РГР - Диференціальне та Інтегральне числення. ПД-11 Гапей М.Ю.

Розрахунково-графічна робота № 2

з дисципліни « Вища математика »

Студента групи ПД-11

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Варіант 20 10 5

Завдання 1. Знайти $y' = \frac{dy}{dx}$ та $y'' = \frac{d^2y}{dx^2}$

а) $y = 5 + 40x + \frac{20}{3}x^3;$

$y' = 40 + 20x^2;$

$y'' = 40x;$

б) $y = e^{x^{10}} \cdot \cos 5x + 20;$

$y' = 10x^9 e^{x^{10}} \cos 5x + e^{x^{10}} \cdot 5(-\sin 5x);$

$y'' = 90x^8 e^{x^{10}} \cos 5x + 100x^{18} e^{x^{10}} \cos 5x +$

$+ 10x^9 e^{x^{10}} \cdot 5(-\sin 5x) + 10x^9 e^{x^{10}} \cdot 5(-\sin 5x) + e^{x^{10}} \cdot 25(-\cos 5x);$

$(f \cdot g \cdot \varphi)' = (fg)'\varphi + (f \cdot g) \cdot \varphi' = f'g\varphi + fg'\varphi + fg\varphi';$

$(fg)'\varphi + fg\varphi' = f'g\varphi + fg'\varphi + fg\varphi';$

$\varphi((f'g) + (fg'));$

в) $y = \frac{20x^2 + 5x - 6}{10x + 20};$

$y' = \frac{10(20x^2 + 5x - 6) - (10x + 20)(40x + 5)}{(10x + 20)^2};$

$y'' = (y')' = \left(\frac{200x^2 + 50x - 60 - 400x^2 - 850x - 100}{(10x + 20)^2}\right)' =$

$= \left(\frac{-200x^2 - 800x - 160}{(10x + 20)^2}\right)' = \frac{(-400x - 800)(10x + 20)^2}{(10x + 20)^4} +$

$+ \frac{20(200x^2 + 800x + 160)(10x + 20)}{(10x + 20)^4};$

г) $y = (20x^2 + 5)\ln(10x^2 - 5);$

$$y' = 40x\ln(10x^2-5) + \frac{20x(20x^2+5)}{10x^2-5} = 40x\ln(10x^2-5) + \frac{80x^3+20x}{2x^2-1}$$

$$y'' = 40\ln(10x^2-5) + \frac{800x^2}{10x^2-5} + (2x^2-1)^{-1}(240x^2+20) -$$

$$- 4x(80x^3+20x)(2x^2-1)^{-2};$$

g) $$y = \frac{20\sin x - 5\cos x}{5\sin x - 10\cos x} = \frac{4\sin x - \cos x}{\sin x - 2\cos x};$$

$$y' = \underbrace{\frac{4\cos x + \sin x}{\sin x - 2\cos x}}_{g} - \underbrace{\frac{(4\sin x - \cos x)(\cos x + 2\sin x)}{(\sin x - 2\cos x)^2}}_{\varphi};$$

$$y'' = g' - \varphi';$$

$$g' = \frac{\cos x - 4\sin x}{\sin x - 2\cos x} - \frac{(4\cos x + \sin x)(2\sin x + \cos x)}{(\sin x - 2\cos x)^2};$$

$$\varphi' = \frac{((4\sin x - \cos x)(\cos x + 2\sin x))'}{(\sin x - 2\cos x)^2} - \frac{(4\cos x + \sin x)(2\sin x + \cos x)((\sin x - 2\cos x)^2)'}{(\sin x - 2\cos x)^4}$$

$$= \frac{(4\cos x + \sin x)(\cos x + 2\sin x)}{(\sin x - 2\cos x)^2} + \frac{(4\sin x - \cos x)(2\cos x - \sin x)}{(\sin x - 2\cos x)^2} -$$

$$- 2\frac{(4\cos x + \sin x)(2\sin x + \cos x)}{(\sin x - 2\cos x)^4}(\sin x - 2\cos x)(2\sin x + \cos x);$$

e) $$y = 20x^{\sqrt{x+5}} - 10;$$

$$y' = (20x^{\sqrt{x+5}})' \Rightarrow (\ln y)' = (\sqrt{x+5}\,\ln 20x)'$$

$$\frac{1}{y}y'_x = \frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x} \Rightarrow y'_x = 20x^{\sqrt{x+5}}\left(\frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x}\right);$$

$$y'' = (y')' = \left(20x^{\sqrt{x+5}}\left(\frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x}\right)\right)'$$

$$(20x^{\sqrt{x+5}})' = y'_x \Rightarrow y'' = y'_x\left(\frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x}\right) +$$

$$+ 20x^{\sqrt{x+5}}\left(\frac{\frac{1}{x}\sqrt{x+5} - \ln 20x\,\frac{1}{2\sqrt{x+5}}}{2x+10} + \frac{\frac{x}{2\sqrt{x+5}} - \sqrt{x+5}}{x^2}\right);$$

$$y'' = 20x^{\sqrt{x+5}}\left(\frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x}\right)\left(\frac{\ln 20x}{2\sqrt{x+5}} + \frac{\sqrt{x+5}}{x}\right) +$$

$$+ 20x^{\sqrt{x+5}}\left(\left(\frac{\sqrt{x+5}}{x} - \frac{\ln 20x}{2\sqrt{x+5}}\right)(2x+10)^{-1} + \left(\frac{x}{2\sqrt{x+5}} - \sqrt{x+5}\right)x^{-2}\right);$$

е) $\begin{cases} x = \ln(1+25t^2) \\ y = t - 10\,\mathrm{arctg}\,20t \end{cases} \Rightarrow y'_x = \dfrac{\psi'(t)}{\varphi'(t)}$, де $\psi = y$, $\varphi = x$

$$\psi'(t) = 1 - \frac{200}{1+400t^2} = \frac{1+400t^2-200}{1+400t^2} = \frac{400t^2-199}{1+400t^2};$$

$$\varphi'(t) = \frac{50t}{1+25t^2};$$

тоді:

$$y'_x = \frac{(400t^2-199)(1+25t^2)}{(1+400t^2)50t} \Rightarrow y''_{xx} = \frac{(y'_x)'}{\varphi'(t)};$$

$$(y'_x)' = \left[\frac{(400t^2-199)(25t^2+1)}{20\,000t^3+50t}\right]' =$$

$$= \frac{800t(25t^2+1)}{20000t^3+50t} + \frac{(400t^2-199)50t}{20000t^3+50t} + \frac{-(400t^2-199)(25t^2+1)}{(20000t^3+50t)^2}\cdot$$

$$\cdot 2(6\cdot10^4t^2+50)(2\cdot10^4t^3+50t);$$

Звідси $y''_{xx} = \dfrac{8\cdot10^2t(25t^2+1)+50t(4\cdot10^2t^2-199)}{2\cdot10^4t^3+50t}\cdot\dfrac{1+25t^2}{50t} -$

$$- \frac{(4\cdot10^2t^2-199)(25t^2+1)^2}{50t(2\cdot10^4t^3+50t)^2};$$

ж)

$$e^x\sin5y - 20e^y\cos11y = 0;$$

$$(e^x\sin5y - 20e^y\cos11y)' = (0)'$$

$$e^x\sin5y + e^x\cos5y\cdot5y'_x - (20e^y y'_x\cos11y + 20e^y(-\sin11y)y'_x\cdot11) = 0$$

