Chapter 3

One-Boson-Exchange Model

Shuzo OGAWA,* Shoji SAWADA,** Tamotsu UEDA,† Wataro WATARI,†† and Minoru YONEZAWA*

* *Department of Physics, Hiroshima University, Hiroshima*

** *Department of Physics, Nagoya University, Nagoya*

† *Department of Applied Mathematics, Faculty of Engineering Science Osaka University, Toyonaka*

†† *Research Institute for Atomic Energy Osaka City University, Osaka*

(Received October 22, 1966)

Contents

§1. Introduction

In 1961, a certain model was proposed by the present authors[1),2)] in an attempt to explain nuclear forces by introducing many kinds of mesons on the basis of the Sakata model for elementary particles. The model has been referred to as the "one-boson-exchange model" (OBE model) or the "one-particle-exchange model". This model has presented a new possibility for the realistic understanding of nuclear forces in the intermediate region.

The OBE model proposed by the present authors has the aim of giving a systematic understanding not only of the nucleon-nucleon interaction but of other various strong reactions. The model was motivated by the investigation of strong interactions on the basis of the Sakata model. In this section a *subjective* approach to this model due to the present authors will be given, though the final justification of the model is a future problem. Similar models have been presented by many authors (R. S. McKean (1962),[3)] D. B. Lichtenberg (1962),[4)] R. A. Bryan, C. R. Dismukes and W. Ramsay (1963),[5)] A. Scotti and D. Y. Wong (1963),[6)] etc.) on somewhat different ground.

1·1 *The Sakata model and the one-hadron-exchange model*

The Sakata model,[7)] proposed in 1955, has presented a clear approach to the study of elementary particle physics. In this model, mesons and baryons are considered as composite systems of the fundamental particles: proton (p), neutron (n) and Λ-particle (Λ), and their antiparticles. In 1958, Ogawa[8)] introduced an idea of "full symmetry" among the fundamental particles p, n and Λ, which is a generalization of the concept of the charge independence in the proton-neutron system. In the following year, Ikeda, Ogawa and Ohnuki[9),10)] formulated Ogawa's idea into the $U(3)$ (unitary group of degree three) theory. Sawada and Yonezawa (1960)[11)] reformulated the semiempirical mass formula given by Matumoto (1956)[12)] so as to incorporate the $U(3)$ theory, and investigated the mass levels of the composite system. These works predicted the existence of many resonance levels including the octet mesons now established and seemed to support the basic features of the Sakata model and it became clear from experiment that the wide-spread existence of these resonance levels is one of the fundamental features of the baryon-meson system.

Through these studies, the following idea has been gradually formed. When we discuss a reaction in the baryon-meson system the resonance should itself be taken as a substance of the same level as the meson and baryon. This idea is quite natural in the Sakata model which has been developed in analogy with the theory of the nucleus. All nuclei stand on the same level of substance because they consist of nucleons. Similarly, in the Sakata model, the resonance states as well as baryon and meson are composed of the fundamental particles: p, n and Λ. This viewpoint has been strengthened again by the recent development of the composite model.*)

The picture of resonances given by the composite model, however, contrasted with the widely accepted interpretation of resonances at that time. In the static meson theory developed by Chew and Low (1956),[17] the nucleon and the pion are assumed to be the fundamental entities and the 3-3 resonance is ascribed to the higher order (or dynamical) effect of the Yukawa interaction between pions and nucleons. Such a theory, however, seems not to have been very successful in giving a systematic explanation of other resonance levels. The existence of such resonance levels was rather naturally anticipated in the composite model.

From analysis of pion production processes, etc., by the isobar model[18] in which the resonance states are regarded as particles, the following points were particularly noted: The treatment of a resonance as an "elementary" particle has considerable applicability and the non-correlated final states play only a minor role.

What does this imply when we consider the dynamics of strong interactions? In the composite theory, the strong interactions should be derived from the fundamental interactions between the fundamental particles, and the Yukawa interaction observed between pion and nucleon is regarded as a "model" Hamiltonian (effective Hamiltonian) which results from the projection of the fundamental interactions including all the higher order corrections on the system of composite particles. This model Hamiltonian may have validity in the region where the structure of the elementary particle (i.e. the composite particle) is not effective.

Higher order effect should not be taken for the Yukawa interaction as they stand in the usual field theory because such effects should be considered in the fundamental interaction, not for the model Hamiltonian. The Yukawa interaction, as a model Hamiltonian, will already contain some of the higher order effects of the fundamental interaction in a certain correlated form. In such a situation, the higher order effects of the model

---

*) In the original Sakata model, p, n and Λ are the constituent particles (fundamental particles) as well as observed baryons and the full equality among the baryons and mesons might be questioned. In the subsequent develoment of the composite model, however, it has become clear that the octet classification[13] is better for the baryon. This has necessitated a revision of the original Sakata model: Various models such as the $U(4)$ theory,[14] the quartet model,[15] the quark mcdel,[16] etc., have been proposed in which the equality of all the baryons and mesons including resonance states is more obvious.

Hamiltonian may have less importance.

A strong supporting argument for the necessity of taking into account the higher order effects of the Yukawa interaction might be given by the success of the Chew-Low theory[17] for the low energy p-wave pion-nucleon scattering. The thing essential for the explanation of the pion-nucleon reaction in the low energy, however, may be considered to be the introduction of the 3-3 resonance[19),*)] or the strong pion-nucleon correlation in the corresponding state, and the necessity of non-correlating higher order correction is not clear. As stated before, the non-correlating final states seem to play a minor role in the production process. Is it unreasonable to consider this fact as an indication that the mechanism of forming a one-particle-exchange mode takes precedence over the higher order effects of the Yukawa interaction?

This idea has been supported by the fact that the usual perturbation theoretical treatment using the Yukawa interaction has not brought unambiguous success in explaining the strong interaction phenomena except for the outer region of the nuclear force where the one-pion-exchange contribution is dominant.

To embody this idea in the study of reaction due to the strong interaction we have proposed the following model.

The hadron reaction is assumed to be well determined by the lowest order process[**)] in the sense of Feynman diagrams. The diagram is composed of vertices and lines which correspond to baryons, mesons and resonance states. This model has been referred to as the one-boson-exchange model (OBE model); this name originates in nucleon-nucleon scattering, and it should be called more generally the one-hadron-exchange model. Of course we cannot logically exclude the possibility that there may remain some parts of the higher order effects of the fundamental interactions not represented by the lowest order diagram and that the higher order of the model interaction might represent such parts. However, we are interested in the OBE model as a zeroth order trial.

Here we compare the model with the reciprocal bootstrap model[20] which is one of the fashionable theories of elementary particles at present. In both models all particles (including resonances) are regarded as standing on the same level. Between the two models, however, there is the essential difference that in the OBE model we assume the existence of fundamental entities and fundamental interactions behind the elementary particles and Yukawa interactions, and hence expect the model to have limitations, while the bootstrap model aims at a self-consistent ultimate theory. The

*) If we introduce the particles corresponding to the 3-3 resonance, etc., *a priori*, then we have to avoid the occurrence of the resonance of "dynamical" origin unless we assume a self-consistent mechanism like that of the bootstrap model. This also led us to ignore the higher order effects of the Yukawa interaction.

**) Here the lowest order means that the diagram involves no hadron closed polygon.

prospect of a more fundamental structure is based on the recognition that there exist endless strata in the structure of matter: molecule→atom→nucleus→elementary particle→fundamental particle→⋯. This point was particularly emphasized by Sakata when he proposed his composite theory of elementary particles.

1·2 *Nuclear force and the one-boson-exchange model*

Now we look at the development of Yukawa's meson theory. Since Taketani, Nakamura and Sasaki (1951)[21] proposed a method of studying nuclear forces, one of the most important achievements along this idea is the quantitative establishment of the one-pion-exchange effect (See Iwadare, Otsuki, Tamagaki and Watari (1956)[22]). It was shown that the one-pion exchange potential (OPEP) is valid in the outer region ($r \gtrsim 1.5\mu_\pi^{-1}$; r is the two nucleon distance and μ_π^{-1} is the pion Compton wave length) of the nucleon-nucleon interaction. They further examined to what extent the intermediate region ($0.7\mu_\pi^{-1} \lesssim r \lesssim 1.5\mu_\pi^{-1}$) of the nuclear force can be explained by the two-pion-exchange potential (TPEP). Taketani, Machida and Ohnuma (1952)[23] derived the TPEP called the TMO potential. Konuma, Miyazawa and Otsuki (1958)[24] calculated the TPEP including the effect of the 3-3 resonance (the KMO potential). The TPEP was calculated also by many other people.[25] In these papers it was shown that the TPEP has a large effect than the OPEP in the intermediate region, and therefore it was hoped that the TPEP would qualitatively explain the major part of the nuclear force in this region. As experimental data at high energies ($E_{\rm lab} \sim 300$ MeV) were accumulated, the existence of a strong LS force in the triplet odd state became apparent; this was difficult to expalin by the TPEP. Hoshizaki and Machida (1962)[26] derived a TPEP which fully includes nonstatic effects. The LS force obtained was, however, too weak compared with the one indicated by experiment.

In the OBE model we assume that the effect of exchange of matter in the form of one boson takes precedence over that in the form of two, three,⋯ uncorrelated pions.*) The large LS force should be understood in terms of the exchange of one composite system (meson) other than pion, but not in terms of higher order effects of the Yukawa interactions (TPEP, etc.). An allowed exchange mode will be provided by the composite theory.

In the history of meson theory, it was already known at the early stage that the LS force can be derived from the exchange of a vector meson or a scalar meson.[29] After evidence for a strong LS force in the intermediate

*) Here we note the difference between the OBE model and the mixed field theory[27] of nuclear forces of past days. In the mixed field theory, the important motive was to eliminate the $1/r^3$ singularity of the nulear potential, which makes the Schrödinger equation insoluble. From the present viewpoint it is meaningless to take up this problem, as was discussed in detail in the previous review article on nuclear forces.[21),28)]

region was found, attempts were made by many authors to attribute this force to bosons othert han the pion. Gupta (1959)[30] considered a neutral scalar meson, Sakurai(1960)[31] and Breit (1960)[32] considered a neutral vector meson and Fujii (1961)[33] introduced the effect of an $I=J=1$ pion-pion resonance state on the nuclear force. These attempts were confined to some special aspects of the nuclear force and have not contributed much things to the comprehensive understanding of the problem.

On the basis of the consideration stated above a new investigation of the nuclear force was begun. In 1961, Hoshizaki, Otsuki, Watari and Yonezawa[1] published the result of their analysis of "phenomenological" nuclear potentials in terms of the contributions from the exchange of mesons in addition to the pion. In their analysis the following two possible cases were examined:

$$\text{(I)} \quad \text{OPEP} + \sum \text{OBEP}$$

$$\text{(II)} \quad \text{OPEP} + \sum \text{OBEP} + \text{TPEP}$$

where $\sum$OBEP denoted the sum of several kinds of one-boson-exchange potentials. Case (I) corresponds to the OBE model and case (II) stands for a more conservative possibility. Their analysis determined the types and properties of the bosons which were required for the nuclear force and clarified the necessity for the existence of the ρ meson and the ω meson before their experimental confirmation. The model of case (I) is referred to as the one-boson-exchange-potential model (OBEP model).

Some parts of the higher order effects of the model Hamiltonian are included in the OBEP model. The potential approach suffers from the defect that it is nonrelativistic and is restricted to problems like that of the nuclear force at low energy. The model which uses a dispersion relation where the contribution from the "left-hand cut" is given by the one-boson-exchange Born amplitudes and that from the "right-hand cut" is given by the iteration of the Born term, is also appropriately classified into the OBEP model by its physical content. In this case, the above mentioned defect does not appear.

As the OBEP model involves some part of the higher order effects, we can consider a more simplified model. This is the one-boson-exchange-contribution model (OBEC model), where the higher order effects of the Yukawa interaction are completely excluded except for damping effects. This is the simplest OBE model and allows us to give a fully relativistic treatment of the problem quite easily. It is also applicable to a wider range of strong reactions such as pion-nucleon scattering and production processes. This model is particularly attractive for its simplicity. The first analysis of the strong interactions by the OBEC model was made on proton-proton scattering below the threshold for pion production ($\lesssim$300 MeV) by Sawada, Ueda, Watari and Yonezawa (1962)[2] and on low energy pion-nucleon scattering by Kikugawa (1964).[34] The OBEC model can be extended to the phenomena in the inelastic region. A preliminary analysis of pion production in proton-proton collisions was made by Ueda (1963)[35] and a more complete analysis satisfying the unitarity requirements has been made by Kikugawa and the present authors.[36]

§2. The one-boson-exchange-contribution model

In this section we give an analysis of nucleon-nucleon scattering by the one-boson-exchange-contribution model.

2·1 *Assumptions of the one-hadron-exchange-contribution model*

Here we summarize the assumptions of the one-hadron-exchange model.

(I) The dynamical behaviour of the reactions of hadrons is determined from the matrix elements corresponding to the lowest order Feynman diagrams with no hadron closed loops in them.

(II) In (I), the hadron lines in the Feynman diagrams designate unstable resonances as well as stable baryons and mesons.

(III) The higher order contributions are omitted except for damping effects, which are necessary from the unitarity requirement.

The one-hadron-exchange model which is based on these assumptions will be called the one-hadron-exchange-contribution model or the one-boson-exchange-contribution model with particular reference to nucleon-nucleon scattering.

2·2 *Calculation of the one-boson-exchange contribution**)

The application of this one-hadron-exchange-contribution model to low energy nucleon-nucleon scattering below the pion production threshold requires us to consider the contributions from the diagram in Fig. 2·1.

In Fig. 2·1, B denotes a meson. The meson which can make contribution to nucleon-nucleon scattering has strangeness zero and isospin $I=0$ or 1. The effective Yukawa interactions of the meson with the nucleon are

$$g_S\bar{\psi}\psi\phi \qquad \text{for a scalar meson,}$$

$$g_P\bar{\psi}i\gamma_5\psi\phi+\frac{f_P}{M}\bar{\psi}i\gamma_5\gamma_\mu\psi\partial_\mu\phi$$

for a pseudoscalar meson,

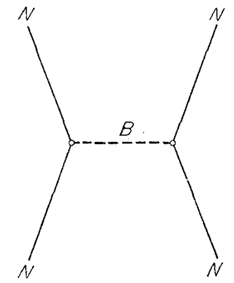

Fig. 2·1. The one-boson-exchange diagram of the nucleon-nucleon scattering.

*) For more details, see also the General Appendix.

$$g_V\bar{\psi}i\gamma_\mu\psi\phi_\mu+\frac{f_V}{2M}\bar{\psi}\sigma_{\mu\nu}\psi F_{\mu\nu} \qquad \text{for a vector meson,} \qquad (2\cdot1)$$

$$g_A\bar{\psi}i\gamma_5\gamma_\mu\psi\phi_\mu \text{ or } \frac{f_A}{2M}\bar{\psi}i\gamma_5\sigma_{\mu\nu}\psi F_{\mu\nu} \qquad \text{for an axial vector meson,}$$

$$\frac{g_T}{2M}\{\bar{\psi}\gamma_\mu\partial_\nu\psi-\partial_\nu\bar{\psi}\gamma_\mu\psi\}\phi_{\mu\nu}+\frac{f_T}{M^2}\partial_\mu\bar{\psi}\partial_\nu\psi\phi_{\mu\nu} \quad \text{for a tensor meson.}$$

Here M is the nucleon mass and is introduced to make the coupling constants dimensionless, and ψ and $\phi(\phi_\mu, \phi_{\mu\nu})$ describe the nucleon and meson with spin 0 (1, 2) respectively, and

$$F_{\mu\nu}=\partial_\mu\phi_\nu-\partial_\nu\phi_\mu\,,$$

$$\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)\,.$$

In Eq. (2·1) we have assumed several invariance principles usually taken for strong reactions such as time reversal, space inversion, charge conjugation, etc., and coupling constants can take real values. We use the following representation for Dirac's γ matrices:

$$\gamma_k=\begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0\end{pmatrix}\ (k=1,2,3),\ \gamma_4=i\gamma_0=\begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix},\ \gamma_5=\begin{pmatrix} 0 & -1 \\ -1 & 0\end{pmatrix}.$$

The nucleon-nucleon scattering process is described by a scattering matrix in nucleon spin space $M(\theta, \varphi)$.[37] This matrix is defined by stationary state wave function for particular spin state a:

$$\begin{aligned}&\Psi_a(\theta,\varphi,r)\xrightarrow[r\to\infty]{}e^{ipr}\chi_a+f_a(\theta,\varphi)e^{ipr}/r,\\ &f_a(\theta,\varphi)=\sum_b M_{ab}(\theta,\varphi)\chi_b\,.\end{aligned} \qquad (2\cdot2)$$

Here χ_a and χ_b are the amplitudes of spin states a and b and $\boldsymbol{p}$ is incident momentum, and $f_a(\theta, \varphi)$ is the scattering amplitude for the spin state a.

