Análisis de sistemas lineales

Oscar Hurtado Esquivel

"Tensión en los componentes de RLC y respuestas ante entradas básicas"

Componente	Frecuencia	Tiempo
Inductor	$V_l = L * S * I_{(s)}$	$V_L = L\frac{di}{dt}$
Capacitor	$V_c = \frac{I_{(s)}}{C * S}$	$V_C = \frac{1}{C}\int_0^t I_C\, dt$
Resistencia	$V_R = I_{(S)} * R$	$V_R = I * R$

Sabiendo que L=1µH, R=1KΩ y C=1µf.

Entradas ($V_{in}(s)$)

Impulso =1

Escalón Unitario= $\frac{1}{s}$

Rampa = $\frac{1}{s^2}$

- **Ecuaciones por utilizar (General de Kirchhoff):**

Para resistencia	$\frac{V_R(S)}{V_{in}(S)} = \frac{S * R}{L * s^2 + R * s + \frac{1}{C}}$
Para inductor	$\frac{V_L(S)}{V_{in}(S)} = \frac{S^2 * L}{L * s^2 + R * s + \frac{1}{C}}$
Para Capacitor	$\frac{V_C(S)}{V_{in}(S)} = \frac{S}{CR * S^2 + CL * S^3 + S}$

- **Resistencia**

$$\frac{\boldsymbol{V_R(S)}}{\boldsymbol{V_{in}(S)}} = \frac{S * R}{L * s^2 + R * s + \frac{1}{C}}$$

  - **Impulso**

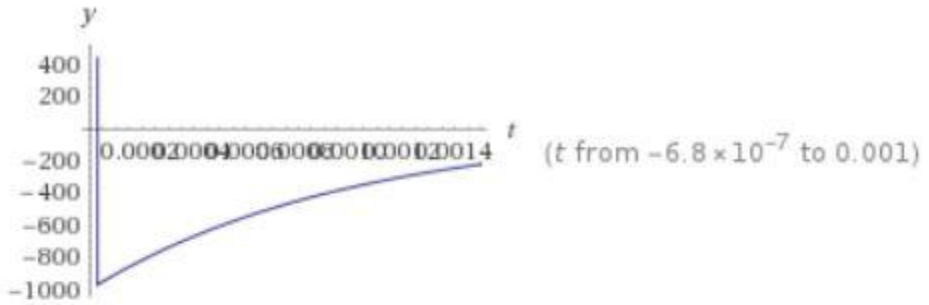
$$\boldsymbol{V_R(s)} = \frac{S * R}{L * s^2 + R * s + \frac{1}{C}} * 1$$

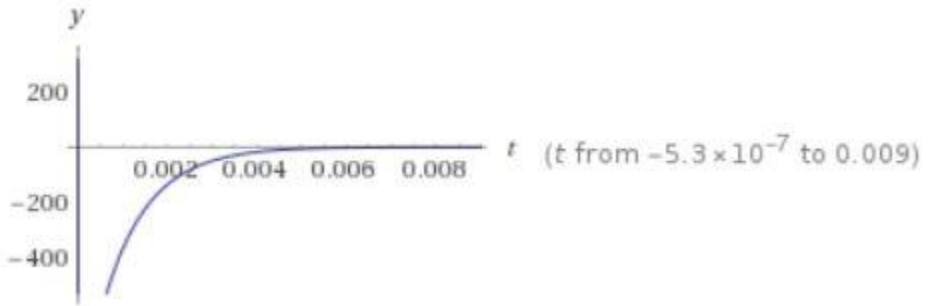
laplace inverse (1*10^3*s)/(1*10^3s+1*10^-6*s^2+(1/1*10^-6))

$$\left(500\,000\,000\, e^{-\frac{1\,000\,000\, t}{500+\sqrt{249\,999}}}\left(499\,999\, e^{\frac{1\,000\,000\, t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}+\right.\right.$$
$$\left.\left.1000\sqrt{249\,999}\, e^{\frac{1\,000\,000\, t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-1\right)\right)/$$
$$\left(249\,999+500\sqrt{249\,999}\right)$$

Input interpretation:

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^3\, s}{1\times10^3\, s+1\times10^{-6}\, s^2+\frac{1}{1\times10^{-6}}}\right](t)$$