$$y'_x(e^x\cos5y\cdot5 - 20e^y\cos11y + 20e^y\sin11y\cdot11) = -e^x\sin5y;$$

$$y'_x = \frac{-e^x\sin5x}{5e^x\cos5y - 20e^y(\cos11y+11\sin11y)};$$

$$y''_{xx} = (y'_x)'$$

Нехай $e^x\sin5x = u$, $e^x\cos5y = v$, $4e^y(\cos11y+11\sin11y) = w$

тоді $(y'_x)' = \dfrac{1}{5}\cdot\left(\dfrac{u}{w-v}\right)' = \dfrac{u'(w-v)-u(w'-v')}{(w-v)^2} =$

$$= \frac{u'}{w-v} - u\frac{(w-v)'}{(w-v)^2};$$

$u' = (e^x \sin 5x)' = e^x \sin 5x + 5e^x \cos 5x = e^x(\sin 5x + 5\cos 5x)$

$v' = (e^x \cos 5y)' = e^x(\cos 5y - 5y'_x \sin 5y)$

$w' = (4e^y(\cos 11y + 11\sin 11y))' = 4y'_x e^y(\cos 11y + 11\sin 11y) +$

$+ y'_x 4e^y(121\cos 11y - 11\sin 11y) = 4e^y(y'_x(\cos 11y + 11\sin 11y) +$

$+(121\cos 11y - 11\sin 11y)y'_x) = 4e^y y'_x(\cos 11y + 121\cos 11y) = \underline{4e^y y'_x\, 122\cos 11y}$

Звідси:

$$y''_{xx} = \frac{1}{5}\Big[\frac{e^x(\sin 5x + 5\cos 5x)}{4e^y(\cos 11y + 11\sin y) - e^x\cos 5y} - \frac{e^x \sin 5x}{(4e^y(\cos 11y + 11\sin y) - e^x\cos 5y)^2}$$

$$\cdot(4e^y y'_x\, 122\cos 11y - e^x(\cos 5y - 5y'_x \sin 5y))\Big], \text{ де } y'_x =$$

$$= \frac{e^x \sin 5x}{20e^y(\cos 11y + 11\sin 11y) - e^x\cos 5y};$$

3) $\begin{cases} x = 5\cos t \\ y = 21\sin 10t \end{cases}$

$$y'_x = \frac{\psi'(t)}{\varphi'(t)}, \text{ де } \psi(t) = 21\sin 10t,\ \varphi(t) = 5\cos t$$

$\psi'(t) = 210\cos 10t$, $\varphi'(t) = -5\sin t$

отже $y'_x = -\dfrac{42\cos 10t}{\sin t}$;

$$y''_{xx} = \frac{(y'_x)'}{\varphi'(t)};\quad (y'_x)' = -\left(\frac{-420\sin 10t \sin t - 42\cos 10t\cos t}{\sin^2 t}\right);$$

$$y''_{xx} = \frac{420\sin 10t \sin t + 42\cos 10t\cos t}{-5\sin^3 t};$$

Завдання 2.

а) $y = \dfrac{x^4}{20} + \dfrac{5}{3}x^3 - 100x^2 + 15$; $\quad$ Нема є критичних точок $x \in \mathbb{R}$

$y' = \dfrac{x^3}{5} - 5x^2 - 200x$; $y' = 0 \Leftrightarrow$ $\quad x\left(\dfrac{x^2}{5} - 5x - 200\right) = 0$;

$x(x^2 - 25x - 1000) = 0$

$x_1 = 0,\ x_2 \approx -22,\ x_3 \approx 47$

$f_{min}(-22) = -55\cdot10^3$

$f_{max}(0) = 15$

$f_{min}(47) = 20\cdot10^4$

f-я спадає при $x\in(-\infty;-22]\cup(0;47)$ і зростає при $x\in(-22;0)\cup(47;+\infty)$.

$$y'' = (y')' = \left(\frac{x^3}{5} - 5x^2 - 200x\right)' = \frac{3}{5}x^2 - 10x - 200$$

$$y'' = 0 \Leftrightarrow \frac{3}{5}x^2 - 10x - 200 = 0 \Rightarrow 3x^2 - 50x - 10^3 = 0$$

$\Rightarrow x_1 = -12,\ x_2 = 28$

точки перегину

при $x\in(-\infty;-12)\cup(28;+\infty)$ – ф-я випукла;

при $x\in(-12;28)$ – ф-я опукла.

б) $y = \dfrac{x^5}{10} - \dfrac{25x^3}{20} - 10x - 100;$ немає критичних точок. $x\in\mathbb{R}$

$y' = \dfrac{x^4}{2} - \dfrac{15}{4}x^2 - 10;$

$y' = 0 \Leftrightarrow 2x^4 - 15x^2 - 40 = 0 \Rightarrow \begin{cases} x_1 = 3 \\ x_2 = -3 \\ x_3 \neq \\ x_4 \neq \end{cases} \in \mathbb{C}$

ф-я: зростає при $x\in(-\infty;-3)\cup(3;+\infty)$

спадає при $x\in(-3;3)$

$f_{max}(-3) = -61;$

$f_{min}(3) = -140;$

$y'' = 2x^3 - \dfrac{15}{2}x;$

$y'' = 0 \Rightarrow x(4x^2 - 15) = 0$

точки перегину: $x = 0,\quad x = \pm\dfrac{\sqrt{15}}{2}$

ф-я: опукла при $x\in\left(-\infty;-\frac{\sqrt{15}}{2}\right)\cup\left(0;\frac{\sqrt{15}}{2}\right)$

випукла при $x\in\left(-\frac{\sqrt{15}}{2};0\right)\cup\left(\frac{\sqrt{15}}{2};+\infty\right)$

в) $y = -\dfrac{10}{x^2-25}$; критичні точки $x = \pm 5$

$\Rightarrow y = \dfrac{10}{25-x^2}$;

$y' = \dfrac{20x}{(25-x^2)^2}$;

$y' = 0 \Rightarrow \dfrac{20x}{(25-x^2)^2} = 0 \Rightarrow \begin{cases} x = 0; \\ x \neq \pm 5. \end{cases}$

$-\quad -\quad +\quad +$

$-5 \quad 0 \quad 5 \quad x$

$\searrow \quad \searrow \quad \nearrow \quad \nearrow$

ф-я зростає при $x \in (0; 5) \cup (5; +\infty)$

ф-я спадає при $x \in (-\infty; -5) \cup (-5; 0)$

$ф_{min}(0) = \dfrac{2}{5}$ (у точці x);

$y'' = \left(\dfrac{20x}{(25-x^2)^2}\right)' = \dfrac{20(25-x^2)^2 + 20x \cdot 4x(25-x^2)}{(25-x^2)^4} =$

$= \dfrac{20}{(25-x^2)^2} + \dfrac{80x^2}{(25-x^2)^3} \Leftrightarrow y'' = 0 \Rightarrow \begin{matrix} x_1 \neq 5 \\ x_2 \neq -5 \end{matrix}$

$\cap \quad \cup \quad \cap$

$- \quad + \quad -$

$-5 \quad 5 \quad x$

ф-я опукла при $x \in (-\infty; -5) \cup (5; +\infty)$;

ф-я випукла при $x \in (-5; 5)$, де -5 і 5 – точки перегину.

