Cluster model of A=7 nuclei revisited, and the astrophysical S factors for ^{3}He(α,γ)^{7}Be and ^{3}H(α,γ)^{7}Li at zero energy

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LETTER TO THE EDITOR

Cluster model of $A = 7$ nuclei revisited, and the astrophysical S factors for $^3He(\alpha, \gamma)^7Be$ and $^3H(\alpha, \gamma)^7Li$ at zero energy

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Abstract. A local potential cluster model treatment of the $A = 7$ nuclei is updated in the light of a recent re-analysis of old experimental data and some new experimental measurements. The model is then used to calculate directly (without any energy extrapolation) the astrophysical S factors for $^3He(\alpha, \gamma)^7Be$ and $^3H(\alpha, \gamma)^7Li$ at zero energy. We find $S_0 = 0.51 \pm 0.03$ keV b for the former reaction and $S_0 = 0.09 \pm 0.03$ keV b for the latter, in good agreement with direct cross section measurements.

The radiative capture reaction, $^3He(\alpha, \gamma)^7Be$, at low energies is of considerable astrophysical significance. Not only is it an important link in the chain of reactions leading to a prediction of the high-energy solar neutrino flux due to ^8B decay, but it also mediates the production of ^7Li (when followed by electron capture) at high densities in the standard Big Bang model of the creation of the universe. At lower baryon densities, this latter role is taken over by the mirror reaction, $^3H(\alpha, \gamma)^7Li$. It is therefore a matter of fundamental importance to know these low-energy reaction rates, or astrophysical S factors $S_E = E \exp(2\pi\eta)\sigma_{cap}(E)$, as accurately as possible.

Unfortunately, both experimental measurements and theoretical estimates of the zero-energy S factors for the above-mentioned reactions show considerable discrepancies. For example, two recent measurements of S_0 for $^3He(\alpha, \gamma)^7Be$ claim values of 0.63 ± 0.04 keV b (Robertson *et al* 1983) and 0.47 ± 0.02 keV b (Alexander *et al* 1984). In particular, it should be noted that activation measurements yield values systematically higher by about 20% than those obtained from direct observations of the cross section. A similar level of disagreement is also evident in calculations of S_0. Using the same resonating group method (RGM), but with different nucleon–nucleon potentials and internal cluster wavefunctions, Walliser *et al* (1983, 1984) find $S_0 = 0.598$ keV b, while Kajino *et al* (1984) (see also Kajino and Arima 1984) find $S_0 = 0.508$ keV b. The situation with regard to the reaction $^3H(\alpha, \gamma)^7Li$ is little better, with measurements ranging from 0.100 ± 0.025 keV b (Griffiths *et al* 1961) to 0.134 ± 0.020 keV b (Schröder *et al* 1987). All of these values are consistent with the limits derived by Kajino *et al* (1988b) from the nuclear matter radii of ^7Be and ^7Li as $0.36 < S_0 < 0.63$ keV b for $^3He(\alpha, \gamma)^7Be$ and $0.083 < S_0 < 0.15$ keV b for $^3H(\alpha, \gamma)^7Li$.

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It is the intention of this letter to try and throw a little more light on the question of the theoretical estimates of S_0 for these two reactions. Part of the difficulty is associated with the generally adopted procedure of calculating the capture cross section at low energies and then extrapolating the results to zero energy. However, Buck *et al* (1985) have presented a simple method of circumventing this source of uncertainty by solving the appropriate zero-energy Schrödinger equation directly and then matching the wavefunction on to a linear combination of limiting forms of the Coulomb wavefunctions. We shall make use of this development and also continue to use their cluster model which is vindicated by its ability to reproduce many independent data on ^{7}Li and ^{7}Be to very high precision.

