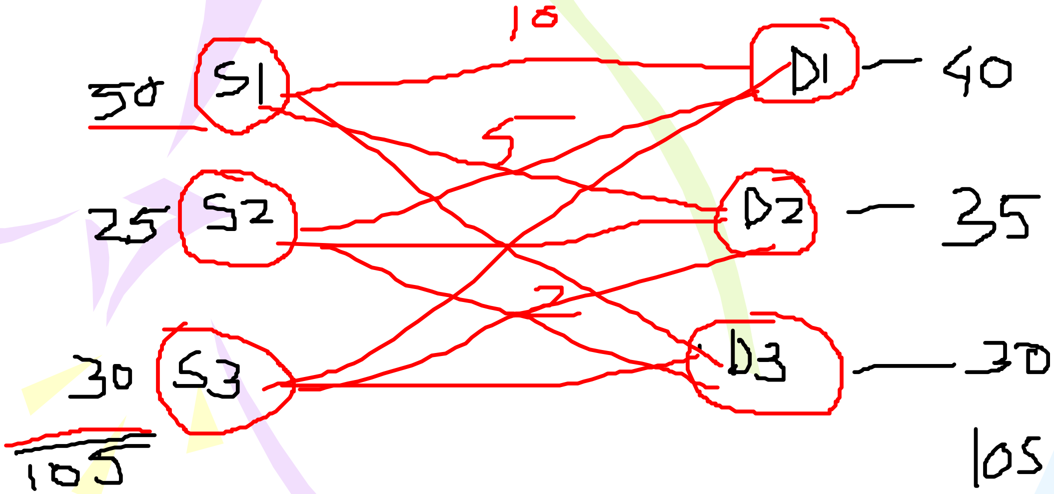
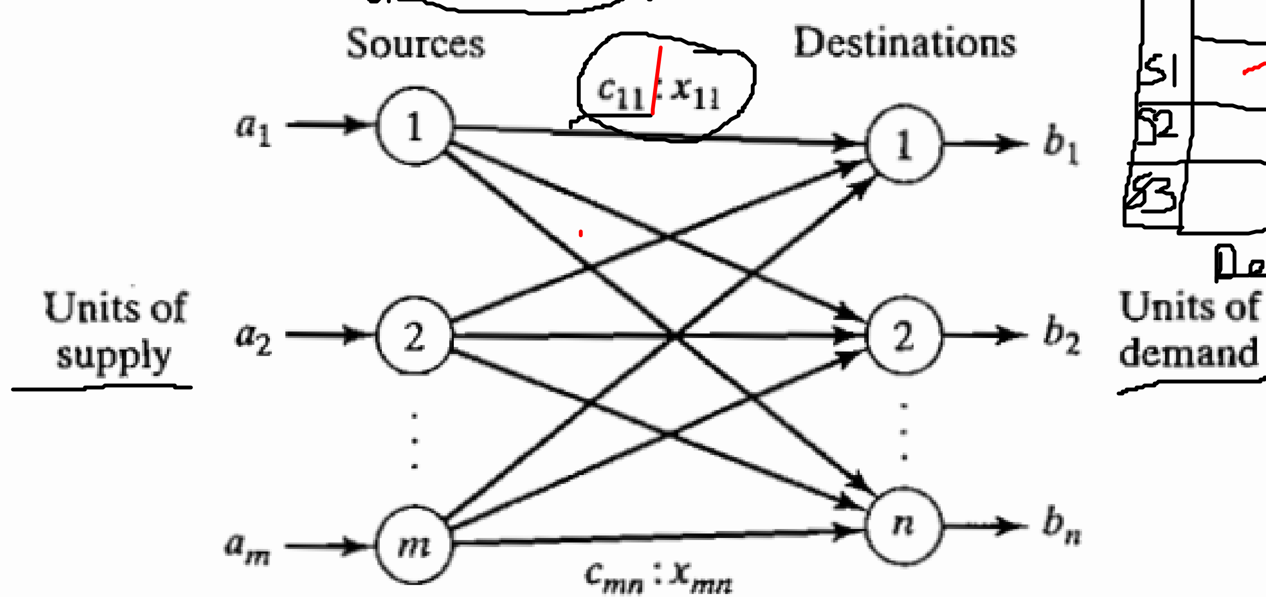
TRANSPORTATION PROBLEM