Асимптоти ф-ї $\lim\limits_{x \to 5^-} \dfrac{10}{25-x^2} = \infty$; $\lim\limits_{x \to -5^-} \dfrac{10}{25-x^2} = \infty$;

$\lim\limits_{x \to 5^+} \dfrac{10}{25-x^2} = \infty$; $\lim\limits_{x \to -5^+} \dfrac{10}{25-x^2} = \infty$;

отже $x = \pm 5$ асимптоти (вертикальні) другого роду.

$y = kx + b = \lim\limits_{x \to \infty} \dfrac{f(x)}{x} \cdot x + \lim\limits_{x \to \infty} \left(f(x) - \lim\limits_{x \to \infty}\left(\dfrac{f(x)}{x}\right) x\right)$

$k = \lim\limits_{x \to \infty} \dfrac{-10}{x(x^2-25)} = 0$

$b = \lim\limits_{x \to \infty} \dfrac{-10}{x^2-25} = 0$ отже $y = 0$ – горизонтальні асимптоти ~~другого~~ першого роду.

г) $y = \frac{10x^2 - 35x - 9}{x-3}$;

точка розриву $x = 3$

$$\lim_{x \to 3^+} \frac{10x^2 - 35x - 9}{x-3} = -\infty;$$

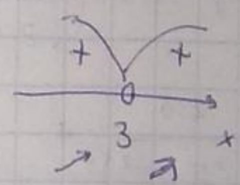
$$\lim_{x \to 3^-} \frac{10x^2 - 35x - 9}{x-3} = +\infty;$$

$$y' = ((20x-35)(x-3) - 10x^2 + 35x + 9)(x-3)^{-2} =$$

$$= (20x^2 - 95x + 105 - 10x^2 + 35x + 9)(x-3)^{-2} =$$

$$= \frac{(10x^2 - 60x + 114)}{(x-3)^2};$$

$$y' = 0 \Rightarrow \frac{10x^2 - 60x + 114}{(x-3)^2} = 0 \Rightarrow \begin{cases} x \notin \mathbb{R} \\ x \neq 3 \end{cases}$$



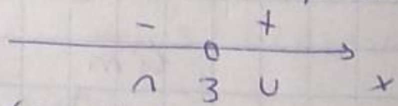
ф-я зростає при $x \in R \setminus \{3\}$;

Немає точок екстремуму функції.

$$y'' = ((20x-60)(x-3)^2 - 2(x-3)(10x^2 - 60x + 114))(x-3)^{-4} =$$

$$= ((20x-60)(\cancel{x-3}) - 20x^2 + 120x - 228)(x-3)^{-3} =$$

$$= \frac{-48}{(x-3)^3}; \quad x = 3 \text{ – точка перегину}$$



ф-я опукла при $x \in (-\infty; 3)$;

ф-я випукла при $x \in (3; +\infty)$.

Завдання 3. Дослідження ф-ї:

а) $y = 20x^3 + 10x$

1. ОДЗ: $x \in \mathbb{R}$;
2. Немає точок розриву

3. $f(-x) = -f(x)$ – ф-я непарна
$-20x^3 - 10x = -(20x^3 + 10x)$

4. OX; $M_1(0;0)$
OY; $M_2 = M_1 = (0;0)$

$y' = 60x^2 + 10$;

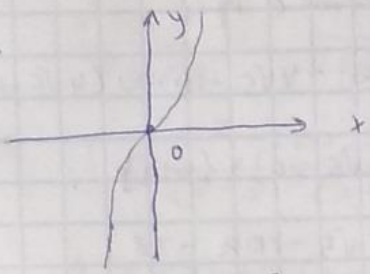
$y' = 0 \Rightarrow x = \pm\sqrt{\frac{-1}{6}} \quad x \notin \mathbb{R}$.

немає нулів ф-ї (точок екстремуму)

5. при $x \in (-\infty; 0)$ – ф-я спадає
при $x \in (0; +\infty)$ – ф-я зростає
немає min і max ф-ї.

6. $y'' = 120x$; при $x \in (-\infty; 0)$ – ф-я випукла
$y''=0 \Rightarrow x=0$ при $x \in (0; +\infty)$ ф-я вгнута

7.



б) $y = \dfrac{x^2-20}{x+10}$;

1. ОДЗ; $x \in \mathbb{R} \setminus \{-10\}$

2. $f(-x) = \dfrac{x^2-20}{-x+10}$ – ф-я ні парна ні не парна.

3. $\lim\limits_{x \to -10^-} \dfrac{x^2-20}{x+10} = -\infty$

$\lim\limits_{x \to -10^+} \dfrac{x^2-20}{x+10} = +\infty$, звідси маємо: $x=-10$ вертикальна асимптота 2-го роду.

$y = kx + b$;

$k = \lim\limits_{x \to \infty} \dfrac{x^2-20}{x(x+10)} = \left[\dfrac{\infty}{\infty}\right] \overset{\text{П.Л.}}{=} \lim\limits_{x \to \infty} \dfrac{2x}{2x+10} = \left[\dfrac{\infty}{\infty}\right] \overset{\text{П.Л.}}{=} \lim\limits_{x \to \infty} \dfrac{2}{2} = 1.$

$b = \lim\limits_{x \to \infty} \left(\dfrac{x^2-20}{x+10} - \dfrac{x^2+10x}{x+10} \right) = \lim\limits_{x \to \infty} \left(\dfrac{-10x-20}{x+10} \right) = \left[\dfrac{\infty}{\infty}\right] =$

$\overset{\text{П.Л.}}{=} \lim\limits_{x \to \infty} \dfrac{-10}{1} = -10$; отже $y = x - 10$ ~~(горизонтальна)~~ похила асимптота.

4. $OX = (\pm\sqrt{20}; 0)$;
$OY = (0; -2)$;

5. $y' = \dfrac{2x(x+10) - x^2 + 20}{(x+10)^2} = \dfrac{x^2 + 20x + 20}{(x^2+10)^2}$;

$y' = 0 \Rightarrow \begin{cases} x^2 + 20x + 20 = 0 \\ x^2 + 10 \neq 0 \end{cases} \begin{cases} x = \pm 4\sqrt{5} - 10; \\ x \neq -10 \end{cases}$.

$+ \quad + \quad - \quad +$

$-4\sqrt{5}-10 \quad -10 \quad 4\sqrt{5}-10 \quad x$

$\nearrow \quad \searrow \quad \searrow \quad \nearrow$

ф-ія зростає при $x \in (-\infty; -4\sqrt{5}-10) \cup (4\sqrt{5}-10; +\infty)$

ф-ія спадає при $x \in (-4\sqrt{5}-10; 4\sqrt{5}-10) \setminus \{-10\}$.