The essence of this model is to describe ^{7}Li and ^{7}Be as α—^{3}H and α—^{3}He systems respectively, with the clusters having essentially unperturbed free space properties. The cluster core interaction is taken to have a Gaussian form, and its parameters are determined unambiguously from the experimentally measured properties of ^{7}Li and ^{7}Be. The relation of this elementary cluster model to the microscopic RGM has recently been investigated by Walliser and Fliessbach (1985), who conclude that as a model of ^{7}Li it is valid and useful, with a similar degree of justification to that of the nucleon picture of the deuteron.

We wish to re-examine this model here, because recent theoretical and experimental developments have cast doubt on the values of some of the quantities used to determine the parameters of the intercluster potential in the earlier work of Buck *et al* (1985). In particular, a careful atomic calculation by Sundholm *et al* (1984) has concluded that the value of the ^{7}Li electric quadrupole moment, Q, deduced by Green (1971) should actually be $-4.06\pm0.03\ e\ \mathrm{fm}^2$ and not $-3.66\pm0.03\ e\ \mathrm{fm}^2$ as previously assumed. Furthermore, two other recent measurements also suggest larger magnitudes with $Q=-4.0\pm1.1\ e\ \mathrm{fm}^2$ (Vermeer *et al* 1984a, b) and $-3.70\pm0.08\ e\ \mathrm{fm}^2$ (Weller *et al* 1985). Since Q was the most accurate experimental input in the determination of the mean square cluster–cluster separation distance, $\langle R^2\rangle$, in the earlier work of Buck *et al* (1985), a significant change in the cluster–cluster potential parameters may be expected as a result of this new information. Therefore, one aim of the present work is to reanalyse the model, taking these new data into account. Another is to apply the method of calculating S_E exactly at zero energy to the $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ reaction, and so calculate the corresponding value of S_0, without introducing any further free parameters.

The elementary cluster model we employ visualises an alpha particle interacting with a ^{3}H or ^{3}He cluster via a local potential. The nuclear part of this potential contains central and spin–orbit terms, and may be conveniently parametrised in Gaussian form as

$$V(R)=-(V+4\alpha V_{\mathrm{SO}}\boldsymbol{L}\cdot\boldsymbol{\sigma})\exp(-\alpha R^2) \tag{1}$$

where R is the distance between the centres of the two clusters, $\boldsymbol{L}$ is their relative orbital angular momentum, $\boldsymbol{\sigma}$ are the Pauli spin matrices associated with ^{3}H or ^{3}He, while the width α, as well as the strengths V and V_{SO}, are free parameters to be fitted to experimental data. The α—^{3}H or α—^{3}He Coulomb interaction is written as

$$\begin{aligned} V_{\mathrm{C}}(R)&=Z_1Z_2e^2/R \qquad R>R_{\mathrm{C}} \\ &=(Z_1Z_2e^2/2R_{\mathrm{C}})[3-(R/R_{\mathrm{C}})^2] \qquad R<R_{\mathrm{C}}. \end{aligned} \tag{2}$$

where Z_1 and Z_2 are the charges of the two clusters, and the Coulomb radius R_{C} is our fourth and final free parameter.

We ensure that the major requirements of the Pauli principle are satisfied by choosing relative motion quantum numbers N (principal quantum number) and L to correspond to the microscopic situation in which the ^{3}H or ^{3}He nucleons occupy shell-model orbitals above the (0s) orbitals filled by the alpha particle. Assuming no internal excitation of either body, this means that $2N+L=3$, so $L=1$, $N=1$ or $L=3$, $N=0$, and the spin–orbit force splits these L-centroids into $3/2^-$–$1/2^-$ and $7/2^-$–$5/2^-$ doublets respectively.