Finding Initial Basic Feasible Solution



DEFINITION OF THE TRANSPORTATION MODEL

The general problem is represented by the network in Figure 5.1. There are m sources and n destinations, each represented by a **node**. The **arcs** represent the routes linking the sources and the destinations. Arc (i, j) joining source i to destination j carries two pieces of information: the transportation cost per unit, c_{ij}, and the amount shipped, x_{ij}. The amount of supply at source i is a_i and the amount of demand at destination j is b_j. The objective of the model is to determine the unknowns x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand restrictions.



	d1	d2	d3	Supp
S1	10	8	9	50
S2	5	6	11	40
S3	15	12	8	30
Dem	20	60	30	

North-West Corner Method

Step1: Select the upper left (north-west) cell of the transportation matrix and allocate the maximum possible value to X_{11} which is equal to $\min(a_1,b_1)$.

Step2:

- If allocation made is equal to the supply available at the first source (a_1 in first row), then move vertically down to the cell (2,1).
- If allocation made is equal to demand of the first destination (b_1 in first column), then move horizontally to the cell (1,2).
- If $a_1=b_1$, then allocate $X_{11}= a_1$ or b_1 and move to cell (2,2).

Step3: Continue the process until an allocation is made in the south-east corner cell of the transportation table.

Example: Solve the Transportation Table to find Initial Basic Feasible Solution using North-West Corner Method.

Total Cost =19*5+30*2+30*6+40*3+70*4+20*14
= Rs. 1015

	D_1	D_2	D_3	D_4	Supply
S_1	19 (5)	30 (2)	50	10	7
S_2	70	30 (6)	40 (3)	60	9
S_3	40	8	70 (4)	20 (14)	18
Demand	5	8	7	14	34

Least Cost Method

Step1: Select the cell having lowest unit cost in the entire table and allocate the minimum of supply or demand values in that cell.

Step2: Then eliminate the row or column in which supply or demand is exhausted. If both the supply and demand values are same, either of the row or column can be eliminated.

In case, the smallest unit cost is not unique, then select the cell where maximum allocation can be made.

Step3: Repeat the process with next lowest unit cost and continue until the entire available supply at various sources and demand at various destinations is satisfied.

	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8 8	70	20	18
Demand	5	8	7	14	34

	D_1	D_3	D_4	Supply
S_1	19	50	10 7	7
S_2	70	40	60	9
S_3	40	70	20	10
Demand	5	7	14	34

	D_1	D_3	D_4	Supply
S_2	70	40	60	9
S_3	40	70	20 7	10
Demand	5	7	7	34

	D_1	D_3	Supply
S_2	70	40 7	9
S_3	40	70	3
Demand	5	7	34

	D_1	Supply
S_2	70 2	2
S_3	40 3	3
Demand	5	34

The total transportation cost obtained by this method

$= 8*8+10*7+20*7+40*7+70*2+40*3$

= Rs.<u>814</u>

Here, we can see that the ***Least Cost Method*** involves a lower cost than the *North-West Corner Method*.

Vogel's Approximation Method

Step1: Calculate penalty for each row and column by taking the difference between the two smallest unit costs. This penalty or extra cost has to be paid if one fails to allocate the minimum unit transportation cost.

Step2: Select the row or column with the highest penalty and select the minimum unit cost of that row or column. Then, allocate the minimum of supply or demand values in that cell. If there is a tie, then select the cell where maximum allocation could be made.

Step3: Adjust the supply and demand and eliminate the satisfied row or column. If a row and column are satisfied simultaneously, only of them is eliminated and the other one is assigned a ***zero*** value.Any row or column having ***zero*** supply or demand, can not be used in calculating future penalties.

Step4: Repeat the process until all the supply sources and demand destinations are satisfied.

	D1	D2	D3	D4	Supply	Row Diff.
S1	19	30	50	10	7	**9**
S2	70	30	40	60	9	**10**
S3	40	8 8	70	20	18	**12**
Demand	5	8	7	14	34	
Col.Diff.	**21**	**22**	**10**	**10**		

	D1	D3	D4	Supply	Row Diff.
S1	19 5	50	10	7	**9**
S2	70	40	60	9	**20**
S3	40	70	20	10	**20**
Demand	5	7	14	34	
Col.Diff.	**21**	**10**	**10**		

	D3	D4	Supply	Row Diff.
S1	50	10	2	**40**
S2	40	60	9	**20**
S3	70	20 10	10	**50**
Demand	7	14	34	
Col.Diff.	**10**	**10**		

	D3	D4	Supply	Row Diff.
S1	50	10 2	2 0	**40**
S2	40	60	9	**20**
Demand	7	4 2	34	
Col.Diff.	**10**	**50**		

	D3	D4	Supply	Row Diff.
S2	40 7	60 2	9	**20**
Demand	7	2	34	
Col.Diff.				

