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Modified two-body potential approach to the peripheral direct capture astrophysical $a + A \to B + \gamma$ reaction and asymptotic normalization coefficients

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Abstract

A modified two-body potential approach is proposed for determination of both the asymptotic normalization coefficient (ANC) (or the respective nuclear vertex constant (NVC)) for the $A + a \to B$ (for the virtual decay $B \to A + a$) from an analysis of the experimental S-factor for the peripheral direct capture $a + A \to B + \gamma$ reaction and the astrophysical S-factor, $S(E)$, at low experimentally inaccessible energy regions. The approach proposed involves two additional conditions which verify the peripheral character of the considered reaction and expresses $S(E)$ in terms of the ANC. The connection between NVC (ANC) and the effective range parameters for Aa-scattering is derived. To test this approach we reanalyse the precise experimental astrophysical S-factors for $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction at energies $E \leqslant 1200$ keV [C.R. Brune et al., Phys. Rev. C 50 (1994) 2205]. The same Wood–Saxon potential form both for the bound $(t + \alpha)$-state wave function and for the αt-scattering wave function is used to guarantee selfconsistency. New estimates have been obtained for the values of the ANC's (the NVC's) for the $\alpha + t \to {}^7\mathrm{Li}$(g.s.), $\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV) and of $S(E)$ at $E \leqslant 50$ keV. These ANC values have been used for getting information about the "indirect" measured values of the effective range parameters and the p-wave phase shift for αt-scattering in the energy range of $100 \lesssim E \lesssim 180$ keV.

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1. Introduction

A reliable estimation of rates of different nuclear astrophysical processes responsible for the light element abundance (for example, ^{6,7}Li, ^{7}Be, ^{8}B, etc.) is one of most important problems of modern nuclear astrophysics [1,2]. A solution of this problem is in turn impossible without obtaining rather low energy cross sections $\sigma(E)$ (or equivalently the astrophysical S-factors $S(E)$) for such reactions as $^{2,3}\mathrm{H} + \alpha \to {}^{6,7}\mathrm{Li} + \gamma$, $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be} + \gamma$, $p + {}^{7}\mathrm{Be} \to {}^{8}\mathrm{B} + \gamma$, etc. For example, reliable information on low energy cross sections $\sigma(E)$ (or $S(E)$) for the direct radiative capture $^{2,3}\mathrm{H} + \alpha \to {}^{6,7}\mathrm{Li} + \gamma$, $^{3}\mathrm{He} + \alpha \to {}^{7}\mathrm{Be} + \gamma$ and $p + {}^{7}\mathrm{Be} \to {}^{8}\mathrm{B} + \gamma$ reactions plays a crucial role for the observed abundances of ^{6,7}Li in big-bang nucleosynthesis calculations [3–8] and of the ^{7}Be and ^{8}B in the solar neutrino problems [9,10].

Despite the impressive improvements in our understanding of these processes made in the past decades (see Refs. [1,2] and references therein), however, some ambiguities connected with both the extrapolation of the measured cross sections for the aforesaid reactions to the stellar energy region and the theoretical predictions for $\sigma(E)$ (or $S(E)$) still exist and they may considerably influence the predictions of the standard solar model [9,10].

As a specific example below we consider the situation concerning the $t + \alpha \to {}^{7}\mathrm{Li} + \gamma$ reaction since the calculations of the astrophysical S-factor, $S(E)$ performed within different methods show considerable spread. The resonating-group method calculations of $S(0)$ show considerable sensitivity to the form of the effective NN-interaction used and the estimates have been obtained within the ranges of $0.10 \leqslant S(0) \leqslant 0.154$ keV b [11–14] and of $0.08 \leqslant S(0) \leqslant 0.19$ keV b [15]. Calculations performed in microscopic single-channel ($(\alpha + t)$) and two-channel ($(\alpha + t)$ and $(n + {}^{6}\mathrm{Li})$) cluster models gave the values of $S(0) \simeq 0.105$ keV b [16] and of $S(0) \approx 0.19$ keV b [17] respectively, that is, the estimation of the value of the $S(0)$ strongly changes when the model space is expanded. Use of data on the experimental electric-dipole polarizability of ^{7}Li led to the value of $S(0) = 0.097 \pm 0.038$ keV b [18]. The calculations performed by authors of Refs. [19] and [20] in the two-body potential model with different forms of the two-body potential gave the same value of $S(0) = 0.1$ keV b, although different values of 1.174 in [19] and 1.0 in [20] have been used for the spectroscopic factor for the $(\alpha + t)$-configuration in ^{7}Li. Calculations performed in the variational Monte Carlo technique (VMCT) with seven-particle wave functions derived from realistic NN-interaction gave $S(0) = 0.095$ keV b [21]. But, as it was emphasized in paper [21], serious problems occur with the low-energy normalization for the calculated astrophysical S-factor $S(E)$ in respect to the high precision in the experimental data [22]. It should be also noted the estimation of $S(0) = 0.093$ keV b [23]. The latter has been obtained within the framework of the asymptotic method developed in [24,25] based on the idea proposed in paper [26]. This idea is based on the assumption about the fact that low-energy direct radiative captures on light nuclei ($a + A \to B + \gamma$) proceed mainly in regions well outside the range of the internuclear interactions. But in Ref. [23] the contribution from the nuclear interior to the amplitude was assumed to be negligibly small. There, in this assumption from the analysis of the experimental astrophysical S-factors for the direct capture the $t + \alpha \to {}^{7}\mathrm{Li(g.s.)} + \gamma$ and the $t + \alpha \to {}^{7}\mathrm{Li}(0.478\ \mathrm{MeV}) + \gamma$ reactions in the energy range $200 \leqslant E \leqslant 400$ keV [22], the values of the asymptotic normalization coefficients (ANC) (or the respective nuclear vertex constants (NVC)) have been obtained, which were used then for calculations of the astrophysical S-factors for the same reactions at $E < 200$ keV, including $E = 0$. However, in [23] the absolute normalization of the calculated astrophysical S-factors for the $t + \alpha \to {}^{7}\mathrm{Li(g.s.)} + \gamma$ and the $t + \alpha \to {}^{7}\mathrm{Li}(0.478\ \mathrm{MeV}) + \gamma$ reactions in the energy range $E < 150$ keV are underestimated in respect to the experimental data [22].

Thus, as it is seen from here, calculations of $S(E)$ for the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction at extremely low energies performed within the framework both of the purely and two-body microscopic models (Refs. [11–18] and [21]) and of the two-body potential approaches (Refs. [19,20] and [23]), where different additional experimental and calculated data (static characteristics of ^{7}Li, scattering phase shifts and spectroscopic factors) were used, still do not clarify the main reason for the existing spread in the predictions of the absolute normalization of the astrophysical S-factor, $S(E)$, for $t + \alpha \to {}^7\mathrm{Li} + \gamma$ at stellar energies.

In this work the modified two-body potential approach is proposed for the direct capture $a + A \to B + \gamma$ reaction, which contains the information on the nuclear vertex constants NVC (or the respective asymptotic normalization coefficients ANC) for the virtual decay $B \to A + a$ and involves two additional conditions verifying the peripheral character of the considered reaction. This method is applied for a reanalysis of the highly precise experimental astrophysical S-factors for the direct capture $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction at energies $55 \leqslant E \leqslant 1200$ keV [22]. This choice is caused by the fact that, firstly, the reaction under consideration is nonresonant for $E < 2.1$ MeV, since there is no energy level between the first and second ($E^* = 4.65$ MeV; $J^\pi = 7/2^-$) states, and, secondly, there is a spread both in the values of the ANC's $\alpha + t \to {}^7$Li(g.s.) and $\alpha + t \to {}^7$Li(0.478 MeV) (see Refs. [21,23] and below too) and, as it is mentioned above, in the value of the $S(0)$ calculated within different theoretical approaches.

The proposed approach has two advantages. Firstly, it may remove the ambiguities inherent for the two-body potential model calculation connected with the choice of the geometric parameters (the radius r_o and the diffuseness a) for the Woods–Saxon potential. Recall that such a potential is frequently used for calculation of both the bound state wave function in the final state and those of the relative motion of colliding particles in the initial state. Secondly, the information on the ANC's obtained from the analysis of the considered reaction is used for the extrapolation of the astrophysical S-factors for the same reaction at extremely low experimentally inaccessible energy regions for which the above mentioned ambiguities are minimal.

The contents of this paper are as follows. In Section 2, the new modified two-body potential approach to the direct radiative capture $a + A \to B + \gamma$ reaction is proposed in which the astrophysical S-factor is expressed in the term of the ANC. In Section 3 the relation between the NVC (or ANC) and the parameters of the effective range expansion is derived. In Sections 4 and 5 the analysis of the direct radiative capture $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction and the calculation of the reaction rates, respectively, are performed. The conclusion is given in Section 6.

2. Modified two-body potential approach to peripheral direct radiative capture $a + A \to B + \gamma$ reaction

In this section we consider the direct radiative capture $a + A \to B + \gamma$ reaction of astrophysical interest, where the a capture proceeds in to loosely bound states of nucleus B within the two-body potential model, and derive the explicit equation for the direct astrophysical S-factor, $S(E)$, expressed in terms of the corresponding ANC for the overlap function in the two-body $(A + a)$ channel [27]. We note that the astrophysical S-factor $S(E)$ is related to the direct cross section $\sigma(E)$ by

$$S(E) = Ee^{2\pi\eta}\sigma(E). \tag{1}$$

Here $\eta = z_A z_a e^2 \mu_{Aa}/k$ is the Coulomb parameter for Aa-scattering, where $z_j e$ is the charge of particle j, μ_{Aa} is the reduced mass of particles A and a, k is the relative momentum of the particles a and A and $E = k^2/2\mu_{Aa}$.

The matrix element of the reaction considered within the two-body potential approach in the long wavelength approximation has the form [7,19,28]

$$M = \langle I_{Aa}(\mathbf{r}_{Aa}) | O(\mathbf{r}_{Aa}) | \psi_{\mathbf{k}}(\mathbf{r}_{Aa}) \rangle, \tag{2}$$

where $I_{Aa}(\mathbf{r}_{Aa})$ is the overlap function of the bound state wave functions ψ_A and ψ_a and ψ_B of the A, a and B nuclei, respectively; $\psi_{\mathbf{k}}(\mathbf{r}_{Aa})$ is the wave function of relative motion of the particles a and A in the initial state; $O(\mathbf{r}_{Aa})$ is the electromagnetic-transition operator and $\mathbf{r}_{Aa}$ is the radius vector connecting the centers of mass of the particles a and A. The overlap function $I_{Aa}(\mathbf{r}_{Aa})$ is given by [27]

$$\begin{aligned} I_{Aa}(\mathbf{r}_{Aa}) &= N_{Aa}^{1/2} \langle \psi_A(\xi_A)\psi_a(\xi_a) | \psi_B(\xi_A, \xi_a; \mathbf{r}_{Aa}) \rangle \\ &= \sum_{l_B \mu_B j_B \nu_B} C^{J_B M_B}_{j_B \nu_B J_A M_A} C^{j_B \nu_B}_{l_B \mu_B J_a M_a} i^{l_B} Y_{l_B \mu_B}(\hat{\mathbf{r}}_{Aa}) I_{Aa;l_B j_B}(r_{Aa}). \end{aligned} \tag{3}$$

Here $J_j(M_j)$ is the spin (its projection) of particle j; $\hat{\mathbf{r}}_{Aa} = \mathbf{r}_{Aa}/r_{Aa}$, j_B and ν_B (l_B and μ_B) are the total angular momentum and its projection (orbital momentum and its projection) of the particle a in the nucleus $B = (A + a)$, respectively; $I_{Aa;l_B j_B}(r_{Aa})$ is the radial overlap function which is not normalized to unity [29]; $C^{a\alpha}_{b\beta c\gamma}$ is the Clebsh–Gordon coefficient, and N_{Aa} is the factor taking into account the nucleons' identity [27], which is absorbed in the $I_{Aa;l_B j_B}(r_{Aa})$. It is noted that the spectroscopic factor $Z_{Aa;l_B j_B}$ is determined by the formula [27]

$$Z_{Aa;l_B j_B} = \int_0^\infty I^2_{Aa;l_B j_B}(r_{Aa}) r^2_{Aa}\, dr_{Aa}. \tag{4}$$

The asymptotic behaviour of the $I_{Aa;l_B j_B}(r_{Aa})$ outside the range of the nuclear interaction $r_{Aa} > r^o_{Aa}$ is given by the relation:

$$I_{Aa;l_B j_B}(r_{Aa}) \simeq C_{Aa;l_B j_B} \frac{W_{-\eta_B;l_B+1/2}(2\kappa_{Aa} r_{Aa})}{r_{Aa}}, \tag{5}$$

where $W_{\alpha;\beta}(r)$ is the Whittaker function, $\eta_B = z_A z_a e^2 \mu_{Aa}/\kappa_{Aa}$ is the Coulomb parameter for the $B = (A + a)$ bound state, $\kappa_{Aa} = \sqrt{2\mu_{Aa}\epsilon_{Aa}}$, ϵ_{Aa} is the binding energy of the B nucleus in respect to the $(A + a)$-channel, r^o_{ij} is the nuclear interaction radius between i and j particles in the $(i + j)$-bound state and $C_{Aa;l_B j_B}$ is the ANC related to the NVC $G_{Aa;l_B j_B}$ for the virtual decay $B \to A + a$ as [27]

$$G_{Aa;l_B j_B} = -i^{l_B+\eta_B} \frac{\sqrt{\pi}}{\mu_{Aa}} C_{Aa;l_B j_B}. \tag{6}$$

In the standard two-body potential model calculation the unknown radial overlap function $I_{Aa;l_B j_B}(r_{Aa})$ is approximated by a model function as

$$I_{Aa;l_B j_B}(r_{Aa}) \approx Z^{1/2}_{Aa;l_B j_B} \varphi_{nl_B j_B}(r_{Aa}). \tag{7}$$

Here, $\varphi_{nl_B j_B}(r_{Aa})$ is the single-particle wave function of the bound $B = A + a$ state, which satisfies the radial Schrödinger equation with the phenomenological Woods–Saxon potential,[1] and has asymptotic behaviour for $r_{Aa} > r^o_{Aa}$ as

$$\varphi_{nl_B j_B}(r_{Aa}) \simeq b_{Aa;l_B j_B} \frac{W_{-\eta_B;l_B+1/2}(2\kappa_{Aa} r_{Aa})}{r_{Aa}}, \tag{8}$$

[1] In paper [20] the potential was taken with the Gaussian form.

where $b_{Aa;l_B j_B}$ is the single-particle ANC and n is the number of nodes of $\varphi_{nl_B j_B}(r_{Aa})$. Then, the astrophysical S-factor $S(E)$ for the direct capture $a + A \to B + \gamma$ reaction within the two-body potential approach is reduced to

$$S(E) = \sum_{j_B} Z_{Aa;l_B j_B} \tilde{S}_{l_B j_B}(E). \tag{9}$$

Here, we assume that only one l_B is admitted in the expansion Eq. (3) and $\tilde{S}_{l_B j_B}(E) = \sum_\lambda \tilde{S}_{l_B j_B \lambda}$ is the single-particle astrophysical S-factor and its explicit form is determined by the expression Eq. (1) and corresponding formulas of Ref. [28], where λ is multipole order of the electromagnetic transition.

