Roll-1605112

(24)(a)

To place 6 non-attacking rooks in forbidden positions on the 6 by 6 the board,

$$R_6(C) = \sum_{k=0}^{6} r_k(C)(-1)^k(6-k)!$$

From the figure, $r_0 = 6!$ and the rook r_1 is the number of forbidden places that is 6.

For r_2, compute neither F_1, F_2 and F_3 each contains two positions,

So, $r_2 = {}^3C_2 \times 2^2 = 12$

For r_3 contains all rooks, $r_3 = 2^3 = 8$

Another rook, $r_4 = r_5 = r_6 = 0$

$$R_6(C) = \sum_{k=0}^{6} r_k(C)(-1)^k(6-k)!$$

$= 6! - (6 \times 5!) + (12 \times 4!) - (8 \times 3!)$

$= 240$ (Ans)

24) (b)

Consider the figure discussed in the textbook, the rook polynomial is,

$$R(C,x) = (1+4x+2x^2)^3$$
$$= 1+12x+54x^2+102x^3+44x^4+48x^5+8x^6$$

From the figure, $r_0 = 6!$ and the rook r_1 is the number of forbidden places that is 12.

For r_2, compute neither F_1, F_2 and F_3 each contains two positions.

$$r_2 = {}^3C_2 \times 4^2 + 3 \times 2 = 54$$

For r_3 contains all rooks and three-time of two in one rook of F_1, F_2 and F_3.

$$r_3 = 4 \times 4 \times 4 + 3 \times (2 \times 8) = 112$$

For r_4 two in each of rook,

$$r_4 = {}^3C_2 \cdot 2^2 + 3\,(2 \cdot 4^2) = 108$$

For $r_5 = 3\left({}^3C_2 \cdot 2^2 + 4\right) = 48$

For $r_6 = 2^3 = 8$

$$R_0(C) = 6! - 12 \times 5! + 54 \times 4! - 112 \times 3!$$
$$+ (108 \times 2!) - (48 \times 1) + (8 \times 0!)$$
$$= 80$$

(Ans)

Hexin

(c) Consider the figure discussed in the textbook.

$$R(C,x) = (1+5x+6x^2+x^3)(1+3x+2x^2)$$
$$= 1+8x+22x^2+24x^3+9x^4+x^5$$

Compare the above expression with

$$R(C,x) = \pi_0 + \pi_1 x + \pi_2 x^2 + \pi_3 x^3 + \pi_4 x^4 + \pi_5 x^5 + \pi_6 x^6$$

So,

$\pi_0 = 1$

$\pi_1 = 8$

$\pi_2 = 22$

$\pi_3 = 24$

And, $\pi_4 = 9$

$\pi_5 = 1$

$\pi_6 = 0$

$$R_6(C) = \pi_0 6! - \pi_1(5!) + \pi_2(4!) - \pi_3(3!) + \pi_4(2!) - \pi_5(1!) + \pi_6(0!)$$

$$= 161$$

(Ans)