Several observables of the $A=7$ nuclei, such as the ft values, magnetic dipole moments and $B(\mathrm{M1};\ 3/2^- \rightarrow 1/2^-)$ transition strengths can now be easily calculated by adding the assumption that the $L=1$ wavefunctions overlap close to unity (i.e. $\langle 1/2|3/2\rangle \simeq 1$) to the above structural hypotheses. These quantities are unchanged by the new experimental input to the model, and remain as given by Buck *et al* (1985). The four free parameters introduced in equations (1) and (2) however, are affected. They are determined from the binding energies of ^{7}Li and ^{7}Be, the $3/2^-$–$1/2^-$ energy difference and the value of $\langle R^2\rangle$ for ^{7}Li. This latter quantity is deduced from the root mean square charge radius, r_c, the electric quadrupole moment, Q, and the $B(\mathrm{E2};\ 3/2^- \rightarrow 1/2^-)$ transition strength using the following formulae:

$$\langle r_c^2\rangle_7 = \tfrac{1}{7}\langle r_c^2\rangle_t + \tfrac{2}{7}\langle r_c^2\rangle_\alpha + \tfrac{34}{147}\langle R^2\rangle \tag{3}$$

$$Q = -\tfrac{68}{245}\langle R^2\rangle \tag{4}$$

$$B(\mathrm{E2};\ 3/2^- \rightarrow 1/2^-) = \left(\frac{17}{49}\right)^2 \frac{|\langle 1/2|R^2|3/2\rangle|^2}{\pi} \tag{5}$$

where the subscripts 7, t and α refer to ^{7}Li, ^{3}H and the alpha particle respectively, and we deviate slightly from Buck *et al* (1985) by writing explicitly $\langle 1/2|R^2|3/2\rangle$ instead of the approximation $\langle 1/2|R^2|3/2\rangle \simeq \langle R^2\rangle$ in equation (5). We then use the empirical result that $\langle 1/2|R^2|3/2\rangle$ is about 3% larger than $\langle R^2\rangle$ for a wide range of potential parameters when we come to include the $B(\mathrm{E2}\uparrow)$ data in our fits.

We take a weighted mean of the experimental measurements of $\langle r_c^2\rangle^{1/2}$, Q and $B(\mathrm{E2}\uparrow)$ presented in table 1 to obtain $\langle R^2\rangle = 13.8 \pm 0.4\ \mathrm{fm}^2$. We have thus included all the data used by Buck *et al* (1985) (with the value of $Q = -3.66 \pm 0.03\ e\ \mathrm{fm}^2$ corrected to $-4.06 \pm 0.03\ e\ \mathrm{fm}^2$) and in addition the recent measurements of Q and $B(\mathrm{E2}\uparrow)$ by Vermeer *et al* (1984a, b) and Weller *et al* (1985). Our result is a little larger than the previous value of $\langle R^2\rangle = 13.5 \pm 0.3\ \mathrm{fm}^2$, and yields an unambiguous determination of our four free parameters as: $V = 83.78$ MeV, $V_{SO} = 1.003$ MeV, $\alpha = 0.157\,47\ \mathrm{fm}^{-2}$ and $R_C = 3.095$ fm, which do not differ very greatly from $V = 86.08$ MeV, $V_{SO} = 0.957$ MeV, $\alpha = 0.163\ \mathrm{fm}^{-2}$ and $R_C = 3.248$ fm found by Buck *et al* (1985). We obtain an excellent description of those observables in ^{7}Li which depend on $\langle R^2\rangle$ (see table 1), including the root mean square magnetisation radius, $\langle r_m^2\rangle_7^{1/2}$, and the magnetic octupole moment, Ω, where we take

$$\mu_7\langle r_m^2\rangle_7 = \tfrac{4}{21}\langle r_c^2\rangle_t + \tfrac{3}{14}\langle r_c^2\rangle_\alpha + \mu_t\langle r_m^2\rangle_t + (\tfrac{209}{3430} + \tfrac{432}{1225}\mu_t)\langle R^2\rangle \tag{6}$$

and

$$\Omega = -\tfrac{48}{245}\mu_t\langle R^2\rangle. \tag{7}$$

We also obtain an excellent description of the widths of the quasibound $7/2^-$ and $5/2^-$ states in ^{7}Li and ^{7}Be, which decay by breaking up into their $\alpha - {}^3$H and $\alpha - {}^3$He components respectively. Adjusting our central potential depth slightly to reproduce the experimental energies of each of the states exactly, we obtain the values shown in

