

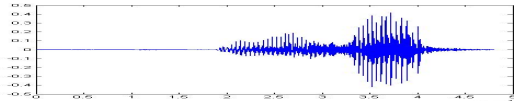
Introduction of Machine / Deep Learning

Hung-yi Lee 李宏毅 机器学习/深度学习介绍

Machine Learning

≈ Looking for Function


- Speech Recognition 语音识别

$$f(\text{  }) = \text{"How are you"}$$

- Image Recognition 图像识别

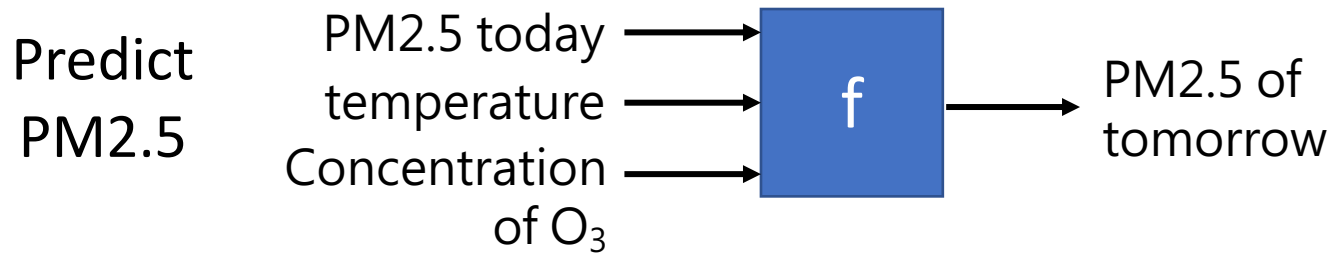
$$f(\text{  }) = \text{"Cat"}$$

- Playing Go 输入：棋盘上黑白棋的位置
输出：下一步的位置

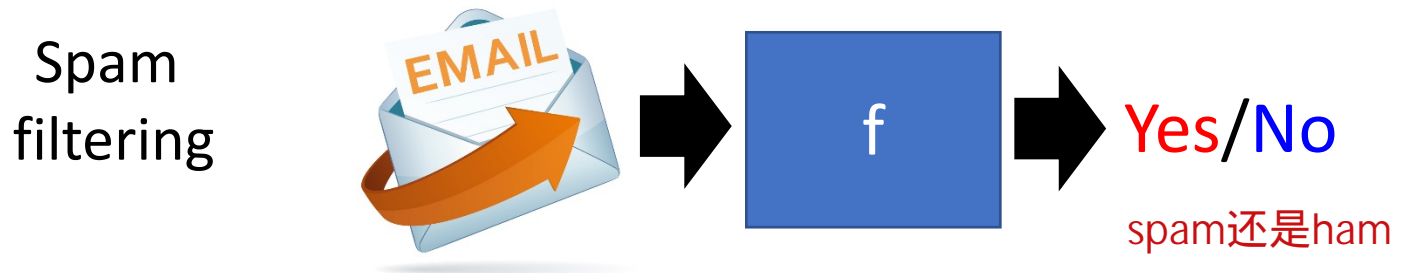
$$f(\text{  }) = \text{"5-5"}_{\text{(next move)}}$$

Different types of Functions

- 1 **Regression**: The function outputs a scalar.



- 2 **Classification**: Given options (classes), the function outputs the correct one.



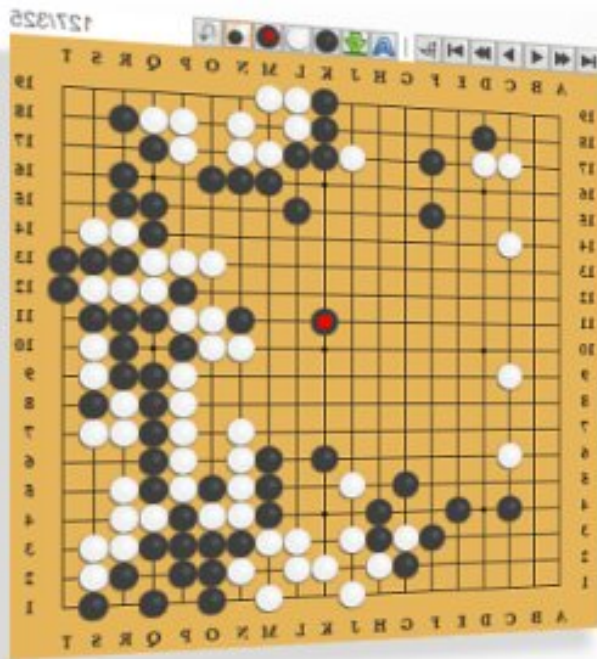
Different types of Functions

Classification: Given options (**classes**), the function outputs the correct one.

Each position
is a class

(19 x 19 classes)

多元分类问题



Playing GO

Alpha Go其实是一个
Classification的问题



a position on
the board

Next move

Structured Learning

create something with
structure (image, document)



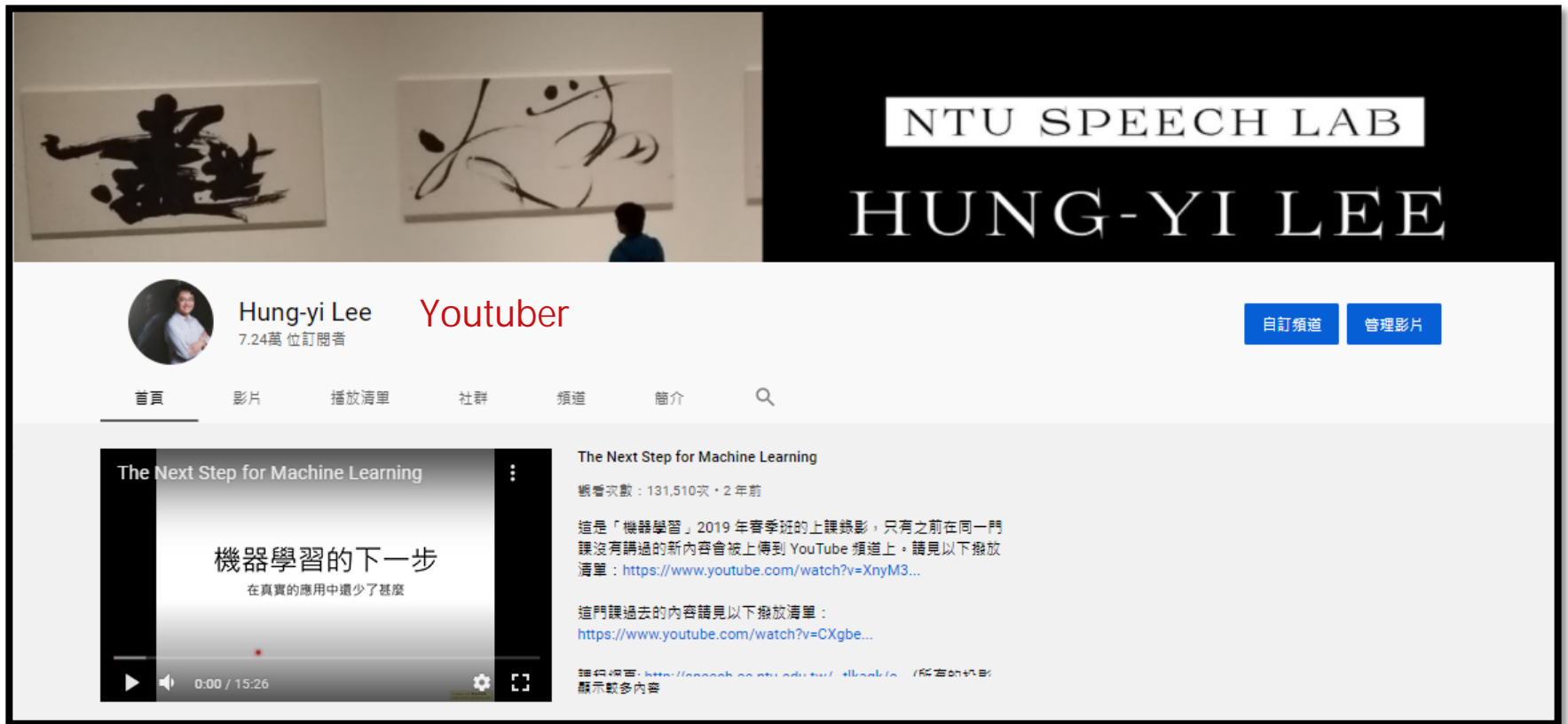
不只是Regression和Classification ,
还有黑暗大陆

Regression,
Classification



How to find a function?
A Case Study


YouTube Channel



<https://www.youtube.com/c/HungyiLeeNTU>

The function we want to find ...

