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THE THREE-CLUSTER STRUCTURES IN ^{7}Li

R. BECK

Kernforschungszentrum Karlsruhe, Institut für Angewandte Kernphysik II, 7500 Karlsruhe, Postfach 3640, Federal Republic of Germany

and

R. KRIVEC and M. V. MIHAILOVIĆ

J. Stefan Institute, E. Kardelj University, POB 199/IV, 61000 Ljubljana, Yugoslavia

and

Kernforschungszentrum Karlsruhe, Institut für Angewandte Kernphysik II, 7500 Karlsruhe, Postfach 3640, Federal Republic of Germany

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Abstract: A cluster model for the description of light nuclei is investigated which includes the interplay of three-cluster structures with the two-cluster ones and allows molecule-like vibrations of clusters. It is applied to the nucleus ^{7}Li in order to study the influence of the three-cluster structures of the type (^{4}He-^{2}H-n) on the low-lying states previously described by two-cluster structures (^{4}He-^{3}H) and (^{6}Li-n). An effective central interaction is used in the calculation.

The space of the wave functions is spanned by antisymmetrized products of two and three shell-model wave functions describing separated clusters. In the case of two-cluster configurations the separation s of the two shell-model potentials is used as the generator coordinate. In the case of three-cluster configurations with fragments A, B and C the separation $s_1 = s_A - s_B$ between the two shell-model potentials for fragments A and B is taken as one of the generator coordinates, the other one being the separation $s_2 = s_C - (As_A + Bs_B)/(A+B)$. The cluster configurations are characterized also by the spins of fragments and their relative angular momenta (L_1 and L_2). The rotational and the space inversion symmetry of the states is obtained by projecting the basic states onto the eigenspace of the operators J, M and the parity operator. An algebraic method based on an algorithm which expresses the rotation of the coordinates in terms of rotation of the generator coordinates is used.

The structure of the nucleus ^{7}Li is described by the two-cluster configuration (^{4}He-^{3}H) and the three-cluster configurations (^{4}He-^{2}H(I_d)-n), with $I_d = 0$, 1, and the total spin $I = \frac{1}{2}, \frac{3}{2}$. In the wave function of three-cluster structure the pair of values $L_1 = 0$, $L_2 = 1$ only is included. The effective nuclear potential V2 of Volkov is used in the calculation. The energy of the ground state described by a single configuration of the two-cluster structure (^{4}He-^{3}H) is lowered by 0.66 MeV when this configuration is coupled to two three-cluster configurations and the molecule-like vibration is allowed through solving the Hill-Wheeler equation. Both mechanisms have approximately equal effects. The ground-state energy (-38.14 MeV) is 0.3 MeV lower than in the model which describes the ^{7}Li by a superposition of two-cluster structures (^{4}He-^{3}H) and (^{6}Li-n).

1. Introduction

Cluster models [1]) of the structure of light nuclei are often used to describe some energy levels which are difficult to access in the traditional shell-model framework. Usually one takes either the cluster structures with nucleons in 1s single-particle shell-model states and with an arbitrary spin [e.g.: (^{4}He-^{4}He), ref. [2]), (^{2}H-^{4}He), ref. [3]), (^{3}H-^{4}He), ref. [4])], or the cluster structures with nucleons in 1s and 1p single-particle shell-model states and with spin equal to zero [e.g.: (^{16}O-^{4}He), ref. [5]), (^{16}O-^{3}H) and (^{16}O-^{3}He), ref. [6])].

As some of these clusters are not tightly bound so that new cluster structures might be formed by transferring some nucleons from one cluster to another, an attempt [7]) has recently been made to include the cluster with nucleons in 1p single-particle shell-model states with an arbitrary spin and to study the interplay of different two-cluster configurations. The nucleus ^{7}Li is treated as the superposition of states corresponding to the cluster configurations (^{3}H-^{4}He) and (^{6}Li-n). The cluster ^{6}Li is described by a superposition of the shell-model wave functions $(1s)^4(1p)^2$. Its bound-state energy comes out too high. This causes troubles in describing the reactions with ^{6}Li in the final channel (e.g.: $^3\mathrm{H}+{}^4\mathrm{He} \rightarrow {}^6\mathrm{Li}+\mathrm{n}$) as well as in describing some states of ^{7}Li. The usual response to this deficiency is that one has to look for a more complicated effective interaction which can describe ^{6}Li in addition to the properties of ^{7}Li, but such an interaction is not proposed yet. Another possibility is to look for a more suitable subspace of the Hilbert space for the ^{7}Li.

The aim of this paper is to investigate the consequences of describing the fragment ^{6}Li as a two-cluster structure (^{4}He-^{2}H). The states of the nucleus ^{7}Li are then described as the interplay of the two-cluster structure (^{4}He-^{3}H) and the three-cluster structure (^{4}He-^{2}H-n), the cluster ^{2}H being in the ground or in the excited state. An effective central nucleon-nucleon interaction [8]) is chosen in order to make possible a comparison with the results of several earlier calculations with that interaction [9,10,7]). We concentrate our attention only on the ground state of the system and the states of the three-cluster structure.

Wave functions of the generator coordinate type are used in the description and their vector space is spanned by two types of Slater determinants:

(i) Two-centre determinants $\mathscr{A}\{\Phi_{^4\mathrm{He}}\Phi_{^3\mathrm{H}_\beta}\}$, where $\Phi_{^4\mathrm{He}}(\Phi_{^3\mathrm{H}_\beta})$ is a product of four (three) 1s single-particle functions of the harmonic oscillator well centred at $s_{^4\mathrm{He}}(s_{^3\mathrm{H}})$. The index β runs over two spin configurations of the ^{3}H. The separation $s = s_{^4\mathrm{He}} - s_{^3\mathrm{H}}$ is the generator coordinate.

