2021 年春季学期/数理统计/第十四周/课后作业解答

龚梓阳

更新：2021 年 6 月 26 日

1 **证明.** (a). 犯第一类错误的概率为

$$\begin{aligned}\alpha =&P\left\{\bar{x} \in W \mid H_0\right\} = P\{\bar{x} \geq 2.6 \mid \mu = 2\} \\ =&P\left\{\frac{\bar{x}-\mu}{1/\sqrt{n}} \geq \frac{2.6-2}{1/\sqrt{20}} = 2.68\right\} = 1-\Phi(2.68) = 0.0037\end{aligned}$$

犯第二类错误的概率为

$$\begin{aligned}\beta =&P\left\{\bar{x} \notin W \mid H_1\right\} = P\{\bar{x} < 2.6 \mid \mu = 3\} \\ =&P\left\{\frac{\bar{x}-\mu}{1/\sqrt{n}} < \frac{2.6-3}{1/\sqrt{20}} = -1.79\right\} = \Phi(-1.79) = 0.0367\end{aligned}$$

(b). 由于

$$\beta = P\{\bar{x} < 2.6 \mid \mu = 3\} = P\left\{\frac{\bar{x}-\mu}{1/\sqrt{n}} < \frac{2.6-3}{1/\sqrt{n}} = -0.4\sqrt{n}\right\} = \Phi(-0.4\sqrt{n}) \leq 0.01$$

则

$$\Phi(0.4\sqrt{n}) \geq 0.99 \Rightarrow 0.4\sqrt{n} \geq 2.33 \Rightarrow n \geq 33.93$$

故 n 至少为 34。

(c).

$$\begin{aligned}\alpha =&P\{\bar{x} \geq 2.6 \mid \mu = 2\} = P\left\{\frac{\bar{x}-\mu}{1/\sqrt{n}} \geq \frac{2.6-2}{1/\sqrt{n}} = 0.6\sqrt{n}\right\} \\ =&1-\Phi(0.6\sqrt{n}) \to 0, \quad n \to \infty \\ \beta =&P\{\bar{x} < 2.6 \mid \mu = 3\} = P\left\{\frac{\bar{x}-\mu}{1/\sqrt{n}} < \frac{2.6-3}{1/\sqrt{n}} = -0.4\sqrt{n}\right\} \\ =&\Phi(-0.4\sqrt{n}) \to 0, \quad n \to \infty\end{aligned}$$

2 **证明.** 因 $X \sim \mathrm{b}(1,p)$，有 $\sum_{i=1}^{10} x_i = 10\bar{x} \sim \mathrm{b}(10,p)$。则

$$\begin{aligned}\alpha =&P\left\{\bar{x} \in W \mid H_0\right\} = P\{\bar{x} \geq 0.5 \mid p = 0.2\} = P\{10\bar{x} \geq 5 \mid p = 0.2\} \\ =&\sum_{k=5}^{10} C_{10}^{k} \cdot 0.2^{k} \cdot 0.8^{10-k} = 0.0328\end{aligned}$$

$$\begin{aligned}\beta =&P\left\{\bar{x} \notin W \mid H_1\right\} = P\{\bar{x} < 0.5 \mid p = 0.4\} = P\{10\bar{x} < 5 \mid p = 0.4\} \\ =&\sum_{k=0}^{4} C_{10}^{k} \cdot 0.4^{k} \cdot 0.6^{10-k} = 0.6331\end{aligned}$$

4 **证明.** 因均匀分布最大顺序统计量 $x_{(n)}$ 的密度函数为

$$p_n(x)=\frac{nx^{n-1}}{\theta^n},\quad 0<x<\theta$$

则

$$\begin{aligned}\alpha=&P\{\bar{x}\in W\mid H_0\}=P\left\{x_{(n)}\leq 2.5\mid\theta=3\right\}\\=&\int_0^{2.5}\frac{nx^{n-1}}{3^n}\,\mathrm{d}x=\left.\frac{x^n}{3^n}\right|_0^{2.5}=\frac{2.5^n}{3^n}=\left(\frac{5}{6}\right)^n\end{aligned}$$

要使得 $\alpha\leq 0.05$，即

$$\left(\frac{5}{6}\right)^n\leq 0.05\Rightarrow n\geq\frac{\ln 0.05}{\ln(5/6)}=16.43$$

故 n 至少为 17。

5 **证明.** 若检验结果是接受原假设，当原假设为真时，是正确的决策，未犯错误；当原假设不真时，则犯了第二类错误。若检飼结果是拒绝原假设，当原假设为真时，则犯了第一类错误；当原假设不真时，是正确的决策，未犯错误。