Інтегральне числення. ПД-11 Гапей М.Ю.

(1) 

1. $\int_1^2 \left( x^3 - \frac{1}{x^4} \right) dx = \int_1^2 (x^3 - x^{-4}) dx = \left( \frac{x^4}{4} + \frac{x^{-3}}{3} \right) \Big|_1^2 =$

$= -\left(\frac{1}{4} + \frac{1}{3}\right) + 4 + \frac{1}{24} = -\frac{7}{12} + 4 + \frac{1}{24} = \frac{83}{24};$

2. $\int_0^8 (\sqrt{2x} + \sqrt[3]{x}) dx = \sqrt{2}\int_0^8 \sqrt{x}\, dx + \int_0^8 \sqrt[3]{x} dx = \sqrt{2}\left(\frac{2}{3} x\sqrt{x}\right)\Big|_0^8 +$

$+ \frac{3}{4} x^{\frac{4}{3}} \Big|_0^8 = \sqrt{2}\, \frac{2}{3} \cdot 8\sqrt{8} + \frac{3}{4} 8^{\frac{4}{3}} = \frac{2}{3} \cdot 2 \cdot 8 \cdot \sqrt{2}^2 + \frac{3}{4}\left(8^{\frac{1}{3}}\right)^4$ $\quad \frac{1}{3} + \frac{3}{3} = \frac{4}{3}$

$= \frac{64}{3} + \frac{3}{4} 2^4 = \frac{64}{3} + 12 = \frac{100}{3};$

3. $\int_0^{\pi/4} \frac{dx}{\cos^2 5x} = \frac{1}{5} \int_0^{\pi/4} \frac{d(5x)}{\cos^2 5x} = \frac{1}{5} \operatorname{tg} 5x \Big|_0^{\frac{\pi}{4}} =$

$\frac{1}{5}\left(\operatorname{tg} \frac{5}{4}\pi - \operatorname{tg} 0\right) = \frac{1}{5};$

$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi$

$\cos^2\varphi = \cos 2\varphi + \sin^2\varphi$

$\cos^2\varphi = \cos 2\varphi + 1 - \cos^2\varphi$

$2\cos^2\varphi = \cos 2\varphi + 1$

$\cos^2\varphi = \frac{1+\cos 2\varphi}{2}$

4. $\int_0^{\frac{\pi}{2}} \cos^2\varphi\, d\varphi = \int_0^{\frac{\pi}{2}} \frac{1+\cos 2\varphi}{2} d\varphi =$

$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2}\cos 2\varphi\right) d\varphi = \frac{\varphi}{2}\Big|_0^{\frac{\pi}{2}} + \frac{1}{4}\sin 2\varphi \Big|_0^{\frac{\pi}{2}} =$

$= \frac{\pi}{4} + \frac{1}{4} \cdot 0 = \frac{\pi}{4};$

5. $\int_0^1 \frac{dx}{x^2+4x+5} = \int_0^1 \frac{dx}{(x+2)^2+1} = \int_0^1 \frac{d(x+2)}{(x+2)^2+1} =$

$= \operatorname{arctg}(x+2)\Big|_0^1 = \operatorname{arctg} 3 - \operatorname{arctg} 2;$

6. $\int_0^2 \frac{dx}{\sqrt{5+4x-x^2}} = \int_0^2 \frac{dx}{\sqrt{-(x^2-4x-5)}} = \int_0^2 \frac{dx}{\sqrt{3^2-(x-2)^2}} =$

$= \arcsin \frac{x-2}{3}\Big|_0^2 = 0 + \arcsin\left(+\frac{2}{3}\right);$

$- \arcsin\left(-\frac{2}{3}\right) = \arcsin\frac{2}{3};$

7. $\int_1^3 2^{2x-3}dx = \frac{1}{2}\int_1^3 2^{2x-3} d(2x-3) = \frac{1}{2}\frac{2^{2x-3}}{\ln 2}\Big|_1^3 =$

$= \frac{1}{2}\frac{2^3}{\ln 2} - \frac{1}{2}\frac{2^{-1}}{\ln 2} = \frac{1}{2\ln 2}\left(2^3 - \frac{1}{2}\right) = \frac{15}{4\ln 2};$

(2) 1. $\int_{\frac{\pi^2}{4}}^{\pi^2} \frac{\sin\sqrt{x}}{\sqrt{x}}dx = \left| \begin{matrix} \sqrt{x} = t & t_1 = \pi \\ 2\sqrt{x}dt = dx & t_2 = \frac{\pi}{2} \end{matrix} \right| = 2\int_{\frac{\pi}{2}}^{\pi} \sin t\, dt =$

$= 2\cos t\Big|_{\frac{\pi}{2}}^{\pi} = -2 - 2\cdot 0 = -2;$

errorbook

2. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx = -\int_1^2 e^{\frac{1}{x}} d\left(\frac{1}{x}\right) = -e^{\frac{1}{x}}\Big|_1^2 = -\left(-e^{1} + e^{\frac{1}{2}}\right) =$

$= e - \sqrt{e};$

3. $\int_0^{\pi/2} \sin x \cos^3 x\, dx = \left| \begin{matrix} \sin x = t \\ dt = \cos x\, dx \end{matrix} \right| = \int_0^{\pi/2} \cos^2 x\, d(\sin x)$

$= \int_0^{\frac{\pi}{2}}$

3. $\int_0^{\frac{\pi}{2}} \sin x \cos^3 x\, dx = -\int_0^{\frac{\pi}{2}} \cos^3 x\, d\cos x = -\frac{\cos^4 x}{4}\Big|_0^{\frac{\pi}{2}} =$

$= \frac{1}{4};$

4. $\int_0^1 \frac{e^x dx}{1+e^{2x}} = \int_0^1 \frac{de^x}{1+(e^x)^2} = \operatorname{arctg} e^x \Big|_0^1 =$

$= \operatorname{arctg} e - \frac{\pi}{4};$ $\operatorname{arctg} 1 = \frac{\pi}{2\cdot 2}$

$1 = \operatorname{tg} \frac{\pi}{2\cdot 2}$

5. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}} = \left| \begin{matrix} 1+\sqrt{2x+1} = t & t_1 = 2 \\ (t-1)^2 = 2x+1 & t_2 = 4 \\ dx = \frac{2(t+(-1))}{2} dt & \end{matrix} \right| =$

$= \int_2^4 \frac{t-1}{t} dt = t \Big|_2^4 - \ln t \Big|_2^4 = 2 - \ln 2;$

ЕВ

6. $\int_0^3 \frac{x dx}{\sqrt{25-x^2}} = -\int_0^3 \frac{\frac{1}{2} d(25-x^2)}{\sqrt{25-x^2}} = -\sqrt{25-x^2} \Big|_0^3 =$

