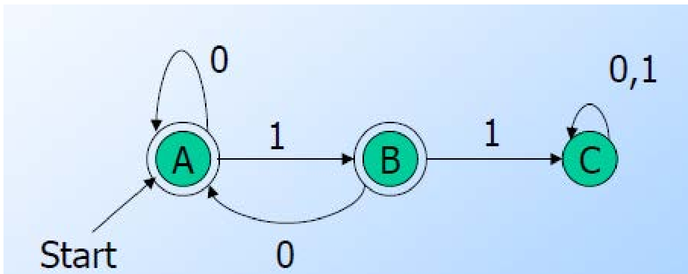
Set Equivalence

L(DFA) = { w | w is in {0, 1}* and w doesn't have two consecutive 1's}

S = { the language of the DFA }



	0	1
→ * A	A	B
* B	A	C
C	C	C

T = { the set of strings of 0's and 1's with no consecutive 1's }

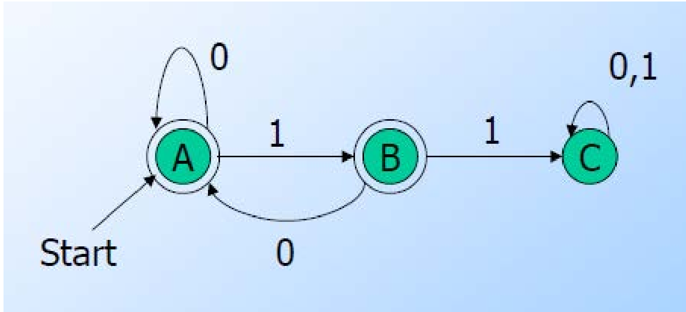
L = {ϵ, 0, 1, 00, 01, 10, 000, 001, 010, 100, 101, … }

Prove that S = T

- To prove S = T, we need to prove

1	$S \subseteq T$	if 'w' is in S then 'w' is in T
2	$T \subseteq S$	if 'w' is in T then 'w' is in S

Part 1: $S \subseteq T \Rightarrow$ if ‘w’ is accepted by DFA then ‘w’ has no consecutive 1’s



	0	1
→ * A	A	B
* B	A	C
C	C	C

Proof:

Proof is an induction on the length of ‘w’

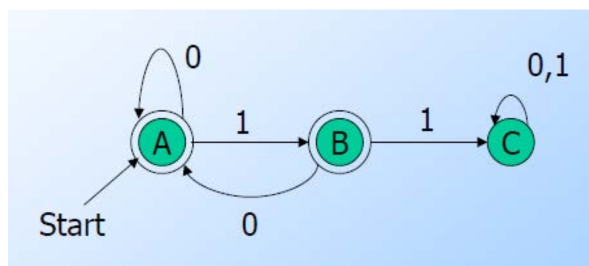
Hypothesis:

- if $\delta(A, w) = A$; then ‘w’ has no consecutive 1’s and doesn’t end with 1
- if $\delta(A, w) = B$; then ‘w’ has no consecutive 1’s and ends in with a single 1’s

Basis:

If $|w| = 0$; then $w = \epsilon$

- ϵ has no 1’s at all
- $\delta(A, \epsilon)$ is not B



Inductive step:

- if Hypothesis (1) and (2) are “True” for the string shorter then ‘w’. So $|w|$ is at least 1 and not empty
- if $w = xa$

  $\delta(A, w) = A$; $\delta(A, x)$ must be A or B then ‘a’ must be 0

- if $w = xa$

  $\delta(A, w) = B$; $\delta(A, x)$ must be A then ‘a’ must be 1

Part 2: T $\subseteq$ S $\Rightarrow$ if ‘w’ has no consecutive 1’s then ‘w’ is accepted by DFA

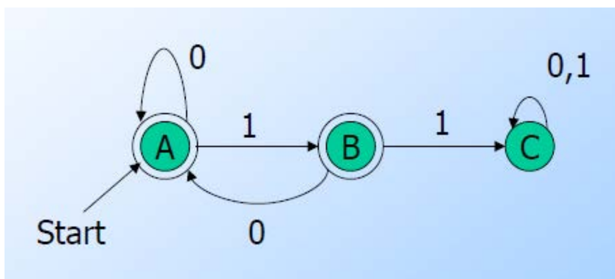
Contrapositive: If ‘w’ is not accepted by the DFA the ‘w’ has 11
(if X then Y is equivalent if not Y then not X)

Proof:

X	‘w’ has no 11’s
Y	‘w’ is accepted by DFA

Contrapositive:

Y	‘w’ is not accepted by DFA
X	‘w’ has 11’s



- The only way ‘w’ is not accepted is if it gets to C
- δ(A, w) = C ; if w = x1y and x gets to B and y is the trail of w
- δ(A, x) = B ; the surely x = z1
- So w = z11y and has 11