Backpropagation

反向传播

Gradient Descent

Network parameters $\theta = \{w_1, w_2, \cdots, b_1, b_2, \cdots\}$

Starting Parameters $\theta^0 \longrightarrow \theta^1 \longrightarrow \theta^2 \longrightarrow \cdots\cdots$

$$\nabla L(\theta) = \begin{bmatrix} \partial L(\theta)/\partial w_1 \\ \partial L(\theta)/\partial w_2 \\ \vdots \\ \partial L(\theta)/\partial b_1 \\ \partial L(\theta)/\partial b_2 \\ \vdots \end{bmatrix}$$

$Compute\ \nabla L(\theta^0)$ $\quad \theta^1 = \theta^0 - \eta \nabla L(\theta^0)$

$Compute\ \nabla L(\theta^1)$ $\quad \theta^2 = \theta^1 - \eta \nabla L(\theta^1)$

Millions of parameters ……

To compute the gradients efficiently, we use ***backpropagation***.

Chain Rule

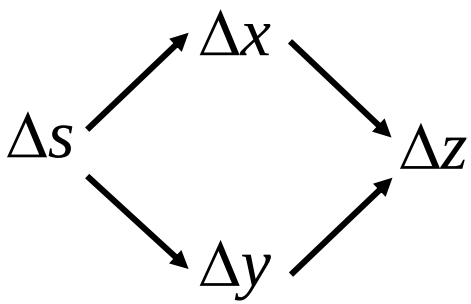
Case 1 $y = g(x) \quad z = h(y)$

$$\Delta x \to \Delta y \to \Delta z$$

$$\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$$

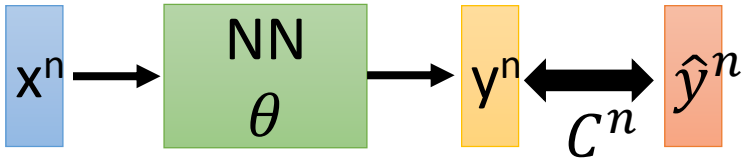
Case 2

$$x = g(s) \qquad y = h(s) \qquad z = k(x, y)$$



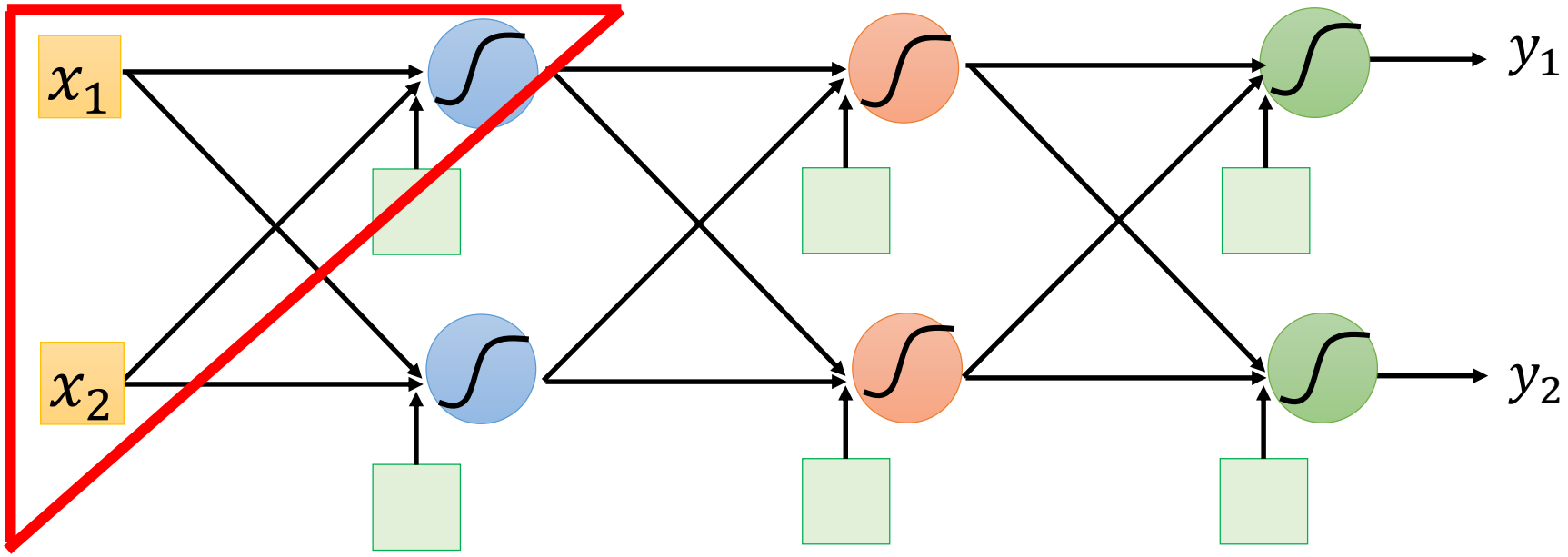
$$\frac{dz}{ds} = \frac{\partial z}{\partial x}\frac{dx}{ds} + \frac{\partial z}{\partial y}\frac{dy}{ds}$$

