.



1. Design the equation of displacement.
2. What will be the velocity at t=0.2 sec?
3. What will be the kinetic energy at t=0.5 sec?

Equation of displacement

$Y = 2\cos 5.2t$