Research paper

Improve searching for a sign by its properties

Tobias Cunnen
585780

Hogeschool Arnhem en Nijmegen
February 2, 2023

Contents

Glossary

Abbreviation	**Description**
NGT	Nederlandse gebarentaal (Dutch sign language).
Corpus	A collection of texts of a certain language.
Ipsilateral	Refers to a location on the body. Ipsilateral is on the same side of the body as the reference point, which is, in most cases, the center of the body.

1 Introduction

"How can the Signbank and Dutch corpus data be used to improve searching for a sign?"

This study aims to examine the most effective method of utilizing the data contained in Signbank and the Dutch corpus to improve searching for a sign based on its properties. The objective is to enable the users to identify a sign they are unfamiliar with, by providing the app with relevant details of the sign they have encountered. By presenting a series of questions related to the sign, the application will aim to determine the intended sign and display a corresponding video demonstrating that sign.

To address the main research question, it has been divided into several sub-questions. These questions are:

1. *"What data is available in the NGT dataset of Signbank?"*
2. *"How can the Dutch corpus data be used to improve searching for a sign?"*
3. *"What is the best set of properties and how can that set be chosen?"*
4. *"What is the optimal way to implement a graph to improve the search functionality?"*

The main research question is decomposed into several sub-questions, each of which will be explored in a dedicated chapter. The final section will present a comprehensive discussion and conclusion that combines and summarizes the answers to these questions.

2 Signbank NGT dataset

"What data is available in the NGT dataset of Signbank?"

This chapter will explore which data is available in Signbank and its relevance for enhancing the search functionality. The Signbank dataset includes various attributes of a sign, including the location of its occurrence and the handshape that is utilized.
Each attribute will be examined and a table will be generated that shows the frequency of the attribute in Signbank. To provide a comprehensive analysis, this tables will present the top ten most frequent properties per attribute, including their representation in terms of percentage. See table 1 for a concrete example of the locations used in the NGT dataset.

This paper will focus on properties that have at least 80% of their data available in the Signbank database, as properties with insufficient data are not reliable to use for the search functionality. A comprehensive list of all properties and their available data is provided in Appendix A.

2.1 Location

The location of a sign refers to the primary area of the body where the sign is performed. For instance, when signing the word *"me"* or *"ik"* in Dutch, the performer typically points to their chest with an index finger. This means the location of this sign is the chest.

Out of the 4,168 signs in the NGT dataset, 3,580 have a specified location. The table below lists the top ten most frequently used locations, with the number of signs and their corresponding percentage of usage. The percentage is not precisely rounded, but the goal is to provide a clear visual representation of the distribution of these locations.

Table 1: Location frequency

Location	**Amount**	**Percentages**
Neutrale ruimte	1742	48
Zwakke hand: palm	207	5
Borst	167	4
Mond	151	4
Kin	143	3
Hoofd	143	3
Zwakke hand: duimkant	97	2
Variabel	81	2
Wang	78	2
Neus	70	1

This table displays the ten most common sign locations, which are present in 2879 out of the 3580 signs that have a recorded location. This represents approximately 80% of all signs with a listed location.

Table 1 shows that the most common location for signs is the neutral space, accounting for about 48% of the signs. Thus, relying on location as a search criterion may not provide optimal results, as it only narrows down the options to half when the neutral space is selected. However, if the sign is not in the neutral space, it significantly reduces the number of potential signs.

The distribution of signs will be analyzed in detail in section 4 to determine the most suitable distribution for the purpose of this study.

2.2 Movement direction

The direction of the movement of a sign refers to the movement relative to the hand of the person executing the sign. For instance, a downward movement is self-explanatory. Ipsilateral, on the other hand, refers to the movement of the hand on the same side of the body, starting from the center and extending outward.

In the NGT dataset, movement direction has been recorded for 3382 of the total 4168 signs.