According to Ref. [28], the function $\tilde{S}_{l_B j_B \lambda}(E)$ contains the radial overlap integral

$$I^{(\lambda)}_{l_b j_b; l_{Aa} j_{Aa}} = \int_0^\infty dr_{Aa}\, \varphi_{nl_B j_B}(r_{Aa}) r_{Aa}^{I+2} \psi_{l_{Aa} j_{Aa}}(r_{Aa}), \tag{10}$$

where $I = \lambda\ (\lambda - 1)$ for the electric (magnetic) transition, $\psi_{l_{Aa} j_{Aa}}$ is the radial wave function for the Aa-scattering in the initial state, and l_{Aa} (j_{Aa}) is the orbital (total) angular momentum of the relative motion of particles A and a in the initial state. The bound state wave function, $\varphi_{nl_B j_B}(r_{Aa})$, is determined by the solution of the radial Schrödinger equation. The solution of this equation is found for the given quantum numbers n, l_B and j_B as well as geometric parameters of r_o and a and with depth adjusted to fit the binding energy ϵ_{Aa}. For the guarantee of self-consistency, the same form of the potential is used for the calculation of the Aa-scattering wave function $\psi_{l_{Aa} j_{Aa}}$. Thus the overall normalization of the cross section of the direct capture $a + A \to B + \gamma$ reaction in the standard two-body potential approach is expressed in terms of the spectroscopic factors $Z_{Aa;l_B j_B}$, which are the only unknown quantities in Eq. (9).

For each fixed value of l_B and j_B the spectroscopic factor, $Z_{Aa;l_B j_B}$, is usually determined from analysis of experimental cross sections on different one-particle transfer reactions [30–32] and the radiative capture $a + A \to B + \gamma$ reactions [7,33,34] by means of normalization of the calculated cross sections to the experimental ones. But, the empirical values of the $Z_{Aa;l_B j_B}$ obtained in such a way depend strongly on model parameters, the radius r_o and the diffuseness a of the Woods–Saxon potential, which are not measured directly. For example, for the direct capture $p + A \to B + \gamma$ reaction the lion's share of the dependence of the cross sections $\sigma(E)$ (or $S(E)$) on the r_o and a parameters comes from the single-particle wave function $\varphi_{nl_B j_B}(r_{Aa})$ which in turn leads to the strong (r_o, a)-dependence of the function $\tilde{S}_{l_b j_B}(E)$. So, these parameters cannot be fixed unambiguously. The theoretical predictions of the spectroscopic factor $Z_{Aa;l_B j_B}$, based on different restrictions on structure of nucleus B [35–38], show a noticeable sensitivity to the form of used potentials [38]. So, the problem of reliable determination of the $Z_{Aa;l_B j_B}$ is still discussed by many authors (see Refs. [30–33] and Refs. [39–41] for example) and is unsolved so far.

But, the normalization of the astrophysical S-factors in terms of spectroscopic factors for the peripheral direct radiative capture $a + A \to B + \gamma$ reaction has another problem. The fact is that the spectroscopic factor determines the probability that the state with the quantum number $(l_B j_B)$ for the nucleus B, which has been created in the direct radiative capture $a + A \to B + \gamma$ reaction, contains the configuration "nucleus A in the ground state and the particle a in the shell orbit $nl_B j_B$". According Eqs. (4) and (5), this probability is determined mainly by the behavior of the radial overlap function in the nuclear interior ($r_{Aa} < r^o_{Aa}$), while the dominant contribution to the peripheral direct radiative capture $a + A \to B + \gamma$ reaction comes from the surface and

external regions of nuclei. It follows from here that the parametrization of the astrophysical S-factor (or the cross section) in terms of the spectroscopic factors (see Eq. (9)) is not justified for the reaction under consideration.

In this case, similar to what has been proposed in [39,41,42] for an analysis of the transfer reaction cross sections, it is expedient to express the astrophysical S-factor $S(E)$, given by Eq. (9), in terms of ANC's, $C_{Aa;l_B j_B}$, rather than the spectroscopic factors $Z_{Aa;l_B j_B}$. Due to the fact that the ANC (or NVC) determines the probability of finding a particle a in the configuration $(A+a)$ at distances $r_{Aa} \gtrsim r^o_{Aa}$, the parametrization of the direct astrophysical S-factor in terms of ANC's for the nucleus B in the $(A+a)$-configuration is adequate also to the physics of the peripheral direct capture $a + A \to B + \gamma$ reaction. Then, by using the relation [27,39]

$$Z_{Aa;l_B j_B} = \frac{C^2_{Aa;l_B j_B}}{b^2_{Aa;l_B j_B}}, \tag{11}$$

which can be obtained by a comparison between Eqs. (5), (7) and (8), we can modify the conventional two-body potential method analysis to take into account the additional condition (11) fixing the correct normalization of the peripheral part of the astrophysical S-factor. Eq. (9) can be rewritten in the following modified form equivalent to Eq. (9) [40]

$$S(E) = \sum_{j_B} C^2_{Aa;l_B j_B} R_{l_B j_B}(E, b_{Aa;l_B j_B}), \tag{12}$$

where

$$R_{l_B j_B}(E, b_{l_B j_B}) = \frac{\tilde{S}_{l_B j_B}(E)}{b^2_{Aa;l_B j_B}}. \tag{13}$$

As shown in the calculation for electric transitions, the function $R_{l_B j_B}(E, b_{l_B j_B})$ does not depend on j_B. Then Eq. (12) can be rewritten as

$$S(E) = \left(\sum_{j_B} C^2_{Aa,l_B j_B}\right) R_{l_B}(E, b_{Aa;l_B j_B}). \tag{14}$$

It is seen from here that the overall normalization of the astrophysical S-factor for the peripheral direct capture $a + A \to B + \gamma$ reaction is determined by the value of the ANC corresponding to the $(A+a)$-configuration in the nucleus B.

Thus, the introduction of the additional condition (11) in to the two-body potential method analysis guarantees the correct absolute normalization of the direct astrophysical S-factor for the reaction under consideration. Consequently, if the ANC's are known, they allow one to calculate the direct (but not the resonant) astrophysical S-factor, $S(E)$, of the reaction under consideration at stellar energies.

To this end in recent years considerable amount of experimental and theoretical studies devoted to the peripheral one-particle (proton and α-particle) transfer reactions were performed within the DWBA approach to determine the ANC's for the $A + p \to B$ and $A + \alpha \to B$ of astrophysics interest (see Refs. [40–51] for example). However, the DWBA approach is the first order perturbation approximation over the Coulomb polarization operator ΔV^C in which the transition operator ΔV^C sandwiched by the initial and final state wave functions is assumed to be small [42]. But as it was shown in Refs. [52–57], when the residual nuclei B are formed in weakly bound states, this assumption is not guaranteed for the peripheral proton (α-particle)

transfer reactions of astrophysical interest and so the obtained values of ANC's for the astrophysical application may not have the necessary accuracy. In this case an inclusion of all orders (the first, second and higher orders) of the power expansion in a series over ΔV^C is required in the transition operator for the DWBA cross section calculations [53,56]. From this point of view one can understand the statement of authors of Ref. [58] on difficulties of the determination of all important systematic errors in the value of the ANC for $^7\mathrm{Be}+p\to{}^8\mathrm{B}$ and of the $S(0)$ for the $p+{}^7\mathrm{Be}\to{}^8\mathrm{B}+\gamma$ reaction obtained in Refs. [44,46] from the DWBA cross section calculations. Note that the obtained ANC for the $^7\mathrm{Be}+p\to{}^8\mathrm{B}$ was used for estimation of the value of $S(0)$ for the $p+{}^7$ Be $\to{}^8\mathrm{B}+\gamma$ reaction [44,46].

However, the required ANC for the $a+A\to B$ can also be obtained from the experimental astrophysical S-factor, $S^{\exp}(E)$, of the direct capture $a+A\to B+\gamma$ reaction at extremely low energies if a measurement of $S^{\exp}(E)$ is done with sufficient precision. In this case Eq. (14) can be used as an independent source of getting information on astrophysical interest value of the ANC, $C^2_{Aa;l_B}=\sum_{j_B}C^2_{Aa;l_Bj_B}$, for the $A+a\to B$. To this end the magnitude $R_{l_B}(E,b_{Aa;l_Bj_B})$ (13) must be considered as a function of the single-particle ANC $b_{Aa;l_Bj_B}(=b_{Aa;l_Bj_B}(r_o,a))$ for each fixed value of energy E from the range ($E_{\min}\leqslant E\leqslant E_{\max}$) in which the direct radiative capture mainly occurs. Therefore, similar to the case of the peripheral nucleon transfer reaction cross section [39,41,42], all of the (r_o,a)-dependence of the direct capture astrophysical S-factor, $S(E)$, enters also through the single-particle ANC $b_{Aa;l_Bj_B}$. The latter can be found from variation over a physically acceptable range of the geometrical parameters (r_o and a) of the Woods–Saxon potential for the bound $(A+a)$ state of the nucleus B, as it was done in Refs. [39,41,42] and [45]. Here we will examine the sensitivity of our results to $\pm10\%$ variantions in r_o and a.

In this case the peripheral character for the direct capture reaction $a+A\to B+\gamma$ must be conditioned by

$$R_{l_B}(E,b_{Aa;l_Bj_B})=f(E) \tag{15}$$

as a function of the $b_{Aa;l_Bj_B}$ (or of the geometry of the Woods–Saxon potential) within the energy range $E_{\min}\leqslant E\leqslant E_{\max}$, where the left side hand of Eq. (15) must not depend on $b_{Aa;l_Bj_B}$ for each fixed E from the aforesaid energy range, and by

$$C^2_{Aa;l_B}=\frac{S(E)}{R_{l_B}(E,b_{Aa;l_Bj_B})}=\mathrm{const} \tag{16}$$

for each fixed E and the function of $R_{l_B}(E,b_{Aa;l_Bj_B})$ from (15). Fulfillment (or weak violation[2]) of the conditions (15) and (16) allows one to obtain valuable information about the experimental value of the ANC $(C^{\exp}_{Aa})^2$ for the $A+a\to B$ by using $S^{\exp}(E)$ instead of the $S(E)$ in the r.h.s. of Eq. (16):

$$\left(C^{\exp}_{Aa;l_B}\right)^2=\frac{S^{\exp}(E)}{R_{l_B}(E,b_{Aa;l_Bj_B})}. \tag{17}$$

Then the value of the ANC, $(C^{\exp}_{Aa;l_B})^2$, obtained from Eq. (17) together with the condition (15) can be used for calculation of $S(E)$ at stellar energies $E<E_{\min}$ by the expression:

$$S(E)=\left(C^{\exp}_{Aa;l_B}\right)^2R_{l_B}(E,b_{Aa;l_Bj_B}). \tag{18}$$

Values obtained in such way for the $(C^{\exp}_{Aa;l_B})^2$ and the $S(E)$ at stellar energies E can be considered as an "indirect measurement" of the ANC (or NVC) for the $A+a\to B$ and of the

[2] In the physical acceptable limit being within the experimental errors for the $S^{\exp}(E)$.

astrophysical S-factor for the direct capture $a + A \to B + \gamma$ reaction at $E < E_{\min}$, including $E = 0$. One notes that Eqs. (14)–(18) are the basic formulae of the modified two-body potential approach.

The main advantage of the proposed modified two-body potential approach is that it allows one to determine both the absolute value of ANC $(C^{\exp}_{Aa;l_B})^2$ (or NVC) for the $A + a \to B$ and the astrophysical S-factor $S(E)$ for the peripheral direct capture $a + A \to B + \gamma$ reaction at stellar energies E by means of the analysis of the same precisely measured values of the experimental astrophysical S-factor, $S^{\exp}(E)$, in a self-consistent way, using the same potential both for the bound $(A + a)$ state and for the Aa-scattering state.

Note once more that the modified two-body potential approach holds only for peripheral direct radiative capture reactions firstly, at the introduction of the information about ANC (or NVC) by (11) into the two-body potential method calculations, the contribution to the radial integral (10) from the nuclear exterior $r \geqslant r^o_{Aa}$ is fixed through the value of ANC, secondly, the electromagnetic operator $O(\mathbf{r})$ can be taken in the form of the two-body approximation, and at last the antisymmetrization effects between nucleons of the incident particle (a) and the nucleons of target A, contributed mainly to the radial integral (10) from the nuclear interior $r < r^o_{Aa}$, are rather small since they are tested by the accuracy of fulfillment of the conditions (15) and (16). So the overall uncertainty for the "indirect" measured value $(C^{\exp}_{Aa;l_B})^2$ and the extrapolated $S(E)$ at stellar energies includes the contributions both from the experimental errors for the $S^{\exp}(E)$ and from the additional uncertainties for values of the calculated function $R_{l_B}(E, b_{Aa;l_B j_B})$ arising because of the possible violation of the condition (15).

It should be emphasized that the NVC (or ANC) for the virtual decay $B \to A + a$ is a fundamental characteristic of the nucleus B in the two-body $(A + a)$-channel and is determined by the dynamics of the strong interactions (see Refs. [15,38] and [59] for example). Consequently, the "indirect" measured value $(C^{\exp}_{Aa;l_B})^2$ for $A + a \to B$ obtained from an analysis of the experimental astrophysical S-factor, $S^{\exp}(E)$, allows one, firstly, to get additional valuable information on the two-particle potential used in the theoretical calculations of ANC performed within different approaches (see Refs. [15,38] and [59] for example) by choosing those forms of the potential which would give the calculated value $(C_{Aa;l_B})^2$ closest to the $(C^{\exp}_{Aa;l_B})^2$. Besides, the latter can be compared with that obtained from independent experimental data on the peripheral one-particle (a) transfer reactions, for example, peripheral proton transfer reactions [41,42,53,56] and elastic Aa-scattering [60]. At last, which is also important, the obtained $(C^{\exp}_{Aa;l_B})^2$ is very useful in getting additional information about the low-energy effective range parameters and the phase shifts for the Aa-scattering [61,62] as well as in the extrapolation of the astrophysical S-factor, $S(E)$, for the peripheral direct radiative capture $a + A \to B + \gamma$ reaction to the energy range of astrophysical interest, where it is difficult to measure the $S(E)$ in direct experiment. These issues are considered below.