$\left[ \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C \right]$ $= -4+5 = 1$

7. $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}} = \int_1^{e^3} \frac{d(\ln x+1)}{\sqrt{1+\ln x}} = 2\sqrt{1+\ln x} \Big|_1^{e^3} =$

ЕВ $= 4 - 2 = 2;$

8. $\int_0^1 \frac{x dx}{(x^2+1)^2} = \frac{1}{2}\int_0^1 \frac{d(x^2+1)}{(x^2+1)^2} = -\frac{1}{2}(x^2+1)^{-1} \Big|_0^1 =$

$= -\frac{1}{2}\left[\frac{1}{2} - 1\right] = \frac{1}{4};$

ЕВ

9. $\int_3^{10} \frac{dx}{(x-1)\sqrt{x+6}} = \left| \begin{matrix} x-1 = \frac{1}{t} \Rightarrow x = \frac{1}{t}+1 = \frac{1+t}{t} \\ dx = -\frac{1+t-t}{t^2} \end{matrix} \right| =$

. . . тупик.

$= \left| \begin{matrix} \sqrt{x+6} = t & t^2 - 6 = x \\ dx = 2t dt & dt = \frac{1}{2t} dx \end{matrix} \right| = \int_3^{10} (t^2-6-1)^{-1} \cdot 2t dt =$

$$= \int_3^{10} \frac{2t\,dt}{x^2-7} = \int_3^{10} \frac{dt^2}{t^2-7} = \ln|t^2-7|\Big|_3^{10} =$$

$$= \ln|x-1|\Big|_3^{10} = \ln 2 - \ln 9;$$

③ 1. $$\int_1^2 \ln(2x+3)\,dx = \frac{1}{2}\int_1^2 \ln(2x+3)\,d(2x+3) = \left| \begin{matrix} 2x+3=t \\ t_1=5,\ t_2=7 \end{matrix} \right|$$

$$= \left| \begin{matrix} u=\ln t & du=\frac{1}{t}dt \\ dv=dt & v=t \end{matrix} \right| = \frac{1}{2} t\ln t \Big|_5^{9-2} - \frac{1}{2}t\Big|_5^7 =$$

$$= \frac{1}{2}\left(t(\ln t-1)\right)\Big|_5^7 = \frac{1}{2}(7\ln 7 - \cancel{1} - 5\ln 5 + \cancel{1}) = \frac{7}{2}\ln 7 - \frac{5}{2}\ln 5;$$

− 2. $$\int_0^1 e^{2y}y\,dy = \left| \begin{matrix} u=e^{2y} & du=2e^{2y}dy \\ \dots & \end{matrix} \right| \dots$$

2. $$\int_0^1 e^{2y}y\,dy = \left| \begin{matrix} y=u & du=dy \\ dv=e^{2y}dy & v=\frac{1}{2}e^{2y} \end{matrix} \right| =$$

$$= \frac{1}{2}e^{2y}\cdot y\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2y}dy = \frac{1}{2}e^{2y}\cdot y\Big|_0^1 - \frac{1}{4}e^{2y}\Big|_0^1 =$$

$$= \frac{1}{2}e^2 - \frac{1}{4}e^2 - \frac{1}{4} = \frac{1}{4}e^2 - \frac{1}{4} = (e^2-1)\frac{1}{4};$$

* 3. $$\int_0^1 x\ln x\,dx = \left| \begin{matrix} \cancel{du=\ln x} & du=\frac{dx}{x} \\ \cancel{dv=x\,dx} & \end{matrix} \right| = \left| \begin{matrix} u=\ln x & du=\frac{dx}{x} \\ dv=x\,dx & v=\frac{x^2}{2} \end{matrix} \right|$$

$$= \frac{x^2}{2}\ln x\Big|_0^1 - \frac{1}{2}\cdot\frac{x^2}{2}\Big|_0^1 = \dots$$

3. $\int_0^1 x \operatorname{arctg} x\,dx = \left| \begin{matrix} u = \operatorname{arctg} x & du = \frac{dx}{1+x^2} \\ dv = x\,dx & v = \frac{x^2}{2} \end{matrix} \right| =$

$= \left[ \frac{x^2}{2} \operatorname{arctg} x - \frac{1}{2} \int \frac{x^2+1-1}{x^2+1} dx \right] \Big|_0^1 = \Big( \frac{x^2}{2} \operatorname{arctg} x +$

$+ \frac{1}{2} \int \frac{1}{x^2+1} dx - \frac{1}{2} \int dx \Big] \Big|_0^1 = \Big( \frac{x^2}{2} \operatorname{arctg} x + \frac{1}{2} \operatorname{arctg} x - \frac{x}{2} \Big] \Big|_0^1 =$

$= -\frac{1}{2} + \frac{\pi}{4};$

4. $\int_0^{\pi/4} x \sin 4x\,dx = \left| \begin{matrix} u = x & du = dx \\ \sin 4x\,dx = dv & v = -\frac{1}{4} \cos 4x \end{matrix} \right| =$

$= -\frac{x}{4} \cos 4x \Big|_0^{\frac{\pi}{4}} + \frac{1}{4} \int \cos 4x\,dx = \Big[ -\frac{x}{4} \cos 4x + \frac{1}{16} \sin 4x \Big] \Big|_0^{\frac{\pi}{4}} =$

$= \frac{\pi}{16};$

5. $\int_0^6 (x+2) \cos 3x\,dx = \left| \begin{matrix} u = x+2 & du = dx \\ dv = \cos 3x & v = \frac{1}{3} \sin 3x \end{matrix} \right| =$

$= \frac{x+2}{3} \sin 3x \Big|_0^6 + \frac{1}{9} \cos 3x \Big|_0^6 = \frac{8}{3} \sin 18 - \frac{1}{9} + \frac{1}{9} \cos 18;$

6. $\int_1^2 \frac{\ln x}{x^5} dx = \left| \begin{matrix} u = \ln x & du = \frac{dx}{x} \\ dv = x^{-5} dx & v = -\frac{x^{-4}}{4} \end{matrix} \right| =$

$= \Big( -\frac{x^{-4} \ln x}{4} \Big) \Big|_1^2 + \int_1^2 \frac{x^{-5}}{4} dx = \Big( -\frac{x^{-4} \ln x}{4} \Big) \Big|_1^2 + \Big( -\frac{1}{16} x^{-4} \Big) \Big|_1^2 =$