Table 2: Movement direction frequency

Movement direction	**Amount**	**Percentages**
Omlaag	482	14
Naar voren	426	12
Ipsilateraal	311	9
Omhoog	201	5
Naar achteren	131	3
Contralateraal	122	3
Heen en weer	77	2
Ipsilateraal en contralateraal	64	1
Naar locatie toe	55	1
Op en neer	51	1

Out of the 4168 signs in the NGT dataset, 10 of the most frequently occurring movement directions are present in 2926 signs, accounting for 87% of all signs that have a movement direction specified.

2.3 Handshape

The handshape is a crucial aspect of a sign and serves as a means of identifying it. It is important to note that the handshape itself has no inherent meaning,

but it is still considered a defining characteristic of a sign.
In sign languages, there is a concept of a dominant/strong hand and a submissive/weak hand. The distinction between these two hands depends on the particular sign language in question. In the NGT, the hand that performs the movement is considered the strong hand ("Dominante Hand (of: Sterke Hand)", n.d.,). If a sign involves symmetrical movements using both hands, in such a scenario there is no weak hand.

In the NGT dataset, 3779 out of 4168 signs have a recorded strong handshape and 3236 out of 4168 have a recorded weak handshape. However, only 253 of the weak handshapes are different from their corresponding strong handshapes. Therefore, using the difference between handshapes of both hands as a criteria for narrowing down the search is not an effective approach as it eliminates only a small number of signs.

The NGT dataset features 79 unique handshapes, with the following being the most frequently used.

Table 3: Handshape frequency

Handshape	**Amount**	**Percentages**
B	580	15
1	403	10
S	293	7
5	285	7
Geld	183	4
T	173	4
C_spreid	162	4
V	141	3
C	132	3
A	131	3

These top ten handshapes are present in 65% of all the signs in the NGT dataset that have a listed dominant handshape.

In this chapter, we have analyzed the properties of signs available in the NGT dataset and concluded that location, movement direction, and handshape can be used as criteria to search for a specific sign. Despite the potential usefulness of other characteristics they cannot be utilized in the search process due to the insufficient data available in the dataset.

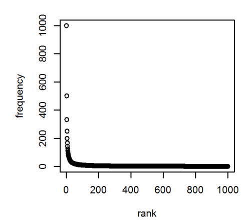
3 Dutch corpus data

"How can the Dutch corpus data be used to improve searching for a sign?"

The Dutch corpus is a substantial dataset of the spoken Dutch language, which enables the creation of a list of Dutch words based on their respective usage in the Dutch language. The information from this list can be utilized to rank the properties on their relevance in the Dutch language instead of their presence in Signbank. This is meant to provide a rough estimate of the significance of a sign and its properties when learning NGT. This ranking is distinct from the one discussed in the previous chapter, which was based on the frequency of all the different words instead of the frequency of one word. In linguistics this is called "types and tokens" (Wetzel, 2006,).

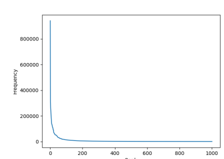
A type refers to a distinct word, and a token refers to the occurrence of that word.
It is important to differentiate between types and tokens because a sign property that is used in many signs may not be frequently used in the language. This is similar to the Dutch language, where there are many words, but only a small number are used in everyday conversation. This distribution of occurrence is referred to as "Zipf's law" in linguistics and states that the most frequent word occurs twice as often as the second-most frequent word (Lestrade, 2017,). Therefore, by utilizing types instead of tokens, the properties are ordered based on their frequency of use. See figure 1 and 2 and Appendix B.1 for a plot of a "Zips curve" and a plot of the Dutch corpus frequency.

Figure 1: Zipf curve plot [1]



A limitation of utilizing the spoken Dutch corpus is that it does not account for words that are more important in the sign language. For instance, the word "deaf" is likely to be used more frequently by individuals who are deaf than those who are not.
There is an NGT corpus that contains frequency information, but the dataset is too limited to be reliable. To overcome this issue, it may be beneficial to incorporate both the NGT and Dutch frequencies and create a balance between the two, see section 3.2 for more information.