$y = f(\text{no. of views on 2/26})$



頻道

Hung-yi Lee

🔍

≡

篩選器

影片

流量來源

地理位置

觀眾年齡

觀眾性別

日期

訂閱狀態

訂閱來源

播放清單

日期 ↓

+

喜歡的人數

訂閱人數

觀看次數

2021年1月26日

54 4.9%

69 5.5%

6,788 5.2%

2021年1月27日

60 5.4%

71 5.6%

6,242 4.7%

2021年1月28日

36 3.2%

63 5.0%

5,868 4.5%

2021年1月29日

27 2.4%

40 3.2%

4,413 3.4%

2021年1月30日

40 3.6%

40 3.2%

4,372 3.3%

2021年1月31日

47 4.2%

51 4.0%

5,135 3.9%

2021年2月1日

61 5.5%

29 2.3%

5,527 4.2%

2021年2月2日

49 4.4%

43 3.4%

5,911 4.5%

2021年2月3日

26 2.3%

44 3.5%

5,248 4.0%

2021年2月4日

43 3.9%

33 2.6%

4,771 3.6%

2021年2月5日

45 4.0%

49 3.9%

3,850 2.9%

2021年2月6日

29 2.6%

42 3.3%

3,828 2.9%

2021年2月7日

26 2.3%

46 3.6%

4,559 3.5%

2021年2月8日

38 3.4%

26 2.1%

4,772 3.6%

2021年2月9日

29 2.6%

25 2.0%

3,847 2.9%

2021年2月10日

31 2.8%

35 2.8%

3,382 2.6%

三个步骤

1. Function with Unknown Parameters

第一步其实就是Model Selection

$$y = f($$



Model
模型

$$y = b + wx_1$$

based on domain knowledge

选择模型时需要domain knowledge

feature

y : no. of views on 2/26, x_1 : no. of views on 2/25 特征

w and b are unknown parameters (learned from data)

参数

weight
权重

bias
偏置

日期	新增影片数	新增订人数	新增订人数	播放次数	新增订数	新增订数 (小时)	平均新增订数
总计	199	17,022	26,011	27,602,732	2,066,634	268,778.0	7.48
2020年1月1日	-	16 0.1%	52 0.2%	57,093	3,977 0.2%	565.6 0.2%	8.32
2020年1月2日	-	33 0.2%	58 0.2%	56,204	4,214 0.2%	589.8 0.2%	8.23
2020年1月3日	-	24 0.1%	89 0.3%	53,321	3,288 0.2%	457.4 0.2%	8.20
2020年1月4日	1 0.5%	27 0.2%	66 0.3%	53,599	3,559 0.2%	483.5 0.2%	8.09
2020年1月5日	-	35 0.2%	85 0.3%	63,001	4,877 0.2%	596.4 0.2%	7.99
2020年1月6日	-	31 0.2%	69 0.3%	60,175	4,682 0.2%	642.0 0.2%	8.13
2020年1月7日	-	40 0.2%	70 0.3%	63,638	4,695 0.2%	618.4 0.2%	7.54
2020年1月8日	-	39 0.2%	59 0.2%	59,900	4,785 0.2%	646.7 0.2%	8.06
2020年1月9日	-	28 0.2%	64 0.3%	54,988	4,911 0.2%	670.9 0.3%	8.11
2020年1月10日	-	17 0.1%	51 0.2%	40,631	3,069 0.2%	372.0 0.1%	7.16
2020年1月11日	-	12 0.1%	54 0.2%	36,168	2,898 0.1%	369.5 0.1%	7.38
2020年1月12日	-	40 0.2%	169 0.7%	53,964	4,477 0.2%	572.9 0.2%	7.40
2020年1月13日	-	29 0.2%	75 0.3%	61,043	5,017 0.2%	661.4 0.3%	7.54
2020年1月14日	-	32 0.2%	83 0.3%	64,968	5,186 0.3%	618.3 0.2%	7.09

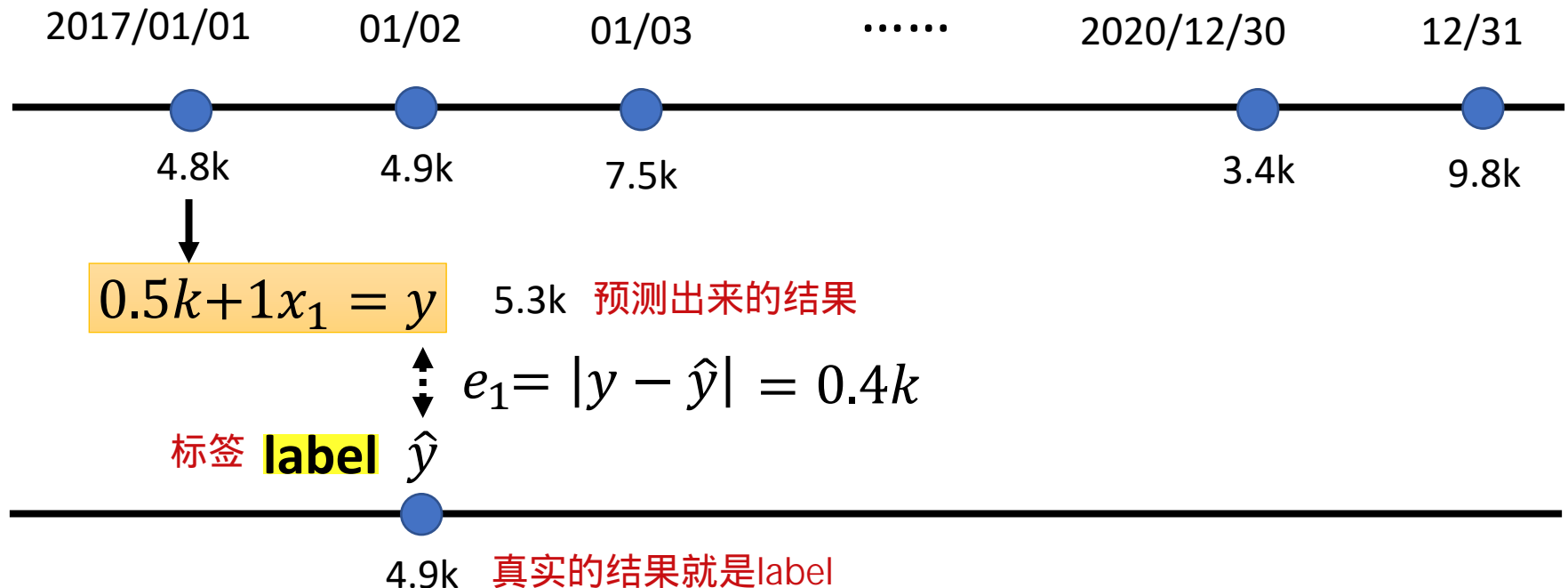
2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.

$L(0.5k, 1)$ $y = b + wx_1 \longrightarrow y = 0.5k + 1x_1$ How good it is?

