

# Workshop 2: Control flow and list comprehensions

FIE463: Numerical Methods in Macroeconomics and Finance using Python

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See GitHub repository for notebooks and data:

<https://github.com/richardfoltyn/FIE463-V25>

## Exercise 1: CRRA utility function

The CRRA utility function (constant relative risk aversion) is the most widely used utility function in macroeconomics and finance. It is defined as

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(c) & \text{else} \end{cases}$$

where  $c$  is consumption and  $\gamma$  is the (constant) risk-aversion parameter, and  $\log(\bullet)$  denotes the natural logarithm.

1. You want to evaluate the utility at  $c = 2$  for various levels of  $\gamma$ .
  1. Define a list `gammas` with the values 0.5, 1, and 2.
  2. Loop over all elements in `gammas` and evaluate the corresponding utility. Use an `if` statement to correctly handle the two cases from the above formula.

*Hint:* Import the `log` function from the `math` module to evaluate the natural logarithm:

```
from math import log
```

*Hint:* To perform exponentiation, use the `**` operator (see the [list of operator](#)).

3. Store the utility in a dictionary, using the values of  $\gamma$  as keys, and print the result.
2. Can you solve the exercise using a single list comprehension to create the result dictionary?

*Hint:* You will need to use a conditional expression we covered in the lecture.

## Exercise 2: Maximizing quadratic utility

Consider the following quadratic utility function

$$u(c) = -A(c - B)^2 + C$$

where  $A > 0$ ,  $B > 0$  and  $C$  are parameters, and  $c$  is the consumption level.

In this exercise, you are asked to locate the consumption level which delivers the maximum utility.

1. Find the maximum using a loop:
  1. Create an array `cons` of 51 candidate consumption levels which are uniformly spaced on the interval  $[0, 4]$ .
  2. Use the parameters  $A = 1$ ,  $B = 2$ , and  $C = 10$ .
  3. Loop through all candidate consumption levels, and compute the associated utility. If this utility is larger than the previous maximum value `u_max`, update `u_max` and store the associated consumption level `cons_max`.
  4. Print `u_max` and `cons_max` after the loop terminates.
2. Repeat the exercise, but instead use vectorized operations from NumPy:
  1. Compute and store the utility levels for *all* elements in `cons` at once (simply apply the formula to the whole array).
  2. Locate the index of the maximum utility level using `np.argmax()`.
  3. Use the index returned by `np.argmax()` to retrieve the maximum utility and the corresponding consumption level, and print the results.

### Exercise 3: Summing finite values

In this exercise, we explore how to ignore non-finite array elements when computing sums, i.e., elements which are either NaN (“Not a number”, represented by `np.nan`),  $-\infty$  (`-np.inf`) or  $\infty$  (`np.inf`). Such situations arise if data for some observations is missing and is then frequently encoded as `np.nan`.

1. Create an array of 1001 elements which are uniformly spaced on the interval  $[0, 10]$ . Set every second element to the value `np.nan`.  
*Hint:* You can select and overwrite every second element using `start:stop:step` array indexing. Using `np.sum()`, verify that the sum of this array is NaN.
2. Write a loop that computes the sum of finite elements in this array. Check that an array element is finite using the function `np.isfinite()` and ignore non-finite elements.  
 Print the resulting sum of finite elements.
3. Since this use case is quite common, NumPy implements the function `np.nansum()` which performs exactly this task for you.

Verify that `np.nansum()` gives the same result and benchmark it against your loop-based implementation.

*Hint:* You’ll need to use the `%timeit` [cell magic](#) (with two %) if you want to benchmark all code contained in a cell.

### Exercise 4: Approximating the sum of a geometric series

Let  $\alpha \in (-1, 1)$ . The sum of the geometric series  $(1, \alpha, \alpha^2, \dots)$  is given by

$$\sigma = \sum_{i=0}^{\infty} \alpha^i = \frac{1}{1 - \alpha}$$

In this exercise, you are asked to approximate this sum using the first  $N$  values of the sequence, i.e.,

$$\sigma \approx s_N = \sum_{i=0}^N \alpha^i$$

where  $N$  is chosen to be sufficiently large.

1. Assume that  $\alpha = 0.9$ . Write a while loop to approximate the sum  $\sigma$  by computing  $s_N$  for an increasing  $N$ . Terminate the computation as soon as an additional increment  $\alpha^N$  is smaller than  $10^{-10}$ . Compare your result to the exact value  $\sigma$ .
2. Now assume that  $\alpha = -0.9$ . Adapt your previous solution so that it terminates when the *absolute value* of the increment is less than  $10^{-10}$ . Compare your result to the exact value  $\sigma$ .

*Hint:* Use the built-in function `abs( )` to compute the absolute value.