- **Parse Tree/ Derivation Tree/ Syntax Tree**
  - Tree representation of derivation
  - Leaves: labeled by a Terminal or $\in$
  - Interior Nodes: labeled by a Variable
  - Root: must be labeled by the start symbol
  - Children (left to right) are labeled by the body of the production rule

  Example 1:

  $A \rightarrow BC$
  $B \rightarrow a \mid b$
  $C \rightarrow \in$

  Example 2: Balanced parenthesis

  $S \rightarrow SS \mid (S) \mid (\,)$

  Example 3:

  $S \rightarrow 0A1 \mid 0S$
  $A \rightarrow A1 \mid \in$

- **Membership checking**
  - If possible, to draw a Parse Tree then the given string is a member of the Language

  Example 4:

  $S \rightarrow 0A1 \mid 0S$
  $A \rightarrow A1 \mid \in$

  w = 0001111

- **Yield of the Parse Tree:**
  - The concatenation of the labels of the leaves in left to right order
  - Preorder traversal

- **Derivation order:**
  - **Left derivation** – if we always derive the left most Variable
  - **Right derivation** – if we always derive the right most Variable

Example 5:

$S \to aB \mid bA$

$A \to a \mid aS \mid bAA$

$B \to b \mid bS \mid aBB$

w = aabbabba

Left derivation			Right Derivation	
S			S	
aB	$S \to aB$		aB	$S \to aB$
aaBB	$B \to aBB$		aaBB	$B \to aBB$
aabB	$B \to b$		aaBbS	$B \to bS$
aabbS	$B \to bS$		aaBbbA	$S \to bA$
aabbaB	$S \to aB$		aaBbba	$A \to a$
aabbabS	$B \to bS$		aabSbba	$B \to bS$
aabbabbA	$S \to bA$		aabbAbba	$S \to bA$
aabbabba	$A \to a$		aabbabba	$A \to a$

Example 6:

$S \to AB \mid \in$

$A \to aB$

$B \to Sb$

w = abb

Left derivation			Right Derivation	
S			S	
AB	$S \to AB$		AB	$S \to AB$
aBB	$A \to aB$		ASb	$B \to Sb$
aSbB	$B \to Sb$		Ab	$S \to \in$
abB	$S \to \in$		aBb	$A \to aB$
abSb	$B \to Sb$		aSbb	$B \to Sb$
abb	$S \to \in$		abb	$S \to \in$

Example 7:

$S \to aSb \mid aX$

$X \to aY \mid a$

$Y \to bY \mid \in$

w1 = aabbb

w2 = bbaab

- **Ambiguous Grammar:**
  - A CFG is ambiguous if there is a string in the Language that is Yield of 2 or more Parse Trees
  - There is a string in the Language that has 2 different Left derivations
  - There is a string in the language that has 2 different Right derivations

Example 7: Balanced parenthesis

$S \rightarrow SS \mid (S) \mid (\,)$

w = ( ) ( ) ( )

** **Unambiguous grammar for Balanced parenthesis:**

$B \rightarrow (\,RB \mid \in$

$R \rightarrow \,) \mid (\,RR$

w = (( ))( )

Example 8:

$E \rightarrow E + E \mid E * E \mid 1 \mid 2 \mid 3$

$w = 2 * 3 + 1$

**** Unambiguous grammar:**

$E \rightarrow E + F \mid F$

$F \rightarrow F * D \mid D$

$D \rightarrow 1 \mid 2 \mid 3$

- $w_1 = 2 * 3 + 1$
- $w_2 = 2 + 3 * 2 + 4 + 1$
- $w_3 = 2 + 3 * 2 + 3 + 1$

- How to generate Grammar Rules:

1. L = $\{0^n1^n \mid n \geq 0\}$
2. L = $\{0^n1^n \mid n \geq 1\}$
3. L = 1(0 + 1)*
4. L = (0 + 1)* 0
5. L = 0n1m |
    a. | n, m ≥ 0
    b. | n ≥ 0, m > 0
    c. | n > 0, m ≥ 0
    d. | n > 0, m > 0
6. L = Even palindrome, Σ = {a, b}
7. L = Odd palindrome, Σ = {a, b}
8. L = string holds at least 3 b's, Σ = {a, b}
9. L = all nonempty strings that start and end with the same symbol