Data from 2017/01/01 – 2020/12/31

这是 Training Data

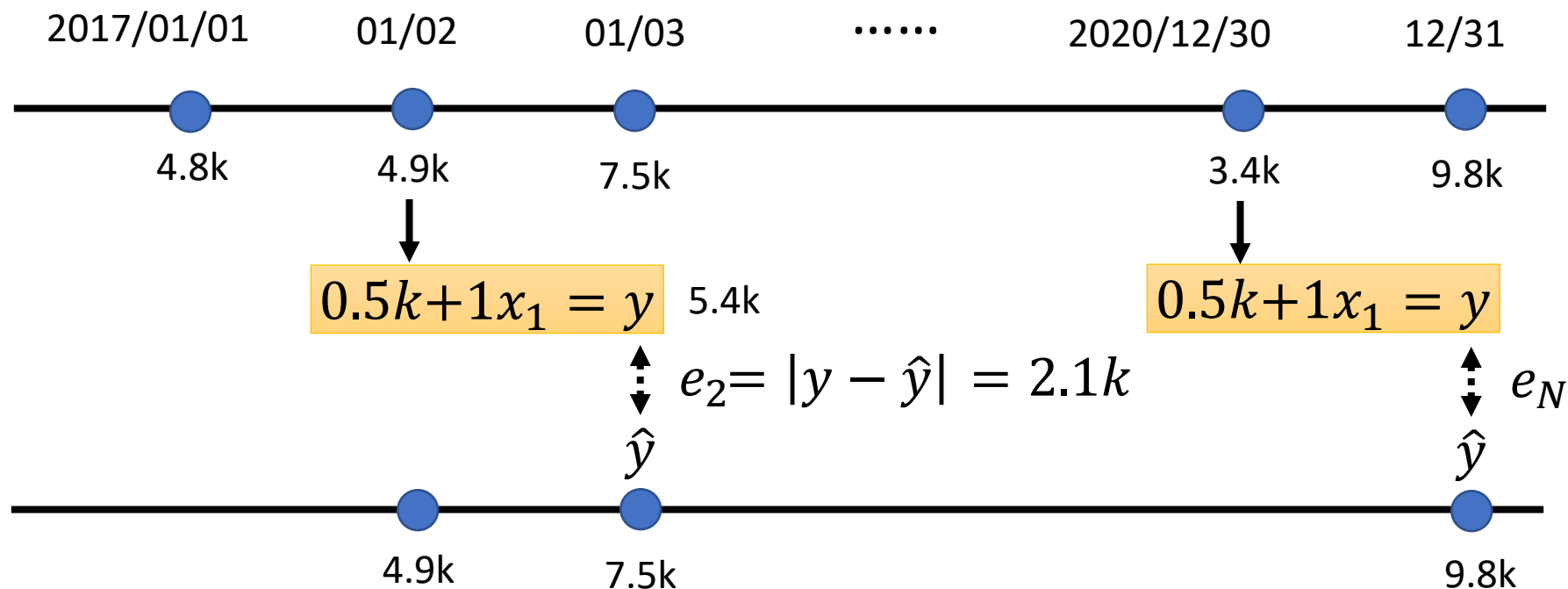


2. Define **Loss** from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.

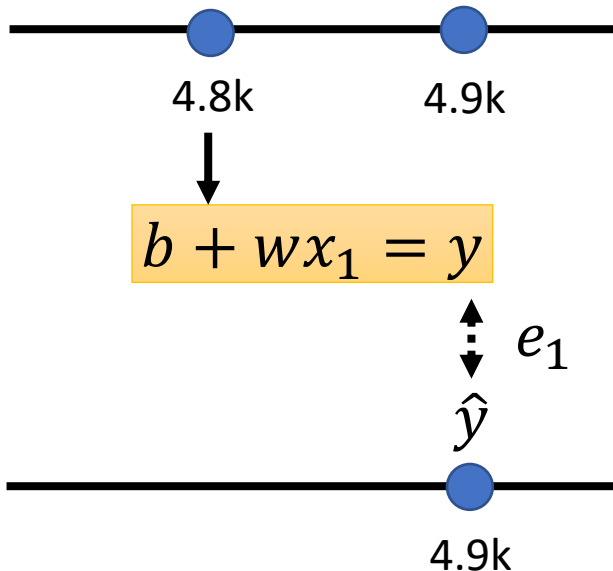
$L(0.5k, 1)$ $y = b + wx_1 \longrightarrow y = 0.5k + 1x_1$ How good it is?

Data from 2017/01/01 – 2020/12/31



2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.



Loss:
$$L = \frac{1}{N} \sum_n e_n$$

$e = |y - \hat{y}|$ L is mean absolute error (**MAE**)

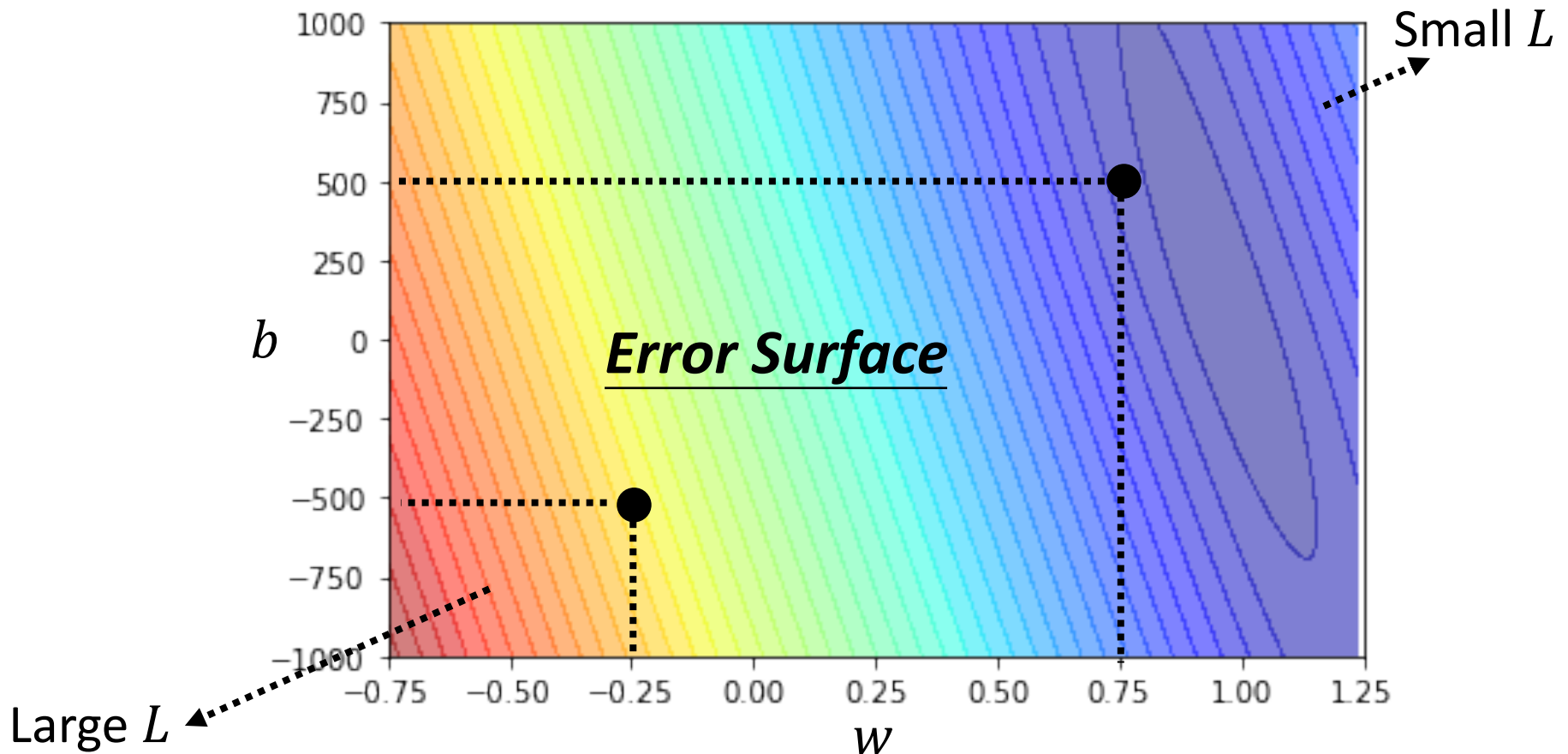
$e = (y - \hat{y})^2$ L is mean square error (**MSE**)

If y and \hat{y} are both probability distributions ➡ **Cross-entropy**

2. Define Loss from Training Data

- Loss is a function of parameters $L(b, w)$
- Loss: how good a set of values is.

