The numerical code used in: Modelling the penetration of subsonic rigid projectile probes into granular materials using the cavity expansion theory

Mechanical Properties and Coefficients

Volcanic ash (Change the properties for other target medium here)

In[1]:=

```
ρ = 1620;
ηs = 0.13;
Ee = 3.192 × 10^9;
k = 2 × 10^9;
ν = (3 k - Ee)/(6 k);
Coh = 4.74342 * 10^6;
ϕf = 25.3769 π / 180; (*Y=-(6 (Coh Cos[ϕf]))/(-3+Sin[ϕf]);*)
Y = 2 Coh;
```

In[3]:=

```
cd = Sqrt[(Ee (1 - ν))/((1 + ν) (1 - 2 ν) ρ)]; (*cd=ce:elastic wave speed*)
β[c_] := c/Sqrt[Ee/ρ]; ρo = ρ;
W = (12 Coh Cos[ϕf])/(Ee (-3 + Sin[ϕf])); X = (3 - Sin[ϕf])/(3 k + 3 k Sin[ϕf]);
term = .01; (*define the acceptable error |cp2-cp1|<term*)
NR = 1000;
(*define the step-size in the Runge-Kutta method*)
```

Equation 17 a & 17 b

```
In[7]:= A[c_] := -((4 c^3 Coh (1 + ν) (-1 + 2 ν) Cos[ϕf]) / ((c - cd) Ee
            (3 cd (c + cd) (-1 + 2 ν) + (cd^2 (1 - 2 ν) + 4 c^2 (1 + ν) + c (cd - 2 cd ν)) Sin[ϕf])));
        B[c_] := (6 cd Coh (1 + ν) (-1 + 2 ν) Cos[ϕf]) / ((c - cd) Ee
            (3 cd (c + cd) (-1 + 2 ν) + (cd^2 (1 - 2 ν) + 4 c^2 (1 + ν) + c (cd - 2 cd ν)) Sin[ϕf]));
```

Assumption 1 : c_p and S_r (hydrostat plastic region)

Solving Equation 30a & 30b

```
In[9]:= Cph = Function[{η, V, ϕ, term},
          clear[out]; error = 1;
          c = V / (η - (2 Coh (-1+η) (1+ν))/Ee)^(1/3);
          If[ϕ == 0, out = c, While[Abs[error] > term, out = V / (-B[c] (c - cd) (c + cd) (-1 + η) + η)^(1/3);
            error = out - c;
            c = out]];
          out];
```

Solving Equation 24 & 25 $\qquad$ S_0: Equation 31 & 32

In[10]:=
```
Sh = Function[{η, V, ϕ, c, ξ}, ψ = V / c;
  If[ϕ ≠ 0, S0 = (1 + B[c] (-c^2 + cd^2))^2 β[c]^2 η + (3 A[c] (1 + ν) + 2 B[c] (cd^2 (1 - 2 ν) + 3 c^2 ν))/(3 (-1 + ν + 2 ν^2)) +
     (Coh Cot[ϕf])/Ee - (β[c]^2 ψ^6 (1 + Sin[ϕf]))/(2 (-1 + η)) - (2 β[c]^2 ψ^3 (1 + Sin[ϕf]))/((-1 + η) (-1 + 3 Sin[ϕf])),
   S0 = (1 + B [c] (-c^2 + cd^2))^2 β[c]^2 η + (3 A[c] (1 + ν) + 2 B[c] (cd^2 (1 - 2 ν) + 3 c^2 ν))/(3 (-1 + ν + 2 ν^2)) -
     (β[c]^2 ψ^3 (-4 + ψ^3))/(2 (-1 + η))];
  If[ϕ ≠ 0, Shy = S0 ξ^(-(4 Sin[ϕf])/(1+Sin[ϕf])) - (Coh (Cos[ϕf] + Cot[ϕf]))/(Ee (1 + Sin[ϕf])) + (β[c]^2 ψ^6 (1 + Sin[ϕf]))/(2 ξ^4 (-1 + η)) +
     (2 β[c]^2 ψ^3 (1 + Sin[ϕf]))/(ξ (-1 + η) (-1 + 3 Sin[ϕf])), Shy = (β[c]^2 ψ^3 (-4 ξ^3 + ψ^3))/(2 ξ^4 (-1 + η)) + S0 - (4 Coh Log[ξ])/Ee];
  Shy];
```

Assumption 1 : Incompressible Elastic

Equation 33a & 33b

In[11]:=
```
CpIn = Function[{η, V, ϕ, term},
   Clear[out, outi]; error = 1;
   c = V / ((3 Coh - 3 Coh η + Ee η)/Ee)^(1/3);
   ceta =
    -(2 V^3 ρ Tan[ϕf])/(3 Coh) + (12 × 2^(1/3) V^6 ρ^2 Tan[ϕf]^2) / (Coh^2 ((6561 Ee V^3 Sec[ϕf])/Coh - (2187 Ee V^3 Tan[ϕf])/Coh -
          (11664 V^9 ρ^3 Tan[ϕf]^3)/Coh^3 + Sqrt[(-(136048896 V^18 ρ^6 Tan[ϕf]^6)/Coh^6 +
             ((6561 Ee V^3 Sec[ϕf])/Coh - (2187 Ee V^3 Tan[ϕf])/Coh - (11664 V^9 ρ^3 Tan[ϕf]^3)/Coh^3)^2)])^(1/3)) +
     1/(27 × 2^(1/3)) ((6561 Ee V^3 Sec[ϕf])/Coh - (2187 Ee V^3 Tan[ϕf])/Coh - (11664 V^9 ρ^3 Tan[ϕf]^3)/Coh^3 +
        Sqrt[(-(136048896 V^18 ρ^6 Tan[ϕf]^6)/Coh^6 +
           ((6561 Ee V^3 Sec[ϕf])/Coh - (2187 Ee V^3 Tan[ϕf])/Coh - (11664 V^9 ρ^3 Tan[ϕf]^3)/Coh^3)^2)])^(1/3);
   If[ϕ == 0, out = c, If[η == 0, out = ceta]];
   If[ϕ ≠ 0, If[η ≠ 0, While[Abs[error] > term, out = V / (η + (9 Coh (-1+η) Cos[ϕf])/(-3 Ee+(Ee+18 c^2 ρ) Sin[ϕf]))^(1/3);

       error = out - c;
       c = out]]];
   out];
```