Figure 2: Dutch corpus data



3.1 Map Dutch to NGT

Dutch is not directly applicable to NGT as it is a distinct language with its own set of words and meanings. Not all Dutch words can be translated into a

[1] Source of Zipf curve plot (Lestrade, 2017,)

sign and vice versa. The context in which a sign is used can also influence its meaning, making a direct comparison between the two languages difficult.

Signbank contains a list of translations for each sign. These translations can be utilized to associate the Dutch frequency information with the signs form Signbank. The three options considered for mapping the Dutch frequency data to the signs are: the sum of all translations, the average of the translations, and the translation with the highest frequency value.

The drawback of the first two options is that the quantity of translations for each sign significantly impacts its properties overall frequency. When the sum of all translations is used a sign with many translations will rank higher on the list. However, using the average will have the opposite effect. As previously discussed, natural languages often have a few high-frequency words and a large number of low-frequency words. This implies that if a sign has numerous translations, the likelihood of most of them having a low frequency is substantial, thus, reducing the overall frequency by half or more per translation.

The last alternative is to consider only the translation with the highest frequency. This method is less impacted by the number of translations compared to the first two options, as it is not subject to any calculations that take into account the number of translations. For this reason it seems to be the best choice for this application.

In the table below, you will find the ten most frequently used locations along with the data from the Dutch corpus. For information regarding movement and handshape, refer to Appendix B.2.

Table 4: Location with spoken Dutch frequencies

Location	**Amount**
Neutrale ruimte	262631
Borst	11851
Zwakke hand: palm	71935
Borst Contra	21617
Schouder	19096
Zwakke hand: vingertoppen	13395
Bovenlichaam	13119
Kin	12517
Hoofd	12222
Zwakke hand: duimkant	7248

By utilizing the data from the Dutch corpus, the ranking of all properties shifts.

3.2 Multiple datasets

To address the difference between Dutch and NGT, it may be advantageous to consider multiple datasets in order to properly determine the importance of words like "deaf", which are more prevalent in sign language. One approach could be to calculate the translations of the NGT corpus and incorporate a search history. To be able to compare these different datasets, the values of each set should be normalized. By doing so, it would be possible to combine the datasets and adjust their relative importance by increasing the normalized values as deemed necessary.

In conclusion, the utilization of Dutch corpus data allows for the arrangement of various sign properties. This particular order is believed to be more relevant and therefore the properties within it are expected to be selected more frequently. The assumption that this ordering will be more relevant and frequently chosen is based on the intuition and expertise of specialists at the Radboud University.

4 Choose the best property set

"What is the best set of properties and how can that set be chosen?"

In the preceding chapters, the properties of the signs in Signbank were analyzed and presented. These properties were separated and organized based on their frequency of occurrence in the spoken Dutch corpus. The current chapter will focus on identifying the most appropriate set of properties to enhance the efficiency of the search functionality.

The goal is to find the sign the user is looking for in a manner that is user-friendly. Within this context, user-friendliness is characterized by minimizing the number of questions posed to the user and presenting options in a logical, relevant order. The ideal outcome is that the user is able to identify the correct option with minimal effort. It is assumed that, on average, a set that is evenly distributed will result in fewer options being presented to the user, thereby facilitating the identification process. Proving this assumption is not within the scope of this study, it is based on the intuition and insights of stakeholders within the Humanities lab.

There are two primary options that are considered when choosing the best distributed set. The first option is to select the set where the most frequent property of that set has the lowest share. For example, the location and movement direction from chapter 2. In this case, the location set has a 48% share, whereas the movement set has a 14% share. Therefore, the movement set would be chosen as the best option to present to the user. However, this approach has a limitation in that it does not take into account the other items within the set. Consider the following example: set $A = [40, 30, 20, 10]$ and $B = [39, 1, 1, 1...n]$ where $n = 1$ until the sum of $B = 100$. Each item represents the spoken Dutch frequency of that property. Therefore, a translation of a sign that contains property A_1 is said 40 times. As stated in chapter 3, a word that is said more often has a higher chance of being chosen. In this situation, set B would be chosen, but set B does not adhere to the definition given at the start of this chapter. This is because it contains too many items that have the same chance of being chosen, which would result in the user having to look through many options.