3. Connection between nuclear vertex constant for the virtual decay $B \to A + a$ and the effective-range expansions parameters

Here we will derive a relation between NVC (ANC) and the effective-range expansions parameters for Aa-scattering with an arbitrary partial wave and an allowance for the Coulomb Aa-interactions. We concentrate on very low energies, i.e., on energies satisfying the condition $kr^o_{Aa} \ll 1$ [63].

The partial amplitudes corresponding to the diagonal matrix element of the Aa-scattering have the form as (see [62] for example)[3]

$$M^I_{l_{Aa}s_{Aa}}(E) = -\frac{2\pi}{\mu_{Aa}} \frac{1}{(k\cot\delta^{(cs)}_{l_{Aa}s_{Aa}} - ik)}, \tag{19}$$

where s_{Aa} and I are a spin and a total angular momentum of the channel, respectively, i.e. $\mathbf{I} = \mathbf{l}_{Aa} + \mathbf{s}_{Aa}$, and $\delta^{(cs)}_{l_{Aa}s_{Aa}}$ is the Coulomb-nuclear scattering phase shift for angular momentum l_{Aa} and spin channel s_{Aa}. One notes that at $E = -\epsilon_{Aa}$ ($\epsilon_{Aa} > 0$), the denominator in the r.h.s. of the expression (19) becomes zero and this point is a pole singularity for the partial amplitudes $M^I_{l_{Aa}s_{Aa}}(E)$. Then the partial amplitudes $M^I_{l_{Aa}s_{Aa}}(E)$ can be presented in the form as

$$M^I_{l_{Aa}s_{Aa}}(E) = \frac{W_{Aa;l_Bs_B}}{E + \epsilon_{Aa}} + g^I_{l_{Aa}s_{Aa}}(E), \tag{20}$$

where $W_{Aa;l_Bs_B}$ is a residue of the partial amplitudes at the point $E = -\epsilon_{Aa}$, $g^I_{l_{Aa}s_{Aa}}(E)$ is a regular at the point $E = -\epsilon_{Aa}$ function and $s_B = s_{Aa}$ for the bound ($B = (A + a)$) state.

The effective-range expansion for low-energy Aa-scattering has the form (see [63] and references therein for example)

$$k^{2l_{Aa}+1} D_l(\eta)\big[C^2_o(\eta)\cot\delta^{(cs)}_{l_{Aa}s_{Aa}} + 2\eta h(\eta)\big] = -\frac{1}{a^{(cs)}_{l_{Aa}s_{Aa}}} + \frac{1}{2} r^{(cs)}_{ef;l_{Aa}s_{Aa}} k^2 + \cdots. \tag{21}$$

Here $a^{(cs)}_{l_{Aa}s_{Aa}}$ and $r^{(cs)}_{ef;l_{Aa}s_{Aa}}$ are the scattering lengths and the effective ranges for angular momentum l_{Aa} and spin channel s_{Aa}, respectively, and

$$C^2_o(\eta) = \frac{2\pi\eta}{e^{2\pi\eta} - 1}, \qquad D_{l_{Aa}}(\eta) = \prod_{j=1}^{l_{Aa}} \big(1 + \eta^2/j^2\big),$$

$$h(\eta) = -\gamma + \eta^2 \sum_{j=1}^{\infty} \frac{1}{j(j^2 + \eta^2)} - \ln\eta, \tag{22}$$

where $\gamma = 0.57721\ldots$ is Euler's constant. The $a^{(cs)}_{l_{Aa}s_{Aa}}$ and $r^{(cs)}_{ef;l_{Aa}s_{Aa}}$ have the dimension of $\mathrm{fm}^{2l_{Aa}+1}$ and $\mathrm{fm}^{-2l_{Aa}+1}$, respectively, and for s-wave states the product $\prod_{j=1}$ for $D_{l_{Aa}}(\eta)$ in Eq. (22) must be replaced by unity, and in this case for the purely nuclear interaction $r^{(cs)}_{ef;0s_{Aa}} \equiv r^{(s)}_{ef;s_{Aa}}$.

Taking into account the Eq. (21), one can rewrite the partial amplitudes $M^I_{l_{Aa}s_{Aa}}(E)$ (19) in the following form

$$M^I_{l_{Aa}s_{Aa}}(E) = -\frac{2\pi}{\mu_{Aa}} \frac{k^{2l} D_{l_{Aa}}(\eta) C^2_o(\eta)}{f_l(k; r^{(cs)}_{ef;l_{Aa}s_{Aa}})}. \tag{23}$$

Here

$$f_l\big(k; r^{(cs)}_{ef;l_{Aa}s_{Aa}}\big) = (-1)^{l+1}\kappa^{2l+1}_{Aa} + \frac{1}{2} r^{(cs)}_{ef;l_{Aa}s_{Aa}} \big(k^2 + \kappa^2_{Aa}\big) - D_{l_{Aa}}(\eta)\big[2\eta h(\eta) + iC^2_o(\eta)\big]k^{2l+1}, \tag{24}$$

[3] Here one notes that, the partial amplitudes (19) differ from that in Ref. [62] by a factor of $-2\pi/\mu_{Aa}$.

where [64]

$$\kappa_{Aa}^{2l+1} = \frac{1}{a_{ef;l_{Aa}s_{Aa}}^{(cs)}} + \frac{1}{2} r_{ef;l_{Aa}s_{Aa}}^{(cs)} \kappa_{Aa}^2. \tag{25}$$

According to Ref. [27], the residue of the partial amplitude, $M^I_{l_{Aa}s_{Aa}}(E)$, corresponding to the diagonal matrix element for elastic Aa-scattering at the pole $E = -\epsilon_{Aa}$ ($k = i\kappa_{Aa}$) with $I = J_B$ is expressed in terms of the NVC for the virtual decay $B \to A + a$ as

$$\operatorname{res}\{M^I_{l_{Aa}s_{Aa}}(E)\} = \lim_{k \to i\kappa_{Aa}} \left(\frac{k^2}{2\mu_{Aa}} + \epsilon_{Aa} \right) M^{J_B}_{l_B s_B}(E) = G^2_{Aa;l_B s_B}, \tag{26}$$

$$G_{Aa;l_B j_B} = \sum_{s_B} \left[(2s_B + 1)(2j_B + 1) \right]^{1/2} W(J_A J_a J_B; s_B j_B) G_{Aa;l_B s_B}, \tag{27}$$

where $I = J_B$, $l_{Aa} = l_B$ and $s_{Aa} = s_B$ for the bound ($B = (A + a)$) state; $\mathbf{s}_B = \mathbf{J}_A + \mathbf{J}_a$ and $\mathbf{J}_B = \mathbf{l}_B + \mathbf{s}_B$ and $W(\cdots)$ is a Racah coefficient. From (27) it follows that [27]

$$|G_{Aa;l_B}|^2 \equiv \sum_{j_B} |G_{Aa;l_B j_B}|^2 = \sum_{s_B} |G_{Aa;l_B s_B}|^2. \tag{28}$$

From (20) and (23) the behavior of the partial amplitudes near the singular point of $E = -\epsilon_{Aa}$ can be presented in the form

$$M^I_{l_{Aa}s_{Aa}}(E) \simeq M^{J_B}_{l_B s_B}(E) = \frac{G^2_{Aa;l_B s_B}}{E + \epsilon_{Aa}}. \tag{29}$$

Here

$$G_{Aa;l_B s_B} = i(-1)^{l_B} \sqrt{\frac{2\pi}{\mu_{Aa}}} \left[\frac{\kappa_{Aa}^{2l} D_{l_B}(-i\eta_B) C_o^2(-i\eta_B)}{f'_{l_B}(i\kappa_{Aa}; r_{ef;l_B s_B}^{(cs)})} \right]^{1/2}, \tag{30}$$

where the prime denotes the derivative of the function f_{l_B} at the point of $k = i\kappa_{Aa}$. It is seen from Eqs. (20) and (26) that $W_{Aa;l_B s_B} = G^2_{Aa;l_B s_B}$.

Differentiating Eq. (24) leads for $k = i\kappa_{Aa}$ to the sought expression for the NVC

$$G^2_{Aa;l_B s_B} = (-1)^{l_B} e^{i\pi\eta_B} \frac{2\pi}{\mu^2_{Aa}} \frac{K(\eta_B) \kappa_{Aa}^{2l_B+1}}{\tilde{f}_{l_B s_B;\eta_B}(r_{ef;l_B s_B}^{(cs)})}. \tag{31}$$

Here

$$\begin{aligned} \tilde{f}_{l_B s_B;\eta_B}\big(r_{ef;l_B s_B}^{(cs)}\big) = (-1)^{l_B} \kappa_{Aa}^{2l_B} \Big\{ & K^2(\eta_B) - 2\eta_B \big[\tilde{h}(\eta_B) \\ & + \big(2\operatorname{Re} h(-i\eta_B) - \pi \cot \pi\eta_B \big) \big(l_B + \eta_B^2 F_{l_B}(\eta_B) \big) \big] \Big\} \\ & - D_{l_B}^{-1}(-i\eta_B) \kappa_{Aa} r_{ef;l_B s_B}^{(cs)} \end{aligned} \tag{32}$$

where $K(\eta_B) = \pi\eta_B / \sin \pi\eta_B$, $F_{l_B}(\eta_B) = (1 - \delta_{l_B 0}) \sum_{j=1}^{l_B} 1/(j^2 - \eta_B^2)$ and

$$\tilde{h}(\eta_B) = 2\eta_B^2 \sum_{j=1}^{\infty} \frac{1}{j(j^2 - \eta_B^2)} + 2\eta_B^4 \sum_{j=1}^{\infty} \frac{1}{j(j^2 - \eta_B^2)^2} + 1. \tag{33}$$

Using Eqs. (6) and (31) one obtains

$$C^2_{Aa;l_Bs_B} = \frac{2K(\eta_B)\kappa_{Aa}^{2l_B+1}}{\tilde{f}_{l_Bs_B;\eta_B}(r^{(cs)}_{ef;l_Bs_B})}. \tag{34}$$

As is seen from Eqs. (31), (32) and (34), the NVC (or ANC) in the effective range approximation is expressed through the binding energy ϵ_{Aa} and the effective range radius $r^{(cs)}_{ef;l_Bs_B}$.

One notes that the expression (34) in the absence of the Coulomb interaction ($\eta_B = 0$) can be reduced to the form

$$C^2_{Aa;l_Bs_B} = \frac{2\kappa_{Aa}^{2l_B+1}}{(-1)^{l_B}(2l_B+1)\kappa_{Aa}^{2l_B} - \kappa_{Aa}r^{(s)}_{ef;l_Bs_B}}, \tag{35}$$

where $r^{(s)}_{ef;l_Bs_B}$ is the effective radius for the purely strong interaction with angular momentum (the channel spin) $l_B(s_B)$. The expression (35) for $l_B = 0$ coincides with the formula (3.12) of Section 3 of Chapter 3 of Ref. [61] obtained for an s-wave state. Besides, the expressions (29) and (31) for $\eta_B = 0$ coincide with that obtained by the authors of Ref. [62] (see Section 131 of Chapter 17 there) for small values of ϵ_{Aa} and $l_B > 0$ since the terms of $\sim \kappa_{Aa}^{2l_B}$ in (35) are negligible with respect to those $\sim \kappa_{Aa}$. In this case the following strong inequality $(2l_B+1)\kappa_{Aa}^{2l_B} \ll \kappa_{Aa}|r^{(sc)}_{ef;l_Bs_B}|$ for $l_B > 0$ occurs and, as it can be easily shown from Eqs. (29), (31) and (32), the partial amplitudes behave as $M^I_{l_{Aa}s_{Aa}}(E) \sim (-1)^{l_{Aa}+1}\epsilon_{Aa}^{l_{Aa}}/b_o(E+\epsilon_{Aa})$, where $b_o = \mu^2_{Aa}r^{(s)}_{ef;l_{Aa}s_{Aa}}/2\pi(2\mu)^{l_B}$.[4]

The expressions (34) and (25) can be used for a determination of values of the effective range parameters, $r^{(cs)}_{ef;l_Bs_B}$ and $a^{(cs)}_{ef;l_Bs_B}$, if values of the ANC $C^2_{Aa;l_Bs_B}$ are known.

4. Analysis of $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction

4.1. The asymptotic normalization coefficients for $t + \alpha \to {}^7\mathrm{Li}$

In this subsection, to determine the ANC values for the $\alpha + t \to {}^7$Li a reanalysis of the experimental astrophysical S-factors, $S^{\exp}(E)$, for the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction populating the ground and first excited ($E^* = 0.478$ MeV; $J^\pi = 1/2^-$) states is carried out on the basis of relation (14) and the conditions (15) and (16) as well as of the relations (17) and (18). For these reactions, the value of l_B ($B = {}^7$Li) is taken to be equal to 1 and the value of j_B is taken to be equal to 3/2 (1/2) for the ground (first excited) state of ^{7}Li, while $l_{\alpha t} = 0$, 2 for the E1-transition and $l_{\alpha t} = 1$ for the E2-transition. The experimental data have been obtained by authors of papers [65–67] with experimental uncertainty δ being within $\delta = 14$–25%. Recently, Brune et al. [22] performed rather precise measurement of the astrophysical S-factor in the energy range of $55 \leqslant E \leqslant 1189$ keV with $\delta = 6$–10%. So in our analysis we naturally use the $S^{\exp}(E)$ measured in [22].

The Woods–Saxon potential split with a parity (l-dependence) for the spin–orbit term proposed by the authors of Refs. [68–70] is used here for the calculations of both the bound state radial wave function $\varphi_{nl_Bj_B}(r_{\alpha t})$ and the scattering wave function $\psi_{l_{\alpha t}j_{\alpha t}}(r_{\alpha t})$. It should be emphasized that the choice of this potential is based on the following considerations. Firstly, this

[4] Recall once more that, the partial amplitudes (19) differ from that in Ref. [62] by a factor of $-2\pi/\mu_{Aa}$.

potential form is justified from the microscopic point of view because it makes it possible to take into account the Pauli principle between nucleons in t-and α-clusters in the $(\alpha + t)$ bound state by means of inclusion of deeply bound states forbidden by the Pauli exclusion principle, i.e. without an explicit introduction of a repulsive core at small distance. The latter imitates the additional node arising in the wave functions of $\alpha - t$ relative motion of in the ^{7}Li. Secondly, this potential describes well the phase shifts for αt-scattering in the wide energy range up to 50 MeV [69,70] and reproduces the energies of low-lying states, the main electromagnetic characteristic of the ^{7}Li [71] and the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ cross section [72].