(ii) Three-centre determinants $\mathscr{A}\{\Phi_{^4\mathrm{He}}\Phi_{^2\mathrm{H}_\beta}\Phi_{\mathrm{n}_\gamma}\}$, where $\Phi_{^4\mathrm{He}}$ is a product of four 1s single-particle functions centred at $s_{^4\mathrm{He}}$, $\Phi_{^2\mathrm{H}_\beta}$ is a product of two 1s single-particle functions of the harmonic oscillator well centred at $s_{^2\mathrm{H}}$ and Φ_{n_γ} is the single-particle function of the neutron centred at s_{n}. The index $\beta(\gamma)$ runs over all spin configurations of the two particles (neutron). The generator coordinates are the separations $s_1 = s_{^4\mathrm{He}} - s_{^2\mathrm{H}}$ and $s_2 = s_{\mathrm{n}} - \frac{1}{6}(2s_{^2\mathrm{H}} + 4s_{^4\mathrm{He}})$.

The rotational and the space inversion symmetry of nuclear states is introduced by projecting the functions of the basis onto the subspaces of eigenfunctions of angular momentum and parity operators. The wave functions of bound states are superpositions of the GC square integrable functions.

The proposed model is used to study the low-lying states of ^{7}Li, performing a calculation similar to that of ref. [7]). The energy levels are calculated first as functions of the GC coordinates s, s_1 and s_2 and then as solutions of the Schrödinger equation in the space of GC wave functions generated by s, s_1 and s_2 separately and in the space which is the direct sum of all spaces.

In sect. 2 we present the model for describing three-cluster configurations and their coupling to the two-cluster configurations. The calculation of some states of ^{7}Li is presented in sect. 3. Sect. 4 contains a discussion of results and open problems.

2. A generator coordinate formalism for coupled two- and three-cluster structures in light nuclei

2.1. THE GC BASIS OF THE TWO- AND THREE-CLUSTER STRUCTURES

The basis of the two-centre functions is described in previous papers [7]) so we shall supply only a few necessary expressions at the end of this section.

In order to construct a basis for the description of a nucleus in which three clusters are formed with mass numbers A, B and C we start with single-particle functions $\psi_i(x_j, s_A)$, $\psi_i(x_j, s_B)$ and $\psi_i(x_j, s_C)$ of the harmonic oscillator potentials centred at s_A, s_B and s_C, respectively. (The spin coordinate of the particle j is comprised in the index i). Then we construct the product of $A(B, C)$ functions ψ which corresponds to the cluster A(B, C):

$$\Phi_{A_\alpha}(x_A, s_A), \qquad \Phi_{B_\beta}(x_B, s_B), \qquad \Phi_{C_\gamma}(x_C, s_C), \tag{2.1}$$

where

$$x_A = (x_1, \ldots x_A), \qquad x_B = (x_{A+1}, \ldots x_{A+B}), \qquad x_C = (x_{A+B+1}, \ldots x_{A+B+C}),$$

and where the symbol $\alpha(\beta, \gamma)$ denotes the spin configuration of nucleons in the cluster A(B, C). From the functions (2.1) we form the set of functions

$$\Phi^{(3)}_{I_i I_v}(x, s_1, s_2) = \{\mathcal{A}[\Phi_{A_\alpha}(x_A, s_A)\Phi_{B_\beta}(x_B, s_B)\Phi_{C_\gamma}(x_C, s_C)]\}_{Iv}, \tag{2.2}$$

where

$$s_1 = s_A - s_B, \qquad s_2 = s_C - (As_A + Bs_B)/(A+B),$$
$$As_A + Bs_B + Cs_C = 0, \qquad x = (x_A, x_B, x_C).$$

The symbols Iv denote the total spin, whereas the symbol I_i stands for all intermediate spin quantum numbers which result from coupling the spins of $A+B+C$ particles to the total spin Iv.

A wave function of the GC type is constructed by superposing the functions (2.2),

$$\Psi^{(3)}(\boldsymbol{x}) = \sum_{I_1 I_V} \int \mathrm{d}s_1 \int \mathrm{d}s_2 f^{(3)}_{I_1 I_V}(s_1, s_2) \Phi^{(3)}_{I_1 I_V}(\boldsymbol{x}, s_1, s_2).$$

We found it useful to express $f^{(3)}$ and $\Phi^{(3)}$ in terms of bipolar harmonics [11]:

$$B_{L_1 L_2 LM}(\hat{s}_1, \hat{s}_2) = \sum_{M_1 M_2} (L_1 M_1, L_2 M_2 | LM)\, Y_{L_1 M_1}(\hat{s}_1) Y_{L_2 M_2}(\hat{s}_2),$$

$$f^{(3)}(s_1, s_2) = \sum_{L_1 L_2 LM} \frac{1}{s_1 s_2} f^{(3)}(L_1 L_2 LM, s_1 s_2) B_{L_1 L_2 LM}(\hat{s}_1, \hat{s}_2),$$

$$\Phi^{(3)}(\boldsymbol{x}, s_1, s_2) = \sum_{L_1 L_2 LM} \frac{1}{s_1 s_2} \Phi^{(3)}(\boldsymbol{x}, L_1 L_2 LM, s_1 s_2) B^*_{L_1 L_2 LM}(\hat{s}_1, \hat{s}_2).$$

This gives

$$\psi^{(3)}(\boldsymbol{x}) = \sum_{I_1 I_V L_1 L_2 LM} \int_0^\infty \mathrm{d}s_1 \int_0^\infty \mathrm{d}s_2 f^{(3)}_{I_1 I_V}(L_1 L_2 LM, s_1 s_2) \Phi^{(3)}_{I_1 I_V}(\boldsymbol{x}, L_1 L_2 LM, s_1 s_2). \quad (2.3)$$

Finally, the basis of the GC functions is obtained by projecting the functions (2.3) onto the eigenspace of the parity operator and the angular momentum operators. The angular momentum projector is

$$P^{JM} = \frac{2J+1}{8\pi^2} \sum_K \int \mathrm{d}\Omega \mathscr{D}^{J*}_{MK}(\Omega) R(\Omega),$$

where $R(\Omega)$ rotates the coordinate system. It has the property

$$(P^{JM})^+ P^{J'M'} = \delta_{JJ'} \delta_{MM'} \sum_{K'K} P^J_{K'K},$$

where

$$P^J_{K'K} = \frac{2J+1}{8\pi^2} \int \mathrm{d}\Omega \mathscr{D}^{J*}_{K'K}(\Omega) R(\Omega).$$

The relevant part of the projected many-body function which enters the calculations of matrix elements is therefore $\sum_{KK'} P^\pi P^J_{K'K} \psi^{(3)}(\boldsymbol{x})$.

