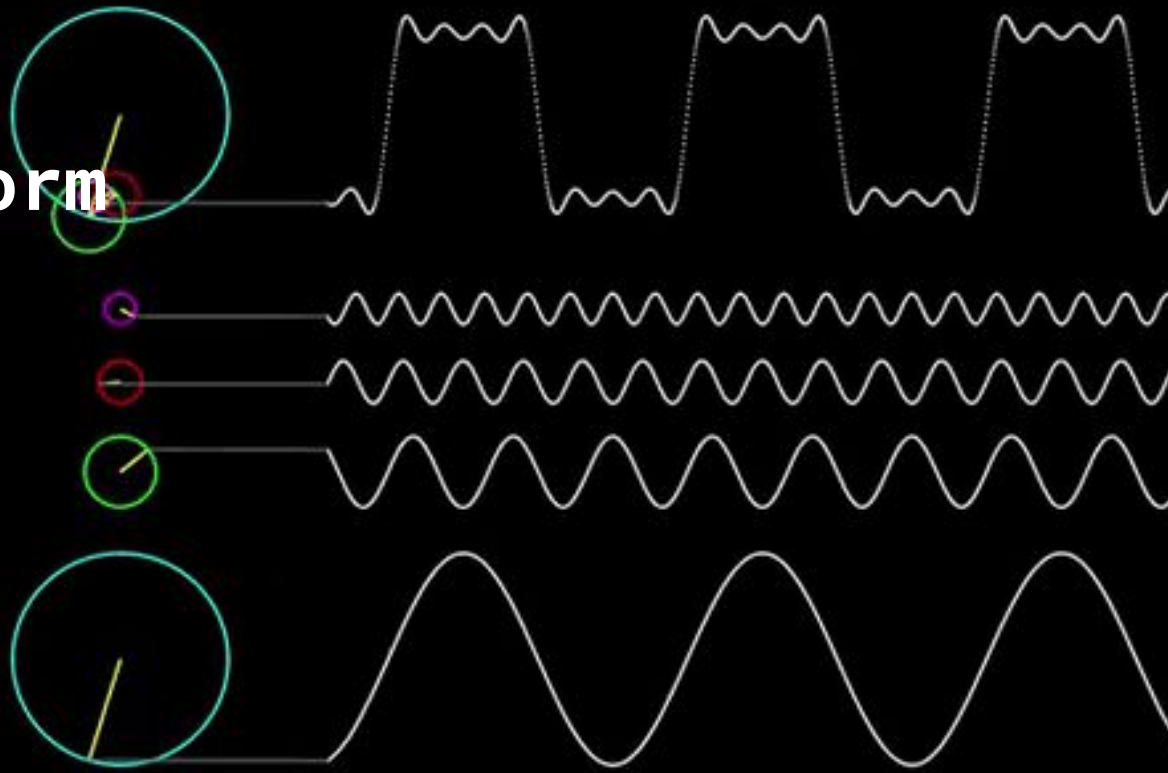
Applications
of the
Fourier Transform



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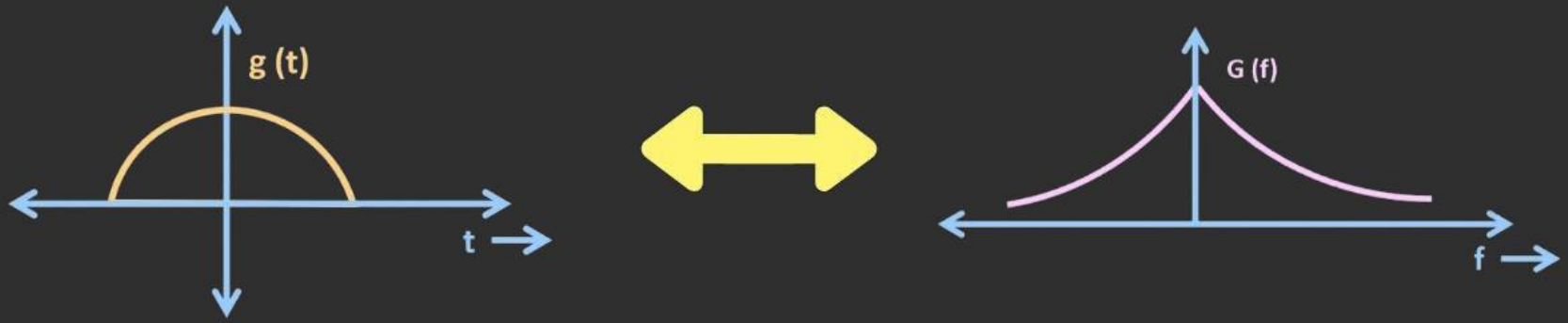
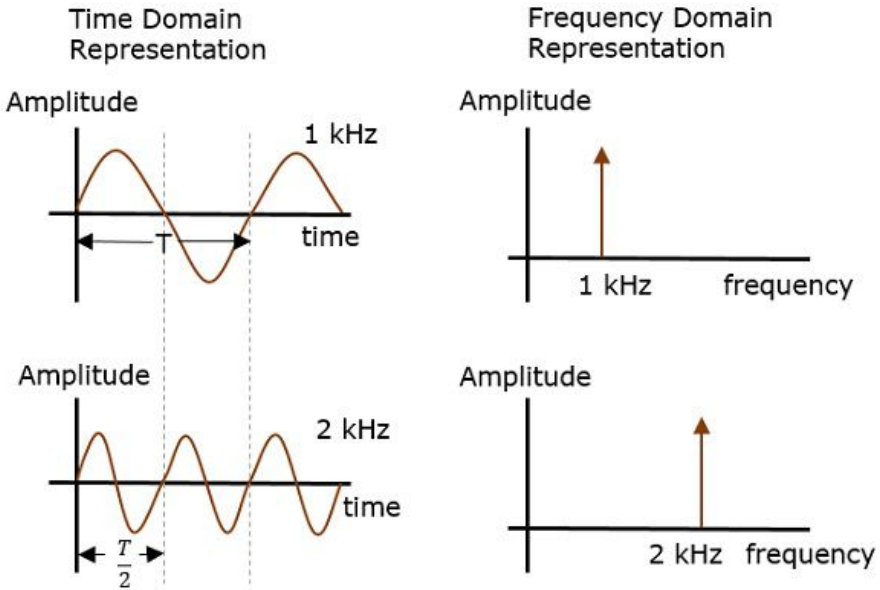
Introduction to the Fourier Transform

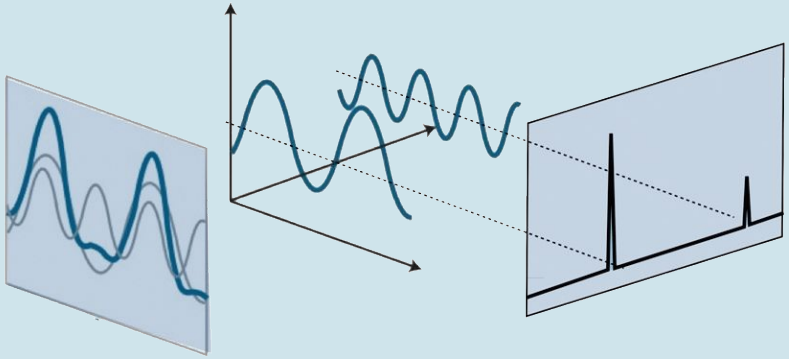


What is Fourier Transform?