$f_{max}(-4\sqrt{5}-10) \Rightarrow$ у точці $x = -8\sqrt{5} - 20 \approx -38$

$f_{min}(4\sqrt{5}-10) \Rightarrow$ у точці $x = 8\sqrt{5} - 20 \approx -2$

6. $y'' = \left(\dfrac{x^2 + 20x + 20}{(x+10)^2}\right)' = \dfrac{(2x+20)(x+10)^2 - 2(x+10)(x^2+20x+20)}{(x+10)^4} =$

$= \dfrac{(2x+20)(x+10) - 2x^2 - 40x - 40}{(x+10)^3} = \dfrac{\cancel{2x^2} - \cancel{40x} + 160}{(x+10)^3}$;

$y'' = 0 \Rightarrow \dfrac{160}{(x+10)^3} = 0 \Rightarrow x = -10$ - точки перегину

$- \quad +$

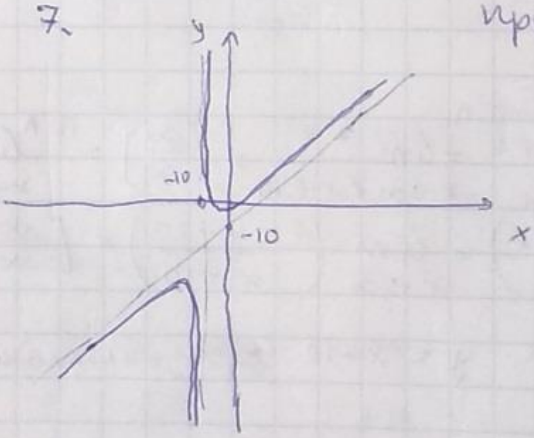
$-10 \quad x$

$\cap \quad \cup$

при $x \in (-\infty; -10)$ - ф-ія опукла

при $x \in (-10; +\infty)$ - ф-ія вигнута

7.



Завдание 4.

1. $\int \frac{(\ln x-10)^{25}}{x}dx = \int (\ln x-10)^{25} d(\ln x-10) = \frac{(\ln x-10)^{26}}{26}+C;$

2. $\int x\,10^{5x}dx = \left| \begin{matrix} u = x & du = dx \\ dv = 10^{5x}dx & v = \frac{10^{5x}}{5\ln 10} \end{matrix} \right| =$

$= \frac{x\,10^{5x}}{5\ln 10} - \frac{1}{5\ln 10}\int 10^{5x}dx = \frac{x\,10^{5x}}{5\ln 10} - \frac{10^{5x}}{25\ln^2 10}+C;$

3. $\int \frac{x^2-22x+5}{(x-3)^2(x+7)}dx = \left| \frac{A}{(x-3)^2}+\frac{B}{x-3}+\frac{C}{x+7} = \frac{x^2-22x+5}{(x-3)^2(x+7)} \right| =$

$= \left| \begin{matrix} A(x+7)+B(x-3)(x+7)+C(x-3)^2 = x^2-22x+5; \\ Ax+7A+B(x^2+4x-21)+C(x^2-6x+9) = x^2-22x+5; \\ x^2(B+C)+x(A+4B-6C)+(7A-21B+9C) = x^2-22x+5; \end{matrix} \right| =$

$= \begin{cases} B+C=1; \\ A+4B-6C=-22; \\ 7A-21B+9C=5; \end{cases} \begin{cases} B=1-C; \\ A+4-4C-6C=-22; \\ 7A-21+21C+9C=5; \end{cases} \begin{cases} B=1-C \\ A=-26+10C \\ -26\cdot 7+70C-21+30C= \\ =5 \end{cases}$

$\begin{cases} C=\frac{52}{25} \\ B=-\frac{27}{25} \\ A=-\frac{26}{5} \end{cases}$ откуда $\frac{x^2-22x+5}{(x-3)^2(x+7)} = \frac{52}{25}\left(\frac{1}{x+7}\right)+\frac{27}{25}\left(\frac{1}{3-x}\right)+$
$+\frac{26}{5}\left(\frac{-1}{(x-3)^2}\right)$

$\int \frac{x^2-22x+5}{(x-3)^2(x+7)}dx = \int\left[\frac{52}{25}\cdot\frac{1}{x+7}-\frac{27}{25}\frac{1}{x-3}-\frac{26}{5}\frac{1}{(x-3)^2}\right]dx =$

$= \frac{52}{25}\ln|x+7| - \frac{27}{25}\ln|x-3| + \frac{26}{5}(x-3)^{-1}+C;$

4. $\int \frac{20x-5}{(x+6)(x^2+8)}dx = \left| \frac{A}{x+6}+\frac{Bx+C}{x^2+8} = \frac{20x-5}{(x+6)(x^2+8)} \right| =$

$\Rightarrow A(x^2+8)+(Bx+C)(x+6) = 20x-5;$
$A(x^2+8)+(Bx^2+6Bx+Cx+6C) = 20x-5;$
$x^2(A+B)+x(6B+C)+(8A+6C) = 20x-5;$

$$\begin{cases} A+B=0 \\ 6B+C=20 \\ 8A+6C=-5 \end{cases} \quad \begin{cases} A=-B \\ C=20-6B \\ -8B+120-36B=-5 \end{cases} \quad \begin{cases} A=-\frac{125}{44} \\ B=\frac{125}{44} \\ C=\frac{65}{22} \end{cases}$$

$$\int \frac{20x-5}{(x+6)(x^2+8)}dx = \int \left[-\frac{125}{44}(x+6)^{-1} + \left(\frac{125}{44}x + \frac{65}{22}\right)(x^2+8)^{-1}\right]dx$$

$$= -\frac{125}{44}\ln|x+6| + \frac{65}{22}\frac{1}{2\sqrt2}\operatorname{arctg}\frac{x}{2\sqrt2} + \frac{125}{44}\int\frac{x}{x^2+8}dx =$$

$$\int\frac{x}{x^2+8}dx = \frac12\int\frac{d(x^2+8)}{x^2+8}$$

$$= -\frac{125}{44}\ln|x+6| + \frac{65}{44\sqrt2}\operatorname{arctg}\frac{x}{2\sqrt2} + \frac{125}{44}\cdot\frac12\ln|x^2+8| + C$$

5. $$\int \sin^2 20x \cdot \cos^2 20x\, dx = \int\left(\frac12 \sin 40x\right)^2 dx = \frac14\int\frac{1-\cos 80x}{2}dx =$$

$$= \frac14\int\left[\frac12 - \frac{\cos 80x}{2}\right]dx = \frac{x}{8} - \frac{\sin 80x}{160} + C;$$

6. $$\int x e^{10x} dx = \left[\begin{matrix} u = x & du = dx \\ dv = e^{10x}dx & v = \frac{e^{10x}}{10}\end{matrix}\right] =$$

$$= \frac{x e^{10x}}{10} - \frac{e^{10x}}{10^2} + C;$$

7. $$\int\frac{20}{5+x^2}dx = \frac{20}{\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} + C;$$

8. $$\int\frac{\sin 10x}{\sin^2 \frac{10}{5}(x)}dx = \int\frac{\sin 10x}{\sin^2 2x}dx = \left|\text{Використаємо універсальну підстановку}\right|$$

$$= \left| \begin{matrix} t = \operatorname{tg}\frac{x}{2} & \sin 10x = \frac{2\operatorname{tg}5x}{1+\operatorname{tg}^2 5x} & ? \\ & \sin 2x = \frac{2\operatorname{tg}x}{1+\operatorname{tg}^2x} & \end{matrix}\right| \ldots$$