The M matrix is related with the S matrix in the center of mass system of two nucleons as

$$(S-1)_{fi}=i(2\pi)^4\delta^4(p_1'+p_2'-p_1-p_2)\left(\frac{4\pi}{E}\right)M_{fi}\,,$$

where p_1 and p_2 (p_1' and p_2') are four momenta of initial (final) nucleons and E is the total energy of a nucleon in the center of mass system.*)

For the contribution from one-boson-exchange diagrams shown in Fig. 2·1, the M matrix elements are expressed as

*) The M matrix corresponds to A (p_1', p_2'; p_1, p_2) used in Chapter 4, which is obtained multiplying M_{fi} by a factor E/M^2.

$$M_{r's'rs}(\boldsymbol{p}',\boldsymbol{p})$$
$$=\frac{M^2}{4\pi E}\sum_B\{\bar{u}_{r'}^{(1)}(\boldsymbol{p}')\bar{u}_{s'}^{(2)}(-\boldsymbol{p}')\Gamma_B u_r^{(1)}(\boldsymbol{p})u_s^{(2)}(-\boldsymbol{p})\}/\{(\boldsymbol{p}-\boldsymbol{p}')^2+\mu_B^2\}. \quad (2\cdot3)$$

Here the summation $\sum_B$ is carried out with respect to the mesons under consideration. In Eq. $(2\cdot3)$ μ_B is the meson mass, and $u_r^{(1)}(\boldsymbol{p})$ is the positive energy Dirac spinor representing the initial nucleon 1 whose momentum is $\boldsymbol{p}$:

$$u_r^{(1)}(\boldsymbol{p})=\left(\frac{E+M}{2M}\right)^{1/2}\begin{pmatrix}\chi_r^{(1)}\\ \dfrac{\boldsymbol{\sigma p}}{E+M}\chi_r^{(1)}\end{pmatrix}. \quad (2\cdot4)$$

The adjoint spinor of $u_{r'}^{(1)}(\boldsymbol{p}')$ is denoted by $\bar{u}_{r'}^{(1)}(\boldsymbol{p}')$ which represents the nucleon 1 after scattering with momentum $\boldsymbol{p}'$:

$$\bar{u}_{r'}^{(1)}(\boldsymbol{p}')=u_{r'}^{*(1)}(\boldsymbol{p}')\gamma_4=\left(\frac{E+M}{2M}\right)^{1/2}\left[\chi_{r'}^{(1)}\quad \frac{-\boldsymbol{\sigma p}'}{E+M}\chi_{r'}^{(1)}\right]. \quad (2\cdot5)$$

Subscripts r, etc., stand for $+$and$-$representing spin up state $\chi_+^{(1)}=\begin{pmatrix}1\\0\end{pmatrix}$ and spin down state $\chi_-^{(1)}=\begin{pmatrix}0\\1\end{pmatrix}$ respectively and $\boldsymbol{\sigma}$ is Pauli's spin matrix. The quantities with superscript (1) and (2) indicate that they are concerned with nucleon 1 and 2 respectively. The quantities Γ_B in Eq. $(2\cdot3)$ are matrices involving Dirac's γ matrices and are given in Table $2\cdot1$. In Table $2\cdot1$, $\mathcal{P}_j(j=1,2,\cdots,5)$ are five independent "perturbative invariants" defined as follows:[38]

$$\mathcal{P}_1=\mathbf{1}^{(1)}\mathbf{1}^{(2)},$$
$$\mathcal{P}_2=i\gamma_\mu^{(1)}P_\mu^{(2)}+i\gamma_\mu^{(2)}P_\mu^{(1)},$$

Table $2\cdot1$.

scalar meson	$\Gamma_S=g_S^2\mathcal{P}_1$
pseudoscalar meson	$\Gamma_P=-G_P^2\mathcal{P}_5$
vector meson	$\Gamma_V=-G_V^2\mathcal{P}_4+(G_Vf_V/M)\mathcal{P}_2+4(f_V/M)^2P_\mu^{(1)}P_\mu^{(2)}\mathcal{P}_1$
axial vector meson	$\Gamma_A=4(g_A/M)^2\{\mathcal{P}_3+P_\mu^{(1)}P_\mu^{(2)}\mathcal{P}_4$
with axial vector coupling	$+(M/\mu_A)^2(Q_\mu^2-\mu_A^2)\mathcal{P}_5\}/Q_\mu^2$
with pseudotensor coupling	$\Gamma_{PT}=4(f_A/M)^2P_\mu^{(1)}P_\mu^{(2)}\mathcal{P}_5$
tensor meson	$\Gamma_T=(g_T/M)^2\{P_\mu^{(1)}P_\mu^{(2)}\mathcal{P}_4+(2M/\mu_T)^2P_\mu^{(1)}P_\mu^{(2)}\mathcal{P}_1+\mathcal{P}_3\}$
	$+(f_T/M^2)^2\{(P_\mu^{(1)}P_\mu^{(2)})^2-\frac{1}{3}P_\mu^{(1)2}P_\mu^{(2)2}\}\mathcal{P}_1$

$$\mathscr{P}_3 = i\gamma_\mu^{(1)} P_\mu^{(2)} i\gamma_\nu^{(2)} P_\nu^{(1)}, \qquad (2\cdot6)^{*)}$$

$$\mathscr{P}_4 = \gamma_\mu^{(1)} \gamma_\mu^{(2)},$$

$$\mathscr{P}_5 = \gamma_5^{(1)} \gamma_5^{(2)}.$$

In Table 2·1 and Eqs. (2·6), the following abbreviations are used:

$$P_\mu^{(1)} = \frac{1}{2}(p'_{1\mu} + p_{1\mu}),$$

$$P_\mu^{(2)} = \frac{1}{2}(p'_{2\mu} + p_{2\mu}),$$

$$Q_\mu = (p'_{1\mu} - p_{1\mu}) = (p_{2\mu} - p'_{2\mu}), \qquad (2\cdot7)$$

$$G_P = g_P + 2f_P$$

and

$$G_V = g_V + 2f_V.$$

Using Eqs. (2·4) and (2·5), the M matrix (2·3) can be written in the form

$$\begin{aligned} M(\boldsymbol{p}', \boldsymbol{p}) = {} & G_0(\boldsymbol{p}', \boldsymbol{p}) + i(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)})\boldsymbol{n} G_1(\boldsymbol{p}', \boldsymbol{p}) \\ & + (\boldsymbol{\sigma}^{(1)}\boldsymbol{l})(\boldsymbol{\sigma}^{(2)}\boldsymbol{l}) G_2(\boldsymbol{p}', \boldsymbol{p}) + (\boldsymbol{\sigma}^{(1)}\boldsymbol{m})(\boldsymbol{\sigma}^{(2)}\boldsymbol{m}) G_3(\boldsymbol{p}', \boldsymbol{p}) \\ & + (\boldsymbol{\sigma}^{(1)}\boldsymbol{n})(\boldsymbol{\sigma}^{(2)}\boldsymbol{n}) G_4(\boldsymbol{p}', \boldsymbol{p}) + (\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}) G_5(\boldsymbol{p}', \boldsymbol{p}) \qquad (2\cdot8)^{**)} \end{aligned}$$

where $G_i(\boldsymbol{p}', \boldsymbol{p})$ are scalar functions of $\boldsymbol{p}$ and $\boldsymbol{p}'$, and $\boldsymbol{l}$, $\boldsymbol{m}$ and $\boldsymbol{n}$ are unit orthogonal vectors:

$$\boldsymbol{l} = \frac{\boldsymbol{p} + \boldsymbol{p}'}{|\boldsymbol{p} + \boldsymbol{p}'|}, \quad \boldsymbol{m} = \frac{\boldsymbol{p} - \boldsymbol{p}'}{|\boldsymbol{p} - \boldsymbol{p}'|} \quad \text{and} \quad \boldsymbol{n} = \boldsymbol{l} \times \boldsymbol{m}.$$

In Eq. (2·8) only five terms are linearly independent because of the relation

$$\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(3)} = (\boldsymbol{\sigma}^{(1)}\boldsymbol{l})(\boldsymbol{\sigma}^{(2)}\boldsymbol{l}) + (\boldsymbol{\sigma}^{(1)}\boldsymbol{m})(\boldsymbol{\sigma}^{(2)}\boldsymbol{m}) + (\boldsymbol{\sigma}^{(1)}\boldsymbol{n})(\boldsymbol{\sigma}^{(2)}\boldsymbol{n}). \qquad (2\cdot9)$$

The M matrix elements in the singlet-triplet representation, $M_{ij}(\theta, \varphi)$, are related with the $G_j(\boldsymbol{p}', \boldsymbol{p})$ by formula given in Table 2·2. In the singlet-triplet representation, the subscripts 1, 0 and -1 refer to the triplet states

$$\chi_1 = \chi_+^{(1)} \chi_+^{(2)},$$

$$\chi_0 = (\chi_+^{(1)} \chi_-^{(2)} + \chi_-^{(1)} \chi_+^{(2)}) / \sqrt{2}$$

and

$$\chi_{-1} = \chi_-^{(1)} \chi_-^{(2)}$$

respectively, and s to the singlet state

---

*) The invariants $\mathscr{P}_j$ defined by Eqs. (2·6) are connected with P_j given by Eq. (3·6) in Chapter 4 as $P_j = u^{(1)}(\boldsymbol{p}')u^{(2)}(-\boldsymbol{p}')\mathscr{P}_j u^{(1)}(\boldsymbol{p})u^{(2)}(-\boldsymbol{p})$ $(j=1, 2, .., 5)$.

**) See the General Appendix.

Table 2·2.

$$M_{11}(\theta,\varphi)=M_{-1-1}(\theta,-\varphi)=G_0(\boldsymbol{p}',\boldsymbol{p})+\frac{1}{2}\{(1+\cos\theta)G_2(\boldsymbol{p}'\boldsymbol{p})+(1-\cos\theta)G_3(\boldsymbol{p}',\boldsymbol{p})\}+G_5(\boldsymbol{p}',\boldsymbol{p})$$

$$M_{00}(\theta,\varphi)=G_0(\boldsymbol{p}',\boldsymbol{p})-\{G_2(\boldsymbol{p}',\boldsymbol{p})-G_3(\boldsymbol{p}',\boldsymbol{p})\}\cos\theta+G_4(\boldsymbol{p}',\boldsymbol{p})+G_5(\boldsymbol{p}',\boldsymbol{p})$$

$$M_{10}(\theta,\varphi)=-M_{-10}(\theta,-\varphi)=\sqrt{2}\,e^{-i\varphi}\left\{G_1(\boldsymbol{p}',\boldsymbol{p})+\frac{1}{2}\sin\theta\, G_2(\boldsymbol{p}',\boldsymbol{p})-\frac{1}{2}\sin\theta\, G_3(\boldsymbol{p}',\boldsymbol{p})\right\}$$

$$M_{01}(\theta,\varphi)=-M_{0-1}(\theta,-\varphi)=\sqrt{2}\,e^{i\varphi}\left\{-G_1(\boldsymbol{p}',\boldsymbol{p})+\frac{1}{2}\sin\theta\, G_2(\boldsymbol{p}',\boldsymbol{p})-\frac{1}{2}\sin\theta\, G_3(\boldsymbol{p}',\boldsymbol{p})\right\}$$

$$\begin{aligned}M_{1-1}(\theta,-\varphi)=M_{-11}(\theta,\varphi)=\frac{1}{2}e^{-2i\varphi}\{&-G_4(\boldsymbol{p}',\boldsymbol{p})\\&+(1-\cos\theta)G_2(\boldsymbol{p}',\boldsymbol{p})+(1+\cos\theta)G_3(\boldsymbol{p}',\boldsymbol{p})\}\end{aligned}$$

$$M_{ss}(\theta,\varphi)=G_0(\boldsymbol{p}',\boldsymbol{p})-G_2(\boldsymbol{p}',\boldsymbol{p})-G_3(\boldsymbol{p}',\boldsymbol{p})-G_4(\boldsymbol{p}',\boldsymbol{p})-3G_5(\boldsymbol{p}',\boldsymbol{p})$$

$$\chi_s=(\chi_+^{(1)}\chi_-^{(2)}-\chi_-^{(1)}\chi_+^{(2)})/\sqrt{2}\,.$$

The θ and φ are the scattering angles. The explicit form of $M_{ij}(\theta,\varphi)$ calculated for the one-boson-exchange contribution is given in Appendix A.

In order to obtain the partial wave transition amplitudes, $M_{ij}(\theta,\varphi)$ are separated into the contributions from the various states with orbital angular momentum l:

$$M_{ij}(\theta,\varphi)=\sum_l M_{ijl}Y_l^m(\theta,\varphi), \tag{2·10}$$

where $Y_l^m(\theta,\varphi)$ is the spherical harmonic function and m is the difference of the z-components of spin of the two-nucleon system between final and initial states.

The partial wave transition matrix T_J, in the state of total angular momentum J, is related to the S matrix by

$$S_J=1+iT_J\,. \tag{2·11}$$

A useful representation of T_J is

$$iT_J=\begin{pmatrix}\alpha_l & 0 & 0 & 0\\ 0 & \alpha_{l=J-1,J} & 0 & \alpha^J\\ 0 & 0 & \alpha_{l=J,J} & 0\\ 0 & \alpha^J & 0 & \alpha_{l=J+1,J}\end{pmatrix} \tag{2·12}$$

here

α_l : transition amplitude between singlet states with orbital angular momentum $l=J$,

$\alpha_{l,J}$: transition amplitude between triplet states with total angular momentum J and orbital angular momentum l,

α^J : transition amplitude between triplet states with total angular momentum J and $\Delta l=\pm 2$ $(l=J\pm 1\rightarrow l=J\mp 1)$.

Now the partial wave transition amplitudes are related to M_{ijl} by equations given in Table 2·3.

Table 2·3.

$$\alpha_l=\frac{ip}{2\sqrt{\pi(2l+1)}}M_{ssl}$$

$$\alpha_{l,l-1}=\frac{ip}{2\sqrt{\pi(2l+1)}}\frac{1}{2l-1}$$
$$\times\left[(l-1)M_{11l}+lM_{00l}-\left\{\frac{2(l+1)!}{(l-1)!}\right\}^{1/2}\left\{\frac{l-1}{l}M_{01l}+M_{10l}\right\}+\frac{1}{l}\left\{\frac{(l+2)!}{(l-2)!}\right\}^{1/2}M_{-11l}\right]$$

$$\alpha_{l,l}=\frac{ip}{2\sqrt{\pi(2l+1)}}\frac{1}{l(l+1)}\left[l(l+1)M_{11l}-\left\{\frac{2(l+1)!}{(l-1)!}\right\}^{1/2}M_{01l}-\left\{\frac{(l+2)!}{(l-2)!}\right\}^{1/2}M_{-11l}\right]$$

$$\alpha_{l,l+1}=\frac{ip}{2\sqrt{\pi(2l+1)}}\frac{1}{2l+3}\left[(l+2)M_{11l}+\left\{\frac{2(l+1)!}{(l-1)!}\right\}^{1/2}\left\{\frac{l+2}{l+1}M_{01l}+M_{10l}\right\}\right.$$
$$\left.+(l+1)M_{00l}+\frac{1}{l+1}\left\{\frac{(l+2)!}{(l-2)!}\right\}^{1/2}M_{-11l}\right]$$

$$\alpha^{J=l\pm 1}=\frac{ip}{2\sqrt{\pi(2l+1)}}\frac{\sqrt{J(J+1)}}{2J+1}$$
$$\times\left[M_{11l}-M_{00l}-\frac{1}{2}\{1\pm(2J+1)\}\left\{\frac{2(l+1)!}{(l-1)!}\right\}^{1/2}\{M_{01l}-M_{10l}\}-\left\{\frac{(l+2)!}{(l-2)!}\right\}^{1/2}M_{-11l}\right]$$

The partial wave transition matrix for the lowest order diagram T_J is obtained from M_{ijl} in Eq. (2·10) by use of Table 2·3. In general, the T_J (α) thus calculated are pure real (imaginary) quantities and we denote them by T_J^B (α^B). Explicit form of the α^B for the various one-boson-exchange contributions are given in Appendix B.

The T_J^B or the α^B do not, in general, satisfy the unitarity requirement. The elastic unitarity requirement, that α must satisfy, is expressed in terms of nuclear bar phase shifts ${}^1\delta_l$ and ${}^3\delta_{l,J}$ and coupling parameter ρ_J as

$$\alpha_l=\exp(2i\,{}^1\delta_l)-1 \tag{2·13}$$

for singlet states,

$$\alpha_{l=J,J}=\exp(2i\,{}^3\delta_{l=J,J})-1 \tag{2·14}$$

for uncoupled triplet states, and

$$\begin{pmatrix}\alpha_{l=J-1,J} & \alpha^J\\ \alpha^J & \alpha_{l=J+1,J}\end{pmatrix}$$
$$=\begin{pmatrix}\sqrt{1-\rho_J^2}\exp(2i\,{}^3\delta_{l=J-1,J})-1 & i\rho_J\exp(i\,{}^3\delta_{l=J-1,J}+i\,{}^3\delta_{l=J+1,J})\\ i\rho_J\exp(i\,{}^3\delta_{l=J-1,J}+i\,{}^3\delta_{l=J+1,J}) & \sqrt{1-\rho_J^2}\exp(2i\,{}^3\delta_{l=J+1,J})-1\end{pmatrix} \tag{2·15}$$

for coupled triplet states.

It is clear from Eqs. (2·13), (2·14) and (2·15) that the unitarity requirement cannot be satisfied with real T_J^B or pure imaginary α^B except for trivial case of $\alpha^B=0$.

It is, however, possible to satisfy this requirement if we take into account the damping effect which arises from the rescattering on the energy shell (the K-matrix method). This effect is introduced by regarding the real transition matrix T_J^B as the K matrix:

$$S_J=\left(1+\frac{i}{2}T_J^B\right)\Big/\left(1-\frac{i}{2}T_J^B\right). \tag{2·16}$$

In terms of the phase shifts, Eq. (2·16) implies, instead of Eqs. (2·13), (2·14) and (2·15),

$$\tan{}^1\delta_l=\alpha_l^B/2i \tag{2·17}$$

for the singlet states,

$$\tan{}^3\delta_{l,J}=\alpha_{l=J,J}^B/2i \tag{2·18}$$

for the uncoupled triplet states, and

$$\begin{pmatrix} (1-\rho_J^2)^{1/2}\exp(2i\,{}^3\delta_{l=J-1,J}) & i\rho_J\exp i({}^3\delta_{l=J-1,J}+{}^3\delta_{l=J+1,J}) \\ i\rho_J\exp i({}^3\delta_{l=J-1,J}+{}^3\delta_{l=J+1,J}) & (1-\rho_J^2)^{1/2}\exp(2i\,{}^3\delta_{l=J+1,J}) \end{pmatrix}$$

$$=\frac{1}{D}\begin{pmatrix} 1+\left(\dfrac{\alpha^{BJ}}{2}\right)^2-\dfrac{1}{4}\alpha_{l=J+1,J}^B\alpha_{l=J-1,J}^B-\dfrac{1}{2}(\alpha_{l=J+1,J}^B-\alpha_{l=J-1,J}^B) & \alpha^{BJ} \\ \alpha^{BJ} & 1+\left(\dfrac{\alpha^{BJ}}{2}\right)^2-\dfrac{1}{4}\alpha_{l=J+1,J}^B\alpha_{l=J-1,J}^B+\dfrac{1}{2}(\alpha_{l=J+1,J}^B-\alpha_{l=J-1,J}^B) \end{pmatrix} \tag{2·19}$$

for the coupled triplet states where

$$D=1-\frac{1}{2}(\alpha_{l=J+1,J}^B+\alpha_{l=J-1,J}^B)+\frac{1}{4}\alpha_{l=J+1,J}^B\alpha_{l=J-1,J}^B-\frac{1}{4}(\alpha^{BJ})^2. \tag{2·20}$$

For simplicity's sake, we have not considered the isospin of the particles so far. Under the assumption of the charge independence, the interaction Hamiltonians of an $I=0$ meson are given by Eqs. (2·1). For an $I=1$ meson, ϕ should be replaced by $\boldsymbol{\tau}\boldsymbol{\phi}$ where $\boldsymbol{\tau}$ are the isospin matrices which operate the nucleon field $\psi=\begin{pmatrix}\psi_p\\ \psi_n\end{pmatrix}$ and $\boldsymbol{\phi}$ is an isovector field. Therefore, for the exchange of an $I=0$ meson the above expressions of the α do not alter. For the contribution of an $I=1$ meson exchange, the α should be multiplied by a factor $\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)}$. This implies only the replacement of the coupling constants of the $I=1$ meson g^2, etc., by $-3g^2$, etc. for $T=0$ nucleon-nucleon scattering amplitudes. By using Eqs. (2·17), (2·18) and (2·19), phase shifts and coupling parameters of the one-boson-exchange contribution which are compared with experimental data are calculated.

2·3 *Comparison of the one-boson-exchange-contribution model with the experimental data*

In 1961 when the analysis of the proton-proton scattering by the OBEC model was begun, there was only very poor experimental evidence for meson resonance states. Sawada, Ueda, Watari and Yonezawa (SUWY) assumed that the pion and heavy mesons having mass $2\mu_\pi \sim 6\mu_\pi$ (μ_π being pion mass) will give significant contributions to nucleon-nucleon scattering below the pion production threshold, and that the masses of these heavy mesons may effectively be represented by $3\mu_\pi$ or $4\mu_\pi$. At that time an almost unique set of phase shifts was determined by the modified phase shift analysis, and this provided experimental data for the investigation of the OBEC model. (See Chapter 5.)