Solving Equation 24 & 25; S_0: Equation 34 & 35

In[12]:=
```
SIn = Function[{η, V, ϕ, c, ξ}, ψ = V / c;
   If[ϕ ≠ 0, S0 = (Coh Cot[ϕf])/Ee - (2 Coh (2 Ee + 9 c^2 ρ) Cos[ϕf])/(Ee (-3 Ee + (Ee + 18 c^2 ρ) Sin[ϕf])) -
      (β[c]^2 ψ^3 (1 + Sin[ϕf]) (4 - ψ^3 + 3 ψ^3 Sin[ϕf]))/(2 (-1 + η) (-1 + 3 Sin[ϕf])) + η (β[c] + (9 Coh β[c] Cos[ϕf])/(-3 Ee + (Ee + 18 c^2 ρ) Sin[ϕf]))^2,
    S0 = (β[c] - (3 Coh β[c])/Ee)^2 η + (2 Coh (2 Ee + 9 c^2 ρ))/(3 Ee^2) - (β[c]^2 ψ^3 (-4 + ψ^3))/(2 (-1 + η))];
   If[ϕ ≠ 0, Sinco = S0 ξ^(-(4 Sin[ϕf])/(1+Sin[ϕf])) - (Coh (Cos[ϕf] + Cot[ϕf]))/(Ee (1 + Sin[ϕf])) + (β[c]^2 ψ^6 (1 + Sin[ϕf]))/(2 ξ^4 (-1 + η)) +
      (2 β[c]^2 ψ^3 (1 + Sin[ϕf]))/(ξ (-1 + η) (-1 + 3 Sin[ϕf])), Sinco = (β[c]^2 ψ^3 (-4 ξ^3 + ψ^3))/(2 ξ^4 (-1 + η)) + S0 - (4 Coh Log[ξ])/Ee];
   Sinco];
```

Assumption 2 : Linear Sloution (for linear pressure – volumetric strain aaumption)

Solving Equation 43

In[13]:=
```
CpLinV = Function[{V},
   Clear[out];
   out = FindRoot[(-Ee^2 W (-1 + c^2 X ρ) (-1 + V^2 X ρ) ArcTanh[c Sqrt[X] Sqrt[ρ]] (3 cd (c + cd) (-1 + 2 ν) +
            (cd^2 (1 - 2 ν) + 4 c^2 (1 + ν) + c (cd - 2 cd ν)) Sin[ϕf]) + Ee^2 W (-1 + c^2 X ρ)
          (-1 + V^2 X ρ) ArcTanh[V Sqrt[X] Sqrt[ρ]] (3 cd (c + cd) (-1 + 2 ν) + (cd^2 (1 - 2 ν) + 4 c^2 (1 + ν) +
             c (cd - 2 cd ν)) Sin[ϕf]) + Sqrt[X] Sqrt[ρ] (12 c^3 cd (c + cd) Coh (-1 + ν + 2 ν^2) ρ
            (-1 + V^2 X ρ) Cos[ϕf] + Ee (Ee (c - V) W + V (2 V^2 + c Ee (c - V) W X) ρ - 2 c^2 V^3 X ρ^2)
            (3 cd (c + cd) (-1 + 2 ν) + (cd^2 (1 - 2 ν) + 4 c^2 (1 + ν) + c (cd - 2 cd ν)) Sin[ϕf]))) ==
      0, {c, (V + cd)/2}, AccuracyGoal → 10, PrecisionGoal → 20][[1, 2]];
   out];
```

Equation 18a & 42d & 41b

In[14]:=

$$\text{SrE}[c\_, \xi\_] := \frac{3\,A[c]\,\xi^3\,(1+\nu) + 2\,B[c]\,\left(cd^2\,(1-2\,\nu) + 3\,c^2\,\xi^2\,\nu\right)}{3\,\xi^3\,\left(-1+\nu+2\,\nu^2\right)};$$

$$\text{SrPL0}[c\_, \xi\_] := \frac{3\,A[c]\,(1+\nu) + 2\,B[c]\,\left(cd^2\,(1-2\,\nu) + 3\,c^2\,\nu\right)}{3\,\left(-1+\nu+2\,\nu^2\right)} + \frac{2\,B[c]\,\text{Ee}\,\beta[c]^4 - \left(W + 2\,B[c]\,cd^2\,\beta[c]^2\right)\rho}{\left(-1+\text{Ee}\,X\,\beta[c]^2\right)\rho} - \frac{1}{2}\,W\,\text{Log}\left[\frac{1}{1-\text{Ee}\,X\,\beta[c]^2}\right];$$

$$\text{SrPL}[c\_, \xi\_] := \frac{W + 2\,B[c]\,\left(-c^2 + cd^2\right)\beta[c]^2}{\left(-1+\text{Ee}\,X\,\beta[c]^2\right)\xi} + \text{SrPL0}[c, \xi] + \left(W\left(2\,\text{ArcTanh}\left[\sqrt{\text{Ee}\,X}\,\beta[c]\right] - 2\,\text{ArcTanh}\left[\sqrt{\text{Ee}\,X}\,\beta[c]\,\xi\right] + \sqrt{\text{Ee}\,X}\,\beta[c]\,\xi\,\text{Log}\left[\frac{\xi^2}{1-\text{Ee}\,X\,\beta[c]^2\,\xi^2}\right]\right)\right) \Big/ \left(2\,\sqrt{\text{Ee}\,X}\,\beta[c]\,\xi\right);$$

Assumption 2 :
Non – Linear Sloution

Solving Equation 44a & 44b with Runge–Kutta method (RK4)