In order to properly evaluate the spread of a dataset, it is necessary to take the entire set into consideration. One method for calculating the spread of a dataset is through the use of the standard deviation (SD). The SD is a measure of how spread out a dataset is, with lower values indicating that the dataset is closely grouped around the mean (Hargrave, 2022,). The SD is commonly used for datasets that have a normal distribution, however, as shown in chapter 3, language frequencies do not typically follow a normal distribution. As such, it is not possible to make further assessments or conclusions based on the SD when analyzing language frequencies. For example, it is not possible to say that 95% of the dataset falls in range of two SD from the mean (Kazmier et al., 2003,) for this dataset. In this context, the primary focus should be on the value of the SD, and any additional evaluations or considerations are not necessary.

The formula for standard deviation is as follows:

$$SD(x) = \sqrt{\frac{(\sum(x_i - \overline{x})^2)}{n}}$$

It is important to note that the formula for calculating the spread of a data set, such as the SD, can be compared to other options, such as the mean absolute deviation (MAD). While both are used to measure the spread of a data set, one key difference is that the SD squares the differences, leading to a higher or equal value in comparison to the MAD. In instances where there are extreme outliers, such as in set B in the aforementioned example, the SD will yield a higher value than the MAD (Zach, 2022,). This is beneficial as it emphasizes the importance of closely grouping the data set.

Due to the distribution of natural languages, relying solely on the SD to determine the optimal set is not effective. Another example utilizing the same data sets as previously discussed can further illustrate this point. In this example, it can be observed that $SD(A) \approx 11.18$ and $SD(B) \approx 4.79$. In this scenario, set B has a lower SD due to the higher quantity of values within the set and the SD being an average of all points within the set. This highlights the importance of considering not only the SD, but also the quantity of items within a set, in order to determine the optimal set.

When determining the most suitable set to use, one approach to consider is the number of items present within the set. However, solely selecting the set with the lowest number of items would not necessarily be the most effective method.

It is possible that a set may possess a property that is frequently utilized in the majority of signs or a property exist with limited options. In the latter scenario, this approach would not be effective.

An alternative approach would be to evaluate the range of the set, which can be defined as the distance between the two furthest points within the set. For example, in set A, the range would be calculated as $A_1 - A_4 = 40 - 10 = 30$. To increase the value of the set with the higher combination, it may be beneficial to multiply the range by the SD. However, it should be noted that this approach may result in a large range for the sets when considering the distribution of spoken Dutch data, as discussed in Chapter 3, because of the large tail end of the data.

Another alternative is to consider only the initial elements of the set. As demonstrated in Chapter 2, the top ten most frequent properties within a set consist of over half of the signs present in Signbank. This implies that the likelihood of a property being within the initial ten values of a set is quite high, and as a result, the probability of a user selecting one of these properties is also high. Therefore, by calculating the SD of the initial ten items, the issue of the number of items within the set is no longer a concern.

Considering all the different options to choose the optimal set, the aforementioned option seems to be the best fit for this project.

5 Graph data structure

"What is the optimal way to implement a graph to improve the search functionality?"

This chapter will address the advantages of utilizing a graph as a means to depict the correlation between various properties and their corresponding signs. Additionally, it will show the methods by which a graph can be used to enhance the search process.
Each property is associated with a specific number of other properties, and the combination of these properties results in one or multiple signs. Thus, the selection of an appropriate graph implementation must take into account the relationships between properties and their corresponding signs.

The reasoning behind a graph data structure is that a graph is best for a set of elements that are connected. It can represent the relationships between the elements more explicitly and efficiently (Gross & Yellen, 2005,). This allows for easy representation and manipulation of the relationships between the elements in the set.

- In a graph, each element is represented as a node and the connections between the elements are represented as edges. This allows for a clear representation of the relationships between the elements in the set.