- It is used to transform signals between time domain and frequency domain.
- Breaks a waveform into an alternate representation



Transforms



Time Domain
s(t)

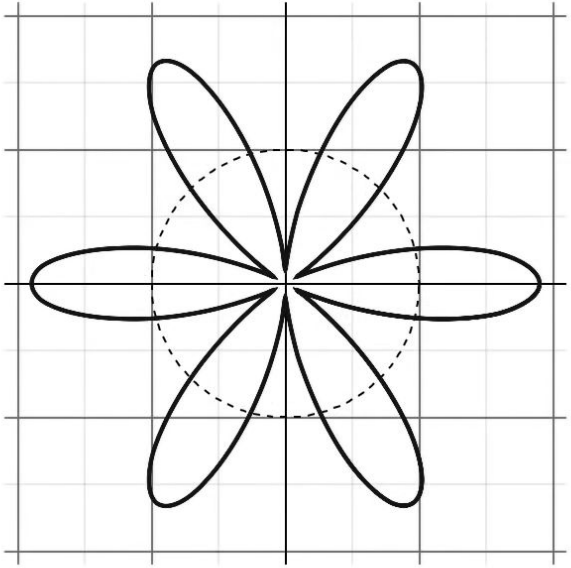
Frequency Domain
S(ω)

The Fourier transform of a function f(x) is given by

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\Pi i k x} \, dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x) e^{-2\Pi i k x} \, dx$$

Where F(k) can be obtained using inverse Fourier transform



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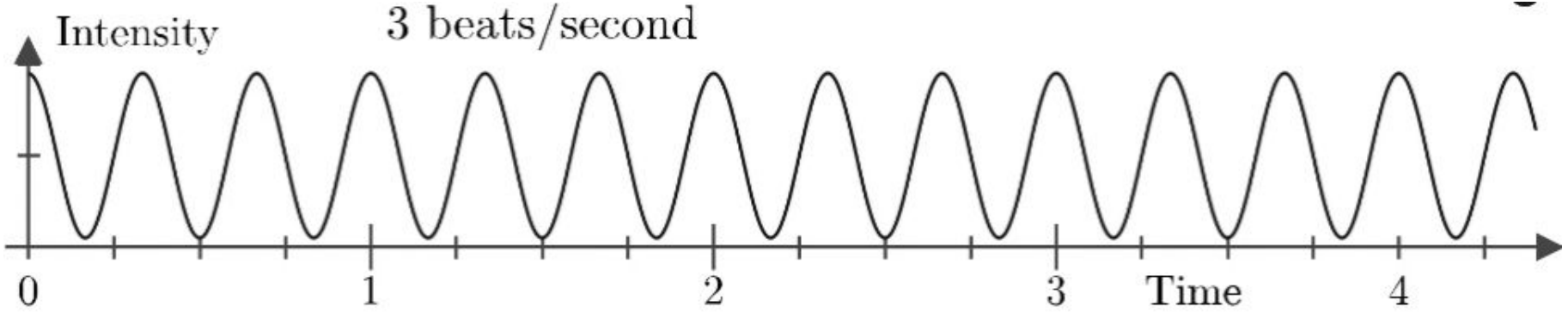
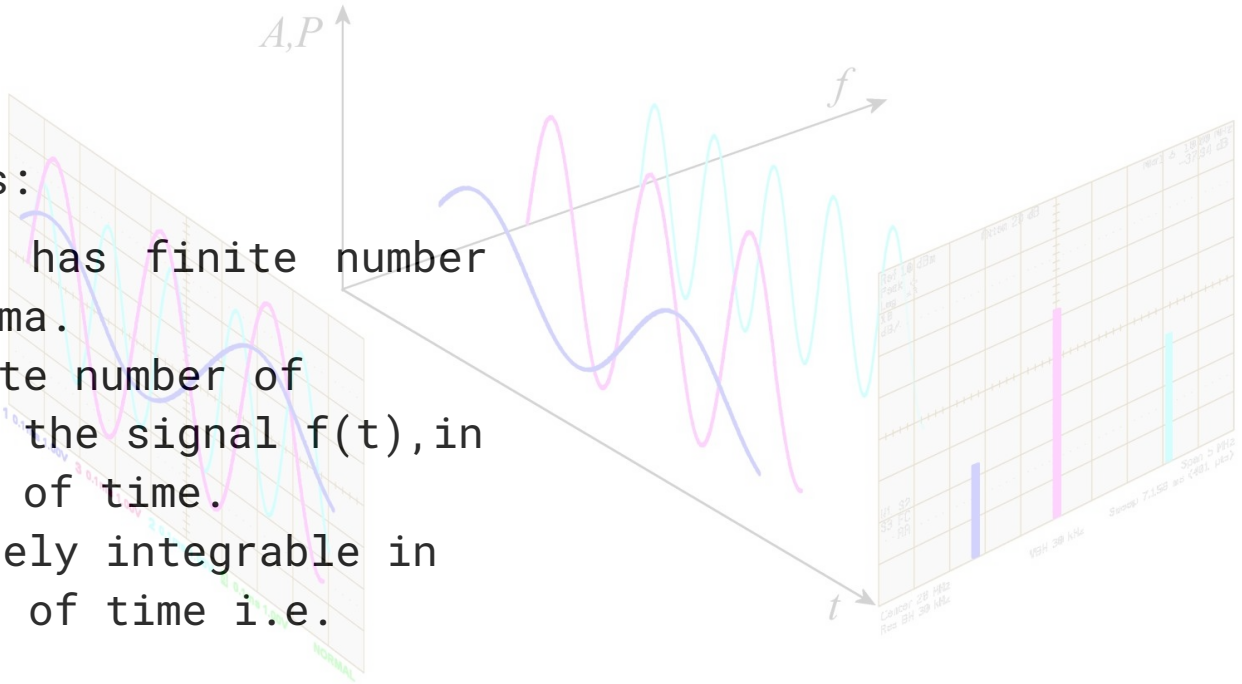


Conditions for Existence of Fourier Transform



Dirichlet's conditions:

- The function f(t) has finite number of maxima and minima.
- There must be finite number of discontinuities in the signal f(t),in the given interval of time.
- It must be absolutely integrable in the given interval of time i.e.