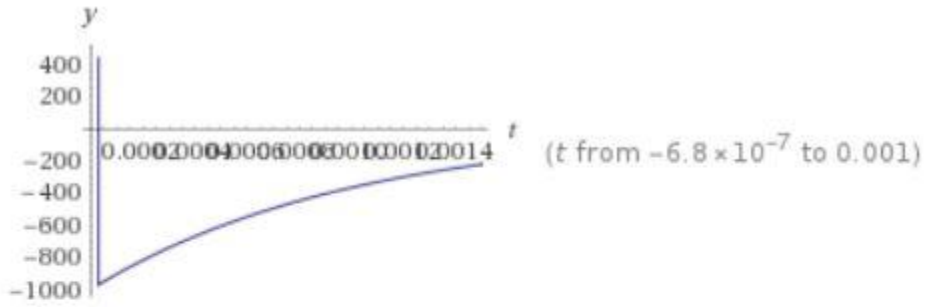
- **Escalón**

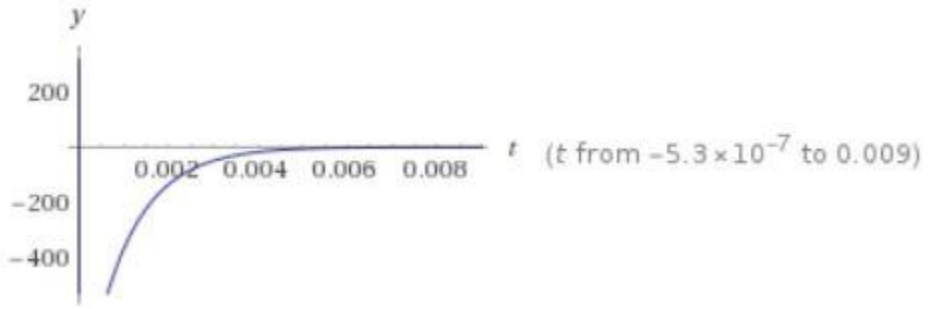
$$V_R(s) = \frac{1000s}{1x10^{-6} * s^2 + 1000 * s + \frac{1}{1x10^{-6}}} * \frac{1}{s}$$

laplace inverse (1*10^3*s)/(1*10^3s+1*10^-6*s^2+(1/1*10^-6))(1/s)

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^3\, s}{1\times10^3\, s + 1\times10^{-6}\, s^2 + \frac{1}{1\times10^{-6}}} \times \frac{1}{s}\right](t)$$

$$-\left(\left(500\left(500+\sqrt{249\,999}\right)e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-1\right)\right)\Big/\left(249\,999+500\sqrt{249\,999}\right)\right)$$





- Rampa

$$V_R(s) = \frac{1000s}{1x10^{-6} * s^2 + 1000 * s + 1x10^6} * \frac{1}{s^2}$$

laplace inverse (1*10^3*s)/(1*10^3s+1*10^-6*s^2+(1/1*10^-6))(1/s^2)

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^3\, s}{1\times10^3\, s + 1\times10^{-6}\, s^2 + \frac{1}{1\times10^{-6}}} \times \frac{1}{s^2}\right](t)$$

$$1000\left(\frac{1}{1\,000\,000} - \left(e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(-e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t} + 499\,999 + 1000\sqrt{249\,999}\right)\right)\Big/\left(2\,000\,000\left(249\,999+500\sqrt{249\,999}\right)\right)\right)$$

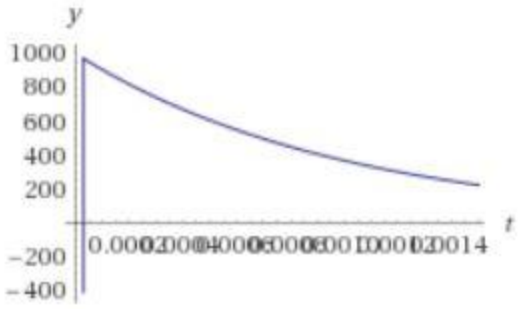
- Capacitor:
  - Impulso

$$V_C(S) = \frac{S}{CR * S^2 + CL * S^3 + S}$$

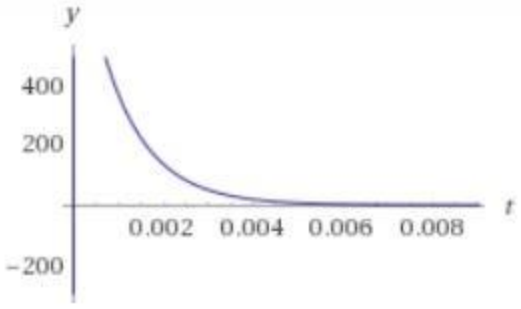
laplace inverse (s)/(1*10^-3*s^2+1*10^-12*s^3+s)

$$\mathcal{L}_s^{-1}\left[\frac{s}{1\times 10^{-3}\, s^2 + 1\times 10^{-12}\, s^3 + s}\right](t)$$

$$-\left(\left(500\,000\left(500+\sqrt{249\,999}\right)e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-1\right)\right)\Big/\left(249\,999+500\sqrt{249\,999}\right)\right)$$



(t from -6.8×10^{-7} to 0.001)



(t from -5.3×10^{-7} to 0.009)

- Escalón Unitario:

$$\boldsymbol{V_C(S)} = \frac{S}{CR * S^2 + CL * S^3 + S} * \frac{1}{s}$$

laplace inverse (s)/(1*10^-3*s^2+1*10^-12*s^3+s)(1/s)

$$\mathcal{L}_s^{-1}\left[\frac{s}{1 \times 10^{-3}\, s^2 + 1 \times 10^{-12}\, s^3 + s} \times \frac{1}{s}\right](t)$$

$$1 - \left(e^{-\frac{1\,000\,000\, t}{500+\sqrt{249\,999}}} \left(-e^{\frac{1\,000\,000\, t}{500+\sqrt{249\,999}} + \left(-500\,000\,000 - 1\,000\,000\sqrt{249\,999}\right)t} + 499\,999 + 1000\sqrt{249\,999}\right)\right) \Big/ \left(2\left(249\,999 + 500\sqrt{249\,999}\right)\right)$$

