Oblig 2: Advection and Diffusion; **Due 12 Oct. 2020**

Problem 1: Tsunami

A tsunami is a surface gravity wave in the ocean that moves quickly and can cause serious damage in coastal regions. Let's model this is using the 1D wave equation in x. The initial disturbance is assumed to be Gaussian and propagates to the left, obeying this equation:

$$\frac{\partial}{\partial t}h - c_0\frac{\partial}{\partial x}h = 0$$

The speed, c_0 is very large in deep water, about 700 km/hr!

If the initial wave is centered at $x = 0$, the amplitude is 1.0 m and the standard deviation is 50 km, so that at $t = 0$:

$$h = 1.0\ exp(-\frac{x^2}{50^2})$$

a) Write the expression for the moving wave.

b) Write the wave equation in finite difference form, using the FTCS scheme.

c) Write this in matrix form, using 7 grid points, two on the boundaries and 5 in the interior. Let the boundary values be for j=0 and j=6, and the interior ones for j=1,2,3,4,5. Use Neumann conditions at the two ends, such that:

$$\frac{\partial}{\partial x}h = 0$$

d) Now code up the FTCS scheme. Let the domain run from $x = -1400$ km to $x = 300$ km, and use a grid spacing so that there are 400 grid points. Set the velocity to $c_0 = 700$ km/hr and the time step, dt, so that the Courant number is $C = c_0 dt/dx = 0.1$.

e) Run the code and plot the results at $t = 0, 0.5, 1.0, 1.5, 2$ hours. Plot the numerical solutions over the analytical ones for the same times. How well does the code do?

f) Now write the finite difference equation using the Crank-Nicholson scheme. Express this in matrix form, using 7 total grid points.

g) Code this up, run it and plot the results at the same time steps as in (e), again with the analytical solutions. Compare the results to the FTCS scheme.

Problem 2: Boundary diffusion

Consider a one dimensional aquifer with oil seeping into one end, as discussed in class. The oil concentration obeys the 1D diffusion equation:

$$\frac{\partial}{\partial t}C = D\frac{\partial^2}{\partial x^2}C$$

a) What is the analytical solution for $C(x,t)$? Assume $C = C_0$ at $x = 0$ and that $C(x,0) = 0$.

b) Assume the diffusivity is given by $D = 1$ cm^2/sec and that the oil concentration at the end is 50%. Plot the concentration as a function of x for $t = 1$ day, $t = 10$ days and 100 days.

c) How long does it take for the concentration to reach 25% at a distance of 100 m from the source? At 1 km from the source? How much longer do you have to wait if the distance is increased by a factor of α?

d) Write the difference equation for this problem in matrix form, using the FTCS scheme. Assume you have 7 grid points, two on the boundaries and 5 interior. Set $C(x = 0) = 0.5$ and $C(x = 100) = 0$.

e) Code this up for the case with N points. Reproduce the plots in (b) numerically. Set s=0.4. Plot the analytical solution over this. How well do they agree?

f) Now set s=0.6. What happens?