When the model was compared with experiment the S states were treated completely phenomenologically. The reason for this was, above all, that mesons with mass higher than those considered in the analysis, which were supposed to exist from the composite theory, have a considerable effect on the S states. Also from the viewpoint of the composite theory the phenomena related to S states would not be independent of the structure of the particle. In fitting the experimental phase shifts with the one-boson-exchange contribution, weight should be put on the higher angular momentum states, since the interaction in these states is mostly determined by the long range part of the nuclear force. Unfortunately, however, these higher wave phase shifts are generally quite small and subject to more ambiguity than the lower partial wave phase shifts. Hence, mainly the P-wave and D-wave phase shifts were used to determine the parameters of the one-boson-exchange contributions. SUWY have shown that the experimental set of phase shifts for proton-proton scattering up to 300 MeV is satisfactorily reproduced if we consider the contribution from a scalar meson and a vector meson besides the pion.

Later their analysis was extended to include the $T=0$ states which made it possible to discuss the isospin of the mesons.[39] In this study they considered an $I=0$ and an $I=1$ scalar meson with arbitrary mass and an $I=1$ heavy pseudoscalar meson with the η-meson mass besides the pion, the η meson, the ω meson and the ρ meson with their observed spin, parity and mass. Results of the analyses are as follows: (1) The experimental data on nucleon-nucleon scattering below the pion-production threshold can be explained by the exchange of the $I=0$ vector meson (ω meson), $I=1$ vector meson (ρ meson), and an $I=0$ scalar meson in addition to the pion. (2) The contribution of the pseudoscalar mesons (η meson, etc.) is not positively required. (3) An $I=0$ scalar meson is necessary and we do not need the contribution from an $I=1$ scalar meson. The mass of the scalar meson has to be $400 \sim 700$ MeV.

The phase shifts and coupling parameters calculated by the one-boson-exchange-contribution model are shown in Figs. 2·2~2·7 with experimental ones.

Ino, Hiroshige, Matsuda and Sawada examined[40] the effect on the nuclear force of the f meson whose spin parity is 2^+, and showed that the introduction of the f meson makes some improvement on the $^3P_0$ phase shift and on the ratio of the vector coupling constant and the tensor coupling constant of the ρ meson, g_ρ/f_ρ, so that they became consistent with the

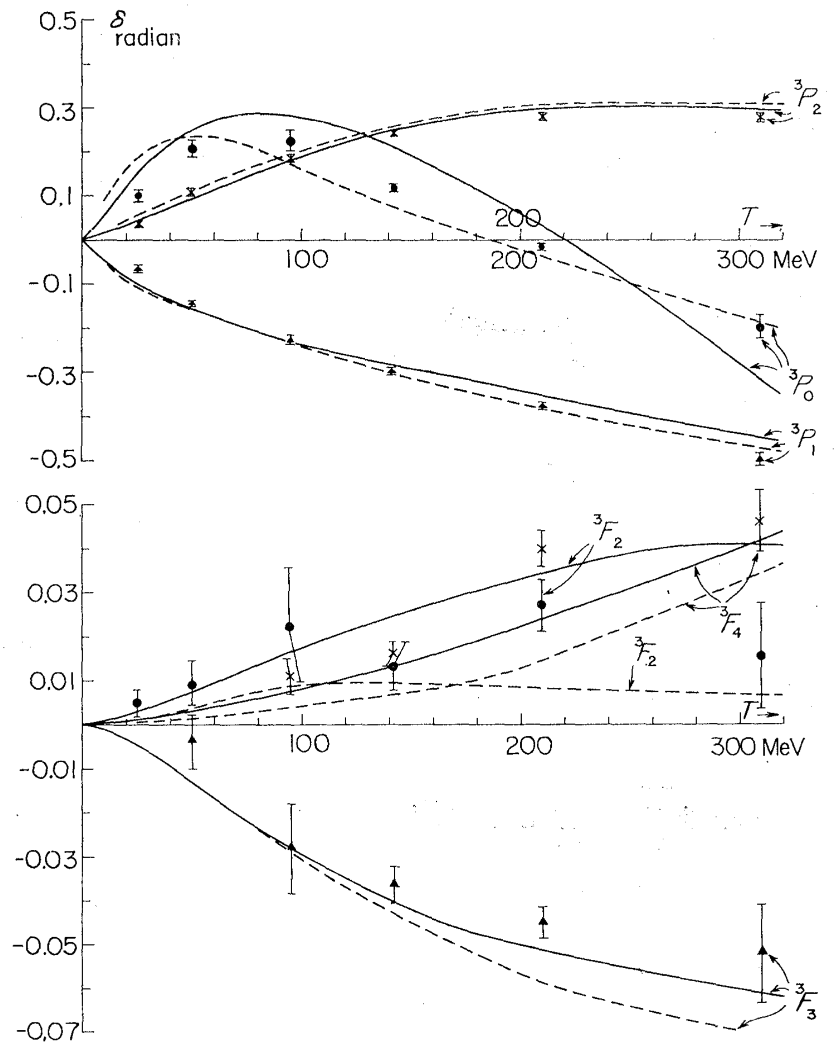

Fig. 2·2. The $T=1$ nuclear bar phase shifts in the triplet odd states (full curves) calculated by the OBEC model. The following parameters are used in Figs. 2·2~2·7:

$G_\pi^2/4\pi=14.4$, $g_\omega^2/4\pi=10.95$, $g_\omega f_\omega/4\pi=0.68$, $f_\omega^2/4\pi=0.04$,
$g_\rho^2/4\pi=1.62$, $g_\rho f_\rho/4\pi=-3.26$, $f_\rho^2/4\pi=6.51$, $g_S^2/4\pi=13.1$,
$\mu_\pi=140$ MeV, $\mu_\omega=\mu_\rho=750$ MeV and $\mu_S=600$ MeV.

Experimental curves (dotted curves) and points are taken from references 73) and 74) respectively.

electromagnetic form factors. A summary of the results, including the coupling constants, is given in Tables 4·1～4·3 in §4.

To be consistent with the assumption (I) of the one-hadron-exchange model we neglect the mass width effect for the unstable mesons. In some analyses by Scotti and Wong[6] such an effect has been taken into account for the ρ meson by assuming an effective mass which is different from the observed one. A preliminary examination of the effect was made by Furuichi, Sawada, Ueda, Watari and Yonezawa.[41] Though the effect is dependent of assumed form of mass distribution, it is possible to define an effective mass if we restrict our consideration to the nucleon-nucleon scattering in elastic region. For the ρ meson of mass 750 MeV with width 100 MeV the effective mass～700 MeV is obtained which is somewhat larger than that of Scotti and Wong (590 MeV).

It will be appropriate here to refer to the works by Ramsay[42] and by Bryan and Arndt[43] and by Köpp and Krammer[44] which have some connection with the calculation using the one-boson-exchange-contribution model. Ramsay tried to determine meson-nucleon coupling constants in the three pole model (the pion, a scalar meson and a vector meson, assuming only vector coupling for the vector meson) from the higher wave proton-proton scattering phase shifts ($^3F_3$, $^3F_4$, $^1G_4$ and $^3H_4$) at 310 MeV. His idea is to avoid the ambiguity due to the short range interaction which is large for the lower angular momentum states. Though this is the best way in principle for determination of the coupling constants, as stated before, the higher wave phase shifts are small and always suffer more ambiguity.

Bryan and Arndt examined a model in which the real part of the scattering amplitude is approximated by the real one-boson-exchange contribution (the real part model). Their intention is to confirm a guess that the essential contribution to the success of the one-boson-exchange model results from the introduction of three types of meson, and that elaborate unitariza-

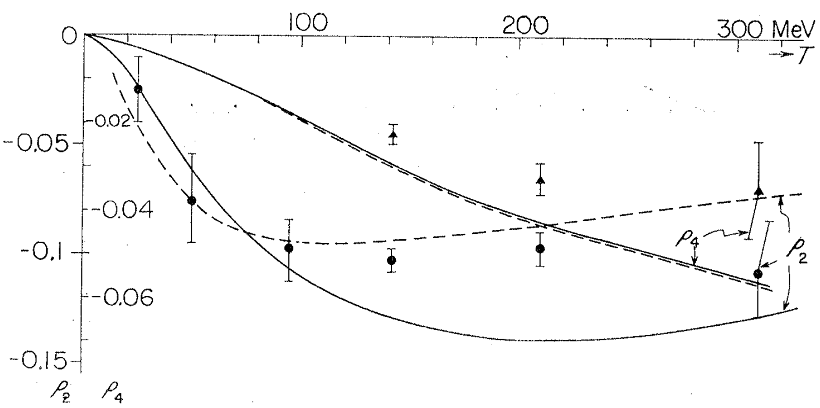

Fig. 2·3. The calculated $T=1$ coupling parameters.

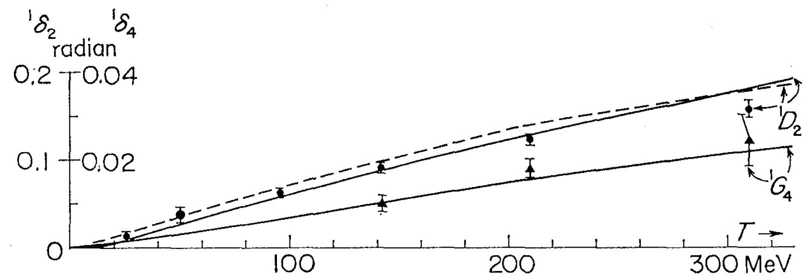

Fig. 2·4. The calculated $T=1$ nuclear phase shifts in the singlet even states.

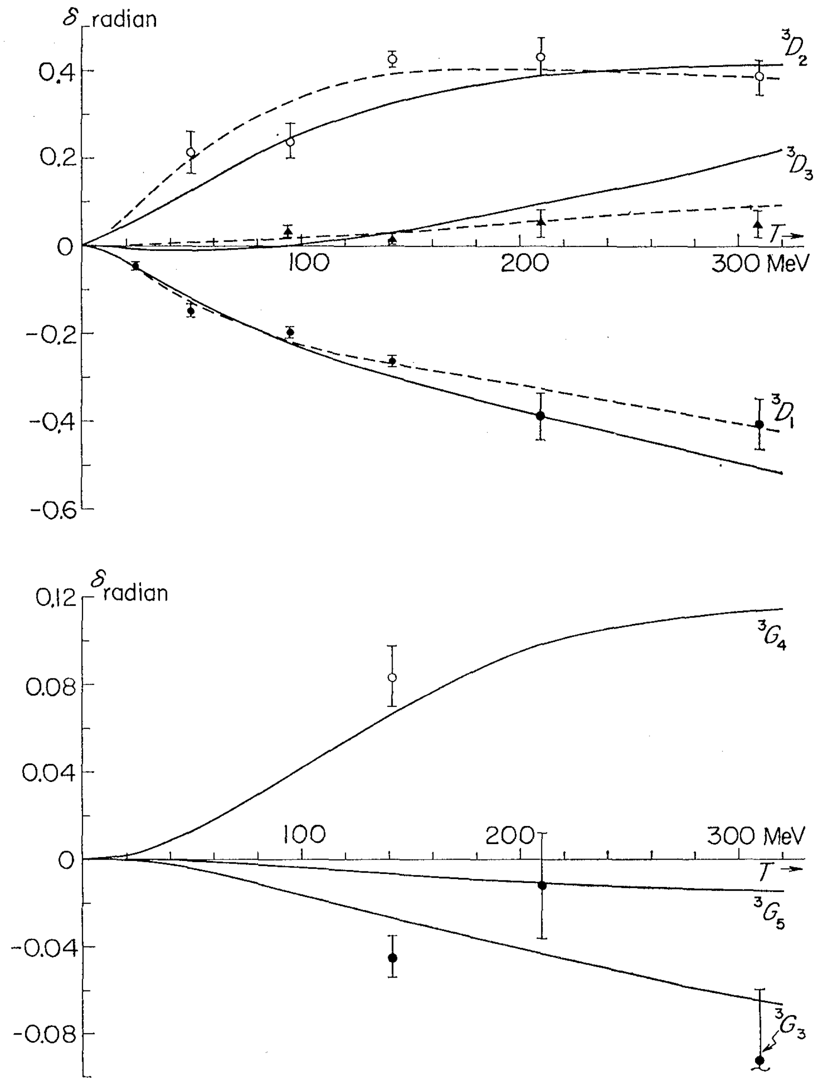

Fig. 2·5. The calculated $T=0$ nuclear bar phase shifts in the triplet even states.

tion methods such as the Schrödinger equation or a dispersion relation, are immaterial. The model is essentially the same as the one-boson-exchange-contribution model for small phase shifts since the former model assumes (for an uncoupled state)

$$\sin 2\delta_l = \alpha_l^B / i, \tag{2·21}$$

while the latter model assumes

$$\sin 2\delta_l = \frac{\alpha_l^B / i}{1 + (\alpha_l^B / 2i)^2}. \tag{2·22}$$

They obtained, however, a somewhat small value for the mass of the scalar meson (~380 MeV) compared with the results of the one-boson-exchange-contribution model (500~700 MeV). We suspect that this is due to the damping effect, i.e. the denominator in (2·22), which is not negligible for

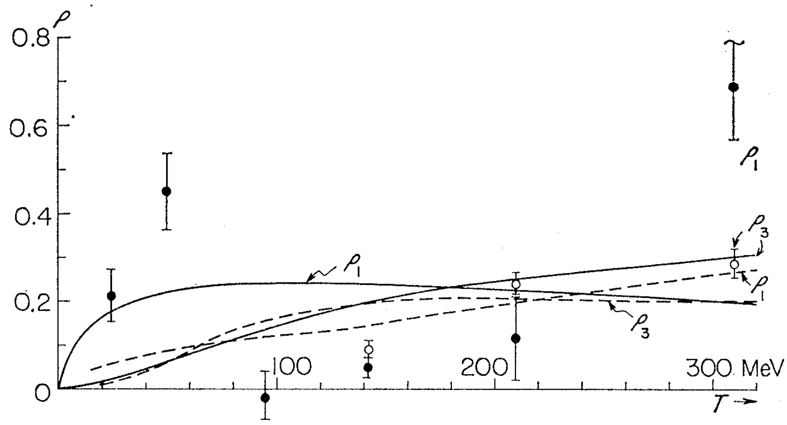

Fig. 2·6. The calculated $T=0$ coupling parameters.

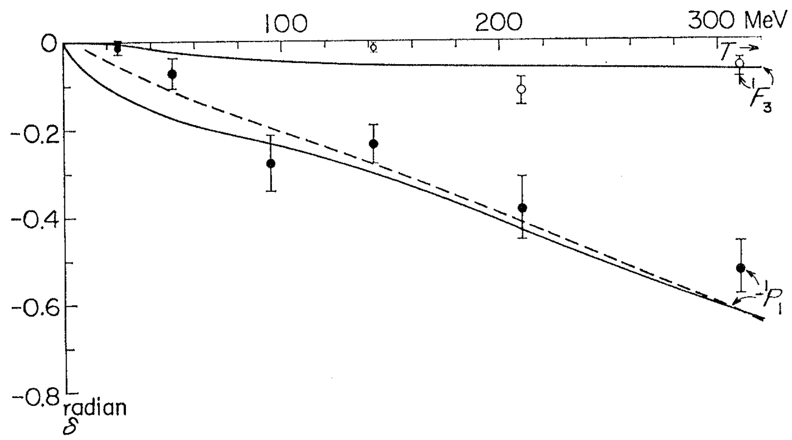

Fig. 2·7. The calculated $T=0$ nuclear phase shifts in the singlet odd states.

the P and D waves. It is obvious from (2·21) that the real part model cannot always be consistent with the unitarity requirements.

Köpp and Krammer recently attempted to determine the coupling constants of the mesons in the one-boson-exchange model with the pion, the η meson, the ρ meson, the ω meson, the φ meson and the ϵ meson from the small phase shifts $J \geq 2$, for the purpose of checking the consistency of the predictions of unitary symmetry theories for the coupling constants. They used the experimental masses for the mesons and the ratio of vector/tensor coupling constants for the ω and ρ mesons determined from the electromagnetic form factors of the nucleon. They found that an acceptable solution for these small phase shifts can be obtained if an additional scalar meson (σ meson) of relatively small mass ($\sim$400 MeV) is introduced. This is not necessarily inconsistent with the results of the OBEC model found by Sawada, Ueda, Watari and Yonezawa.[38] The P waves were also taken into account in the latter analysis, while in the former only $J \geq 2$ waves are considered. It is to be noted that the use of the $^3P_2$, $^1D_2$ and $^3D_2$ phase shifts by Köpp and Krammer does not seem to be really justified, in the light of the necessity of avoiding the uncertainty associated with rescattering effects.

Usually, in the modified phase shift analysis, high angular momentum states (e.g. $l > 6$ at 300 MeV) are approximated by the one-pion-exchange contribution and the lower phase shifts are determined as free parameters. In view of the non-negligible contributions to high angular momentum states from mesons other than the pion, it is desirable to make a "re-modified" phase shift analysis in order to obtain a more reliable phase shift solution.

The results for the mesons are discussed in §4 in connection with the experimental evidence for the mesons.