Table 1. A comparison between the experimental and calculated observables in ^{7}Li which depend only on $\langle R^2 \rangle$, and between the experimental and calculated zero-energy astrophysical S factors, S_0, and their derivatives with respect to energy, $dS/dE|_0$, for the radiative capture reactions ^{3}He$(\alpha, \gamma)^7$Be and ^{3}H$(\alpha, \gamma)^7$Li at zero energy.

Quantity	Experimental values	This calculation	BBR[a]	RGM[b]
$\langle r_c^2 \rangle^{1/2}$ (fm)	2.41 ± 0.10[c]	2.43 ± 0.02	2.42 ± 0.015	2.55
Q (e fm^2)	-4.06 ± 0.03[d] -3.70 ± 0.08[e] -4.0 ± 1.1[f]	-3.83 ± 0.13	-3.74 ± 0.08	-4.41
B(E2; ↑) (e^2 fm^4)	8.9 ± 0.6[d] 8.3 ± 0.5[e] 7.42 ± 0.14[f] 6.7 ± 0.2[g] 7.4 ± 0.1[h] 8.3 ± 0.6[i]	7.75 ± 0.50	7.4 ± 0.3	10.57
$\langle r_m^2 \rangle^{1/2}$ (fm)	2.98 ± 0.05[g] 2.70 ± 0.15[j]	2.78 ± 0.03	2.75 ± 0.02	3.04
Ω/μ_7 (fm)^2	$\pm 2.9 \pm 0.1$[g] $\pm 2.8 \pm 0.5$[j]	-2.48 ± 0.08	-2.42 ± 0.05	-3.22
S_0 ^{3}He$(\alpha, \gamma)^7$Be (keV b)	0.47 ± 0.02[k] 0.53 ± 0.03[l] 0.63 ± 0.04[m] 0.51 ± 0.05[n] 0.47 ± 0.05[o]	0.51 ± 0.03	0.47 ± 0.02	0.508
$dS/dE\|_0$ (mb)	-0.28 ± 0.04[n]	-0.27 ± 0.04	-0.23 ± 0.03	
S_0 ^{3}H$(\alpha, \gamma)^7$Li (keV b)	0.100 ± 0.025[p] 0.134 ± 0.020[q]	0.089 ± 0.030	0.084 ± 0.020	0.097
$dS/dE\|_0$ (mb)		-0.18 ± 0.04	-0.15 ± 0.03	

[a]Buck *et al* (1985).
[b]Kajino *et al* (1984, 1988a) and Kajino and Arima (1984).
[c]de Vries *et al* (1987).
[d]Green (1971), but see Sundholm *et al* (1984).
[e]Weller *et al* (1985).
[f]Vermeer *et al* (1984a, b).
[g]van Niftrik *et al* (1971).
[h]Bamberger *et al* (1972).
[i]Häusser *et al* (1973).
[j]Rand *et al* (1966).
[k]Alexander *et al* (1984).
[l]Osborne *et al* (1984).
[m]Robertson *et al* (1983).
[n]Nagatani *et al* (1969).
[o]Parker and Kavanagh (1963).
[p]Griffiths *et al* (1961).
[q]Schröder *et al* (1987).

table 2. We can also give a good account of the s-wave elastic scattering of ^{3}He by ^{4}He by reducing our potential depth to 66.10 MeV (cf 67.67 MeV in Buck *et al* 1985). This rather large change in depth is a manifestation of the strong parity dependence of the effective potentials for this system, which has also been encountered in RGM calculations.