Properties of transforms:

- Linearity -

  Case I:If h(x) -> H(f) then ah(x) -> aH(f)

  Case II:

  If h(x) -> H(f) and g(x) -> G(f) then h(x)+g(x) -> H(f)+G(f)

- Time Shift -

  If f(t) -> F(w) then $f(t-t') \rightarrow F(w)e^{-jwt'}$

- Frequency Shift -

  If f(t) -> F(w) then $f(t)e^{-jw't} \rightarrow F(w-w')$

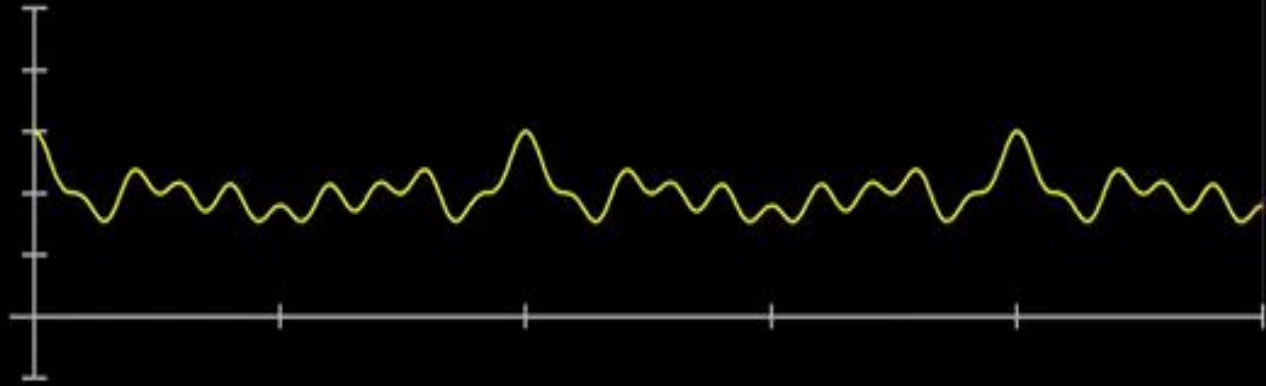
Properties of Transforms:

- Differentiation -
  If f(t) -> F(w) then f'(t) -> jwF(w)

- Scaling -
  If f(t) -> F(w) then f(at) -> (1/|a|)F(w/a)

- Convolution -
  If f(t) -> F(w) and g(t) -> G(w)

Signal Processing



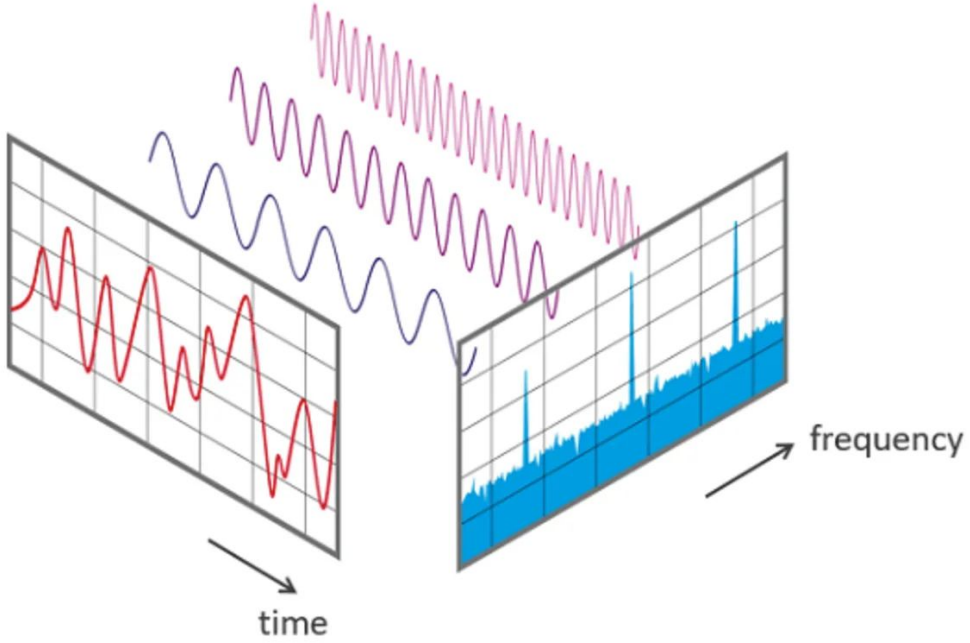
What’s a Signal

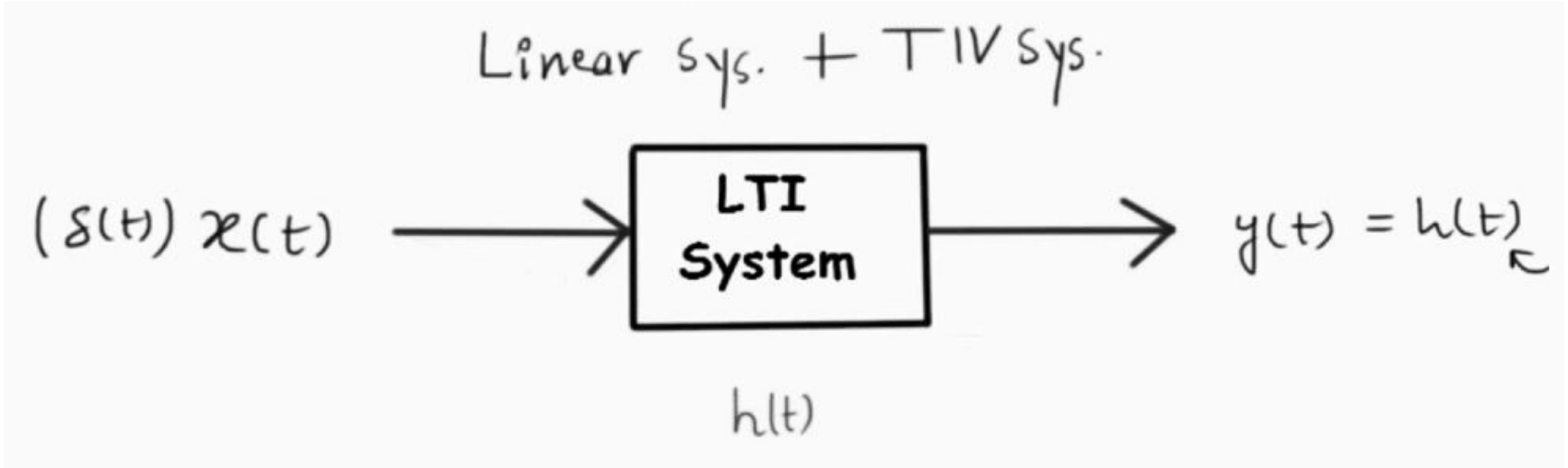
In signal processing, a signal is defined as a function that conveys information about a phenomenon.