The test of the peripheral character of the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction for the energy range of $55 \leqslant E \leqslant 1189$ keV has been made by means of verifying the conditions (15) and (16) and by changing the geometric parameters (radius r_o and diffuseness a) of the adopted Woods–Saxon potential using the procedure of the depth adjusted to fit the binding energies, as was done in Refs. [39,41] and [42] for the nucleon transfer reactions. The calculation shows that for each energy E the lion's share of dependence of the function $\tilde{S}(E)$ on the parameters r_o and a enters mainly through the single-particle ANC $b(= b(r_o, a))$.[5] It should be noted that if one varies only one parameter r_o or a fixing another of them, then b changes strongly. But if one varies r_o and a with the condition $b = b(r_o, a) = \text{const}$, then the extremely weak dependence of the $\tilde{S}(E)$ function on r_o and a for each $b(r_o, a) = \text{const}$ (the "residual" (r_o, a)-dependence [39]) also occurs. According to Refs. [41] and [42], we vary r_o and a in the physically acceptable ranges (r_o in 1.62–1.98 fm and a in 0.63–0.77 fm) in respect to the standard values ($r_o = 1.80$ fm and $a = 0.70$ fm [69,70]). Such a choice of the limit of variation of the parameters r_o and a allows us to supply fulfillment of the conditions (15) and (16) in the energy range of $55 \leqslant E \leqslant 1189$ keV within the experimental errors for the $S^{\exp}(E)$. The "residual" (r_o, a)-dependence of the single-particle astrophysical S-factor, $\tilde{S}(E)$, on r_o and a for each $b(r_o, a) = \text{const}$ turns out to be extremely weak up to about $\pm 2\%$. So, $\tilde{S}(E)$ is a rapidly varying function of b with the extremely weak "residual" (r_o, a)-dependence for each $b = \text{const}$. However, for each fixed experimental point of energy E a quantity $R(E, b)$ depends weakly (up to $\pm 8.5\%$) on the variation of b, and its "residual" (r_o, a)-dependence on r_o and a for each $b = \text{const}$ is also extremely weak (up to $\pm 2\%$). Besides, it should be emphasized that the uncertainty in the $R(E, b)$ values is decreased with a decrease of the energy E.

As an illustration, Fig. 1 shows plots of the dependence of the $R(E, b)$ on the single-particle ANC, b, only for the three energies E. The width of each band for the curves is the result of the weak "residual" (r_o, a)-dependence of the $R(E, b)$ on the parameters r_o and a (up to $\pm 2\%$) for the $b = b(r_o, a) = \text{const}$. The same dependence is observed at other energies E ($\leqslant 1200$ keV). It is seen that for the calculated values of the $R(E, b)$ the dependence on the b-value is rather weak (no more than $\pm 8.5\%$) in the interval of $2.73 \leqslant b \leqslant 3.76$ fm$^{-1/2}$ ($2.30 \leqslant b \leqslant 3.10$ fm$^{-1/2}$) for the $t + \alpha \to {}^7\mathrm{Li}(\mathrm{g.s.}) + \gamma$ ($t + \alpha \to {}^7\mathrm{Li}(0.478\ \mathrm{MeV} + \gamma)$ reaction. It follows from here that the condition (15) is satisfied for the considered reaction within the uncertainties not exceeding the experimental errors of $S^{\exp}(E)$.

This circumstance allows us to test the condition (16), which is no less essential for the peripheral character of these reactions. For the same energies E as in Fig. 1 we present in Fig. 2 the results of calculation of the quantity of

[5] Here and below for simplicity all indexes specifying the singe-particle ANC, the quantum numbers specifying the ANC's, NVC's and spectroscopic factors as well as the functions $R_{l_B}(E, b_{Aa;l_B j_B})$ and $\tilde{S}_{l_B j_B}(E)$ have been dropped.

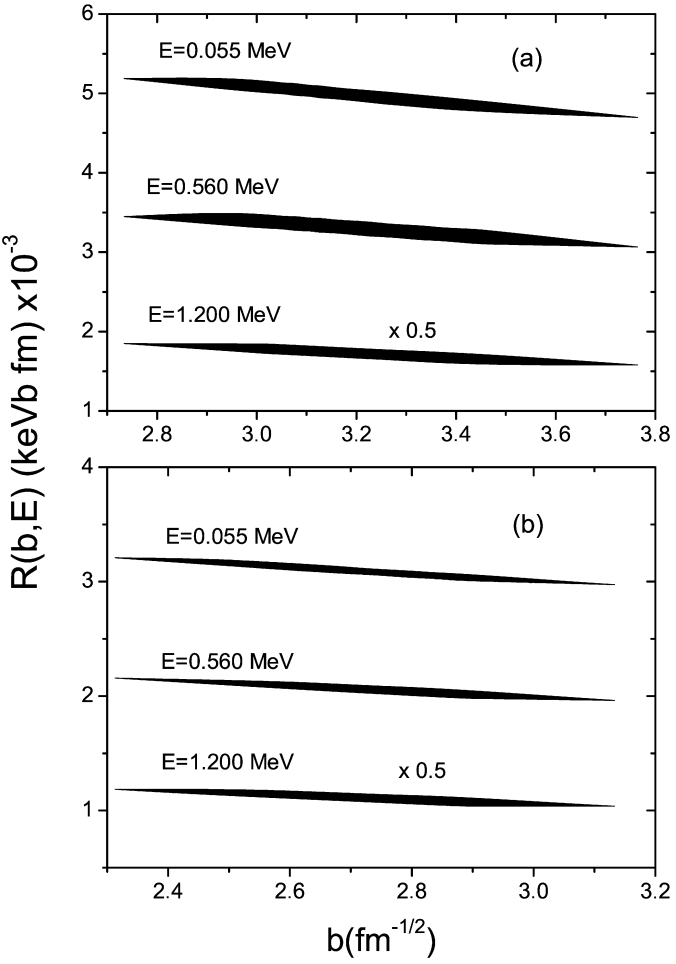

Fig. 1. The dependence of $R(E,b)$ as a function of the single-particle ANC, b, for the $t+\alpha \to {}^7\mathrm{Li(g.s.)}+\gamma$ (a) and $t+\alpha \to {}^7\mathrm{Li(0.478\ MeV)}+\gamma$ (b) reactions at different center of mass energies E.

$$C^2_{\alpha t} = \frac{S(E)}{R(E,b)}, \tag{36}$$

where instead of the $S(E)$ the experimental S-factors for the $t+\alpha \to {}^7\mathrm{Li}+\gamma$ reaction populating the ground and first excited states of ${}^7\mathrm{Li}$ were taken. It is also noted that the same dependence occurs for other considered energies. It is seen from the figure that the obtained values of the $C^2_{\alpha t}$ are also weakly dependent on the b-value, which corresponds to the parameters of the adopted Woods–Saxon potential r_o ranging from 1.62–1.98 fm and a in the range of 0.63–0.77 fm. However, the values of the spectroscopic factors, $Z_{\alpha t}$ and $Z^*_{\alpha t}$ corresponding to the $(\alpha+t)$-configuration for ${}^7\mathrm{Li(g.s.)}$ and ${}^7\mathrm{Li(0.478\ keV)}$, respectively, change strongly (see, the lower panels in Fig. 2). For visual clarity Table 1 illustrates the dependence of b, $C^2_{\alpha t}$, $(C^*_{\alpha t})^2$, $Z_{\alpha t}$, $Z^*_{\alpha t}$, the single-particle astrophysical S-factors, $\tilde{S}(E)$, and the $R(E,b)$ functions on the parameters of r_o and a in the regions of $1.62 \leqslant r_o \leqslant 1.98$ fm and of $0.63 \leqslant a \leqslant 0.77$ fm for the reactions under consideration at three energies E. As it is seen from Table 1 the uncertainty in

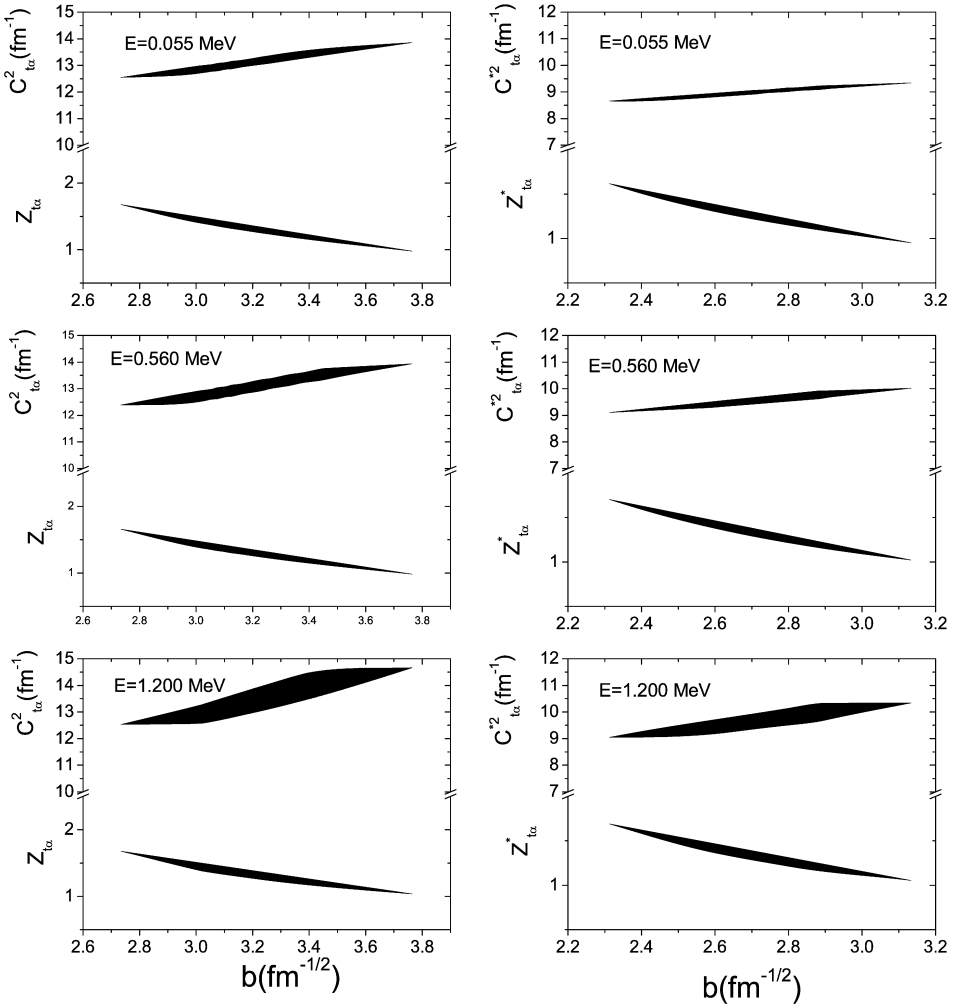

Fig. 2. The dependence of the ANC's $C^2_{t\alpha}$ and $C^{*2}_{t\alpha}$ (upper band) and the spectroscopic factors $Z_{t\alpha}$ and $Z^*_{t\alpha}$ (lower band) on the single-particle ANC b for the $t+\alpha \to {}^7\mathrm{Li(g.s.)}+\gamma$ (the left column) and $t+\alpha \to {}^7\mathrm{Li(0.478\ MeV)}+\gamma$ (the right column) reactions at different energies E.

$R(E,b)$, $C^2_{\alpha t}$ and $(C^*_{\alpha t})^2$ is up to $\pm 8.5\%$ relative to the central values of $R(E,b)$, $C^2_{\alpha t}$ and $(C^*_{\alpha t})^2$, obtained for the standard values of $r_o = 1.80$ fm and $a = 0.60$ fm, for the (r_o, a)-pair varying in the above mentioned intervals for r_o and a, while the uncertainty in the $Z_{\alpha t}$ and $Z^*_{\alpha t}$ is about $\pm 35\%$. It should be noted that the uncertainty in the $C^2_{\alpha t}$ and $(C^*_{\alpha t})^2$ values becomes even less when one uses in (36) the experimental astrophysical S-factors corresponding to smaller energies. Thus, the peripheral character of the reactions under consideration allows one to determine the $C^2_{\alpha t}$ and $(C^*_{\alpha t})^2$ values for the $\alpha + t \to {}^7\mathrm{Li(g.s.)}$ and $\alpha + t \to {}^7\mathrm{Li(0.478\ keV)}$, respectively, with a maximal uncertainty of about $\pm 8.5\%$ when the geometric parameters r_o and a are varied within the aforesaid ranges and the experimental data from the full energy region ($55 \leqslant E \leqslant 1189$ keV) for an analysis are used.

Table 1
The dependence of the single-particle astrophysical S-factor $\tilde{S}(E)$, the function $R(E,b)$, ANC's ($C_{t\alpha}$ and $C^*_{t\alpha}$) and spectroscopic factors ($Z_{t\alpha}$ and $Z^*_{t\alpha}$) on the single-particle ANC b

(a) for the $t+\alpha \to {}^7\mathrm{Li(g.s.)}+\gamma$ reaction

r_o (fm)	a (fm)	b ($\mathrm{fm}^{-1/2}$)	$C^2_{t\alpha}$ ($\mathrm{fm}^{-1/2}$)			$Z_{t\alpha}$			$\tilde{S}\times 10^{-1}$ (MeV b)			$R(E,b)\times 10^{-2}$ (MeV b fm)		
1.62	0.63	2.734	12.546	12.447	12.533	1.678	1.665	1.677	0.388	0.230	0.276	0.519	0.351	0.369
1.67	0.67	2.959	12.867	12.819	13.048	1.470	1.464	1.490	0.443	0.298	0.311	0.506	0.341	0.355
1.8	0.70	3.219	13.171	13.153	13.479	1.271	1.269	1.301	0.512	0.344	0.356	0.494	0.332	0.344
1.94	0.73	3.501	13.501	13.518	13.977	1.102	1.103	1.140	0.591	0.396	0.406	0.482	0.323	0.331
1.98	0.77	3.764	13.858	13.931	14.661	0.978	0.983	1.035	0.665	0.444	0.447	0.470	0.313	0.316
E(MeV) →			0.055	0.506	1.189	0.055	0.506	1.189	0.055	0.506	1.189	0.055	0.506	1.189

(b) for the $t+\alpha \to {}^7\mathrm{Li(0.478\ MeV)}+\gamma$ reaction

r_o (fm)	a (fm)	b ($\mathrm{fm}^{-1/2}$)	$C^{*2}_{t\alpha}$ ($\mathrm{fm}^{-1/2}$)			$Z^*_{t\alpha}$			$\tilde{S}\times 10^{-1}$ (MeV b)			$R(E,b)\times 10^{-2}$ (MeV b fm)		
1.62	0.63	2.313	8.658	8.625	9.046	1.618	1.612	1.691	0.178	0.121	0.131	0.321	0.219	0.237
1.67	0.67	2.495	8.829	8.833	9.364	1.418	1.419	1.504	0.196	0.133	0.143	0.315	0.214	0.229
1.8	0.70	2.701	8.990	9.018	9.625	1.232	1.236	1.319	0.226	0.153	0.163	0.309	0.210	0.223
1.94	0.73	2.924	9.161	9.216	9.926	1.072	1.078	1.161	0.259	0.175	0.185	0.303	0.205	0.216
1.98	0.77	3.133	9.342	9.438	10.34	0.952	0.962	1.054	0.292	0.197	0.204	0.298	0.200	0.208
E(MeV) →			0.055	0.506	1.189	0.055	0.506	1.189	0.055	0.506	1.189	0.055	0.506	1.189