- Rampa

$$\boldsymbol{V_C(S)} = \frac{S}{CR * S^2 + CL * S^3 + S} * \frac{1}{s^2}$$

laplace inverse (s)/(1*10^-3*s^2+1*10^-12*s^3+s)(1/s^2)

$$\mathcal{L}_s^{-1}\left[\frac{s}{1 \times 10^{-3}\, s^2 + 1 \times 10^{-12}\, s^3 + s} \times \frac{1}{s^2}\right](t)$$

$$\left(e^{-\frac{1\,000\,000\, t}{500+\sqrt{249\,999}}} \left(-500\, e^{\frac{1\,000\,000\, t}{500+\sqrt{249\,999}} + \left(-500\,000\,000 - 1\,000\,000\sqrt{249\,999}\right)t} + \sqrt{249\,999}\; e^{\frac{1\,000\,000\, t}{500+\sqrt{249\,999}} + \left(-500\,000\,000 - 1\,000\,000\sqrt{249\,999}\right)t} + 499\,998\,500 + 999\,999\sqrt{249\,999}\right)\right) \Big/ \left(2\,000\,000\left(249\,999 + 500\sqrt{249\,999}\right)\right) + t - \frac{1}{1000}$$

- Inductor

$$V_L(S) = \frac{S^2 * L}{L * s^2 + R * s + \frac{1}{C}} * 1$$

  - Impulso

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^{-6}\,s^2}{1\times10^{3}\,s+1\times10^{-6}\,s^2+\frac{1}{1\times10^{-6}}}\right](t)$$

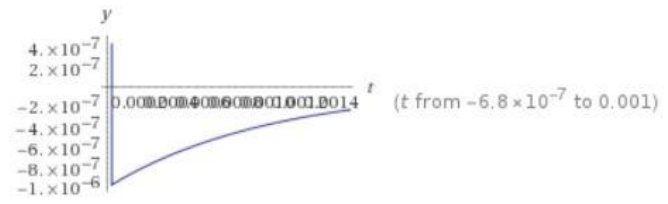
$$\frac{1}{1\,000\,000}\left(1\,000\,000\,\delta(t)-\left(500\,000\,000\,000\,e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(499\,998\,500\,e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}+999\,999\sqrt{249\,999}\;e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-500+\sqrt{249\,999}\right)\right)\Big/\left(249\,999+500\sqrt{249\,999}\right)\right)$$

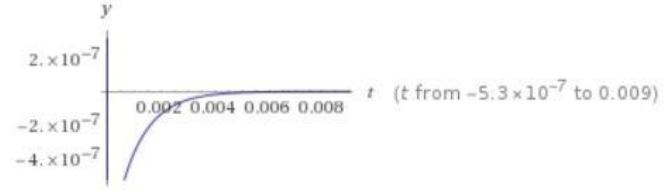
  - Escalón Unitario:

laplace inverse (s)/(1*10^-3*s^2+1*10^-12*s^3+s)(1/s)

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^{-6}\,s^2}{1\times10^{3}\,s+1\times10^{-6}\,s^2+\frac{1}{1\times10^{-6}}}\times\frac{1}{s}\right](t)$$

$$\left(e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(499\,999\,e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}+1000\sqrt{249\,999}\;e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-1\right)\right)\Big/\left(2\left(249\,999+500\sqrt{249\,999}\right)\right)$$



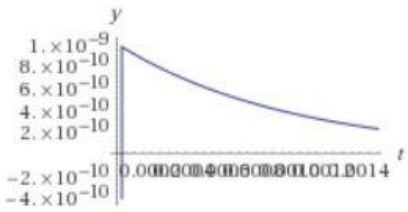


- Rampa

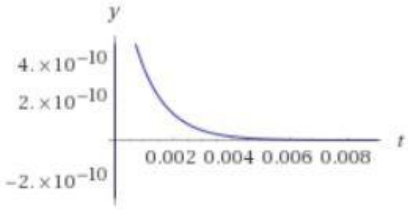
laplace inverse (1*10^-6*s^2)/(1*10^3s+1*10^-6*s^2+(1/1*10^-6))(1/s^2)

$$\mathcal{L}_s^{-1}\left[\frac{1\times10^{-6}\,s^2}{1\times10^{3}\,s+1\times10^{-6}\,s^2+\frac{1}{1\times10^{-6}}}\times\frac{1}{s^2}\right](t)$$

$$-\frac{(500+\sqrt{249\,999})\,e^{-\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}}\left(e^{\frac{1\,000\,000\,t}{500+\sqrt{249\,999}}+\left(-500\,000\,000-1\,000\,000\sqrt{249\,999}\right)t}-1\right)}{2\,000\,000\,(249\,999+500\sqrt{249\,999})}$$



(t from -7.3×10^{-7} to 0.001)



(t from -5.3×10^{-7} to 0.009)