In electronics and communications, it refers to any time varying voltage, current or electromagnetic wave that carries information.

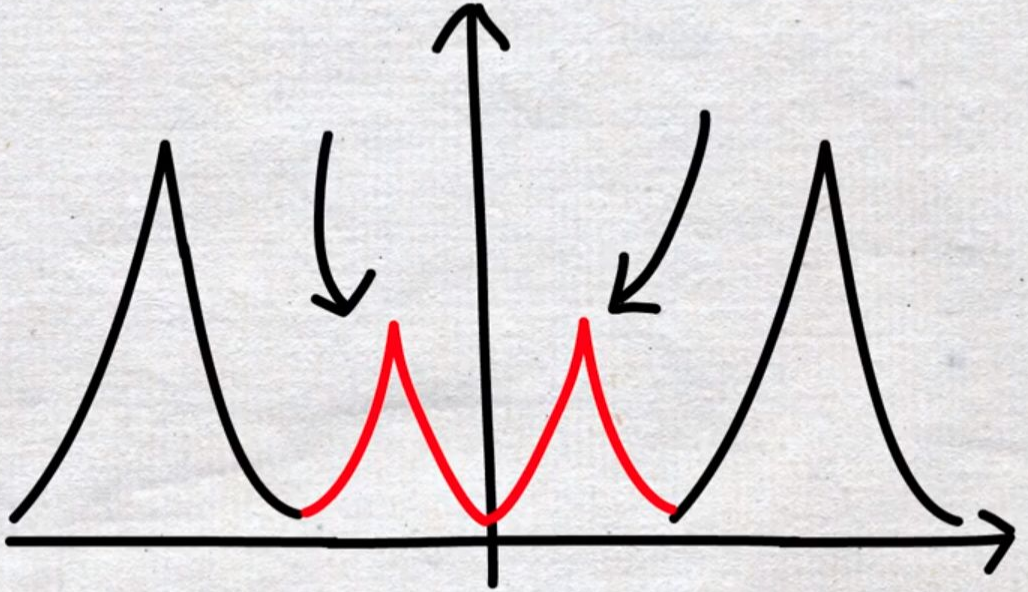
Spectral Analysis

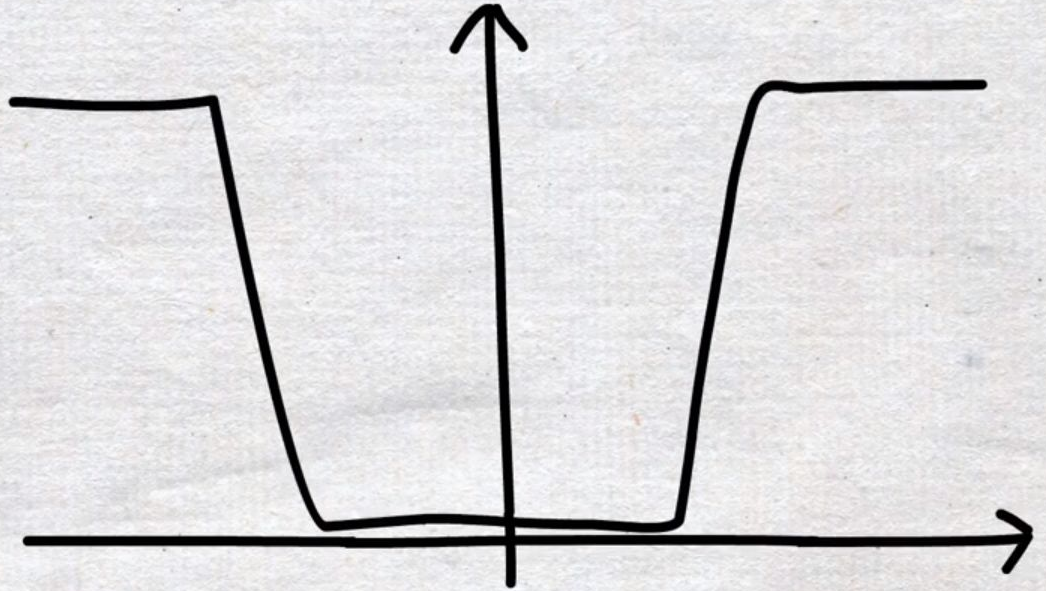
- Equalization of audio recordings with a series of bandpass filters
- Cross correlation of similar images for co-alignment
- X-ray crystallography to reconstruct a crystal structure from its diffraction pattern
- Passive sonar used to classify targets based on machinery noise.