For different energies E we also estimate a relative contribution of the nuclear interior ($r \leqslant r^o_{\alpha t}$) to the astrophysical S-factors for the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction populating the ground and first excited states in dependence on the variation b (or r_o and a) introducing the cutoff radius $r_{\rm cut}$ ($r_{\rm cut} \approx r^o_{\alpha t}$) in the lower limit of integration of the radial integral (10). With this aim one considers the ratio

$$\Delta(E, b; r_{\rm cut}) = \frac{|R(E, b) - \tilde{R}(E, b; r_{\rm cut})|}{R(E, b)}, \tag{37}$$

where $\tilde{R}(E, b; r_{\rm cut})$ is given by Eqs. (10) and (13) but in the radial integral (10) the integration over r is performed in the interval $r_{\rm cut} \leqslant r \leqslant \infty$, i.e. $\tilde{R}(E, b; 0) = R(E, b)$. The $R(E, b)$ and $\tilde{R}(E, b; r_{\rm cut})$ functions were calculated for different values of the single-particle ANC b. A value of the cutoff radius is taken as in Ref. [33], that is $r_{\rm cut} = r^o_{\alpha t} = 1.36(4^{1/3} + 3^{1/3}) = 4.12$ fm, as well as $r_{\rm cut} = 4.00$ fm and 4.25 fm. The results of calculation of $\Delta(E, b; r_{\rm cut})$ (in percent) for different energies are presented in Table 2. As it is seen from Table 2 the quantities of $\Delta(E, b; r_{\rm cut})$ change from 0.1% up to 9.2% under variation of b and $r_{\rm cut}$, and a change of $\Delta(E, b; r_{\rm cut})$ is up to about 7% for $r_{\rm cut} = 4.12$ fm. One can see from Table 2 that the contribution of the nuclear interior ($r < r^o_{\alpha t}$) to the astrophysical S-factors calculated for different sets of geometric parameters r_o and a of the Woods–Saxon potential and values of $r_{\rm cut}$ does not exceed about 9% and this small quantity is due mainly to oscillations observed in the integrand of the radial integral (10). As an illustration of this fact, we show a dependence of the bound state wave function on b (Fig. 3) and of the αt-scattering wave function (Fig. 4a) as well as of the integrand of the radial integral (10) (Fig. 4b) on geometric parameters r_o and a of the Woods–Saxon potential and values

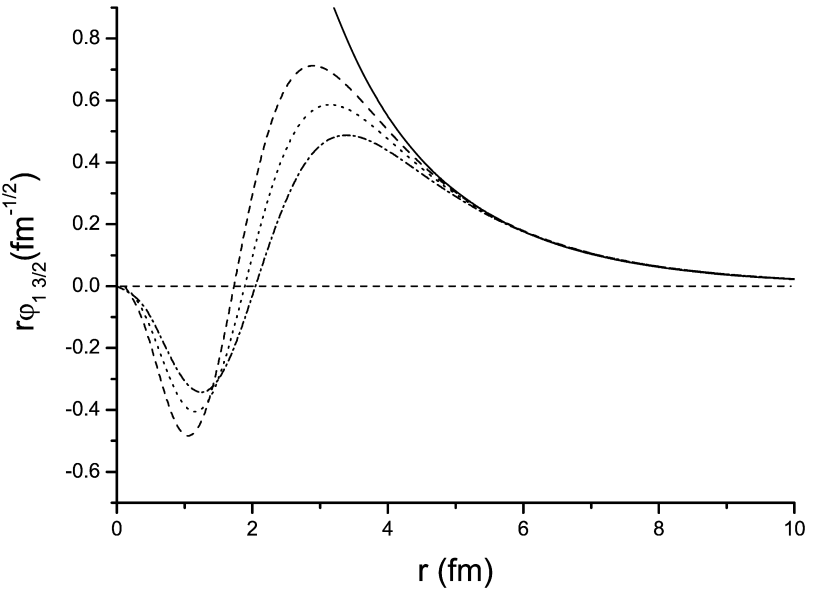

Fig. 3. The radial behavior of the single particle radial bound state wave functions $r\varphi_{l_B j_B}(r)$, $l_B = 1$, $j_B = 3/2$, in ^{7}Li(g.s.) in the $(\alpha + t)$-channel calculated for Woods–Saxon potentials with different sets of $(r_o; a)$-pair: (1.62 fm; 0.63 fm) (dashed line), (1.80 fm; 0.70 fm) (dotted line), (1.98 fm; 0.77 fm) (dashed-dotted line). The Coulomb radius r_c is taken $r_c = r_o$. The solid line is the tail of $bW_{-\eta_B;3/2}(2\kappa_{t\alpha}r)$ for $b = 3.219$ fm$^{-1/2}$ corresponding to the standard values of $r_o = 1.80$ fm and $a = 0.70$ fm. The two bound state wave functions other than that using (1.80 fm and 0.70 fm) are normalized to that tail of that standard.

Table 2
The dependence of the relative contribution of the nuclear interior ($0 \leqslant r \leqslant r^o_{\alpha,t}$ and $r^o_{\alpha t} = r_{\rm cut}$) $\Delta(E, b; r_{\rm cut})$ (in percent) to the calculated astrophysical S-factors of the reaction on the single-particle ANC b at different energies E

(a) for the $t + \alpha \to {}^7{\rm Li}({\rm g.s.}) + \gamma$ reaction

b (fm$^{-1/2}$)	$\Delta(E, b, r_{\rm cut})\%$														
	$r_{\rm cut} = 4.0$ fm					$r_{\rm cut} = 4.12$ fm					$r_{\rm cut} = 4.25$ fm				
2.734	1.0	0.9	0.9	1.4	2.4	2.1	2.2	2.3	4.1	7.1	3.5	3.5	3.7	5.8	9.2
2.959	3.0	3.0	3.0	2.4	0.5	0.1	0.1	2.4	1.4	4.2	1.2	1.3	1.4	3.1	6.3
3.219	4.9	4.9	4.9	4.5	2.6	2.1	2.1	2.1	1.0	1.8	0.8	0.8	0.7	0.7	3.8
3.501	6.7	6.7	6.8	6.8	4.8	4.2	4.2	4.2	3.4	1.9	3.0	3.0	3.0	1.9	1.3
3.764	8.7	8.9	8.8	9.1	7.4	6.5	6.5	6.5	6.1	3.6	5.4	5.4	5.4	4.7	1.8
E (MeV)	0.010	0.025	0.055	0.500	1.20	0.010	0.025	0.055	0.500	1.20	0.010	0.025	0.055	0.500	1.20

(b) for the $t + \alpha \to {}^7{\rm Li}(0.478\ {\rm MeV}) + \gamma$ reaction

b (fm$^{-1/2}$)	$\Delta(E, b, r_{\rm cut})\%$														
	$r_{\rm cut} = 4.0$ fm					$r_{\rm cut} = 4.12$ fm					$r_{\rm cut} = 4.25$ fm				
2.313	0.3	0.3	0.2	0.7	2.7	2.1	2.2	2.3	4.0	6.8	3.2	3.3	3.4	5.4	8.6
2.495	1.9	1.9	1.9	1.4	0.3	0.4	0.5	0.5	1.8	4.3	1.5	1.5	1.6	3.2	6.1
2.701	3.3	3.3	3.3	3.1	1.5	1.1	1.1	1.1	0.1	2.3	0.1	0.1	0.0	1.2	4.1
2.924	4.8	4.8	4.8	4.9	3.3	2.8	2.8	2.8	2.1	0.3	1.8	1.8	1.8	0.9	1.9
3.133	6.2	6.2	6.3	6.8	5.4	4.5	4.5	4.5	4.3	2.2	3.6	3.6	3.6	3.1	0.6
E (MeV)	0.010	0.025	0.055	0.500	1.20	0.010	0.025	0.055	0.500	1.20	0.010	0.025	0.055	0.500	1.20

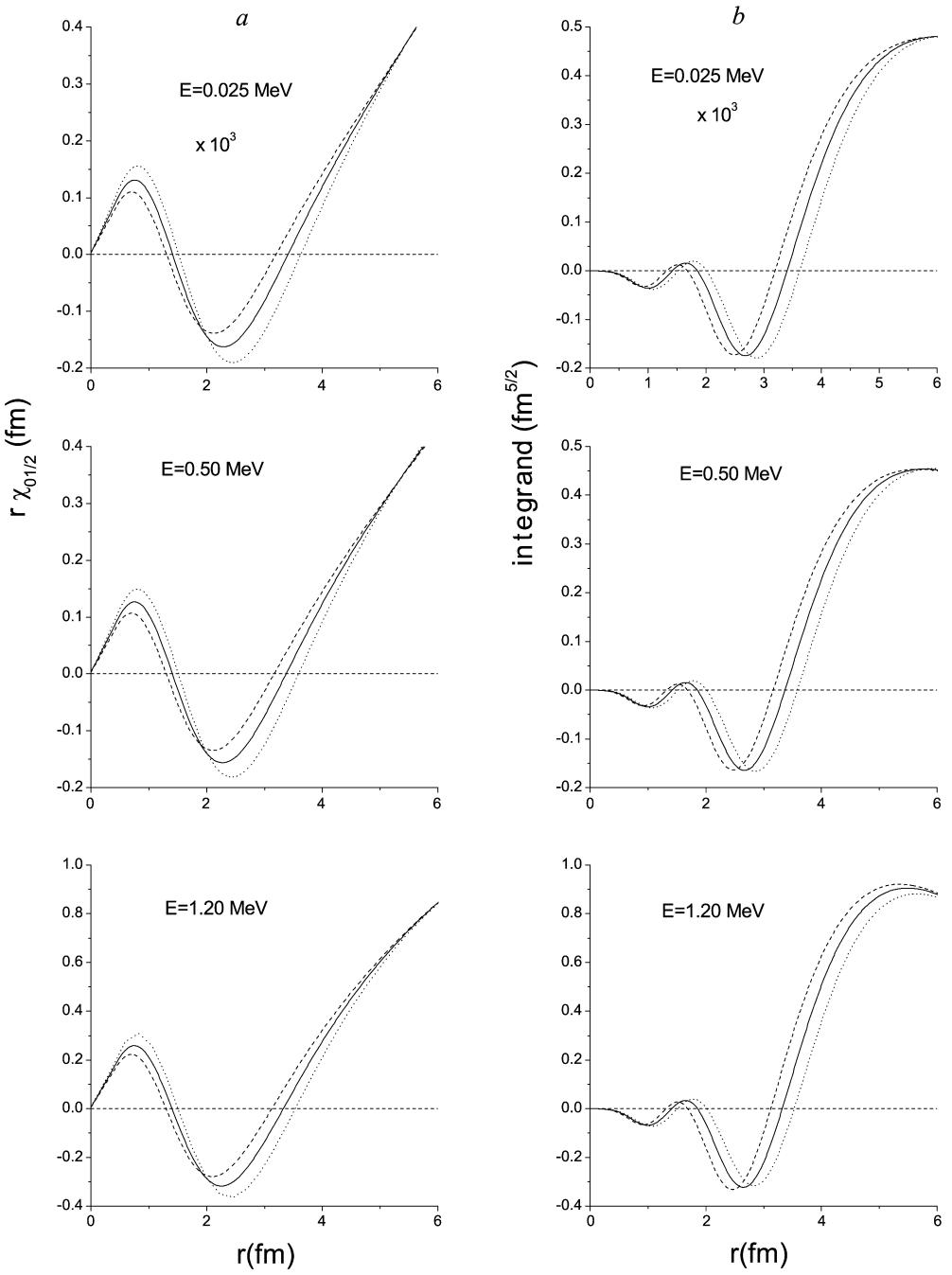

Fig. 4. Radial αt-scattering wave functions for partial s-wave (a) and the integrand of the radial integral (10) (b) for the $\alpha + t \to {}^{7}\mathrm{Li(g.s.)} + \gamma$ reaction at energies $E = 0.025$ and 0.050 and 1.2 MeV for different sets of $(r_o; a)$-pairs: (1.62 fm; 0.63 fm) (dashed line), (1.80 fm; 0.70 fm) (solid line as our standard) and (1.98 fm; 0.77 fm) (dotted line).

of $r_{\rm cut}$ for the $\alpha + t \to {}^7\mathrm{Li(g.s.)} + \gamma$ reaction at different energies. As one can see from Figs. 3 and 4 for the calculated bound state wave function, the calculated αt-scattering wave function and the integrand of the radial integral (10) change with the variation of the geometric parameters r_o and a. Such a change leads to calculated $\tilde{S}(E)$ that vary by 1.75 times over the energy region $55 \leqslant E \leqslant 1189$ keV, while the calculated values of the function $R(E, b)$ change by only $\pm 8.5\%$ with respect to the value of $R(E, b)$ corresponding to the standard values of $r_o = 1.80$ fm and $a = 0.70$ fm. We also see in Fig. 3 that all of the bound state wave functions indeed reach their asymptotic form (8) for $r > 4.5$ fm. Besides, it is seen from Fig. 4(b) that the behavior of the integrand in the radial integral (10) over a wide energy range, including the stellar ones, provides a strong suppression of the contribution only from the part of the nuclear interior with $0 \leqslant r \lesssim 2.0$ fm to the integral (10). But a noticeable change of the integrand of the aforesaid integral is observed with the variation of the parameters r_o and a in the range $2.0 \lesssim r \lesssim 6.0$ fm. The similar situation occurs for the $\alpha + t \to {}^7\mathrm{Li}(0.478\ \mathrm{MeV}) + \gamma$ reaction. Therefore, the choice of the cutoff radius $r_{\rm cut}$ becomes unambiguous since a fitted value of $r_{\rm cut}$ becomes dependent also on the parameters r_o and a (or on the single-particle ANC b). In this connection one would like to note the following. In paper [23] the calculation of the astrophysical S-factors for the reactions under consideration has been carried out using the expression (9) but introducing the cutoff radius $r_{\rm cut}$ in the lower limit of integration in the radial integral (10) and replacing on it the bound state wave function $\varphi_{nl_B j_B}(r_{Aa})$ by its asymptotic form (8) beginning with $r = r_{\rm cut}$. There the same Woods–Saxon potential with the geometric parameters $r_o = 1.80$ fm and $a = 0.70$ fm was used for calculating the αt-scattering wave function and the cutoff radius parameter $r_{\rm cut}$ was chosen by the best fitting of the calculated $S(E)$ to the experimental ones within the energy range $200 \leqslant E \leqslant 400$ keV, which is reached when the cutoff radius is $r_{\rm cut} = 4.0$ fm. From Figs. 3, 4 and Table 2 it is seen that in paper [23] indeed the contribution of the nuclear interior to the calculated astrophysical S-factors was underestimted,[6] since a contribution of the nuclear interior $0 < r \leqslant 4.0$ fm to the calculated astrophysical S-factors, which indeed is up to about 9%, has not been taken into account. Here, firstly, the contribution of the nuclear interior ($r \leqslant r^o_{\alpha t}$) to the calculated astrophysical S-factors is taken into account in a correct way by means of the appropriate choice of the adopted potential both for the initial state and for final state of the reactions under consideration. Secondly, the problem of the ambiguity connected with the strong (r_o, a)-dependence of the calculated astrophysical S-factors is removed by inclusion of the information about ANC (or NVC). The latter reduces this ambiguity to minimum.

