Chapter-5 Roll-1605112

(10) $2\binom{m}{2}+\binom{m}{1}=m(m-1)+m=m^2$

$\binom{n}{k}=\binom{n-1}{k}$ $\binom{n-1}{k-1}$

Therefore,

$$\sum_{k=1}^{n} k^2=\sum_{k=0}^{n} k^2$$

$$=2\sum_{k=0}^{n}\binom{k}{2}+\sum_{k=0}^{n}\binom{k}{1}$$

$$=n\binom{n+1}{3}+\binom{n+1}{2}$$

$$=\frac{n(n+1)(2n+1)}{6}$$

(11) Given integers $n\geq 3$ and $1\leq k\leq n$, We show,

$$\binom{n}{k}-\binom{n-3}{k}=\binom{n-1}{k-1}+\binom{n-2}{k-1}+\binom{n-3}{k-1}$$

Let, S denote the set of k-subsets of $\{1,2,\dots,n\}$

Let, S_1 consist of the elements in S that contain 1.

Let, S_2 " " " " " S " " 2 but not contain 1

Let, S_3 " " " " " S " " 3 but do not contain 1 or 2

Let, S_4 " " " " " S " " 4 but do not contain 1 or 2 or 3

Note that $\{S_i\}_{i=1}^{4}$ partition S so

$|S|=\sum_{i=1}^{4}|S_i|$. We have,

$|S|=\binom{n}{k}$, $|S_1|=\binom{n-1}{k-1}$, $|S_2|=\binom{n-2}{k-1}$, $|S_3|=\binom{n-3}{k-1}$

$|S_4|=\binom{n-3}{k}$