$= -\frac{\ln 2}{64} + \frac{12}{256};$

④ 1. $\int\limits_1^\infty \frac{dx}{x^5} = \lim\limits_{x\to\infty} \int\limits_1^x x^{-5} dx = \lim\limits_{x\to\infty}\left(-\frac{1}{4}x^{-4}\right)\Big|_1^x =$

$= \lim\limits_{x\to\infty}\left(\frac{1}{4x^4}\right) - \lim\limits_{x\to\infty}\left(-\frac{1}{4}\right) = \frac{1}{4};$

2. $\int\limits_1^\infty \frac{dx}{\sqrt[3]{x}} = \lim\limits_{A\to\infty}\int\limits_1^A x^{\frac{1}{3}} dx = \left(\lim\limits_{A\to\infty} \frac{3}{4}x^{\frac{4}{3}}\right)\Big|_1^A =$

$= \lim\limits_{A\to\infty} \frac{3}{4} x^{\frac{4}{3}} -$

$\int\limits_1^\infty x^{\frac{1}{3}} dx = \lim\limits_{\alpha\to\infty}\int\limits_1^\alpha x^{\frac{1}{3}-1+1} dx = \lim\limits_{\alpha\to\infty}\left(\frac{3}{4}x^{\frac{4}{3}}\Big|_1^\alpha\right) =$

$= \lim\limits_{\alpha\to\infty}\left(\frac{3}{4}\alpha^{\frac{4}{3}} - \frac{3}{4}\right) = \infty;$

$\int\limits_1^\infty x^{-\frac{1}{3}} dx = \lim\limits_{\varphi\to\infty}\int\limits_1^\varphi x^{-\frac{1}{3}} dx = \lim\limits_{\varphi\to\infty}\left(\frac{3}{2}x^{\frac{2}{3}}\Big|_1^\varphi\right) =$

$= \lim\limits_{\varphi\to\infty}\left(\frac{3}{2}\sqrt[3]{\varphi^2} - \frac{3}{2}\right) = \infty - \frac{3}{2} = \infty;$

3. $\int\limits_0^\infty \sin x\, dx = \lim\limits_{\varphi\to\infty}\int\limits_0^\varphi \sin x\, dx = \lim\limits_{\varphi\to\infty}\left(\cos x\Big|_0^\varphi\right) =$

$= \lim\limits_{\varphi\to\infty}(\cos\varphi - 1) = \infty;$

EB

4. $\int\limits_1^\infty e^{-2x} dx = \lim\limits_{x\to\infty}\int\limits_1^x e^{-2x} dx = -\frac{1}{2}\lim\limits_{x\to\infty}\left(e^{-2x}\Big|_1^x\right) =$

$= -\frac{1}{2}\lim\limits_{x\to\infty}\left(e^{-2x} - e^{-2}\right) = -\frac{1}{2}\lim\limits_{x\to\infty}\left(\frac{1}{e^{2x}} - \frac{1}{e^2}\right) =$

$= \frac{1}{2e^2};$

5. $$\int_0^\infty x e^{-x^2} dx = -\frac{1}{2}\lim_{\varphi\to\infty}\int_0^\varphi e^{-x^2} d(-x^2) =$$

$$= -\frac{1}{2}\lim_{\varphi\to\infty}\left(\frac{1}{e^{x^2}}\Big|_0^\varphi\right) = \frac{1}{2};$$

6. $$\int_{-\infty}^{\infty}\frac{dx}{x^2+4} = 2\int_0^\infty \frac{dx}{x^2+4} = 2\lim_{x\to\infty}\int_0^x \frac{dx}{x^2+4} =$$

$$= \frac{2}{2}\lim_{x\to\infty}\operatorname{arctg}\frac{x}{2}\Big|_0^x = \lim_{x\to\infty}\left(\operatorname{arctg}\frac{x}{2} - 0\right) = \underline{\frac{\pi}{2}}$$

$\operatorname{arctg}\infty = \frac{\pi}{2}$ Because $\operatorname{tg} 90^\circ =$ невозможно.

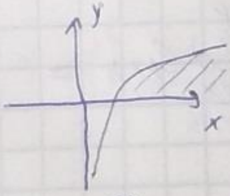
7. $$\int_0^\infty \frac{\operatorname{arctg} x\, dx}{x^2+1} = \lim_{x\to\infty}\int_0^x d(\operatorname{arctg} x) =$$

$$= \lim_{x\to\infty}\left(\operatorname{arctg} x\Big|_0^x\right) = \frac{\pi}{2};$$

8. $$\int_e^\infty \frac{dx}{x\ln x} = \lim_{x\to\infty}\int_e^x \frac{1}{\ln x}\, d\ln x = \lim_{x\to\infty}\ln\ln x\Big|_e^x =$$

$$= \lim_{x\to\infty}(\ln\ln x - 0) = \lim_{x\to\infty}\ln\ln x = \ln\lim_{x\to\infty}\ln x =$$

$$= \infty;$$




9. $$\int_0^\infty \frac{dx}{\sqrt[3]{(x+1)^5}} = \lim_{x\to\infty}\int_0^\infty \frac{d(x+1)}{(x+1)^{\frac{5}{3}}} = \lim_{x\to\infty}\left(\left(-\frac{5}{3}+1\right)^{-1}(x+1)^{-\frac{5}{3}+1}\Big|_0^x\right)$$

$$= \lim_{x\to\infty}\left(\left(-\frac{3}{2}\right)(x+1)^{-\frac{2}{3}}\Big|_0^x\right) = \left(-\frac{3}{2}\right)\lim_{x\to\infty}\left(\frac{1}{(x+1)^{\frac{2}{3}}} - 1\right) =$$

$$= \frac{3}{2};$$

(5)

