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### Chapter-3

#### Problem 41:

Workers 1, ..., n are currently idle. Suppose that each worker, independently, has probability  $p$  of being eligible for a job, that a job is equally likely to be assigned to any of the workers that are eligible for it (if none are eligible, the job is rejected). Find the probability that the next job is assigned to worker 1.

Solution:

Worker 1 gets the job if,  
# 5.

The probability is

$$\begin{aligned} & p \sum_{k=0}^{n-1} \binom{n-1}{k} p^k (1-p)^{n-1-k} \frac{1}{k+1} \\ &= p \sum_{k=0}^{n-1} \frac{(n-1)!}{(k+1)!(n-k-1)!} p^k (1-p)^{n-1-k} \\ &= \frac{1}{n} \sum_{k=0}^{n-1} \binom{n}{k+1} p^{k+1} (1-p)^{n-(k+1)} \\ &= \frac{1}{n} \sum_{k=1}^n \binom{n}{k} p^k (1-p)^{n-k} \\ &= \frac{1}{n} \left( 1 - \binom{n}{0} p^0 (1-p)^{n-0} \right) = \frac{1 - (1-p)^n}{n} \end{aligned}$$