Penalty Curve Derivation

T. Hulsmans, A. Radulescu, R. Haggins

Sigmadex Foundation

August 2021

Abstract

The Sigmadex Yield Farm Penalty Curve is derived, users stake liquidity points in a yield farm over a set timeframe which are then penalized for premature withdrawal by an amount provided by the curve in exchange for a reward at the end. Estimators for a modified CAPM form asset specific discount rates, which inform the NPV of a liquidity pool. Portions of this NPV staked in yield farms forms the basis of the penalty curve - which undergoes convex/concavification based on the harmonic mean of the relative risks of the assets.

1 Introduction

Sigmadex provides two unique features that differentiates itself from the rest of the ecosystem; a yield farm that timelocks liquidity to a penalty curve and a reward system the compensates users for farming yield against this curve. The subject of this document concerns the mechanics of the penalty curve - specifically how Sigmadex builds this curve. We begin through the quantification of individual assets discount rate and risk betas through a modified Capital Asset Pricing Model. This followed by using these values to calculate the "Net Present Value" of a yield farm, then portioning off sections of this curve by staked liquidity points to form a base penalty curve. This function then undergoes convex/concavification based on the weighted risks of the assets to form the penalty curve.

2 The Sigmadex Risk Index

To begin formulating the penalty curve, Sigmadex requires estimators for deriving the Net Present Value of the assets in that yield farm. Calculating these values is the job of the Sigmadex risk engine, which creates the Sigmadex risk index. Specifically, Sigmadex utilizes a modified capital asset pricing model.

$$r_a = r_{riskfree} + \beta(r_{market} - r_{riskfree}) \tag{1}$$

where $r_{riskfree}$ is the risk free interest rate derived from the 10 year treasury bond, r_{market} is the average rate of return for the market of cryptocurrency and β is

$$\beta = \frac{CoVar(r_a, r_{market})}{Var(r_{market})} \tag{2}$$

the relative volatility of an asset compared to the market volatility. These factors derive the discount rate for a particular asset r_a.

3 The Value of a Yield Farm

To determine the value of a Yield Farm, take a pool of assets of A and B, (i.e. ETH and USDT). When adding equivalent amounts to the pool, one receives in return Liquidity Points (LP) which denotes ones share of the pool. Liquidity points can then be staked into yield farms, to incentivize liquidity accumulation for extra rewards in the form of a platform utility token Y, for example SDEX.

To determine the value of a yield farm we can use a discounted cash flow analysis in perpetuity.

$$NPV_{farm} = A + B + \sum_{n=1}^{\infty} \frac{D}{(1+r)^n} \tag{3}$$

Where r is the discount rate derived in the prior section and D is the dividend per period. Furthermore, let the dividend be the expected fees earned from swappers and the amount of yield earned from staking.

$$NPV_{farm} = NPV_A + NPV_B + NPV_Y = A + B + \sum_{n=1}^{\infty} \frac{tx_{fee}^A}{(1+r_A)^n} + \sum_{n=1}^{\infty} \frac{tx_{fee}^B}{(1+r_B)^n} + \sum_{n=1}^{\infty} \frac{Y}{(1+r_Y)^n} \tag{4}$$

Where the tx fees are the amount earned from swaps on the liquidity position and Y is the amount of yield earned from staking. In other words the value of a yield farm is equivalent to its total assets plus the discounted cash flows arising from accumulating transaction fees and utility tokens.

Equivalently we can express the total value of the farm as the sum of individual liquidity positions

$$NPV_{farm} = \sum_{j=1}^{k} NPV_{farm}^j \tag{5}$$

Where k is the total amount of liquidity positions, and j is the individuals liquidity position. The NPV of a persons share is the same as the farm, with each variable now denoting the proportion of what they've contributed

$$NPV_{farm}^j = NPV_a^j + NPV_b^j + NPV_y^j = \frac{a_j}{A} + \frac{b_j}{B} + \sum_{n=1}^{\infty} \frac{tx_{fee}^a}{(1+r_A)^n} + \sum_{n=1}^{\infty} \frac{tx_{fee}^b}{(1+r_B)^n} + \sum_{n=1}^{\infty} \frac{Y \frac{lp_{staked}}{lp}}{(1+r_Y)^n} \tag{6}$$

where

$$\sum_{j=1}^{k} a_j = A, \sum_{j=1}^{k} b_j = B \tag{7}$$

Where lower case letters a and b denote the j^{th} liquidity position in the pool.