After some straightforward algebra one gets

$$\sum_{KK'} P^\pi P^J_{K'K} \psi^{(3)}(\boldsymbol{x}) = \sum_{I_1 I_V L_1 L_2 LMK'} (Iv, LM | JK') \int_0^\infty \mathrm{d}s_1 \int_0^\infty \mathrm{d}s_2 f^{(3)J\pi}_{I_1 I}(L_1 L_2 L, s_1 s_2) \times \Phi^{(3)}_{I_1 I_V}(\boldsymbol{x}, L_1 L_2 LM, s_1 s_2), \quad (2.4)$$

with

$$f^{(3)J\pi}_{I_1 I}(L_1 L_2 L, s_1 s_2) = \sum_{vMK} f^{(3)}_{I_1 I_V}(L_1 L_2 LM, s_1 s_2)(Iv, LM | JK) \tfrac{1}{2}[1 + (-)^{(1-\pi)/2 - L_1 - L_2}]. \quad (2.5)$$

For later use we write the corresponding two-centre GC functions for a nucleus with clusters A and B. The expression corresponding to (2.2) is

$$\Phi^{(2)}_{I_1 I_V}(\boldsymbol{x}, s) = \{\mathscr{A}[\Phi_{A_\alpha}(\boldsymbol{x}_A, s_A)\Phi_{B_\beta}(\boldsymbol{x}_B, s_B)]\}_{I_V}, \tag{2.2'}$$

with

$$\boldsymbol{x}_A = (\boldsymbol{x}_1, \ldots \boldsymbol{x}_A), \qquad \boldsymbol{x}_B = (\boldsymbol{x}_{A+1}, \ldots \boldsymbol{x}_{A+B}), \qquad s = s_A - s_B,$$

while that corresponding to eq. (2.3) is

$$\psi^{(2)}(\boldsymbol{x}) = \sum_{I_1 I_V LM} \int_0^\infty \mathrm{d}s f^{(2)}_{I_1 I_V}(LM, s)\Phi^{(2)}_{I_1 I_V}(\boldsymbol{x}, LM, s), \tag{2.3'}$$

and those corresponding to eqs. (2.4) and (2.5) are

$$\sum_{KK'} P^\pi P^J_{K'K}\psi^{(2)}(\boldsymbol{x}) = \sum_{I, I_V LMK'} (I_V, LM|JK') \int_0^\infty \mathrm{d}s f^{(2)J\pi}_{I_1 I}(L, s)\Phi^{(2)}_{I_1 I_V}(\boldsymbol{x}, LM, s), \tag{2.4'}$$

$$f^{(2)J\pi}_{I_1 I}(L, s) = \sum_{\nu MK} f^{(2)}_{I_1 I_V}(LM, s)(I_V, LM|JK)\tfrac{1}{2}[1 + (-)^{(1-\pi)/2-L}]. \tag{2.5'}$$

2.2. THE MODEL HAMILTONIAN

We use in our calculation the microscopic hamiltonian of the form

$$H = \left(-\frac{\hbar^2}{2m}\sum_i \nabla_i^2 - T_{\text{c.m.}}\right) + \sum_{i<j} V^{\text{N}}_{ij}. \tag{2.6}$$

Here V^{N}_{ij} stands for the nucleon-nucleon potential which we assume to be expressible by a series of appropriate gaussian potentials.

For the effective central nuclear interaction we chose the effective nuclear potential V2 of Volkov [8]) which was already used in Hartree-Fock calculations [9]), in calculations with cluster orbitals [10]) and in the model with interplay of cluster structures [7]).

2.3. THE EQUATION OF MOTION IN THE MODEL SPACE

To determine the eigenfunctions of the hamiltonian (2.6) we use the variational principle

$$\delta \frac{\langle \psi^{JM\pi}|H|\psi^{JM\pi}\rangle}{\langle \psi^{JM\pi}|\psi^{JM\pi}\rangle} = 0, \tag{2.7}$$

with the trial function

$$\psi^{JM\pi}(\boldsymbol{x}) = P^{JM\pi}[\psi^{(2)}(\boldsymbol{x}) + \psi^{(3)}(\boldsymbol{x})],$$

where $\psi^{(2)}(x)$ and $\psi^{(3)}(x)$ are given in eqs. (2.3) and (2.3′). In the GC basis eq. (2.7) has the following form

$$\sum_{I_i} \int_0^\infty \mathrm{d}s' f^{(2)J\pi}_{I_i I}(L, s') \langle \Phi^{(2)}_{I_i I}(L, s) | \bar{H} | \Phi^{(2)}_{I_i I}(L, s') \rangle$$

$$+ \sum_{I_i L_1 L_2} \int_0^\infty \mathrm{d}s_1' \int_0^\infty \mathrm{d}s_2' f^{(3)J\pi}_{I_i I}(L_1' L_2' L, s_1' s_2') \langle \Phi^{(2)}_{I_i I}(L, s) | \bar{H} | \Phi^{(3)}_{I_i I}(L_1' L_2' L, s_1' s_2') \rangle = 0,$$

$$\sum_{I_i} \int_0^\infty \mathrm{d}s' f^{(2)J\pi}_{I_i I}(L, s') \langle \Phi^{(3)}_{I_i I}(L_1 L_2 L, s_1 s_2) | \bar{H} | \Phi^{(2)}_{I_i I}(L, s') \rangle$$

$$+ \sum_{I_i L_1 L_2} \int_0^\infty \mathrm{d}s_1' \int_0^\infty \mathrm{d}s_2' f^{(3)J\pi}_{I_i I}(L_1' L_2' L, s_1' s_2') \langle \Phi^{(3)}_{I_i I}(L_1 L_2 L, s_1 s_2) | \bar{H} | \Phi^{(3)}_{I_i I}(L_1' L_2' L, s_1' s_2') \rangle = 0, \tag{2.8}$$

which has to be satisfied for all values of I_i, L_1, L_2, s, s_1 and s_2. Here $\bar{H} = H - EN$, where H is the hamiltonian operator (2.6) and N is the identity operator.