- Graphs can be used to represent various types of data such as directed, undirected, weighted and unweighted graphs.
- Graphs are more flexible than other data structures like arrays and linked lists, because they can handle complex relationships between elements.
- Many graph algorithms exist for solving a variety of problems, such as finding the shortest path between two nodes, detecting cycles, and traversing the graph (Cormen & Leiserson, 2009,).

These characteristics make a graph an ideal data structure for this application.

5.1 Graph type

There are many types of graphs. In order to determine the most appropriate one for this application, an analysis of the property connections is necessary.
As mentioned in chapter 2 there are, at the moment, three different properties of a sign. The combination of these three properties creates a sign.
For each connection a weight can be added to represent the importance of that connection. For example the weight can be set to the frequency of the spoken Dutch translation of that sign or the amount of times that a property is used in Signbank.

Two options are considered for the graph structure: a connected graph and a tree. Both options will be analyzed and the reasoning behind each option will be explained.

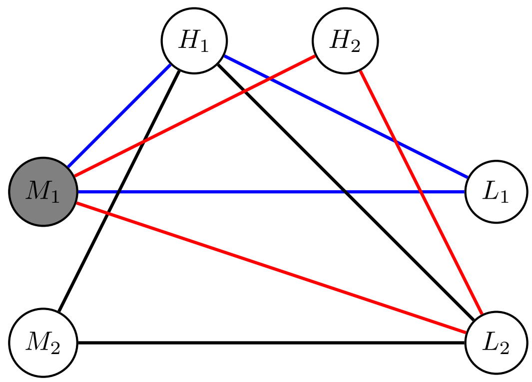
5.1.1 Connected graph

First the connected graph will be covered. The idea behind a connected graph is to store each property as a separate node in a list, and establish connections or edges between these nodes. The connections are established based on their respective signs. An illustration of this type of graph can be found in Appendix C.2.
The representation of different properties is indicated by letters, while the weights associated with each sign are depicted by numbers on the edges. This illustration is a cyclic, weighted, and undirected graph. Additionally, it is a multi-parted graph as a property is not connected to a node of the same property. When a specific set of properties is used in only one sign, these properties are not connected to any other properties, leading to a disjointed graph.

The advantage of the connected graph approach is the absence of duplicate nodes, thus reducing the graph's space complexity. However, the drawback is the difficulty in identifying the unique combination of a sign when all nodes are interconnected. See figure 3 for an example. This is a graph with three different signs, each represented by a distinct color. The gray node serves as the starting point. The first step in this approach is to evaluate all the outgoing edges from

M_1, which results in four options, divided into two sets: the first set, consisting of $[H_1, H_2]$, and the second set, consisting of $[L_1, L_2]$.

Figure 3: Connected graph example



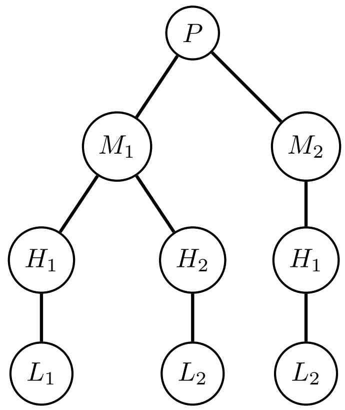
The difficulty in identifying the correct edge from a node lies in the fact that not all outgoing edges may lead to a combination that is a sign. For example, when H_1 is the second node chosen, one must determine which outgoing edge from H_1 leads to a valid combination. Which in this case is the blue edge to L_1.
One approach to solving this issue is to examine if the edges form a cycle. However, this method is not foolproof, as a node with a matching cycle may not necessarily represent a sign. See for example the sign between M_1, H_1 and L_2. Another solution could be to analyze the weight of the edges and determine if they match. However, the problem with this approach is that the weights are not unique, which means that different edges could have the same value.
Two alternative options for resolving this issue are to assign a color or a value to the edges, or to maintain a list of signs for each node and compare them to see if there are any matches.

It may be possible to utilize a connected graph with a combination of the aforementioned options or an alternative approach, however, it would likely be complex and difficult to implement.