For each experimental point of E ($E = E_i$, $i = 1$–17) from the interval of $55 \leqslant E \leqslant 1189$ keV using the corresponding experimental astrophysical S-factor in the r.h.s. of the relation (17) instead of the $S(E)$ and the central values of $R(E, b)$ corresponding to the adopted values of the parameters r_0 and a, the values of the ANC's are obtained for the $\alpha + t \to {}^7\mathrm{Li(g.s.)}$ and $\alpha + t \to {}^7\mathrm{Li}(0.478\ \mathrm{MeV})$. The results of the ANC's, $(C^{\rm exp}_{\alpha t})^2$ and $(C^{*\rm exp}_{\alpha t})^2$ for the seventeen experimental points of energy E are displayed in Fig. 5. The uncertainties pointed in this figure correspond to those found from (36) (averaged square errors (a.s.e.), which includes both the statistics experimental errors in the corresponding experimental astrophysical S-factor and the aforesaid uncertainty in the $R(E, b)$, and the systematic uncertainty of $\pm 6\%$ [22] added to the a.s.e.). It is seen from Fig. 5 that the ratio in the r.h.s. of the relation (17) practically does not depend on the energy E although absolute values of the corresponding experimental astrophysical

[6] It should be noted that there is a misprint in the line 36 upper of Section 3 of [23]. There the phrase "no more than 1% to" must be written as "no more than 10% to".

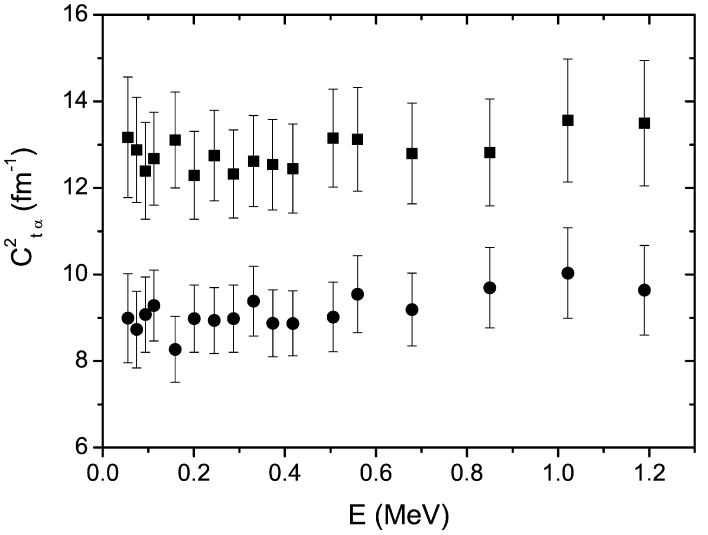

Fig. 5. The values of the ANC's, $C^2_{t\alpha}$ and $(C^*_{t\alpha})^2$, for the $\alpha + t \to {}^7\mathrm{Li}$ at all experimental energies E_i. The square (circle) symbols are for the ground (excited ($E^* = 0.478$ MeV)) state of ^{7}Li.

S-factors for the reactions under consideration depend noticeably on the energy and change by up to about 1.6 times in changing E from 55 keV to 1189 keV. This fact allows us to conclude that the energy dependence of the experimental astrophysical S-factors [22] is well determined by the calculated function $R(E, b)$ and, hence, the corresponding experimental astrophysical S-factors can be used as an independent source of getting information about the ANC's for the $\alpha + t \to {}^7\mathrm{Li}$(g.s.) and $\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV).

To test self-consistency we also calculated the αt-elastic scattering phase shifts by variation of the parameters r_o and a in the same range for the adopted Woods–Saxon potential. The results of calculations corresponding to s-, p-and d-waves are presented in Fig. 6 in which the width of the bands corresponds to a change of values of the phase shifts with respect to a variation of values of the parameters r_o and a. As is seen from Fig. 6, the experimental phase shifts [73–75] are well reproduced within an uncertainty of 10%.

The weighted means of the ANC-values obtained from the data presented in Fig. 5 for $\alpha + t \to {}^7\mathrm{Li}$(g.s.) and $\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV) are equal to $(C^{\mathrm{exp}}_{\alpha t})^2 = 12.74 \pm 1.10$ fm^{-1} and $(C^{*\mathrm{exp}}_{\alpha t})^2 = 9.00 \pm 0.90$ fm^{-1} ($C^{\mathrm{exp}}_{\alpha t} = 3.57 \pm 0.15$ fm$^{-1/2}$ and $C^{*\mathrm{exp}}_{\alpha t} = 3.00 \pm 0.15$ fm$^{-1/2}$). As shown in Refs. [15,21] and Fig. 3, one notes that $C_{\alpha t} > 0$ and $C^*_{\alpha t} > 0$, and the values of $C_{\alpha t}$ and $C^*_{\alpha t}$ should not be equal, in contrast with the assumption made in Ref. [76]. The corresponding values of the NVC's are $|G^{\mathrm{exp}}_{\alpha t}|^2 = 0.60 \pm 0.05$ fm and $|G^{*\mathrm{exp}}_{\alpha t}|^2 = 0.42 \pm 0.04$ fm. As noted earlier in paper [23], the values ANC's (NVC's) $C^2_{\alpha t} = 10.40$ fm^{-1} and $(C^*_{\alpha t})^2 = 7.87$ fm^{-1} ($|G_{\alpha t}|^2 = 0.48$ fm and $|G^*_{\alpha t}|^2 = 0.37$ fm) were obtained from an analysis of the same experimental data [22]. But as it was mentioned by us above, in paper [23], firstly, the contribution of the nuclear interior ($r \leqslant 4.0$ fm) to the calculated astrophysical S-factors was not included and, secondly, the values of ANC's were obtained from the best fitting only within the energy range $200 \leqslant E \leqslant 400$ keV. It is seen that, taking into account the contribution of the nuclear interior and using the experimental data in a wider energy region ($55 \leqslant E \leqslant 1189$ keV) than in Ref. [23] influences strongly the extracted values of the ANC's for $\alpha + t \to {}^7\mathrm{Li}$(g.s.) and $\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV). A comparison of the present result and that obtained in paper [23]

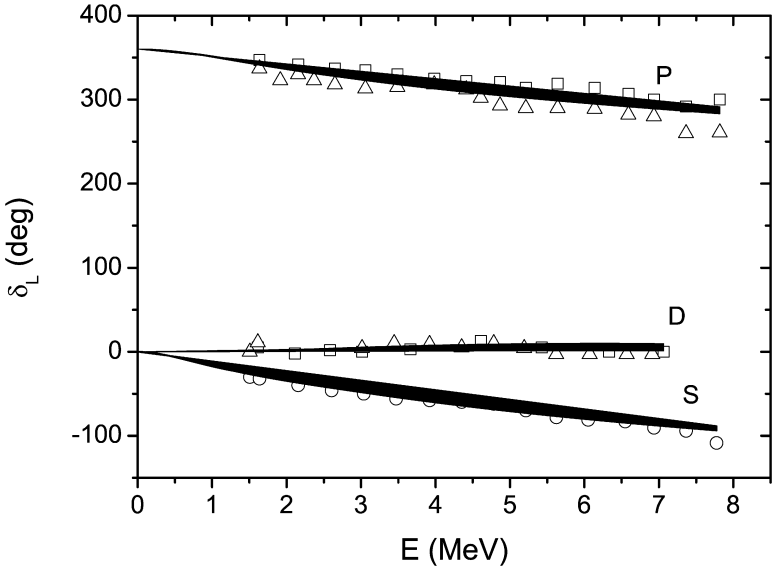

Fig. 6. The energy dependence of the $t\alpha$-elastic scattering phase shifts for different partial waves. The experimental data are from [73–75]. The bands are our calculated data. The width of the bands for fixed energies corresponds to the variation of the parameters r_o and a of the adopted Woods–Saxon potential within the intervals of $r_o = 1.62$ to 1.98 fm and $a = 0.63$ to 0.77 fm.

shows that, indeed the underestimation of the contribution both of the nuclear interior and of the nuclear exterior occurs in [23] since the present values of ANC's obtained from a more detailed analysis of the experimental astrophysical S-factors in the energy range $55 \leqslant E \leqslant 1189$ keV are larger than those obtained in [23] from the energy range of $200 \leqslant E \leqslant 400$ keV. Perhaps, this is one of the main reasons of the discrepancy observed between the astrophysical S-factors calculated in [23] and the experimental data [22] for $E < 120$ keV.

The resulting ANC (NVC) values obtained by us are in a good agreement with the value $C^2_{\alpha t} = 12.6 \pm 1.9$ fm^{-1} ($|G_{\alpha t}|^2 = 0.59 \pm 0.08$ fm) recommended in Ref. [50], which has been compiled from the results of Refs. [15,19,23] and has slightly larger uncertainty than our result, with the values $C^2_{\alpha t} = 11.47$ fm^{-1} and $(C^*_{\alpha t})^2 = 8.07$ fm^{-1} ($|G_{\alpha t}|^2 = 0.54$ fm and $|G^*_{\alpha t}|^2 = 0.38$ fm) [15,23] and with the values $(C_{\alpha t})^2 = 11.56 \pm 0.69$ fm^{-1} ($C_{\alpha t} = 3.4 \pm 0.1$ fm$^{-1/2}$ and $|G_{\alpha t}|^2 = 0.54 \pm 0.03$ fm) [21]. However, the results obtained by us for these ANC's differ noticeably from the values of $(C_{\alpha t})^2 = 9.76$ fm^{-1} ($|G_{\alpha t}|^2 = 0.46$ fm) [52] and $(C^*_{\alpha t})^2 = 7.02 \pm 0.54$ fm^{-1} ($C^*_{\alpha t} = 2.65 \pm 0.10$ fm$^{-1/2}$ and $|G^*_{\alpha t}|^2 = 0.33 \pm 0.03$ fm) [21].

Besides, one would like to note also the recent result of Ref. [76] obtained for $C_{\alpha t}$ and $C^*_{\alpha t}$ from the analysis of the same experimental data [22] performed within the R-matrix method. But there to reduce the number of free parameters, which are $C_{\alpha t}$, $C^*_{\alpha t}$ and the channel radius (r_c) as well as the R-matrix parameters (E^r_1, Γ_i andΓ_f) simulating the background for a single-pole at E^r_1, the assumption about equality of the ANC's ($C_{\alpha t} = C^*_{\alpha t}$) was used. The best fitting of the data was reached at $C_{\alpha t} = -3.49$ fm$^{-1/2}$, $r_c = 3.0$ fm, $E^r_1 = 10$ MeV, $\Gamma_i = 4.87$ MeV and $\Gamma_f = 1.71 \times 10^{-2}$ MeV for the $\alpha + t \to {}^7\mathrm{Li(g.s.)} + \gamma$ reaction. The same values for these parameters were obtained for the $\alpha + t \to {}^7\mathrm{Li}(0.478\ \mathrm{MeV}) + \gamma$ reaction, except the parameter Γ_f, which is equal to $\Gamma_f = 3.70 \times 10^{-3}$ MeV. But as it is mentioned above, firstly, in reality the sign of the ANC's is positive but not negative and, secondly, as it can be seen from Figs. 3 and 4, the

asymptotic behaviors of the bound $(\alpha + t)$ state and αt-scattering wave functions are reached simultaneously only at $r_c \gtrsim 5.0$ fm and, so, at $r_c \geqslant 3.0$ fm their substitution for these wave functions in the external part of the amplitude in Ref. [76] is not correct. Additionally, as it was shown in Section 3, the ANC (or NVC) for the $\alpha + t \to {}^7\mathrm{Li}$ in the effective range expansion depends both on the binding energy $\epsilon_{\alpha t}$ and on the effective range "radius" $r^{(cs)}_{ef;1s_B}$, where $s_B = 3/2$ (1/2) for the ground (the first excited) state of ^{7}Li. It follows from here that, in a reality the values of the ANC's, $C_{\alpha t}$ and $C^*_{\alpha t}$, should be not equal.

4.2. The effective range parameters and the phase shifts for the αt-scattering

Here one applies the expressions (34) and (25) for determination of values of the effective range parameters, $r^{(cs)}_{ef;1s_B}$ and $a^{(cs)}_{ef;1s_B}$, for the p-wave αt-scattering, where $s_B = 3/2$ and 1/2, since the values of the ANC's, $(C^{\mathrm{exp}}_{\alpha t})^2$ and $(C^{*\mathrm{exp}}_{\alpha t})^2$, are known. Using the weighed means of the ANC-values obtained by us one obtains the following values of the effective range parameters $r^{(cs)}_{ef;13/2} = -0.670 \pm 0.003$ fm^{-1} and $a^{(cs)}_{ef;13/2} = -42.03 \pm 0.52$ fm^3 for $s_B = 3/2$ and $r^{(cs)}_{ef;11/2} = -0.576 \pm 0.004$ fm^{-1} and $a^{(cs)}_{ef;11/2} = -51.71 \pm 0.85$ fm^3 for $s_B = 1/2$. As it is seen from here, differences between the values of the effective range parameters for $s_B = 3/2$ and $s_B = 1/2$ are comparatively (about a factor of 1.20) smaller, than those in the values of the ANC's (NVC's) (about a factor of $\mathcal{R}^{\mathrm{exp}} = |G^{\mathrm{exp}}_{\alpha t}|^2/|G^{*\mathrm{exp}}_{\alpha t}|^2 = (C^{\mathrm{exp}}_{\alpha t})^2/(C^{*\mathrm{exp}}_{\alpha t})^2 = 1.42 \pm 0.20$). One also notes that values of the effective range parameters obtained from Eqs. (34) and (25) (or Eq. (35)) without taking into account the Coulomb αt-interactions ($\eta_B = 0$) are equal to $r^{(s)}_{ef;13/2} = -1.387 \pm 0.003$ fm^{-1} and $a^{(s)}_{ef;13/2} = 20.26 \pm 0.12$ fm^3 for $s_B = 3/2$ and $r^{(s)}_{ef;11/2} = -1.254 \pm 0.004$ fm^{-1} and $a^{(s)}_{ef;11/2} = 27.48 \pm 0.23$ fm^3 for $s_B = 1/2$. It is seen that an inclusion of the Coulomb interaction influences noticeably the values of the effective range parameters. The results given here for $r^{(cs)}_{ef;1s_B}$ and $a^{(cs)}_{ef;1s_B}$ ($s_B = 3/2$ and 1/2) can be considered as "indirect measurements" of these parameters for p-wave αt-scattering since they are also determined by the "indirectly measured" values of the ANC's, $(C^{\mathrm{exp}}_{\alpha t})^2$ and $(C^{*\mathrm{exp}}_{\alpha t})^2$, in the relation (31) and (32) (or (34)).

