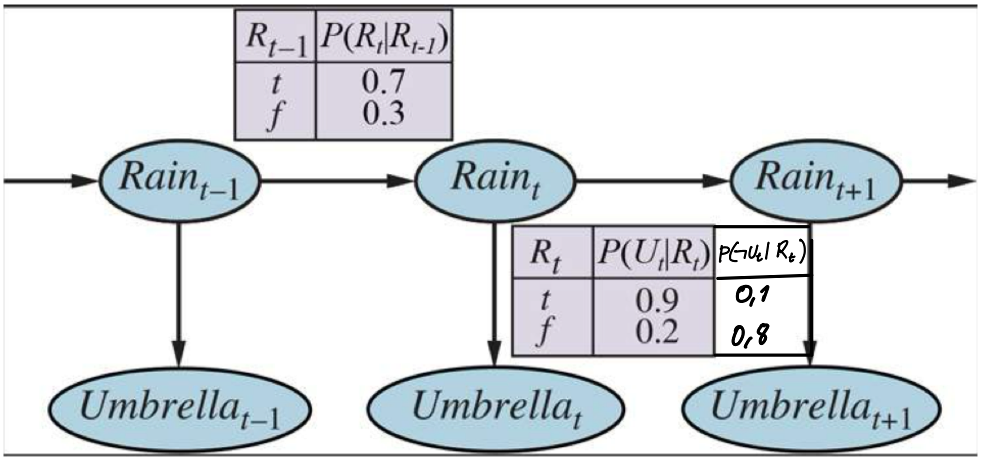
Exercise 1:

Description of the umbrella world as an HMM.



- The set of unobserved variable(s) for a given time-slice t, denoted X_t, represents whether it is raining or not on day t.

- The set of observed variable(s) for a given time slice t, denoted E_t, represents whether someone (the director) carries an umbrella on day t.

Let; r = rainy $\qquad u$ = umbrella

$\neg r$ = not rainy $\qquad \neg u$ = no umbrella

Dynamic model;

$P(X_t | X_{t-1})$

$$= \begin{bmatrix} P(X_t = r \mid X_{t-1} = r) & P(X_t = r \mid X_{t-1} = \neg r) \\ P(X_t = \neg r \mid X_{t-1} = r) & P(X_t = \neg r \mid X_{t-1} = \neg r) \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}7 & 0{,}3 \\ 0{,}3 & 0{,}7 \end{bmatrix}$$

Observation model;

$P(E_t \mid X_t)$

$$= \begin{bmatrix} P(E_t = u \mid X_{t-1} = r) & P(E_t = u \mid X_{t-1} = \neg r) \\ P(E_t = \neg u \mid X_{t-1} = r) & P(E_t = \neg u \mid X_{t-1} = \neg r) \end{bmatrix}$$

- **Encoded Assumptions;**

- The probability of the current state only depends on the previous state (Markov Assumption)
- The transition and observation probabilities stay constant over time. (Stationarity Assumption)
- Initial State is unknown (We need $P(X_0)$)

Reasonableness of Assumptions;

Real world weather systems probably has more complex dynamics, which violates the Markov assumption.
Also, the climate changes over time, which violates the stationarity assumption.

Exercise 2;

I am to implement filtering using the equations;

(14.5)

$$\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{e}_{1:t+1}) = \alpha \mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1}) \sum_{\mathbf{X}_t} \mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t, \mathbf{e}_{1:t}) P(\mathbf{x}_t|\mathbf{e}_{1:t})$$

$$= \alpha \underbrace{\mathbf{P}(\mathbf{e}_{t+1}|\mathbf{X}_{t+1})}_{\text{sensor model}} \sum_{\mathbf{x}_t} \underbrace{\mathbf{P}(\mathbf{X}_{t+1}|\mathbf{x}_t)}_{\text{transition model}} \underbrace{P(\mathbf{x}_t|\mathbf{e}_{1:t})}_{\text{recursion}} \quad \text{(Markov assumption).}$$

(14.12)

$$\mathbf{f}_{1:t+1} = \alpha \mathbf{O}_{t+1} \mathbf{T}^{\top} \mathbf{f}_{1:t}$$

From the attached Assignment3_E2.py file we get;

```
All normalized forward messages with the 5 days worth of observations:
 [array([0.81818182, 0.18181818]), array([0.88335704, 0.11664296]), array([0.3068472, 0.6931528])
, array([0.76719456, 0.23280544]), array([0.87416355, 0.12583645])]
```