PROBLEM SET 1: GEO2300:
DUE: 16 SEPT. 2020

GEO2300: FYSISKE PROSESSER I GEOFAG

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1. Problem 1: Matricies

In this section we will study the properties of matricies. How they are used to solve a system of linear equations, inverting matricies and finding eiegenvalues and eigenvectors, and lastly expressing matricies as a sum of eigenvectors and exponentials with corresponding eigenvalues.

First we will look at solving the following system of linear equations as a matrix equation

$$\begin{aligned} x + 2y + z &= -1 \\ 2x - y + 3z &= -5 \\ -x + 3y - z &= 6 \end{aligned} \tag{1}$$

We can rewrite Eq: 1 on the form $\mathbf{A}\vec{x} = \vec{b}$

$$\begin{bmatrix} 1 & 2 & 1 & -1 \\ 2 & -1 & 3 & -5 \\ -1 & 3 & -1 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -5 \\ 6 \end{bmatrix} \tag{2}$$

We can rewrite this system on the form $[\mathbf{A} : \vec{b}]$ and solve the augmented matrix by finding the row reduced echelon form. We then have

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & -1 & 3 & -5 \\ -1 & 3 & -1 & 6 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -5 & 1 & -7 \\ 0 & 5 & 0 & 7 \end{array}\right] \tag{3}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 7/5 \\ 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -9/5 \\ 0 & 1 & 0 & 7/5 \\ 0 & 0 & 1 & 0 \end{array}\right] \tag{4}$$

giving us that the solution to the set of linear equations given in Eq: 1 is given by

$$\begin{aligned} x &= -9/5 \\ y &= 7/5 \\ z &= 0 \end{aligned}$$

This can easily be confirmed numerically by doing $rref([A : b])$.

Second, we will look at inversing matrixes, both analytically for the 2x2-matrix and numerically for the others. Looking at the 2x2-matrix we have that in the general case

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

We then have for our 2x2-matrix as follows

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, \quad A^{-1} = \frac{1}{3 \cdot 1 - (-1 \cdot 2)} \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & -2 \\ 1 & 1 \end{bmatrix} \tag{5}$$

For the two next matricies being a 3x3 and 4x4 matrix, which are fairly tedious to solve by hand, we will use functions in Python 3 to do the inverting for us, spesifically the Numpy library.

```python
import numpy as np

A = np.Matrix([[1, 2, 5], [-1, 3, -1], [2, 1, -2]])  # 3x3 matrix
B = np.Matrix([[1, 2, 5, 2], [-1, 3, -1, -1],
               [2, 1, -2, 1], [1, -1, 1, -1]])  # 4x4 matrix
A = np.linalg.inv(A)
B = np.linalg.inv(B)
```

Which gives the following output

2. Problem 2: Poisueille Flow

3. Problem 3: More finite differences