(11) Suppose, that S be the set of all permutations of {1,2, ..., 8} since the even integers in the set {1,2, ..., 8} are 2,4,6,8. Assume A_1, A_2, A_3, A_4 be the sets of permutations that 2,4,6,8 are fixed.
The total number of integers are,
$|S| = 8!$
Similarily for A_1, A_2, A_3, A_4
The fixed position of an even number is
$|A_1| = |A_2| = |A_3| = |A_4| = 7!$
Now find the intersection of ~~$|A_i \cap A_j| = 6!$~~.
$|A_i \cap A_j| = 6!$
where the range is $1 \leq i \leq j$ and $j \leq 8$.
Similarly find the intersection of $|A_1 \cap A_2 \cap A_3|$, $|A_1 \cap A_2 \cap A_4|$, $|A_1 \cap A_3 \cap A_4|$, $|A_2 \cap A_3 \cap A_4|$ is:
$|A_1 \cap A_2 \cap A_3| = |A_1 \cap A_2 \cap A_4| = |A_1 \cap A_3 \cap A_4| = |A_2 \cap A_3 \cap$
$= 5!$
Similarly find the intersection of
$|A_1 \cap A_2 \cap A_3 \cap A_4|$ is: $4!$
By the inclusion-exclusion formulas, $|A \cup B| = |A| + |B| -$
So,
$8! - 4(7!) + 6(6!) - 4(5!) + 4! = 24024$
(Ans)

Hexin

(12) Since $\{1, 2, \ldots, 8\}$

If first pick up the 4 of the 8 numbers to fix

Then dearrange the other 4

Then each permutation arises once in such a way

So by which the number of such permutations

are follows as, $\binom{8}{4} |D_4| = \binom{8}{4} 4 \sum_{i=0}^{4} \frac{(-1)^i}{i!}$

$= 70\left(24 \times \left(1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right)\right)$

$= 630$

(Ans)

(13) Consider a set S be a set of whole set, there are total 9 number, 5 numbers are odd other number are even.

For select the first odd number, the ways of choosen is, $\binom{5}{1} = {}^5C_1 = \frac{5!}{1!4!} = 5$

The rest of number is $9 - 1 = 8$

So the total ways are: $5 \times 8! = 201600$

For second odd numbers, $\binom{5}{2} = {}^5C_2 = 10$

The rest of number is $9 - 2 = 7$

So total ways are $10 \times 7! = 50400$

For three odd numbers, $\binom{5}{3} = 10$

The rest of numbers are $= 9 - 3 = 6$

So total ways are $10 \times 6! = 7200$

For four odd numbers, $\binom{5}{4} = 5$

The rest of number is $9-4=5$

$\therefore$ Total ways are $5 \times 5! = 600$

For five odd numbers, $\binom{5}{5} = 1$

The rest of the numbers are $9-5=4$

So the total ways are $= 1 \times 4! = 24$

So the number of permutations at least one odd number is in its natural position is,

$(5\times 8!) - (10\times 7!) + (10\times 6!) - (5\times 5!) + 4! = 157824$

(Ans)