2·4 *Non-static effects involved in the one-boson-exchange contribution*

It was expected that the non-static effects would become important as the energy increased. The experimental evidence for the LS force gave a strong motivation for introducing a heavy meson contribution to the nucleon-nucleon interaction. Here it is interesting to discuss the non-static effects involved in the solution of the one-boson-exchange model.[45] For this purpose it will be convenient to express the transition matrix of the one-boson-exchange contribution as*)

*) In Eq. (2·23) only five terms are independent as in Eq. (2·8). This is not the case, however, when energy is not conserved between the initial and final two nucleon states, and the most general potential in momentum space can be expressed by six independent terms which correspond to various terms in Eq. (2·23). In order to discuss the relation with the potential we separate the transition matrix of the one-boson exchange into six terms in disregard of (2·28). For a general discussion of non-static forces see Chapter 2,

$$T(\boldsymbol{p}',\boldsymbol{p})=\sum_B\Big[A_0+\frac{i(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\boldsymbol{k}\times\boldsymbol{q}}{\mu_B^2}A_1+\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{k})(\boldsymbol{\sigma}^{(2)}\boldsymbol{k})}{\mu_B^2}A_2$$

$$+\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\boldsymbol{q})}{\mu_B^2}A_3+\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{k}\times\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\boldsymbol{k}\times\boldsymbol{q})}{\mu_B^4}A_4+(\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)})A_5\Big]\frac{-pE}{\boldsymbol{k}^2+\mu_B^2},\tag{2·23}$$

where $\boldsymbol{k}=\boldsymbol{p}'-\boldsymbol{p}$, $\boldsymbol{q}=(1/2)(\boldsymbol{p}+\boldsymbol{p}')$, and the A_i are scalar functions of $\boldsymbol{p}$ and $\boldsymbol{p}'$. The A_0, A_5 and A_2 terms, which give the central force, the spin-spin force and the tensor force plus the spin-spin force respectively, can be further expanded with respect to $\boldsymbol{q}^2/M^2$ and $\boldsymbol{k}^2/M^2$:

$$A_i(\boldsymbol{p}',\boldsymbol{p})=a_{i0}+[a_{i1}(\boldsymbol{k}^2/M^2)+\cdots]+[b_{i1}(\boldsymbol{q}^2/M^2)+\cdots].\tag{2·24}$$

This can be rewritten as

$$A_i(\boldsymbol{p}',\boldsymbol{p})=a'_{i0}+[a'_{i1}(\mu_B^2/M^2)+\cdots]+[b_{i1}(p^2/M^2)+\cdots].\tag{2·25}$$

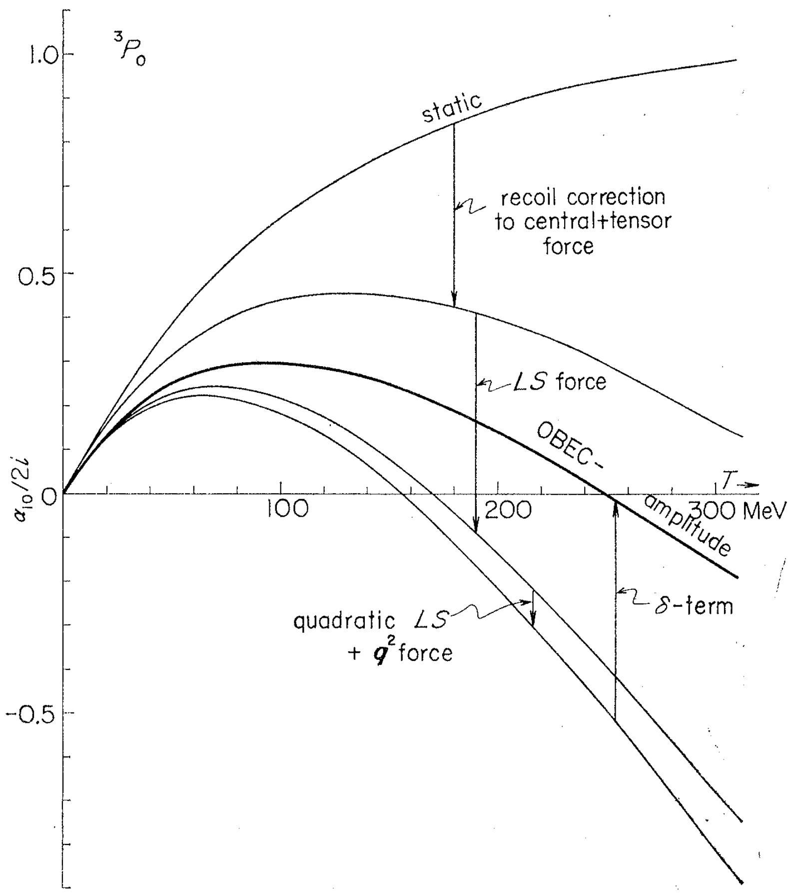

Fig. 2·8. The non-static effects involved in the partial transition amplitudes of the one-boson-exchange contribution.

where the relations*)

$$\boldsymbol{k}^2 = -\mu_B^2 \quad \text{and} \quad \boldsymbol{q}^2 = \mu_B^2/4 + p^2 \tag{2·26}$$

have been used. The constant terms a_{i0} give a static central force (including a spin-spin force) and a static tensor force. The terms in the second bracket in (2·25) give energy-independent recoil corrections. The third terms cause energy-dependent non-static effects.

The A_1, A_3 and A_4 terms give the LS force, the q-square force and the quadratic LS force respectively.

In Figs. 2·8~2·10, we illustrate the decomposition of the amplitudes into the contribution from (1) the static parts of the central+tensor force, (2)

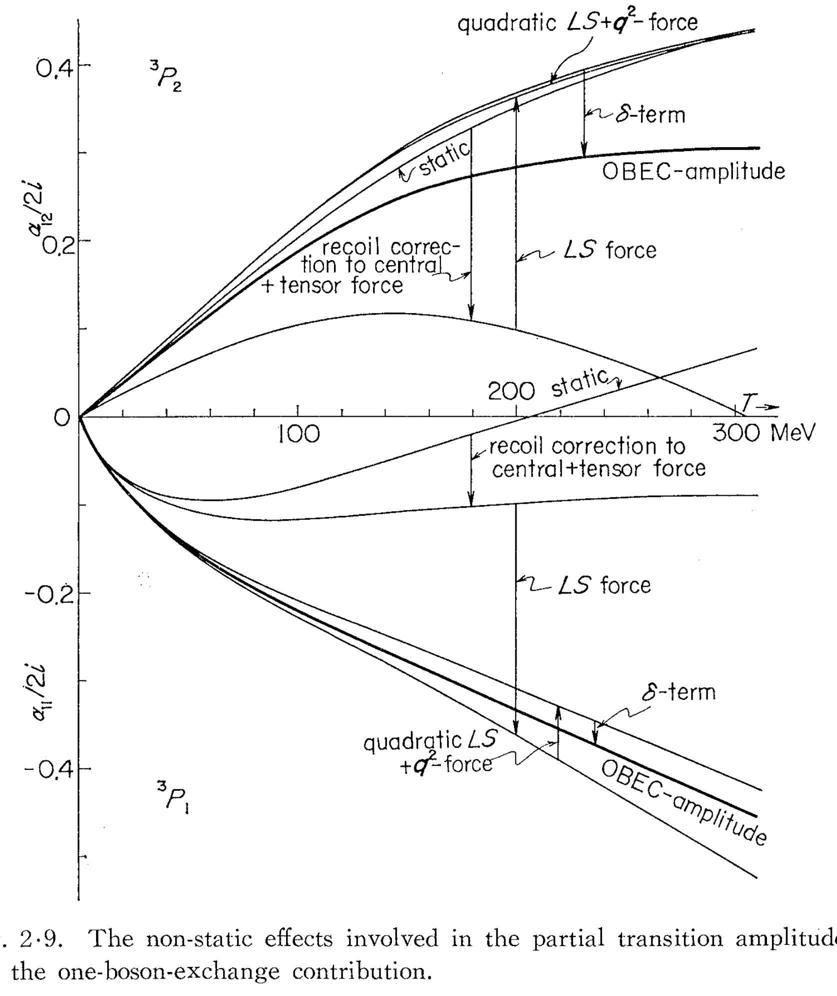

Fig. 2·9. The non-static effects involved in the partial transition amplitudes of the one-boson-exchange contribution.

*) When (2·23) is transformed into configuration space, a term $(\boldsymbol{k}^2/M^2)^n$ is transformed as

$$\int (\boldsymbol{k}^2/M^2)^n \frac{1}{\boldsymbol{k}^2+\mu_B^2} e^{i\boldsymbol{k}\boldsymbol{r}} d^3\boldsymbol{k} = (-\mu_B^2/M^2)^n \frac{e^{-\mu_B r}}{r}$$

$$+(\delta(\boldsymbol{r}) \text{ function or/and its derivatives}).$$

In (2·25) we omitted $\delta(\boldsymbol{r})$ function and its derivatives as in the usual derivation of potentials.

recoil corrections to the central + tensor force, (3) the LS force, (4) the quadratic LS force + q^2 force and (5) δ terms, for several states. The δ terms come from contact interactions whose Fourier transforms into the coordinate space are singular potentials such as $\delta(\boldsymbol{r})$. The contributions from these contact interactions are particulary important in the S and $^3P_0$ states even in the low energy region ($\sim$50 MeV). Some qualitative features of the non-static effects involved in the one-boson-exchange contribution are summarized in Table 2·4.

There is some apparent difference with regard to the non-static effects between the one-boson-exchange contribution and the phenomenological potential. As stated in Chapter 1, a quadratic LS force and L^2 force ($W_{12}V_W$-term and L^2V_{LL}-term) have been introduced in the phenomenological potential analysis. The reason is to reduce the amplitudes of $^1D_2$, $^1G_4$, $^3D_2$ and $^3F_4$ states at high energy[46),47),48)] ($\gtrsim$200 MeV). Though the contribu-

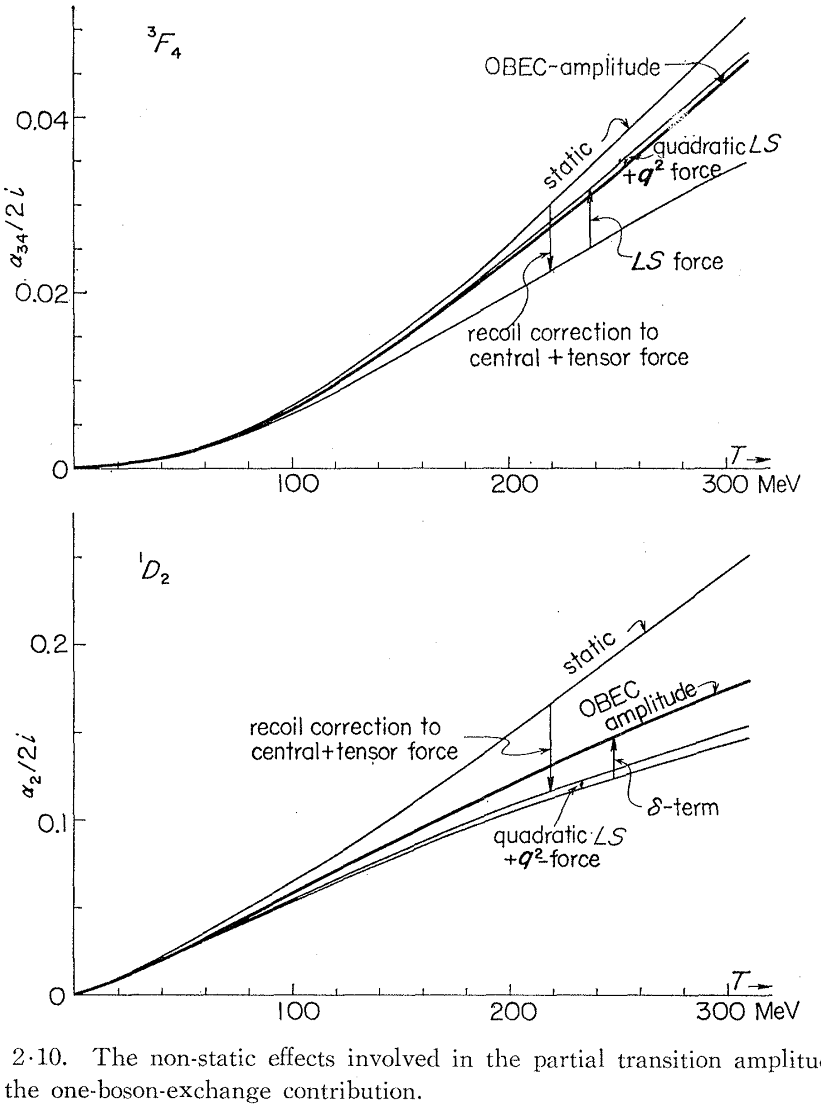

Fig. 2·10. The non-static effects involved in the partial transition amplitudes of the one-boson-exchange contribution.

Table 2·4. Non-static Effects in the OBEC Model

		^1E-states	^3O-states
LS force $\frac{i(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\boldsymbol{k}\times\boldsymbol{q}}{\mu_B^2}A_1$		—	LS splitting in ^3F-states smaller than that given by the potential model
quadratic LS force $\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{k}\times\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\boldsymbol{k}\times\boldsymbol{q})}{\mu_B^2}A_2$		~ 0	very weak
q-square force $\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{q})(\boldsymbol{\sigma}^{(2)}\boldsymbol{q})}{\mu_B^2}A_3$		~ 0	very weak
non-static correction to central and tensor force	momentum independent recoil effects, μ_B/M, $\mu_B^2/M^2,\cdots$	large in $^1S_0$ state	large in ^3P states
	momentum dependent effect, $p^2/M^2,\cdots$	important for all waves when the energy goes over 200 MeV	

tion of the quadratic LS force is very small in the OBEC model, the fit of this model to the experimental phase shifts is good. (See Figs. 2·2 and 2·4). This is due to the non-static corrections to the central and tensor forces. Thus, as stated in Chapter 1, the $W_{12}V_W$ and L^2V_{LL} terms can be regarded as effective substitutes for non-static corrections. Another interesting result is that the A_1 term which generates the LS force is much weaker in the ^3F states than that given by calculations based on the potential model (cf. Chapter 1). This may be taken as the reason why we can obtain a good fit to the $^3F_4$ phase shifts with a very weak quadratic LS force in the OBEC model.

§3. The one-boson-exchange potential model

In this section we summarize works based on the one-boson-exchange model which utilize the non-relativistic Schrödinger equation or the dispersion relation to generate unitarity. In these models the off-energy-shell rescattering effects, or the contributions from the right-hand cut, are added to the one-boson-exchange amplitudes. It is clear that these modifications have little effect on the small phase shifts. The necessity of such modifications to the OBEC model will be discussed in §6.

A detailed discussion of the dispersion theoretic version of the OBE model is given by Furuichi in Chapter 4, §6 and here we mainly discuss the results of the potential approach based on the Schrödinger equation.

3·1 *Approach using the potential*

There are two ways of treating the OBEP model of nucleon-nucleon scattering. One is an indirect way in which the successful phenomenological two nucleon potentials are analyzed in terms of the one-boson-exchange potentials and the other is to calculate the phase shifts by directly solving the Schrödinger equation with the one-boson-exchange potential.

The first systematic analysis was undertaken by Hoshizaki, Otsuki, Watari and Yonezawa (HOWY).[1] They examined whether the difference between the phenomenological nuclear potentials and the pion theoretical potential in the range larger than about $0.5\,\mu_\pi^{-1}$ can be understood in terms of the one-boson-exchange potential (OBEP). They did not consider the interaction in the inner region, especially "hard core", which they thought to be explained by more complicated mechanism. They considered two possibilities:

(I) The first is the one-boson-exchange potential model. Exchange of two or more pions does not take place in contradiction to what would be expected from the conventional field theory with the Yukawa interaction. Then in this case

$$\text{phenomenological potential} = \text{OPEP} + \sum \text{OBEP}$$

is assumed, where the OPEP is written separately to stress the fact that it had already been well established.

(II) The second is a more conservative model: the OPEP, the TPEP (particularly the non-correlating two-pion-exchange potential), etc., are added to the one-boson-exchange potentials. That is,

$$\text{phenomenogical potential} = \text{OPEP} + \text{TPEP} + \cdots + \sum \text{OBEP}.$$

They used the Taketani-Machida-Ohnuma (TMO)[23] potentials and the Konuma-Miyazawa-Otsuki (KMO)[24] potentials, neglecting three and more pion exchange potentials and two and more boson exchange potentials. It was found that for both cases the $I=0$ and $I=1$ vector mesons are necessary besides the pion to reproduce Hamada's potentials[46] in the intermediate region. In addition, an $I=0$ scalar meson is needed for the case (I). (The existence of other types of meson is not necessarily excluded.)

While the investigation by Hoshizaki, Otsuki, Watari and Yonezawa was much stimulated by the fact that the composite particle model predicts the existence of many new meson states, there were published similar papers which were mostly initiated by the experimental discovery of new meson resonance states.

McKean[3] examined the effects of two vector mesons (the ρ meson with mass $4.5\mu_\pi$ and ω meson with mass $3\mu_\pi$) on the nucleon-nucleon potentials using resonance parameters obtained from the nucleon electromagnetic form factors. He found that the qualitative features of the central, spin-orbit, and tensor potentials in all states can be reproduced except that the central repulsive core in his model has too long a range. He hoped that the last point would be improved if we consider the contribution of the exchange of an uncorrelated $T=0$, $J=0$ two pion state or the ABC-particle and take the observed mass $5.6\mu_\pi$ for the ω meson. Lichtenberg[4] interpreted the nucleon-nucleon interaction by assuming contributions from the pion, the η meson, the ρ meson, the ω meson and the ζ meson and pointed out the important role of the ζ meson if it is an $I=1$ scalar meson. Babikov[49] analysed the phenomenological Hamada-Johnston[47] potential in terms of a OBEP due to the ρ meson, the ω meson and an $I=0$ scalar meson with mass $2.5\mu_\pi$, simultaneously attempting to explain the hard core by the repulsive force due to the ω meson. This was qualitatively attained by taking a large coupling constant for the ω meson: $g_\omega^2=12$.

Very extensive analysis by potential calculations was carried out by Bryan, Dismukes and Ramsay and by Bryan and Scott. Bryan, Dismukes and Ramsay[5] (BDR) analysed the low energy proton-proton scattering by taking into account recoil effects containing μ_B/M and its powers. They showed that the Gammel-Thaler potential[50] can be reproduced by the pion, the ρ meson, the ω meson and a scalar meson with mass $3.5\mu_\pi$ to $5\mu_\pi$, taking $f_\omega^2=f_\rho^2=0$. For the determination of the coupling constants they calculated the phase shifts solving the Schrödinger equation with zero cutoff at internucleon distance about $0.35\mu_\pi^{-1}$. Bryan and Scott[51] (BS) analysed low energy nucleon-nucleon scattering following the work of Bryan, Dismukes and Ramsay. They considered the η meson and an $I=1$ scalar meson in addition to those mesons considered in reference 5). The mass of the $I=1$ scalar meson was 770 MeV and that of the $I=0$ scalar meson was 560 MeV in good agreement with the phenomenological potentials. The necessity of the $I=1$ scalar meson (more generally 0^+, 2^+, etc.) is a special feature of the results. The arguments by Bryan and Scott are based on the fact that there is a strong attractive interaction in the isovector part of the central potential deduced from the Hamada-Johnston potentials[47] and the Yale potentials.[52] This strong attractive short range interaction, however, cannot be considered very definite, since it depends sensitively on the assumed shape of the phenomenological potentials, i.e. the force range and singularity.*) In this connection we think that an extensive analysis using the potentials which are represented by a superposition of some OBE potentials is necessary.

