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The sums of the binomial coefficients along the diagonals of Pascal's triangle running upward from the left are Fibbonacci numbers.

$$f_n = \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \cdots + \binom{n-t}{t-1}$$

where $t = \lfloor \frac{n+1}{2} \rfloor$ is the floor of $\frac{n+1}{2}$

Proof: Define $g_0 = 0$ and

$$g_n = \binom{n-1}{0} + \binom{n-2}{1} + \cdots + \binom{n-t}{t-1} ; (n \geq 1)$$

since $\binom{m}{p} = 0$ for each integer $p > m$, we also have.

$$g_n = \binom{n-1}{0} + \binom{n-2}{1} + \binom{n-3}{2} + \cdots + \binom{0}{n-1} ; (n \geq 1)$$

or using summation notation,

$$g_n = \sum_{p=0}^{n-1} \binom{n-1-p}{k}$$

To prove the theorum it will suffice to show that g_n satisfies the Fibonacci recurrence relation and has the same initial values as the Fibbonacci sequence.

We have. $g_0 = \binom{0}{-1} = 0$, $g_1 = \binom{0}{0} = 1$, $g_2 = \binom{1}{0} + \binom{0}{1}$
$= 1 + 0 = 1$

Using Pascal's formula, we see that,

$$g_{n-1}+g_{n-2} = \sum_{k=0}^{n-2}\binom{n-2-k}{k} + \sum_{j=0}^{n-3}\binom{n-3-j}{j}$$

$$= \binom{n-2}{0} + \sum_{k=1}^{n-2}\binom{n-2-k}{k} + \sum_{k=1}^{n-2}\binom{n-2-k}{k-1}$$

$$= \binom{n-2}{0} + \sum_{k=1}^{n-2}\left(\binom{n-2-k}{k} + \binom{n-2-k}{k-1}\right)$$

$$= \binom{n-2}{0} + \sum_{k=1}^{n-2}\binom{n-1-k}{k}$$

$$= \binom{n-1}{0} + \sum_{k=1}^{n-2}\binom{n-1-k}{k} + \binom{0}{n-1}$$

$$= \sum_{k=0}^{n-1}\binom{n-1-k}{k} = g_n$$

Here we have used the facts that,

$$\binom{n-1}{0} = 1 = \binom{n-2}{0} \quad \text{and} \quad \binom{0}{n-1} = 0 \quad (n \geq 2)$$

We conclude that $g_0, g_1, g_2 \cdots g_n \cdots$ is the Fibonacci sequence and this proves the theorum.