N 22

Задача 1.

x_i	-3	-1	1	4
p_i	p	$\frac{3}{2}p$	$\frac{5}{2}p$	$3p$

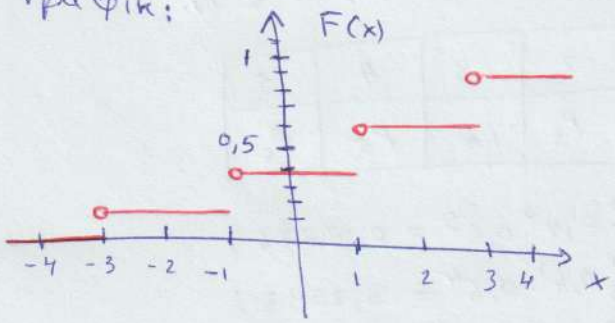
$\because \sum_i p_i = 1$

$\therefore 8p = 1 \Rightarrow p = \frac{1}{8}$

x_i	-3	-1	1	4
p_i	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{3}{8}$

$$F(x) = \begin{cases} 0, & x \le -3 \\ 1/8, & -3 < x \le -1 \\ 5/16, & -1 < x \le 1 \\ 10/16, & 1 < x \le 4 \\ 1, & x > 4 \end{cases}$$

График:



$D(x) = M(X^2) - M^2(X) = 7,625 - 1,25^2 = 6,0625$

$P(x < 2) = F(2) - F(-\infty) = \frac{5}{8} = 0,625$

Завдання 2.

$$f(x) = \frac{a}{1+9x^2}, \quad x \in (0; \infty)$$

$$1 + 9x^2 = 1 + (3x)^2$$

$$\therefore \quad dx = \frac{1}{3} d(3x) \Rightarrow \frac{1}{3} \int_0^{\infty} \frac{a\, d(3x)}{1+9x^2} \Rightarrow \frac{a}{3} \operatorname{arctan}(3x)$$

$$I = \frac{a}{3} \operatorname{arctan}(3x) \Big|_0^{\infty} = \frac{a}{3} \cdot \frac{\pi}{2} = 1 \Rightarrow a = \frac{6}{\pi}$$

$$\therefore \quad f(x) = \frac{6}{\pi} \cdot \frac{1}{1+9x^2}, \quad x \in (0; \infty)$$

Завдання 3.

$n = 5$, $p = 0{,}4$, $q = 0{,}6$

$$P_n^m = C_n^m p^m q^{n-m}, \quad n \geq m \in \mathbb{N}$$

X	0	1	2	3	4	5
P_i	P_1	P_2	P_3	P_4	P_5	P_6

$P_1 = P_5^0 = C_5^0 0{,}4^0 \cdot 0{,}6^5 = 0{,}07776$;

$P_2 = P_5^1 = C_5^1 0{,}4^1 \cdot 0{,}6^4 = 0{,}2592$;

$P_3 = P_5^2 = C_5^2 0{,}4^2 \cdot 0{,}6^3 = 0{,}3456$;

$P_4 = 0{,}2304$; $P_5 = 0{,}0768$; $P_6 = 0{,}01024$

X	0	1	2	3	4	5	Σ
P_i	0,07776	0,2592	0,3456	0,2304	0,0768	0,01024	1

Завдання 4

$$f(x) = \begin{cases} 0, & x \notin (0;1] \\ a x^{\frac{1}{3}}, & x \in (0;1] \end{cases} \qquad Y = 3x^2$$

$$a = \left(\int_0^1 x^{\frac{1}{3}} dx \right)^{-1} = \left(\frac{3}{4} x^{\frac{4}{3}} \Big|_0^1 \right)^{-1} = \frac{4}{3}$$

оскільки $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\psi(y) = \frac{\sqrt{y}}{\sqrt{3}}, \quad g(y) = f(\psi(y)) \cdot |\psi'(y)|$$

$$\psi'(y) = \frac{1}{2\sqrt{3}} \cdot \frac{1}{\sqrt{y}}$$

$$\therefore g(y) = \begin{cases} 0, & y \notin (0;3] \\ \frac{4}{3} \cdot \left(\frac{y}{3}\right)^{\frac{1}{6}} \cdot \frac{1}{2\sqrt{3}} \cdot \frac{1}{\sqrt{y}}, & y \in (0;3] \end{cases}$$

$$y^{\frac{1}{6} - \frac{1}{2}} = y^{\frac{1}{6} - \frac{3}{6}} = y^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{y}}$$

$$3^{\frac{1}{6} + 1 + \frac{1}{2}} = 3^{\frac{7}{6} + \frac{3}{6}} = 3^{\frac{5}{3}}$$

Перепишемо:

$$g(y) = \begin{cases} 0, & y \notin (0;3] \\ 2 \cdot 3^{-\frac{5}{3}} \cdot y^{-\frac{1}{3}}, & y \in (0;3] \end{cases}$$

Завдання 5.

Y_i \ X_j	1	3	4
2	0,2	0,2	0,15
5	0,25	0,1	P

$$1 - \sum_{i,j} P_{ij} \Big|_{ij \neq 23} = P$$

$\therefore P = 0,1$

X_j	1	3	4
P_j	0,45	0,3	0,25

Y_i	2	5
P_i	0,55	0,45

$M(X) = \sum_j X_j P_j = 2,35$ $\quad M(Y) = \sum_i Y_i P_i = 3,35$

$D(X) = M(X^2) - M^2(X)$ $\quad D(Y) = M(Y^2) - M^2(Y)$

$\therefore D(X) = 1,6275$ $\quad \therefore D(Y) = 2,2275$

$$M(XY) = \sum_i \sum_j X_j Y_i P_{ij} = 7,55$$

Оскільки $r_{XY} = \dfrac{M(XY) - M(X)M(Y)}{\sqrt{D(X)D(Y)}}$

Звідси маємо: $r_{XY} = \dfrac{7,55 - 2,35 \cdot 3,35}{\sqrt{1,6275 \cdot 2,2275}} \approx -0,17$