The equation of motion (2.8) is solved approximately:

(i) the integrations are replaced by finite sums:

$$f^{(2)}(s) = \sum_{i=1}^{N^{(2)}} f^{(2)}(s_i) \delta(s - s_i),$$

$$f^{(3)}(s_1, s_2) = \sum_{i=1}^{N^{(3)}} \sum_{j=1}^{N^{(3)}} f^{(3)}(s_{1i}, s_{2j}) \delta(s_1 - s_{1i}) \delta(s_2 - s_{2j}),$$

so that eq. (2.8) transforms into a generalized eigenvalue problem in a finite basis for the hamiltonian;

(ii) only a few pairs (L_1, L_2) are chosen for a given set (J, π, I).

3. The low-lying states of ^{7}Li

3.1. THE MODEL FOR MOLECULE-LIKE STATES OF ^{7}Li

The structure of ^{7}Li is described as a superposition of the following cluster structures: (^{3}H-^{4}He), (^{4}He-^{2}H($I_d = 1$)-n) and (^{4}He-^{2}H($I_d = 0$)-n). The secular equation is then of the form

$$\begin{pmatrix} \bar{H}^{(22)} & \bar{H}^{(23)}_0 & \bar{H}^{(23)}_1 \\ \bar{H}^{(32)}_0 & \bar{H}^{(33)}_{00} & \bar{H}^{(33)}_{01} \\ \bar{H}^{(32)}_1 & \bar{H}^{(33)}_{10} & \bar{H}^{(33)}_{11} \end{pmatrix} \begin{pmatrix} f^{(2)} \\ f^{(3)}(I_d' = 0) \\ f^{(3)}(I_d' = 1) \end{pmatrix} = 0, \tag{3.1}$$

where the following notation is used: the amplitudes f are vectors

$$f^{(2)} = \begin{pmatrix} f^{(2)}(s_1) \\ \cdot \\ \cdot \\ \cdot \\ f^{(2)}(s_{N^{(2)}}) \end{pmatrix}, \qquad f^{(3)}(I_d) = \begin{pmatrix} f^{(3)}_{I_d}(s_{11}, s_{21}) \\ \cdot \\ \cdot \\ \cdot \\ f^{(3)}_{I_d}(s_{1N^{(3)}}, s_{2N^{(3)}}) \end{pmatrix}, \qquad I_d = 1, 0, \tag{3.3}$$

and the quantities $\bar{H}^{(22)}$, $\bar{H}^{(23)}_{I_d}$ and $\bar{H}^{(33)}_{I_d I'_d}$ are matrices with matrix elements

$$\langle \Phi^{(2)}_I(L, s_i)|\bar{H}|\Phi^{(2)}_I(L, s_{i'})\rangle, \qquad (i, i' = 1,\ldots N^{(2)}), \tag{3.3'}$$

$$\langle \Phi^{(2)}_I(L, s_i)|\bar{H}|\Phi^{(3)}_{I'_d I}(01L, s_{1i'_1}s_{2i'_2})\rangle, \qquad (i = 1,\ldots N^{(2)}, i'_1, i'_2 = 1,\ldots N^{(3)}),$$

$$I'_d = 1,0, \tag{3.3''}$$

$$\langle \Phi^{(3)}_{I_d I}(01L, s_{1i_1}s_{2i_2})|\bar{H}|\Phi^{(3)}_{I'_d I}(01L, s_{1i'_1}s_{2i'_2})\rangle,$$

$$(i_1, i_2, i'_1, i'_2 = 1,\ldots N^{(3)}),\ I_d, I'_d = 1, 0, \tag{3.3'''}$$

respectively.

3.2. RESULTS OF THE CALCULATION

The calculation is performed in two steps. In the first step the nucleus ^{7}Li is described by the three-cluster configuration (^{4}He-^{2}H(I_d = 1)-n) and (^{4}He-^{2}H(I_d = 0)-n) separately and then by coupling both configurations. We present the expectation values of the hamiltonians $H^{(33)J^\pi}_{I_d I_d}(s_1 = s'_1, s_2 = s'_2)/N^{(33)J^\pi}_{I_d I_d}(s_1 = s'_1, s_2 = s'_2)$ as functions of distances s_1 and s_2 of the shell-model potentials, as well as the eigensolutions of matrices $H^{(33)}_{I_d I'_d}$ and the matrix

$$\begin{pmatrix} \bar{H}^{(33)}_{00} & \bar{H}^{(33)}_{01} \\ \bar{H}^{(33)}_{10} & \bar{H}^{(33)}_{11} \end{pmatrix}.$$

In the second step the states of the nucleus ^{7}Li are described as a superposition of the two-cluster structure (^{4}He-^{3}H) and three-cluster structures (^{4}He-^{2}H(I_d)-n), I_d = 1, 0. The information on suitable parameters of the structure (^{4}He-^{3}H) is taken from a previous calculation [7]).

3.2.1. Three-cluster structures in ^{7}Li.

(*i*) *The single structures* (^{4}*He-*^{2}*H*(I_d = *1*)*-n*). There are two three-cluster structures of this type, one with the eigenvalue $\frac{1}{2}$ and another with the eigenvalue $\frac{3}{2}$ of the total spin I, eq. (2.2). The first structure has the total angular momentum and the parity $J^\pi = \frac{3}{2}^-, \frac{1}{2}^-$ and the second one $J^\pi = \frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$.