Backpropagation

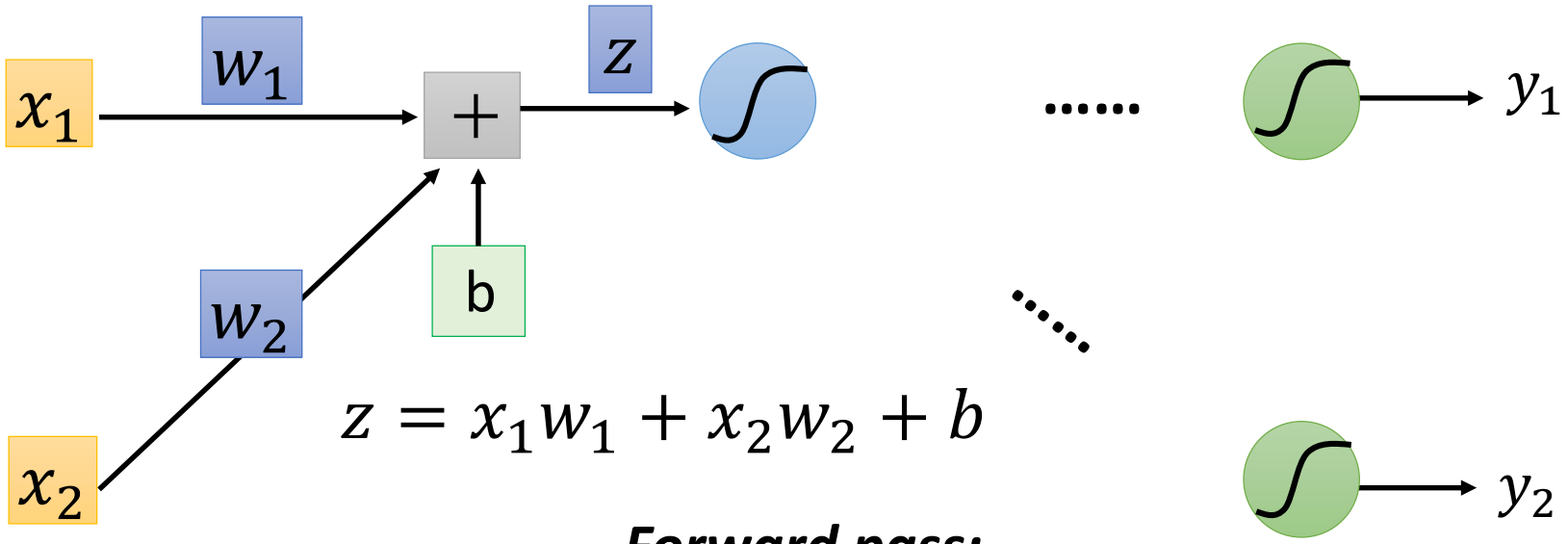


$$L(\theta) = \sum_{n=1}^{N} C^n(\theta) \quad \Rightarrow \quad \frac{\partial L(\theta)}{\partial w} = \sum_{n=1}^{N} \frac{\partial C^n(\theta)}{\partial w}$$

先考虑某个neuron



Backpropagation



$$z = x_1 w_1 + x_2 w_2 + b$$

$$\frac{\partial C}{\partial w} = ?$$

$$\frac{\partial z}{\partial w}\frac{\partial C}{\partial z}$$

(Chain rule)

Forward pass:

Compute $\partial z/\partial w$ for all parameters

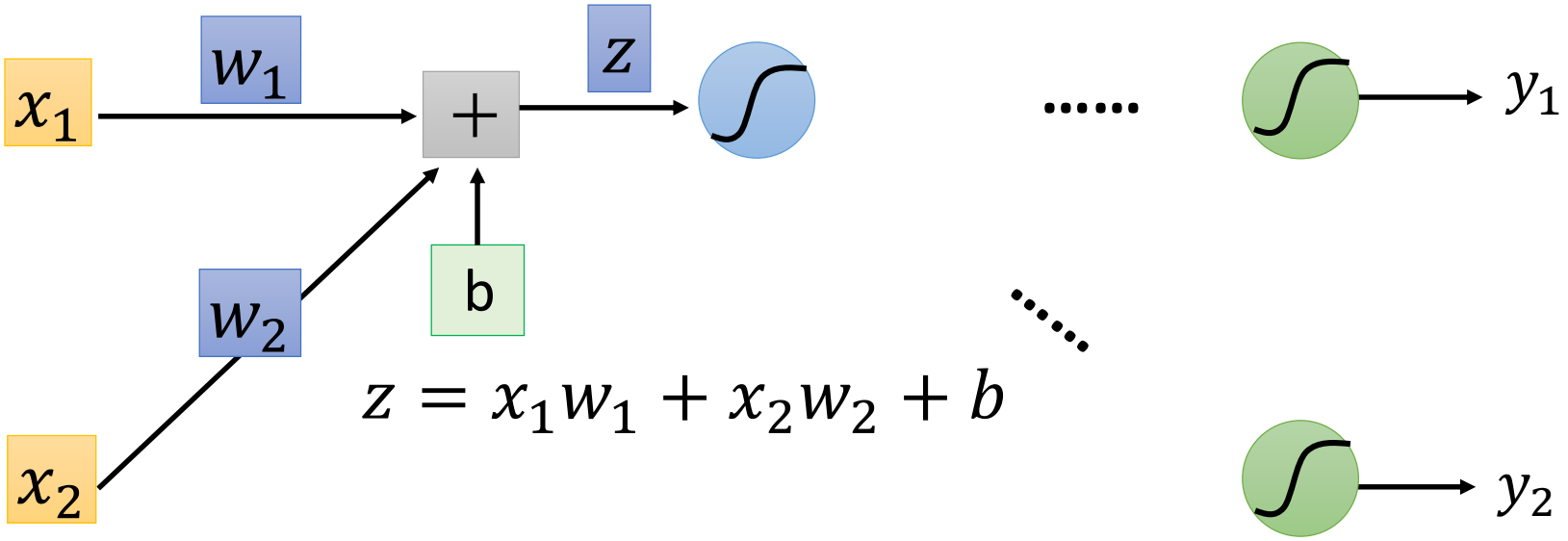
（简单）

Backward pass:

Compute $\partial C/\partial z$ for all activation function inputs z （复杂）

Backpropagation – Forward pass

Compute $\partial z/\partial w$ for all parameters



$\partial z/\partial w_1 =?\ x_1$

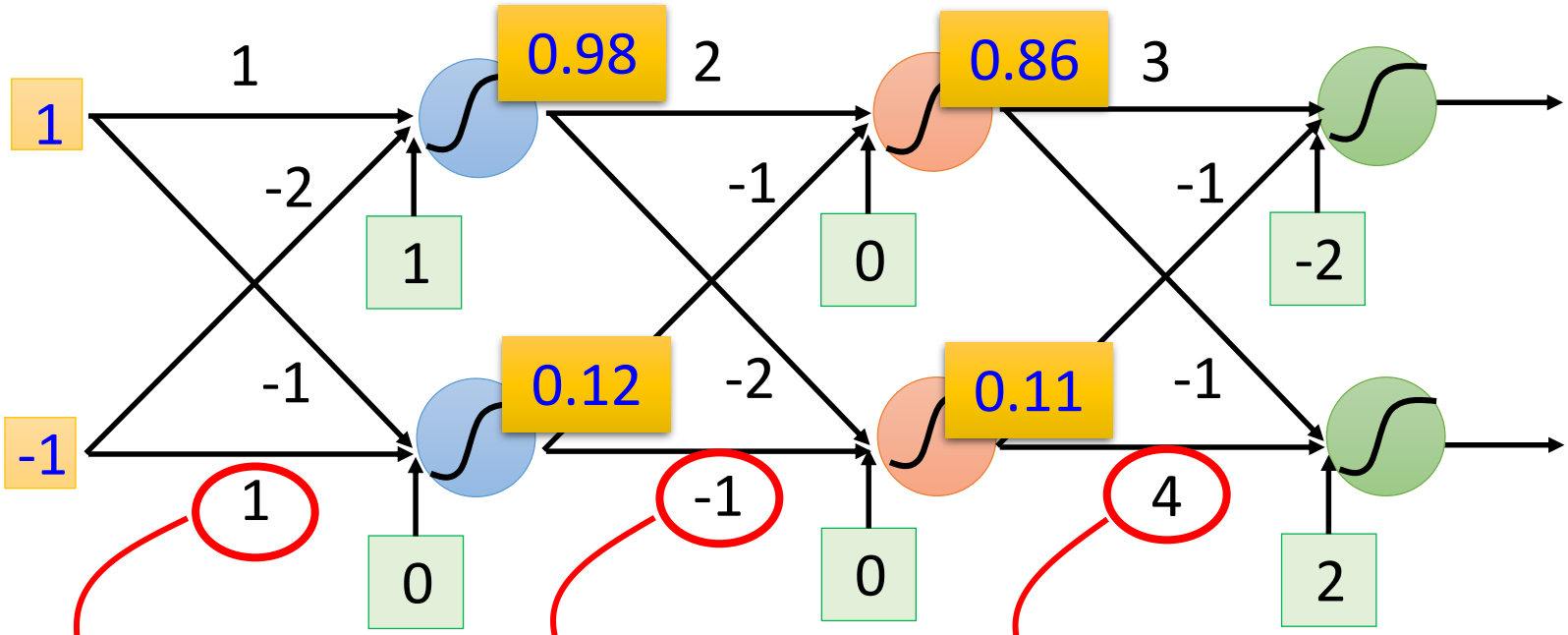
$\partial z/\partial w_2 =?\ x_2$

The value of the input connected by the weight

Backpropagation – Forward pass

Compute $\partial z/\partial w$ for all parameters

算出每个neuron的值就可以算出来forward pass的结果了



$$\frac{\partial z}{\partial w} = -1$$

前面接的input是-1

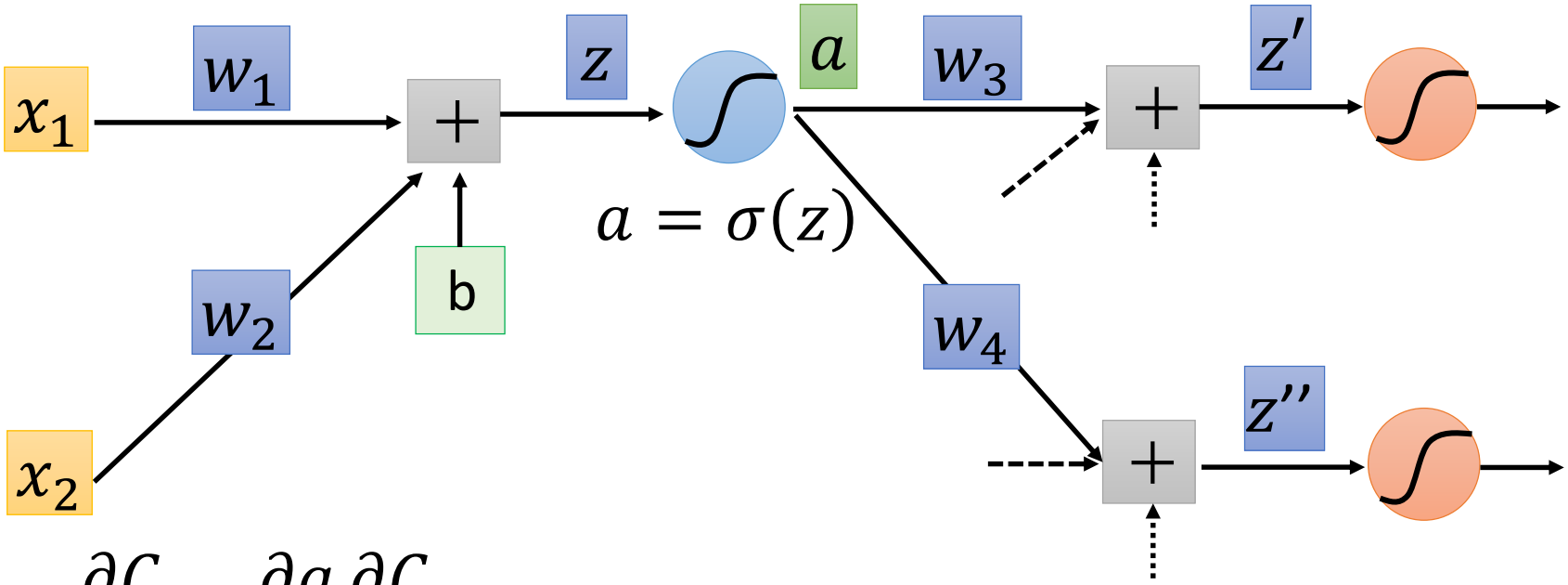
$$\frac{\partial z}{\partial w} = 0.12$$

前面接的input是0.12

$$\frac{\partial z}{\partial w} = 0.11$$

Backpropagation – Backward pass

Compute $\partial C/\partial z$ for all activation function inputs z



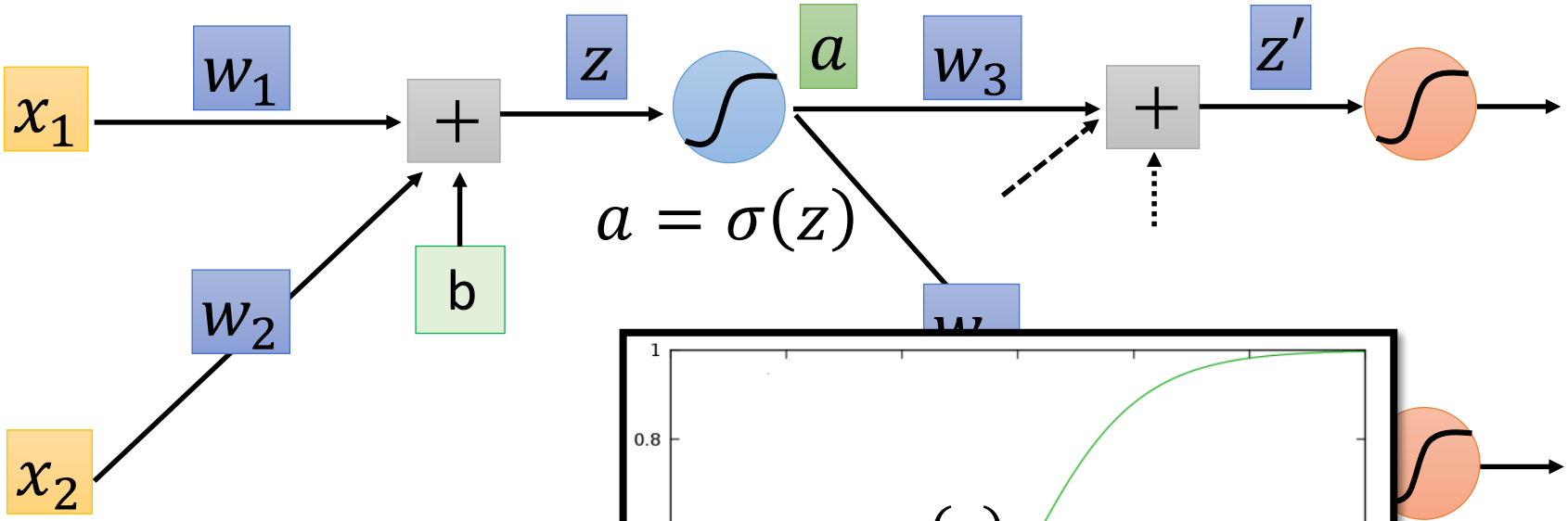
$$\frac{\partial C}{\partial z} = \frac{\partial a}{\partial z}\frac{\partial C}{\partial a}$$