3.1 Timelocking

A core feature in Sigmadex is that a user can timelock their liquidity. Intuitively the benefits of creating a commitment scheme that enforces a liquidity provider to stake for a certain period of time may seem benign. However they become a lot more salient in considering low volume, new crypto pairs. Generally the game of decentralized exchanges and yield farming are a game of virtuous cycles. Volume reduces slippage which increases capital efficiency attracting traders generating fees that attracts more volume. To incentivize this positive cycle, yield farms offer returns beyond trading fees by offering platform tokens over time. While these approaches are generally successful in their stated objective, temporal volatility of the underlying assets, and large changes in APYs can trigger capital outflow resonant cascades. To provide market stability in bootstrapping pairs, Sigmadex offers an instrument that reduces the liquidity (defined as ease of trade) of the pair over a period of time in exchange for greater rewards at the end by offering the ability to timelock a stake.

When a user wishes to stake their liquidity points in the yield farm to earn extra alpha in terms of SDEX tokens, they are prompted for how long they wish to commit these liquidity points. If the current time is denoted t then the time staked will be t_n. In doing so they expose themselves to a penalty curve for a reward at the end of that term.

3.2 Linear Penalty Curve

In essence this liquidity provider becomes responsible for maintaining the NPV^j_{farm} over the period they commit to. If for example one chooses to timelock for 3 years. Their commitment to the farm is then:

$$NPV^j_{farm} = NPV^j_a + NPV^j_b + NPV^j_y = \frac{a_j}{A} + \frac{b_j}{B} + \sum_{n=1}^{3} \frac{tx^a_{fee}}{(1+r_A)^n} + \sum_{n=1}^{3} \frac{tx^b_{fee}}{(1+r_B)^n} \tag{8}$$

A user commits a_j and b_j into the pool, then stakes these Liquidity points, they become 'on the hook' for the NPV of the transaction fees. The penalty function then, dockets

$$P(t_n) = \sum_{n=t_n}^{3} \frac{tx^a_{fee}}{(1+r_A)^n} + \sum_{n=1}^{3} \frac{tx^b_{fee}}{(1+r_B)^n} \tag{9}$$

$$P(t_n) \mapsto lp_{penalty}, lp_j - lp_{penalty} = refund \tag{10}$$

From the liquidity points staked in the yield farm. At the end of the vestige period the farmer earns the yield

$$reward = lp_j + \sum_{n=1}^{3} \frac{y^j}{(1+r_Y)^n} + \epsilon \tag{11}$$

where ϵ is a non tangible gift, and the sum is the total SDEX tokens earned.

4 Convex/Concavification

When Sigmadex derives the discount rates of the various cryptocurrencies through a modified CAPM model generates relative risks of each asset compared to each other called Betas. This collection is the Sigmadex risk index. We use these betas to further strengthen or loosen the penalty curve by having them inform the shape of the curve. Higher risk pairs are given a convex function so that the penalty decreases slowly over time, while low risk pairs are given a concave function over time. Mathematically, the mean of the two beta's map to a parameter A, that controls the degree of curvature, up to the point d_{limit}. Beginning with the base curve the y and x intercepts, y_0 and t_0, are taken to provide a linearized and continuous base penalty function:

$$p\left(t\right)=\sum_{n=t}^{t_f}\left(\frac{t_{usdt}}{\left(1\ +\ r_{usdt}\right)^n}\right)+\sum_{n=t}^{t_f}\left(\frac{t_{eth}}{\left(1\ +\ r_{eth}\right)^n}\right) \tag{12}$$

the y intercept is found

$$y_0\ =\ p\left(0\right) \tag{13}$$

The slope is found

$$m=\frac{-y_0}{t_0+1} \tag{14}$$

Sigmadex utilizes the wave equation for this process, l pegs half a wave length to the intercepts

$$l=2\sqrt{y_0^2+\left(t_0+1\right)^2} \tag{15}$$

The wave equations is rotated to the slope by s

$$s=\arctan\left(m\right) \tag{16}$$

d is the derivative of the wave equation at 0, d_limit undergoes the same rotation and translation to ensure the amplitude doesn't cause the function to go into negative penalty space.

$$d_{limit}=\frac{l(-y_0)cos(s)-sin(s)}{2\pi t_0(0*sin(s)+(0-(t_0+1))cos(s))} \tag{17}$$