1. $$\int_0^2 \frac{dx}{\sqrt{4-x^2}} = \lim_{\delta\to 2} \int_0^{\delta} \frac{dx}{\sqrt{2^2-x^2}} = \left[\arcsin\frac{x}{2}\right] =$$

$$= \lim_{\delta\to 2}\left(\arcsin\frac{x}{2}\Big|_0^{\delta}\right) = \lim_{\delta\to 2}\left(\arcsin\frac{\delta}{2} - 0\right) =$$

$$= \left[\arcsin 1 = x \quad \sin x = 1 \Rightarrow x = \frac{\pi}{2} + 2\pi n, n\in Z\right]$$

$$= \frac{\pi}{2};$$

2. $$\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}} = -\frac{1}{2}\lim_{\delta\to 1}\int_0^{\delta}\frac{d(1-x^2)}{\sqrt{1-x^2}} = -\lim_{\delta\to 1}\left(\sqrt{1-x^2}\Big|_0^{\delta}\right)$$

$$= 1;$$

3. $$\int_0^1 x\ln x\,dx = \lim_{\delta\to 0}\int_{\delta}^1 x\ln x\,dx = \left|\begin{matrix} u=\ln x & du=\frac{dx}{x} \\ dx = dv & v=\frac{x^2}{2}\end{matrix}\right|$$

$$= \lim_{\delta\to 0}\left(\frac{x^2}{2}\ln x\Big|_{\delta}^1 - \frac{1}{2}\frac{x^2}{2}\Big|_{\delta}^1\right) = \lim_{\delta\to 0}\left(\frac{x^2}{2}\left(\ln x - \frac{1}{2}\right)\Big|_{\delta}^1\right) =$$

$$= \infty;$$

4. $$\int_1^e \frac{dx}{x\ln x} = \lim_{\delta\to 1}\int_{\delta}^e \frac{dx}{x\ln x} = \lim_{\delta\to 1}\int_{\delta}^e \frac{1}{\ln x}\,d\ln x =$$

$$= \lim_{\delta\to 1} \ln\ln x\Big|_{\delta}^e = -\infty;$$

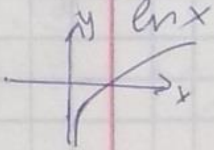
5. $$\int_{-1}^2 \frac{dx}{x} = \lim_{\delta\to 0}\int_{\delta}^2 d\ln x = \lim_{\delta\to 0}\ln x\Big|_{\delta}^2 =$$

$$= \ln 2 - \infty = -\infty, \quad \lim_{\delta\to 0}\left(\int_{-1}^{\delta}\frac{dx}{x} + \int_{\delta}^2\frac{dx}{x}\right) =$$

$$= \lim_{\delta\to 0}\left[\cancel{\ln\delta} - \ln(-1) + \ln 2 - \cancel{\ln\delta}\right] = +\infty$$

6. $\int_1^2 \frac{dx}{(x-1)^2} = \lim_{\delta\to 1}\int_\delta^2 \frac{d(x-1)}{(x-1)^2} = -\lim_{\delta\to 1}\left(\frac{1}{x-1}\Big|_\delta^2\right) =$

$= +\infty;$




7. $\int_0^{\pi/4} ctg\,x\,dx = \left| \begin{matrix} \frac{\cos x}{\sin x} = ctg\,x \\ \sin x = t \quad dt = \cos x\,dx \end{matrix} \right| =$

$= \lim_{\delta\to 0}\int_\delta^{\pi/4} \frac{dt}{t} = \lim_{\delta\to 0}\left(\ln|\sin x|\right)\Big|_\delta^{\frac{\pi}{4}} = \ln\frac{\sqrt2}{2} - \ln 0$

$= +\infty;$ $\quad \ln 0 = -\infty$

8. $\int_1^2 \frac{x\,dx}{\sqrt{x-1}} = \lim_{\delta\to 1}\int_\delta^2 \frac{x\,dx}{\sqrt{x-1}} = \left| \begin{matrix} t=\sqrt{x-1} \\ t^2+1 = x \\ dx = 2t\,dt \end{matrix} \right| =$

$= \lim_{\delta\to 1}\int_\delta^2 \frac{t^2+1}{\cancel{t}}\cdot 2t\,dt = 2\lim_{\delta\to 1}\int_\delta^2 (t^2+1)\,dt = \left[\frac{t^3}{3}+t\right] =$

$= 2\lim_{\delta\to 1}\left(\left(\frac{\sqrt{x-1}^{\,3}}{3} + \sqrt{x-1}\right)\Big|_\delta^2\right) = 2\lim_{\delta\to 1}\left(\frac13 + 1 - 0 + 0\right) =$

$= +\frac{8}{3};$

9. $\int_3^6 \frac{dx}{x^2-7x+10} = \left| \begin{matrix} x^2-7x+10=0 & x_1 = +\sqrt{\frac94}+\frac72 = 5 \\ (x-\frac72)^2+10-(\frac72)^2 = 0 & x_2 = -\sqrt{\frac94}+\frac72 = 2 \\ x = \pm\sqrt{\frac{49}{4}-10}+\frac72 & \end{matrix} \right|$

$= \lim_{\delta\to 5}\int_3^\delta \frac{dx}{(x-\frac72)^2+10-(\frac72)^2} + \lim_{\delta\to 5}\int_\delta^6 \frac{dx}{(x-\frac72)^2+10-(\frac72)^2} =$

$x-5$

$= \lim_{\delta\to 5}\frac13\ln\left|\frac{x-\frac72-\frac32}{x-\frac72+\frac32}\right|\Big|_3^\delta + \lim_{\delta\to 5}\frac13\ln\left|\frac{x-\frac72-\frac32}{x-\frac72+\frac32}\right|\Big|_\delta^6$ $\quad \frac{40}{4}-\frac{49}{4} = -\frac94$

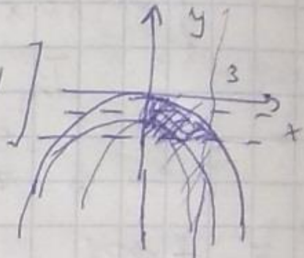
$x-2$

$= \frac13\lim_{\delta\to 5}\ln\left|\frac{x-5}{x-2}\right|\Big|_3^\delta + \frac13\lim_{\delta\to 5}\ln\left|\frac{x-5}{x-2}\right|\Big|_\delta^6 =$

$= \frac{1}{3} \lim_{\delta \to 5} \left(-\infty + \infty\right) + \frac{1}{3} \ln \frac{1}{4} + \infty = \infty;$

(6). Обчислити площу фігури, обмеженої лініями:

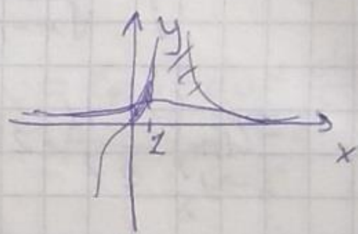
1. $\left[\, y = -\frac{x^2}{9},\ x = 0,\ x = 3,\ y = -1 \,\right]$




$$S = -\int_0^3 \left(\frac{x^2}{9} - 1\right) dx = -\left(\frac{x^3}{27} - x\right)\Big|_0^3 =$$