9. $$\int 10x\ln 20x\,dx = 10\int x\ln 20x\,dx = \left|\begin{matrix} u = \ln 20x & du = \frac{dx}{x} \\ dv = x\,dx & v = \frac{x^2}{2}\end{matrix}\right|$$

$$= 5x^2\ln 20x - \frac52 x^2 + C;$$

10. $$\int \operatorname{ctg}^2(15x)dx = \int\left(\frac{1}{\sin^2 15x} - 1\right)dx = -\frac{\operatorname{ctg}15x}{15} - x;$$

11. $\int \frac{x^3-2x^2-10x-20}{x^3+5x}dx = \left| \frac{A}{x} + \frac{Bx+C}{x^2+5} = \frac{x^3-2x^2-10x-20}{x(x^2+5)} \right|$

$A(x^2+5)+Bx^2+Cx = x^3-2x^2-10x-20;$

$x^2(A+B)+Cx+A5 \neq x^3-2x^2-10x-20$

$\int \frac{x^3+5x-15x-2x^2-20}{x^3+5x}dx = \int \frac{-2x^2-15x-20}{x^3+5x}dx + x =$

$= x - \int \frac{2x^2+15x+20}{x^3+5x}dx = x - \int \frac{\frac{2}{3}d(x^3+5x) - \frac{10}{3} + 20 + 15x\,dx}{x^3+5x} =$

$= x - \left(\frac{2}{3}\ln|x^3+5x| + \frac{15}{\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} + \frac{50}{3}\underbrace{\int\frac{dx}{x(x^2+5)}}\right)$

$\delta = \int\frac{dx}{x(x^2+5)} = \left| \frac{A}{x} + \frac{Bx+C}{x^2+5} = \frac{1}{x(x^2+5)} \right| =$

$= x^2(A+B)+Cx+A5 = 1 \Rightarrow$

$\begin{cases} A+B=0 \\ C=0 \\ 5A=1 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{5} \\ B=-\frac{1}{5} \end{cases}$ отже $\frac{1}{x(x^2+5)} = \frac{1}{5}\frac{1}{x} - \frac{1}{5}\frac{x}{x^2+5}$

$\delta = \frac{1}{5}\int\left(\frac{1}{x} - \frac{x}{x^2+5}\right)dx = \frac{1}{5}\ln x - \frac{1}{10\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} + C$

$I = x - \left(\frac{2}{3}\ln|x^3+5x| + \frac{15}{\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5} + \frac{50}{3}\left(\frac{1}{5}\ln x - \frac{1}{10\sqrt5}\operatorname{arctg}\frac{x}{\sqrt5}\right)\right) + C$

12. $\int\frac{20}{\sqrt{5x^2-10}}dx = \frac{20}{\sqrt5}\int\frac{dx}{\sqrt{x^2-2}} = \frac{20}{\sqrt5}\ln|x+\sqrt{x^2-2}| + C$

$\int 2\sqrt{x}\,dx = \frac{4}{3}x\sqrt{x} + C$

Завдання 5.

1. $S = \int_1^4 \frac{10x-5}{\sqrt x}dx = 5\int_1^4\left(2\sqrt x - \frac{1}{\sqrt x}\right)dx = \left(\frac{20}{3}x\sqrt x - 10\sqrt x\right)\Big|_1^4 =$

$= \frac{20}{3}\cdot 2\cdot 4 - 10\cdot 2 - \left(\frac{20}{3} - 10\right) = \frac{20}{3}\cdot 7 - 10 = \frac{140-30}{3} = \frac{110}{3};$

2. $\int_0^{0,5}\frac{\arccos^{-5}x}{\sqrt{1-x^2}}dx = \int_{0,5}^{0}\arccos^{-5}x\,d\arccos x = \frac{\arccos^{-4}x}{-4}\Big|_{0,5}^{0}$

$= \frac{\arccos^{-4}0}{-4} + \frac{\arccos^{-4}0,5}{4} = \frac{\frac{\pi}{2}^{-4}}{-4} + \frac{\frac{\pi}{3}^{-4}}{4};$

3. $\int_0^{\pi} \sin^{10} 5x \cos^{30} 5x\,dx = \int \left(\frac{1}{2}\sin 5x \cos 5x\right)^{10} \cos^{20} 5x\,dx$

$= \int \frac{1}{2^{10}} \sin^{10} 10x (\cos^2 5x)^{10} dx = 2^{-10} \int \left(\frac{1-\cos 5x}{2}\right)^5 \dots$

$\int_0^{\pi} \sin^{10} 5x \cos^{30} 5x\,dx = \int_0^{\pi} \left(\frac{1-\cos 10x}{2}\right)^5 \left(\frac{1+\cos 10x}{2}\right)^{15} dx =$

$= \int_0^{\pi} \left(\frac{1-\cos^2 10x}{2}\right)^5 \left(\frac{1+\cos 10x}{2}\right)^{10} dx =$

$= \int_0^{\pi} \left(\frac{1-\cos 10x}{2}\right)^5 \left(\frac{1+\cos 10x}{2}\right)^5 \left(\frac{1+\cos 10x}{2}\right)^5 \left(\frac{1+\cos 10x}{2}\right)^5 dx =$

$\lim\limits_{x \to N} \sum_{i=1}^{N} \xi(i) \Delta x_i \Longrightarrow \int_0^{\pi} \left(\frac{1-\cos^2 10x}{2}\left(\frac{1+\cos 10x}{2}\right)^2\right)^5 dx \approx 7{,}2 \cdot 10^{-6}$

4. $\int_1^e \frac{\sqrt[5]{10+5\ln x}}{20x} dx = \frac{\sqrt[5]{5}}{20} \int_1^e \frac{\sqrt{2+\ln x}}{x} dx =$

$= \frac{\sqrt[5]{5}}{20} \int_1^e \sqrt[5]{2+\ln x}\, d(\ln x + 2) = \frac{\sqrt[5]{5}}{20} \left(\frac{5}{6} \sqrt[5]{(2+\ln x)^6}\right)\Big|_1^e =$

$= \frac{\sqrt[5]{5}}{24} \cdot \sqrt[5]{3^6} - \frac{\sqrt[5]{5}}{24} \sqrt[5]{2^6}$;

5. $\int_0^{e-5} \ln(x+20)\,dx = \left| \begin{matrix} u = \ln(x+20) & du = \frac{dx}{x+20} \\ dv = dx & v = x \end{matrix} \right| =$

$= x\ln(x+20)\Big|_0^{e-5} - \int_0^{e-5} \frac{x\,dx}{x+20} = x\ln(x+20)\Big|_0^{e-5} - \int_0^{e-5} \left(1 - \frac{20}{x+20}\right) dx =$

$\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$ $\quad = (e-5)\ln(e+15) - e + 5 + 20\ln(e+15) - 20\ln 20 =$

$= \ln(e+15)(e+15) - 20\ln 20 - e + 5$;

$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$ $\quad$ 6. $\int_1^{64} \frac{dx}{20\sqrt{x} + 10\sqrt[3]{x}} = \left| \begin{matrix} x^{\frac{1}{2}} + x^{\frac{1}{3}} = x^{\frac{1}{3}}(x^{\frac{1}{6}} + 1) \\ x^{2^{-1}} + x^{3^{-1}} = x^{3^{-1}}(x^{6^{-1}} + 1) \end{matrix} \right| =$