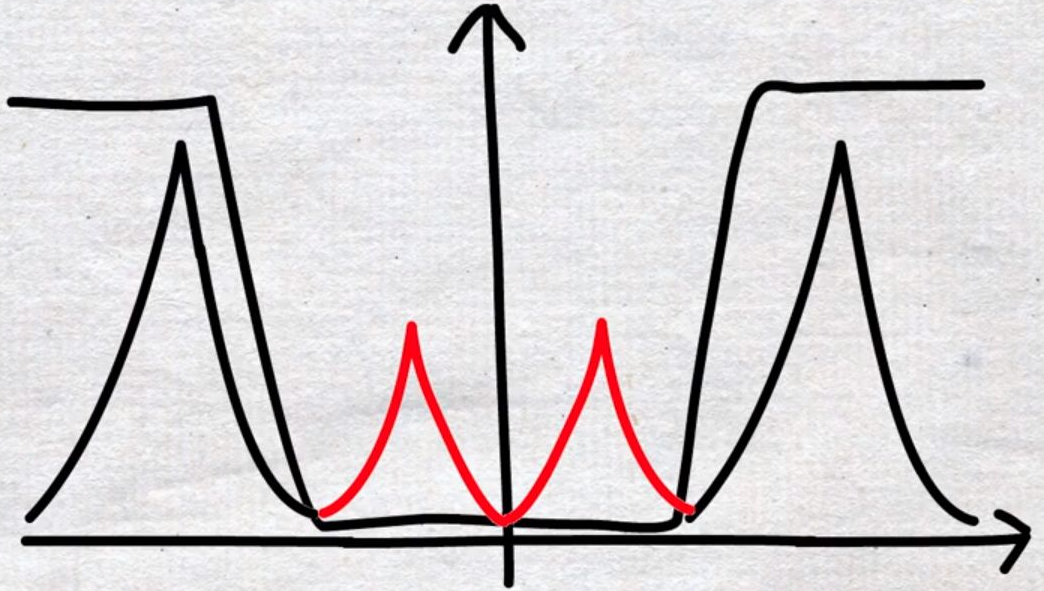


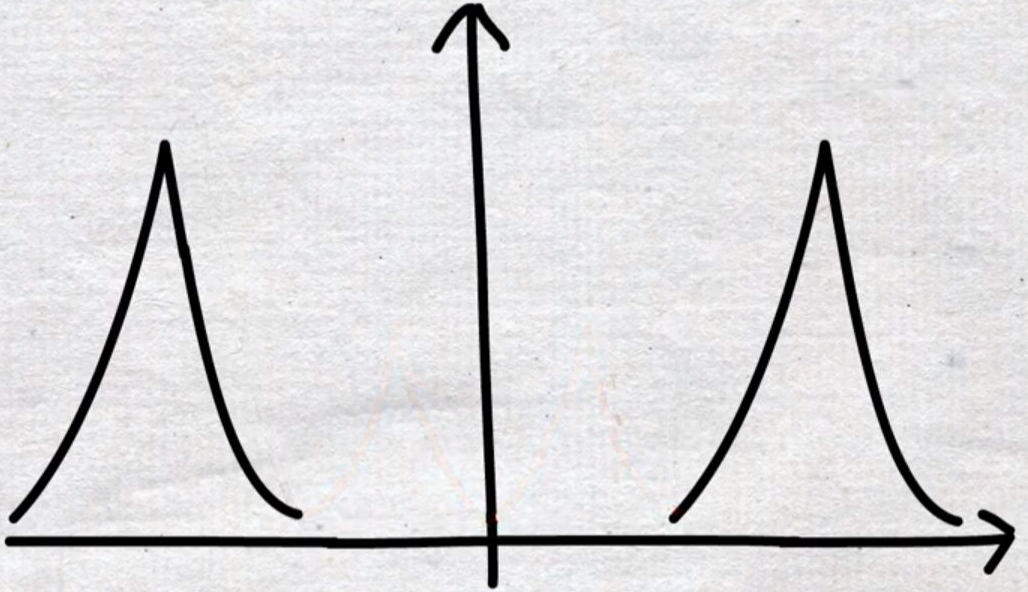


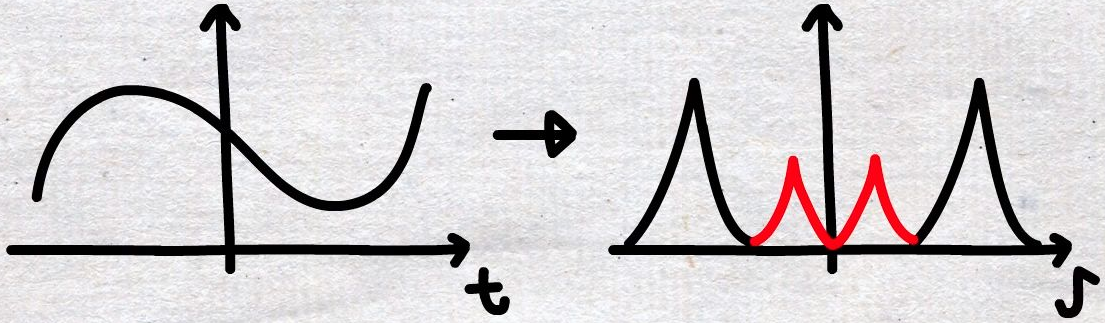
1. Linear
    a. Superposition Principle
    b. Scaling
2. Time Invariant
    Regardless of when it is measured










t
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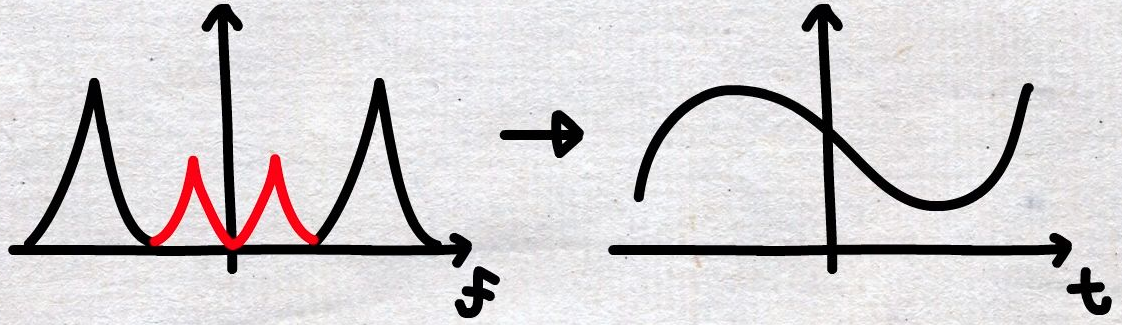
f
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Image
Processing

What does an Image mean?

Pixel intensity value

f(1,1) = 103

Pixel location

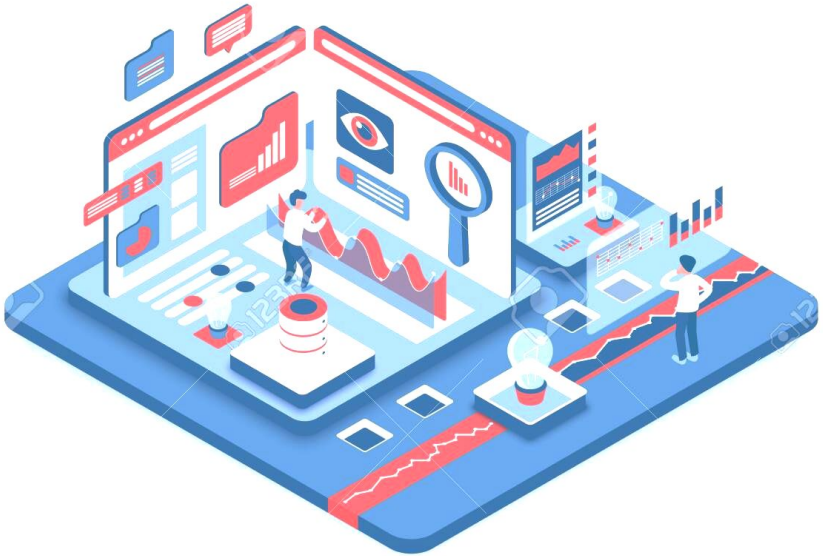
f(2724,2336) = 88

rows columns

f(645:650,1323:1328) =

83	82	82	82	82	82
82	82	82	81	81	81
82	82	81	81	80	80
82	82	81	80	80	79
80	79	78	77	77	77
80	79	78	78	77	77

Image Processing



Definition: Image processing is a method to perform some operations on an image, in order to get an enhanced image or to extract some useful information from it

