Exercise 1;

a)

S: Strawberry

A: Anchovy

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

$P(S) = 0{,}7$

$P(A) = 0{,}3$

$P(\text{Shape}=\text{Round}|S) = 0{,}8 \rightarrow P(\text{Shape}=\text{Square}|S) = 0{,}2$

$P(\text{Shape}=\text{Square}|A) = 0{,}9 \rightarrow P(\text{Shape}=\text{Round}|A) = 0{,}1$

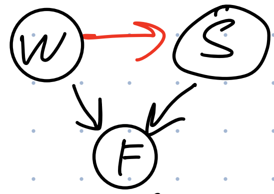
$P(\text{Wrap}=\text{Red}|S) = 0{,}8 \rightarrow P(\text{Wrap}=\text{Brown}|S) = 0{,}2$

$P(\text{Wrap}=\text{Brown}|A) = 0{,}9 \rightarrow P(\text{Wrap}=\text{Red}|A) = 0{,}1$

Both Wrapping (colour) and shape says something about the chance of a flavour. They do not directly affect the other, but since Wrapping affects the chance of getting a flavour, it will affect the probability for the different shapes. Same goes for Shape to Wrapper.

$\Rightarrow$ (i)
$$\begin{tikzcd} W \arrow[dr] & & S \arrow[dl] \\ & F & \end{tikzcd}$$
and (iii)
$$\begin{tikzcd} & F \arrow[dl] \arrow[dr] & \\ W & & S \end{tikzcd}$$

It cannot be ii)



because wrapper are choosen randomly, independant of shape directly, but only through flavor if flavor is not known.

b) Since Flavour is supposed to be "the surprise", model (i) makes the most sense. Since we have the probabilities for Wrapper and Shape given Flavour, model (iii) would be easier to make.
Both models have an equal ammount of connections / equal size of representation

c) When flavor is known, then wrapper and shape are independant (in (i)). When flavor is not known, they are dependant through flavour, as explaind in a) and b).

d) $P(Wrapper = Red)$

$= P(Wrapper = Red \mid S) \cdot P(S)$
$+ P(Wrapper = Red \mid A) \cdot P(A)$

$= 0{,}8 \cdot 0{,}7 + 0{,}1 \cdot 0{,}3 = \underline{\underline{0{,}59}}$

e)

$P(S \mid Shape = Round, Wrapper = Red)$

$$= \frac{P(Shape = Round, Wrapper = Red \mid S) \cdot P(S)}{P(Shape = Round, Wrapper = Red)}$$

$P(Round, Red \mid S) = P(Round \mid S) \cdot P(Red \mid S)$
$= 0{,}8 \cdot 0{,}8 = 0{,}64$

$P(Round, Red \mid A) = P(Round \mid A) \cdot P(Red \mid A)$
$= 0{,}1 \cdot 0{,}1 = 0{,}01$

$$P(\text{Round}, \text{Red}) = P(\text{Round}, \text{Red} \mid S) \cdot P(S) + P(\text{Round}, \text{Red} \mid A) \cdot P(A)$$

$$= 0{,}64 \cdot 0{,}7 + 0{,}01 \cdot 0{,}3$$

$$= 0{,}451$$

$$P(S \mid \text{Shape} = \text{Round}, \text{Wrapper} = \text{Red})$$

$$= \frac{0{,}64 \cdot 0{,}7}{0{,}451} = \underline{\underline{0{,}99334811753}}$$

f) $E[\,] = s \cdot P(S) + a \cdot P(A)$

$$= 0{,}7s + 0{,}3a$$

Exercise 2:

$$U = -e^{-\frac{x}{R}}$$

a) $R = 500 \Rightarrow U(x) = -e^{-\frac{x}{500}}$

Choice 1: $P_{500}(\text{Price}) = 1$, $P_{500}(\text{No Price}) = 0$

2: $P_{5000}(\text{Price}) = 0{,}6$, $P_{5000}(\text{No Price}) = 0{,}4$

Rational → Maximize utility

$$EU\_option1 = U(500) \cdot P_{500}(\text{Price}) + U(0) \cdot P_{500}(\text{No Price})$$

$$= -e^{-\frac{500}{500}} \cdot 1 - 1 \cdot 0 = \underline{-0{,}36789}$$

$$EU\_Lottery = U(5000) \cdot P_{5000}(\text{Price}) + U(0) \cdot P_{5000}(\text{No Price})$$

$$= -e^{-\frac{5000}{500}} \cdot 0{,}6 - e^{0} \cdot 0{,}4$$

$$= -0{,}000027 - 0{,}4 = \underline{-0{,}400027}$$

$EU\_option1 > EU\_Lottery$

⇒ If Mary acts rationally she would choose Option 1 (Receive \$500 guaranteed) b)

Choice 1 ; $P_{100}(\text{Price}) = 1$, $P_{100}(\text{No Price}) = 0$

2 ; $P_{500}(\text{Price}) = 0,5$, $P_{500}(\text{No Price}) = 0,5$

Approximate R such that ;

$$U(100) \cdot P_{100}(\text{Price}) = U(500) \cdot P_{500}(\text{Price}) + U(0) \cdot P_{500}(\text{No Price})$$

$$\Rightarrow -e^{-\frac{100}{R}} = -e^{-\frac{500}{R}} \cdot 0,5 - e^{-\frac{0}{R}} \cdot 0,5$$

$$e^{-\frac{100}{R}} = e^{-\frac{500}{R}} \cdot 0,5 + 0,5$$

$R=100$: LHS $\approx 0,367879$, RHS $\approx 0,503369$

$R=120$: LHS $\approx 0,434598$, RHS $\approx 0,507752$

$R=160$: LHS $\approx 0,535261$, RHS $\approx 0,521968$

$R=150$: LHS $\approx 0,513417$, RHS $\approx 0,517837$

$R=151$: LHS $\approx 0,515689$, RHS $\approx 0,518235$

$R=152$: LHS $\approx 0,517941$, RHS $\approx 0,518637$

$R=153$; LHS $\approx 0,520173$, RHS $\approx 0,519042$

$\Rightarrow$ $R \approx 152$ makes individual indifferent