5.1.2 Tree

Another option that was considered was a tree structure, as illustrated in Figure 4. The advantage of this structure is that it simplifies the process of searching for the correct options, as only the child nodes of the selected node need to be returned. However, the disadvantage of this approach is that it increases the space complexity due to the presence of duplicate nodes, as demonstrated by the duplication of nodes H_1 and L_2 in Figure 4.

Figure 4: Example in tree structure



In the tree structure, the determination of the parent and child nodes should be considered. Unlike the connected graph, where starting from any node was possible, in the tree structure, the options can only be obtained from the top down. When starting from the middle or end of the tree, it may not be clear if duplicate nodes with the same value exist and are connected to other nodes.

The challenge lies in determining the order in which nodes should be added to the tree. In the previous chapter we discussed which set of properties would be the most advantageous to present to the user first. This information can be utilized to determine the which property set is to be added and designated as the parent node. Subsequently, the best options for child nodes can be evaluated in the same way. This process can be repeated until the entire tree structure is completed.

It may be concluded that this solution is the most optimal among the options considered, as it allows for efficient property searches through retrieval of the children of a selected node, although it may result in an increase in space complexity. This option does lack the ability to dynamically adjust the order of properties based on real time data, should such a requirement be desired in the future.

6 Discussion

In this paper, we have discussed the limitations of using the Dutch and NGT corpus for sign language search functionality. One major disadvantage is that searching for other datasets is no longer possible. Additionally, we have identified that handshape change and handedness could be useful search options, but currently, there is no available data to support this.
Furthermore, we have acknowledged that spoken Dutch frequencies may not align with NGT frequencies, as certain words such as 'deaf' are more prevalent in sign languages. To address this issue, it would be beneficial to conduct further research on NGT frequencies and strive to achieve a balance between the two.
Additionally, we have suggested that a study could be conducted to determine the most recognizable elements of a sign for someone who does not know a sign language and use those elements to search for a sign instead of relying on available data.
In addition, we have noted that the movement direction (and potentially other options) is entered as 0 or null, but it is unclear which of these options indicate that there is no movement in the sign or that the Signbank user did not fill it in.
Furthermore, we have acknowledged that choosing the best set is based on an assumption that may not be accurate. Lastly, we have suggested that there might be more suitable data structures than a graph, but this was not covered in this paper.

7 Conclusion

"How can the Signbank and Dutch corpus data be used to improve searching for a sign?"

In this study, the aim was to determine a method of utilizing Signbank and Dutch corpus data to enhance the search process for signs by its properties. A thorough examination of the available data in Signbank revealed that the properties of location, movement direction, and handshape were the most suitable. The properties were then combined with Dutch spoken frequency data to form a list of properties ordered by their relevance in the Dutch languages. It was assumed that these properties would be the most commonly selected.

To determine the optimal property set to present to the user, the ordered lists were analyzed using the standard deviation to measure their spread. This analysis showed that the spread of the first ten properties needed to be calculated to determine the best set of properties. Based on this data, the decision was made to implement a tree data structure for its simplicity in obtaining a new property set during the search process. The trade-off for this choice is an increase in space complexity.

Ultimately, the chosen solutions allow for a more efficient and faster search process through the utilization of Signbank and Dutch corpus data.

8 References

Cormen, T. H., & Leiserson, C. E. (2009). *Introduction to algorithms, 3rd edition*. MIT press.

Dominante hand (of: Sterke hand). (n.d.). https://www.gebareninzicht.nl/termview/55

Gross, J. L., & Yellen, J. (2005). *Graph theory and its applications*. CRC Press.

Hargrave, M. (2022). Standard deviation formula and uses vs. variance. https://www.investopedia.com/terms/s/standarddeviation.asp

Kazmier, L. J., Fulks, D. L., & Staton, M. K. (2003). *Business statistics*. McGraw-Hill.