The total transportation cost obtained by this method
= 8*8+19*5+20*10+10*2+40*7+60*2
= Rs $\boxed{779}$
Here, we can see that ***Vogel's Approximation Method*** involves the lowest cost than *North-West Corner Method* and *Least Cost Method* and hence is the most preferred method of finding initial basic feasible solution.

THE TRANSSHIPMENT MODEL

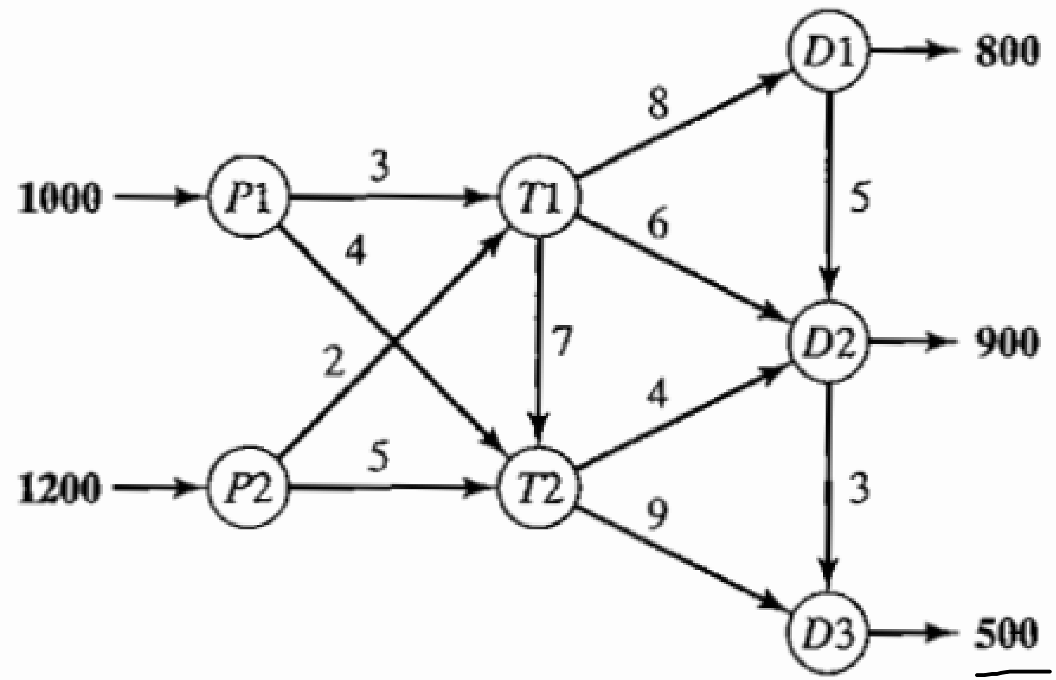
The transshipment model recognizes that it may be cheaper to ship through intermediate or *transient* nodes before reaching the final destination. This concept is more general than that of the regular transportation model, where direct shipments only are allowed between a source and a destination.

This section shows how a transshipment model can be converted to (and solved as) a regular transportation model using the idea of a **buffer**.

Example 5.5-1

Two automobile plants, *P*1 and *P*2, are linked to three dealers, *D*1, *D*2, and *D*3, by way of two transit centers, *T*1 and *T*2, according to the network shown in Figure 5.7. The supply amounts at plants *P*1 and *P*2 are 1000 and 1200 cars, and the demand amounts at dealers *D*1, *D*2, and *D*3, are 800, 900, and 500 cars. The shipping costs per car (in hundreds of dollars) between pairs of nodes are shown on the connecting links (or arcs) of the network.

Transshipment occurs in the network in Figure 5.7 because the entire supply amount of 2200 (= 1000 + 1200) cars at nodes *P*1 and *P*2 could conceivably pass through any node of the



network before ultimately reaching their destinations at nodes $D1$, $D2$, and $D3$. In this regard each node of the network with both input and output arcs ($T1$, $T2$, $D1$, and $D2$) acts as both a source and a destination and is referred to as a **transshipment node**. The remaining nodes are either **pure supply nodes** ($P1$ and $P2$) or **pure demand nodes** ($D3$).

pure supply nodes ($P1$ and $P2$) or pure demand nodes ($D3$).

