Nondeterministic Finite Automata (NFA)

- It has the ability to be in several states at once.
- Transitions from a state on an input symbol can be to any set of states.
- Starts in one state
- Accept if any sequence of choices leads to a final state

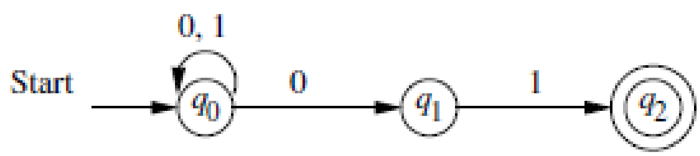


Figure 2.9: An NFA accepting all strings that end in 01

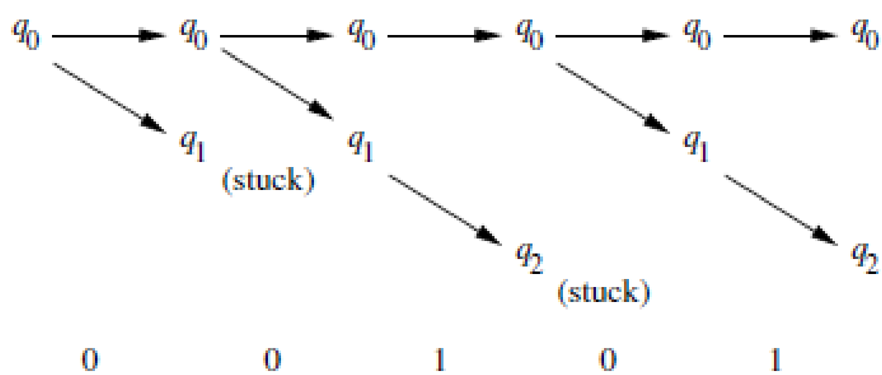
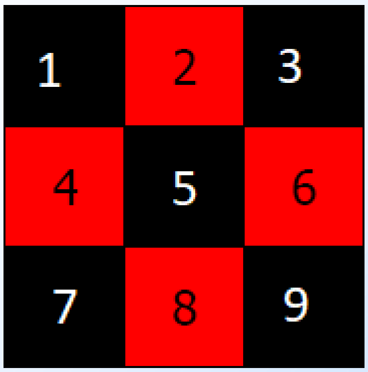
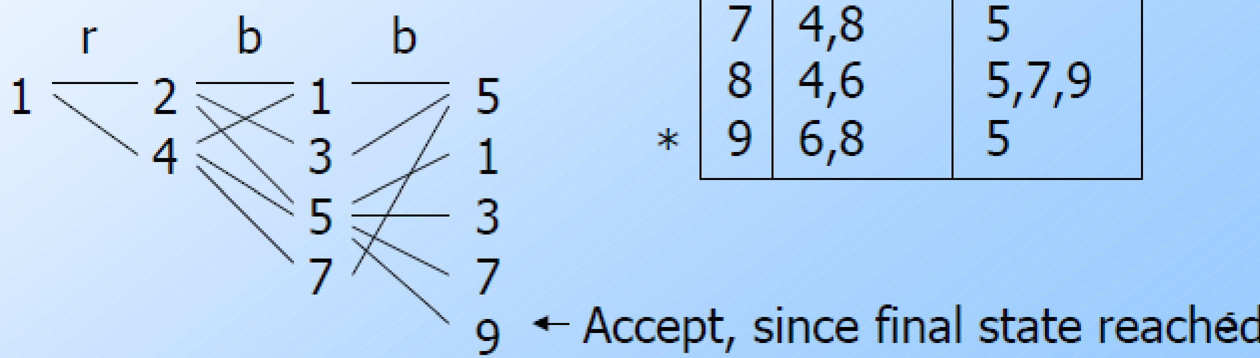


Figure 2.10: The states an NFA is in during the processing of input sequence 00101

Chessboard



		r	b
→	1	2,4	5
	2	4,6	1,3,5
	3	2,6	5
	4	2,8	1,5,7
	5	2,4,6,8	1,3,7,9
	6	2,8	3,5,9
	7	4,8	5
	8	4,6	5,7,9
*	9	6,8	5



