Workshop 3: Functions and modules

FIE463: Numerical Methods in Macroeconomics and Finance using Python

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See GitHub repository for notebooks and data:

https://github.com/richardfoltyn/FIE463-V25

Exercise 1: Standard deviation of a sequence of numbers

The standard deviation σ characterizes the dispersion of a sequence of data $(x_1, x_2, \dots, x_N)$ around its mean $\overline{x}$. It is computed as the square root of the variance σ^2, defined as

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} \Big(x_i - \overline{x}\Big)^2$$

where N is the number of elements (we ignore the degrees-of-freedom correction), and the mean $\overline{x}$ is defined as

$$\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$$

The above formula for the variance can be rewritten as

$$\sigma^2 = \left(\frac{1}{N} \sum_{i=1}^{N} x_i^2 \right) - \overline{x}^2$$

This suggests the following algorithm to compute the standard deviation:

1. Compute the mean $\overline{x} = \frac{1}{N} \sum_{i=1}^{N} x_i$
2. Compute the mean of squares $S = \frac{1}{N} \sum_{i=1}^{N} x_i^2$
3. Compute the variance $\sigma^2 = S - \overline{x}^2$
4. Compute the standard deviation $\sigma = \sqrt{\sigma^2}$

In this exercise, you are asked to implement the above algorithm and compare your function with NumPy's implementation `np.std()`.

1. Create a module `my_stats.py` and add the function

   ```python
   def my_std(x):
       """
       Compute and return the standard deviation of the sequence x.
       """
   ```

   which implements the above algorithm to compute the standard deviation of a given sequence `x` (this could be a tuple, list, array, etc.). Your implementation should *only use built-in functions* such as `len()`, `sum()`, and `sqrt()` from the `math` module.

2. Import this function into the Jupyter notebook. Using an array of 11 elements which are uniformly spaced on the interval $[0, 10]$, confirm that your function returns the same value as `np.std()`.
3. Benchmark your implementation against `np.std()` for three different arrays with 11, 101, and 10001 elements which are uniformly spaced on the interval $[0, 10]$.

   Hint: Use the cell magic `%timeit` to time the execution of a statement.

Exercise 2: Locating maximum values

In this exercise, you are asked to write a function that returns the position of the largest element from a given sequence (list, tuple, array, etc.).

1. Write a function `my_argmax()` that takes as argument a sequence and returns the (first) index where the maximum value is located. Only use built-in functionality in your implementation (no NumPy).
2. Create an array with 101 values constructed using the sine function,

   ```
   arr = np.sin(np.linspace(0.0, np.pi, 101))
   ```

   and use it to test your function.
3. Compare the result returned by your function to NumPy's implementation `np.argmax()`.

Exercise 3: Two-period consumption-savings problem

This exercise asks you to find the utility-maximizing consumption levels using grid search, an algorithm that evaluates all possible alternatives from a given set (the "grid") to locate the maximum.

Consider the following standard consumption-savings problem over two periods with lifetime utility $U(c_1, c_2)$ given by

$$\begin{aligned} \max_{c_1,\, c_2} \quad & U(c_1, c_2) = u(c_1) + \beta u(c_2) \\ \text{s.t.} \quad & c_1 + \frac{c_2}{1+r} = w \\ & c_1 \geq 0,\ c_2 \geq 0 \end{aligned}$$

where β is the discount factor, r is the interest rate, w is initial wealth, (c_1, c_2) is the optimal consumption allocation to be determined. The second line is the budget constraint which ensures that the chosen consumption bundle (c_1, c_2) is feasible. The per-period CRRA utility function $u(c)$ is given by

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(c) & \text{if } \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion (RRA) and $\log(\bullet)$ denotes the natural logarithm.

1. Write a function `util(c, gamma)` which evaluates the per-period utility $u(c)$ for a given consumption level c and the parameter γ. Make sure to take into account the log case!

   Hint: You can use the `np.log()` function from NumPy to compute the natural logarithm.
2. Write a function `util_life(c_1, c_2, beta, gamma)` which uses `util()` from above to compute the lifetime utility $U(c_1, c_2)$ for given consumption levels (c_1, c_2) and parameters.
3. Assume that $r = 0.04$, $\beta = 0.96$, $\gamma = 1$, and $w = 1$.

   - Create a candidate array (grid) of period-1 consumption levels with 100 grid points with are uniformly spaced on the on the interval $[\epsilon, w - \epsilon]$ where $\epsilon = 10^{-5}$.

     Note that we enforce a minimum consuption level ϵ, as zero consumption yields $-\infty$ utility for the given preferences which can never be optimal.

- Compute the implied array of period-2 consumption levels from the budget constraint.
- Given these candidate consumption levels, use the function `util_life()` you wrote earlier to evaluate lifetime utility for each bundle of consumption levels (c_1, c_2).

4. Use the function `np.argmax()` to locale the index at which lifetime utility is maximized. Print the maximizing consumption levels (c_1, c_2) as well as the associated maximized utility level.