Model $y = b + wx_1$



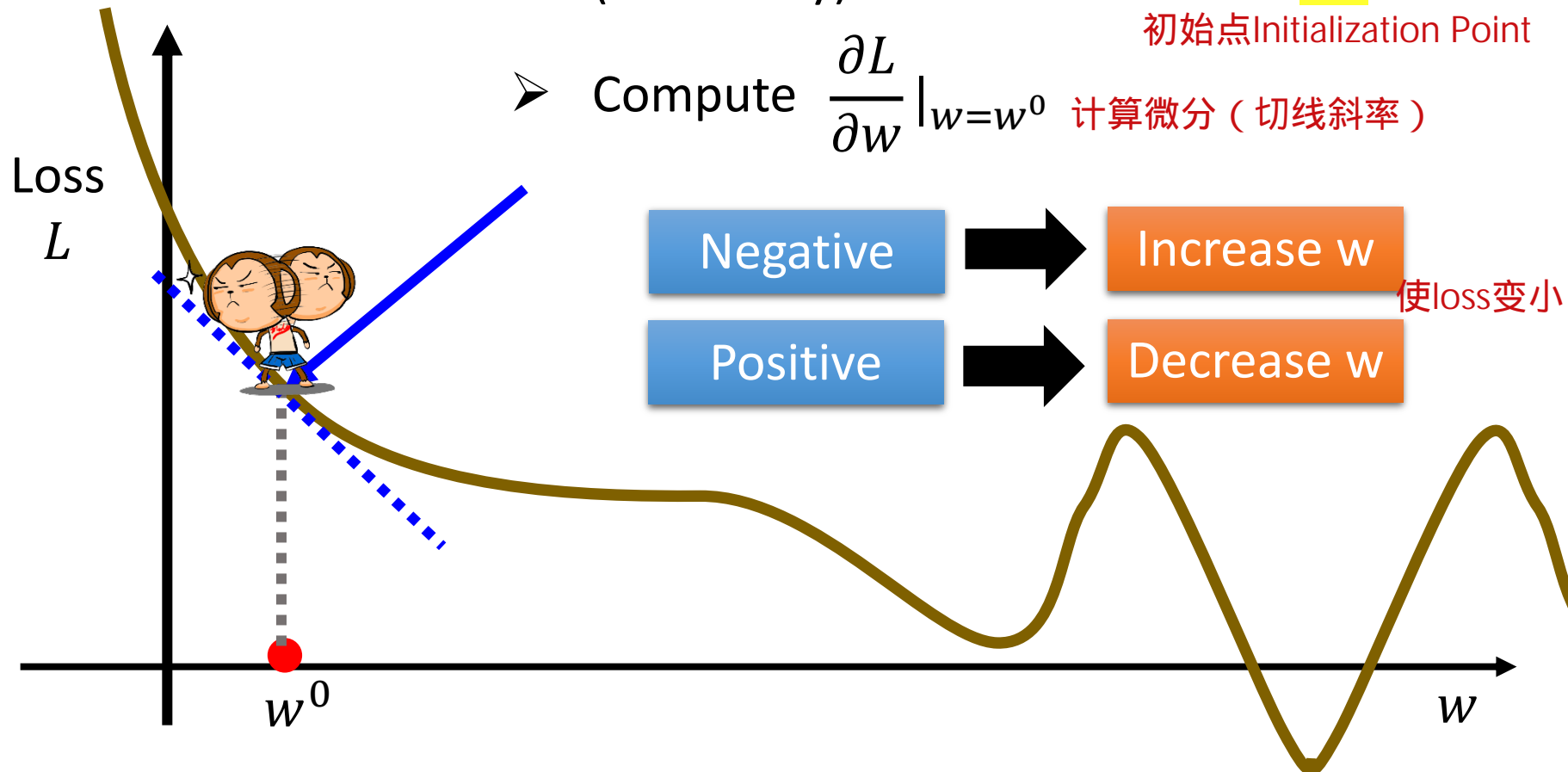
3. Optimization

$$w^* = \arg \min_w L$$

简化起见，假设我们只有一个参数

Gradient Descent

- (Randomly) Pick an initial value w^0
初始点 Initialization Point
- Compute $\frac{\partial L}{\partial w} \big|_{w=w^0}$ 计算微分 (切线斜率)



3. Optimization

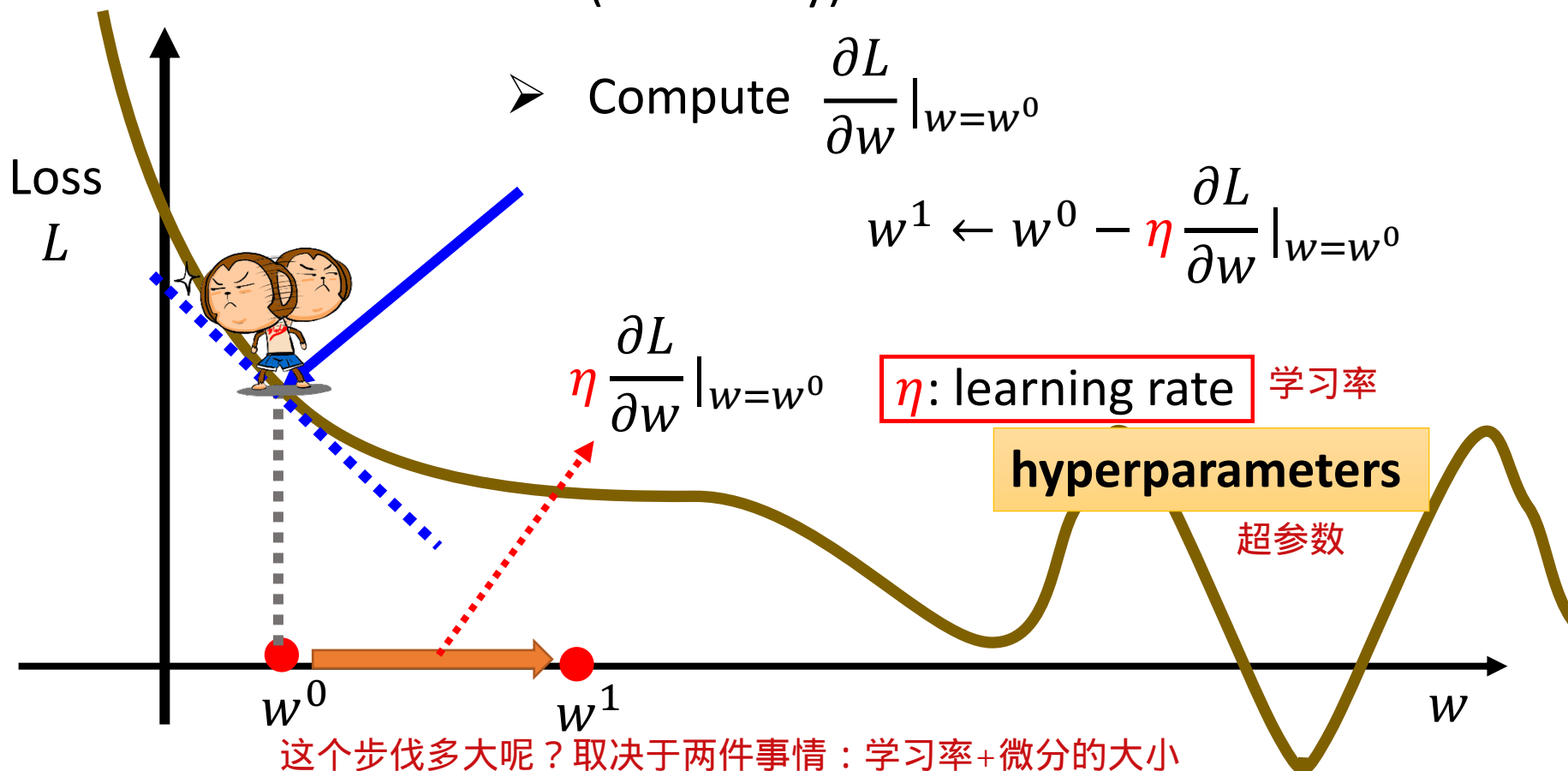
$$w^* = \arg \min_w L$$

Gradient Descent

➤ (Randomly) Pick an initial value w^0

➤ Compute $\frac{\partial L}{\partial w} \big|_{w=w^0}$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \big|_{w=w^0}$$