$= \frac{1}{10} \int_1^{64} \frac{dx}{x^{\frac{1}{3}}(2x^{\frac{1}{6}} + 1)} = \left| \begin{matrix} t = x^{\frac{1}{6}} & t^3 = x^{\frac{1}{2}} \\ dt = \frac{1}{6} x^{-\frac{5}{6}} dx & dx = 6x^{\frac{5}{6}} dt \end{matrix} \right| =$

$$= \frac{1}{10}\int_1^{64} \frac{6x^{\frac{5}{6}}}{x^{\frac{1}{3}}(2t+1)}dt = \frac{3}{5}\int_1^{64}\frac{t^3}{2t+1}dt = \qquad x^{\frac{5}{6}-\frac{1}{3}} = x^{\frac{5-2}{6}} = \frac{1}{2}$$

$$= \left| \begin{matrix} 2t+1 = w & \frac{dw}{2} = dt \\ t = \frac{w-1}{2} & \end{matrix} \right| = \frac{3}{80}\int_1^{64} \frac{(w-1)^3}{w} dw =$$

$$= \frac{3}{80}\int_1^{64} [w^2 - 3w + 3 - \frac{1}{w}]dw = \frac{3}{80}[\frac{w^3}{3} - \frac{3}{2}w^2 + 3w -$$

$$- \ln w] \Big|_1^{64} = \frac{3}{80}[3(2\sqrt[6]{x}+1) + \frac{1}{3}(2\sqrt[6]{x}+1)^3 - \frac{3}{2}(2\sqrt[6]{x}+1)^2 -$$

$$- \ln(2\sqrt[6]{x}+1)]\Big|_1^{64} \approx 0{,}53.$$

7. $$\int_1^5 \frac{x+5}{\sqrt{2x-10}+3}dx = \underbrace{\int_1^5 \frac{1}{2}\frac{5\,d(2x-10)}{\sqrt{2x-10}+3}}_{\delta_1} + \underbrace{\int_1^5 \frac{x}{\sqrt{2x-10}+3}dx}_{\delta_2} =$$

$$= \delta_1 + \delta_2;$$

$$\delta_1 = \frac{5}{2}\int_1^5 \frac{dx}{\sqrt{2x-10}+3} = \left| \begin{matrix} \sqrt{2x-10}+3 = t & dx = t-3\,dt \\ \frac{(t-3)^2+10}{2} = x & t_1 \neq \\ & t_2 = 0 \end{matrix} \right| =$$

$$= \frac{5}{2}\int_1^5 \frac{t-3}{t}dt = [\frac{5}{2}t - \frac{15}{2}\ln|t|]\Big|_1^5 = [\frac{5}{2}(\sqrt{2x-10}+3) - \frac{15}{2}$$

$\ln(\sqrt{2x-10}+3)] \Big|_1^5 =$ інтеграл розходиться.

$$\delta_2 = \int_1^5 \frac{x}{\sqrt{2x-10}+3}dx = \left| \begin{matrix} \sqrt{2x-10}+3 = t & x = \frac{(t-3)^2+10}{2} \\ dx = t-3\,dt & \end{matrix} \right| =$$

$$= \frac{1}{2}\int_1^5 \frac{(t-3)^3 + 10(t-3)}{t}dt = \frac{1}{2}\int_1^5 \frac{((t-3)^2+10)(t-3)}{t}dt =$$

$$= \frac{1}{2}\int_1^5 [\frac{(t^2-6t+19)t}{t} + \frac{-3t^2 + 18t - 57}{t}]dt =$$

$$= \frac{1}{2}\int_1^5 [t^2 - 6t + 19 - 3t + 18 - \frac{57}{t}]dt =$$

$$= \frac{1}{2}\int_1^5 [t^2 - 9t - \frac{57}{t} + 37]dt = \frac{1}{2}[\frac{t^3}{3} - \frac{9}{2}t^2 - 57\ln t + 37t]\Big|_1^5$$

$\Rightarrow$ інтеграл розходиться.

8. $\int_0^{\pi/6} \frac{\sin^3 25x}{\cos^6 25x} dx = \left| \begin{matrix} t = 25x & dx = \frac{dt}{25} \\ dt = 25dx & \end{matrix} \right| =$

$= \frac{1}{25} \int_0^{\frac{\pi}{6}} \frac{\sin^3 x}{\cos^6 t} dt = \frac{1}{25} \int_0^{\frac{\pi}{6}} tg^2 t \cdot tg\, t \cdot \frac{1}{\cos^3 t} dt =$

$= \frac{1}{25} \int_0^{\frac{\pi}{6}} \left(\frac{1}{\cos^2 t} - 1\right)\left(\frac{tg\, t}{\cos t} \frac{1}{\cos^2 t}\right) dt = \left| \begin{matrix} \frac{1}{\cos x} = t \\ dt = \frac{\sin x}{\cos^2 x} dx \end{matrix} \right| =$

$= \frac{1}{25} \int_0^{\frac{\pi}{6}} \left(\frac{1}{\cos^2 t} - 1\right)\left(\frac{\sin t}{\cos^2 t} \cdot \frac{1}{\cos^2 t}\right) du =$

$= \frac{1}{25} \int_0^{\frac{\pi}{6}} (u^2 - 1) u^2 du = \frac{1}{25} \int_0^{\pi/6} (u^4 - u^2) du =$

$= \left[\frac{1}{25} \cdot \frac{u^5}{5} - \frac{1}{25} \frac{u^3}{3}\right] \Big|_0^{\frac{\pi}{6}} = \left[\frac{1}{125} \cdot \frac{1}{\cos 25x} - \frac{1}{75} \cdot \frac{1}{\cos 25x}\right]_0^{\frac{\pi}{6}} =$

= інтеграл розходиться у точці $x = \frac{\pi}{50}$ $\left(0 < \frac{\pi}{50} < \frac{\pi}{6} \right)$

9. $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{10x}{\sin^2 5x} dx = \left| \begin{matrix} 5x = t & \\ 2t = 10x & \frac{dt}{5} = dx \\ t_1 = \frac{5}{6}\pi & t_2 = \frac{5}{2}\pi \end{matrix} \right| = \frac{2}{5} \int_{\frac{5}{6}\pi}^{\frac{5}{2}\pi} \frac{t}{\sin^2 t} dt$

$= \left| \begin{matrix} u = t & du = dx \\ dv = \frac{dt}{\sin^2 t} & v = -ctg\, t \end{matrix} \right| = \left[-t\, ctg\, t + \ln|ctg\, t|\right]_{\frac{5}{2}\pi}^{\frac{5}{6}\pi} =$

$\Rightarrow \approx -1,53$

10. $\int_0^1 \frac{tg^5 x}{\cos^2 x} dx = \int_0^1 tg^5 x\, d\, tg\, x = \frac{tg^6 x}{6} \Big|_0^1 =$

$= \frac{tg^6 1}{6}$;

11. $\int_1^3 \frac{(x-20)dx}{\sqrt{10 - 2x - x^2}} = \int_1^3 \frac{x - 20}{\sqrt{11 - (x+1)^2}} dx = \int_1^3 \frac{-20\, d(x+1)}{\sqrt{11 - (x+1)^2}} +$

$+ \int_1^3 \frac{-\frac{1}{2} d(x^2 + 2x - 10) - dx}{\sqrt{10 - 2x - x^2}} = -\int_1^3 \frac{20\, d(x+1)}{\sqrt{11^2 - (x+1)^2}} + \frac{-1}{2} \int_1^3 \frac{d(-x^2 - 2x + 10)}{\sqrt{10 - 2x - x^2}} -$

$- \int \frac{d(x+1)}{\sqrt{11 - (x+1)^2}} = \left[-21 \arcsin \frac{x+1}{\sqrt{11}} + \left(-\sqrt{10 - 2x - x^2}\right)\right] \Big|_1^3 =$

$= -21 \arcsin \frac{4}{\sqrt{11}} - \sqrt{-5} + 21 \arcsin \frac{2}{\sqrt{11}} + \sqrt{7}$ інтеграл розходиться.