We now calculate S_0 for the capture reactions ^{3}He$(\alpha, \gamma)^7$Be and ^{3}H$(\alpha, \gamma)^7$Li by considering only electric dipole capture from incident s waves as follows. We integrate the zero-energy Schrödinger equation (involving the s-wave potential mentioned above)

$$\left(\frac{-\hbar^2}{2\mu}\frac{d^2}{dR^2} + V(R) + V_C(R)\right)U_0(R) = 0 \tag{8}$$

and match $U_0(R)$ to a linear combination of the limiting forms of the Coulomb functions

$$U_0(R\to\infty)\to R\left(\frac{I_1(2\sqrt{2\varepsilon R})}{\sqrt{2\varepsilon R}} - 4\varepsilon a\frac{K_1(2\sqrt{2\varepsilon R})}{\sqrt{2\varepsilon R}}\right) \tag{9}$$

at some arbitrary distance, R_m, beyond the range of the nuclear forces. This matching of the logarithmic derivatives of the two expressions for $U_0(R)$ immediately yields the scattering length, a. The other quantities in equation (9) are: the modified Bessel functions I_1 and K_1 and $\varepsilon = \mu Z_1 Z_2 e^2/\hbar^2$, where μ is the reduced mass of the cluster–cluster system. The required S factors are then found as $S_0 = S_0(3/2) + S_0(1/2)$ where

$$S_0(j) = \tfrac{32}{441}\pi^2\frac{\varepsilon e^2}{(\hbar c)^3}(2j+1)(B_j)^3|\langle U_j|R|U_0\rangle|^2 \tag{10}$$

for both reactions. B_j is the binding energy and U_j the corresponding normalised wavefunction for the bound state of spin j. Our results are compared with experiment in table 1 (where we also include our calculation of S_0 for ^{3}H$(\alpha, \gamma)^7$Li resulting from the earlier parametrisation of Buck *et al* (1985)).

We have also calculated the energy derivative near to zero energy of the S factor, $dS/dE|_0$, for both reactions using our new potential and that of Buck *et al* (1985). This was done by explicity evaluating the expression for the capture cross section (Christy and Duck 1961) at a few low energies (~ 10 keV) and extracting the corresponding variation of S. The incoming scattering wavefunction was generated by an optical model code employing the Coulomb wavefunction subroutine of Barnett *et al* (1974). We find a small negative slope at the origin for both reactions, which is in good qualitative agreement with the experimental trends. It is also in good quantitative

Table 2. A comparison of the calculated and measured widths, Γ, for break-up of the $7/2^- - 5/2^-$ doublet states of ^{7}Li and ^{7}Be into their α—^{3}H and α—^{3}He components. Experimental data are taken from Ajzenberg-Selove (1984).

	Γ (keV)		
J^π (nucleus)	Experiment	This calculation	BBR[a]
$7/2^-$ (^{7}Li)	93 ± 8	91.9 ± 1.4	84.8 ± 1.3
$5/2^-$ (^{7}Li)	875^{+200}_{-100}	900 ± 35	836 ± 33
$7/2^-$ (^{7}Be)	175 ± 7	167 ± 11	144 ± 10
$5/2^-$ (^{7}Be)	1200	1220 ± 80	1131 ± 77

[a]Results obtained using the cluster–cluster potential parameters of Buck *et al* (1985).

agreement with the value extracted by Nagatani *et al* (1969) for the case of ^{3}He(α, γ)^{7}Be (see table 1).

We see that the four-parameter model gives a consistent description of many independent data on ^{7}Li and ^{7}Be as well as describing accurately S_0 for both ^{3}He(α, γ)^{7}Be and ^{3}H(α, γ)^{7}Li with the same parameter set. The S_0 values agree within errors with those values deduced from direct cross section determinations of Osborne *et al* (1984), Alexander *et al* 1984), Nagatani *et al* (1969) and Parker and Kavanagh (1963) for ^{3}He(α, γ)^{7}Be and of Griffiths *et al* (1961) and Schröder *et al* (1987) for ^{3}H(α, γ)^{7}Li.

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