3. Optimization

$$w^* = \arg \min_w L$$

Gradient Descent

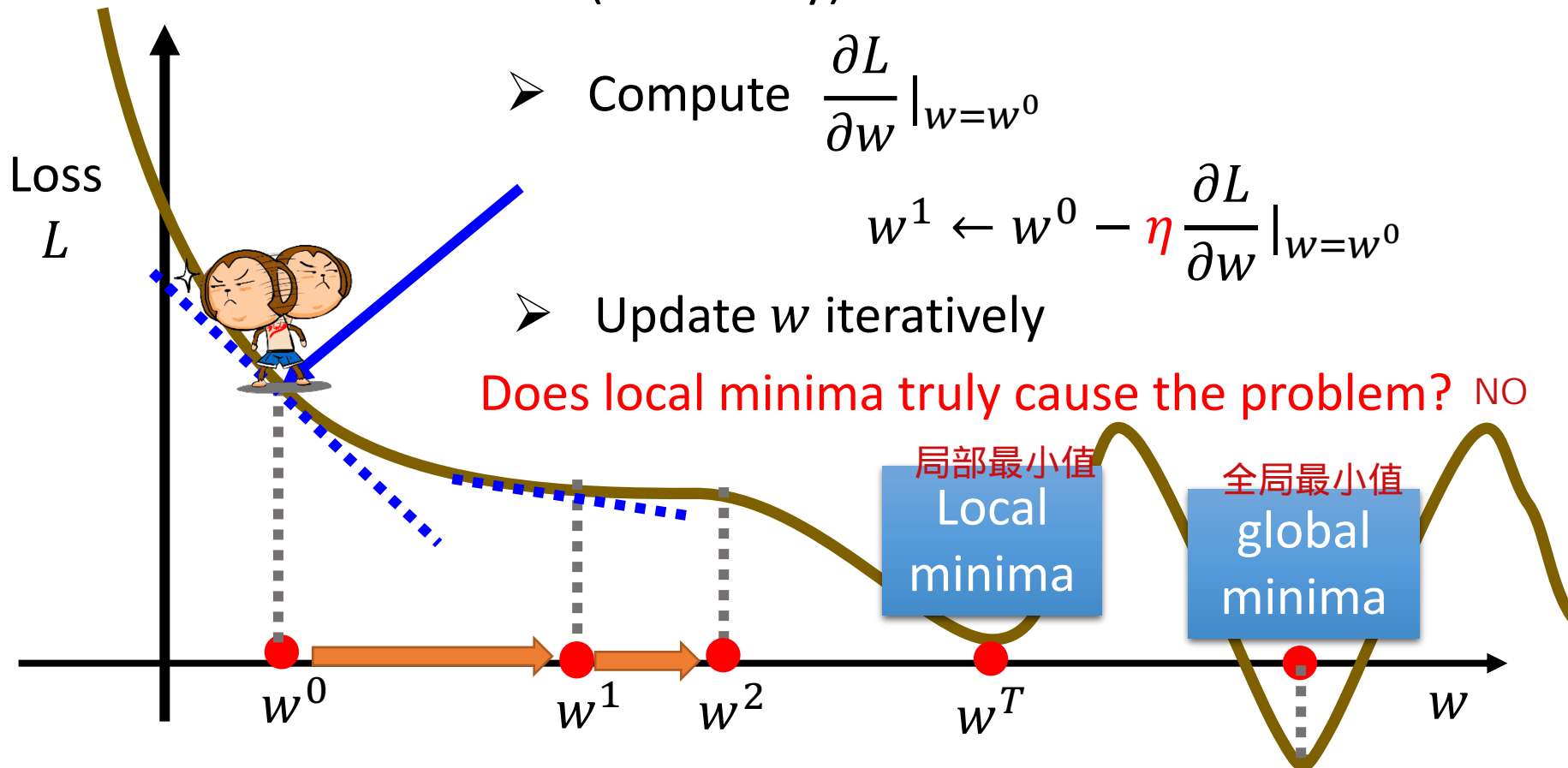
➤ (Randomly) Pick an initial value w^0

➤ Compute $\frac{\partial L}{\partial w} \big|_{w=w^0}$

$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \big|_{w=w^0}$$

➤ Update w iteratively


Does local minima truly cause the problem? NO



3. Optimization

$$w^*, b^* = \arg \min_{w, b} L$$

- (Randomly) Pick initial values w^0, b^0
- Compute

$$\begin{aligned} \frac{\partial L}{\partial w} \Big|_{w=w^0, b=b^0} \\ \frac{\partial L}{\partial b} \Big|_{w=w^0, b=b^0} \end{aligned}$$


$$w^1 \leftarrow w^0 - \eta \frac{\partial L}{\partial w} \Big|_{w=w^0, b=b^0}$$

$$b^1 \leftarrow b^0 - \eta \frac{\partial L}{\partial b} \Big|_{w=w^0, b=b^0}$$

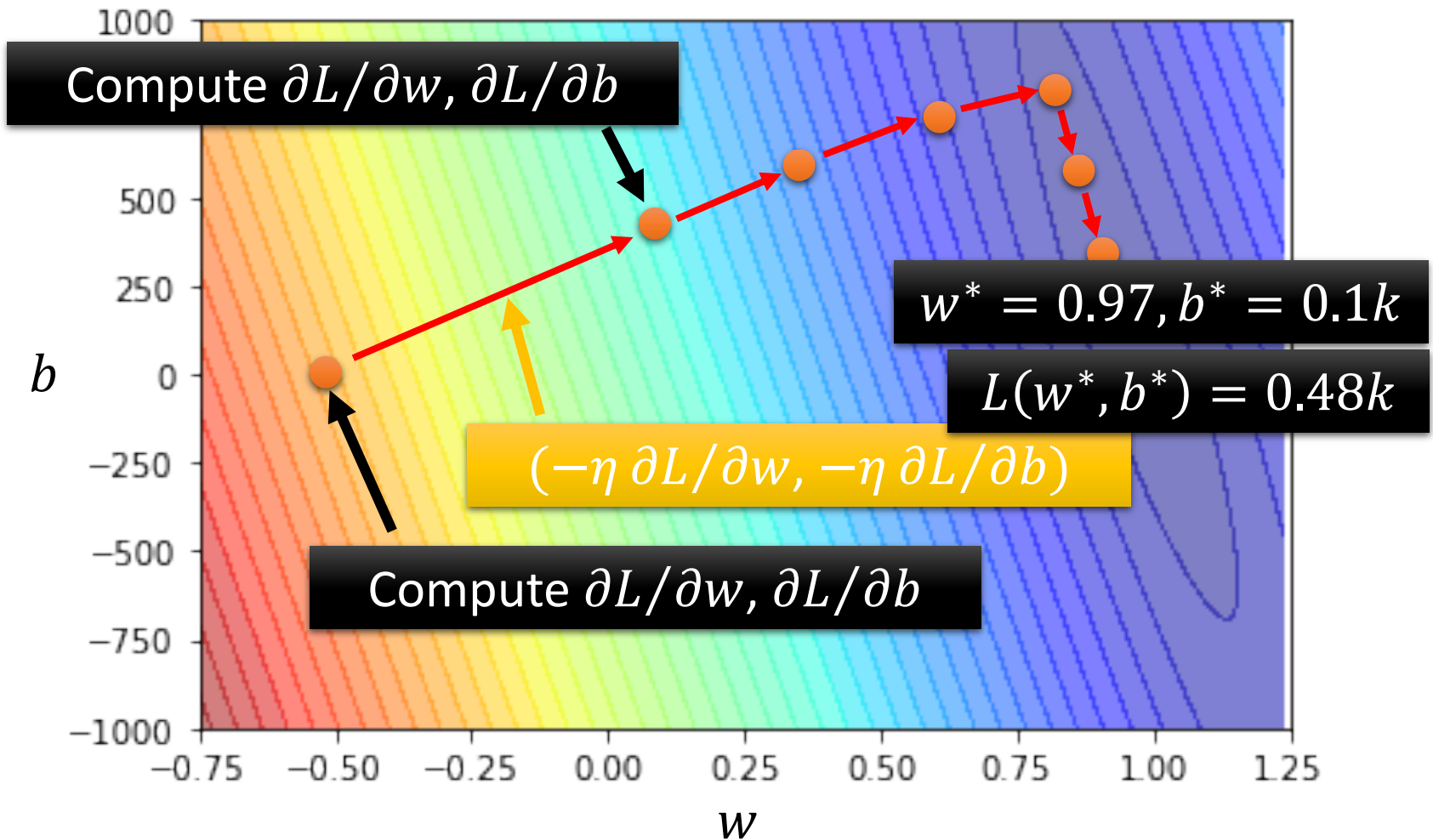
Can be done in one line in most deep learning frameworks