(a) The energy surface

$$E^{(33)\frac{3}{2}^-,\frac{1}{2}^-}_{11}(s_1, s_2) = H^{(33)\frac{3}{2}^-,\frac{1}{2}^-}_{11}(s_1, s_2; s_1, s_2)/N^{(33)\frac{3}{2}^-,\frac{1}{2}^-}_{11}(s_1, s_2; s_1, s_2)$$

is shown in fig. 1. The minimum -29.3 MeV is below both energies [7]) that correspond to any single cluster configuration (^{6}Li-n) (~ -28.0 MeV) as well as that

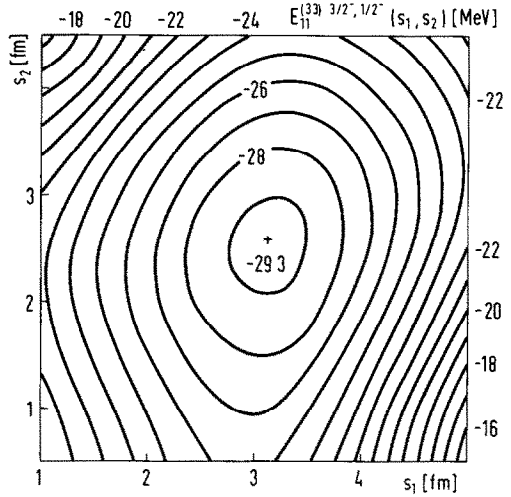

Fig. 1. The energy surface $E_{11}^{(33)3/2^-,1/2^-}(s_1, s_2)$ is the expectation value of the hamiltonian which corresponds to the three-cluster configuration (^{4}He-^{2}H(I_d = 1)-n) with the total internal spin $I = \frac{1}{2}$ in the states $J^\pi = \frac{3}{2}^-$, $\frac{1}{2}^-$. The effective nuclear potential V2 of Volkov [8] is used. The single-particle eigenfunctions of the harmonic oscillator with the oscillator parameter $\beta = 0.5$ fm^{-2} are used to construct the GC basis.

corresponding to the cluster configuration (^{6}Li-n) (~ -28.8 MeV) in which the fragment ^{6}Li is described by a shell-model wave function composed of $(1s)^4(1p)^2$ determinants.

The dependence of $E_{11}^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$ on the oscillator parameter β is shown in fig. 2 for two choices of s_1 and s_2 near the values corresponding to the minimum.

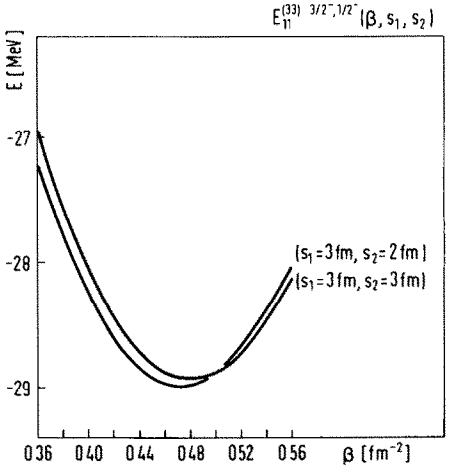

Fig. 2. The energy shown in fig. 1 is presented as a function of the oscillator parameter of the single-particle functions for the two pairs of values of s_1 and s_2 near the minimum of the energy surface $E_{11}^{(33)3/2^-,1/2^-}(s_1, s_2)$.

An optimal β is about 0.48 fm^{-2} which is very close to the optimal value $\beta = 0.5$ fm^{-2} for the cluster structure (^{4}He-^{3}H). This is quite different from the optimal $\beta = 0.38$ fm^{-2} for the cluster structure (^{6}Li-n).

The Hill-Wheeler equation $\bar{H}_{11}^{(33)} f_1^{(3)} = 0$ is solved for the ground-state quantum numbers by using a basis of 25 GC functions. The results is -31.17 MeV for the ground state and -18.08 MeV for the first excited state.

(b) The energy surface $E_{11}^{(33)\frac{5}{2}^-,\frac{3}{2}^-,\frac{1}{2}^-}(s_1, s_2)$ is shown in fig. 3. The minimum energy is -19.3 MeV, its position is approximately equal to that in fig. 1.

Solving the Hill-Wheeler equation $\bar{H}_{11}^{(33)} f_1^{(3)\frac{5}{2}^-,\frac{3}{2}^-,\frac{1}{2}^-} = 0$ in a basis of 25 GC functions we obtained -22.70 MeV and -12.54 MeV for the lowest and the

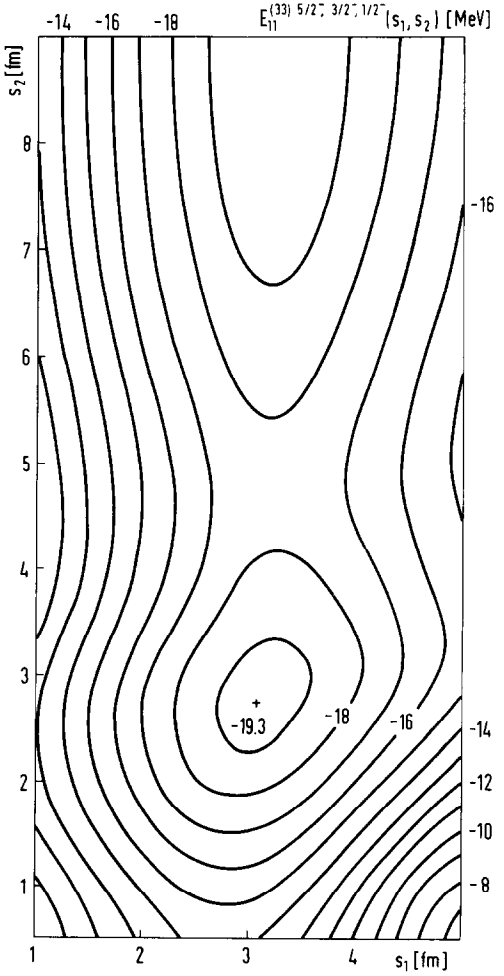

Fig. 3. The energy surface $E_{11}^{(33)5/2^-,3/2^-,1/2^-}(s_1, s_2)$ is the expectation value of the hamiltonian which corresponds to the three-cluster configuration (^{4}He-^{2}H($I_d = 1$)-n) with $I = \frac{3}{2}$ in the states $J^\pi = \frac{5}{2}^-$, $\frac{3}{2}^-$, $\frac{1}{2}^-$. In this and the following figs. the interaction and the oscillator parameter are the same as in fig. 1.

first excited state with the above quantum numbers. Both states represent excited states of the nucleus ^{7}Li.