Average risk

$$\beta_{average}=\frac{\beta_a+\beta_b}{2} \tag{18}$$

The amplitude of the wave equation is the parameter that informs the degree of concave/convexification

$$A=2d_{limit}(\frac{1}{1+e^{\beta average-1}}-0.5) \tag{19}$$

In the same intuition of a pulled guitar string this equation is then stretched by A to the desired curve:

$$y=(mt+y_0) \tag{20}$$

$$f\left(t\right)\ =\ At\cdot\sin\left(\frac{2\pi}{l}\left(t\right)\right) \tag{21}$$

$$\left(y-y_0\right))\cos\left(s\right)\ -\ t\sin\left(s\right)=f\left(y\sin\left(s\right)+\left(t-\left(t_0+1\right)\right)\cos\left(s\right)\right) \tag{22}$$

Figure 1: A confexified penalty curve, A¡0, riskier tokens penalties decrease less rapidly

5 Example

A Liquidity pool for ETH-USDT with 0.3% transaction fees is created. The current price of Ethereum in USDT is $3000 . 0.0667 ETH and 200 USDT was provided to the pool. Let's say 1 Liquidity Point was emitted in return to symbolize the ownership of the pool. The expected fees earned by swappers per day is $2.

The user then takes that liquidity point and deposits it in a yield farm. Let us assume the following parameters:

Figure 2: A concavified penalty curve, A¿0, stabler tokens penalties decrease more rapidly

Value	Amount	Description
a_{usdt}	200	Amount of USDT in Pool (in USD)
a_{eth}	200	Amount of ETH in Pool (in USD)
lp	1	Liquidity Points staked
tx_{eth}	1	fees earned from swaps per period ETH in (USD)
tx_{usdt}	1	fees earned from swaps per period USDT in (USD)
r_{usdt}	0.0134%	discount rate USDT per period (daily)
r_{eth}	0.02%	discount rate ETH per period (daily)
t_n	90	days timelocked
β_{eth}	1	Relative risk of ETH compared to market
β_{usdt}	1/39	Relative risk of USDT compared to market

The penalty function takes on the character

$$P(t_n) = \sum_{n=t_n}^{90} \frac{1}{(1+0.0002)^n} + \sum_{n=1}^{90} \frac{1}{(1+0.000134)^n} \tag{23}$$

We then find the y and x intercepts in order to find the base penalty curve:

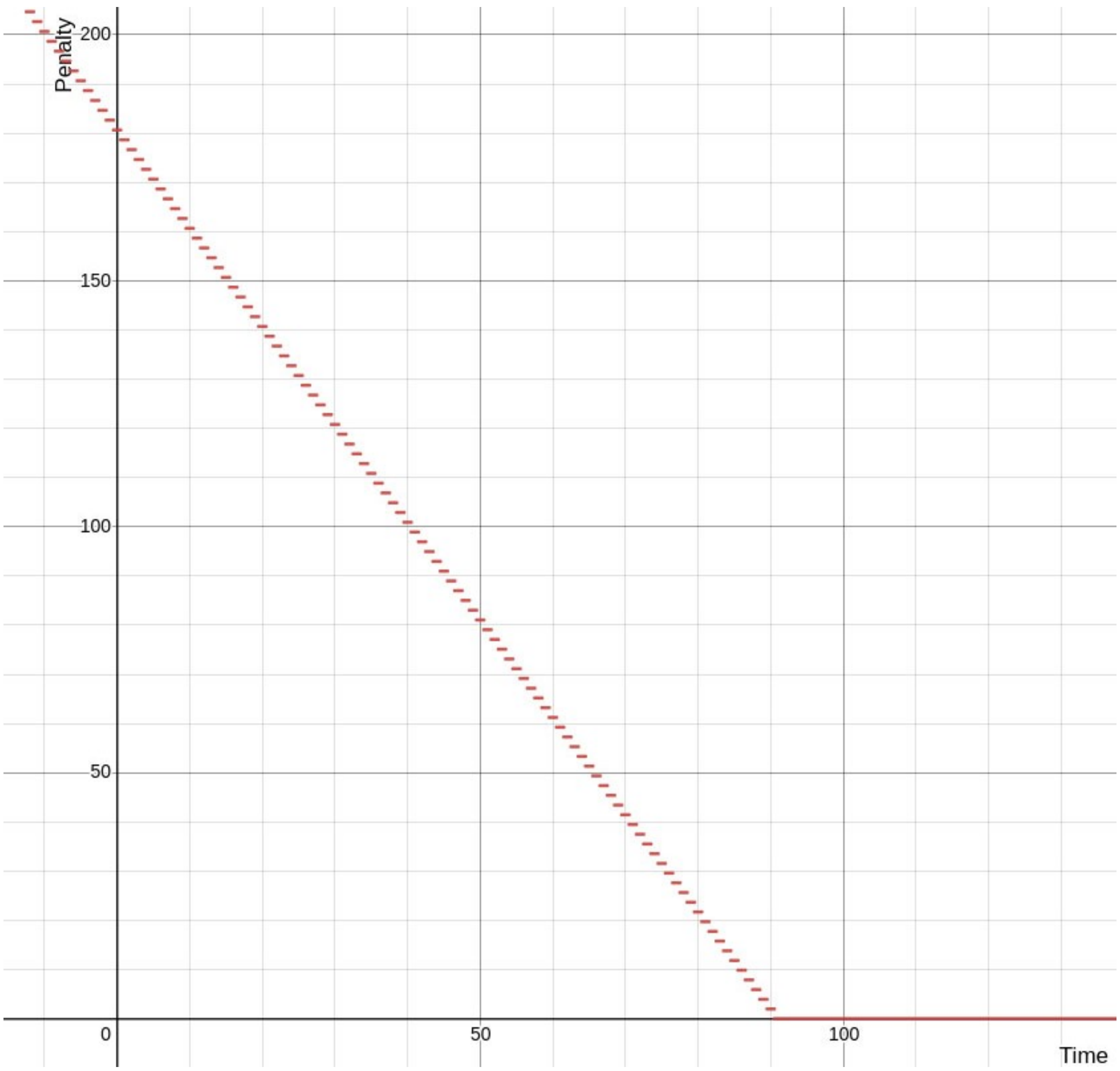

Figure 3: The NPV curve

$$P(t_n) = 1.98504964656t_n + 180.639517837 \tag{24}$$

We then find the mean of the betas to infer the degree of convex/concavification of the base curve:

$$\beta_{avg} = \frac{\beta_{eth} + \beta_{usdt}}{2} = \frac{1 + (1/39)}{2} = 0.512 \tag{25}$$

Given the average β is less than the average market risk, we slightly concavify the curve.