Formal Definition of NFA:

- A finite set of states Q
- An input alphabet Σ
- A transition function δ
- A start state in Q, typically q_0
- A set of final state $F \subseteq Q$

Transition Function of an NFA

- δ(q, a) is a set of states
- Extended to strings as follows
  - Basis: δ(q, ϵ) = {q}
  - Induction: δ(q, wa) = the union over all states p in δ(q, w) of δ(p, a)

Language of the NFA

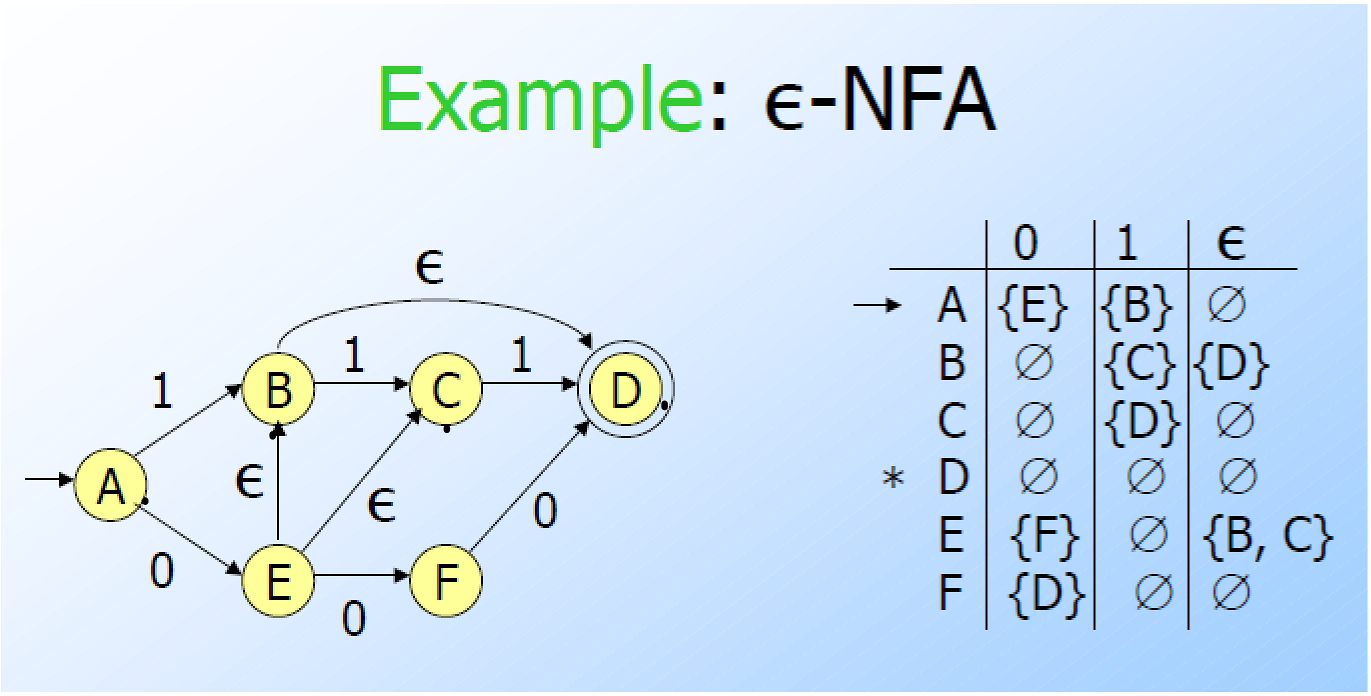
- A string w is accepted by an NFA if δ(q0, w) contains at least one final state
- The language of the NFA is the set of strings it accepts

Example:

1. Design an NFA with ∑ {0, 1} accepts all string ending with 01
2. Design an NFA with ∑ {0, 1} in which double '1' is followed by double '0'
3. Design an NFA with ∑ {0, 1} in which all the string contains a substring 1110
4. Design an NFA with ∑ {0, 1} accepts all string in which the third symbol from the right end is always 0

ϵ - NFA (NFA with ϵ):

- We can allow state-to-state transitions on ϵ input
- These transitions are done spontaneously, without looking at the input string