In[17]:=
```
Rungekutta = Function[{V, c, zetaend, zetabeg, NR, y0, y00},
   Clear[out];
   dydt1[ξ_, u_, s_] := (2 (4 Coh Cos[ϕf] + Ee s (-3 + Sin[ϕf]) + 3 k (1 + Sin[ϕf]))
        (6 k X ξ (Coh Cos[ϕf] + Ee s Sin[ϕf]) +
          u (2 Coh (2 - 3 k X) Cos[ϕf] + 3 k (1 + Sin[ϕf]) + Ee s (-3 + Sin[ϕf] - 6 k X Sin[ϕf])))) /
      (ξ (-Ee^2 s^2 (-3 + Sin[ϕf])^2 - 2 Ee s (-3 + Sin[ϕf]) (4 Coh Cos[ϕf] + 3 k (1 + Sin[ϕf])) +
          (1 + Sin[ϕf]) (-16 Coh^2 + 9 k^2 (-1 + c^2 X ξ^2 ρo) - 24 Coh k Cos[ϕf] +
             (16 Coh^2 + 9 k^2 (-1 + c^2 X ξ^2 ρo)) Sin[ϕf] - 9 c^2 k^2 u X (-u + 2 ξ) ρo (1 + Sin[ϕf]))));
   dydt2[ξ_, u_, s_] := -((2 (4 Coh Cos[ϕf] + Ee s (-3 + Sin[ϕf]) + 3 k (1 + Sin[ϕf]))
          (-Ee^2 s^2 (-1 + Cos[2 ϕf] + 6 Sin[ϕf]) + (1 + Sin[ϕf])
             (2 Coh (3 k Cos[ϕf] - 4 Coh (-1 + Sin[ϕf])) + 3 c^2 k u (-u + ξ) ρo (1 + Sin[ϕf])) +
            Ee s (3 k - 6 Coh Cos[ϕf] - 3 k Cos[2 ϕf] + 6 k Sin[ϕf] + 5 Coh Sin[2 ϕf]))) /
       (Ee ξ (1 + Sin[ϕf]) (Ee^2 s^2 (-3 + Sin[ϕf])^2 + 2 Ee s (-3 + Sin[ϕf])
             (4 Coh Cos[ϕf] + 3 k (1 + Sin[ϕf])) +
            (1 + Sin[ϕf]) (16 Coh^2 - 9 k^2 (-1 + c^2 X ξ^2 ρo) + 24 Coh k Cos[ϕf] + (-16 Coh^2 -
                 9 k^2 (-1 + c^2 X ξ^2 ρo)) Sin[ϕf] + 9 c^2 k^2 u X (-u + 2 ξ) ρo (1 + Sin[ϕf])))));
   h = (zetabeg - zetaend)/NR;
   ans = {{zetaend, y0, y00}};
   Do[
    k1 = {{dydt1[zet, ans[[1, 2]], ans[[1, 3]]], dydt2[zet, ans[[1, 2]], ans[[1, 3]]]}};
    k2 = {{dydt1[zet + h/2, ans[[1, 2]] + h/2 k1[[1, 1]], ans[[1, 3]] + h/2 k1[[1, 2]]],
       dydt2[zet + h/2, ans[[1, 2]] + h/2 k1[[1, 1]], ans[[1, 3]] + h/2 k1[[1, 2]]]}};
    k3 = {{dydt1[zet + h/2, ans[[1, 2]] + h/2 k2[[1, 1]], ans[[1, 3]] + h/2 k2[[1, 2]]],
       dydt2[zet + h/2, ans[[1, 2]] + h/2 k2[[1, 1]], ans[[1, 3]] + h/2 k2[[1, 2]]]}};
    k4 = {{dydt1[zet + h, ans[[1, 2]] + h k3[[1, 1]], ans[[1, 3]] + h k3[[1, 2]]],
       dydt2[zet + h, ans[[1, 2]] + h k3[[1, 1]], ans[[1, 3]] + h k3[[1, 2]]]}};
    ynp1 = {{ans[[1, 2]], ans[[1, 3]]}} + h/6 (k1 + 2 k2 + 2 k3 + k4);
    ans = Prepend[ans, {zet, ynp1[[1, 1]], ynp1[[1, 2]]}], {zet, zetaend + h, zetabeg, h}];


   ans];
```

Probe

S_r and c_p for differnt assumptions

In[18]:=

```
ηse = ηs;
listCph = {}; (*cp: Assumption 1*)
listCpInE = {};(*cp: Assumption 1 incompressible elastic*)
listCpLinV = {}; (*cp: Assumption 2: Linear*)
listCpFV = {}; (*cp: Assumption 2: Non-Linear*)
listSh = {}; (*Sr: Assumption 1*)
listSInE = {};(*Sr: Assumption 1 incompressible elastic*)
listSLinV = {}; (*Sr: Assumption 2: Linear*)
listSFV = {}; (*Sr: Assumption 2: Non-Linear*)
```

$$\text{Do}\left[\text{Vi} = \text{i}\sqrt{\frac{\text{Y}}{\rho}};\ \text{c} = \text{CpLinV[Vi]};\right.$$

$$\text{listCpLinV} = \text{Append}\left[\text{listCpLinV},\ \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}},\ \text{c}\sqrt{\frac{\rho}{\text{Y}}}\right\}\right];$$

$$\text{listSLinV} = \text{Append}\left[\text{listSLinV},\ \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}},\ \text{SrPL}\left[\text{c},\ \frac{\text{Vi}}{\text{c}}\right]\frac{\text{Ee}}{\text{Y}}\right\}\right];$$

```
err = 1; If[ArrayDepth[listCpFV] > 5, c = cp]; errlist = {};
While[Abs[err] > term, y0 = B[c] (c - cd) (c + cd);
```

$$\text{y00} = \frac{3\,\text{A[c]}\,(1+\nu) + 2\,\text{B[c]}\,(\text{cd}^2\,(1-2\,\nu) + 3\,\text{c}^2\,\nu)}{3\,(-1+\nu+2\,\nu^2)};$$

$$\text{zetaend} = 1;$$

$$\text{zetabeg} = \frac{\text{Vi}}{\text{c}};$$

```
ansFR = Rungekutta[Vi, c, zetaend, zetabeg, NR, y0, y00];
```

$$\text{cp} = 0.01\,\frac{\text{Vi}}{\text{ansFR[[1, 2]]}} + 0.99\,\text{c};$$

$$\text{err} = \text{c} - \frac{\text{Vi}}{\text{ansFR[[1, 2]]}};$$

```
errlist = Append[errlist, {err, c}];
If[Length[errlist] > 2, errlistf = Interpolation[errlist, InterpolationOrder → 1];
 cp = errlistf[0]];
c = cp];
```

$$\text{listCpFV} = \text{Append}\left[\text{listCpFV},\ \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}},\ \text{cp}\sqrt{\frac{\rho}{\text{Y}}}\right\}\right];$$

$$\text{listSFV} = \text{Append}\left[\text{listSFV}, \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}}, \text{ansFR}[[1,3]]\,\frac{\text{Ee}}{\text{Y}}\right\}\right];$$

errh = 1;
While[Abs[errh] > term, c = Cph[ηs, Vi, ϕf, term];

$$\sigma r = \text{Sh}\left[\eta s, \text{Vi}, \phi f, c, \frac{\text{Vi}}{c}\right] \text{Ee};$$

$$p = \frac{3\,\sigma r - 4\,\text{Coh}\,\text{Cos}[\phi f] - \sigma r\,\text{Sin}[\phi f]}{3 + 3\,\text{Sin}[\phi f]};$$

$$\eta sm = \frac{p}{k};$$

$$\text{errh} = \frac{\eta sm - \eta s}{\eta s}\,100;$$

ηs = ηsm];
If[ηs < 0.12, ηs = ηsm, ηs = 0.12]; c = Cph[ηs, Vi, ϕf, term];

$$\text{listCph} = \text{Append}\left[\text{listCph}, \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}}, c\sqrt{\frac{\rho}{\text{Y}}}\right\}\right];$$