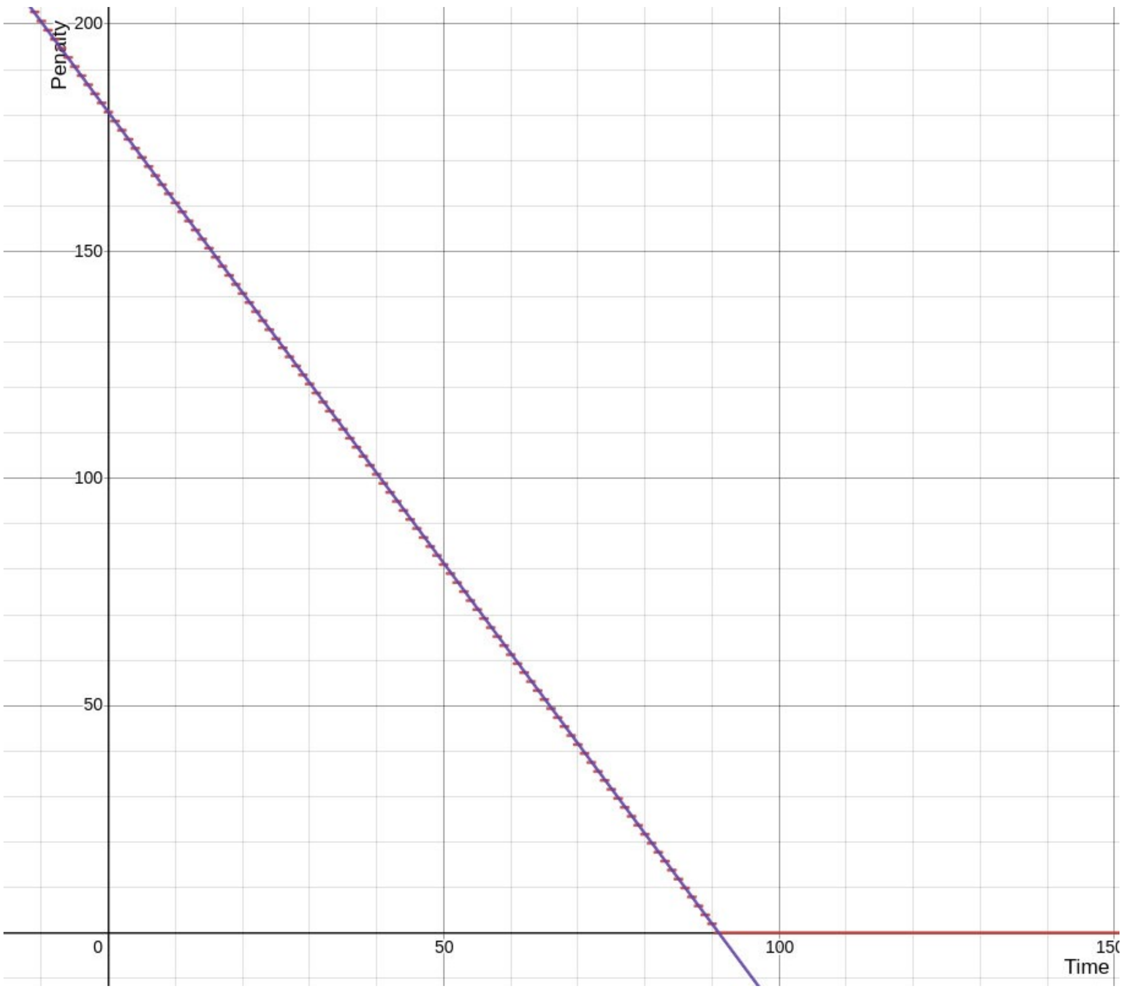

Figure 4: The linearized base curve

$$d_{limit} = \frac{l\,(0 - y_0)\cos(s) \;-\; 0\sin(s)}{2\pi t_0\left((0)\sin(s) + (0 - (t_0 + 1))\cos(s)\right)} = 1.42004599424 \tag{26}$$

$$l = 2\sqrt{y_0^2 + (t_0 + 1)^2} = 404.532497603 \tag{27}$$

$$A \;=\; 2d_{limit}\left(\frac{1}{1 + e^{\beta_{average} - 1}} - 0.5\right) = 0.280282032847 \tag{28}$$

$$(y - y_0)\cos(s) \;-\; t\sin(s) = f\left(y\sin(s) + (t - (t_0 + 1))\cos(s)\right) \tag{29}$$

$$(y - 180.639) * 0.45 \;-\; t * -0.893 = f\left(y * -0.893 + (t - (91)) * 0.45)\right) \tag{30}$$

6 Reward Valuation

The final piece of this algorithm is the fun part, the reward one receives at the end of a vesting period. Emission rates of rewards usually in the platform governance token are typically set manually by farm operators. Sigmadex operates in a similar fashion, tracking competing farm emission rates to remain competitive in the ecosystem. However, as this liquidity protocol asks the yield farmer to forgo the marginal benefit of switching freely between liquidity pools, as protocols such as yearn finance automate, the protocol must provide an additional incentive to platform users to farm yield.