$= +2;$

2. $y = \frac{1}{1+x^2},\ y = \frac{x^3}{2},\ x = 0$

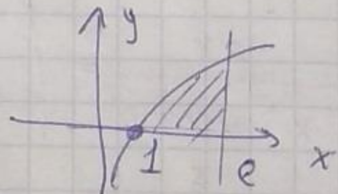



$$\frac{x^3}{2} = \frac{1}{1+x^2} \Rightarrow x = 1$$

$$S = \int_0^1 \left(\frac{1}{1+x^2} - \frac{x^3}{2}\right) dx = \operatorname{arctg} x \Big|_0^1 - \frac{1}{8}x^4 \Big|_0^1 =$$

$= \frac{\pi}{4} - \frac{1}{8};$

3. $y = \ln x,\ x = e,\ y = 0$




$$S = \int_1^e \ln x \, dx = (x\ln x - x)\Big|_1^e =$$

$= e - e + 1 = 1;$

$$\int \ln x \, dx = \left| \begin{matrix} u = \ln x & du = \frac{dx}{x} \\ dv = dx & v = x \end{matrix} \right| = x\ln x - x;$$

(2) обчислити методом заміни

1. $\int \frac{t^3}{t^4-9}\,dt = \frac{1}{3}\int \frac{d(t^4-9)}{t^4-9} = \frac{1}{3}\ln|t^4-9| + C;$

2. $\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{d\cos x}{\cos x} = -\ln|\cos x| + C;$

3. $\int \frac{dx}{x\ln^2 x} = \int \frac{d\ln x}{\ln^2 x} = -\frac{1}{\ln x} + C;$

4. $\int \frac{e^x}{e^x-8}\,dx = \int \frac{d(e^x-8)}{e^x-8} = \ln|e^x-8| + C;$

5. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^{\sqrt{x}}\,d\sqrt{x} = 2e^{\sqrt{x}} + C;$

6. $\int x\sqrt{x^2-3}\,dx = \frac{1}{2}\int \sqrt{x^2-3}\,d(x^2-3) = \frac{1}{3}\sqrt{(x^2-3)^3} + C$

7. $\int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{\ln x}\,d\ln x = \frac{2}{3}\sqrt{\ln^3 x} + C;$

8. $\int 5^{\sin x}\cos x\,dx = \int 5^{\sin x}\,d\sin x = \frac{5^{\sin x}}{\ln 5} + C;$

9. $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx = \int \frac{de^x}{\sqrt{4-e^{2x}}} = \arcsin\frac{e^x}{2} + C;$

10. $\int \frac{tg\,7x}{\cos^2 7x}\,dx = \frac{1}{7}\int tg\,7x\,d\,tg\,7x = \frac{tg^2 7x}{14} + C;$

11. $\int \frac{\ln(x-1)}{x-1}\,dx = \int \ln(x-1)\,d\ln(x-1) = \frac{\ln^2(x-1)}{2} + C;$

12. $\int \frac{1}{(1+x^2)\,arctg\,x}\,dx = \int \frac{d\,arctg\,x}{arctg\,x} = \ln|arctg\,x| + C;$

13. $\int x\sqrt{x-1}\,dx = \left| \begin{array}{ll} \sqrt{x-1} = t & dx = 2t\,dt \\ t^2+1 = x & \end{array} \right| = 2\int (t^3 + t)\,dt =$

$$\cancel{\Rightarrow 2\left(\frac{t^4}{4}+\frac{t^2}{2}\right)+C=(x-1)^2}$$

$$=2\int(t^4+t^2)\,dt = 2\frac{t^5}{5}+2\frac{t^3}{3}+C = \frac{2}{5}\sqrt{(x-1)^5}+\frac{2}{3}\sqrt{(x-1)^3}+C$$

14. $$\int\frac{dx}{1+\sqrt{x+3}} = \left|\begin{array}{ll} 1+\sqrt{x+3}=t & dx=2(t-1)\,dt \\ (t-1)^2-3=x & \end{array}\right| =$$

$$=\int\frac{2(t-1)\,dt}{t} = \int\left(2-\frac{2}{t}\right)dt = 2t\,\cancel{\pm\frac{2}{t^2}+C} =$$

$$\cancel{\Rightarrow 2+2\sqrt{x+3}+\frac{2}{(1+\sqrt{x+3})^2}}\;=2t-2\ln|t|+C=$$

$$=2+2\sqrt{x+3}-2\ln\left(1+\sqrt{x+3}\right)+C = 2\left(\sqrt{x+3}-\ln\left(1+\sqrt{x+3}\right)\right)+C$$

15. $$\int\frac{(2x-3)\,dx}{\sqrt[3]{(2x+5)^2}+2\sqrt[3]{2x+5}+4} = \int\frac{(2x-3)\,dx}{(\sqrt[3]{2x+5}+1)^2+3} =$$

$$=\left[\begin{array}{ll} \sqrt[3]{2x+5}=t & dx=\frac{1}{2}(3t^2)\,dt=\frac{3}{2}t^2\,dt \\ \frac{t^3-5}{2}=x & \end{array}\right] =$$

$$=\int\frac{t^3-8}{(t+1)^2+3}\,dt = \int\frac{t^3\,dt}{(t+1)^2+3}-8\int\frac{dt}{(t+1)^2+3} =$$

$$=-\frac{8}{\sqrt{3}}\operatorname{arctg}\frac{t+1}{\sqrt{3}}+\ldots \qquad a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$\int\frac{(\sqrt[3]{2x+5}-2)(2x-3)}{\sqrt[3]{2x+5}^{\,3}-2^3}\,dx = \int\frac{(\sqrt[3]{2x-5}-2)\cancel{(2x-3)}}{\cancel{2x-3}}\,dx =$$

$$=\int\left(\sqrt[3]{2x-5}-2\right)dx = \frac{1}{2}\int\sqrt[3]{2x-5}\,d(2x-5)-2\int dx =$$

$$= \frac{1}{2}\cdot\frac{(2x-5)^{\frac{4}{3}}}{\frac{4}{3}} - 2x + C;$$

16. $\int \sin^3 x \cos x\,dx = \int \sin^3 x\, d\sin x = \frac{\sin^4 x}{4} + C;$

17. $\int \sqrt[3]{\cos^2 x}\,\sin x\,dx = -\int \cos^{\frac{2}{3}} x\, d\cos x = -\frac{\cos^{\frac{5}{3}} x}{\frac{5}{3}} + C;$

18.
a) $\int \frac{\sin x}{1+\cos x}dx = -\int \frac{d(\cos x+1)}{1+\cos x} = -\ln|1+\cos x| + C;$

б) $\int \frac{\sin x}{1+\cos x}dx = -\int \frac{d(1+\cos x)}{1+\cos x} = -\ln|1+\cos x| + C;$

3) інтегрування частинами;

1. $\int x\cos 2x\,dx = \left| \begin{matrix} u = x & du = dx \\ dv = \cos 2x & v = \frac{1}{2}\sin 2x \end{matrix} \right| =$