(ii) The single structure (^{4}He-^{2}H($I_d = 0$)-n). The energy surface $E_{00}^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$ is shown in fig. 4. The values of s_1 and s_2 which correspond to the minimum -23.8 MeV are approximately equal to those for the previous curve $E_{11}^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$.

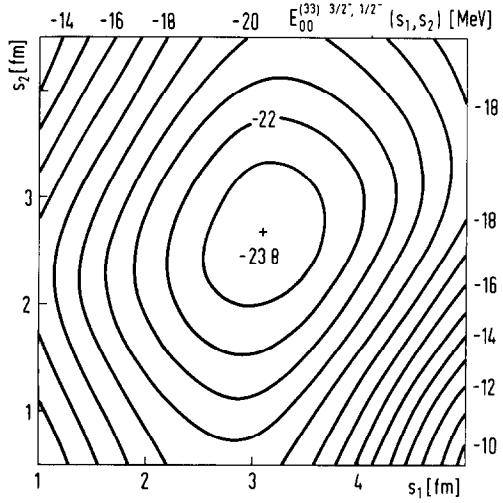

Fig. 4 The same as in fig. 1 except $I_d = 0$.

The solution of the equation $\bar{H}_{00}^{33} f_0^{(3)} = 0$ in a basis of 25 GC functions gives the energies -26.23 MeV and -14.75 MeV for the lowest and first excited state, respectively.

(iii) Superposition of three-cluster structures (^{4}He-^{2}H($I_d = 1$)-n and (^{4}He-^{2}H ($I_d = 0$)-n). The energy surface $E^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$ shown in fig. 5 is obtained by solving the equations

$$\begin{pmatrix} \bar{H}_{00}^{(33)} & \bar{H}_{01}^{(33)} \\ \bar{H}_{10}^{(33)} & \bar{H}_{11}^{(33)} \end{pmatrix} \begin{pmatrix} f_0^{(3)} \\ f_1^{(3)} \end{pmatrix} = 0 \tag{3.4}$$

separately for each of 35 pairs of values $(s_1, s_2)_{(^4\mathrm{He}-^2\mathrm{H}(I_d=1)-n)} = (s_1, s_2)_{(^4\mathrm{He}-^2\mathrm{H}(I_d=0)-n)}$. The position of the minimum is approximately at the same place as the minima of the surfaces $E_{11}^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$ and $E_{00}^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)$. The energy $[E^{(33)\frac{1}{2}^-,\frac{1}{2}^-}(s_1, s_2)]_{\min} = -31.2$ MeV.

This energy surface should be compared with the energy curve corresponding to the two-cluster structure (^{6}Li-n), when the fragment ^{6}Li is described by a symmetric combination of spherical shell model configurations [ref. [7]), fig. 5]. There, for the interaction V2 of Volkov [8]) $[E^{\frac{1}{2}^-\ \frac{1}{2}^-}(s)]_{\min} = -30.8$ MeV for $\beta = 0.38$

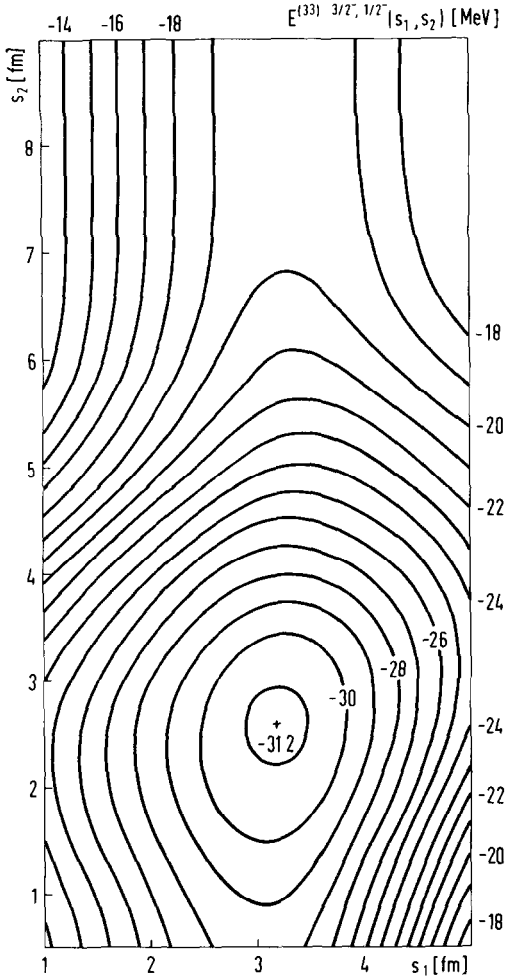

Fig 5. The effect of the superposition of the three-cluster structures (^{4}He-^{2}H(I_d = 1)-n) and (^{4}He-^{2}H(I_d = 0)-n) is shown by drawing the eigenenergy of the eq (3.4) calculated for several pairs $(s_1, s_2)_{I_d=1} = (s_1, s_2)_{I_d=0}$.

fm^{-2} and about 2 MeV higher for the value of β = 0.50 fm^{-2} which is used in coupling different cluster structures.

Solving the Hill-Wheeler equation (3.4) in a GC basis of 2×25 vectors we obtain -32.89 MeV and -22.70 MeV for the lowest and the first excited state, respectively.

In order to estimate the threshold of the reaction ^{6}Li+n, eq. (3.4) is solved in a GC basis spanned by 10 functions corresponding to a value of s_2 which is sufficiently large for this estimate (s_2 = 9.0 fm). We obtained -21.40 MeV. This corresponds to -26.58 MeV for the binding energy of the fragment ^{6}Li when the spuriousness of the relative motion of the neutron [7]) has been taken into account.

3.2.2. Interplay of two-cluster structure and three-cluster structures. The effect of the superposition of the structure (^{4}He-^{3}H) and the structures (^{4}He-^{2}H(I_d = 1)-n) and (^{4}He-^{2}H(I_d = 0)-n) on the expectation values of the hamiltonian for the ground state is shown in figs. 6 and 7.

