with variables $y \in \mathbf{R}^n$, $t \in \mathbf{R}$.

Power assignment in a wireless communication system. We consider n transmitters with powers $p_1, \ldots, p_n \geq 0$, transmitting to n receivers. These powers are the optimization variables in the problem. We let $G \in \mathbf{R}^{n \times n}$ denote the matrix of *path gains* from the transmitters to the receivers; $G_{ij} \geq 0$ is the path gain from transmitter j to receiver i. The *signal power* at receiver i is then $S_i = G_{ii} p_i$, and the *interference power* at receiver i is $I_i = \sum_{k \neq i} G_{ik} p_k$. The *signal to interference plus noise ratio*, denoted SINR, at receiver i, is given by $S_i/(I_i + \sigma_i)$, where $\sigma_i > 0$ is the (self-) noise power in receiver i. The objective in the problem is to maximize the minimum SINR ratio, over all receivers, *i.e.*, to maximize

$$\min_{i=1,\ldots,n} \frac{S_i}{I_i + \sigma_i}.$$

There are a number of constraints on the powers that must be satisfied, in addition to the obvious one $p_i \geq 0$. The first is a maximum allowable power for each transmitter, *i.e.*, $p_i \leq P_i^{\max}$, where $P_i^{\max} > 0$ is given. In addition, the transmitters are partitioned into groups, with each group sharing the same power supply, so there is a total power constraint for each group of transmitter powers. More precisely, we have subsets $K_1, \ldots, K_m$ of $\{1, \ldots, n\}$ with $K_1 \cup \cdots \cup K_m = \{1, \ldots, n\}$, and $K_j \cap K_l = 0$ if $j \neq l$. For each group K_l, the total associated transmitter power cannot exceed $P_l^{\mathrm{gp}} > 0$:

$$\sum_{k \in K_l} p_k \leq P_l^{\mathrm{gp}}, \quad l = 1, \ldots, m.$$

Finally, we have a limit $P_k^{\mathrm{rc}} > 0$ on the total received power at each receiver:

$$\sum_{k=1}^{n} G_{ik} p_k \leq P_i^{\mathrm{rc}}, \quad i = 1, \ldots, n.$$

(This constraint reflects the fact that the receivers will saturate if the total received power is too large.)

Formulate the SINR maximization problem as a generalized linear-fractional program.

SINR maximization. Solve the following instance of problem 4.20: We have $n = 5$ transmitters, grouped into two groups: $\{1, 2\}$ and $\{3, 4, 5\}$. The maximum power for each transmitter is 3, the total power limit for the first group is 4, and the total power limit for the second group is 6. The noise σ is equal to 0.5 and the limit on total received power is 5 for each receiver. Finally, the path gain matrix is given by

$$G = \begin{bmatrix} 1.0 & 0.1 & 0.2 & 0.1 & 0.0 \\ 0.1 & 1.0 & 0.1 & 0.1 & 0.0 \\ 0.2 & 0.1 & 2.0 & 0.2 & 0.2 \\ 0.1 & 0.1 & 0.2 & 1.0 & 0.1 \\ 0.0 & 0.0 & 0.2 & 0.1 & 1.0 \end{bmatrix}.$$

Find the transmitter powers $p_1, \ldots, p_5$ that maximize the minimum SINR ratio over all receivers. Also report the maximum SINR value. Solving the problem to an accuracy of 0.05 (in SINR) is fine.

Hint. When implementing a bisection method in CVX, you will need to check feasibility of a convex problem. You can do this using `strcmpi(cvx_status, 'Solved')`.