- Update w and b iteratively

Model $y = b + wx_1$

3. Optimization

$$w^*, b^* = \arg \min_{w, b} L$$



Machine Learning is so simple

$$w^* = 0.97, b^* = 0.1k$$

$$L(w^*, b^*) = 0.48k$$

$$y = b + wx_1$$

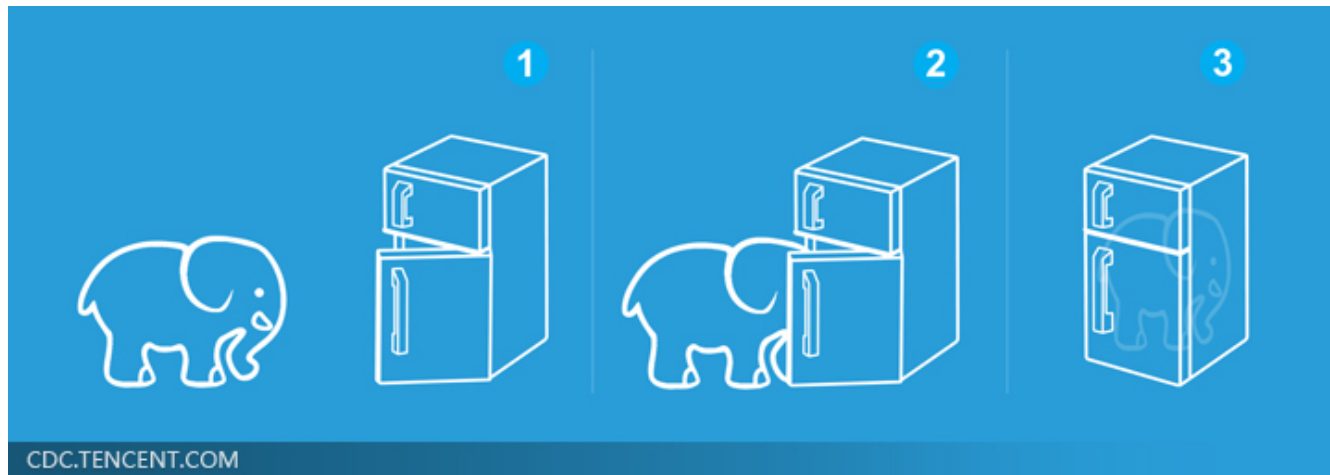
Step 1:
function with
unknown



Step 2: define
loss from
training data



Step 3:
optimization



Machine Learning is so simple

$$w^* = 0.97, b^* = 0.1k$$

$$L(w^*, b^*) = 0.48k$$



$y = 0.1k + 0.97x_1$ achieves the smallest loss $L = 0.48k$ on data of 2017 – 2020 (**training data**) 训练集

How about data of 2021 (**unseen during training**)?

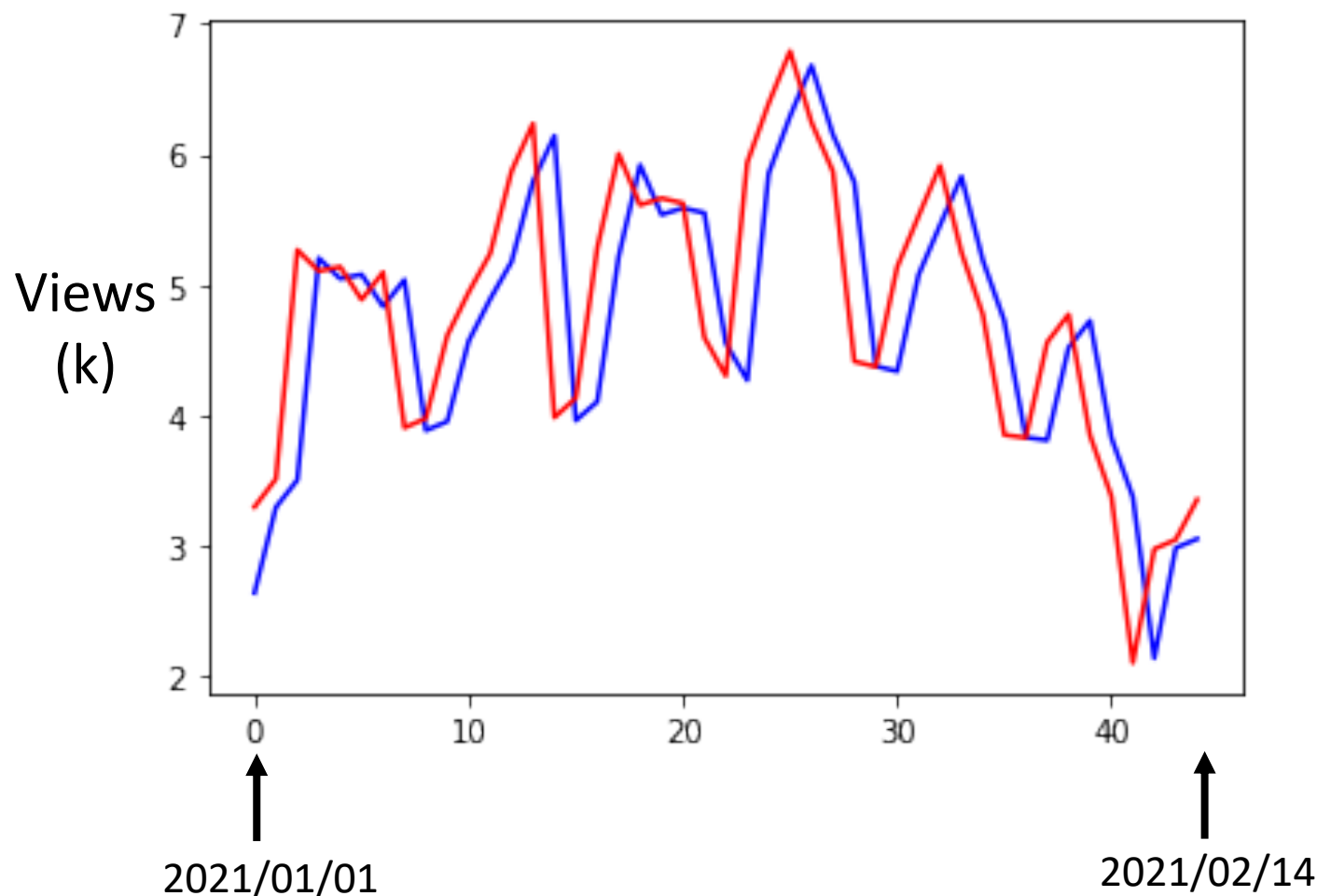
testing data 测试集

$$L' = 0.58k$$

$$y = 0.1k + 0.97x_1$$

Red: real no. of views

blue: estimated no. of views



规律：每周循环，而且周五和周六观看人数少

对于model的修改，涉及到对于问题的理解，也就是domain knowledge

$$y = b + wx_1$$

2017 - 2020

2021

$$L = 0.48k$$

$$L' = 0.58k$$

7 考虑前七天

$$y = b + \sum_{j=1}^7 w_j x_j$$

2017 - 2020

2021

$$L = 0.38k$$

$$L' = 0.49k$$

b	w_1^*	w_2^*	w_3^*	w_4^*	w_5^*	w_6^*	w_7^*
0.05k	0.79	-0.31	0.12	-0.01	-0.10	0.30	0.18

