Lab 2 - Fourier Transform and LTI systems

Objectives: In this lab, we will

- numerically compute Fourier transform (FT) of some common continuous-time signals we have seen in class and plot them, verify some properties of FT;
- process periodic signals (using FS coefficients) with LTI systems acting as filters (given their frequency response), plot and compare input & output signals.

2.1 Continuous-time Fourier transform

(a) Write a matlab function `X = continuousFT(xt,t,a,b,ω)` to numerically compute continuous-time FT of the given signal $x(t)$ which has finite support in [a, b] and is zero outside. The inputs to this function are

1. xt – signal whose FT is to be computed (function of symbolic variable t)
2. t – symbolic variable
3. a,b – the signal is equal to xt in the interval [a, b] and zero outside
4. ω – the vector ω contains the values of frequency where FT is to be computed.

>> The function should return a vector X which contains the FT of $x(t)$ for each of the frequencies in the input vector ω.

(b) Write a matlab script that calls the function `continuousFT` for a rectangular pulse of unit amplitude in [-T, T] where T = 2 and ω = -5:0.1:5. In a single figure, using `subplot()` commands to get a 2x2 grid of subplots, plot the real part, imaginary part, absolute value and phase of the computed FT as function of ω.

>> For phase use the command `angle()` in matlab. Can you explain each of the subplots? What about phase?

>> Optional: You can try using a finer spacing (ex. 0.05) of frequency ω for smoother plots, but it will slow down the code execution.

(c) Repeat part (b) for T = 1 and T = 4. Use ω = -5:0.1:5. What FT property supports your observations when T is changed?

(d) Repeat part (b) for $x(t) = \mathrm{e}^{jt}$, $x(t) = \cos(t)$ and $x(t) = \sin(t)$. Limit signals to the interval [-T, T] where T = π and ω = -5:0.1:5. What is the expected FT? What are the shapes you are observing? Make use of theory to explain your observations.

(e) Repeat part (b) for a triangle pulse of height 1 and base/support [-1,1]. What is the expected FT?

(f) Optional: play with some more signals $x(t)$ to test your function and verify whether standard properties of FT are satisfied as expected.

2.2 Filtering of periodic signals with LTI systems

(a) (Theory) You are given an LTI system with frequency response $H(\omega)$. A continuous-time periodic signal $x(t)$ has Fourier series (FS) coefficients a_k. If $x(t)$ is input to the LTI system, what are Fourier series coefficients of the output signal? What about the periodicity of the output signal?

(b) Write a script which calls the function `x = partialfouriersum(A,T,t)` you wrote in previous lab session with inputs T = 2π, t = -2T:0.01:2T and 'A' chosen such that it returns the signal $x(t) = \cos(t)$. What should be the input 'A'? Plot x as a function of t and verify.
>> Use following instructions for plotting: as done in the first part, use `subplot()` commands to get a 2x2 grid of subplots and `plot(t,x)` in the top-left panel.
>> Other panels of this figure will be filled below. Continue coding in the same matlab script file for the tasks below (except when asked to write functions).

(c) Recall that in (b), the input A consists of Fourier series coefficients $a_k, k = -N{:}N$. What are the frequencies corresponding to these coefficients? Create a vector 'F' containing these frequencies.

(d) Ideal low pass filter (LPF): let $\omega_c > 0$ be its cut-off frequency. We wish to find FS coefficients $b_k, k = -N{:}N$, of the output signal when the input signal with coefficients $a_k, k = -N{:}N$, is passed through an Ideal LPF.
>> Write a code for the matlab function `B = LPF(A,F,wc)` which takes input signal FS coefficents A, corresponding frequencies F, cut-off frequency wc and returns the output signal FS coefficients in the vector B. Note that your code should be written for general N.
>> In your main script call this function for 'A' in (b) and for an LPF with $\omega_c = 2$.
>> Use the function `partialfouriersum` to obtain the time domain signal corresponding to B and plot in the top-right panel of the figure. Also plot the original signal in same panel for reference. Give appropriate title and legend to each of the subplots.
>> What happens when we change cut-off to $\omega_c = 0.5$?

(e) Ideal high pass filter (HPF): let $\omega_c > 0$ be its cut-off frequency. Repeat part (d) for an ideal HPF and write the function `B = HPF(A,F,wc)`. In your main script call this function for 'A' in (b) and for an HPF with $\omega_c = 2$.
>> Use the function `partialfouriersum` to obtain the time domain signal corresponding to B and plot in the bottom-left panel of the figure. Also plot the original signal in same panel for reference.
>> What happens when we change cut-off to $\omega_c = 0.5$?

(f) Non-ideal filter: let the frequency response be $H(\omega) = \frac{G}{a+j\,\omega}$ where G and a are positive real constants. What is the nature of this filter? Write a code for the matlab function `B = NonIdeal(A,F,G,a)`. In your main script call this function for 'A' in (b) and G = 1, a = 1.
>> Use the function `partialfouriersum` to obtain the time domain signal corresponding to B and plot in the bottom-right panel of the figure. Also plot the original signal in same panel for reference.
>> How is the complex-valued nature of the LTI system frequency response manifested in the output signal?
>> For what values of G and a is the frequency response of this filter equal to that of the RC filter we studied in class?

(g) Repeat the above when the input signal is $x(t) = \sin(2t) + \cos(3t)$. Note that you must appropriately modify A, F, and T for this example. For this input set ideal LPF and ideal HPF filter cut-offs to be $\omega_c = 2.5$.

(h) Optional: repeat when 'A' corresponds to FS coefficients of the periodic square wave.