12. $\{\int_1^2 \frac{dx}{\sqrt[3]{x^2+5}} = \left| \begin{matrix} \sqrt[3]{x^2+5} = t & dx = \frac{3t^2}{2\sqrt{t^3-5}}dt \\ \sqrt{t^3-5} = x & \end{matrix} \right| =$

$= \int_1^2 \frac{3t}{2\sqrt{t^3-5}} dt = \dots \}$

$\int_1^2 \frac{dx}{(\sqrt{20+5x^2})^3} = \left| \begin{matrix} \sqrt{5x^2+20} = t & \sqrt{\frac{t^2-20}{5}} = x \\ dt = \frac{5x}{\sqrt{5x^2+20}}dx & \end{matrix} \right| =$

$= \int_1^2 \frac{\sqrt{5x^2+20}}{\sqrt{5x^2+20}^3} \cdot \frac{1}{5x} dt = \frac{1}{5}\int_1^2 \frac{1}{5x^2+20} \cdot \frac{\sqrt{5}}{\sqrt{t^2-20}} dt =$

$= \frac{1}{5}\int_1^2 \frac{1}{t^2} \frac{\sqrt{5}}{\sqrt{t^2-20}} dt = \left| \begin{matrix} t = \sqrt{20}\frac{1}{\cos u} \\ dt = \frac{\sqrt{20}\sin u}{\cos^2 u}du \end{matrix} \right| =$

$u = \arccos\frac{\sqrt{20}}{t}$

$= \int_1^2 \frac{20}{\cos^2 u} \cdot \frac{\sqrt{5}}{20 tg\, u} \cdot \sqrt{20}\,\frac{tg\, u}{\cos u} du$

$= \frac{\sqrt{5}}{5}\int_1^2 \frac{\cos^2 u}{20} \cdot \frac{1}{\sqrt{20} tg\, u} \frac{\sqrt{20}\, tg\, u}{\cos u} du = \frac{\sqrt{5}}{100}\int_1^2 \cos u\, du =$

$= \frac{\sqrt{5}}{20} \sin u \Big|_1^2 = \frac{\sqrt{5}}{20} \sin \arccos \frac{\sqrt{20}}{t}\Big|_1^2 = \frac{\sqrt{5}}{20} \sin\arccos \frac{\sqrt{20}}{\sqrt{5x^2+20}}\Big|_1^2$

$= \frac{\sqrt{5}}{100} \frac{x}{\sqrt{x^2+4}}\Big|_1^2 = \frac{\sqrt{5}}{100}\left(\frac{2}{2\sqrt{2}} - \frac{1}{\sqrt{5}} \right);$

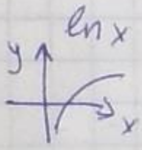
Завдання 6.

1. $\int_{-\infty}^{\infty} \frac{dx}{5+20x^2} = \left| \begin{matrix} f(-x) = f(x) \text{ тоді} \\ \int_{-\infty}^{\infty} f(x)dx = 2\int_0^{\infty} f(x)dx \end{matrix} \right| =$

$= \frac{2}{5}\int_0^{\infty} \frac{dx}{1+4x^2} = \frac{2}{5}\lim_{\delta\to\infty}\frac{1}{4}\int_0^{\delta} \frac{dx}{\frac{1}{4}+x^2} = \frac{1}{10}\lim_{\delta\to\infty} 2\,arctg\, 2x\Big|_0^{\delta} =$

$= \lim_{\delta\to\infty}\frac{2}{10}(arctg\, 2\delta - arctg\, 0) = \frac{\pi}{10};$

2. $\int_{-15}^{4} \frac{dx}{x+15} = \lim_{\delta\to -15}\int_{\delta}^{4} \frac{dx}{x+15} = \lim_{\delta\to -15}\left(\ln|x+15| \Big|_{\delta}^{4}\right) =$



$= \lim_{\delta\to -15}(\ln 19 - \ln|\delta+15|) = +\infty$ Інт. розходиться.

3. $\int_e^\infty \frac{\ln 5x}{10x}dx = \frac{1}{10}\lim_{\delta\to\infty}\int_e^\delta \ln 5x\, d\ln 5x =$

$\frac{1}{20}\lim_{\delta\to\infty}\left(\ln^2 5x\Big|_e^\delta\right) = \frac{1}{20}\lim_{\delta\to\infty}\left(\ln^2 5\delta - \ln^2 5e\right) =$

$=\infty$ – інтеграл розходиться.

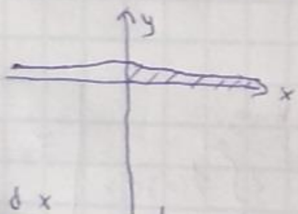
4. $\int_{-2}^\infty \frac{dx}{x^2+40x+5} = \lim_{\delta\to\infty}\int_{-2}^\delta \frac{d(x+20)}{(x+20)^2 - 395} = \left(\frac{1}{2\sqrt{395}}\ln\left|\frac{x+20-\sqrt{395}}{x+20+\sqrt{395}}\right|\right)\Big|_{-2}^\delta = \frac{1}{2\sqrt{395}}\lim_{\delta\to\infty}\left(\ln\left|\frac{x+20-\sqrt{395}}{x+20+\sqrt{395}}\right|\Big|_{-2}^\delta\right) =$ інтеграл розходиться.

5. $\int_{20}^3 \frac{5dx}{(x-20)^3} = 5\lim_{\delta\to 20}\int_\delta^3 \frac{d(x-20)}{(x-20)^3} = 5\lim_{\delta\to 20}\left(\frac{(x-20)^{-2}}{-2}\Big|_\delta^3\right) =$

$= -\frac{5}{2}\lim_{\delta\to 20}\left(\frac{1}{23^2} - \frac{1}{(\delta-20)^2}\right) =$ інтеграл розходиться.

6. $\int_0^\infty \frac{dx}{\sqrt[3]{x^2+5}} = \lim_{\delta\to\infty}\int_0^\delta \frac{dx}{(x^2+5)^{\frac{1}{3}}} = \left|\begin{array}{l}\sqrt[3]{x^2+5}=t\\ t^3-5=x^2\\ x\,dx=\frac{3}{2}t^2dt\end{array}\right| =$

...



$+\frac{2}{3} < 0$ інтеграл ~~розходиться~~ сходиться.