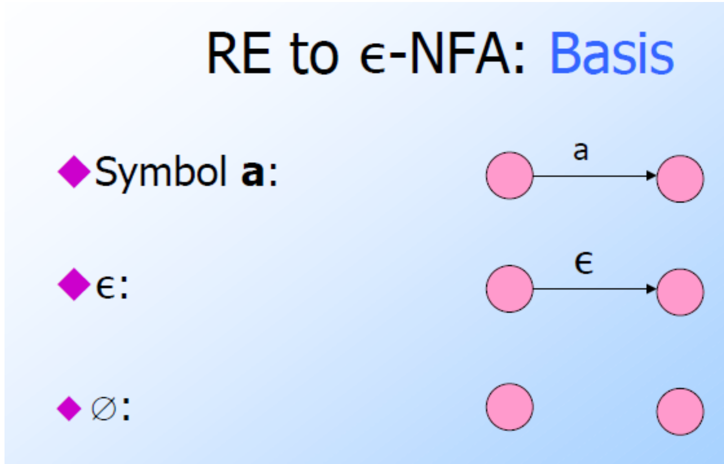
Closure of States:

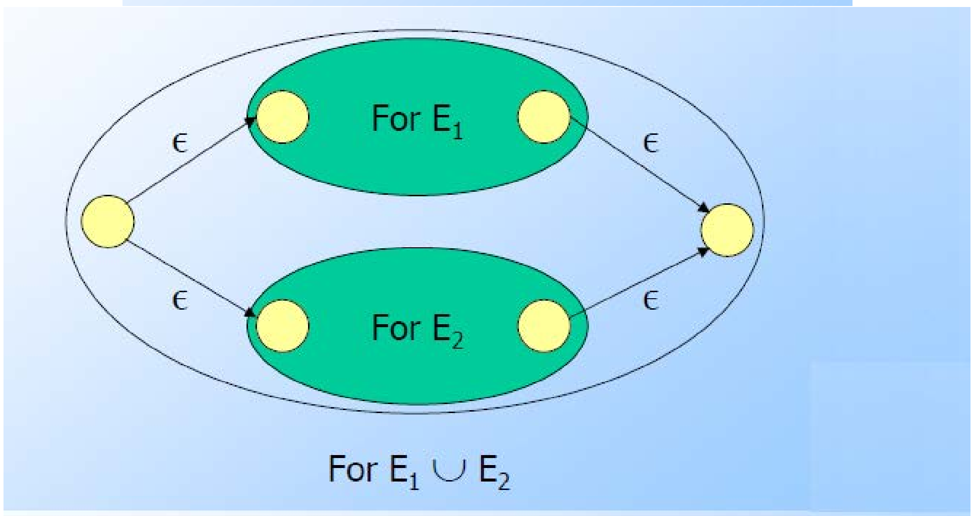
- Closure of a set of states = Union of the closure of each state
- CL(q) = set of states you can reach from state q following only arcs labeled ϵ
- CL(A) = {A}; CL(E) = {B, C, D, E}

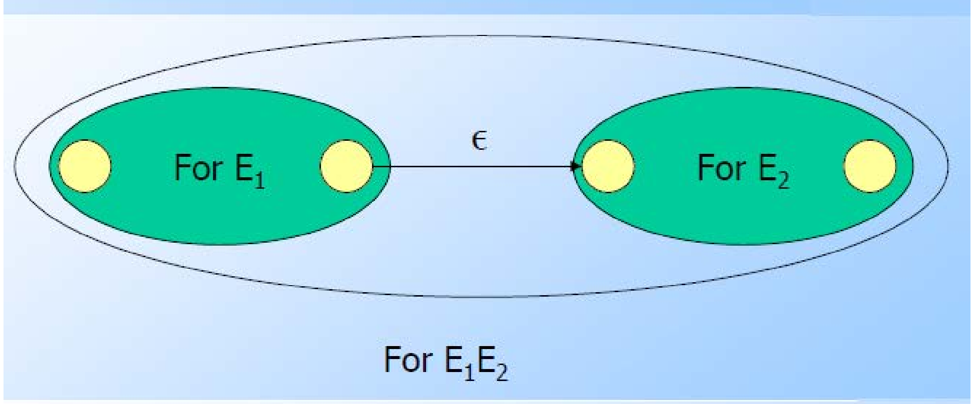
Extended Delta:

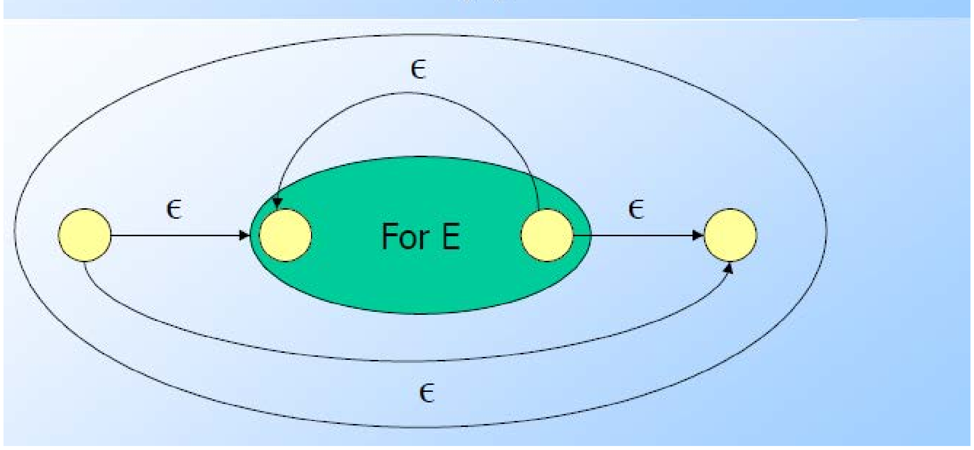
- $\hat{\delta}(A, \epsilon)$ = CL(A) = {A}
- $\hat{\delta}(A, 0)$ = CL({E}) = {B, C, D, E}
- $\hat{\delta}(A, 01)$ = CL({C, D}) = {C, D}

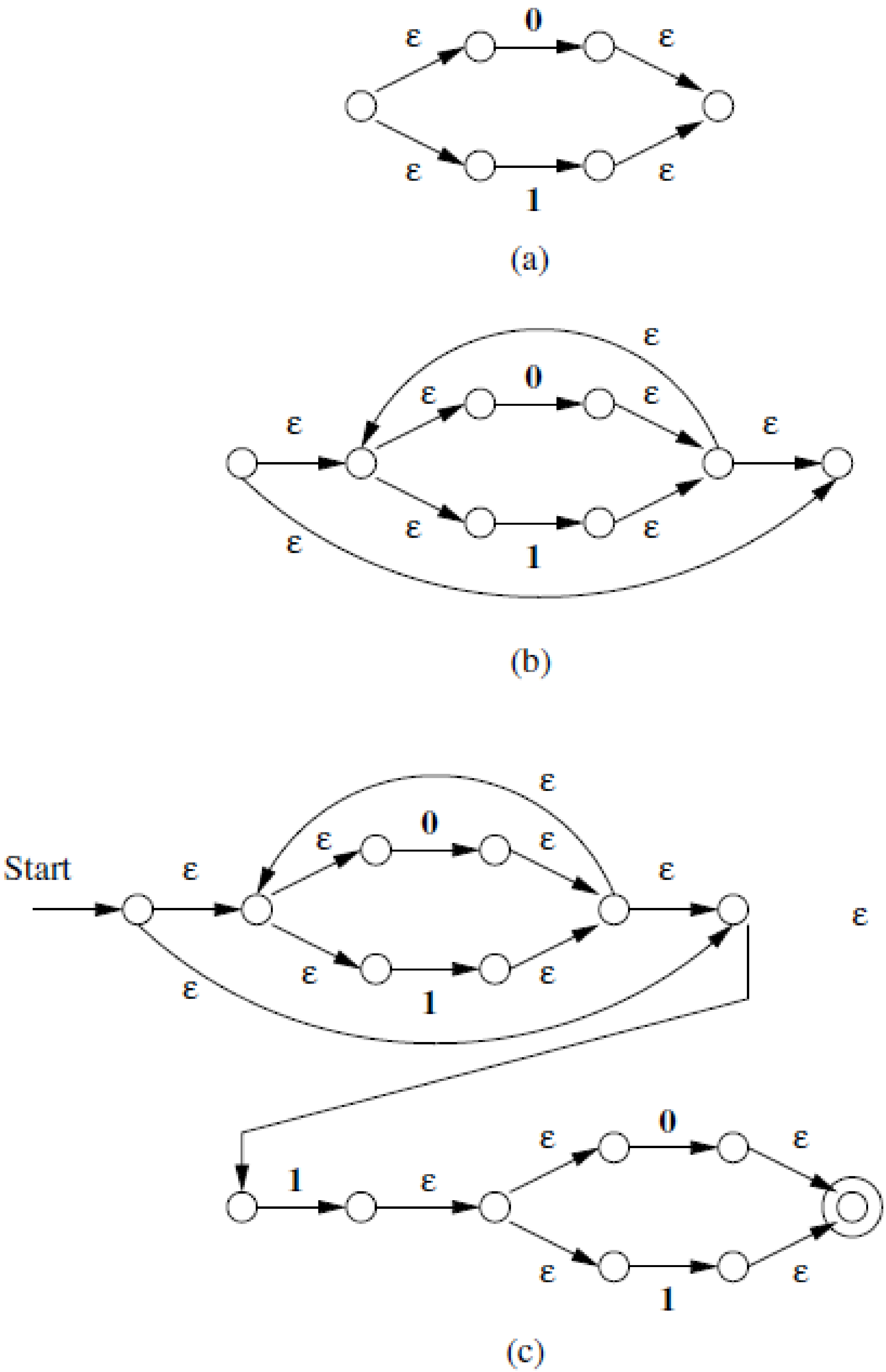
Regular Expression (RE) to ϵ - NFA:





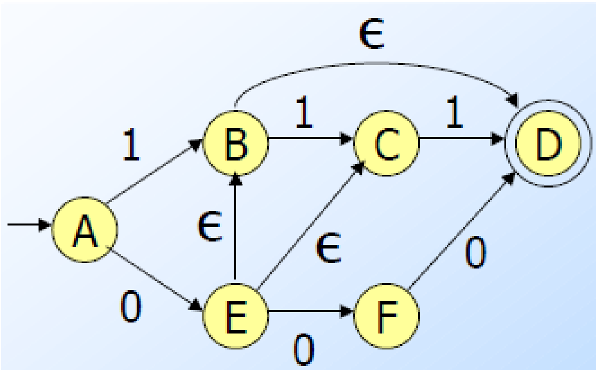






Automata constructed for $(\mathbf{0}+\mathbf{1})^*\mathbf{1}(\mathbf{0}+\mathbf{1})$

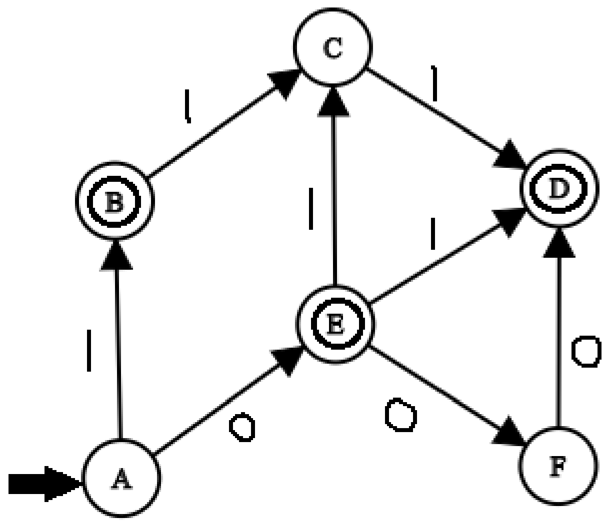
Converting ϵ - NFA to NFA:



		0	1	ϵ
→	A	{E}	{B}	∅
	B	∅	{C}	{D}
	C	∅	{D}	∅
*	D	∅	∅	∅
	E	{F}	∅	{B, C}
	F	{D}	∅	∅

		0	1
→	A	{E}	{B}
*	B	∅	{C}
	C	∅	{D}
*	D	∅	∅
*	E	{F}	{C, D}
	F	{D}	∅

		0	1
→	A	{E}	{B}
*	B	∅	{C}
	C	∅	{D}
*	D	∅	∅
*	E	{F}	{C, D}
	F	{D}	∅

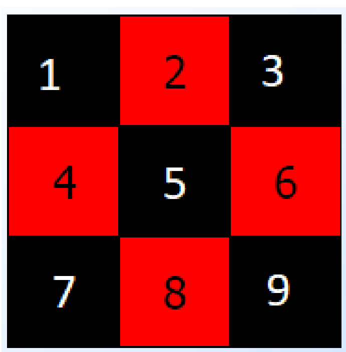


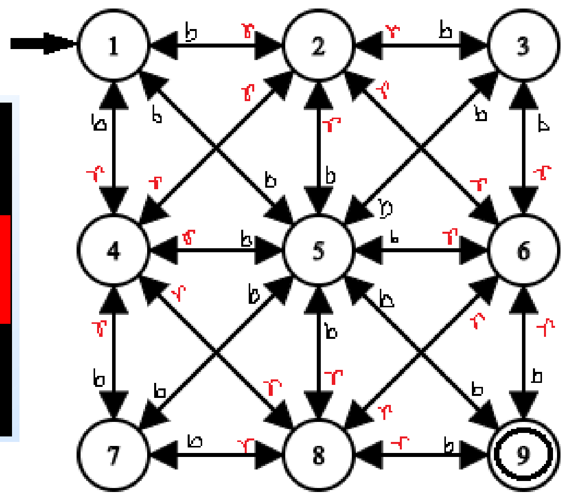
Converting NFA to DFA:

Given a NFA with state Q, input Σ, transition function δ_N, start state q_0, and and final states F, the equivalent DFA is:

- State 2^Q (Set of subset of Q)
- Input Σ
- Start state $\{q_0\}$
- Final states = all those with a member of F

Subset Construction:





		r	b
→	1	2,4	5
	2	4,6	1,3,5
	3	2,6	5
	4	2,8	1,5,7
	5	2,4,6,8	1,3,7,9
	6	2,8	3,5,9
	7	4,8	5
	8	4,6	5,7,9
*	9	6,8	5

	r	b
→ 1	2, 4	5
2	4, 6	1, 3, 5
3	2, 6	5
4	2, 8	1, 5, 7
5	2, 4, 6, 8	1, 3, 7, 9
6	2, 8	3, 5, 9
7	4, 8	5
8	4, 6	5, 7, 9
*** 9**	6, 8	5

	r	b
→ {1}	{2, 4}	{5}
{2, 4}	{2, 4, 6, 8}	{1, 3, 5, 7}
{5}	{2, 4, 6, 8}	{1, 3, 7, 9}
{2, 4, 6, 8}	{2, 4, 6, 8}	{1, 3, 5, 7, 9}
{1, 3, 5, 7}	{2, 4, 6, 8}	{1, 3, 5, 7, 9}
*** {1, 3, 7, 9}**	{2, 4, 6, 8}	{5}
*** {1, 3, 5, 7, 9}**	{2, 4, 6, 8}	{1, 3, 5, 7, 9}

	r	b
→ {1}	{2, 4}	{5}
{2, 4}	{2, 4, 6, 8}	{1, 3, 5, 7}
{5}	{2, 4, 6, 8}	{1, 3, 7, 9}
{2, 4, 6, 8}	{2, 4, 6, 8}	{1, 3, 5, 7, 9}
{1, 3, 5, 7}	{2, 4, 6, 8}	{1, 3, 5, 7, 9}
* {1, 3, 7, 9}	{2, 4, 6, 8}	{5}
* {1, 3, 5, 7, 9}	{2, 4, 6, 8}	{1, 3, 5, 7, 9}

	r	b
→ 1	2	3
2	4	5
3	4	6
4	4	7
5	4	7
* 6	4	3
* 7	4	7

	r	b
→ 1	2	3
2	4	5
3	4	6
4	4	7
5	4	7
* 6	4	3
* 7	4	7