$\sigma'(z)$

就是sigmoid函数的微分

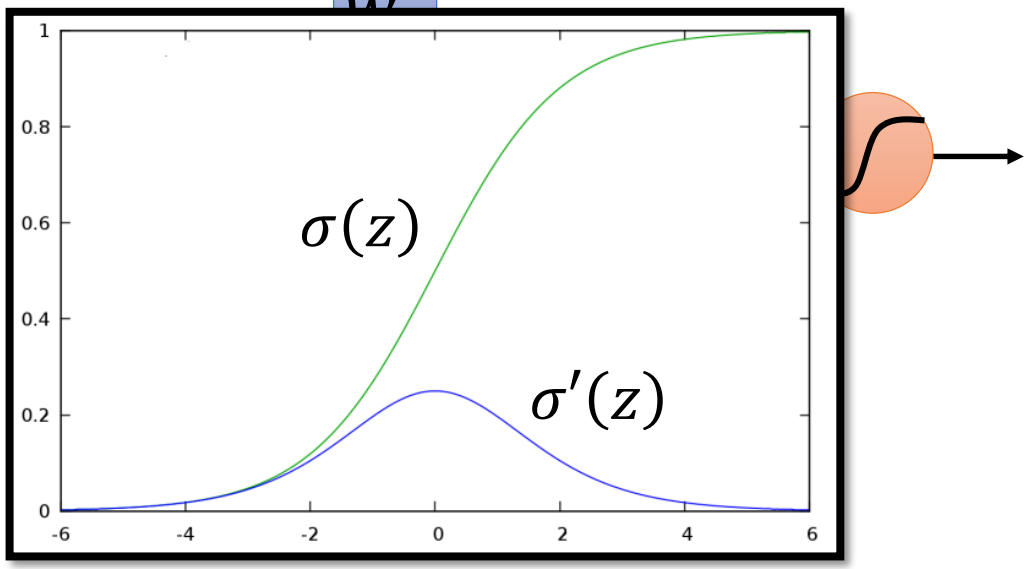
Backpropagation – Backward pass

Compute $\partial C / \partial z$ for all activation function inputs z



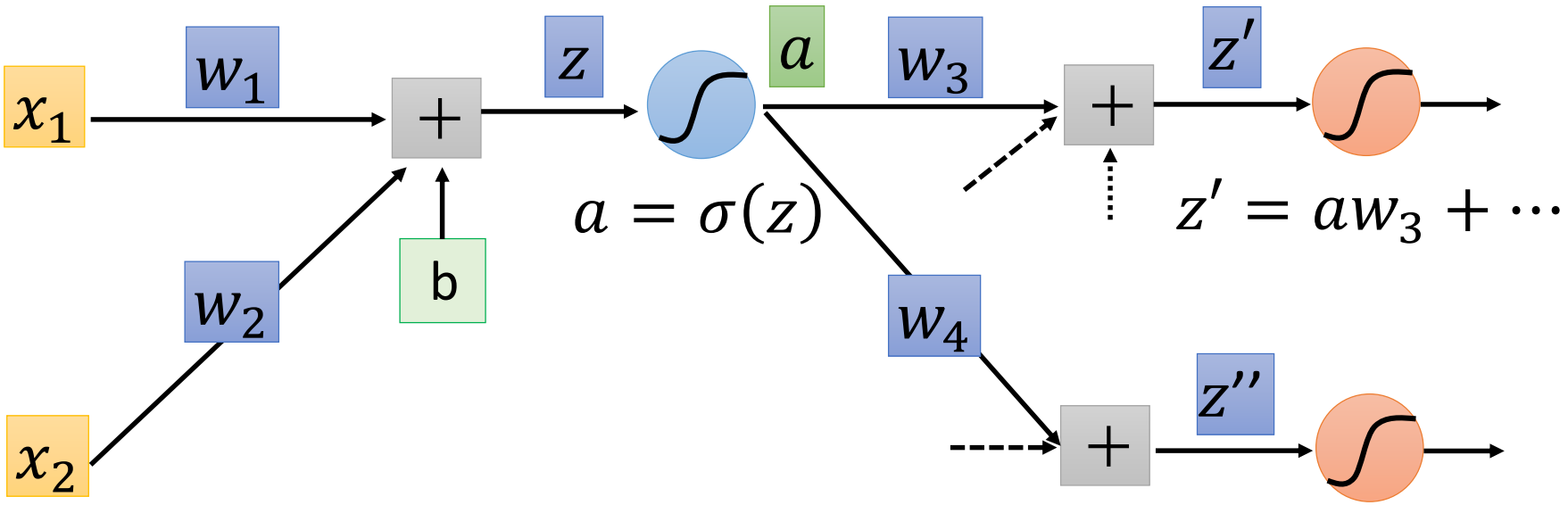
$$\frac{\partial C}{\partial z} = \frac{\partial a}{\partial z}\frac{\partial C}{\partial a}$$

$\Rightarrow \sigma'(z)$



Backpropagation – Backward pass

Compute $\partial C/\partial z$ for all activation function inputs z



$$\frac{\partial C}{\partial z} = \frac{\partial a}{\partial z}\frac{\partial C}{\partial a} \qquad \frac{\partial C}{\partial a} = \frac{\partial z'}{\partial a}\frac{\partial C}{\partial z'} + \frac{\partial z''}{\partial a}\frac{\partial C}{\partial z''}$$

(Chain rule)