The expression (21) allows also one to obtain values of the phase shifts, $\delta^{(cs)}_{13/2}$ and $\delta^{(cs)}_{11/2}$, for the p-wave αt-scattering at rather low energies, if the effective range parameters obtained above are used there. The found values of the phase shifts are presented in Table 3 (the fourth and sixth columns). In this table the results of calculation of these phase shifts, which were obtained from the radial Schrödinger equation with the adopted Woods–Saxon potential for the standard values of the geometric parameters ($r_o = 1.80$ fm, $a = 0.70$ fm), are also presented (the fifth and seventh columns). As it is seen from Table 3, the values of the p-wave phase shifts found in the energy range of $100 \lesssim E \lesssim 180$ keV within the effective range expansion ($0.375 \lesssim kr^o_{\alpha t} \lesssim 0.502$, $r^o_{\alpha t} = 4.12$ fm), using the information about the ANC's for the $\alpha + t \to {}^7\mathrm{Li}$ (or corresponding to them, the effective range parameters), are also in a good agreement with those obtained from the calculations. Besides, Table 3 shows the noticeable discrepancy between the calculated and obtained from the effective range expansion values of the p-wave phase shifts in the energy ranges of $E < 100$ keV, where an influence of the Coulomb αt-interaction increases as the values of E decrease (i.e. the value η becomes rather large), and of $E > 190$ keV, where the next terms of the expansion in the r.h.s. of (21) should be taken into account with an increase of a value of E (or k). Apparently, an application of the effective range expansion for the p-wave αt-scattering for those energy ranges is hardly justified.

Table 3
The Coulomb-nuclear αt-scattering phase shifts $\delta^{(cs)}_{l_{\alpha t}s_{\alpha t}}$ obtained from Eq. (21) within the effective range approximation (ERA) and from the Schrödinger equation (SE) with our adopted Woods–Saxon potential with the standard values of the geometric parameters ($r_o = 1.80$ fm and $a = 0.70$ fm). Here, k is the relative momentum of the α and t and η is the Sommerfeld parameter for the αt-scattering

E MeV	k fm^{-1}	η	$\delta^{(cs)}_{l_{\alpha t}s_{\alpha t}} \times 10^3$ (rad)			
			$l_{\alpha t}=1, s_{\alpha t}=3/2$		$l_{\alpha t}=1, s_{\alpha t}=1/2$	
			ERA	SE	ERA	SE
0.04	0.0575	2.070	0.001298	0.001721	0.001603	0.002143
0.05	0.0643	1.851	0.005439	0.007043	0.006723	0.009200
0.06	0.0704	1.690	0.01588	0.02008	0.01965	0.002529
0.07	0.0760	1.567	0.03694	0.04560	0.04577	0.05680
0.08	0.0813	1.464	0.07366	0.08805	0.09138	0.1096
0.10	0.0909	1.309	0.2164	0.2479	0.2691	0.3063
0.12	0.0996	1.195	0.4911	0.5354	0.6126	0.6601
0.14	0.1076	1.106	0.9470	0.9807	1.185	1.207
0.16	0.1150	1.035	1.636	1.606	2.055	1.972
0.18	0.1220	0.976	2.610	2.428	3.293	2.972
0.20	0.1285	0.926	3.930	3.454	4.980	4.217
0.21	0.1317	0.903	4.739	4.045	6.020	4.940

Thus, as it is seen from here that, the measured ANC's for the $t + \alpha \to {}^7\mathrm{Li}$ can also be used to get independent information about the "measured" values of the effective range parameters and the p-wave phase shifts for the αt-scattering at the aforesaid energy range.

4.3. Astrophysical S-factor for the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction at stellar energies

Eq. (18) and the weighed means of the ANC's obtained can be used for calculating the $t + \alpha \to {}^7\mathrm{Li} + \gamma$ astrophysical S-factor for capture to the ground and first excited states as well as the total astrophysical S-factor at stellar energies ($E \leqslant 50$ keV). At first, we tested again the fulfilment of the condition (15) in the same way as it is done above for $E \geqslant 50$ keV. Similar results plotted in Fig. 1 are also observed for a dependence of the $R(E, b)$ function on the single-particle ANC b, at stellar energies of $E \leqslant 50$ keV. The results of extrapolation of the astrophysical S-factor for six values of E ($E = 0$, 10, 20, 30, 40 and 50 keV) obtained by us are displayed in Fig. 7. In this figure circles and squares correspond to the astrophysical S-factors $S_{\mathrm{ex}}(E)$ and $S_{\mathrm{g.s.}}(E)$ for the capture to the excited (0.478 MeV) and ground states of ^{7}Li, respectively, triangles correspond to the total astrophysical S-factor $S(E)$. The closed symbols in the Fig. 7 are experimental data from [22], while the open symbols are the result of extrapolation of the S-factor at the above mentioned energies. The solid lines present our calculations performed with the standard values of geometric parameters $r_o = 1.80$ fm and $a = 0.70$ fm both for the bound $(\alpha + t)$ state and for αt-scattering state. These curves can be presented by the polynomial formulae as the following:

$$S_{\mathrm{g.s.}}(E) = 0.06826 - 0.0804E + 0.07727E^2 - 0.02769E^3 + 0,00488E^4 - 3.06721 \times 10^{-4}E^5, \tag{38}$$

$$S_{\mathrm{ex}}(E) = 0.02910 - 0.03514E + 0.03473E^2 - 0.01267E^3 + 0.00226E^4 - 1.4539 \times 10^{-4}E^5, \tag{39}$$

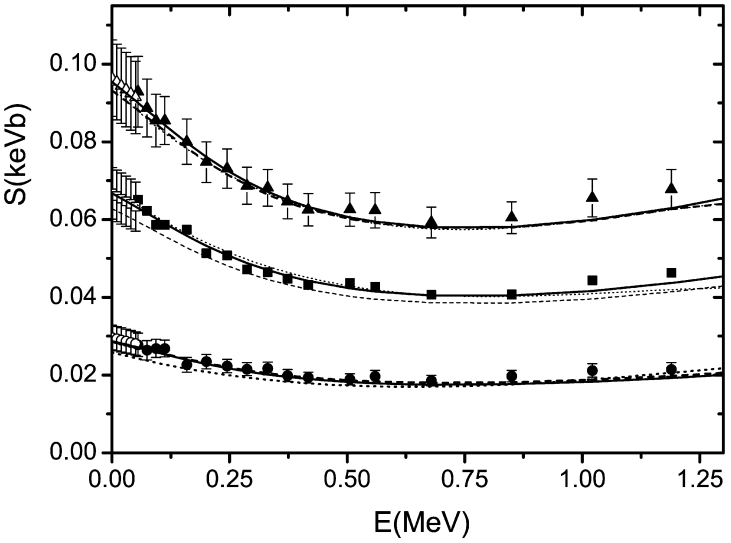

Fig. 7. The astrophysical S-factors for the $t + \alpha \to {}^7\mathrm{Li}(\mathrm{g.s.}) + \gamma$ (square), $t + \alpha \to {}^7\mathrm{Li}(0.478\ \mathrm{MeV}) + \gamma$ (circles) and $t + \alpha \to {}^7\mathrm{Li}(\mathrm{g.s.} + 0.478\ \mathrm{MeV}) + \gamma$ (triangles) reactions. The filled symbols are experimental data taken from [22] and the blank points are our results. The solid lines are the results of our calculation with the standard values of $r_o = 1.80$ fm and $a = 0.70$ fm, the dashed lines are the results of Ref. [23] and the dotted lines are the results of Ref. [21].

$$S(E) = 0.09736 - 0.11554E + 0.112E^2 - 0.04036E^3 + 0.00714E^4 - 4.52111 \times 10^{-4}E^5 \quad (40)$$

in units of keV b, and E in MeV, where $S(E) = S_{\mathrm{g.s.}}(E) + S_{\mathrm{ex}}(E)$.

These polynomial formulae were obtained by fitting to our calculated astrophysical S-factors given by (18) both for the $S_{\mathrm{g.s.}}(E)$ and for the $S_{\mathrm{ex}}(E)$ and also for the total $S(E)$ at low energies up to 2.0 MeV by using the corresponding values of ANC's of our data and the standard values of geometric parameters $r_o = 1.80$ fm and $a = 0.70$ fm. One notes that the ratio of $S(E)$ obtained from formula (40) in the energy range $1.2 < E \lesssim 2.0$ MeV ($0 \leqslant E \lesssim 0.4$ MeV) to that obtained from the linear extrapolation formula $S(E) = 0.0337 + 0.0285E$ (keV b) ($S(E) = 0.1 - 0.5E + 0.13E^2$ (keV b)), which was recommended in Ref. [22] (Ref. [2]) for a calculation of the reaction rate at the energy range of $E > 1.20$ MeV ($E < 0.4$ MeV), changes from 0.94 to 1.02 (from 0.97 to 1.10) times with increase of E, i.e. the linear extrapolation formula proposed in [22] ([2]) in the energy range $1.2 < E \lesssim 2.0$ MeV ($0 \leqslant E \lesssim 0.4$ MeV) has an accuracy not exceeding 8% (13%) in respect to our results. The values of the total astrophysical S-factor $S(E)$ recommended by us for the reaction under consideration at stellar energies are $S(0) = 0.0974 \pm 0.0100$ keV b, $S(10\ \mathrm{keV}) = 0.0954 \pm 0.0097$ keV b, $S(20\ \mathrm{keV}) = 0.0944 \pm 0.0097$ keV b, $S(30\ \mathrm{keV}) = 0.0934 \pm 0.0095$ keV b, $S(40\ \mathrm{keV}) = 0.0924 \pm 0.0094$ keV b and $S(50\ \mathrm{keV}) = 0.0914 \pm 0.0093$ keV b. Each of the quoted uncertainties in the $S(E)$ is the a.s.e., which involves the uncertainties both for the ANC's adopted and that in $R(E, b)$. As it is seen from Fig. 7, Eq. (18) allows us to perform a correct extrapolation of the corresponding astrophysical S-factor at stellar energies practically in an independent way when the corresponding ANC values are known. For comparison, in Fig. 7 the results of Ref. [23] (dashed lines) and Ref. [21] (dotted lines) are also displayed. The figure shows that at extremely low energies a discrepancy between the present results and that of Ref. [23] at $E \lesssim 150$ keV occurs. Recall that the result of

Ref. [23] has been obtained from analysis of the same $S_{\rm g.s.}^{\rm exp}(E)$ and $S_{\rm ex}^{\rm exp}(E)$ performed in the two-body potential approach, but introducing the cutoff radius ($r_{\rm cut} = 4.0$ fm) in lower limit of the radial integral (10) and replacing the single-particle wave function, $\varphi_{nl_B j_B}(r_{\alpha t})$, by its asymptotic expression (8), where, as it was mentioned above, the underestimation of the contributions both from the nuclear interior ($r < r_{\alpha t}^{o}$) and the external region ($r \geqslant r_{\alpha t}^{o}$) occurred. Besides, the noticeable underestimation in absolute values of the calculated in Ref. [21] ratio $S_{\rm ex}(E)/S_{\rm g.s.}(E)$ with the experimental data also occurs (see Fig. 7 in [21]). This circumstance is apparently connected with the underestimated value of $C_{\alpha t}^2$ and $(C_{\alpha t}^*)^2$ (or $|G_{\alpha t}|^2$ and $|G_{\alpha t}^*|^2$) obtained in Ref. [21] in respect to ours. This conclusion is confirmed by the fact that there is rather noticeable discrepancy between the $S_{\rm ex}(E)$ calculated in Ref. [21] and the corresponding experimental data (see Fig. 7). In this connection it should be noted that the bound state wave functions and the initial state wave functions in Ref. [21] were computed with different potentials and, so, these calculations were not self-consistent. Besides, the values of the binding energies for the bound states of ^{7}Li calculated in Ref. [21] differ from the experimental ones. Therefore, the calculated values of the binding energies ($\epsilon_{\alpha t}$) for the bound states of ^{7}Li in the $(\alpha + t)$-channel do not agree with the experimental ones also (see Table I in Ref. [21]). Since the ANC's (or NVC's) for the $\alpha + t \to {}^7$Li and the $S(0)$ are sensitive to the form of the NN potential [15], it is desirable, firstly, to calculate the wave functions of the bound state using other forms of the NN-potential, and, secondly, to guarantee the self-consistency the same forms of the NN-potential should be used for a calculation of the initial wave functions. This would allow to choose a NN-potential that yields ANC's (or NVC's) values and a $S(0)$ value closest to those obtained in the present work since the ambiguity arising due to the model dependence of the ANC's (or NVC's) and of the calculated astrophysical S-factors on the geometric parameters of the adopted Woods–Saxon potential used for the calculations of the bound $(\alpha + t)$ state and the αt-scattering state, is reduced to minimum here.

In this context we would like to note especially the results of calculation of $S(0)$ obtained in Ref. [15] within the microscopical single-channel $(\alpha + t)$ resonating group method using seven different forms of the NN-potential. One of those values of $S(0) = 0.098 \pm 0.006$ keV b calculated by the modified Hasegawa–Nigata (MHN) form of the NN-potential is the closest to that obtained in the present work. One of the possible reason of this coincidence is the fact that the wave function of $\alpha - t$ relative motion in ^{7}Li, calculated in Ref. [15] using the MHN form of the NN-potential, has the correct asymptotic behavior, which in turn gives the ANC (NVC) value equal $C_{\alpha t}^2 \approx 12.6$ fm^{-1} ($|G_{\alpha t}|^2 \approx 0.59$ fm) [15,50]. The latter figures are in a good agreement with those obtained in the present work too. It is worth noting here that the MHN potential reproduces rather well other observed static nuclear properties of ^{7}Li [15] and, as shown in the present work, the data obtained here for the ANC's (NVC's) should also be included as an observable nuclear property of ^{7}Li. Besides, we observe that the values of $S(0) \approx 0.105$ keV b [16] and $S(0) \approx 0.1$ keV b [19] obtained within the microscopical $(\alpha + t)$-cluster approach and within the two-body cluster model using αt-potential deduced by a double-folding procedure, respectively, are also in a good agreement with our results. It is seen from here that the mutual agreement between the results obtained here and other authors within the aforesaid different methods [15,16,19], which are based on the common approximation about the cluster $(\alpha + t)$ structure of the ^{7}Li, confirms also the conclusion made by Kajino in paper [15] of the dominant contribution of the $(\alpha + t)$ clusterization to the low-energy $t + \alpha \to {}^7\text{Li} + \gamma$ cross section both in the absolute

normalization and in the energy dependence [22].[7] Therefore, a single-channel ($\alpha + t$) approximation is quite appropriate for this reaction in the considered energy range. Our result is also in a good agreement with that recommended in Ref. [76], $S(0) = 0.095 \pm 0.005$ keV b.