In fig. 6 the energy curve C2 represents the expectation value of the hamiltonian

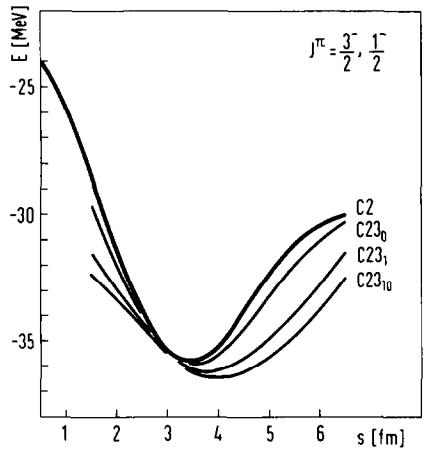

Fig. 6. The effect of the interplay of two- and three-cluster structures is shown by presenting how the expectation value of the energy for the two-cluster structure (C2) is changed upon coupling this structure to three-cluster configurations (^{4}He-^{2}H(I_d)-n). The curve $C23_0$ (23_1) corresponds to the spin of ^{2}H I_d = 0(1). It is obtained from eq. (3.5) by fixing the generator coordinates $s_1 = s_1'$ and $s_2 = s_2'$ at the minimum of the surface in fig. 4(1). The curve $C23_{10}$ corresponds to coupling of both three-cluster configurations to the two-cluster configuration (^{4}He-^{3}H) by using eq. (3.1) of dimension 3.

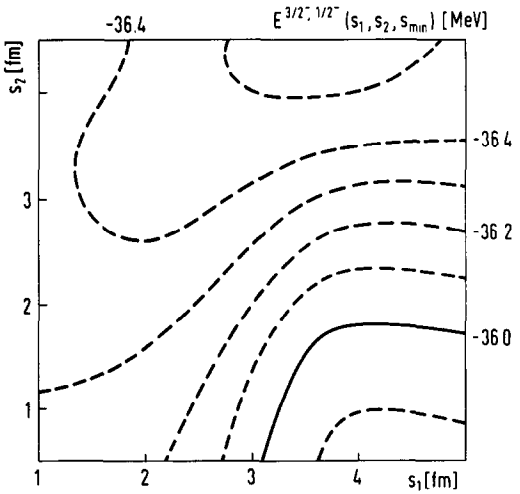

Fig. 7. The energy surface $E^{3/2^-,1/2^-}(s_1, s_2; s_{min})$ is calculated from eqs. (3.1) in the space spanned by three wave functions, the first corresponding to the two-cluster configuration with the lowest energy ($s = s_{min}$) and the other two wave functions corresponding to the three-cluster configurations (^{4}He-^{2}H(I_d)-n), I_d = 0, 1.

$E^{(22)\frac{3}{2}^-,\frac{1}{2}^-}(s) = H^{(22)}(s, s)/N^{(22)}(s, s)$. The energy curve $C23_1$ is obtained by solving the equation

$$\begin{pmatrix} \bar{H}^{(22)} & \bar{H}_1^{(23)} \\ \bar{H}_1^{(32)} & \bar{H}_{11}^{(33)} \end{pmatrix} \begin{pmatrix} f^{(2)} \\ f_1^{(3)} \end{pmatrix} = 0 \tag{3.5}$$

in the basis of two wave functions which correspond to the states $s = s'$ and $s_1 = s_1' = s_{1\min}$, $s_2 = s_2' = s_{2\min}$, where $s_{1\min}$ and $s_{2\min}$ are the coordinates of the minimum in fig. 1. The curve $C23_0$ is obtained similarly by coupling the cluster structure (^{4}He-^{3}H) with (^{4}He-^{2}H($I_d = 0$)-n) and using for $s_{1\min}$ and $s_{2\min}$ the values from the surface in fig. 2. The energy curve $C23_{10}$ is obtained by solving the system (3.1) in a GC basis of three vectors obtained by setting $s = s'$ and choosing for $s_1 = s_1'$ and $s_2 = s_2'$ the corresponding pairs of $s_{1\min}$ and $s_{2\min}$. The minimum energy of this three-component model for ^{7}Li is -36.6 MeV with a "distance" between ^{4}He and ^{3}H shifted to ~ 4 fm.

In fig. 7 we present the energy surface obtained by solving eq. (3.1) in a space of three GC functions obtained by fixing $s = s' = 3.5$ fm for a chosen pair of $s_1 (= s_1')$ and $s_2 (= s_2')$. The rather flat surface expresses the dominance of the two-cluster structure (^{4}He-^{3}H) in the ground state.

Finally, eq. (3.1) is solved in the GC basis

$$\{\Phi^{JM\pi}_{(^4\mathrm{He}-^3\mathrm{H})}(\boldsymbol{x}, s),\ \Phi^{JM\pi}_{(^4\mathrm{He}-^2\mathrm{H}(I_d=1)-n)}(\boldsymbol{x}, s_1, s_2),\ \Phi^{JM\pi}_{(^4\mathrm{He}-^2\mathrm{H}(I_d=0)-n)}(\boldsymbol{x}, s_1, s_2)\},$$

$$s = 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 6.5 \text{ fm},$$

$$s_1 = 1.0, 2.0, 3.0, 4.0, 5.0 \text{ fm},$$

$$s_2 = 0.5, 1.5, 2.5, 3.5, 4.5 \text{ fm}.$$

The energy of the ground state $E^{\frac{3}{2}^-,\frac{1}{2}^-}_{\mathrm{g.s}} = -38.14$ MeV and of the first excited state $E^{\frac{3}{2}^-,\frac{1}{2}^-}_{\mathrm{exc}} = -29.67$ MeV.

4. Discussion

In this paper we examined the role of three-cluster configurations in the nucleus ^{7}Li. In the calculations we used a model for light nuclei which allows the interference of two-cluster and three-cluster configurations.

(I) The two main characteristics of the model are:

(i) it includes the molecular-like vibrations of fragments and

(ii) it includes the superposition of different configurations of two- and three-cluster structures.