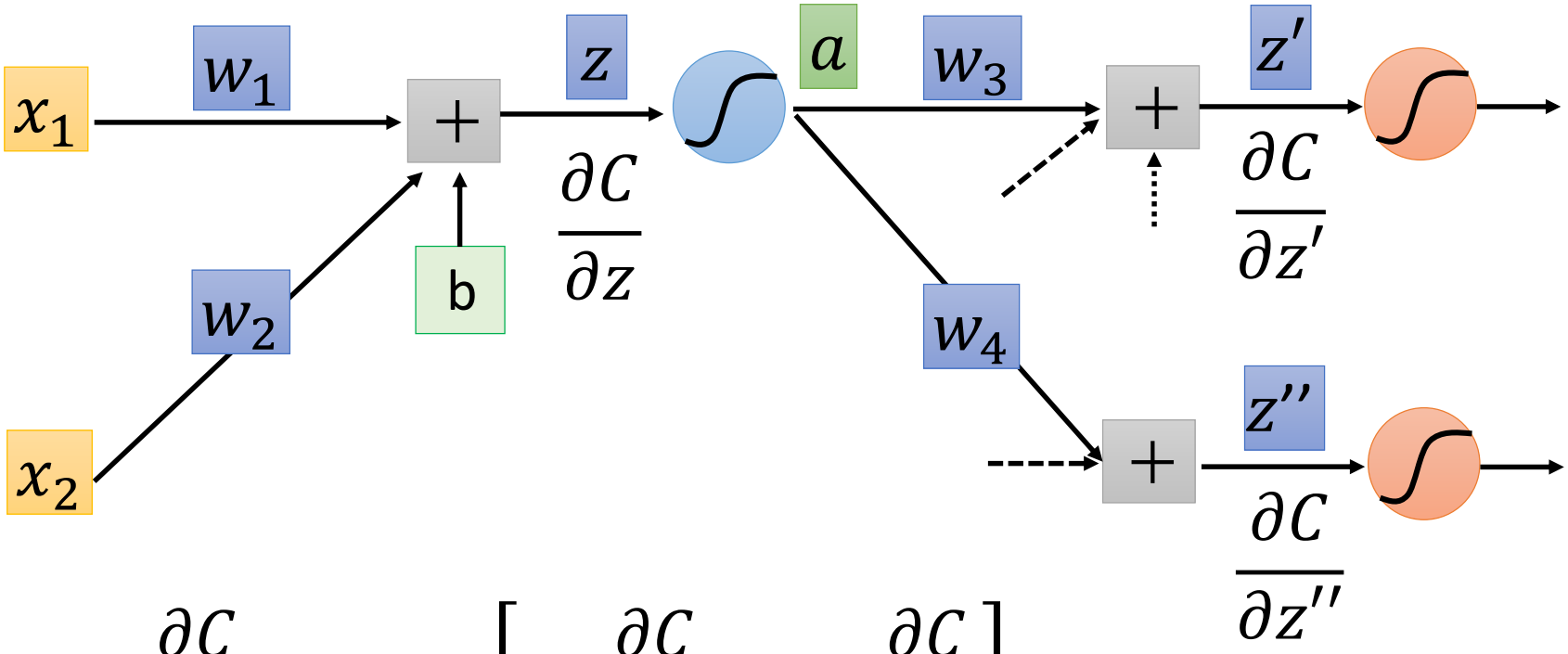
$\frac{\partial z'}{\partial a} = w_3$, $\frac{\partial C}{\partial z'}$ = ?, $\frac{\partial z''}{\partial a} = w_4$, $\frac{\partial C}{\partial z''}$ = ?

Assumed it's known

假设我们知道这两项的值

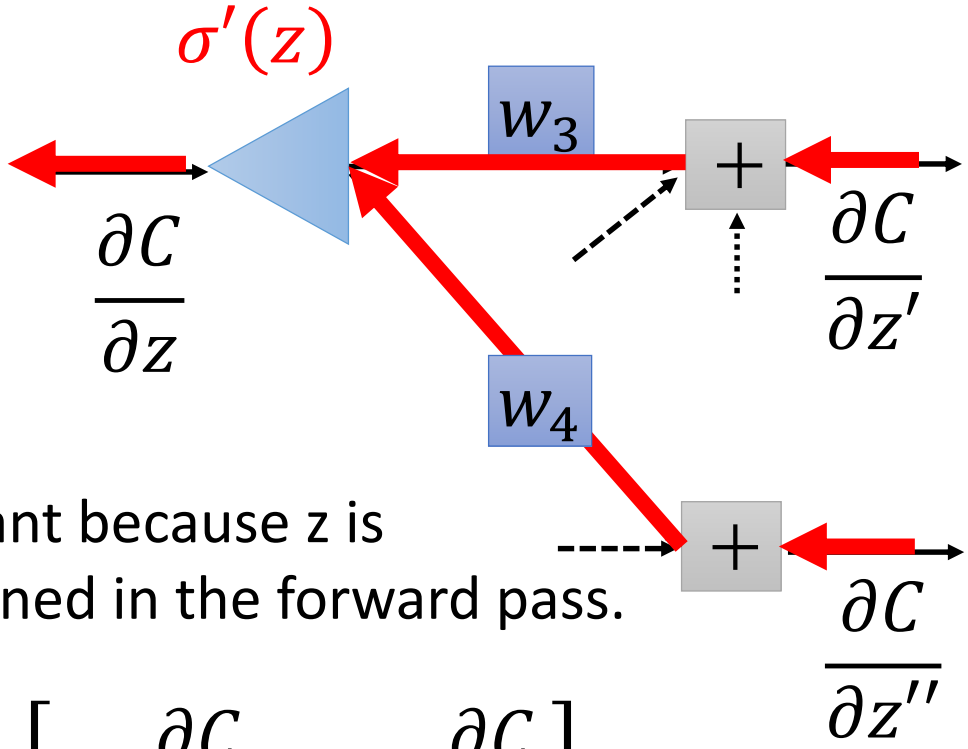
Backpropagation – Backward pass

Compute $\partial C / \partial z$ for all activation function inputs z



$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3 \frac{\partial C}{\partial z'} + w_4 \frac{\partial C}{\partial z''}\right]$$

Backpropagation – Backward pass



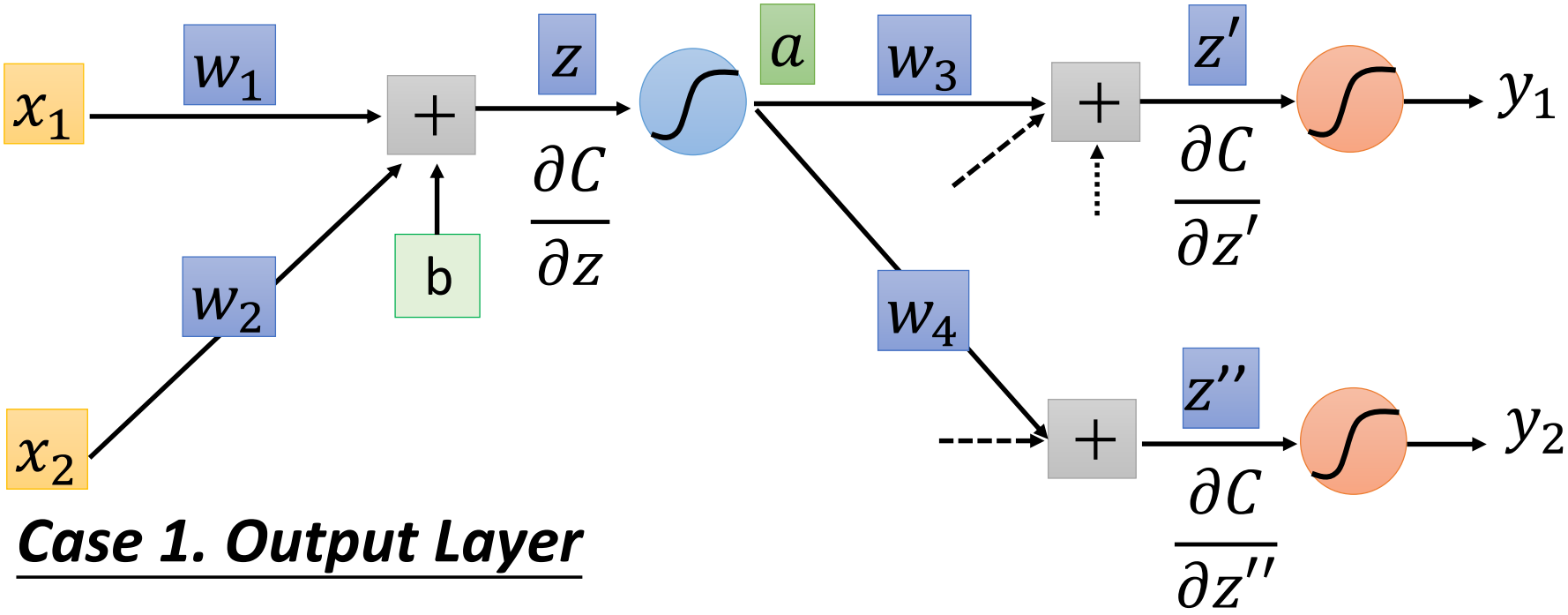
$\sigma'(z)$ is a constant because z is already determined in the forward pass.