$= \frac{x}{2}\sin 2x - \frac{1}{2}\int \sin 2x\,dx = \frac{x}{2}\sin 2x + \frac{1}{4}\cos 2x + C;$

2. $\int e^{\frac{x}{3}} x^3 dx = \left| \begin{matrix} u = x^3 & du = 3x^2dx \\ dv = e^{\frac{x}{3}}dx & v = 3e^{\frac{x}{3}} \end{matrix} \right| =$

$= 3x^3 e^{\frac{x}{3}} - 9\int x^2 e^{\frac{x}{3}}dx = \left| \begin{matrix} u = x^2 & du = 2x\,dx \\ dv = e^{\frac{x}{3}}dx & v = 3e^{\frac{x}{3}} \end{matrix} \right| =$

$= 3x^3 e^{\frac{x}{3}} - 27x^2 e^{\frac{x}{3}} + 54\int x e^{\frac{x}{3}}dx = \left| \begin{matrix} u = x & du = dx \\ dv = e^{\frac{x}{3}}dx & v = 3e^{\frac{x}{3}} \end{matrix} \right| =$

$= 3x^3 e^{\frac{x}{3}} - 27x^2 e^{\frac{x}{3}} + 54\cdot 3\cdot x\cdot e^{\frac{x}{3}} - \overset{54}{9} e^{\frac{x}{3}} + C;$

3. $\underbrace{\int \arcsin t\,dt}_{I_1} = \left| \begin{matrix} u = \arcsin t & du = \frac{dt}{\sqrt{1-t^2}} \\ dv = dt & v = t \end{matrix} \right| =$

$= t\arcsin t - \underbrace{\int \frac{t\,dt}{\sqrt{1-t^2}}}_{I_2} = t\arcsin t - I_2;$

$$I_2 = \int \frac{t\,dt}{\sqrt{1-t^2}} = -\frac{1}{2}\int \frac{d(1-t^2)}{\sqrt{1-t^2}} = -\sqrt{1-t^2} + C$$

$$I_1 = t \arcsin t + \sqrt{1-t^2} + C;$$

4. $$\int x \arccos x\,dx = \left| \begin{matrix} u = \arccos x & du = \frac{-dx}{\sqrt{1-x^2}} \\ dv = x\,dx & v = \frac{x^2}{2} \end{matrix} \right| =$$

$$= \frac{x^2}{2}\arccos x + \frac{1}{2}\int \frac{x^2\,dx}{\sqrt{1-x^2}} = \frac{x^2}{2}\arccos x + \frac{1}{2}\int \frac{-x^2-1+1}{\sqrt{1-x^2}}dx$$

$$= \frac{x^2}{2}\arccos x - \frac{1}{2}\left(\int \frac{1-x^2}{\sqrt{1-x^2}}dx - \int \frac{dx}{\sqrt{1-x^2}}dx\right) = \frac{x^2}{2}\arccos x - \frac{1}{2}\int \sqrt{1-x^2}\,dx +$$

$$+ \frac{1}{2}\arcsin x = \frac{x^2}{2}\arccos x + \frac{1}{2}\arcsin x - \frac{1}{2}\underbrace{\int \sqrt{1-x^2}\,dx}$$

$$\frac{1}{2}\delta = \int \sqrt{1-x^2}\,dx = \left| \begin{matrix} x = \sin t \\ dx = \cos t\,dt \end{matrix} \right| = \int \frac{1+\cos 2t}{2}\,dt =$$

$$= \frac{t}{2} + \frac{1}{4}\sin 2t + C = \frac{\arcsin x}{2} + \frac{1}{4}\sin(2\arcsin x) + C$$

$$\int x \arccos x\,dx = \frac{x^2}{2}\arccos x + \frac{1}{2}\arcsin x - \frac{1}{4}\arcsin x -$$

$$\frac{1}{8}\sin(2\arcsin x) + C = \boxed{\frac{x^2}{2}\arccos x + \frac{1}{4}\arcsin x - \frac{x\sqrt{1-x^2}}{4} + C}$$

$$\sin(2\arcsin x) = 2\cos(\arcsin x)\sin(\arcsin x) =$$

$$= 2\sqrt{1-x^2}\cdot x;$$

5. $$\int x^2 \ln x\,dx = \left| \begin{matrix} u = \ln x & du = \frac{dx}{x} \\ dv = x^2 dx & v = \frac{x^3}{3} \end{matrix} \right| =$$

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2\,dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C;$$

6. $$\int x^2 \sin x\,dx = \left| \begin{matrix} u = x^2 & du = 2x\,dx \\ dv = \sin x\,dx & v = -\cos x \end{matrix} \right| =$$

$$= -x^2\cos x + 2\int x\cos x\,dx = \left| \begin{matrix} u = x & du = dx \\ dv = \cos x\,dx & v = \sin x \end{matrix} \right| =$$

$= -x^2\cos x + 2x\sin x - \cos x + C;$

7. $\int \frac{\ln x}{x^2}\,dx = \left| \begin{matrix} u=\ln x & du=\frac{dx}{x} \\ dv\ \ \frac{dx}{x^2} & v=-\frac{1}{x} \end{matrix} \right| =$

$= -\frac{\ln x}{x} + \int \frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x} + C;$

8. $\int \cos x \ln \sin x\,dx = \int \ln \sin x\, d\sin x = \left| \text{let } \sin x = t \right| =$

$= \int \ln t\,dt = t\ln t - t + C = \sin x(\ln \sin x - 1) + C;$

9. $\int \ln^2 x\,dx = \left| \begin{matrix} \ln^2 x = u & du = \frac{2\ln x}{x}dx \\ dv = dx & v = x \end{matrix} \right| =$

$= x\ln^2 x - 2\int \ln x\,dx = x\ln^2 x - 2x(\ln x - 1) + C;$

10. $\int \frac{\cos 2x}{e^{3x}}\,dx = \left| \begin{matrix} u = \frac{1}{e^{3x}} = e^{-3x} & du = -3e^{-3x}dx \\ dv = \cos 2x\,dx & v = \frac{1}{2}\sin 2x \end{matrix} \right| =$

$= \frac{1}{2}e^{-3x}\sin 2x + \frac{3}{2}\int e^{-3x}\sin 2x\,dx = \left| \begin{matrix} u = e^{-3x} & du = -3e^{-3x}dx \\ dv = \sin 2x\,dx & v = -\frac{1}{2}\cos 2x \end{matrix} \right|$