3·2 *The one-boson-exchange potential*

The one-boson-exchange potentials were calculated by Hoshizaki, Lin

*) Moreover the coupling constant of this meson is effectively 1/5~1/10 of that of the $I=0$ scalar meson, and such a small contribution is within the uncertainty of the analyses and there will be no serious inconsistency between the results of Bryan and Scott and others.

and Machida[53] for spin zero and unity. They are given for the interaction Hamiltonian (2·1) as follows:*)

$$V_S(\boldsymbol{r})=\frac{\mu_S g_S^2}{4\pi}\left[-\left(1-\frac{\mu_S^2}{8M^2}\right)^2 Y(\mu_S r)-\frac{\mu_S^2}{2M^2}\left(1-\frac{\mu_S^2}{8M^2}\right)\boldsymbol{LS}Z(\mu_S r)\right],$$

$$V_P(\boldsymbol{r})=\frac{\mu_P F_P^2}{4\pi}\frac{1}{3}\left[\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\ Y(\mu_P r)+S_{12}X(\mu_P r)\right],$$

where

$$F_P=f_P+\frac{\mu_P}{2M}g_P=\frac{\mu_P}{2M}G_P.$$

$$\begin{aligned}V_V(\boldsymbol{r})=&\frac{\mu_V g_V^2}{4\pi}\left[\left(1+\frac{\mu_V^2}{2M^2}+\frac{\mu_V^4}{64M^4}+\frac{\mu_V^2}{6M^2}\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\right)Y(\mu_V r)\right.\\&\left.-\frac{\mu_V^2}{12M^2}S_{12}X(\mu_V r)-\frac{\mu_V^2}{2M^2}\left(3+\frac{\mu_V^2}{8M^2}\right)\boldsymbol{LS}Z(\mu_V r)\right]\\&+\frac{\mu_V g_V f_V}{4\pi}\left[\left(\frac{\mu_V}{M}+\frac{\mu_V^3}{4M^3}+\frac{2\mu_V}{3M}\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\right)Y(\mu_V r)\right.\\&\left.-\frac{1}{3}\frac{\mu_V}{M}S_{12}X(\mu_V r)-\frac{4\mu_V}{M}\left(1+\frac{\mu_V^2}{4M^2}\right)\boldsymbol{LS}Z(\mu_V r)\right]\\&+\frac{\mu_V f_V^2}{4\pi}\left[\left\{\frac{\mu_V^2}{4M^2}+\frac{\mu_V^4}{64M^4}+\frac{2}{3}\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\left(1+\frac{\mu_V^2}{8M^2}\right)^2\right\}Y(\mu_V r)\right.\\&\left.-\frac{1}{3}\left(1+\frac{\mu_V^2}{8M^2}\right)^2 S_{12}X(\mu_V r)-\frac{\mu_V^2}{2M^2}\left(3+\frac{\mu_V^2}{8M^2}\right)\boldsymbol{LS}Z(\mu_V r)\right],\end{aligned}\tag{3·1}$$

$$\begin{aligned}V_{AV}(\boldsymbol{r})=&\frac{\mu_A g_A^2}{4\pi}\left[-\frac{2}{3}\left(1-\frac{\mu_A^2}{4M^2}\right)\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}Y(\mu_A r)\right.\\&\left.+\frac{1}{3}\left(1-\frac{\mu_A^2}{4M^2}\right)S_{12}X(\mu_A r)-\frac{\mu_A^2}{2M^2}\boldsymbol{LS}Z(\mu_A r)\right],\end{aligned}$$

$$V_{AT}(\boldsymbol{r})=-\frac{\mu_A f_A^2}{4\pi}\frac{1}{3}\left[\left(1+\frac{\mu_A^2}{4M^2}\right)\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}Y(\mu_A r)+\left(1+\frac{\mu_A^2}{4M^2}\right)S_{12}X(\mu_A r)\right].$$

Here

$$\begin{aligned}&Y(x)=e^{-x}/x,\\&X(x)=(1+3/x+3/x^2)\,Y(x),\\&Z(x)=(1/x+1/x^2)\,Y(x).\end{aligned}\tag{3·2}$$

In (3·1), S_{12} and $\boldsymbol{LS}$ are the usual tensor and spin-orbit operators:

$$S_{12}=3\frac{(\boldsymbol{\sigma}^{(1)}\boldsymbol{r})(\boldsymbol{\sigma}^{(2)}\boldsymbol{r})}{r^2}-\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}\tag{3·3}$$

and

$$\boldsymbol{LS}=\frac{i}{2}\boldsymbol{r}\times\boldsymbol{\nabla}(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)}),$$

*) The potentials listed here are obtained from those given by Hoshizaki, Lin and Machida by replacing $\boldsymbol{q}^2/M^2=\boldsymbol{p}^2/M^2-\boldsymbol{k}^2/4M^2$ by $\mu_B^2/4M^2$, which is the Born approximation value when p^2/M^2 is neglected.

where $\boldsymbol{\sigma}^{(1)}$ and $\boldsymbol{\sigma}^{(2)}$ are Pauli's spin matrices for nucleons 1 and 2, $\boldsymbol{r}$ is the relative coordinate of two nucleons and $r=|\boldsymbol{r}|$. Hoshizaki, Otsuki, Watari and Yonezawa examined the OBEP model by taking only the leading terms in the above potentials and tried to reproduce Hamada's phenomenological potentials[46] as "experimental" data.

In Figs. 3·1∼3·3 we show the OBEP calculated from (3·1) neglecting higher order correction terms than μ/M and μ^3/M^3 for central and tensor potential and for LS potential, respectively with the following set of mesons:

$$\begin{array}{ll} \omega \text{ meson:} & g_\omega^2/4\pi=10,\ g_\omega f_\omega/4\pi=3.1,\quad f_\omega^2/4\pi=1, \\ \rho \text{ meson:} & g_\rho^2/4\pi=0.4,\ g_\rho f_\rho/4\pi=1.6,\ f_\rho^2/4\pi=6.5, \\ I=0 \text{ scalar meson} & \\ (\text{mass } 540\,\text{MeV}): & g_S^2/4\pi=5.2, \\ \text{pion:} & G_\pi^2/4\pi=14.4. \end{array} \tag{3·4}$$

The Hamada-Johnston phenomenological potential,[47] which is a new version of Hamada's potentials and reproduces the experimental data well below 300 MeV, is also plotted in Figs. 3·1∼3·3 for comparison. As is seen from these figures the OBEP qualitatively coincides with the Hamada-Johnston potential for the greater part of the region $r \gtrsim 0.5\ \mu_\pi^{-1}$.

Here we mention the hard core. There are two possibilities. One is that the hard core is strongly related to the structure of the composite particles and the fundamental interaction which constructs composite systems.[1] The other is that the greater part of the hard core effect can be

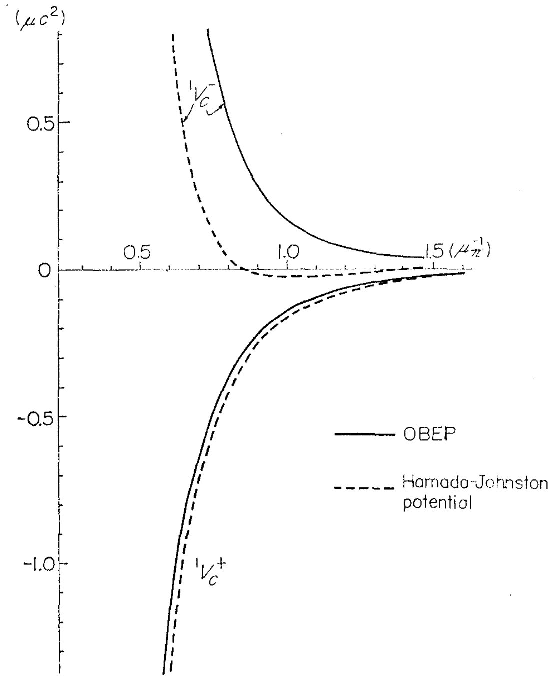

Fig. 3·1.

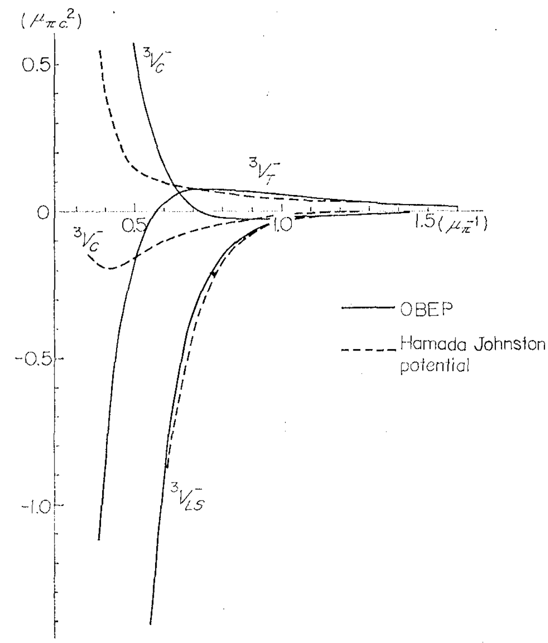

Fig. 3·2.

Figs. 3·1∼3·2. The one-boson-exchange potentials using (3·1) and (3·4).

Table 3·1. The roles of mesons in the OBE potential

		meson: type	meson: coupling	role of the OBEP in the intermediate region
(I) OPEP+ΣOBEP		$I=1$ vector	mainly tensor coupling	a tensor potential weaker than that of the OPEP.
		$I=0$ vector	mainly vector coupling	$\begin{cases} V_C>0 \\ V_{LS}<0 \end{cases}$
		$I=0$ scalar	scalar coupling	$V_C<0$ ($\lvert V_C(\text{scalar})\rvert > \lvert V_C(\text{vector})\rvert$)
(II) OPEP +TPEP +ΣOBEP	TPEP =TMO	$I=1$ vector	mainly vector coupling	$V_{LS}<0$
		$I=0$ vector	mainly vector couling	
		$I=0$ scalar		
		$I=1$ pseudo-scalar	small	
	TPEP =KMO	$I=1$ vector	mainly tensor coupling	$V_C>0$, $V_{LS}<0$
		$I=0$ vector	mainly vector coupling	
		$I=0$ and/or $I=1$ pseudo-scalar		

understood by the one-boson-exchange. Some authors[31),32)] have considered that the hard core effects are explainable by an $I=0$ vector meson such as the ω meson. Even if the repulsive core is due to the one-boson-exchange mechanism, it may be too simple to consider that the ω meson or the φ meson is a sole agent for it, as many mesons have been observed with higher masses than those of the ω meson and the φ meson.

More extensive discussion of the hard core will be given in Chapter 7.

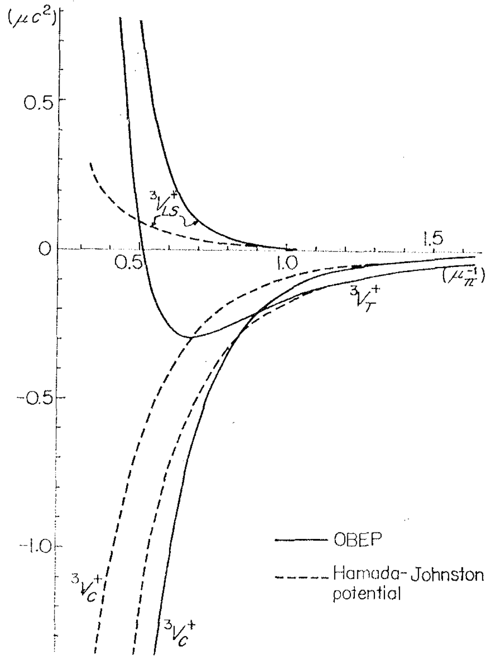

Fig. 3·3. The one-boson-exchange potentials using (3·1) and (3·4).

3·3 *Approach using the dispersion relation*

The dispersion theoretic treatment of the OBE model will be discussed in detail in Chapter 4, §6 and we will here only briefly touch on the analyses of this type. In these analyses the partial scattering amplitude $T_l(p^2)$ is assumed to be given by the OBE pole terms due to the left-hand-cut (l. h. c.) plus the right-hand-cut (r. h. c.) contributions. A standard form for the contribution from the r. h. c. is

$$\text{r. h. c.} = \frac{1}{2\pi} \int_0^\infty \frac{dp'^2}{p'^2 - p^2 - i\epsilon} \, | T_l(p'^2) |^2$$

where p is the center of mass momentum of nucleon. This simple form, however, does not work well for the threshold behaviour, so that we have to use some modified form such as that obtained by introducing an unphysical pole (Scotti-Wong) or by l-times subtraction (Kantor).

Scotti and Wong[6] (SW) assumed a dispersion relation which involves an unphysical pole at s_0 in order to preserve a reasonable threshold and high energy behaviour of the scattering amplitude. They solved it by the N/D method for the S, P and D waves and approximated the r. h. c. contributions by the one-pion-exchange amplitudes for the state with $2 < l \leq 5$ while ignoring such effects for the higher waves $l \geq 6$. For vector mesons, they replaced the usual Born amplitudes by amplitudes which have high energy behaviour of the Regge type introducing parameters c_ρ, c_ω and c_φ. Taking s_0 and the scattering lengths for the $^1S_0$ and $^3S_1$ states and c_ρ, c_ω and c_φ as free parameters in addition to the usual parameters of the OBE model, i. e. the masses and coupling constants of mesons, they showed that good agreement is obtained for the low energy experimental data with a relatively small mass for the scalar meson.

Kantor[54] used an l-times-subtracted form for the r. h. c. Instead of solving the dispersion relation, he proposed to replace the r. h. c. by the experimental data. Assuming complete absorption for the unknown high energy amplitudes, he could fit the amplitudes of the singlet even proton-proton states with $l \leq 4$ with the contribution of the pion, the η meson, the ρ meson, the ω meson and the f meson. This result is characteristically different from the results of other one-boson-exchange calculation in that he did not require a scalar meson and that a large coupling constant was obtained for the η meson. This is not surprising, however, since he only fitted the singlet states and the contributions from the pseudoscalar meson to these states are attractive so that the η meson with a large coupling constant can be used in place of a scalar meson as far as these states are concerned.

Kantor's method was criticized by MacGregor[55] in connection with the treatment of the threshold behaviour and some alternative methods were proposed; later these methods were further criticized by Moravcsik,[56] who showed the non-uniqueness of the results from the methods proposed by MacGregor.

§4. Experimental evidence for the mesons required by the OBE model

In this section we discuss the experimental evidence for the mesons which contribute to the nucleon-nucleon interaction. The mesons have to be of strangeness zero and isospin 0 or 1.

4·1 *Scalar meson*

One of the important results of the analysis of nucleon-nucleon interactions based on the OBE model is that an $I=0$ scalar meson (or strong attractive pion-pion interaction which has the same effect on nucleon-nucleon scattering) is necessary. This conclusion has been obtained in all the analyses except in a few attempts where the discussion has been confined to some particular states. The masses and coupling constants of the meson obtained are summarized in Table 4·1 and Fig. 4·1.

Table 4·1. Mass and coupling constant of scalar meson.

Mass (μ_S/μ_π)	Coupling constant $g_S^2/4\pi$	Reference
4	5.0	HOWY[1] case (I)
3 4	2.4 5.2	SUWY[2]
2.5	1.2	Babikov[49]
3 3.5 4	3.6 8.8 21	Ramsay[42]
4 5	15.4 40	BDR[5]
3.6 4.3 5	5 13 35	SUWY[39]
4	9.4	BS[51]
3.1	3.05	SW[6]

As is seen from this table and Fig. 4·1 though the values of coupling constant are much different in various analyses, the coupling constant vs mass plot falls near on a smooth curve. A few points deviate from the smooth curve. But they can be ignored since there are more detailed later analyses which give coupling constants on the curve. The mass of the scalar meson is not definitely determined. The reasons for this are that (1) the effect of the mass variation of the scalar meson can be replaced by that of renormalization of its coupling constant to a certain degree, (2) the mass of the scalar meson is strongly related to those of the vector mesons (the ω meson, the ρ meson and the φ meson) assumed in each analysis and (3) the mass also depends on the models which differ mainly in the treatment of the rescattering effect (i. e. the Schrödinger equation, the K-matrix method, the dispersion theoretic treatment, real part model, etc.).

With increasing mass of the scalar meson the magnitude of the coupling constant of the scalar meson has to be increased rapidly, in particular when the mass is near that of the vector meson. This is due to a large mutual cancellation of the contributions from the scalar and the vector mesons.

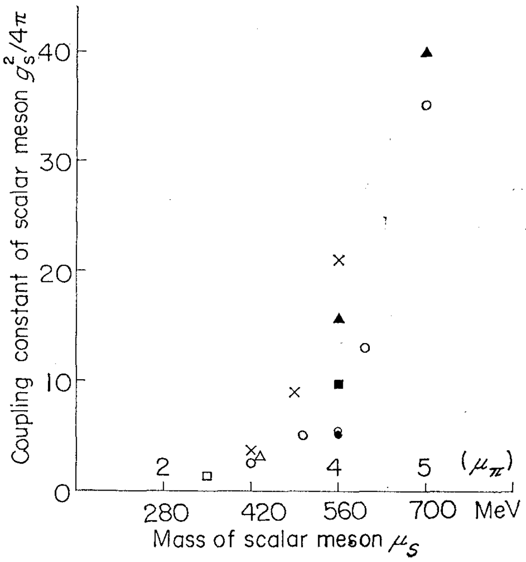

Fig. 4·1. Mass and coupling constant of the $I=0$ scalar meson obtained by various authors.

In 1960, Abashian, Booth and Crowe[57] reported evidence for an $I=0$ scalar resonance with mass about 310 MeV. This resonance (ABC particle), however, has not so far been observed in any process other than the reaction $H+d \to He^3+?$ and it is usually regarded not as a resonance but as the effect of a strong attractive force in the $I=J=0$ state of the two-pion system at low energies.

The mass of the scalar meson has to be between 400 MeV and the vector meson mass according to most analyses based on the OBE model. Thus the ABC particle is too light to be a candidate. In addition to the ABC particle, an $I=J=0$ even parity resonance with mass$\sim$400 MeV, the σ meson, has been suggested by both experiment and theoretical investigations.[58]

While these mesons have relatively small mass, there is some evidence for a scalar meson of high mass. From the asymmetry of the angular distribution of $\pi^+-\pi^-$ scattering near the ρ-meson peak compared with $\pi^\pm-\pi^0$ scattering, the existence of a strong $I=0$ S-wave pion-pion interaction has long been suspected at these energies, and recent experimental and theoretical analyses have indicated some evidence of a resonance with $I=J=0$ with mass$\sim$720 MeV (the ϵ meson).[59] This can be identified with that required by the nuclear force, though its mass may be too high for some models.[60]

As to the $I=1$ scalar meson we have no definite data. If this meson exists, dominant decay model will be $\pi+\eta$ for $\mu_S \gtrsim 5\mu_\pi$ or $2\pi+\gamma$ for $\mu_S \lesssim 5\mu_\pi$ violating G-parity conservation. Lichtenberg[4),61] has considered the ζ meson (mass 560 MeV) as a candidate for this meson, but no confirming experimental data have been obtained so far. Bryan and Scott[51] have indicated the necessity for the $I=1$ scalar meson, while this meson was not positively required from other analyses based on the OBE model.

It is a very interesting problem to the composite particle model what multiplet structure is realized for scalar particles if they exist. It is temptating to consider the low-lying 0^+ nonet with the σ meson and the ϵ meson as the unitary octet and singlet components in connection with the observed 0^-, 1^-, 2^+,$\cdots$ nonets.

4·2. *Pseudoscalar meson*

The η meson with mass 550 MeV is firmly established as the $I=0$ pseudoscalar meson and is expected to be the isosinglet constituent of a pseudoscalar octet of the $U(3)$ theory together with the pion and the kaon. The coupling constant of this meson with nucleons is not definitely determined from the analysis of the nucleon-nucleon scattering. The coupling constants of the η meson are given in Table 4·2. A relatively large value was given by Scotti and Wong,[6] $G_\eta^2/4\pi=10\sim12$. Bryan and Scott[51] gave $G_\eta^2/4\pi=7$ and a small value $G_\eta^2/4\pi\lesssim10$ was obtained by the OBEC model.[39] The extraordinary large value $G_\eta^2/4\pi=34$ given by Kantor is due to the fact that the η meson has been made to play the role of the scalar meson of other analyses for singlet states.

