Lab 6 – Discrete-time Fourier transform

Objectives: In this lab we will

- numerically compute DTFT and inverse DTFT
- study LTI systems using DTFT analysis

6.1 Discrete-time Fourier transform (DTFT) (function + script)

(a) Write a function `X = DTFT(x,N0,w)` that takes as inputs

- x, a discrete-time signal of finite duration (assume that the signal is zero elsewhere)
- N_0, location of the sample x[0] in the given input signal x (note that $1 \leq N_0 \leq length(x)$ because matlab indexing starts from 1)
- ω, a vector of frequencies at which to compute the DTFT

The function should return X, a complex vector corresponding to the DTFT computed at the frequencies in ω. Write the function using at most one for-loop. The DTFT of a discrete-time signal x[n] is given by

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

(b) Write a script which calls the `DTFT()` function. Set $\omega = -10{:}\,0.01{:}\,10$ for computing the DTFT. Your script should compute DTFT for each of the following discrete-time signals:

1. unit impulse $\delta[n]$
2. shifted unit impulse $\delta[n-5]$
3. rectangular pulse from -5 to 5
4. rectangular pulse from 0 to 10
5. sinusoid $\sin(\omega_0 n)$ with $\omega_0 = \frac{\pi}{3}$

Appropriately choose x and N_0 for each of the signals. Restrict the sinusoid signal for finite duration (say n = -500 to 500). For each signal, plot the DTFT spectrum (i.e. magnitude, phase, real, imaginary parts) in a 2x2 figure. Compare your plots with the analytical answers worked out in class. Is the DTFT periodic? What is the period?

(c) In the same script, compute DTFT for the signal $a^n u[n]$. We will do this for $a = b$ and $a = -b$ where $b \in (0,1)$ (values of b are given below). Restrict your signal (n = 0 to 500) for finite computations. Set $\omega = -10{:}\,0.01{:}\,10$. In a 2x2 figure, plot the two time domain signals (corresponding to $a = b$ and $a = -b$) in the top panels and their DTFT magnitude spectrum in the bottom panels. Do this for b = 0.1, 0.5, 0.9 and note your observations as b changes.

6.2 Inverse DTFT (script)

(a) Write a matlab script which numerically computes inverse DTFT using the `int()` command. Recall that the inverse DTFT is given by the expression

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{j\omega n}\, d\omega.$$

You must compute the inverse DTFT for the frequency domain rectangular wave which in the interval $[-\pi, \pi]$ is given by

$$X\left(e^{j\omega}\right) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

Let $\omega_c = \frac{\pi}{16}$. Compute the signal $x[n]$ numerically for n = -100 to 100 and plot it as function of time n. Is $x[n]$ expected to be real or complex valued? Plot accordingly.

(b) Repeat above when $\omega_c = \frac{\pi}{8}$, $\omega_c = \frac{\pi}{4}$ and $\omega_c = \frac{\pi}{2}$ and compare your observations. What happens when $\omega_c = \pi$. Can you explain this observation using theory?

(c) Repeat part (a) when the DTFT is given by the band-pass signal of the form

$$X\left(e^{j\omega}\right) = \begin{cases} 1, & \omega_1 \le |\omega| \le \omega_2 \\ 0, & |\omega| < \omega_1 \ \ and\ \omega_2 < |\omega| < \pi \end{cases}$$

Let $\omega_1 = \frac{\pi}{8}$ and $\omega_2 = \frac{\pi}{4}$. Try another set of values for ω_1 and ω_2 and note your observations.

6.3 LTI systems corresponding to difference equations (script)

(a) Consider a discrete-time LTI system with input x[n] and output y[n] given by the relation

$$y[n] = x[n] - x[n-2]$$

Find the impulse response of this LTI system. Use the `DTFT()` function written above to find the frequency response of this system. Plot the magnitude and phase response in a 2x1 figure for $\omega = -10{:}\,0.01{:}\,10$.

(b) Repeat part (a) for the following LTI systems

1. $y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$
2. $y[n] = \frac{1}{4}(2x[n] - x[n+1] - x[n-1])$

(c) The LTI systems above can be interpreted as filters as well. Comment on the nature of the filters from the plots of the magnitude response above, i.e. low-pass, high-pass, band-pass, etc.

(d) From the plots obtained in 6.1 (c), if $h[n] = a^n u[n]$ is the impulse response of the LTI system, how do these systems/filters behave for positive and negative values of a? Comment on the behaviour of these filters as the value of b is changed.