$$\frac{\partial C}{\partial z} = \sigma'(z)\left[w_3\frac{\partial C}{\partial z'} + w_4\frac{\partial C}{\partial z''}\right]$$

这一项是常数

Backpropagation – Backward pass

Compute $\partial C/\partial z$ for all activation function inputs z



Case 1. Output Layer

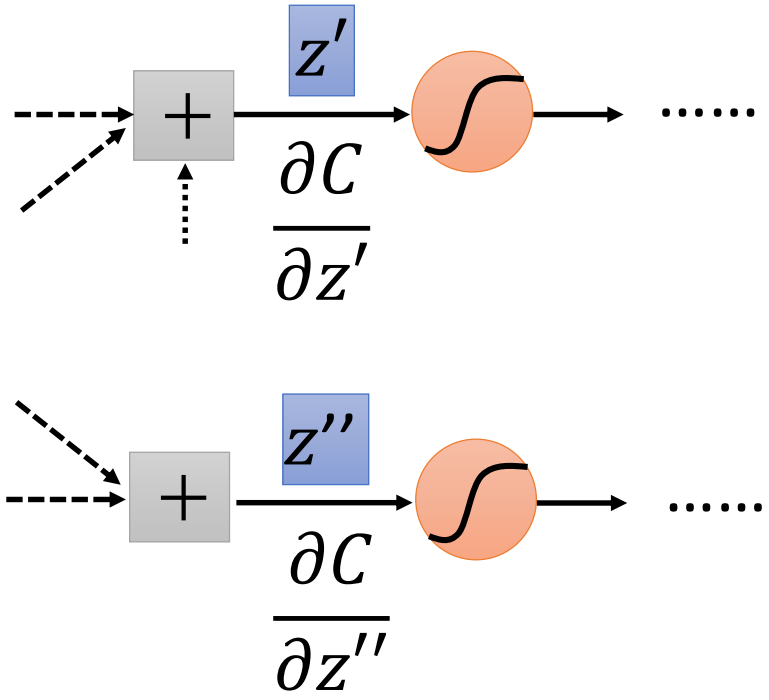
$$\frac{\partial C}{\partial z'} = \frac{\partial y_1}{\partial z'}\frac{\partial C}{\partial y_1} \qquad \frac{\partial C}{\partial z''} = \frac{\partial y_2}{\partial z''}\frac{\partial C}{\partial y_2}$$

Done!