Table 4·2. Coupling constant of the η meson.

$G_\eta^2/4\pi$	Author
4	Lichtenberg[4]
$\lesssim10$	SUWY[39]
12.1 10.4	SW[6]
33.9	Kantor[54]
7.0	BS[51]

The contribution to the nuclear force of an $I=0$ pseudoscalar meson, the χ_0 meson, with mass 959 MeV is not clear. The meson is considered to be unitary singlet meson. Whether an $I=1$ pseudoscalar meson heavier than the pion exists or not is also interesting in connection with the pion production process in nucleon-nucleon collisions, as discussed by Ueda.[35] At present the experimental evidence is negative. From the analysis of the low energy nuclear force by the OBEC model a contribution from an $I=1$ pseudoscalar meson is not positively required.

4·3 *Vector mesons*

Both the $I=1$ and the $I=0$ vector mesons are required by the analysis of the nuclear force. To date, the ω meson ($I=0$), the ρ meson ($I=1$) and the φ meson ($I=0$) have been firmly established. No vector meson whose mass is smaller than that of the ρ meson or the ω meson has been observed.*) The masses of the vector mesons are not definitely determined

*) The ζ meson having mass 550 MeV might be an $I=1$ vector meson but experimental evidence for it is very scanty and doubtful.

from the analysis of only the low energy nucleon-nucleon scattering for reasons similar to the case of the scalar meson. Assuming that the ω meson and ρ meson are vecter mesons having lowest masses which contribute to nuclear forces, their coupling constants with nucleon are well determined from the analysis of nucleon-nucleon scattering at low energies. As to the φ meson the contribution is not clear from these analysis. The coupling constants of these vector meson are particulary interesting in view of group theoretical consideration and model of elementary particles. The coupling constants of ρ and ω mesons obtained from the analysis of nucleon-nucleon scattering by the OBE model are summarized in Table 4·3.

Table 4·3. Coupling constants of vector mesons†)

ω meson ($I=0$ vector meson)			ρ meson ($I=1$ vector meson)			Author and Remarks
$\frac{g_\omega^2}{4\pi}$	$\frac{f_\omega^2}{4\pi}$	$\frac{f_\omega}{g_\omega}$	$\frac{g_\rho^2}{4\pi}$	$\frac{f_\rho^2}{4\pi}$	$\frac{f_\rho}{g_\rho}$	
2.06	3.1	1.2	0.31	3.1	3.2	HOWY[1] case(I) $\mu_\rho=\mu_\omega=4\mu_\pi$
11	0.04	0	1∼2	6∼7	$\lesssim 0$	SUWY[39]
21.5	0††)	0††)	0.68	1.87	2.2	BS[51]
2.77	0††)	0††)	1.27	$\begin{cases}2.85\\3.05\end{cases}$	$\begin{cases}1.5\\2.4\end{cases}$	SW[6] $\mu_\rho=4.3\mu_\pi$, $g_\varphi^2/4\pi=\begin{cases}2.26\\2.65\end{cases}$

†) The tensor type coupling constants f_V are made dimensionless using the nucleon mass M.

††) From the analysis of nucleon electromagnetic form factors.[62]

4·4 *Other types of meson*

Among the mesons having spin zero and one we have not so far taken into account the axial vector meson. We have no a priori reason to believe that there exists no contribution from such a meson, though this meson is not positively required to explain nucleon-nucleon scattering. Recently some experimental evidence for the axial vector meson was reported. The mass of the meson is larger than 1 GeV. Thus the contribution to low energy nucleon-nucleon scattering is probably small.

The existence of mesons having spin larger than one is naturally expected from the composite particle theory. The f meson (mass 1240 MeV) seems to be well established, with $I=0$ and $J^P=2^+$. The coupling constant of this meson is not well determined from the low energy data. Hiroshige, Ino, Matsuda and Sawada[40] obtained $g_I^2/4\pi=10\sim20$ and Kantor gave $g_I^2/4\pi=5.71$ and $g_{II}^2/4\pi=4.45$. As is stressed in reference 40) the effect of tensor meson exchange is to cause a repulsive LS force for triplet odd states and to improve the fit of the ${}^3P_0$ phase shift.

The list of mesons which have zero strangeness and may contribute to the nuclear force is given in Table 4·4.

Table 4·4. Mesons having zero strangeness.*)

Spin-parity	Isospin	Symbol	Mass(MeV)	Nuclear Force	Remarks
0^-	1	π	140	The OPE is firmly established in the outer region of the nuclear force.	
0^-	0	η	549	The contribution is not positively evidenced in nucleon-nueleon scattering.	
0^-	0	χ	959	Contribution to the nuclear force is entirely unknown.	
0^+	0	ABC	310	The mass is smaller than those required from the OBE model.	Experimental evidence for the existence is doubtful.
0^+	0	σ	400	Strongly required by the analysis of the nuclear force in the mass range 400∼700 MeV.	Not yet well established.
0^+	0	ϵ	720		
1^-	1	ρ	765	Give important contribution to the nuclear force.	
1^-	0	ω	783		
1^-	0	φ	1020	Contribution is not clear.	
1^+?	1	A_1	1070	Not positively required by the nuclear force.	Not yet well established.
1^+?	0?	D	1286		Spin and parity is not established.
2^+	1	A_2	1300	Give a repulsive LS force for the ^3O states and improve the $^3P_0$ phase shift.	Spin and parity is not well established.
2^+	0	f	1250		
2^+	0	f'	1500		
?	1	B	1220		Existence is not well established.
?	0?	E	1420		

*) The experimental evidence for the meson and their assignment is summarized in reference 63).

§5. The one-boson-exchange-contribution model of high energy nucleon-nucleon collisions

The application of the OBE model to the single pion production process in proton-proton collision was made by Ueda.[35] It is experimentally well confirmed that the single pion production takes place mainly through 3-3 resonance production when the incident proton energy is in the energy region of about 1 GeV:

$$p+p \to N_{33}+n$$
$$\quad\quad \hookrightarrow p+\pi^+$$

Here N_{33} denotes the 3-3 resonance. Ueda showed that the one-pion-exchange contribution (B=pion in Fig. 5·1.) gives too large a cross section even below 1 GeV*) and that if a heavy meson having the same quantum numbers as the pion ($I=1$, $J^P=0^-$) exists, its contribution can interfere destructively with that from the pion and can reduce the cross section. In the analysis given by Ueda, rescattering effects both of elastic and inelastic processes were not take into account. The huge one-pion-exchange contribution implies that the unitarity requirement is severely violated. Therefore, it is desired to involve these rescattering effects into account. This is possible if we consider the transitions between the two-proton states and neutron plus 3-3 resonance states as a many-channel problem:

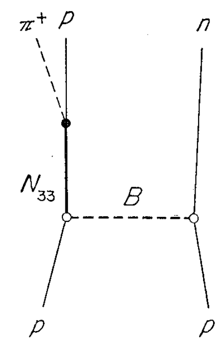

Fig. 5·1. The one-boson-exchange diagarm of the single pion production process via 3-3 resonance in the proton-proton collision.

$$\begin{array}{ccc} p+p & \longrightarrow & p+p \\ & \times & \\ N_{33}+N & \longrightarrow & N_{33}+N \end{array}$$

and a procedure similar to that stated in §2 is applied:

$$\boldsymbol{S}_A^B=\mathbf{1}+i\boldsymbol{T}_A^B \Rightarrow \boldsymbol{S}_A=\mathbf{1}+i\boldsymbol{T}_A=\frac{1+\dfrac{i}{2}\boldsymbol{T}_A^B}{1-\dfrac{i}{2}\boldsymbol{T}_A^B}, \tag{5·1}$$

where

$$\boldsymbol{T}_A^B=(T_{Aij}^B)$$

is the lowest order T matrix between the j-th and i-th channels which are specified by common conserved quantum numbers A (isospin, total angular momentum, parity, etc.). An analysis of the proton-proton interaction near 1 GeV based on this method was made by Kikugawa, Sawada, Ueda, Watari and Yonezawa.[36] They assumed for T_{A11}^B, which corresponds to elastic proton-proton scattering, the one-boson-exchange amplitude that reproduces well the experimental proton-proton scattering below 400 MeV. For T_{A12}^B (T_{A21}^B) which corresponds to $p+p\to N+N_{33}$ ($N+N_{33}\to p+p$) they assumed the one-pion-exchange and the one-ρ-meson-exchange amplitude. T_{A22}^B which corresponds to $N+N_{33}\to N+N_{33}$ is treated as free parameter.

*) To reduce the cross section, Ferrari and Selleri[64] introduced a purely phenomenological pionic form factor.

The main results of the analysis are the following.

i) For $p+p \rightarrow p+\pi^{+}+n$

(1) The unitarization makes a remarkable suppression and a forward peaking for differential cross section in comparison with that results from ununitarized one-boson-exchange amplitudes $\boldsymbol{T}_{A}^{B}$. By the unitarization the cross section from the ununitarized one-pion-exchange amplitude is suppressed to about twice of the experimental cross section. Adding one-ρ-meson-exchange contribution to the one-pion-exchange contribution and unitarizing the one-boson-exchange amplitude, we can get a fit to experimental angular distribution.

(2) The matrix element T_{A22}^{B} is not so much influencing to the differential cross section of $p+p \rightarrow N+N_{33}$, when the matrix element T_{A22}^{B} is varied within the range of the magnitude of T_{A11}^{B}.

ii) For $p+p \rightarrow p+p$

(1) A characteristic result is that central absorption is small in the sense that an absorption coefficient of ${}^{1}S_{0}$ state $r({}^{1}S_{0})$ is in the range

$$1 \gtrsim r({}^{1}S_{0}) \gtrsim 0.8\,.$$

(2) Real parts of the proton-proton scattering phase shifts depend considerably on amplitudes T_{A22}^{B} and we can find an appropriate value of T_{A22}^{B} which gives a fit of the real part of phase shifts to the experiment.[65]

The multi-channel treatment of high energy reactions above 1 GeV by the OBE model or more generally by the OHE model is possible but only a few things have been made in this energy region because of a large number of participating channels. There are many attempts to explain the inelastic process at energies about 1 to 5 GeV by the dominance of a *particular* one-particle-exchange process which may be considered to be an approximation to the OHE model. In these calculations it is usually assumed that some one-boson-exchange diagram such as pion-exchange or ρ-meson-exchange, etc., gives a dominant contribution. This may be too simplified assumption from the viewpoint of the OHE model, though worth trying. These calculations, however, often give too large cross sections, as in the case of pion production in nucleon-nucleon collisions.

This is essentially due to the fact that the unitarity requirements are not satisfied in these calculations. To overcome this difficulty and to introduce a correction due to the unitarity, the following replacement of the Born amplitude T_{A12}^{B} has been employed in many calculations:

$$T_{A12}^{B} \rightarrow T_{A12} = (S_{A11})^{1/2} T_{A12}^{B} (S_{A22})^{1/2}. \tag{5·2}$$

Here the S_{A11} and S_{A22} are the S matrices for the elastic channel 1 and 2 and T_{A12}^{B} is the uncorrected Born amplitude. This was derived by Sopkovich and by Gottfried and Jackson.[66]

This expression is probably correct when T^B_{A12} is small with some conditions on S_{A11} and S_{A22}. But in the case we meet in the actual calculation these requirements seem not to be satisfied, and in many cases the inelastic process under discussion is even the dominant one. It should be interesting to use also another type of replacement:

$$T^B_{A12} \to T_{A12} = \frac{2}{T^B_{A12}}\left[1-\left\{1-\frac{1}{4}(S_{A11}+1)(T^B_{A12})^2(S_{A22}+1)\right\}^{1/2}\right], \tag{5·3}$$

where T^B_{A12} may be a Born amplitude or modified in some way. This expression is an identity in the two-channel case of the damping theory and holds in the many-channel case if there is no direct interaction between the final states.

An approximation to (5·3) is

$$T_{A12} = \frac{1}{4}(1+S_{A11})\,T^B_{A12}(1+S_{A22}). \tag{5·4}$$

The same expression has been derived by Ross and Shaw.[67] It should be interesting to use (5·2)～(5·4) for the analysis of the inelastic process above 1 GeV where the number of channels will become too large for the straightforward application of a multi-channel OHEC model which takes all possible one-hadron-exchange contributions into account.

§6. Problems of the one-boson-exchange model

In this chapter we have been discussing the OBE model mainly in connection with the OBEC model. The reason for this is that the OBEC model bears the characteristic features of the OBE model in the simplest form and should be taken as a first attempt to treat the strongly interacting system in "Korrespondenzmässig" based on the composite model. Its simplicity will make it more widely applicable and be helpful in clarifying the merits and difficulties involved in the model.

6·1 *Difficulties of the OBEC model connected with the S wave and the necessity of non-OBEC effects*

First we discuss some difficulties expected for the OBEC model. These difficulties appear when we attempt to discuss the S wave states of the two-nucleon system by using the OBEC model. In the analysis of the S wave in nucleon-nucleon scattering, the basic standpoint taken by the present authors was that we should treat this completely phenomenologically. This is chiefly because we have not enough information on (a) high mass mesons and (b) the inner structure of the particles which may exist and contribute to the nucleon-nucleon interaction at short distance. Although this standpoint need not be altered, there is a difficulty if we try to treat the S waves by straightforward application of the OBEC model.

This difficulty may be related to the formalism of the OBEC model and appear in the ${}^1S_0$ and ${}^3S_1$ phase shifts at low energies. The observed energy dependence of the ${}^3S_1$ phase shift is that it starts from π at zero energy and passes through $\pi/2$ at $\sim$20 MeV. Such energy dependence, however, is hard to explain by the OBEC model even if a particle corresponding to the deuteron is introduced in addition to the one-boson-exchange effect.*)

A similar situation is also seen in the ${}^1S_0$ state where the phase shift starts from 0 at zero energy and reaches its maximum value of almost 60° at about 10 MeV making very sharp peak. The ${}^1S_0$ phase shift obtained by the OBEC model is round and does not show the very sharp peak characteristic of the experimental one. It is not easy to reproduce such an energy dependence. Strictly speaking one cannot rule out the possibility of giving such an abrupt change by introducing as many mesons as one likes. However, our experience suggests that this is practically impossible in terms of a limited number of one-boson-exchange contributions.

Thus we may have to introduce some other effects in addition to the one-boson-exchange contributions. For this purpose, the most natural extension of the OBEC model will be to include some part of the higher order effects of the Yukawa interactions. Here in order to overcome these difficulties, retaining the basic idea of the OBEC model, the preference should go to those parts which (a) are closely related with or completely determined by the one-boson-exchange contribution and (b) have a physical effect different from the one-boson-exchange contribution.

Furuichi and Yonezawa[68] analysed the two-pion-exchange effect using the dispersion relation and showed that it is possible to make a unique separation of the two-pion exchange into the iterated part of the one-pion exchange and the "proper" two-pion exchange by using a suitable defined dispersion relation. They showed that the iterated effect of the one-pion exchange and the "proper" two-pion-exchange effect correspond to the contribution from the right-hand cut and that from the left-hand cut respectively, the former contribution shows behaviour quite different from that of the one-boson-exchange contribution and has the desired feature required to improve the fit of the S states, while the latter resembles the one-boson-ex-

*) In connection with this we note the simple fact that in the OBEC model the phase shifts of uncoupled state cannot go through $\pi/2$ at a certain energy unless a resonance particle with corresponding mass is introduced beforehand (no dynamical resonance). Although this is no longer the case for the coupled states, the energy dependence of the undamped amplitudes calculated from the experiments seems to require non-OBE-type contribution.

change. The energy dependence of the S wave phase shifts can be reproduced also by solving the Schrödinger equation for the OBE type potentials.[69]

These analyses indicate that the contribution from the right-hand cut in the dispersion relation, or from rescattering effects off the energy shell in the Schrödinger equation, will satisfy the requirements (a) and (b) stated above.

This implies that the OBEP model is a natural generalization of the OBEC model to cope with such difficulties. Several attempts have already been made which may be categorized in this model.

Of these OBEP models the dispersion theoretic version may be attractive from the formalistic viewpoint and because of its wider applicability. The attempts so far made, however, have encountered various difficulties as discussed in Chapter 4. It is possible to fit the S waves by a dispersion relation as done by Scotti and Wong[6] and by Yamada[70] if the scattering lengths are introduced as free parameters. But this does not imply that the S wave problem has been solved, and it is questionable whether these analyses could add something new to the information obtained by the OBEC model as far as the nucleon-nucleon scattering is concerned.

6·2 *Limitations on the one-boson-exchange model implied by the high energy behaviour of the scattering amplitudes*

Other limitations on the OBEC model was discussed by Hoshizaki and Hama.[71] They investigated to what extent the OBEC model remains valid, with the intention of using the OBEC model for the phase shift analysis of nucleon-nucleon scattering in the inelastic region[65] ($\gtrsim$400 MeV). Their argument is based on the Levinson theorem which states that $\lim_{E\to\infty}\delta(E)=0$ for $\delta(0)=n\pi$ where n is the number of bound states. The asymptotic limit of the phase shifts calculated using the OBEC model for large energy E is generally $\pm\pi/2$. Hama and Hoshizaki showed that the phase shifts for given l reach some large value which is common to various l, when the corresponding impact parameter defined by

$$b=\sqrt{l(l+1)}\,/p$$

exceeds a critical value, b_c, which is almost independent of the value of l. When the parameters determined by the OBEC analysis[38] are used, the largest phase shift with the same l passes through $\pm 50°$ for the impact parameter b between 0.4 and 0.6 μ_π^{-1}. Thus they concluded that the OBEC model is applicable to the nuclear forces in the region where the nucleon-nucleon distance is larger than $b_c\sim 0.5$.

Their results should be understood as follows. The parameters obtained by the OBEC model are based on the analysis of nucleon-nucleon scattering below 300 MeV. In this analysis the S states are treated completely phenomenologically and higher angular momentum states with $l \geq 1$ were analysed. The incident laboratory nucleon energy $T = 300$ MeV corresponds to the impact parameter 0.5 μ_π^{-1} for the P state. This set of parameters for the OBEC model is therefore reliable only for the nuclear force at internucleon distance larger than ~ 0.5 μ_π^{-1}, and it is expected that for the nuclear force in the region interior to 0.5 μ_π^{-1} the fit to experiment becomes worse. In principle it is necessary to take into account the contributions from the exchange of all existing mesons and inelastic processes in order to discuss the high energy behaviours.

From the contents of the OBEC model itself, however, we cannot deduce the conclusion that the analysis should be cut off at a particular distance. We will be able to make the OBEC analysis in energy regions higher than that considered so far if we take into account the contribution of further heavier bosons, though this may not be continued endlessly. We think that the applicability of the OBEC model should be determined not internally, from the model itself, but rather from knowledge external to it. This knowledge might be obtained from the analysis of the experiments on the nuclear force in the inner region. From the viewpoint of the composite theory the most decisive information will be the mechanism of composition and decomposition of composite systems since the Yukawa interaction used in the calculation by the OBEC model may be regarded as a model Hamiltonian whose applicability is determined by a more fundamental interaction. The limitation will be clarified by investigating a wide range of strong interactions both from the experimental and the theoretical sides on the basis of the composite theory. For this purpose a comprehensive analysis of nucleon-nucleon scattering in the S states or in the hard core region, inelastic processes in the sub-GeV region and very high energy phenomena will be very helpful.

