Oppenheimer Assignment 2

Sumanth N R

Contents

1 **Question 2.40** 1

2 **Solution** 1

***Abstract*—Oppenheim and Schafer Discrete Time Signal Processing Prentise Hall 2nd Edition, Solution for Question 2.40**

1 Question 2.40

Consider a linear time invariant system with impulse response

$$h[n] = \left(\frac{j}{2}\right)^n u[n] \qquad \text{where } j = \sqrt{-1} \tag{1.1}$$

Determine the steady-state response, i.e., the response for large n, to the excitation

$$x[n] = \cos(n\pi)\, u[n] \tag{1.2}$$

2 Solution

We know that,

$$\begin{aligned} y[n] &= h[n] * x[n] = x[n] * h[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k] \\ &= \sum_{k=-\infty}^{\infty} \left(\frac{j}{2}\right)^k u[k] \cos((n-k)\pi)\, u[n-k] \\ &= \sum_{k=0}^{n} \left(\frac{j}{2}\right)^k (-1)^{n-k} \\ &= (-1)^n \sum_{k=0}^{n} \left(\frac{j}{-2}\right)^k \end{aligned}$$

$$y[n] = (-1)^n \left(\frac{1-\left(-\frac{j}{2}\right)^{n+1}}{1+\frac{j}{2}}\right) \tag{2.1}$$

For large n, we have,

$$n \to \infty \implies \left(-\frac{j}{2}\right)^{n+1} \to 0 \tag{2.2}$$

Using (2.2) on (2.1), we get,

$$y[n] = (-1)^n \left(\frac{1}{1+\frac{j}{2}}\right)$$

$$\implies y[n] = \frac{\cos(n\pi)}{1+\frac{j}{2}} \tag{2.3}$$

This, is the steady state response of the system to the excitation given by (2.3).