$= \frac{1}{2}e^{-3x}\sin 2x + \frac{3}{2}\left(-\frac{1}{2}e^{-3x}\cos 2x - \frac{3}{2}\underbrace{\int e^{-3x}\cos 2x\,dx}_{I}\right) =$

$I = \underbrace{\frac{1}{2}e^{-3x}\sin 2x - \frac{3}{4}e^{-3x}\cos 2x}_{F(x)} - \frac{9}{4}I$, then:

$I = F(x) - \frac{3}{2}I \Rightarrow I + \frac{9}{4}I = F(x) \Rightarrow I = \frac{4}{13}F(x);$

$\int \frac{\cos 2x}{e^{3x}}\,dx = \frac{2}{13}e^{-3x}\sin 2x - \frac{3}{13}e^{-3x}\cos 2x + C;$

11. $\int e^{2x}\sin 5x\,dx = \left| \begin{matrix} u = e^{2x} & du = 2e^{2x}dx \\ dv = \sin 5x\,dx & v = -\frac{1}{5}\cos 5x \end{matrix} \right| =$

$= -\frac{1}{5}e^{2x}\cos 5x + \frac{2}{5}\int e^{2x}\cos 5x\,dx = \left| \begin{matrix} u = e^{2x} & du = 2e^{2x}dx \\ dv = \cos 5x\,dx & v = \frac{1}{5}\sin 5x \end{matrix} \right|$

$\frac{29}{25}I = F(x)$

$= -\frac{1}{5}e^{2x}\cos 5x + \frac{2}{25}e^{2x}\sin 5x - \frac{4}{25}\int e^{2x}\sin 5x\,dx;$

$\int e^{2x}\sin 5x\,dx = -\frac{5}{29}e^{2x}\cos 5x + \frac{2}{29}e^{2x}\sin 5x + C;$

12. $\int (x+2)e^{-x}dx = \left| \begin{matrix} u = x+2 & du = dx \\ dv = e^{-x}dx & v = -e^{-x} \end{matrix} \right| =$

$= -(x+2)e^{-x} - e^{-x} + C;$

5)

1. $\int \frac{5}{x^2-4x+3}dx = 5\int \frac{d(x-2)}{(x-2)^2-1} = \frac{5}{2}\ln\left|\frac{x-3}{x-1}\right| + C$

2. $\int \frac{x+1}{x^2+x-6}dx = \int \frac{\frac{1}{2}d(x^2+x-6)+\frac{1}{2}dx}{x^2+x-6} =$

$\frac{24}{4}+\frac{1}{4} = \frac{25}{4} = \left(\frac{5}{2}\right)^2$

$= \frac{1}{2}\ln|x^2+x-6| + \frac{1}{2}\int \frac{dx}{(x+\frac{1}{2})^2-(6+\frac{1}{4})} =$

$= \frac{1}{2}\ln|x^2+x-6| + \frac{1}{2}\cdot\frac{2}{5}\cdot\frac{1}{2}\ln\left|\frac{x-2}{x+3}\right| + C$

$\frac{1}{2}-\frac{5}{2} = -2$

$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$

$\frac{1}{2}+\frac{5}{2} = 3$

3. $\int \frac{3x^3-2x-3}{x^3-x}dx = \int \frac{3x^3-3x+x-3}{x^3-x}dx =$

$= \int\left(3 + \frac{x-3}{x^3-x}\right)dx = 3x + \int\left(\frac{1}{x^2-1} - \frac{3}{x^3-x}\right)dx =$

$= 3x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| - 3\int\frac{dx}{x(x^2-1)} \Rightarrow$ Let $\int\frac{dx}{x(x^2-1)} = \delta$

$$\delta = \int \frac{dx}{x(x^2-1)} = \left| \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} = \frac{1}{x(x-1)(x+1)} \right.$$

$$\left. \frac{A(x-1)(x+1) + Bx(x+1) + Cx(x-1)}{x(x-1)(x+1)} \right|$$

$$\Rightarrow \quad A(x^2-1) + B(x^2+x) + C(x^2-x) = 1;$$

$$x^2(A+B+C) + x(B-C) - A = 1;$$

$$\begin{cases} A+B+C=0 \\ B-C=0 \\ -A=1 \end{cases}, \quad \begin{cases} B+C-1=0 \\ B-C=0 \end{cases} \Rightarrow 2B = 1 \Rightarrow B = \frac{1}{2};$$

then $C = \frac{1}{2}$.

$$\delta = \int \left(-\frac{1}{x} + \frac{\frac{1}{2}}{x-1} + \frac{\frac{1}{2}}{x+1}\right) dx = -\ln x + \frac{1}{2}\ln|x^2-1| + C$$

$$\int \frac{3x^3-2x-3}{x^3-x} dx = 3x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + 3\ln x - \frac{3}{2}\ln|x^2-1| + C$$

4. $$\int \frac{5x\,dx}{2+x^2} = \int \frac{\frac{5}{2}\,d(x^2+2)}{x^2+2} = \frac{5}{2}\ln|x^2+2| + C;$$

5. $$\int \frac{dx}{x^2-4x+5} = \int \frac{d(x-2)}{(x-2)^2+1} = \operatorname{arctg}(x-2) + C;$$

6. $$\int \frac{x}{x^2+8x+20} dx = \int \frac{\frac{1}{2}d(x^2+8x+20) - 4dx}{x^2+8x+20} =$$

$$= \frac{1}{2}\ln|x^2+8x+20| - 4\int \frac{d(x+4)}{(x+4)^2+4} = \frac{1}{2}\ln|x^2+8x+20| - 2\operatorname{arctg}\frac{x+4}{2} + C$$

7. $$\int \frac{(x+5)\,dx}{x^2-2x+5} = \int \frac{\frac{1}{2}d(x^2-2x+5) + 6\,d(x-1)}{(x-1)^2+4} =$$

$\frac{1}{2}\ln|x^2-2x+5| + 3\,\mathrm{arctg}\,\frac{x-1}{2} + C;$

8. $\int \frac{5x^3+10x^2+x+4}{x^2+2x+10}\,dx = \left[\begin{array}{l} 5x(x^2+2x+10)- \\ 49x+4 = \\ 5x^3+10x^2+x+4 \end{array}\right.$

$= \int \frac{5x(x^2+2x+10)-49x+4}{x^2+2x+10}\,dx = \frac{5}{2}x^2 + \int \frac{-49x+4}{x^2+2x+10}\,dx$

$= \frac{5}{2}x^2 + \int \frac{-\frac{49}{2}\,d(x^2+2x+10)+53\,d(x+1)}{(x+1)^2+9} =$