Backpropagation – Backward pass

Compute $\partial C / \partial z$ for all activation function inputs z

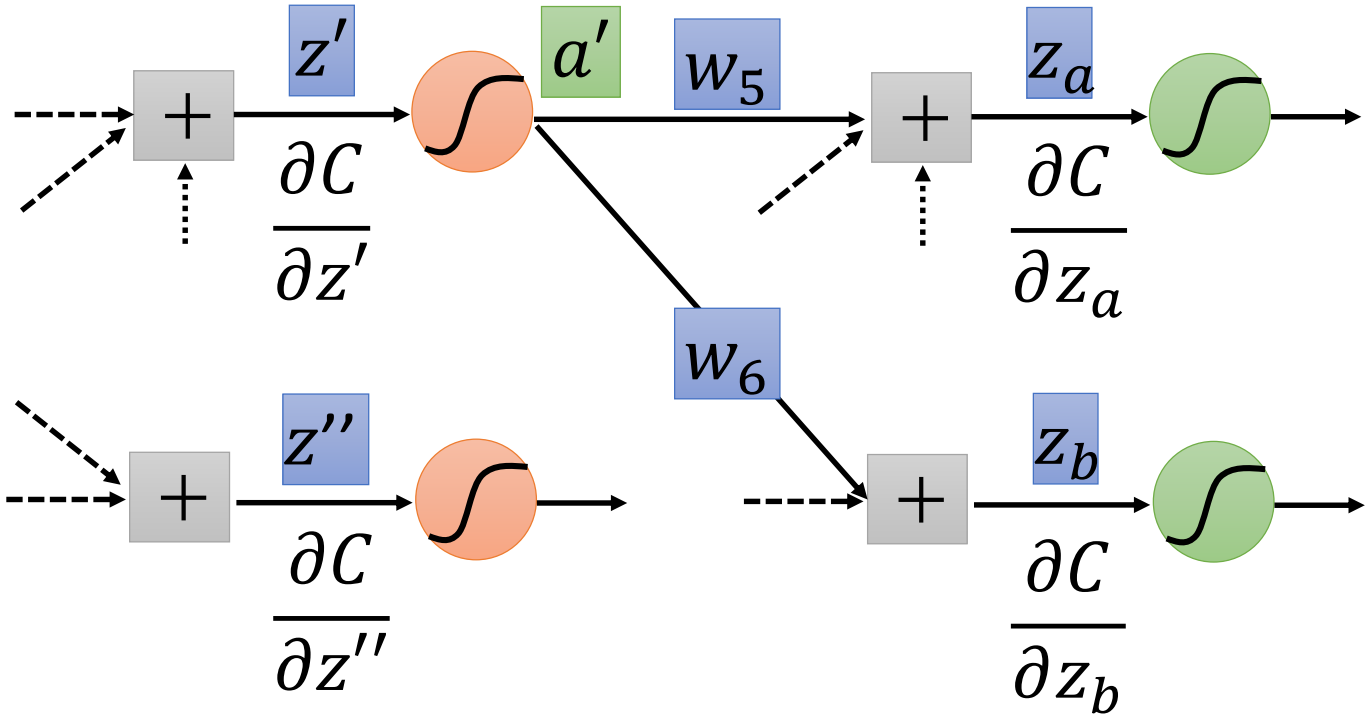
Case 2. Not Output Layer



Backpropagation – Backward pass

Compute $\partial C / \partial z$ for all activation function inputs z

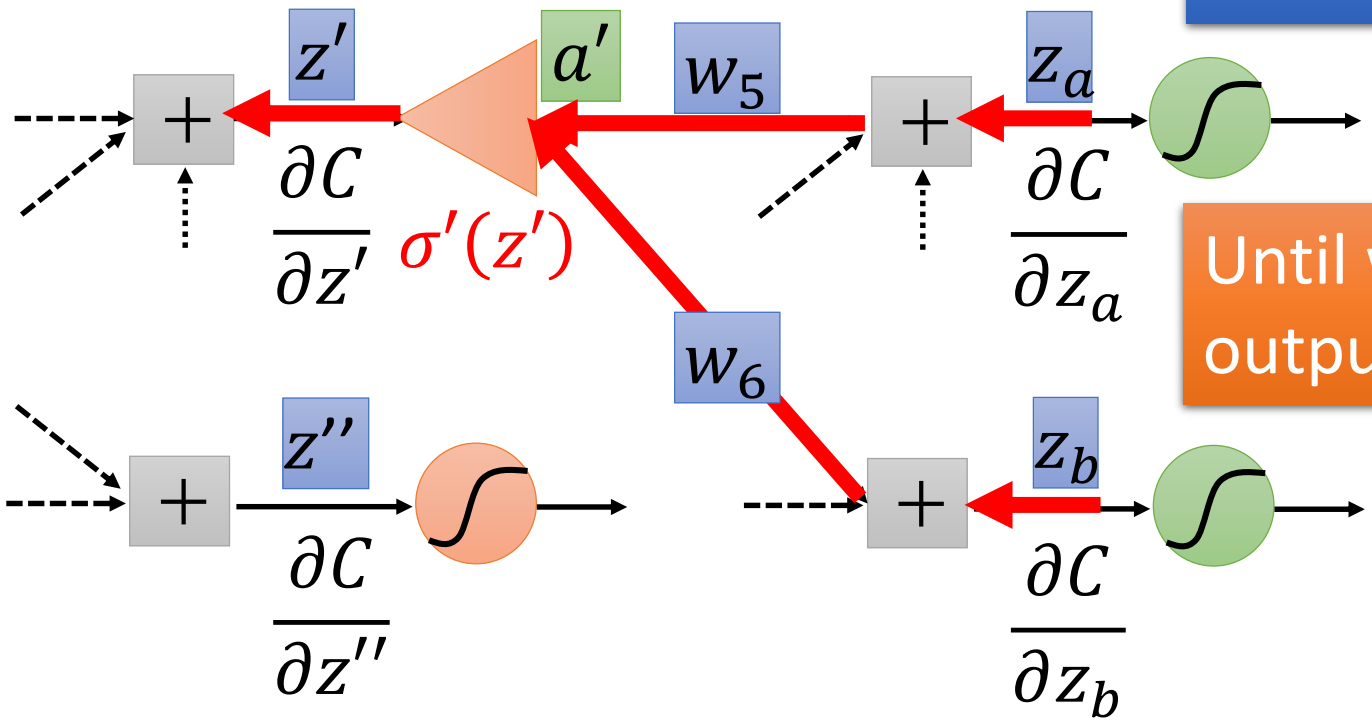
Case 2. Not Output Layer



Backpropagation – Backward pass

Compute $\partial C/\partial z$ for all activation function inputs z

Case 2. Not Output Layer



Compute $\partial C/\partial z$ recursively

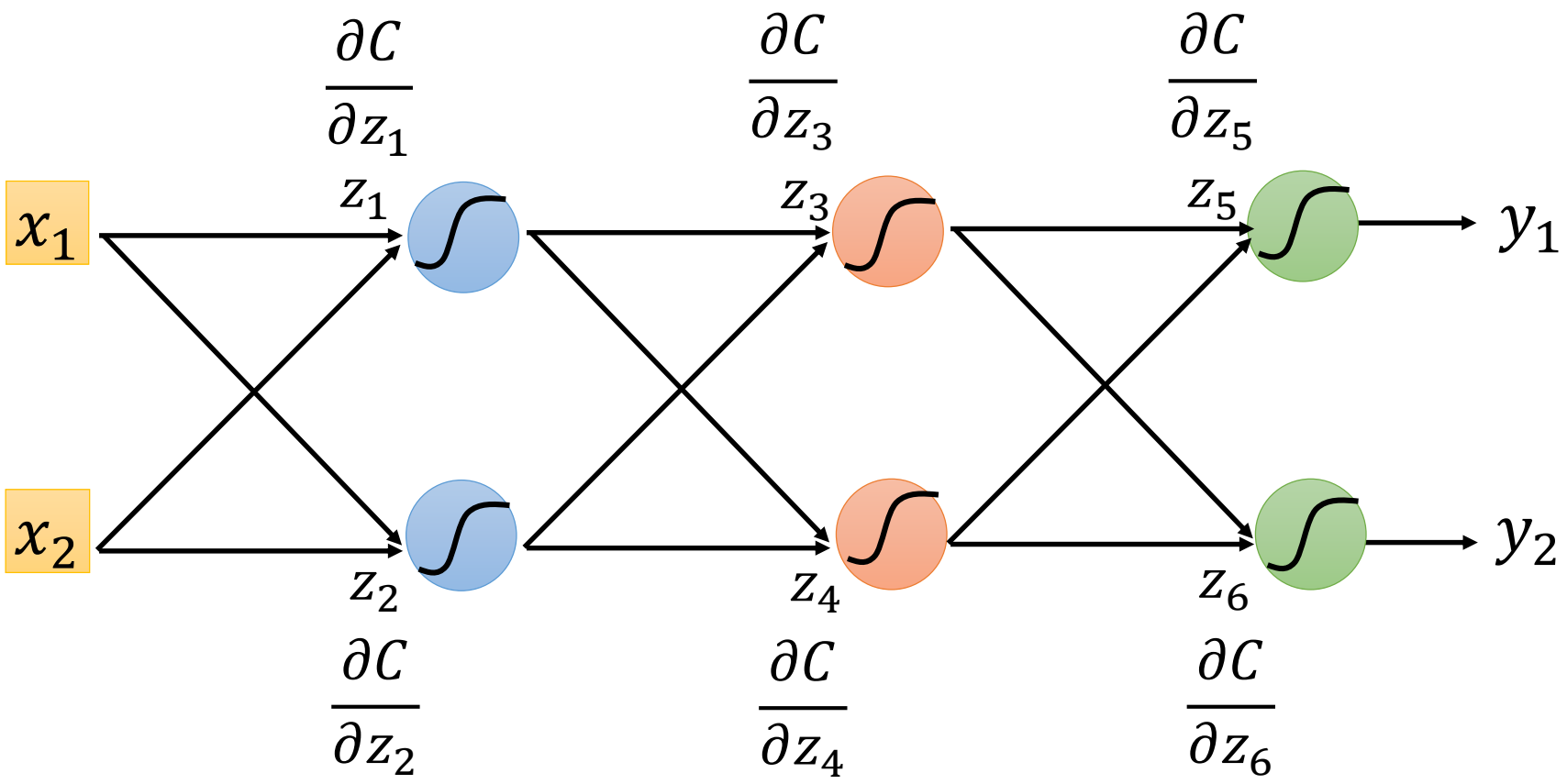
递归直到输出层

Until we reach the output layer ……