$$\text{listSh} = \text{Append}\left[\text{listSh}, \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}}, \text{Sh}\left[\eta s, \text{Vi}, \phi f, c, \frac{\text{Vi}}{c}\right]\frac{\text{Ee}}{\text{Y}}\right\}\right];$$

errhE = 1;
While[Abs[errhE] > term, c = CpIn[ηse, Vi, ϕf, term];

$$\sigma r = \text{SIn}\left[\eta se, \text{Vi}, \phi f, c, \frac{\text{Vi}}{c}\right] \text{Ee};$$

$$p = \frac{3\,\sigma r - 4\,\text{Coh}\,\text{Cos}[\phi f] - \sigma r\,\text{Sin}[\phi f]}{3 + 3\,\text{Sin}[\phi f]};$$

$$\eta sme = \frac{p}{k};$$

$$\text{errhE} = \frac{\eta sme - \eta se}{\eta se}\,100;$$

```
 ηse = ηsme];
If[ηse < 0.12, ηse = ηsme, ηse = 0.12];
Print["i=", i, "     Assumption I η=",
 ηs, "     Assumption I: Incompressible Elastic η=", ηse];
c = CpIn[ηse, Vi, ϕf, term];
```

$$\text{listCpInE} = \text{Append}\left[\text{listCpInE}, \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}}, c\sqrt{\frac{\rho}{\text{Y}}}\right\}\right];$$

$$\text{listSInE} = \text{Append}\left[\text{listSInE}, \left\{\text{Vi}\sqrt{\frac{\rho}{\text{Y}}}, \text{SIn}\left[\eta se, \text{Vi}, \phi f, c, \frac{\text{Vi}}{c}\right]\frac{\text{Ee}}{\text{Y}}\right\}\right], \{i, 0.01, 5.01, .1\}]$$

InterpolatingFunction: Input value {0} lies outside the range of data in the interpolating function. Extrapolation will be used.

```
i=0.01     Assumption I η=0.0180708     Assumption I: Incompressible Elastic η=0.017831
```

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

InterpolatingFunction: Input value {0} lies outside the range of data in the interpolating function. Extrapolation will be used.

i=0.11 Assumption I η=0.0181592 Assumption I: Incompressible Elastic η=0.0179296

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

InterpolatingFunction: Input value {0} lies outside the range of data in the interpolating function. Extrapolation will be used.

General: Further output of InterpolatingFunction::dmval will be suppressed during this calculation.

i=0.21 Assumption I η=0.0183953 Assumption I: Incompressible Elastic η=0.0181933

i=0.31 Assumption I η=0.0187748 Assumption I: Incompressible Elastic η=0.018618

FindRoot: The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient decrease in the merit function. You may need more than MachinePrecision digits of working precision to meet these tolerances.

General: Further output of FindRoot::lstol will be suppressed during this calculation.