$= \frac{5}{2}x^2 - \frac{49}{2}\ln|x^2+2x+10| + \frac{53}{3}\,\mathrm{arctg}\,\frac{x+1}{3} + C;$

9. $\int \sin^2 x\,dx = \int \frac{1-\cancel{2}\cos 2x}{2}\,dx = \frac{x}{2} - \frac{1}{4}\sin 2x + C$

$\sin^2 x = \frac{1-\cancel{2}\cos 2x}{2} \Rightarrow \frac{1-\cancel{2}(\cos^2 x-\sin^2 x)}{2} = \frac{2\sin^2 x}{2}$

10. $\int \sin 2x \sin 4x\,dx = \int (2\sin x\cos x)(2\sin 2x\cos 2x)\,dx$

$= \int 2\sin x\cos x \cdot 2\sin x\cos x\,\underbrace{(\cos^2 x-\sin^2 x)}_{\cos 2x}\,dx =$

$= 4\int \sin^2 x\cos^2 x(\cos^2 x-\sin^2 x)\,dx =$

$\left(\frac{1-\cos 2x}{2}\cdot\frac{1+\cos 2x}{2}\right)\underbrace{\left(\frac{1+\cos 2x}{2}-\frac{1-\cos 2x}{2}\right)}_{\cos 2x} =$

$= \frac{1-\cos^2 2x}{4}\cos 2x$

$\frac{\cos 2x-\cos^3 2x}{4} = \frac{\cos 2x}{4} - \frac{\cos^3 2x}{4}$

$\int \sin 2x\sin 4x\,dx = \int \frac{\cos 2x}{2}\,dx - \int \frac{\cos 6x}{2}\,dx$

$$\sin 2x \sin 4x = \frac{1}{2}(\cos 2x - \cos 6x).$$

$$\int \sin 2x \sin 4x \, dx = \frac{1}{4}\sin 2x - \frac{1}{12}\sin 6x + C$$

11. $$\int \frac{dx}{\sin x + \cos x} = \left| \begin{array}{l} \cancel{2\,tg\,x = x \quad t = arctg\frac{x}{2}} \\ x = 2\,arctg\,t \qquad dx = \frac{2\,dt}{1+t^2} \end{array} \right| =$$

$$= \int \frac{\frac{2}{1+t^2}dt}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} = -2\int \frac{d(t-1)}{(t-1)^2-2} = -\frac{\sqrt{2}}{2}\ln\left|\frac{tg\frac{x}{2} - \ldots}{tg\frac{x}{2} \ldots}\right|$$

12. $$\int \frac{(1+\sqrt{x})^2}{\sqrt[3]{x}}dx = \int \frac{x+2\sqrt{x}+1}{\sqrt[3]{x}}dx = \int \left(x^{\frac{2}{3}} + 2x^{\frac{1}{6}} + x^{-\frac{1}{3}}\right)dx$$

$$= \frac{3}{5}x^{\frac{5}{3}} + 2\frac{x^{\frac{7}{6}}}{\frac{7}{6}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C;$$

13. $$\int \frac{dx}{\sqrt{3-2x-x^2}} = \int \frac{d(x+1)}{\sqrt{4-(x+1)^2}} = \arcsin\frac{x+1}{2} + C;$$

14. $$\int \frac{dx}{\sqrt{x^2+2x+5}} = \int \frac{d(x+1)}{\sqrt{(x+1)^2+4}} = \ln\left|x+1+\sqrt{x^2+2x+5}\right| + C$$

15. $$\int \frac{\sqrt{2x+1} + \sqrt[3]{2x+1}}{\sqrt{2x+1}}dx = \int dx + \int (2x+1)^{-\frac{1}{6}}dx =$$

$$= x + \frac{6}{5}(2x+1)^{\frac{5}{6}} + C;$$

11. $$\int \frac{dx}{\sin x + \cos x} = \left| \begin{array}{l} \frac{x}{2} = arctg\,t \Rightarrow t = tg\frac{x}{2} \\ \sin(2\,arctg\,t) = 2\sin(arctg\,t)\cos(arctg\,t) = \\ = \frac{2t}{1+t^2} \\ \cos(2\,arctg\,t) = \frac{1-t^2}{1+t^2} \\ \cos 2\alpha = \cos^2\alpha - \sin^2\alpha \end{array} \right| =$$

$$= \int \frac{\frac{2}{1+t^2}\,dt}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} = -2\int \frac{d(t-1)}{(t-1)^2-2} = -\frac{1}{\sqrt{2}} \ln\left|\frac{t-1-\sqrt{2}}{t-1+\sqrt{2}}\right| + C =$$

$$= -\frac{1}{\sqrt{2}} \ln\left|\frac{tg\frac{x}{2}-1-\sqrt{2}}{tg\frac{x}{2}-1+\sqrt{2}}\right| + C;$$

$< \sin x = \pm\sqrt{1-\cos^2 x}$

$< \cos x = \pm\sqrt{1-\sin^2 x}$

$1 + tg^2 x = \frac{1}{\cos^2 x}$

$1 + ctg^2 x = \frac{1}{\sin^2 x}$

$\Downarrow$

$\sin x = \frac{1}{\sqrt{1+ctg^2 x}}$

$\cos x = \frac{1}{\sqrt{1+tg^2 x}}$

$|\sin(\arcsin x) = x$

$|\cos(\arccos x) = x$

$|\sin \arccos x = \sqrt{1-x^2}$

$|\cos \arcsin x = \sqrt{1-x^2}$

$< tg\, x = \frac{\sqrt{1-\cos^2 x}}{\sqrt{1-\sin^2 x}}$

$< ctg\, x = \frac{\sqrt{1-\sin^2 x}}{\sqrt{1-\cos^2 x}}$

$\Rightarrow tg^2 x = \frac{1}{ctg^2 x}$

$ctg^2 x = \frac{1}{tg^2 x}$

$|tg \arcsin x = \frac{x}{\sqrt{1-x^2}}$

$|tg \arccos x = \frac{\sqrt{1-x^2}}{x}$

$|ctg \arcsin x = \frac{\sqrt{1-x^2}}{x}$

$|ctg \arccos x = \frac{x}{\sqrt{1-x^2}}$

$\Downarrow$

$$\arcsin x = arctg\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\sin(\arcsin x) = \sin\left(arctg\left(\frac{x}{\sqrt{1-x^2}}\right)\right)$$

$$x = ?\quad tg \arcsin x \cdot \sqrt{1-x^2}$$

$$\sin x = tg\, x \cos x \Rightarrow \left[\frac{\sin x}{\cos x}\cos x\right]$$

$$\cos x = \frac{1}{tg\, x}\sin x$$