The transshipment model can be converted into a regular transportation model with six sources ($P1, P2, T1, T2, D1$, and $D2$) and five destinations ($T1, T2, D1, D2$, and $D3$). The amounts of supply and demand at the different nodes are computed as

$$\text{Supply at a } \textit{pure supply node} = \text{Original supply}$$

$$\text{Demand at a } \textit{pure demand node} = \text{Original demand}$$

$$\text{Supply at a } \textit{transshipment node} = \text{Original supply} + \text{Buffer amount}$$

$$\text{Demand at a } \textit{transshipment node} = \text{Original demand} + \text{Buffer amount}$$

The buffer amount should be sufficiently large to allow all of the *original* supply (or demand) units to pass through any of the *transshipment* nodes. Let B be the desired buffer amount; then

$$\begin{aligned} B &= \text{Total supply (or demand)} \\ &= 1000 + 1200 \text{ (or } 800 + 900 + 500) \\ &= 2200 \text{ cars} \end{aligned}$$

Using the buffer B and the unit shipping costs given in the network, we construct the equivalent regular transportation model as in Table 5.44.

TABLE 5.44 Transshipment Model

	T1	T2	D1	D2	D3	
P1	3	4	M	M	M	1000
P2	2	5	M	M	M	1200
T1	0	7	8	6	M	B
T2	M	0	M	4	9	B
D1	M	M	0	5	M	B
D2	M	M	M	0	3	B
	B	B	800 + B	900 + B	500	

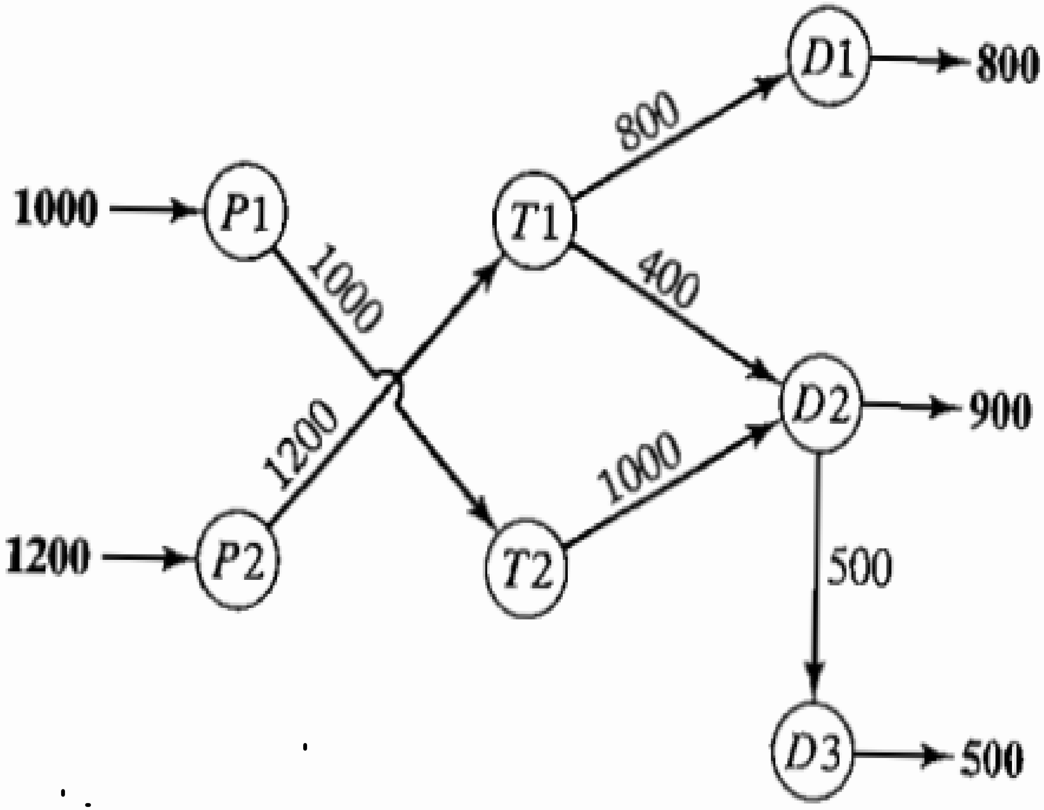


FIGURE 5.8

Solution of the transshipment model