i=0.41 Assumption I η=0.0192986 Assumption I: Incompressible Elastic η=0.0192038

i=0.51 Assumption I η=0.0199641 Assumption I: Incompressible Elastic η=0.019947

i=0.61 Assumption I η=0.0207691 Assumption I: Incompressible Elastic η=0.020844

i=0.71 Assumption I η=0.0217111 Assumption I: Incompressible Elastic η=0.0218908

i=0.81 Assumption I η=0.0227872 Assumption I: Incompressible Elastic η=0.0230837

i=0.91 Assumption I η=0.0239946 Assumption I: Incompressible Elastic η=0.0244163

i=1.01 Assumption I η=0.0253299 Assumption I: Incompressible Elastic η=0.0258846

i=1.11 Assumption I η=0.0267898 Assumption I: Incompressible Elastic η=0.0274837

i=1.21 Assumption I η=0.0283706 Assumption I: Incompressible Elastic η=0.0292088

i=1.31 Assumption I η=0.0300687 Assumption I: Incompressible Elastic η=0.0310539

i=1.41 Assumption I η=0.0318803 Assumption I: Incompressible Elastic η=0.0330167

i=1.51 Assumption I η=0.0338017 Assumption I: Incompressible Elastic η=0.0350917

i=1.61 Assumption I η=0.0358293 Assumption I: Incompressible Elastic η=0.0372746

i=1.71 Assumption I η=0.0379592 Assumption I: Incompressible Elastic η=0.0395616

i=1.81 Assumption I η=0.0401881 Assumption I: Incompressible Elastic η=0.0419489

i=1.91 Assumption I η=0.0425124 Assumption I: Incompressible Elastic η=0.0444329

i=2.01 Assumption I η=0.0449303 Assumption I: Incompressible Elastic η=0.0470103

i=2.11 Assumption I η=0.0474357 Assumption I: Incompressible Elastic η=0.0496782

i=2.21 Assumption I η=0.050027 Assumption I: Incompressible Elastic η=0.0524335

i=2.31 Assumption I η=0.0527014 Assumption I: Incompressible Elastic η=0.0552737

i=2.41 Assumption I η=0.055456 Assumption I: Incompressible Elastic η=0.0581963

i=2.51 Assumption I η=0.0582883 Assumption I: Incompressible Elastic η=0.0611991

i=2.61 Assumption I η=0.0611959 Assumption I: Incompressible Elastic η=0.0642799

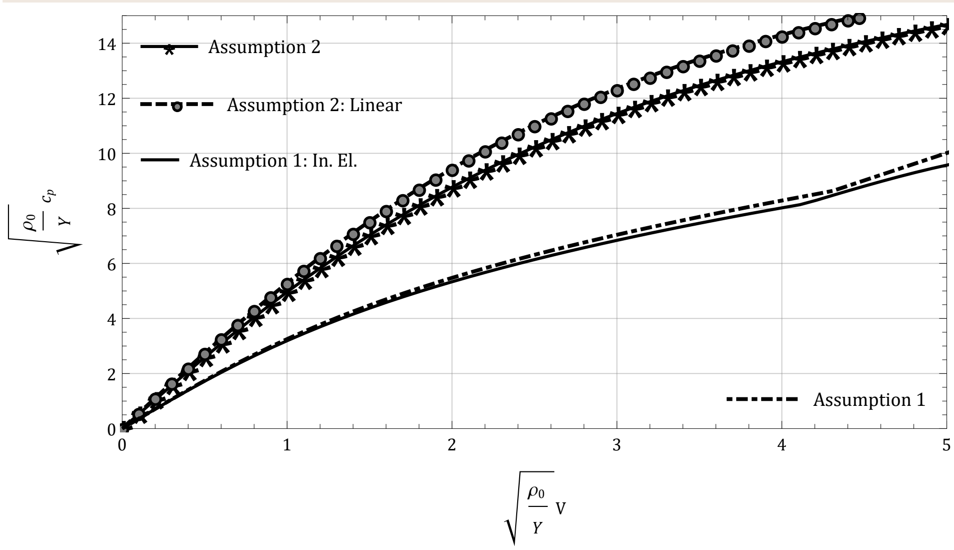
i=2.71 Assumption I η=0.0641763 Assumption I: Incompressible Elastic η=0.0674367

```
i=2.81    Assumption I η=0.0672274    Assumption I: Incompressible Elastic η=0.070668
i=2.91    Assumption I η=0.0703472    Assumption I: Incompressible Elastic η=0.073972
i=3.01    Assumption I η=0.0735336    Assumption I: Incompressible Elastic η=0.0773472
i=3.11    Assumption I η=0.0767847    Assumption I: Incompressible Elastic η=0.0807924
i=3.21    Assumption I η=0.0800988    Assumption I: Incompressible Elastic η=0.0843063
i=3.31    Assumption I η=0.0834743    Assumption I: Incompressible Elastic η=0.087888
i=3.41    Assumption I η=0.0869096    Assumption I: Incompressible Elastic η=0.0915365
i=3.51    Assumption I η=0.0904031    Assumption I: Incompressible Elastic η=0.0952512
i=3.61    Assumption I η=0.0939534    Assumption I: Incompressible Elastic η=0.0990353
i=3.71    Assumption I η=0.0975593    Assumption I: Incompressible Elastic η=0.102881
i=3.81    Assumption I η=0.101219    Assumption I: Incompressible Elastic η=0.106791
i=3.91    Assumption I η=0.104932    Assumption I: Incompressible Elastic η=0.110766
i=4.01    Assumption I η=0.108697    Assumption I: Incompressible Elastic η=0.114805
i=4.11    Assumption I η=0.112513    Assumption I: Incompressible Elastic η=0.118909
i=4.21    Assumption I η=0.116378    Assumption I: Incompressible Elastic η=0.12
i=4.31    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.41    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.51    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.61    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.71    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.81    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=4.91    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
i=5.01    Assumption I η=0.12    Assumption I: Incompressible Elastic η=0.12
```

In[22]:=

```
CPLinV =
  ListPlot[listCpLinV , PlotLegends → Placed[{"Assumption 2: Linear"}, {Left, Top}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 15}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → {Black, Dashed, Thick},
   FrameLabel → {{"Sqrt[ρ0/Y] cp", None}, {"Sqrt[ρ0/Y] V", None}}, PlotMarkers → { "●", Medium}];
CPFV = ListPlot[listCpFV , PlotLegends → Placed[{"Assumption 2"}, {Left, Top}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 15}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → Black,
   FrameLabel → {{"Sqrt[ρ0/Y] cp", None}, {"Sqrt[ρ0/Y] V", None}}, PlotMarkers → { "*", Large}];
CPH = ListPlot[listCph , PlotLegends → Placed[{"Assumption 1"}, {Right, Bottom}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 15}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → {Black, DotDashed, Thick},
   FrameLabel → {{"Sqrt[ρ0/Y] cp", None}, {"Sqrt[ρ0/Y] V", None}}];
CPHINE = ListPlot[listCpInE, PlotLegends → Placed[{"Assumption 1: In. El."},
     {Left, Top}], LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 15}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → Black,
   FrameLabel → {{"Sqrt[ρ0/Y] cp", None}, {"Sqrt[ρ0/Y] V", None}}];
Show[CPFV, CPLinV, CPH, CPHINE]
```

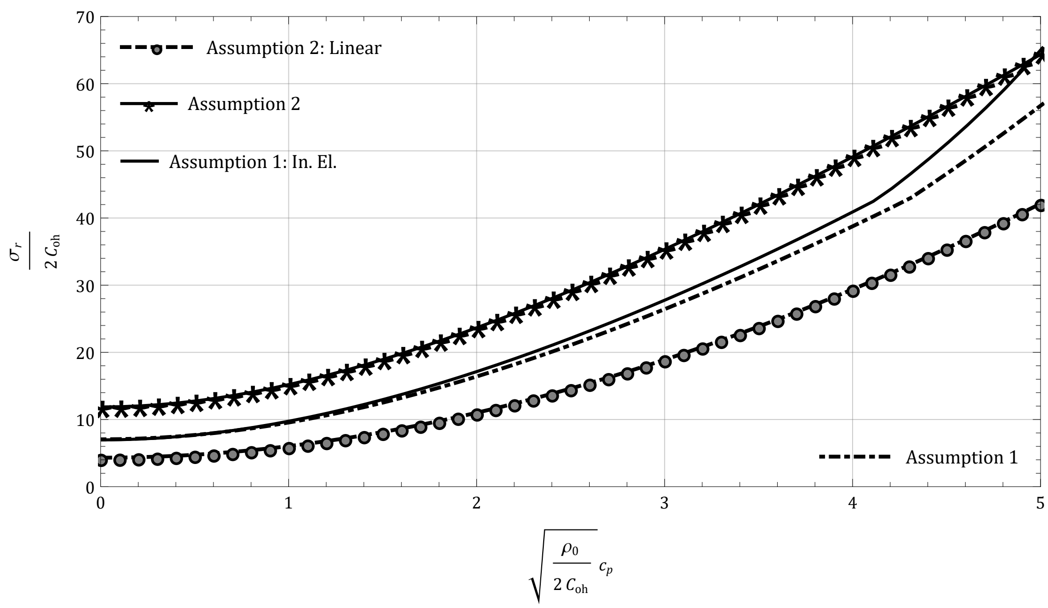
Out[26]=



In[27]:=

```
SLinV = ListPlot[listSLinV , PlotLegends → Placed[{"Assumption 2: Linear"}, {Left, Top}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 70}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → {Black, Dashed, Thick},
   FrameLabel → {{"σr/(2 Coh)", None}, {"Sqrt[ρ0/(2 Coh)] cp", None}}, PlotMarkers → { "●", Medium}];
SFV = ListPlot[listSFV , PlotLegends → Placed[{"Assumption 2"}, {Left, Top}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 70}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → Black,
   FrameLabel → {{"σr/(2 Coh)", None}, {"Sqrt[ρ0/(2 Coh)] cp", None}}, PlotMarkers → { "*", Large}];
SH = ListPlot[listSh , PlotLegends → Placed[{"Assumption 1"}, {Right, Bottom}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True,
   PlotRange → {{0, 5}, {0, 70}}, GridLines → Automatic, AspectRatio → .5,
   Frame → {{True, True}, {True, True}}, PlotStyle → {Black, DotDashed, Thick},
   FrameLabel → {{"σr/(2 Coh)", None}, {"Sqrt[ρ0/(2 Coh)] cp", None}}];
SHINE = ListPlot[listSInE, PlotLegends → Placed[{"Assumption 1: In. El."}, {Left, Top}],
   LabelStyle → (FontFamily → "Cambria"), Joined → True, PlotRange → {{0, 5}, {0, 70}},
   GridLines → Automatic, AspectRatio → .5, Frame → {{True, True}, {True, True}},
   PlotStyle → Black, FrameLabel → {{"σr/(2 Coh)", None}, {"Sqrt[ρ0/(2 Coh)] cp", None}}];
Show[SLinV, SFV, SH, SHINE]
```