The above discussion of the limitations holds equally for the other OBE models which use potentials or dispersion relations.

6·3 *The $I=0$ scalar meson predicted from the OBE model*

The OBE model predicts the existence of an $I=0$ scalar meson with mass 400~700 MeV. This is common to most analyses except in a few papers where the analysis is not comprehensive.

The view has often been expressed that the necessary scalar meson is a phenomenological stand-in for the strong pion-pion interaction in the $I=J=0$ state. This is due to the situation that we have no established evidence for a meson having the required properties, while it is likely from various experiments that the pion-pion interaction in the $I=J=0$ states is strong and attractive at low energy. In fact Scotti and Wong demonstrated that nucleon-nucleon scattering can be explained by an S wave pion-pion interaction of appropriate strength instead of the scalar meson.

In order to have an interpretation consistent with such an pion-pion interaction model, however, it is necessary to consider also the contributions from the uncorrelated two-pion, three-pion, etc., since these contributions are by no means smaller than those from the correlated many-pion states, in so far as we calculate them by the conventional methods. Moreover in this case it might be no longer necessary even to relate the scalar meson with the S wave pion-pion correlation. As is suggested from Table 3·1 and shown by the detailed investigation of two-pion-exchange effects by Furuichi and Watari[72] (cf. Chapter 4), the 3-3 resonance state gives a contribution to the nucleon-nucleon interaction which behaves like that from a scalar meson.

The existence or non-existence of the iso-singlet scalar meson seems to have a critical significance for the further development of the theory of strong interactions and the deeper understanding of the nature of the hadrons.

If there exists no scalar meson in nature, we have to consider the required scalar meson as a phenomenological substitute for the S wave pion-pion interaction and/or the 3-3 resonance, etc. In this case it may not be impossible to explain the nuclear force by the one-pion, two-pion, three-pion, $\cdots$ contributions and other hadron reactions by the Yukawa interaction and its higher order effects.

On the other hand, if there exists a scalar meson, we have the possibility that the contributions from the two-pion, three-pion, $\cdots$ continuum do not exist, or they are damped, at least, for some reason. In such a situation a property which is foreign to the particle picture based on the conventional field theory might be required for the hadrons since a contribution from the two-pion, three-pion, $\cdots$continuum should exist as far as we adhere to the current field theoretical picture. Extensive experimental investigation of this problem is to be desired.

Acknowledgements

Among the authors S. Sawada is indebted to the Fujyukai Foundation and T. Ueda to the Sakkokai Foundation for financial aids. The numerical calculations were carried out with HITAC 5020 at Computer Center, Tokyo University.

Appendix A

In this appendix we give the M matrices (2·9) for each meson contribution. The following abbreviations are used:

$$x=\cos\theta,$$
$$x_0=1+\mu_B^2/2p^2,$$
$$\varepsilon=p^2/(M+E)^2,$$

$$\left.\begin{aligned}
&\kappa=3p^2+2M^2,\\
&X_a=\frac{1}{4}\left(\frac{1}{x_0-x}-\frac{1}{x_0+x}\right),\\
&X_b=\frac{1}{4}\left(\frac{1}{x_0-x}+\frac{1}{x_0+x}\right),\\
&Y_a=\frac{\kappa+p^2x}{x_0-x}-\frac{\kappa-p^2x}{x_0+x},\\
&Y_b=\frac{\kappa+p^2x}{x_0-x}+\frac{\kappa-p^2x}{x_0+x}
\end{aligned}\right\}\text{ for } T=1 \text{ states.}$$

For the $T=0$ states the definitions of X_a and Y_a are interchanged with those of X_b and Y_b:

$$\left.\begin{aligned}
&X_a=\frac{1}{4}\left(\frac{1}{x_0-x}+\frac{1}{x_0+x}\right),\\
&X_b=\frac{1}{4}\left(\frac{1}{x_0-x}-\frac{1}{x_0+x}\right),\\
&Y_a=\frac{\kappa+p^2x}{x_0-x}+\frac{\kappa-p^2x}{x_0+x},\\
&Y_b=\frac{\kappa+p^2x}{x_0-x}-\frac{\kappa-p^2x}{x_0+x}
\end{aligned}\right\}\text{ for } T=0 \text{ states.}$$

(1) Scalar meson

$$M^S_{11}(\theta,\phi)=C_S\{(1+\varepsilon^2x^2)X_a-2\varepsilon xX_b\},$$

$$M^S_{00}(\theta,\phi)=C_S\{(1-\varepsilon^2+2\varepsilon^2x^2)X_a-2\varepsilon xX_b\},$$

$$M^S_{01}(\theta,\phi)=\sqrt{2}\,C_S\{X_b-\varepsilon xX_a\}\,\varepsilon\sin\theta\, e^{i\phi},$$

$$M^S_{10}(\theta,\phi)=-M^S_{01}(\theta,-\phi),$$

$$M^S_{1-1}(\theta,\phi)=C_SX_a\varepsilon^2\sin^2\theta\, e^{-2i\phi},$$

$$M^S_{ss}(\theta,\phi)=C_S\{(1+\varepsilon^2)X_b-2\varepsilon xX_a\},$$

$$C_S=\frac{g_S^2}{4\pi}\frac{(E+M)^2}{2p^2E}.$$

(2) Pseudoscalar meson

$$M^P_{11}(\theta,\phi)=C_P\{(1+x^2)X_a-2xX_b\},$$

$$M^P_{00}(\theta,\phi)=2C_P\{-x^2X_a+xX_b\},$$

$$M^P_{01}(\theta,\phi)=\sqrt{2}\,C_P\{xX_a-X_b\}\sin\theta\, e^{i\phi},$$

$$M^P_{10}(\theta,\phi)=M^P_{01}(\theta,-\phi),$$
$$M^P_{1-1}(\theta,\phi)=C_P X_a \sin^2\theta\, e^{-2i\phi},$$
$$M^P_{ss}(\theta,\phi)=-2C_P\{X_b-xX_a\},$$
$$C_P=\frac{G_P^2}{4\pi}\frac{1}{4E},\quad G_P=g_P+2f_P.$$

(3) Vector meson

$$M^V_{ij}(\theta,\phi)=M^{VV}_{ij}(\theta,\phi)+M^{VT}_{ij}(\theta,\phi)+M^{TT}_{ij}(\theta,\phi).$$

Vector-vector coupling:

$$M^{VV}_{11}(\theta,\phi)=-C_{VV}\{(1+\varepsilon(1+(1+\varepsilon)x^2))X_a+4\varepsilon xX_b\},$$
$$M^{VV}_{00}(\theta,\phi)=-C_{VV}\{(1-\varepsilon^2-2\varepsilon(1-\varepsilon)x^2)X_a+8\varepsilon xX_b\},$$
$$M^{VV}_{01}(\theta,\phi)=-\sqrt{2}\,C_{VV}\{2\varepsilon X_b+\varepsilon(1+\varepsilon)xX_a\}\sin\theta\, e^{i\phi},$$
$$M^{VV}_{10}(\theta,\phi)=\sqrt{2}\,C_{VV}\{4\varepsilon X_b-\varepsilon(1-\varepsilon)xX_a\}\sin\theta\, e^{-i\phi},$$
$$M^{VV}_{1-1}(\theta,\phi)=-2C_{VV}\{\varepsilon(1+\varepsilon)X_a\}\sin^2\theta\, e^{-2i\phi},$$
$$M^{VV}_{ss}(\theta,\phi)=-C_{VV}(1+6\varepsilon+\varepsilon^2)X_b,$$
$$C_{VV}=\frac{G_V^2}{4\pi}\frac{(E+M)^2}{2p^2E},\quad G_V=g_V+2f_V.$$

Vector-tensor coupling:

$$M^{VT}_{11}(\theta,\phi)=C_{VT}\{(A-B\varepsilon^2x^2)X_a+2\varepsilon xX_b\},$$
$$M^{VT}_{00}(\theta,\phi)=C_{VT}\{(A+B\varepsilon^2-2B\varepsilon^2x^2)X_a+2\varepsilon xX_b\},$$
$$M^{VT}_{01}(\theta,\phi)=\sqrt{2}\,C_{VT}\{\varepsilon X_b-B\varepsilon^2xX_a\}\sin\theta\, e^{i\phi},$$
$$M^{VT}_{10}(\theta,\phi)=-\sqrt{2}\,C_{VT}\{\varepsilon X_b-B\varepsilon^2xX_a\}\sin\theta\, e^{-i\phi},$$
$$M^{VT}_{1-1}(\theta,\phi)=-C_{VT}B\varepsilon^2X_a\sin^2\theta\, e^{-2i\phi},$$
$$M^{VT}_{ss}(\theta,\phi)=C_{VT}\{(A-B\varepsilon^2)X_b+2\varepsilon xX_a\},$$
$$C_{VT}=\frac{2G_Vf_V}{4\pi}\frac{(E+M)^2}{p^2E},\quad A=2E-M,\quad B=2\mathrm{E}+M.$$

Tensor-tensor coupling:

$$M^{TT}_{11}(\theta,\phi)=-C_{TT}\{(1+\varepsilon^2x^2)Y_a-2\varepsilon xY_b\},$$
$$M^{TT}_{00}(\theta,\phi)=-C_{TT}\{(1-\varepsilon^2+2\varepsilon^2x^2)Y_a-2\varepsilon xY_b\},$$
$$M^{TT}_{01}(\theta,\phi)=\sqrt{2}\,C_{TT}\{\varepsilon Y_b-\varepsilon^2xY_a\}\sin\theta\, e^{i\phi},$$
$$M^{TT}_{10}(\theta,\phi)=-\sqrt{2}\,C_{TT}\{\varepsilon Y_b-\varepsilon^2xY_a\}\sin\theta\, e^{-i\phi},$$
$$M^{TT}_{1-1}(\theta,\phi)=-C_{TT}\varepsilon^2Y_a\sin^2\theta\, e^{-2i\phi},$$

$$M_{ss}^{TT}(\theta,\phi)=-C_{TT}\{(1+\varepsilon^2)Y_b-2\varepsilon xY_a\},$$

$$C_{TT}=\frac{f_V^2}{4\pi}\frac{(E+M)^2}{4p^2E}.$$

(4) Axial vector meson

Axial vector coupling:

$$M_{11}^A(\theta,\phi)=C_A\left\{(1+\varepsilon)(1+\varepsilon x^2)X_a+4\varepsilon xX_b+\left(\frac{2M}{\mu}\right)^2\varepsilon((1+x^2)X_a-2xX_b)\right\},$$

$$M_{00}^A(\theta,\phi)=C_A\left\{(1-\varepsilon)(1+\varepsilon-2\varepsilon x^2)X_a-8\varepsilon xX_b-2\left(\frac{2M}{\mu}\right)^2\varepsilon(x^2X_a-xX_b)\right\},$$

$$M_{01}^A(\theta,\phi)=\sqrt{2}\,C_A\left\{2\varepsilon X_a+(1+\varepsilon)\varepsilon xX_b+\left(\frac{2M}{\mu}\right)^2\varepsilon(xX_b-X_a)\right\}\sin\theta\, e^{i\phi},$$

$$M_{10}^A(\theta,\phi)=\sqrt{2}\,C_A\left\{4\varepsilon X_a+(1-\varepsilon)\varepsilon xX_b+\left(\frac{2M}{\mu}\right)^2\varepsilon(xX_b-X_a)\right\}\sin\theta\, e^{-i\phi},$$

$$M_{1-1}^A(\theta,\phi)=C_A\left\{(1+\varepsilon)\varepsilon X_a+\left(\frac{2M}{\mu}\right)^2\varepsilon X_a\right\}\sin^2\theta\, e^{-2i\phi},$$

$$M_{ss}^A(\theta,\phi)=-C_A\left\{(3+2\varepsilon+3\varepsilon^2)X_b-2\left(\frac{2M}{\mu}\right)^2\varepsilon(xX_a-X_b)\right\},$$

$$C_A=\frac{g_A^2}{4\pi}\frac{(E+M)^2}{2p^2E}.$$

Pseudotensor coupling:

$$M_{11}^{PT}(\theta,\phi)=-C_{PT}\{(1+x^2)Y_a-2xY_b\},$$

$$M_{00}^{PT}(\theta,\phi)=-2C_{PT}\{-x^2Y_a+xY_b\},$$

$$M_{01}^{PT}(\theta,\phi)=-\sqrt{2}\,C_{PT}\{-Y_b+xY_a\}\sin\theta\, e^{i\phi},$$

$$M_{10}^{PT}(\theta,\phi)=-\sqrt{2}\,C_{PT}\{xY_a-Y_b\}\sin\theta\, e^{-i\phi},$$

$$M_{1-1}^{PT}(\theta,\phi)=-C_{PT}Y_a\sin^2\theta\, e^{-2i\phi},$$

$$M_{ss}^{PT}(\theta,\phi)=2C_{PT}\{Y_b-xY_a\},$$

$$C_{PT}=\frac{f_A^2}{4\pi}\frac{1}{2E}.$$

Appendix B

In this Appendix the partial transition amplitudes α^B defined by (2·12) are given. The function $Q_l(x_0)$ is the Legendre function of the second kind and $K=(E+M)^2/2p^2E$.

(1) Scalar meson

$$\alpha_{l,l+1}^S=\frac{ig_S^2Kp}{4\pi\cdot 2(2l+3)^2}[4(l+1)(l+2)\varepsilon^2Q_{l+2}(x_0)$$
$$-2(2l+3)^2\varepsilon Q_{l+1}(x_0)+\{(2l+3)^2+\varepsilon^2\}Q_l(x_0)],$$

$$\alpha^S_{l,l}=\frac{ig_S^2Kp}{4\pi\cdot 2(2l+1)}[-2l\varepsilon Q_{l+1}(x_0)$$
$$+(2l+1)(1+\varepsilon^2)Q_l(x_0)-2(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^S_{l,l-1}=\frac{ig_S^2Kp}{4\pi\cdot 2(2l-1)^2}[\{(2l-1)^2+\varepsilon^2\}Q_l(x_0)$$
$$-2(2l-1)^2\varepsilon Q_{l-1}(x_0)+4l(l-1)\varepsilon^2 Q_{l-2}(x_0)],$$

$$\alpha^{SJ}=\frac{ig_S^2Kp}{4\pi\cdot(2J+1)^2}[J(J+1)]^{1/2}\varepsilon^2[Q_{J+1}(x_0)-Q_{J-1}(x_0)],$$

$$\alpha^S_l=\frac{ig_S^2Kp}{4\pi\cdot 2}[(1+\varepsilon^2-2\varepsilon x_0)Q_l(x_0)+2\varepsilon\delta_{l0}].$$

(2) Pseudoscalar meson

$$\alpha^P_{l,l+1}=\frac{iG_P^2Kp\varepsilon}{4\pi\cdot(2l+3)}[Q_{l+1}(x_0)-Q_l(x_0)],$$

$$\alpha^P_{l,l}=-\frac{iG_P^2Kp\varepsilon}{4\pi\cdot(2l+1)}[lQ_{l+1}(x_0)-(2l+1)Q_l(x_0)+(l+1)Q_{l-1}(x_0)],$$

$$\alpha^P_{l,l-1}=-\frac{iG_P^2Kp\varepsilon}{4\pi\cdot(2l-1)}[Q_l(x_0)-Q_{l-1}(x_0)],$$

$$\alpha^{PJ}=-\frac{iG_P^2Kp\varepsilon}{4\pi\cdot(2J+1)}[J(J+1)]^{1/2}[Q_{J+1}(x_0)-2Q_J(x_0)+Q_{J-1}(x_0)],$$

$$\alpha^P_l=\frac{iG_P^2Kp\varepsilon}{4\pi}[(x_0-1)Q_l(x_0)-\delta_{0l}].$$

(3) Vector meson

Vector-vector coupling:

$$\alpha^{VV}_{l,l+1}=-\frac{iG_V^2Kp}{4\pi\cdot 2(2l+3)^2}[4(l+1)(l+2)\varepsilon^2Q_{l+2}(x_0)$$
$$+4(2l+3)(3l+4)\varepsilon Q_{l+1}(x_0)+\{(2l+3)+\varepsilon\}^2Q_l(x_0)],$$

$$\alpha^{VV}_{l,l}=-\frac{iG_V^2Kp}{4\pi\cdot 2(2l+1)}[4l\varepsilon Q_{l+1}(x_0)$$
$$+(2l+1)(1+\varepsilon)^2Q_l(x_0)+4(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha^{VV}_{l,l-1}=-\frac{iG_V^2Kp}{4\pi\cdot 2(2l-1)^2}[\{(2l-1)-\varepsilon\}^2Q_l(x_0)$$
$$+4(2l-1)(3l-1)\varepsilon Q_{l-1}(x_0)+4l(l-1)\varepsilon^2Q_{l-2}(x_0)],$$

$$\alpha^{VVJ}=\frac{iG_V^2Kp}{4\pi\cdot(2J+1)^2}[J(J+1)]^{1/2}[\{(2J+1)-\varepsilon\}\varepsilon Q_{J+1}(x_0)$$
$$-2(2J+1)\varepsilon Q_J(x_0)+\{(2J+1)+\varepsilon\}\varepsilon Q_{J-1}(x_0)],$$

$$\alpha_l^{VV} = -\frac{iG_V^2 Kp}{4\pi\cdot 2}(1+6\varepsilon+\varepsilon^2)Q_l(x_0).$$

Vector-tensor coupling:

$$\alpha_{l,l+1}^{VT} = \frac{2iG_V f_V Kp}{4\pi\cdot(2l+3)^2}[-4(l+1)(l+2)B\varepsilon^2 Q_{l+2}(x_0)$$
$$+2(2l+3)^2\varepsilon Q_{l+1}(x_0)+\{(2l+3)^2 A - B\varepsilon^2\}Q_l(x_0)],$$

$$\alpha_{l,l}^{VT} = \frac{2iG_V f_V Kp}{4\pi\cdot(2l+1)}[2l\varepsilon Q_{l+1}(x_0)$$
$$+(2l+1)(A-B\varepsilon^2)Q_l(x_0)+2(l+1)\varepsilon Q_{l-1}(x_0)],$$

$$\alpha_{l,l-1}^{VT} = \frac{2iG_V f_V Kp}{4\pi(2l-1)^2}[\{(2l-1)^2 A - B\varepsilon^2\}Q_l(x_0)$$
$$+2(2l-1)^2\varepsilon Q_{l-1}(x_0)-4l(l-1)B\varepsilon^2 Q_{l-2}(x_0)],$$

$$\alpha^{VTJ} = -\frac{4iG_V f_V Kp}{4\pi(2J+1)^2}[J(J+1)]^{1/2}B\varepsilon^2[Q_{J+1}(x_0)-Q_{J-1}(x_0)],$$

$$\alpha_l^{VT} = \frac{2iG_V f_V Kp}{4\pi}[(A+2\varepsilon x_0-B\varepsilon^2)Q_l(x_0)-2\varepsilon\delta_{0l}].$$