28 考虑28天 (1月)

$$y = b + \sum_{j=1}^{28} w_j x_j$$

2017 - 2020

2021

$$L = 0.33k$$

$$L' = 0.46k$$

56 考虑56天 (2月)

$$y = b + \sum_{j=1}^{56} w_j x_j$$

2017 - 2020

2021

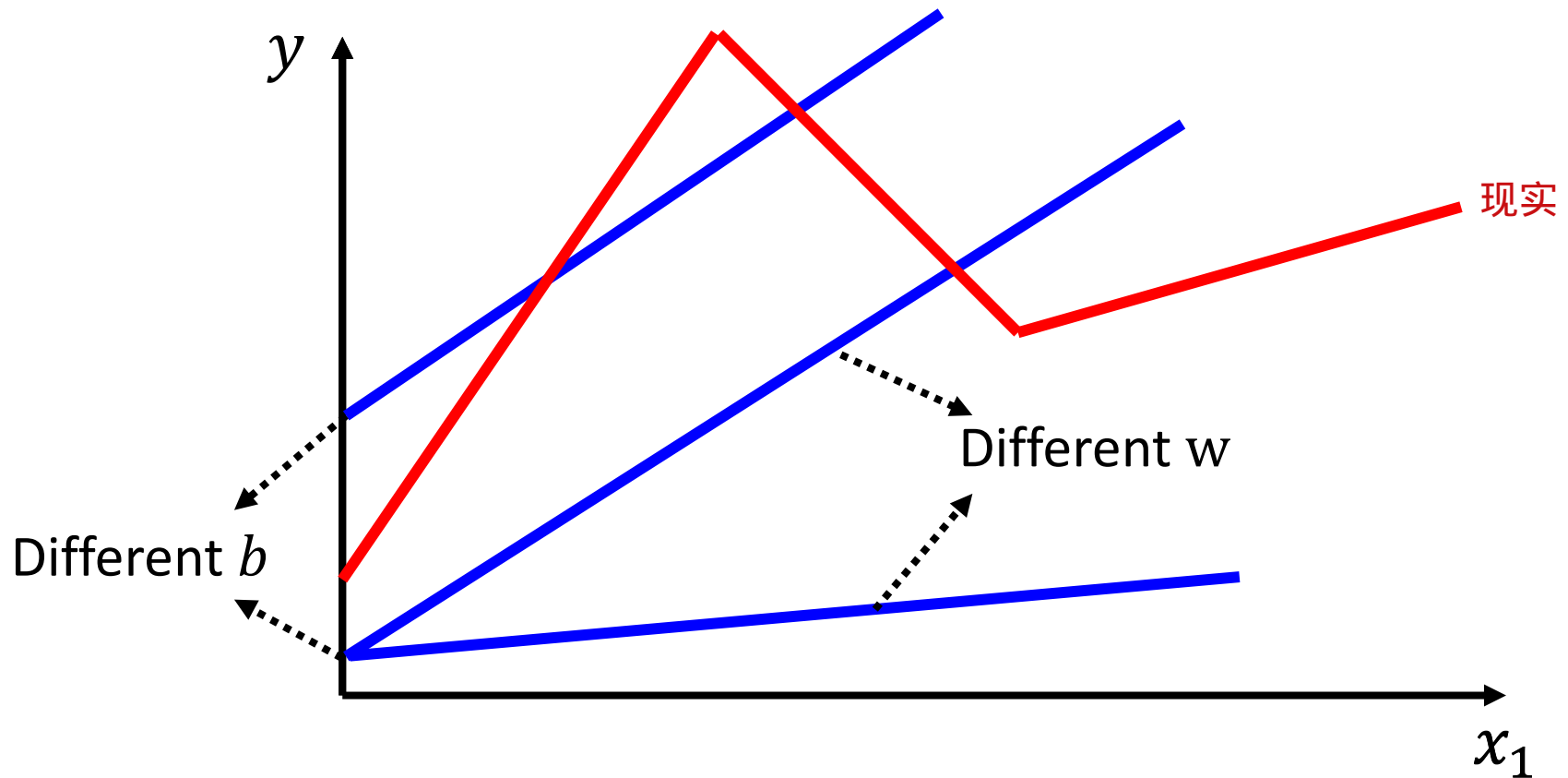
$$L = 0.32k$$

$$L' = 0.46k$$

提升不大，
考虑天数这件事情
可以停下了

Linear models


Linear models are too simple ... we need more sophisticated modes.

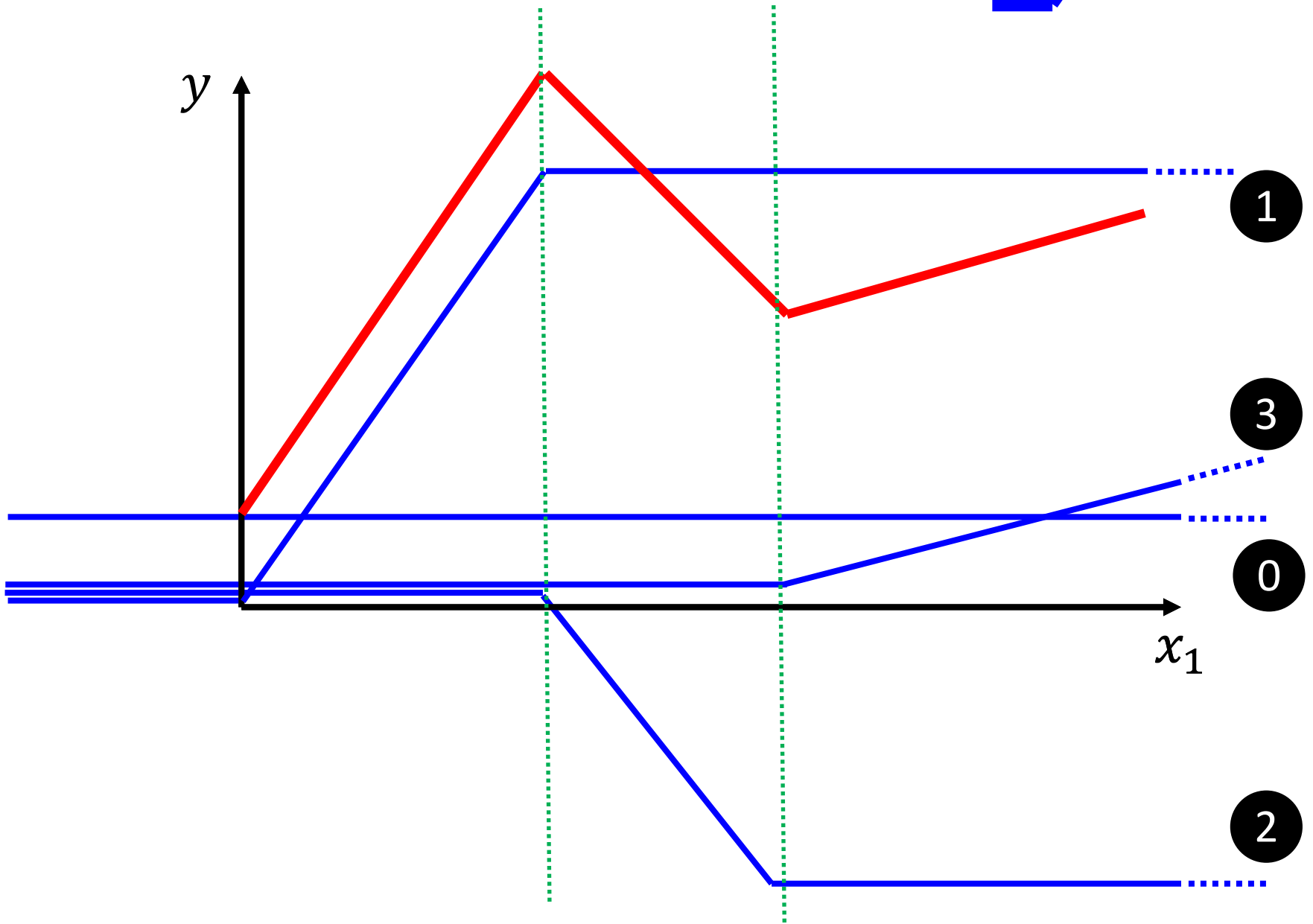


Linear models have severe limitation. **Model Bias**

来自model的限制

We need a more flexible model!