Out[31]=



Penetration

Geometry of the probe

In[32]:=
```
a = 0.156/2; (*M= 162+55.3;*); M = 162;
Vp = 520; μ = 0;
CRH = 6;
s = 2 a CRH;
θo = ArcSin[(s - a)/s];
```

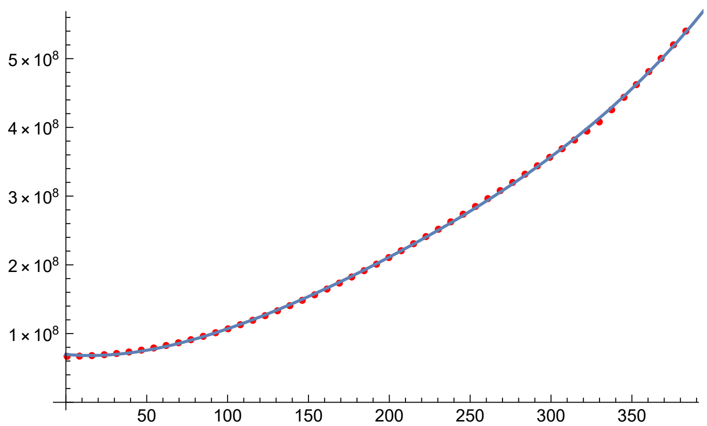
Assumption 1

Curve fitting for Assumption 1 (Hydrostat model)

In[36]:=

```
listSh2 = listSh;
listSh2[[All, 1]] = listSh2[[All, 1]] / Sqrt[ρ/Y];
listSh2[[All, 2]] = listSh2[[All, 2]] Y;
fith = Fit[listSh2, {1, x, x^2, x^3, x^4}, x];
g[x_] := fith
Show[ListPlot[listSh2, PlotStyle → Red], Plot[ g[x] , {x, 0, 500}]]
SIGMAFh[vp_, ϕ_] := fith /. x → vp Cos[ϕ]; (*V=vp Cos[ϕ]*)
```

Out[41]=



Curve fitting maximum error and coefficient of determination

In[43]:=

```
R2 = 0;
sstot = 0;
ssres = 0;
sum = 0;
error = {};
Do[sum = sum + listSh2[[i, 2]], {i, 1, Dimensions[listSh2][[1]]}];
ave = sum / Dimensions[listSh2][[1]];
Do[sstot = sstot + (listSh2[[i, 2]] - ave)^2;
 val = (fith /. x → listSh2[[i, 1]]);
 error = Append[error, Abs[listSh2[[i, 2]] - val] / Abs[listSh2[[i, 2]]]];
 ssres = ssres + (listSh2[[i, 2]] - val)^2, {i, 1, Dimensions[listSh2][[1]]}];
R2 = 1 - ssres / sstot;
maxerror = Max[error] 100;
Print["coefficient of determination=", R2, "      Maximum Error=", maxerror, "%"]
```

```
coefficient of determination=0.999867      Maximum Error=3.86776%
```

Penetration Time (Tf) and Penetration depth (Zf) without the pusher plate with different friction

In[47]:=
```
M = 162; Clear[μ];
Do[dah = (-SIGMAFh[vp, ϕ])/M (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
 (*Equation 50*)
 azh = Integrate[dah, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify; (*Equation 51*)
 azhy[v_] := azh /. vp → v;
 dzh = vp/azhy[vp] // Simplify;
 Zhvp = Integrate[dzh, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 Zh[v_, v0_] := Zhvp /. Vph → v /. Vp0 → v0;
 tvp = Integrate[1/azhy[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 th[v_, v0_] := tvp /. Vph → v /. Vp0 → v0;
 tfhy = th[0, Vp];
 zfhy = Zh[0, Vp];
 Print["μ=", μ, "      Tf=", Round[1000 tfhy, 0.1],
  "ms", "      Zf=", Round[zfhy, 0.01], "m"], {μ, 0, .1, 0.01}]
```

```
μ=0.      Tf=53.1ms      Zf=12.39m
μ=0.01      Tf=50.1ms      Zf=11.71m
μ=0.02      Tf=47.4ms      Zf=11.1m
μ=0.03      Tf=44.9ms      Zf=10.55m
μ=0.04      Tf=42.8ms      Zf=10.06m
μ=0.05      Tf=40.8ms      Zf=9.61m
μ=0.06      Tf=39.ms      Zf=9.19m
μ=0.07      Tf=37.3ms      Zf=8.82m
μ=0.08      Tf=35.8ms      Zf=8.47m
μ=0.09      Tf=34.4ms      Zf=8.14m
μ=0.1      Tf=33.1ms      Zf=7.85m
```

Adding effects of the pushing plate between t = 0 to 0.003 s

In[49]:=
```
M = 162 + 55.3;
Do[dah = (-SIGMAFh[vp, ϕ]/M) (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
 azh = Integrate[dah, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify;
 azhy[v_] := azh /. vp → v;
 dzh = vp/azhy[vp] // Simplify;
 tvp = Integrate[1/azhy[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 th[v_, v0_] := tvp /. Vph → v /. Vp0 → v0;
 v3ms = FindRoot[th[vv, Vp] - 0.003, {vv, Vp}][[1, 2]];
 Zhvp = Integrate[dzh, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 Zh[v_, v0_] := Zhvp /. Vph → v /. Vp0 → v0;
 tfhy = th[0, v3ms]/(162 + 55.3) (162) + 0.003;
 zfhy = Zh[v3ms, Vp] + Zh[0, v3ms] 162/(162 + 55.3);
 Print["μ=", μ, "     Tf=", Round[1000 tfhy, 0.1],
  "ms", "     Zf=", Round[zfhy, 0.01], "m"], {μ, 0, .1, 0.01}]
```

```
μ=0.     Tf=53.8ms     Zf=12.77m
μ=0.01     Tf=50.8ms     Zf=12.09m
μ=0.02     Tf=48.1ms     Zf=11.49m
μ=0.03     Tf=45.7ms     Zf=10.94m
μ=0.04     Tf=43.5ms     Zf=10.44m
μ=0.05     Tf=41.5ms     Zf=9.99m
μ=0.06     Tf=39.7ms     Zf=9.57m
μ=0.07     Tf=38.1ms     Zf=9.2m
μ=0.08     Tf=36.6ms     Zf=8.85m
μ=0.09     Tf=35.2ms     Zf=8.52m
μ=0.1     Tf=33.9ms     Zf=8.22m
```