Three-cluster configurations are described by two sets of data: (a) the quantities describing fragments, e.g. their spins (I_1, I_2, I_3) and isospins, and (b) orbital angular momenta between fragments (L_1 = the orbital angular momentum of the relative motion between the first and the second fragment, and L_2 = the orbital angular

momentum between the third fragment and two other fragments). In principle, one has to include all possible clusterings, i.e. triplets (I_1, I_2, I_3) and their relative angular momenta (L_1, L_2). However, it is one of properties of light nuclei ($A \leqq 12$) that there are not many pairs or triplets of fragments which can contribute to the formation of a particular nuclear state. Also some very simple fragments (^{4}He, ^{3}H) are present very often. A similar situation occurs with relative angular momenta L_1 and L_2. In general one has to make a priority list of (I_1, I_2, I_3) and (L_1, L_2) and procede step by step. In practice, however, one has to restrict the set of combinations by studying experimental and (or) theoretical data on fragments.

The motion of fragments is described by GC wave functions, the angular momentum part is expressed conveniently in terms of bipolar harmonics and the radial part is generated by two Jacobi GC coordinates, eqs. (2.4), (2.4′). The radial part of the hamiltonian and identity operator kernels in the Hill-Wheeler equation contains sums of products of modified spherical Bessel functions,

$$\sum_{l_1 \ldots l_6 = 0}^{\tau} \prod_{i=1}^{6} i_{l_i}(\alpha_i s'_j s_j),$$

and similar products containing four Bessel functions. It proved, however, that these sums converge reasonably fast for all values of GC parameters s_1 and s_2 which have to be included in a useful basis.

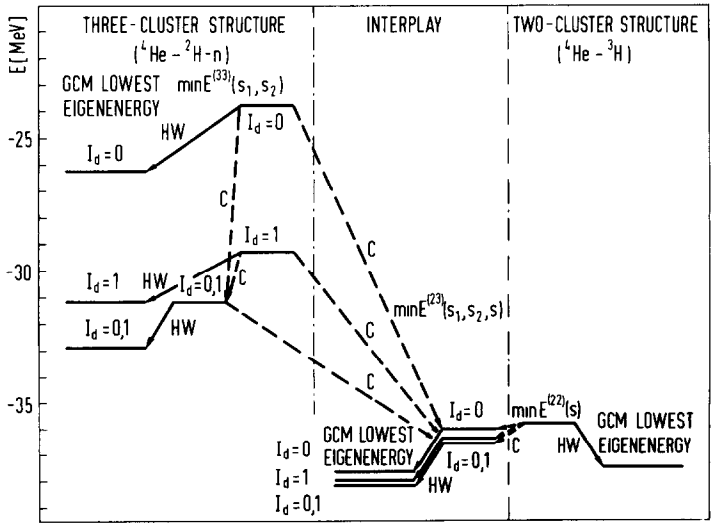

Fig. 8. A diagram presenting the influence of the two mechanisms, the molecule-like vibrational degree of freedom and the interplay of different cluster structures, on the ground state of ^{7}Li. The symbol HW means that the solution is obtained by solving the corresponding Hill-Wheeler equation, and the coupling of different cluster configurations and/or structures is indicated by the symbol C. The symbol $I_d = 0, 1$ means that the state corresponds to the mixture of both three-cluster configurations (^{4}He-^{2}H(I_d)-n), $I_d = 0, 1$.

(II). In the application of the proposed model to the study of the influence of three-cluster configurations on energy levels of the nucleus ^{7}Li we restricted ourselves to the three-cluster configurations: (^{4}He-^{2}H(I_d)-n), $I_d = 1, 0$, with the total spin $I = \frac{1}{2}$ and (^{4}He-^{2}H($I_d = 1$)-n) with $I = \frac{3}{2}$, and the orbital angular momenta $L_1 = 0, L_2 = 1$.

In fig. 8 we present a diagram which shows how the two mechanisms of the proposed model influence the energy of the ground state of ^{7}Li. Our conclusion is that both mechanisms, the interplay of two- and three-cluster structures and the molecule-like vibrational degree of freedom, are of equal importance.

A comparison of the two-cluster models for ^{7}Li, the one described in this paper and the other [7]) with two cluster structures, can be summarized in the following way:

(i) In both models the dominant structure in the ground state is the two-cluster structure (^{4}He-^{3}H). The admixture of three-cluster structures (^{4}He-^{2}H-n) seems to be more effective than that of the structure (^{6}Li-n) since already in the first approximation, taking only the term with $L_1 = 0$, $L_2 = 1$, one gets better binding.

(ii) The optimal binding energy corresponds to a value of the oscillator parameter β which is equal to those which provide the lowest energy values of the structure (^{4}He-^{3}H) and that of (^{4}He-^{2}H-n). This value of β differs considerably from the value which corresponds to the optimal configuration (^{6}Li-n) (ref. [7]), figs. 5 and 6). The energies of states having three-cluster structure are for about 2 MeV lower than those of the states of the structure (^{6}Li-n).

(iii) From the computational point of view each model has its own difficulties. In the model with two-cluster configurations only, the use of the shell-model description of ^{6}Li has as the consequence that one has to calculate a rather large number of different kernels and rather complicated bookkeeping is necessary in order to reduce the computational effort. In the model which contains two- and three-cluster configurations of 1s fragments the overlap kernels are less numerous, but they contain more terms which result from the coupling of angular momenta of three clusters. In addition one has to solve approximately the two-dimensional Hill-Wheeler equation instead of a one-dimensional. Hopefully, it will not often be necessary to work with 3-cluster structures with 1p fragments or with 4-cluster configurations.

The calculation in this paper is performed in order to examine whether a coupling of two- and three-cluster configurations is feasible with reasonable effort for a realistic system and to see what the influence of the three-cluster configurations on some low-lying states of ^{7}Li is. For a more detailed calculation of energy levels one has to use an effective interaction which contains the tensor parts. The inclusion of three-cluster configurations will hopefully help us in determining an effective interaction which will be suitable for describing levels of ^{7}Li and the thresholds of related reactions.

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