Lestrade, S. (2017). Unzipping zipf's law. *PLOS ONE*, *12*(8). https://doi.org/10.1371/journal.pone.0181987

Wetzel, L. (2006). Types and tokens. https://plato.stanford.edu/entries/types-tokens/#DisBetTypTok

Zach. (2022). Mean absolute deviation vs. standard deviation: What's the difference? https://www.statology.org/mean-absolute-deviation-vs-standard-deviation/

A Signbank data

A.1 Handshape Change

2993 of the 4168 signs have a handshape change.

A.2 Relative orientation movement

2574 of the 4168 signs have a relative orientation movement.

A.3 Movement shape

461 of the 4168 signs have a movement shape.

A.4 Orientation change

541 of the 4168 signs have an orientation change.

A.5 Relative orientation location

707 of the 4168 signs have a relative orientation location.

A.6 Relation between articulators

392 of the 4168 signs have a relation between articulators.

A.7 Absolute orientation: palm

738 of the 4168 signs have a absolute orientation of the palm.

A.8 Contact Type

1471 of the 4168 signs have a contact type.

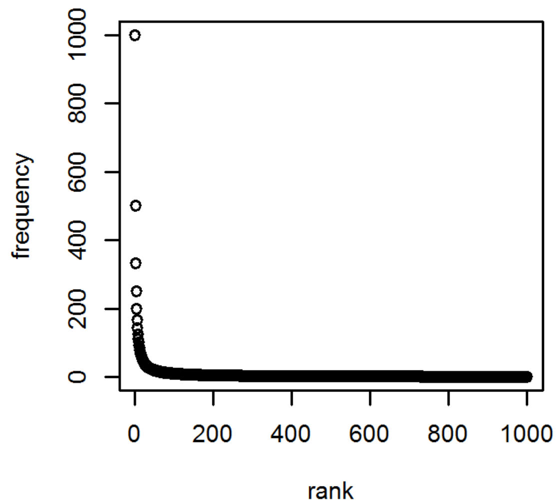
A.9 Named Entity and Semantic meaning

Is not relevante because the assumption is that the user does not know the meaning of the sign.

B Dutch corpus data

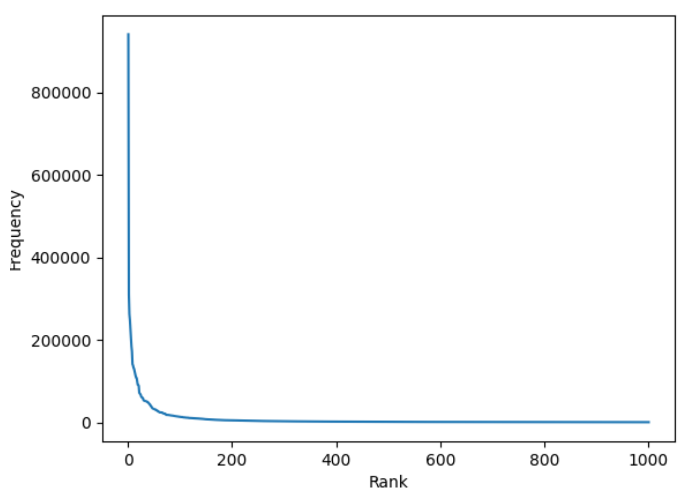
B.1 Data plots

Figure 5: Zipf curve



Source: (Lestrade, 2017,)

Figure 6: Plot of first 1000 items of the Dutch corpus data



B.2 Sum of frequencies

Location	**Amount**
Neutrale ruimte	2681746
Zwakke hand: palm	145312
Kin	49553
Mond	49553
Zwakke hand: duimkant	38752
Borst	33218
Schouder	30809
Borst Contra	26488
Variabel	21149
Bovenlichaam	17478

Movement direction	**Amount**
Naar voren	711479
Variabel	669674
Ipsilateraal	542608
Omlaag	450874
Omlaag en ipsilateraal	229560
Omhoog	181633
Contralateraal	71410
Naar voren en ipsilateraal	55915
Heen en weer	55003
Naar achteren	44117

Handshape	**Amount**
1	1420412
B	614491
5	203438
T	178787
V	166565
S	105630
W	79907
Y	55297
K	53718
B_hoek	45263