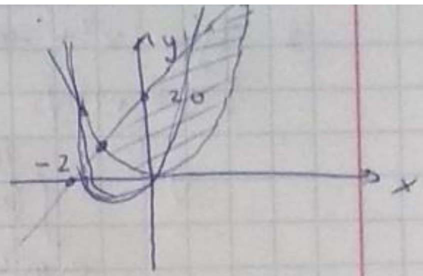
інт. виду $= \int_1^\infty \frac{dx}{x^\alpha} =$ | при $\alpha > 1$ ~~розходиться~~ сходиться; при $\alpha \le 1$ розходиться

$\int_1^\infty x^{-\alpha}dx =$ | $=\infty$ при $\alpha \le 1$; ~~розходиться~~, при $\alpha > 1$ сходиться при $\alpha \neq 1$: $\displaystyle\int_1^\infty \frac{1}{x^\alpha}\,dx = \lim_{x\to\infty}\int_1^x x^{-\alpha}\,dx = \lim_{x\to\infty}\left(\frac{x^{1-\alpha}}{1-\alpha}\Big|_1^x\right) \overset{(1)}{=}$

$$\lim_{x\to\infty}\left(\frac{x^{1-\alpha}}{1-\alpha} - \frac{1}{1-\alpha}\right) = \begin{cases} \text{если } \alpha = 1 \Rightarrow \ln(\infty) = \infty \\ \left.\begin{array}{l}\text{если } \alpha > 1\end{array}\right\} \quad \dfrac{1}{\alpha-1} \\ \text{если } \alpha < 1 \quad \Rightarrow \infty \end{cases}$$

(1) $$\left[\frac{x^{1-\alpha}-1}{1-\alpha}\right] = \begin{cases} \alpha > 1 & \dfrac{\frac{1}{x^{\alpha-1}}-1}{\alpha-1} \Rightarrow x\to\infty \Rightarrow -\dfrac{1}{1-\alpha} = \dfrac{1}{\alpha-1} \\ \alpha < 1 & \dfrac{x^{1-\alpha}-1}{1-\alpha} \Rightarrow x\to\infty \Rightarrow \infty \end{cases}$$

Завдання 7.



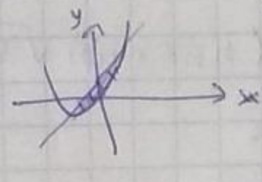
1. $y = x^2 + 5x$, $y = 10x + 20$

$x^2 + 5x = 10x + 20$

$x^2 - 5x - 20 = 0$

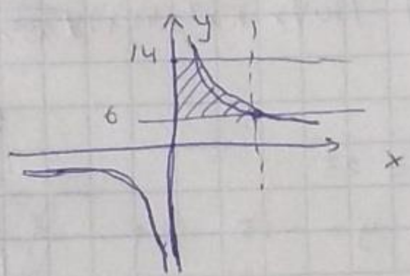
$x_1 = \frac{5}{2} \pm \sqrt{\frac{25}{4} + 20} = \pm\sqrt{\frac{105}{4}} + \frac{5}{2} = \frac{\pm\sqrt{105} + 5}{2}$;

$$S = \int_{\frac{5-\sqrt{105}}{2}}^{\frac{5+\sqrt{105}}{2}} (10x + 20 - x^2 - 5x)\,dx = \left(5x^2 + 20x - \frac{x^3}{3} - \frac{5}{2}x^2\right)\Big| = \ldots$$

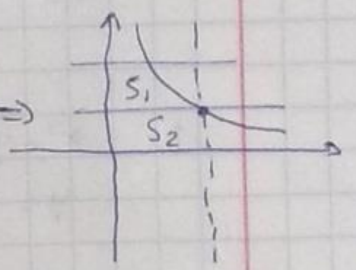


$= \frac{35\sqrt{105}}{2}$ кв.од.

2. $\begin{cases} y = \frac{15}{x}; \\ y = 6; \\ y = 14; \\ x = 0. \end{cases}$



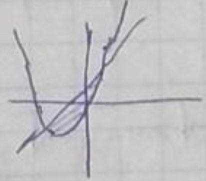
$\Rightarrow$



$S_1 = S - S_2$

$$S_1 = \int_6^{14} \frac{15}{y}\,dy = 15\ln|y|\Big|_6^{14} = 15(\ln 14 - \ln 6) \text{ кв. од.}$$

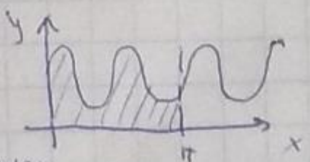
* 1. $\begin{cases} y = x(x+5) \\ y = 10(x+2) \end{cases}$



$S = \frac{35\sqrt{105}}{2} \approx 179,3$

Завдання 8.

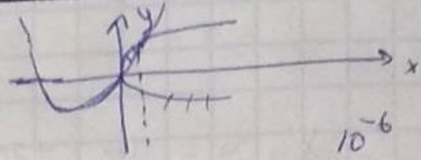
$y = \sin 5x + 10$, $x = 0$, $x = \pi$, $y = 0$



$$V = \pi \int_0^{\pi} (\sin 5x + 10)^2\,dx = \pi \int_0^{\pi} \left(\frac{1 - \cos 10x}{2} + 20\sin 5x + 100\right) dx =$$

$$= \pi\left[\frac{x}{2} - \frac{\sin 10x}{20} - \frac{\cos 5x}{100} + 100x\right]_0^{\pi} = \frac{201}{2}\pi^2 \text{ (кв. од.)}$$

2. $\begin{cases} y = x^2 + 40x \\ y = +10\sqrt{x} \end{cases}$



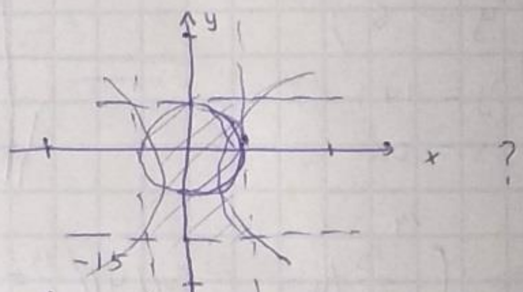
$$V = \pi \int_{0}^{10^{-6}} (10\sqrt{x} + x^2 + 40x)^2 dx = \int_0^{10^{-6}} (100x + x^4 + 16\cdot10^2 x^2 + 20x^2\sqrt{x} + 800x\sqrt{x} + 80x^3) dx = \left(50x^2 + \frac{x^5}{5} + \frac{1600x^3}{3} + \frac{40\sqrt{x^7}}{7} + 320\sqrt{x^5} + 20x^4\right)\Big|_0^{10^{-6}} \approx 5\cdot10^{-11}$$

1. $$S_{т.об} = \int_0^{\pi} (\sin 5x + 10)\sqrt{5\cos 5x + 1}\, dx = 54,4 \text{ кв.оди.}$$

2. $$S_{т.об} = \int_0^{10^{-6}} (10\sqrt{x} + x^2 + 40x)\sqrt{2x + \frac{5}{\sqrt{x}} + 41} \approx 5\cdot10^{-7} \text{ кв. оди.}$$

3. $x^2 - y^2 = 25$

$y = 5, \; y = -15$



$$V = \pi\int_{-15}^{5}(+25 + y^2)\,dy = \pi\left[25y + \frac{y^3}{3}\right]_{-15}^{5} = 5235,98 = \frac{5000}{3}\pi \text{ кв.оди.}$$

$$S = \int_{-15}^{5} \left(\sqrt{25+y^2}\sqrt{\frac{y}{\sqrt{25+y^2}} + 1}\right)dy \approx 98 \text{ кв.оди.}$$