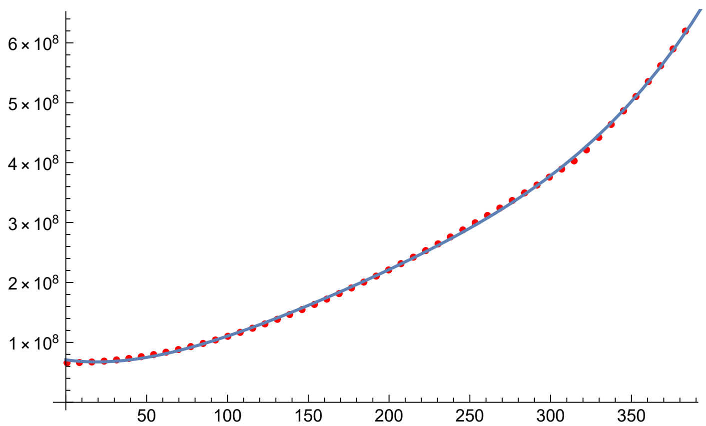
Assumption 1:Incompressible Elastic Assumption

Curve fitting for Assumption 1 (incompressible elastic)

In[51]:=

```
listSInE2 = listSInE;
listSInE2[[All, 1]] = listSInE2[[All, 1]] / Sqrt[ρ/Y];
listSInE2[[All, 2]] = listSInE2[[All, 2]] Y;
fitInE = Fit[listSInE2, {1, x, x^2, x^3, x^4}, x];
gInE[x_] := fitInE
Show[ListPlot[listSInE2, PlotStyle → Red], Plot[ gInE[x] , {x, 0, 500}]]
SIGMAFInE[vp_, ϕ_] := fitInE /. x → vp Cos[ϕ];(*V=vp Cos[ϕ]*)
```

Out[56]=



Curve fitting maximum error and coefficient of determination

In[58]:=

```
R2 = 0;
sstot = 0;
ssres = 0;
sum = 0;
error = {};
Do[sum = sum + listSInE2[[i, 2]], {i, 1, Dimensions[listSInE2][[1]]}];
ave = sum / Dimensions[listSInE2][[1]];
Do[sstot = sstot + (listSInE2[[i, 2]] - ave)^2;
  val = (fitInE /. x → listSInE2[[i, 1]]);
  error = Append[error, Abs[listSInE2[[i, 2]] - val] / Abs[listSInE2[[i, 2]]]];
  ssres = ssres + (listSInE2[[i, 2]] - val)^2, {i, 1, Dimensions[listSInE2][[1]]}];
R2 = 1 - ssres / sstot;
maxerror = Max[error] 100;
Print["coefficient of determination=", R2, "      Maximum Error=", maxerror, "%"]
```

```
coefficient of determination=0.999733      Maximum Error=6.85462%
```

Penetration Time (Tf) and Penetration depth (Zf) without the pusher plate with different friction

In[62]:=
```
M = 162; Clear[μ];
Do[daInE = (-SIGMAFInE[vp, ϕ]/M) (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
 (*Equation 50*)
 azInE = Integrate[daInE, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify;
 (*Equation 51*)
 azIncE[v_] := azInE /. vp → v; dzIncE = vp/azIncE[vp] // Simplify;
 ZIncEvp = Integrate[dzIncE, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 ZIncE[v_, v0_] := ZIncEvp /. Vph → v /. Vp0 → v0;
 tIncE = Integrate[1/azIncE[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 tFtIncE[v_, v0_] := tIncE /. Vph → v /. Vp0 → v0;
 tfFInE = tFtIncE[0, Vp]; zfFInE = ZIncE[0, Vp];
 Print["μ=", μ, "      Tf=", Round[1000 tfFInE, 0.1],
  "ms", "      Zf=", Round[zfFInE, 0.01], "m"], {μ, 0, .1, 0.01}]
```

```
μ=0.      Tf=52.8ms      Zf=12.23m
μ=0.01      Tf=49.8ms      Zf=11.56m
μ=0.02      Tf=47.1ms      Zf=10.96m
μ=0.03      Tf=44.7ms      Zf=10.42m
μ=0.04      Tf=42.5ms      Zf=9.94m
μ=0.05      Tf=40.6ms      Zf=9.49m
μ=0.06      Tf=38.8ms      Zf=9.09m
μ=0.07      Tf=37.1ms      Zf=8.71m
μ=0.08      Tf=35.6ms      Zf=8.37m
μ=0.09      Tf=34.2ms      Zf=8.05m
μ=0.1      Tf=32.9ms      Zf=7.76m
```

Adding effects of the pushing plate between t = 0 to 0.003 s

In[64]:=
```
M = 162 + 55.3;
Do[daInE = (-SIGMAFInE[vp, ϕ]/M) (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
 azInE = Integrate[daInE, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify;
 azIncE[v_] := azInE /. vp → v; dzIncE = vp/azIncE[vp] // Simplify;
 tIncE = Integrate[1/azIncE[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 tFtIncE[v_, v0_] := tIncE /. Vph → v /. Vp0 → v0;
 v3ms = FindRoot[tFtIncE[vv, Vp] - 0.003, {vv, Vp}][[1, 2]];
 ZIncEvp = Integrate[dzIncE, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 ZIncE[v_, v0_] := ZIncEvp /. Vph → v /. Vp0 → v0;
 tfFInE = tFtIncE[0, v3ms]/(162 + 55.3) (162) + 0.003;
 zfFInE = ZIncE[v3ms, Vp] + ZIncE[0, v3ms] 162/(162 + 55.3);
 Print["μ=", μ, "      Tf=", Round[1000 tfFInE, 0.1],
  "ms", "      Zf=", Round[zfFInE, 0.01], "m"], {μ, 0, .1, 0.01}]
```