B.3 Translation with the highest frequency

Location	Amount
Neutrale ruimte	262631
Borst	11851
Zwakke hand: palm	71935
Borst Contra	21617
Schouder	19096
Zwakke hand: vingertoppen	13395
Bovenlichaam	13119
Kin	12517
Hoofd	12222
Zwakke hand: duimkant	7248

Movement direction	Amount
Variabel	262631
Omlaag en ipsilateraal	226189
Ipsilateraal	141417
Naar voren	132643
Omlaag	71935
Omhoog	58845
Naar voren en ipsilateraal	52947
Naar locatie toe	22560
Heen en weer	14983
Omlaag en contralateraal	12839

Handshape	Amount
1	262631
V	141417
B	132643
K	58845
5	52947
S	51873
W	27356
Y	21617
Snavel2 open spreid	18224
Snavel open	18092

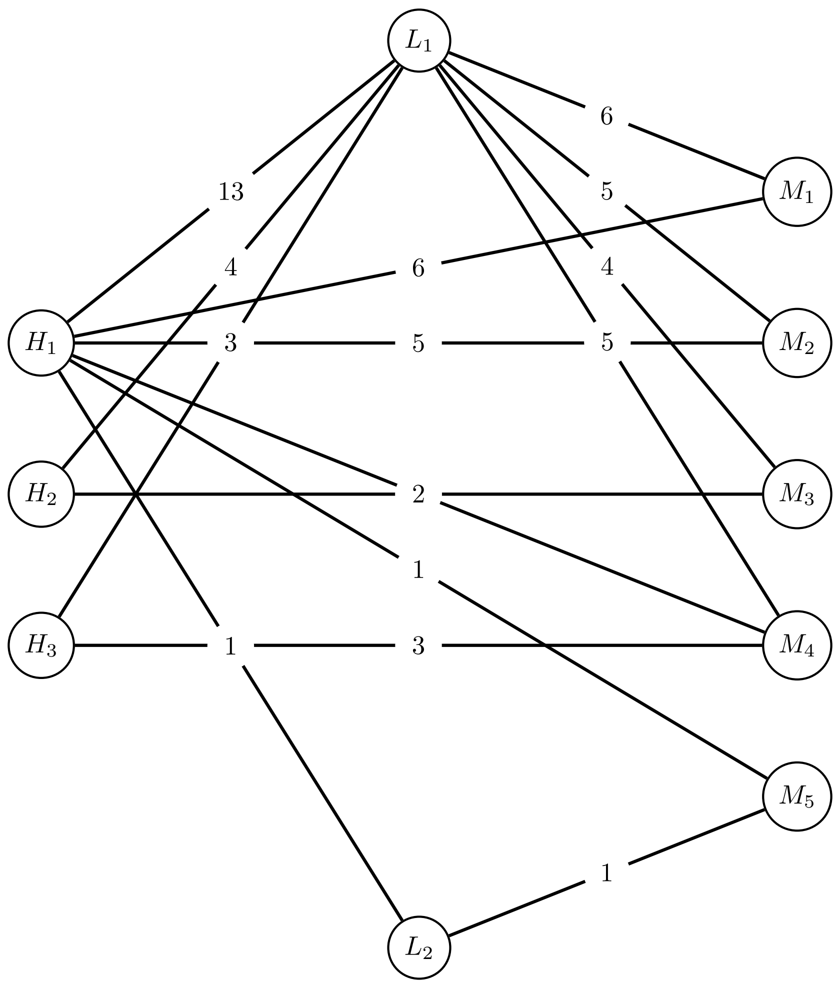
C Graph implementation

C.1 Used signs

1. L_1, M_1, H_1
2. L_1, M_2, H_1
3. L_1, M_3, H_2
4. L_1, M_4, H_3
5. L_1, M_4, H_1
6. L_2, M_5, H_1

C.2 Connected graph

Figure 7: Connected graph



C.3 Tree

Figure 8: Tree configuration