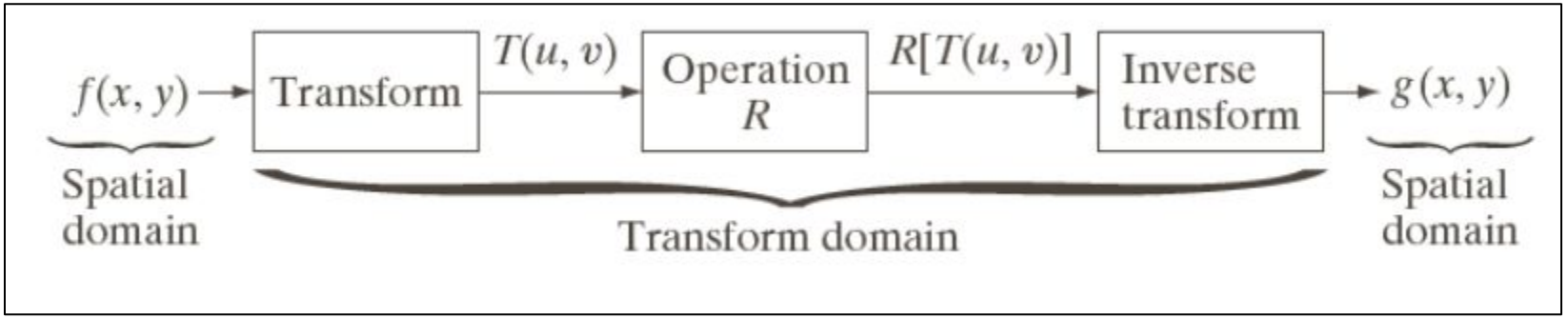
Types of Image Processing:

1. Analog Image Processing
2. Digital Image Processing

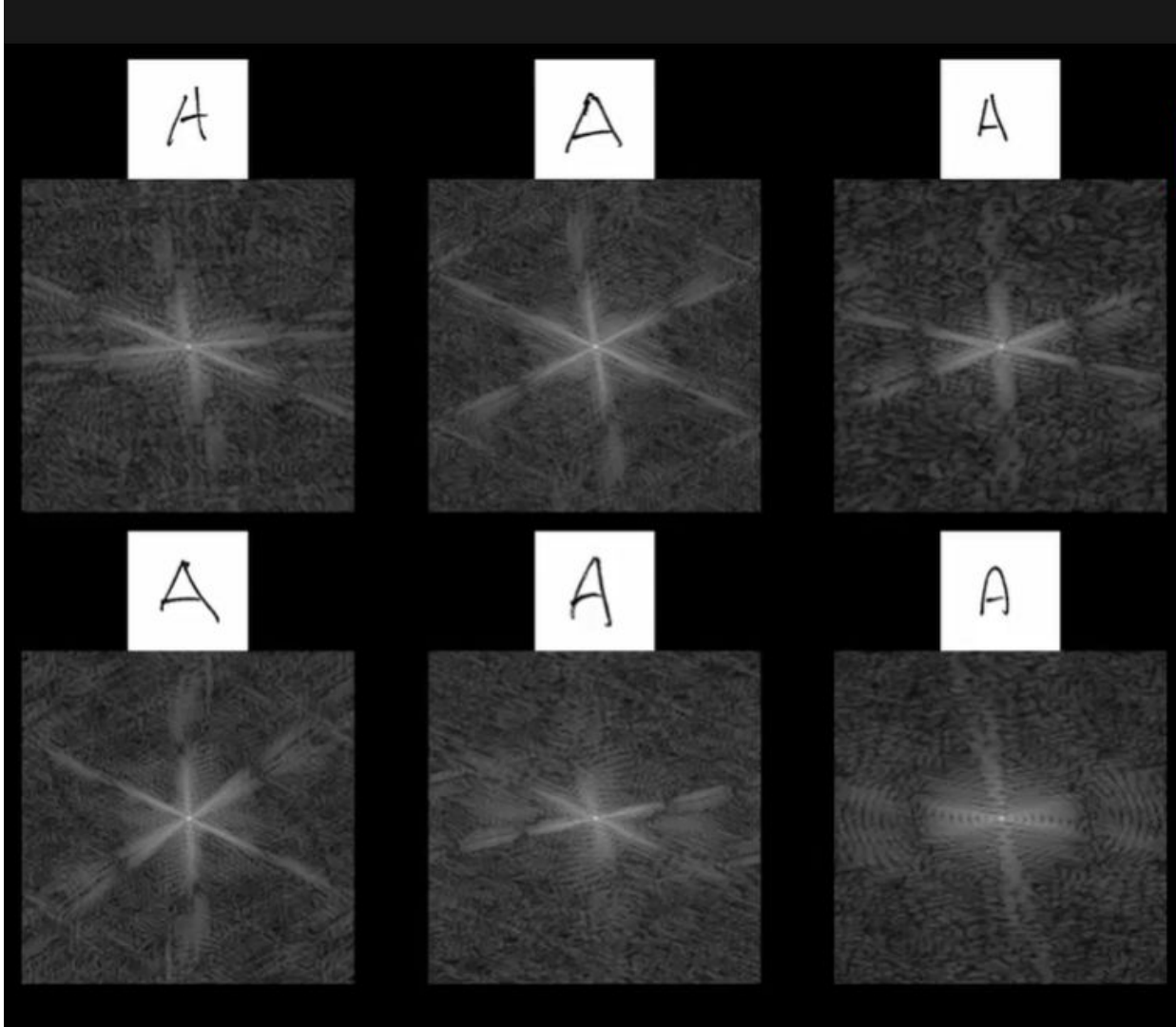
Fourier Transform in Image Processing

Image Transform Key steps:-

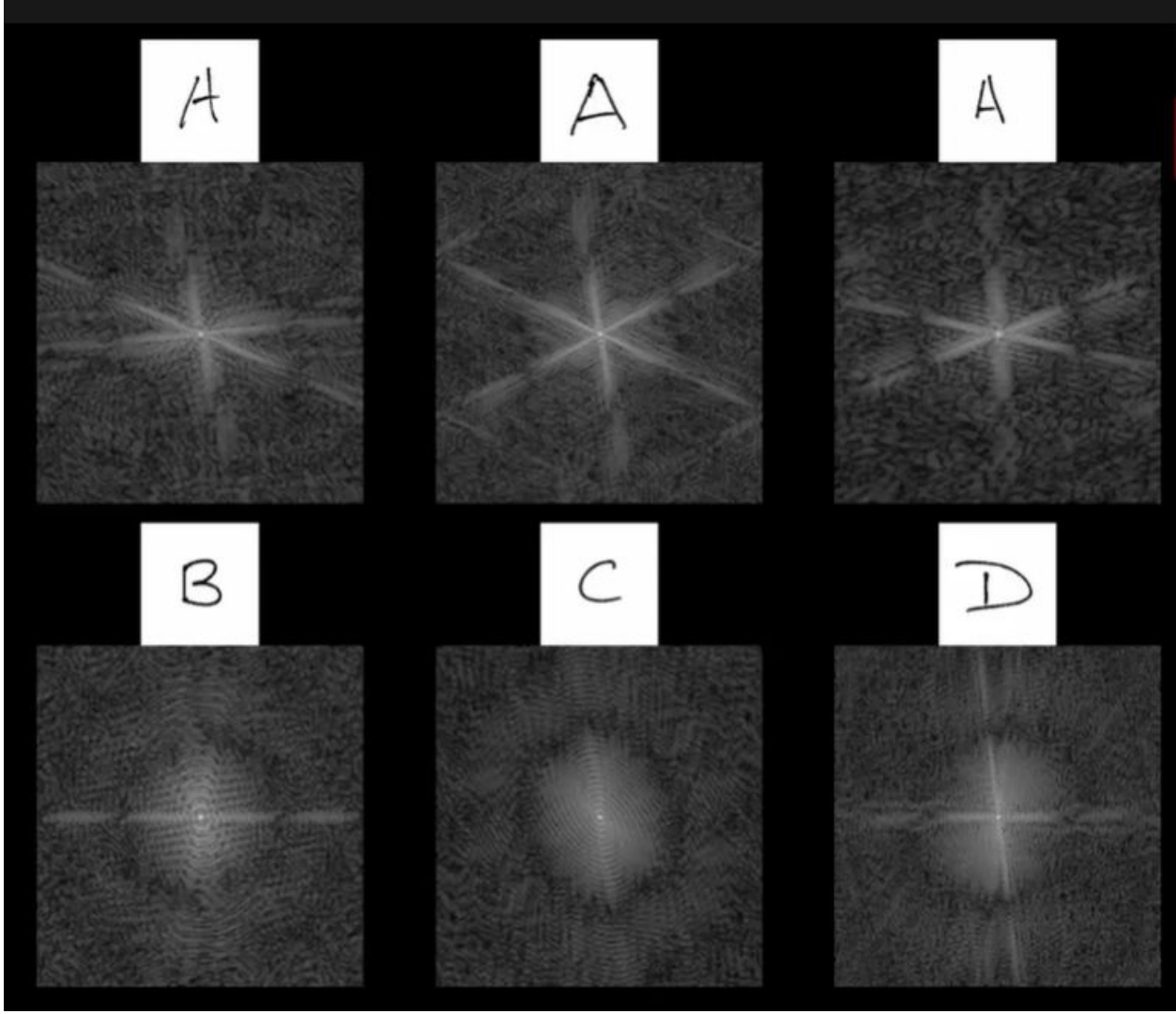
(1) Transform the image
(2) Carry the task(s) in the transformed domain.
(3) Apply inverse transform to return to the spatial domain.



Images in Frequency Domain

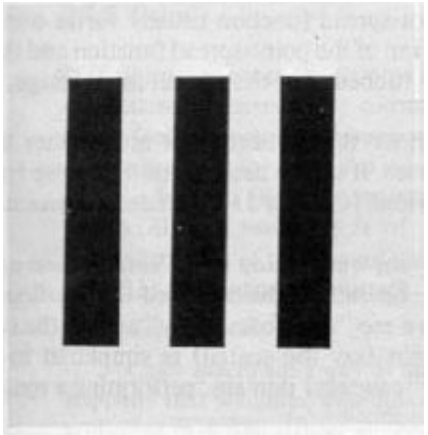


Images in Frequency Domain

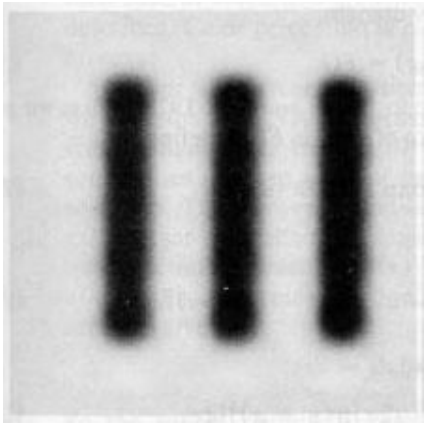


How do frequencies show up in an image?

- Low frequencies correspond to slowly varying pixel intensities (e.g., continuous surface).
- High frequencies correspond to quickly varying pixel intensities (e.g., edges)

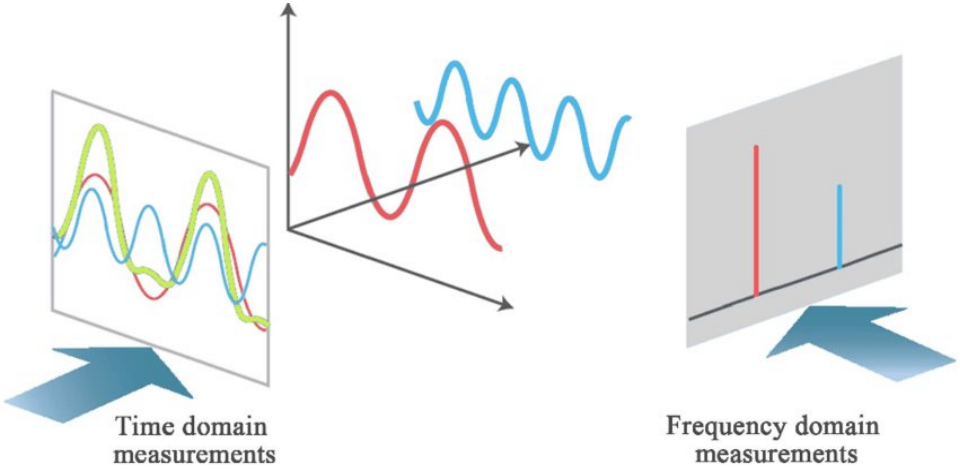


Original Image



Low-passed image (i.e., high frequencies removed)

Application of Fourier Transform in Image Processing



- Image Filtering
- Image Analysis
- Image Reconstruction
- Image Compression
- Image Enhancement