Backpropagation – Backward Pass

Compute $\partial C/\partial z$ for all activation function inputs z

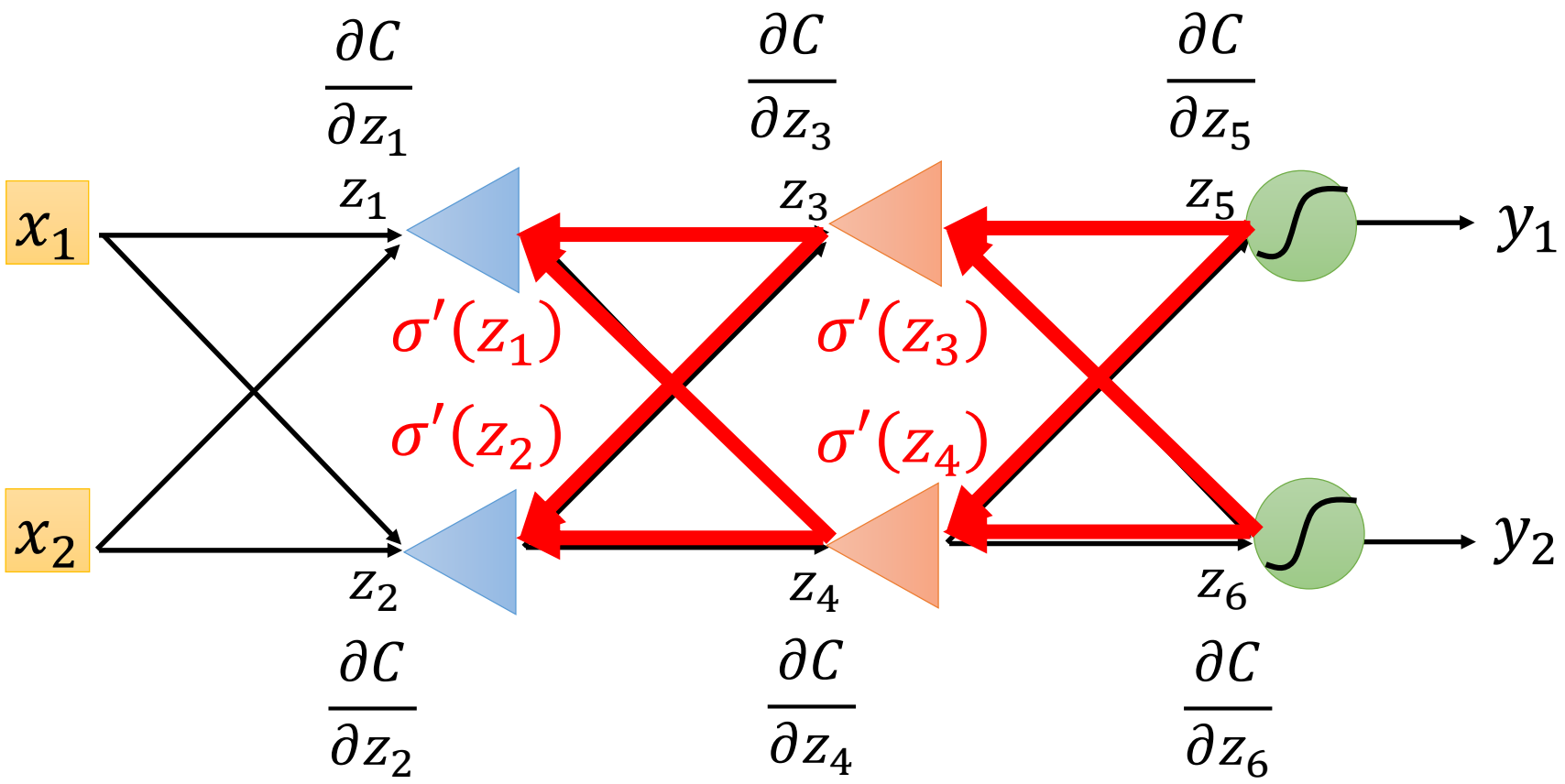
Compute $\partial C/\partial z$ from the output layer



Backpropagation – Backward Pass

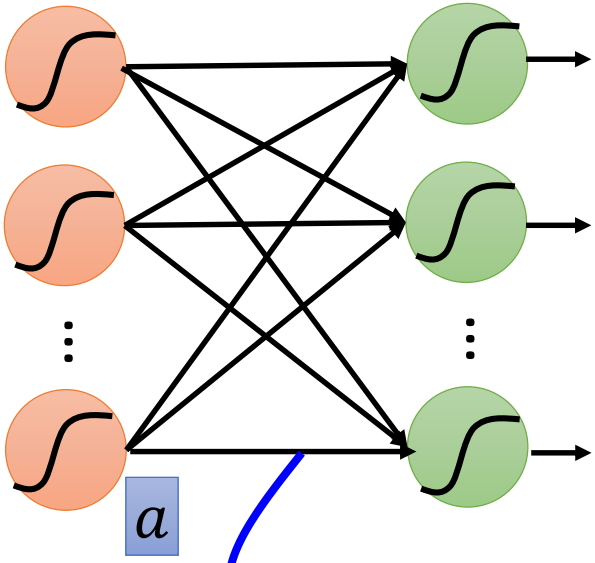
Compute $\partial C/\partial z$ for all activation function inputs z

Compute $\partial C/\partial z$ from the output layer



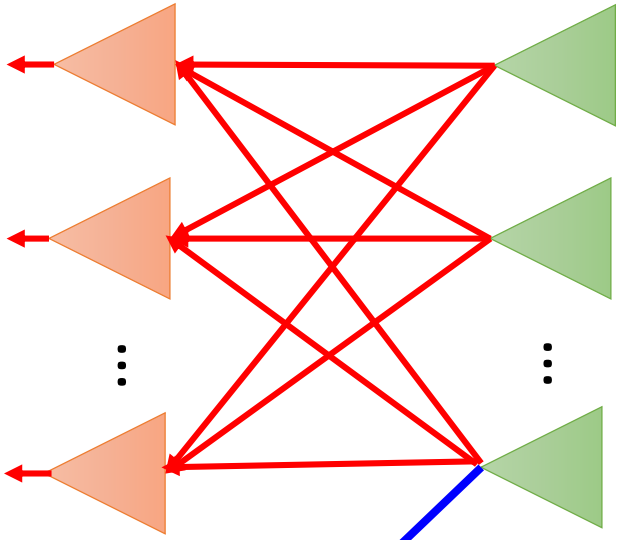
Backpropagation – Summary

Forward Pass



$$\frac{\partial z}{\partial w} = a$$

Backward Pass



$$\times \quad \frac{\partial C}{\partial z} = \frac{\partial C}{\partial w}$$

for all w