Tensor-tensor coupling:

$$\alpha_{l,l+1}^{T} = -\frac{if_V^2 Kp}{4\pi M^2(2l+3)^2}\Bigg[\frac{4(l+1)(l+2)(l+3)}{2l+5}\varepsilon^2 p^2 Q_{l+3}(x_0)$$
$$+\{4(l+1)(l+2)\kappa\varepsilon^2-2(l+2)(2l+3)\varepsilon p^2\}Q_{l+2}(x_0)$$
$$+\Bigg\{\frac{(l+1)(2l+3)[(2l+3)^2-2]}{(2l+1)(2l+5)}\varepsilon^2 p^2+\frac{(l+1)(2l+3)^2}{2l+1}p^2$$
$$-2(2l+3)^2\kappa\varepsilon\Bigg\}Q_{l+1}(x_0)$$
$$+\{(2l+3)^2\kappa+\varepsilon^2\kappa-2(l+1)(2l+3)\varepsilon p^2\}Q_l(x_0)$$
$$+\frac{l}{2l+1}\{(2l+3)^2+\varepsilon^2\}p^2 Q_{l-1}(x_0)\Bigg],$$

$$\alpha_{l,l}^{T} = -\frac{if_V^2 Kp}{4\pi M^2(2l+1)}\Bigg[-\frac{2l(l+2)}{2l+3}\varepsilon p^2 Q_{l+2}(x_0)+\{(l+1)(1+\varepsilon^2)p^2$$
$$-2l\kappa\varepsilon\}Q_{l+1}(x_0)+(2l+1)\Bigg\{\kappa(1+\varepsilon^2)-\frac{4l(l+1)}{(2l-1)(2l+3)}\varepsilon p^2\Bigg\}Q_l(x_0)$$
$$-\{2(l+1)\varepsilon\kappa-l(1+\varepsilon^2)p^2\}Q_{l-1}(x_0)-\frac{2(l-1)(l+1)}{2l-1}\varepsilon p^2 Q_{l-2}(x_0),$$

$$\alpha_{l,l-1}^{T} = -\frac{if_V^2 Kp}{4\pi M^2(2l-1)^2}\Bigg[\frac{l+1}{2l+1}\{(2l-1)^2+\varepsilon^2\}p^2 Q_{l+1}(x_0)$$
$$+\{(2l-1)^2\kappa+\varepsilon^2\kappa-2l(l-1)\varepsilon p^2\}Q_l(x_0)$$

$$+\left\{\frac{l[(2l-1)^2-2]}{(2l+1)(2l-3)}\varepsilon^2p^2+\frac{l(2l-1)^2}{2l+1}p^2-2(2l-1)^2\kappa\varepsilon\right\}Q_{l-1}(x_0)$$

$$+\{4(l-1)l\kappa\varepsilon^2-2(l-1)(2l-1)\varepsilon p^2\}Q_{l-2}(x_0)$$

$$+\frac{4(l-2)(l-1)l}{2l-3}\varepsilon^2p^2Q_{l-3}(x_0)\Big],$$

$$\alpha^{TJ}=-\frac{if_V^2Kp}{4\pi M^2(2J+1)^2}[J(J+1)]^{1/2}\Big[\frac{2(J+2)}{2J+3}\varepsilon^2p^2Q_{J+2}(x_0)+2\kappa\varepsilon^2Q_{J+1}(x_0)$$

$$-\frac{2(2J+1)}{(2J-1)(2J+3)}\varepsilon^2p^2Q_J(x_0)-2\varepsilon^2\kappa Q_{J-1}(x_0)$$

$$-\frac{2(J-1)}{2J-1}\varepsilon^2p^2Q_{J-2}(x_0)\Big],$$

$$\alpha_l^T=-\frac{if_V^2Kp}{4\pi M^2}[(\kappa+p^2x_0)(1+\varepsilon^2-2\varepsilon x_0)Q_l(x_0)$$

$$+\{2(\kappa+p^2x_0)\varepsilon-(1+\varepsilon^2)p^2\}\delta_{0l}].$$

(4) Axial vector meson

Axial vector coupling:

$$\alpha_{l,l+1}^A=\frac{ig_A^2Kp}{4\pi\cdot 2(2l+3)^2}\Big[4(l+1)(l+2)\varepsilon^2Q_{l+2}(x_0)$$

$$-\left\{4l(2l+3)+2(2l+3)\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{l+1}(x_0)$$

$$+\left\{(2l+3+\varepsilon)^2+2(2l+3)\left(\frac{2M}{\mu_A}\right)^2\varepsilon\right\}Q_l(x_0)\Big],$$

$$\alpha_{l,l}^A=\frac{ig_A^2Kp}{4\pi\cdot 2(2l+1)}\Big[2l\left\{2-\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{l+1}(x_0)$$

$$+(2l+1)\left\{(1+\varepsilon)^2+2\left(\frac{2M}{\mu_A}\right)^2\varepsilon\right\}Q_l(x_0)$$

$$+2(l+1)\left\{2-\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{l-1}(x_0)\Big],$$

$$\alpha_{l,l-1}^A=\frac{ig_A^2Kp}{4\pi\cdot 2(2l-1)^2}\Big[\left\{(2l-1-\varepsilon)^2-2(2l-1)\left(\frac{2M}{\mu_A}\right)^2\varepsilon\right\}Q_l(x_0)$$

$$+\left\{-4(l+1)(2l-1)-2(2l-1)\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{l-1}(x_0)$$

$$+4(l-1)l\varepsilon^2Q_{l-2}(x_0)\Big],$$

$$\alpha^{AJ}=-\frac{ig_A^2Kp}{4\pi\cdot 2(2J+1)}[J(J+1)]^{1/2}\Big[2\left\{1-\frac{\varepsilon}{2J+1}+\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{J+1}(x_0)$$

$$+4\left\{3-\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_J(x_0)+2\left\{1+\frac{\varepsilon}{2J+1}+\left(\frac{2M}{\mu_A}\right)^2\right\}\varepsilon Q_{J-1}(x_0)\Big],$$

$$\alpha_l^A = -\frac{ig_A^2 Kp}{4\pi\cdot 2}\left[\left\{3+2\varepsilon+3\varepsilon^2-2\left(\frac{2M}{\mu_A}\right)^2\varepsilon(x_0-1)\right\}Q_l(x_0)+2\left(\frac{2M}{\mu_A}\right)^2\varepsilon\delta_{l0}\right].$$

Pseudotensor coupling:

$$\alpha_{l,l+1}^{PT}=\frac{2if_A^2Kp\varepsilon}{4\pi M^2(2l+3)}\left[\frac{l+3}{2l+3}p^2Q_{l+2}(x_0)+\left(\kappa-\frac{l+1}{2l+1}p^2\right)Q_{l+1}(x_0)\right.$$
$$\left.-\left(\kappa-\frac{l+1}{2l+3}p^2\right)Q_l(x_0)-\frac{l}{2l+1}p^2Q_{l-1}(x_0)\right],$$

$$\alpha_{l,l}^{PT}=\frac{2if_A^2Kp\varepsilon}{4\pi M^2(2l+1)}\left[\frac{l(l+2)}{2l+3}p^2Q_{l+2}(x_0)+\{l\kappa-(l+1)p^2\}Q_{l+1}(x_0)\right.$$
$$-\left\{(2l+1)\kappa-l(l+1)\left(\frac{1}{2l+3}+\frac{1}{2l-1}\right)p^2\right\}Q_l(x_0)$$
$$\left.+\{(l+1)\kappa-lp^2\}Q_{l-1}(x_0)+\frac{(l-1)(l+1)}{2l-1}p^2Q_{l-2}(x_0)\right],$$

$$\alpha_{l,l-1}^{PT}=\frac{2if_A^2Kp\varepsilon}{4\pi M^2(2l-1)}\left[\frac{l+1}{2l+1}p^2Q_{l+1}(x_0)+\left(\kappa-\frac{l}{2l-1}p^2\right)Q_l(x_0)\right.$$
$$\left.-\left(\kappa-\frac{l}{2l+1}p^2\right)Q_{l-1}(x_0)-\frac{l-1}{2l-1}p^2Q_{l-2}(x_0)\right],$$

$$\alpha^{PTJ}=\frac{2if_A^2Kp\varepsilon}{4\pi M^2(2J+1)}[J(J+1)]^{1/2}\left[\frac{J+2}{2J+3}p^2Q_{J+2}(x_0)\right.$$
$$+\left\{\kappa-\frac{2(J+1)}{2J+1}p^2\right\}Q_{J+1}(x_0)-\left\{2\kappa-\left(\frac{J+1}{2J+3}+\frac{J}{2J-1}\right)p^2\right\}Q_J(x_0)$$
$$\left.+\left\{\kappa-\frac{2J}{2J+1}p^2\right\}Q_{J-1}(x_0)+\frac{J-1}{2J-1}p^2Q_{J-2}(x_0)\right],$$

$$\alpha_l^{PT}=-\frac{2if_A^2Kp\varepsilon}{4\pi M^2}[(\kappa+p^2x_0)(x_0-1)Q_l(x_0)-(\kappa+p^2x_0-p^2)\delta_{l0}].$$

References

1) N. Hoshizaki, S. Otsuki, W. Watari and M. Yonezawa, Soryushiron Kenkyu (mimeographed circular in Japanese) **23** (1961), 539; Prog. Theor. Phys. **27** (1962), 1199.
2) S. Sawada, T. Ueda, W. Watari and M. Yonezawa, Prog. Theor. Phys. **28** (1962), 991.
3) R. S. McKean, Jr., Phys. Rev. **125** (1962), 1399.
4) D. B. Lichtenberg, Nuovo Cim. **25** (1962), 1106.
5) R. A. Bryan, C. R. Dismukes and W. Ramsay, Nucl. Phys. **45** (1963), 353.
6) A. Scotti and D. Y. Wong, Phys. Rev. Letters **10** (1963), 142; Phys. Rev. **138** (1964), B145.
7) S. Sakata, Prog. Theor. Phys. **16** (1956), 686; Prog. Theor. Phys., Suppl. No. 19 (1961).
8) S. Ogawa, Prog. Theor. Phys. **21** (1959), 209.
9) M. Ikeda, S. Ogawa and Y. Ohnuki, Prog. Theor. Phys. **22** (1959), 715; **23** (1960), 1073.
M. Ikeda, Y. Miyachi, S. Ogawa, S. Sawada and M. Yonezawa, Prog. Theor, Phys. **25** (1961), 1.

10) Y. Yamaguchi, Prog. Theor. Phys., Suppl. No. 11 (1959), 1.
11) S. Sawada and M. Yonezawa, Prog. Theor. Phys. **23** (1960), 662.
12) K. Matumoto, Prog. Theor. Phys. **16** (1956), 583.
13) Y. Ne'eman, Nucl. Phys. **26** (1961), 222.
M. Gell-Mann, Phys. Rev. **125** (1962), 1068.
14) P. Tarjanne and V. L. Teplitz, Phys. Rev. Letters **11** (1963), 447.
15) Z. Maki, Prog. Theor. Phys. **31** (1964), 331, 333.
Z. Maki and Y. Ohnuki, Prog. Theor. Phys. **32** (1964), 114.
Y. Hara, Phys. Rev. **134** (1964), B701.
16) M. Gell-Mann, Phys. Letters **8** (1964), 214. G. Zweig, CERN, preprint.
17) G. F. Chew and F. E. Low, Phys. Rev. **101** (1956), 1570.
18) R.B. Sternheimer and S. J. Lindenbaum, Phys. Rev. **109** (1958), 1723.
S. J. Lindenbaum and R. B. Sternheimer, Phys. Rev. Letters **5** (1960), 24.
S. Sawada, Prog. Theor. Phys. **25** (1961), 83.
S. Sawada, T. Ueda and M. Yonezawa, Prog. Theor. Phys. **25** (1961), 873.
19) Y. Fujimoto and H. Miyazawa, Prog. Theor. Phys. **5** (1950), 1052.
20) G. F. Chew, *Proceedings of the International Conference on High Energy Nuclear Physics*, Geneva (1962), 525.
R. H. Capps, Phys. Rev. Letters **10** (1963), 312.
21) M. Taketani, S. Nakamura and M. Sasaki, Prog. Theor. Phys. **6** (1951), 578.
22) J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, Prog. Theor. Phys., Suppl. No. 3 (1956), Part II.
23) M. Taketani, S. Machida and S. Ohnuma, Prog. Theor, Phys. **7** (1952), 45.
24) T. Konuma, H. Miyazawa and S. Otsuki, Prog. Theor. Phys. **19** (1958), 17.
25) S. Okubo and S. Sato, Prog. Theor. Phys. **21** (1959), 383.
K. A. Brueckner and K. M. Watson, Phys. Rev. **90** (1953), 699.
S. Gartenhaus, Phys. Rev. **100** (1955), 900.
See also Prog. Theor. Phys., Suppl. No. 3 (1956).
26) N. Hoshizaki and S. Machida, Prog. Theor. Phys. **27** (1962), 288.
27) C. Møller and L. Rosenfeld, Proc. Phys. Soc. (Copenhagen), **17** (1940), 8.
28) M. Taketani, Prog. Theor. Phys., Suppl. No. 3 (1956), Part I.
29) L. Rosenfeld, *Nuclear Force* (North-Holland Publishing Company, Amsterdam, 1948).
30) S. N. Gupta, Phys. Rev. Letters **2** (1959), 124.
31) J. J. Sakurai, Phys. Rev. **119** (1960), 1784.
32) G. Breit, Proc. Natl. Acad. Sci. (U. S. A.) **46** (1960), 764; Phys. Rev. **120** (1960), 278.
33) Y. Fujii, Prog. Theor. Phys. **25** (1961), 441.
34) M. Kikugawa, Prog. Theor. Phys. **31** (1964), 654.
35) T. Ueda, Prog. Theor. Phys. **29** (1963), 829.
36) M. Kikugawa, S. Sawada, T. Ueda, W. Watari and M. Yonezawa, Prog. Theor. Phys., Suppl. Extra Number (1965), 564 and Prog. Theor. Phys. **37** (1967), 88.
37) L. Wolfenstein, Phys. Rev. **96** (1954), 1654.
H. P. Stapp, T. J. Ypsilantis and N. Metropolis, Phys. Rev. **105** (1957), 302.
38) D. Amati, E. Leader and B. Vitale, Nuovo Cim. **17** (1960), 68.
S. Furuichi, Prog. Theor. Phys. **29** (1963), 235.
39) S. Sawada, T. Ueda, W. Watari and M. Yonezawa, Prog. Theor. Phys. **32** (1964), 380.
40) N. Hiroshige, T. Ino and M. Matsuda, Prog. Theor. Phys. **31** (1964), 719.
T. Ino, M. Matsuda and S. Sawada, Prog. Theor. Phys. **33** (1965), 489.
41) S. Furuichi, S. Sawada, T. Ueda, W. Watari and M. Yonezawa, Prog. Theor. Phys. **32** (1964), 966.
42) W. Ramsay, Phys. Rev. **130** (1963), 1552.
43) R. A. Bryan and R. A. Arndt, preprints.
For the later similar analysis, see also R. A. Arndt, R. A. Bryan and M. H. MacGregor, Phys. Letters **21** (1966), 314.

44) G. Köpp and G. Kramer, Phys. Letters **19** (1965), 593.
See also G. Köpp, Z. Physik **191** (1966), 273, and G. Köpp and P. Söding, Phys. Letters **23** (1966), 494.
45) M. Matsuda and S. Sawada, Prog. Theor. Phys. **31** (1964), 804.
46) T. Hamada, Prog. Theor. Phys. **24** (1960), 220, 1033.
47) T. Hamada and I. D. Johnston, Nucl. Phys. **34** (1962), 382.
48) R. Tamagaki, M. Wada and W. Watari, Prog. Theor. Phys. **31** (1964), 632; **33** (1965), 55.
49) V. V. Babikov, Prog. Theor. Phys. **29** (1963), 712.
50) J. L. Gammel and R. M. Thaler, Phys. Rev. **107** (1957), 291.
51) R. A. Bryan and B. L. Scott, Phys. Rev. **135** (1964), B434.
52) K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald and G. Breit, Phys. Rev. **126** (1962), 881.
53) N. Hoshizaki, I. Lin and S. Machida, Prog. Theor. Phys. **24** (1961), 480.
54) P. B. Kantor, Phys. Rev. Letters **12** (1964), 52.
55) M. H. MacGregor, Phys. Rev. Letters **12** (1964), 403.
56) M. J. Moravcsik, Ann. of Phys. **30** (1964), 10.
57) A. Abashian, N. E. Booth and K. M. Crowe, Phys. Rev. Letters **5** (1960), 258.
58) P. G. Thurnauer, Phys. Rev. Letters **14** (1965), 985.
L. Brown and P. Singer, Phys. Rev. **133** (1964), B812.
59) V. Hagopian and W. Selove, Phys. Rev. Letters **10** (1963), 533.
G. Wolf, Phys. Letters **19** (1965), 328.
60) M. Kikugawa, S. Sawada, M. Yonezawa, T. Ueda and W. Watari, Nuovo Cim. **40** (1965), 647.
61) D. B. Lichtenberg and G. C. Summerfield, Phys. Rev. **127** (1962), 1806.
62) For example, see L. N. Hand, D. G. Miller and R. Wilson, Rev. Mod. Phys. **35** (1963), 335.
63) A. H. Rosenfeld, A. Barbaro-Galtieri, W. H. Barkas, P. L. Bastien, J. Kirz and M. Roos, Rev. Mod. Phys. **37** (1965), 633.
64) E. Ferrari and F. Selleri, Phys. Rev. Letters **7** (1961), 387.
65) Y. Hama and N. Hoshizaki, Prog. Theor. Phys. **34** (1965), 584.
Y. Hama, Prog. Theor. Phys. **35** (1966), 261.
66) N. J. Sopkovich, Nuovo Cim. **26** (1962), 186.
R. Gottfried and J. D. Jackson, Nuovo Cim. **34** (1964), 735.
67) M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12** (1964), 627.
68) S. Furuichi and M. Yonezawa, Prog. Theor. Phys. **33** (1965), 238.
69) S. Otsuki, S. Sawada, R. Tamagaki and W. Watari, Prog. Theor. Phys. **35** (1966), 181.
70) M. Yamada, Prog. Theor. Phys. **31** (1964), 1094.
71) Y. Hama and N. Hoshizaki, Prog. Theor. Phys. **34** (1965), 455.
72) S. Furuichi and W. Watari, Prog. Theor. Phys. **34** (1965), 594; **36** (1966), 348.
73) G. Breit, M. H. Hull, Jr., K. E. Lassila, K. D. Pyatt, Jr. and H. M. Ruppel, Phys. Rev. **128** (1962), 826.
M. H. Hull, Jr., K. E. Lassila, H. M. Ruppel, F. A. McDonald and G. Breit, Phys. Rev. **128** (1962), 830.
74) H. P. Noyes, D. S. Bailey, R. A. Arndt and M. H. MacGregor, Phys. Rev. **139** (1965), B380.