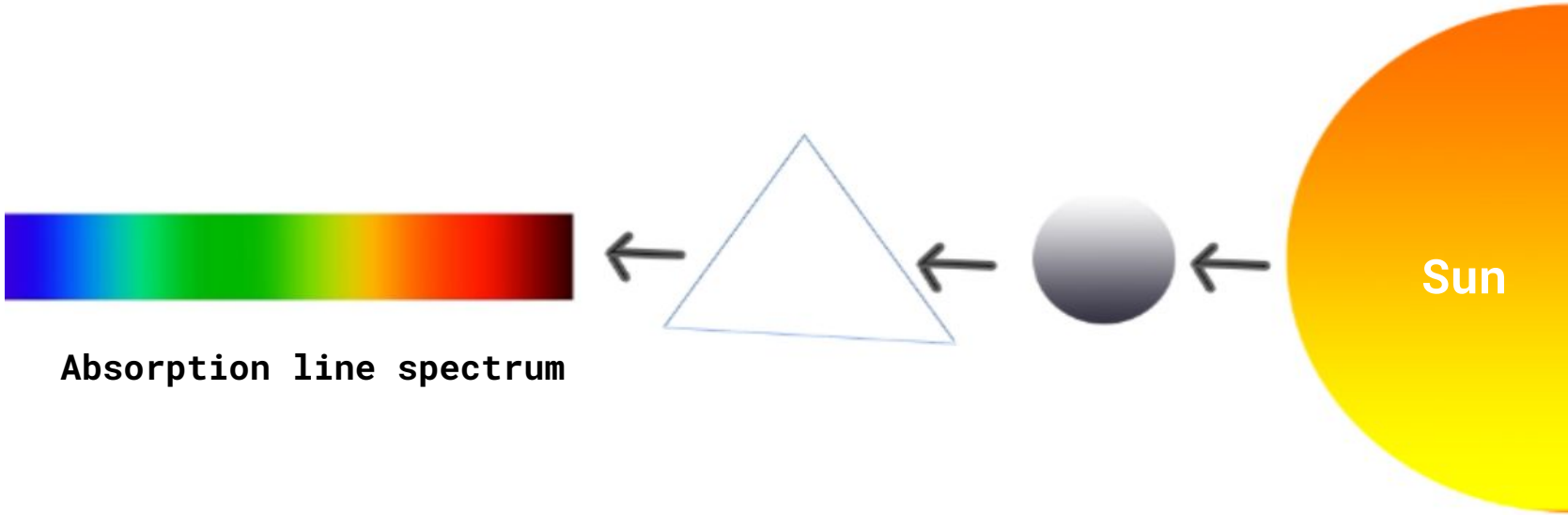
Fourier Transform Infrared Spectroscopy

FTIR

Fourier-transform infrared spectroscopy (FTIR) is a technique used to obtain an infrared spectrum of absorption or emission of a solid, liquid or gas

Absorption Spectroscopy

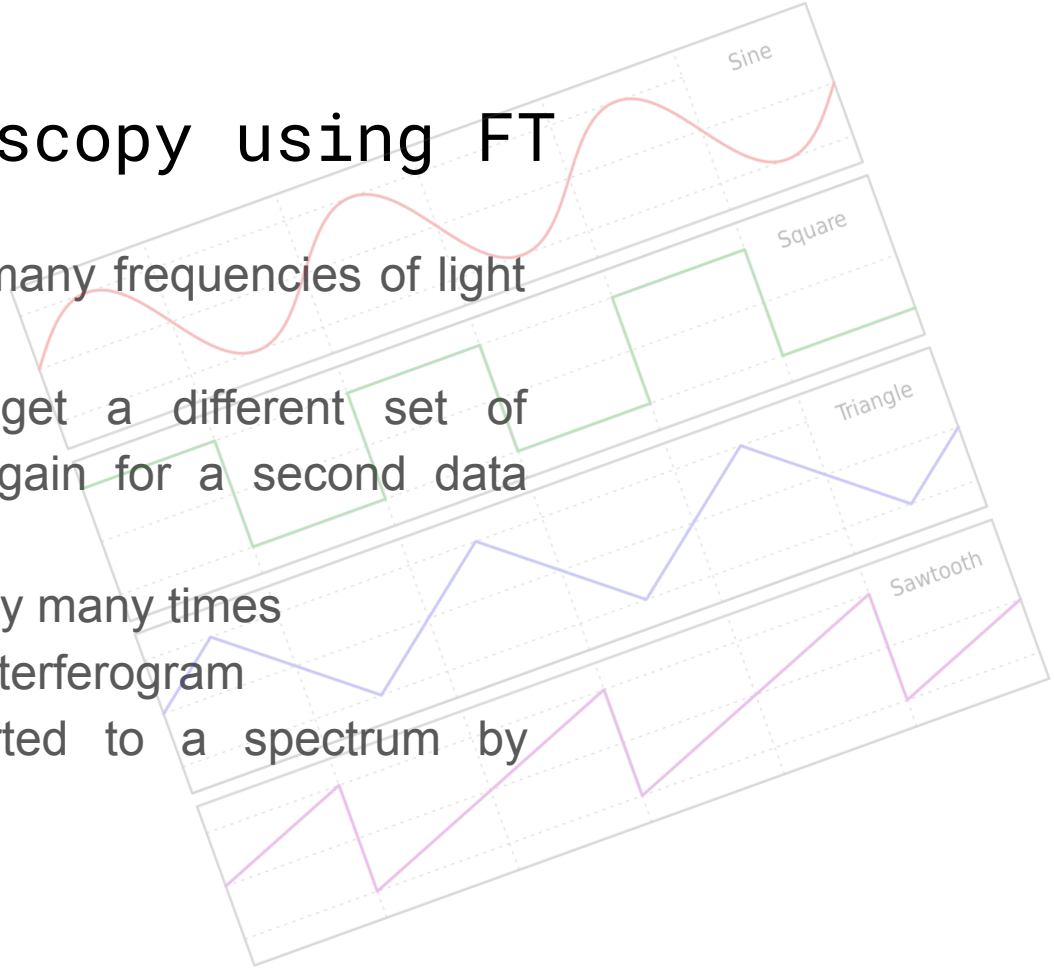
The goal of absorption spectroscopy techniques (FTIR, ultraviolet-visible spectroscopy, etc.) is to measure how much light a sample absorbs at each wavelength.



Absorption line spectrum

Absorption Spectroscopy using FT

- A beam is shined containing many frequencies of light at once and measured.
- The beam is modified to get a different set of frequencies and measured again for a second data point
- This process is repeated rapidly many times
- Data collected here is called interferogram
- The interferogram is converted to a spectrum by Fourier transformation



Conclusion

We have covered the introduction of the fourier transform, overview of properties of the transform.

Then we have mentioned some of its applications in signal processing, some use in image processing and finally Fourier Transform Infrared Spectroscopy

References

1. “But what is the Fourier Transform? A visual introduction” by 3Blue1Brown, https://www.youtube.com/watch?v=spUNpyF58BY

2. “INTRODUCTION TO FOURIER TRANSFORMS FOR IMAGE PROCESSING”, https://www.cs.unm.edu/~brayer/vision/fourier.html

3. Griffiths, P.; de Hasseth, J. A. (18 May 2007). Fourier Transform Infrared Spectrometry (2nd ed.). Wiley-Blackwell. ISBN 978-0-471-19404-0.

4. “Signal Processing: Continuous and Discrete”, MIT OpenCourseWare

Thank You!