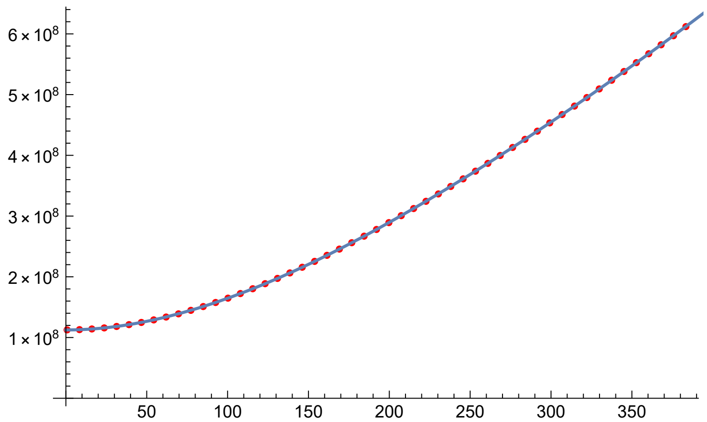
```
μ=0.      Tf=53.5ms      Zf=12.61m
μ=0.01      Tf=50.5ms      Zf=11.94m
μ=0.02      Tf=47.9ms      Zf=11.34m
μ=0.03      Tf=45.5ms      Zf=10.81m
μ=0.04      Tf=43.3ms      Zf=10.32m
μ=0.05      Tf=41.3ms      Zf=9.87m
μ=0.06      Tf=39.5ms      Zf=9.47m
μ=0.07      Tf=37.9ms      Zf=9.09m
μ=0.08      Tf=36.4ms      Zf=8.75m
μ=0.09      Tf=35.ms      Zf=8.43m
μ=0.1      Tf=33.7ms      Zf=8.14m
```

Assumption 2

Curve fitting for Linear Volumetric model

In[66]:=
```
listSFV2 = listSFV;
listSFV2[[All, 1]] = listSFV2[[All, 1]] / Sqrt[ρ/Y];
listSFV2[[All, 2]] = listSFV2[[All, 2]] Y;
fitFV = Fit[listSFV2, {1, x, x^2, x^3, x^4}, x];
gFV[x_] := fitFV
Show[ListPlot[listSFV2, PlotStyle → Red], Plot[gFV[x] , {x, 0, 500}]]
SIGMAFFV[vp_, ϕ_] := fitFV /. x → vp Cos[ϕ]; (*V=vp Cos[ϕ]*)
```

Out[71]=



Curve fitting maximum error and coefficient of determination

In[73]:=
```
R2 = 0;
sstot = 0;
ssres = 0;
sum = 0;
error = {};
Do[sum = sum + listSFV2[[i, 2]], {i, 1, Dimensions[listSFV2][[1]]}];
ave = sum / Dimensions[listSFV2][[1]];
Do[sstot = sstot + (listSFV2[[i, 2]] - ave)^2;
 val = (fitFV /. x → listSFV2[[i, 1]]);
 error = Append[error, Abs[listSFV2[[i, 2]] - val] / Abs[listSFV2[[i, 2]]]];
 ssres = ssres + (listSFV2[[i, 2]] - val)^2, {i, 1, Dimensions[listSFV2][[1]]}];
R2 = 1 - ssres / sstot;
maxerror = Max[error] 100;
Print["coefficient of determination=", R2, "      Maximum Error=", maxerror, "%"]
```

```
coefficient of determination=1.      Maximum Error=0.106333%
```

Penetration Time (Tf) and Penetration depth (Zf) without the pusher plate with different friction

In[77]:=

```
M = 162; Clear[μ];
Do[daFV = (-SIGMAFFV[vp, ϕ]/M) (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
  (*Equation 50*)
  azFV = Integrate[daFV, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify;
  (*Equation 51*)
  azFVN[v_] := azFV /. vp → v; dzFV = vp/azFVN[vp] // Simplify;
  ZFV = Integrate[dzFV, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
  ZFVN[v_, v0_] := ZFV /. Vph → v /. Vp0 → v0;
  tFV = Integrate[1/azFVN[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
  tFVN[v_, v0_] := tFV /. Vph → v /. Vp0 → v0;
  tfFV = tFVN[0, Vp]; zfFV = ZFVN[0, Vp];
  Print["μ=", μ, "      Tf=", Round[1000 tfFV, 0.1],
   "ms", "      Zf=", Round[zfFV, 0.01], "m"], {μ, 0, .1, 0.01}]
```

```
μ=0.      Tf=32.7ms      Zf=7.79m
μ=0.01      Tf=30.9ms      Zf=7.36m
μ=0.02      Tf=29.2ms      Zf=6.97m
μ=0.03      Tf=27.7ms      Zf=6.62m
μ=0.04      Tf=26.3ms      Zf=6.31m
μ=0.05      Tf=25.1ms      Zf=6.02m
μ=0.06      Tf=24.ms      Zf=5.76m
μ=0.07      Tf=23.ms      Zf=5.52m
μ=0.08      Tf=22.ms      Zf=5.3m
μ=0.09      Tf=21.2ms      Zf=5.09m
μ=0.1      Tf=20.4ms      Zf=4.9m
```

Adding effects of the pushing plate between t = 0 to 0.003 s

In[79]:=
```
M = 162 + 55.3;
Do[daFV = (-SIGMAFFV[vp, ϕ])/M (μ Sin[ϕ] + Cos[ϕ]) (Sin[ϕ] - (s - a)/s) 2 π s^2 // FullSimplify;
 azFV = Integrate[daFV, {ϕ, ϕ0, π/2}] /. ϕ0 → θo // Simplify;
 azFVN[v_] := azFV /. vp → v;
 dzFV = vp/azFVN[vp] // Simplify;
 tFV = Integrate[1/azFVN[vp], {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 tFVN[v_, v0_] := tFV /. Vph → v /. Vp0 → v0;
 v3ms = FindRoot[tFVN[vv, Vp] - 0.003, {vv, Vp}][[1, 2]];
 ZFV = Integrate[dzFV, {vp, Vp0, Vph}, Assumptions → 1000 > Vp0 > Vph > 0];
 ZFVN[v_, v0_] := ZFV /. Vph → v /. Vp0 → v0;
 tfFV = tFVN[0, v3ms]/(162 + 55.3) (162) + 0.003;
 zfFV = ZFVN[v3ms, Vp] + ZFVN[0, v3ms] 162/(162 + 55.3);
 Print["μ=", μ, "      Tf=", Round[1000 tfFV, 0.1],
  "ms", "      Zf=", Round[zfFV, 0.01], "m"], {μ, 0, .1, 0.01}]
```

```
μ=0.      Tf=33.5ms      Zf=8.17m
μ=0.01      Tf=31.6ms      Zf=7.73m
μ=0.02      Tf=30.ms      Zf=7.35m
μ=0.03      Tf=28.5ms      Zf=7.m
μ=0.04      Tf=27.1ms      Zf=6.68m
μ=0.05      Tf=25.9ms      Zf=6.39m
μ=0.06      Tf=24.8ms      Zf=6.13m
μ=0.07      Tf=23.7ms      Zf=5.89m
μ=0.08      Tf=22.8ms      Zf=5.67m
μ=0.09      Tf=21.9ms      Zf=5.46m
μ=0.1      Tf=21.1ms      Zf=5.27m
```