Thus, it follows from here that the overall normalization of the astrophysical S-factors at stellar energies for the reactions under consideration is mainly determined by the ANC-values for the $\alpha + t \to {}^7\mathrm{Li}$(g.s.) and $\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV), which can be determined rather well from an analysis of the corresponding experimental astrophysical S-factor [22] a model independent way, and the values of the ANC's allow us to perform correct extrapolation of the astrophysical S-factors for the direct radiative capture $t + \alpha \to {}^7\mathrm{Li} + \gamma$ reaction at stellar energies, including $E = 0$.

5. Thermonuclear reaction rate

The obtained total astrophysical S-factor, $S(E)$, has been used to calculate the reaction rates as a function of stellar temperature within the range of $10^{-3} \leqslant T_9 \leqslant 10$, where T_9 is a temperature in unit of 10^9 K. The Maxwellian-averaged reaction rates $N_A\langle\sigma v\rangle$ are given by [78,79]

$$N_A\langle\sigma v\rangle = N_A\left(\frac{8}{\pi\mu_{t\alpha}}\right)^2 (k_B T)^{-3/2} \int_0^\infty \sigma(E)\exp[-E/k_B T]E\,dE \tag{41}$$

as a function of the temperature T. Herein N_A is the Avogadro number, k_B is the Boltzmann constant, $v = \sqrt{2E/\mu_{t\alpha}}$, and E is in MeV.

To provide a high accuracy of the numerical integration in Eq. (41) for each fixed T_9, the cross sections $\sigma(E)$ were calculated within the R-matrix method similar as it has been performed in work of Ref. [76], i.e. the internal (resonance) contribution is simulated by the background with a single pole at $E_1^r = 10$ MeV. But differently from Ref. [76] we fixed here the contribution from the external (nonresonance) part by using the values of the ANC's obtained in the present work and of the channel radius equals to $r_c = 5.0$ fm. The best fitting of the data was reached at the values of the resonance parameters $\Gamma_i = 4.87$ MeV and $\Gamma_f = 3.65 \times 10^{-3}$ MeV ($\Gamma_f = 1.74 \times 10^{-3}$ MeV) for the $\alpha + t \to {}^7\mathrm{Li}$(g.s.)$+\gamma$ ($\alpha + t \to {}^7\mathrm{Li}$(0.478 MeV)$+\gamma$) reaction with the reduced χ^2 value equal to 0.22. It is seen that the values of Γ_f obtained by us differ noticeably from those obtained in Ref. [76] (see Section 4.1 also). However, one notes that the astrophysical S-factors calculated by us in the R-matrix method in the energy range of $0 \leqslant E \leqslant 5$ MeV are in excellent agreement with those obtained from Eq. (18) (see Fig. 7 also). But at energies $E \gtrsim 5.0$ MeV a noticeable influence of the aforesaid background resonance is observed. The good accuracy for the reaction rate calculation at each temperature is reached by limiting the numerical integration in Eq. (41) to the energy range $0 \leqslant E \leqslant 12$ MeV. The accuracy is such that at least 4 or 5 digits on the reaction rate are significant.

In Fig. 8 we present the ratio of result of our calculation of the reaction rate $N_A\langle\sigma\vartheta\rangle$ to those generated by the recent result recommended in Ref. [76] (solid line), to the result of Ref. [2] (dashed line) and that recommended in Ref. [22] (dash-dotted lines). As it is seen the reaction rate obtained by us is in fairly good agreement with that given in [22] at the temperatures $T_9 \lesssim 6$. However, for the range of $T_9 > 6$ the given rates are noticeably larger (up to a factor of 1.15 times) than those obtained in Ref. [22]. This is caused by the fact that the present astrophysical S-factors

[7] We also wish to point out the need to understand the $t + \alpha$ structure of ${}^7\mathrm{Li}$ in order to treat reactions, such as ${}^{34}S({}^7\mathrm{Li}, t\alpha){}^{34}S$ [77] for investigating the cluster nature of states in heavier nuclei.

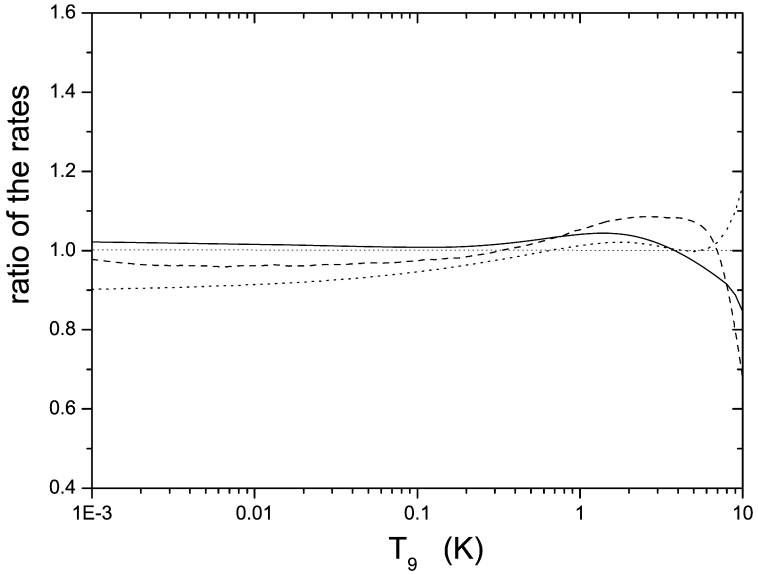

Fig. 8. The ratio of the $t + \alpha \to {}^{7}\mathrm{Li} + \gamma$ reaction rate $N_A\langle\sigma\vartheta\rangle$ of the present work to that taken from Ref. [76] (solid line), to that taken from Ref. [2] (dashed line) and to that taken from Ref. [22] (dash-dotted line).

and those calculated in the Ref. [22] are rather close to each other up to energies about 2.0 MeV. Note that in work of Ref. [22] the rate of the reaction under consideration has been calculated by using the $S(E)$ calculated by authors of Ref. [80] for energies $E \leqslant 1.2$ MeV and the aforesaid linear extrapolation formula for energies $E > 1.2$ MeV, while in the present calculation we use the single expression for all energies E up to 12 MeV. From Fig. 8 one can also see that our results are in good agreement with the rates given in Ref. [2] at the temperatures $T_9 \lesssim 1$ (dashed line). However, for the range of $1 < T_9 < 6$ the given rates are noticeably larger (up to a factor of about 1.1 times), while for $T_9 > 7$ this ratio rapidly decreases. This difference is explained by the fact that the total astrophysical S-factors within the range of $0.2 \leqslant E \leqslant 1.2$ MeV, which was used in Ref. [2] for calculation of the reaction rate, are smaller than ours, but as the energy E increases, this situation becomes vice versa. There is also seen a good agreement between our result and that obtained in [76] at $T_9 < 6$. However, the noticeably distinction (up to a factor of 1.15) between our result and those recommended in Ref. [76] occurs for the range of $1 < T_9 < 6$. Apparently, this is caused by the fact that the present astrophysical S-factors $S(E)$ are noticeably smaller than those obtained in [76] at the energy range of $E > 1.0$ MeV.

6. Conclusion

The modified two-body potential approach for analysis of the astrophysical S-factor $S(E)$ for the peripheral direct radiative capture $a + A \to B + \gamma$ reaction of astrophysical interest is proposed in which two additional conditions of verification of the peripheral character of the $a + A \to B + \gamma$ reaction are also formulated. The proposed approach involves information on the nuclear vertex constant (NVC) (or the respective asymptotic normalization coefficient (ANC)) for the virtual decay $B \to A + a$ and allows one to reduce to a minimum the model dependence of the calculated direct astrophysical S-factor, $S(E)$, on the geometric parameters (radius r_o and diffuseness a) of the adopted Woods–Saxon potential, which is usually used for calculations both

for the two-body $(A + a)$ bound state and for the Aa-scattering state. The connection between the NVC (ANC) and the parameters of the effective range parameters for the Aa-scattering for an arbitrary partial wave and with an allowance for the Coulomb Aa-interaction has been derived.

The proposed approach has been applied to the reanalysis of the experimental astrophysical S-factor, $S^{\rm exp}(E)$, precisely measured in the energy range $55 \leqslant E \leqslant 1189$ keV [22] for the $t + \alpha \to {}^7{\rm Li} + \gamma$ reaction populating to the ground and first excited ($E^* = 0.478$ MeV) states of ^{7}Li. It is shown that at extremely low energies these reactions are peripheral and the contribution of the nuclear interior ($r < r^o_{\alpha t}$) to the astrophysical S-factors is up to 9%. It is demonstrated that the experimental astrophysical S-factors of the reactions under consideration measured in the energy region $55 \leqslant E \leqslant 1189$ MeV can be used as an independent source of getting the information about the ANC's (or NVC's) for α-particle removed from the ground and first excited states of ^{7}Li. It was also demonstrated that the obtained values of the ANC's (NVC's) have fairly weak dependence on variation of the geometric parameters (r_o and a) of the adopted Woods–Saxon potential both for the bound $(\alpha + t)$ state and for the αt-scattering state, while the extracted values of spectroscopic factors, $Z_{\alpha t}$ and $Z^*_{\alpha t}$, change rather strongly. The weighed means of the ANC's (NVC's) for α-particle removed from the ground and first excited states of ^{7}Li are obtained and they have to be $C^2_{\alpha t} = 12.74 \pm 1.10$ fm^{-1} and $(C^*_{\alpha t})^2 = 9.00 \pm 0.90$ fm^{-1} ($|G_{\alpha t}|^2 = 0.60 \pm 0.05$ fm and $|G^*_{\alpha t}|^2 = 0.42 \pm 0.04$ fm), respectively. The uncertainties in the ANC's (NVC's)-values involve both the experimental errors for the experimental astrophysical S-factors, $S^{\rm exp}_{\rm g.s.}(E)$ and $S^{\rm exp}_{\rm ex}(E)$, and that of the proposed approach. It is shown that the parametrization of the direct astrophysical S-factors in terms of ANC's for the nucleus ^{7}Li in the $(\alpha + t)$-configuration is quite justified for the under consideration reaction.

The obtained values of the ANC's were used for the estimation of values the effective range parameters and the phase shift for the p-wave αt-scattering as well as an extrapolation of the astrophysical S-factors of the reaction under consideration at energies less than 50 keV, including $E = 0$. In particular, for the full astrophysical S-factor $S(0)$ the value of $S(0) = 0.097 \pm 0.010$ keV b has been obtained, which is in a good agreement with the recent result of the NACRE compilation $S(0) = 0.10 \pm 0.02$ keV b [2] and that of $S(0) = 0.095 \pm 0.005$ keV b [76] as well as with $S(0) = 0.098 \pm 0.006$ keV b [15], $S(0) \approx 0.105$ keV b [16] and $S(0) \approx 0.1$ keV b [19] obtained within the microscopic single-channel $(\alpha + t)$ resonating-group method using the MHN form for the NN-interaction, the microscopic cluster $(\alpha + t)$ model and the two-body $(\alpha + t)$ cluster model using the double-folded αt-potential, respectively.

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Erratum

Erratum to "Modified two-body potential approach to the peripheral direct capture astrophysical $a + A \to B + \gamma$ reaction and asymptotic normalization coefficients" [Nucl. Phys. A 781 (2007) 247]

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In our publication in [Nucl. Phys. A 781 (2007) 247] the relation (25), where due to our misprint the expression $\kappa_{Aa}^{2l_{Aa}+1}$ on the left-hand side should be replaced by $(-1)^{l_{Aa}}\kappa_{Aa}^{2l_{Aa}+1}$, is valid only for the neutral case ($\eta = 0$).[1] In this connection, the first term ($(-1)^{l_{Aa}+1}\kappa_{Aa}^{2l_{Aa}+1}$) on the right-hand side of Eq. (24) should be replaced by $-1/a_{ef;l_{Aa}s_{Aa}}^{(cs)} - r_{ef;l_{Aa}s_{Aa}}^{(cs)}\kappa_{Aa}^2/2$. Nevertheless, the final results of Section 3 given by Eqs. (31)–(35) as well as the values of the effective radii in Section 4.2 are correct. But, the values of the scattering lengths $a_{ef;13/2}^{(cs)}$ and $a_{ef;11/2}^{(cs)}$ for αt-scattering presented in Section 4.2 are not correct since they have been obtained by using the relation (25) valid for the neutral case.

The relation (25) in the charged case ($\eta \neq 0$) should be replaced by the relation

$$(-1)^{l_{Aa}}\kappa_{Aa}^{2l_{Aa}+1} = \frac{1/a_{ef;l_{Aa}s_{Aa}}^{(cs)} + r_{ef;l_{Aa}s_{Aa}}^{(cs)}\kappa_{Aa}^2/2}{\eta_B D_{l_{Aa}}(-i\eta_B)[\pi\cot\pi\eta_B - 2\,\mathrm{Re}\,h(-i\eta_B)]}, \tag{25}$$

which is derived from the condition $f_{l_{Aa}}(i\kappa_{Aa}; r_{ef;l_{Aa}s_{Aa}}^{(cs)}) = 0$ and this condition corresponds to the pole singularity of the S matrix located at the point $k = i\kappa_{Aa}$. In the absence of the Coulomb interaction ($\eta_B = 0$) this relation is reduced to $(-1)^{l_{Aa}}\kappa_{Aa}^{2l_{Aa}+1} = 1/a_{ef;l_{Aa}s_{Aa}}^{(s)} + r_{ef;l_{Aa}s_{Aa}}^{(s)}\kappa_{Aa}^2/2$.

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The correct values of the scattering lengths $a^{(cs)}_{ef;13/2}$ and $a^{(cs)}_{ef;11/2}$ are equal to 29.95 ± 0.47 fm^3 and 28.76 ± 0.40 fm^3, respectively. Due to this, the values of the phase shifts ($\delta^{(cs)}_{l_{\alpha t} s_{\alpha t}}$) for αt-scattering presented in the fourth and sixth columns of Table 3 are modified and they differ noticeably from those given in the fifth and seventh columns of Table 3.

We have spotted two minor misprints as well: (1) the function $f_l(k; r^{(cs)}_{ef;l_{Aa}s_{Aa}})$ in the right-hand side of Eq. (23) should be replaced by $f_{l_{Aa}}(k; r^{(cs)}_{ef;l_{Aa}s_{Aa}})$ and (2) the phrase "the cutoff radius r_{cut} becomes unambiguous..." in the line 14 (upper) of p. 265 should be replaced by "the cutoff radius r_{cut} becomes ambiguous...".