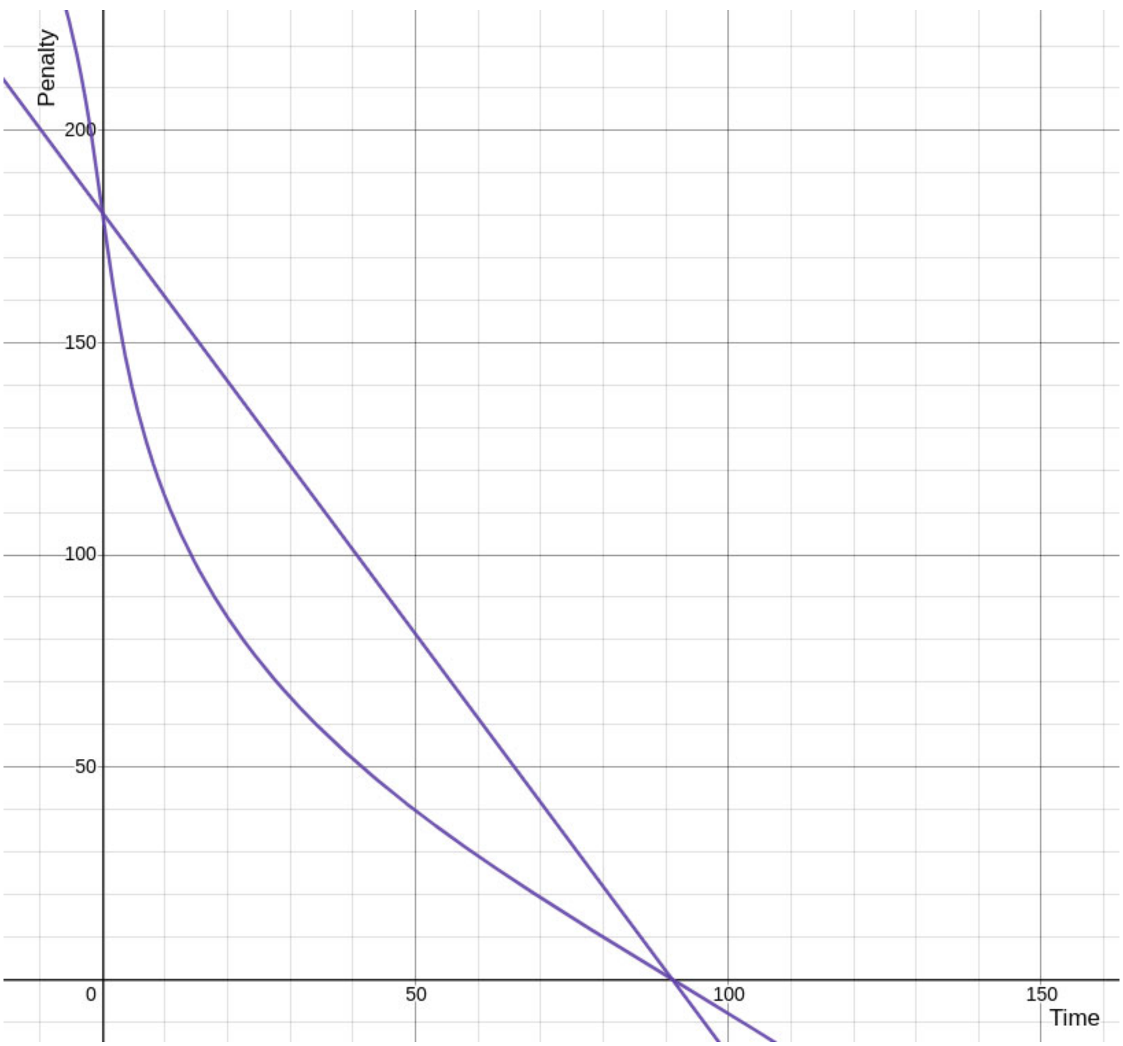

Figure 5: The Concavified base curve

From the prior reward equation, the user receives their LP back, the emission of governance tokens, plus a value epsilon, which is in the form of one more reward NFT. These are described in detail in another paper, but for now we focus on where their monetary value comes from and how its quantified:

$$reward = lp_j + \sum_{n=1}^{3} \frac{y^j}{(1 + r_Y)^n} + \epsilon \tag{31}$$

The source of ϵ comes from a pool of funds that are fueled by a small platform transaction fee, fee_{sdex} and in greater proportion, from penalties that have been accrued from previous farmers $fee_{penalties}$. To determine how much one receives in eplison we use the proportion of an individuals area under their penalty curve to the sum total of all areas under everyone else's penalty curves, take this percentage from the penalty pool and mint reward NFTs of this expected value.

$$\lambda = \frac{\int_{t_0}^{t_n} P(t)dt}{\sum_{a=1}^{p} \int_{t_0}^{t_{an}} P_a(t)dt} \tag{32}$$

where λ is the proportion of the penalty pool that is claimed by completing the liquidity position and p is the total amount of active yield farm penalty curves.

7 Conclusion

The Sigmadex yield farm penalty curve and its monetary rewards was derived. The Sigmadex risk index forms discount rates and risk β's for each asset, alongside estimators for the fees earned through swaps to derive the Discounted Cash flow analysis of a yield farm. A user can then become liable for a proportion of this value according to a penalty function when they stake liquidity points in a yield farm. This function is convexified if the assets are risky, steepening the penalty and increasing disincentives to deviate. While less risky assets are concavified, increasing competitiveness's to other yield farms. The proportion of the area under a penalty curve per the sum of all areas under all the other penalty curves is the proportion of the penalty pool one receives upon successful completion of a vestige period. An example was constructed.