red curve = constant + sum of a set of 



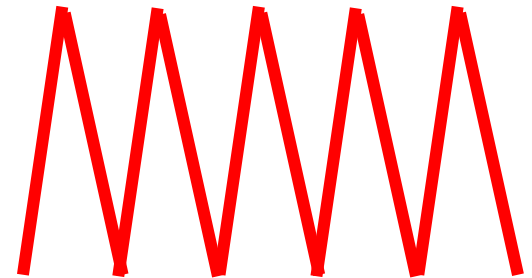
分段

All Piecewise Linear Curves

= constant + sum of a set of



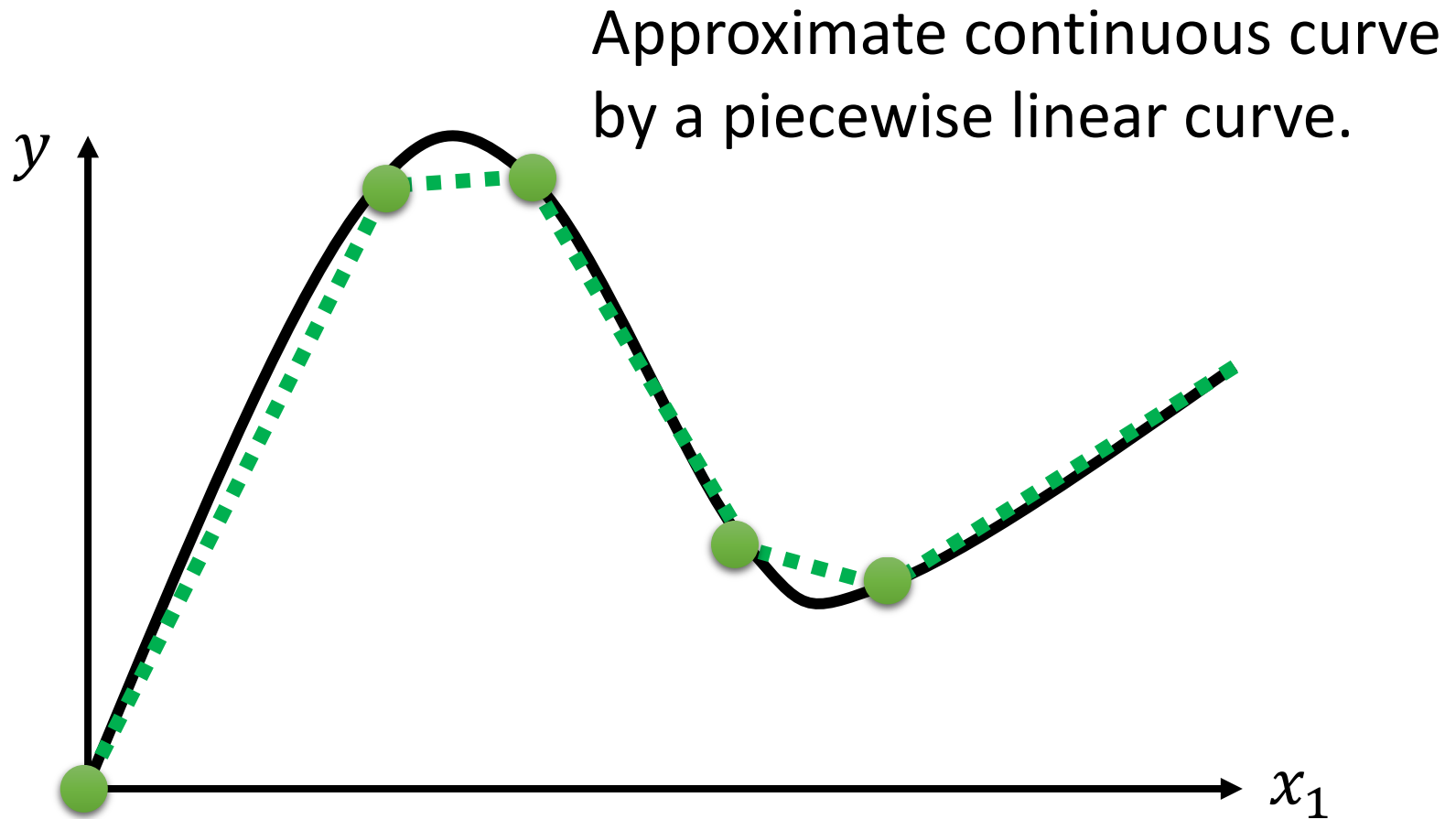
选择不一样的蓝色的function



More pieces require more

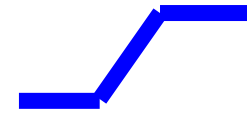


Beyond Piecewise Linear?

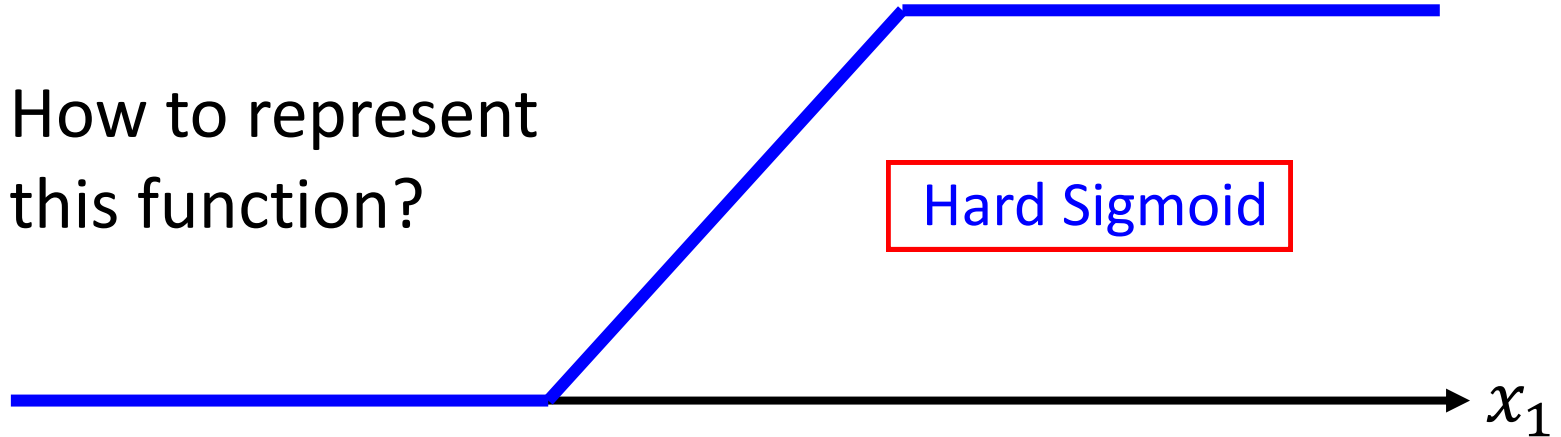


To have good approximation, we need sufficient pieces.

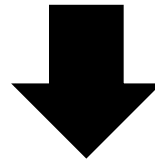
red curve = constant + sum of a set of



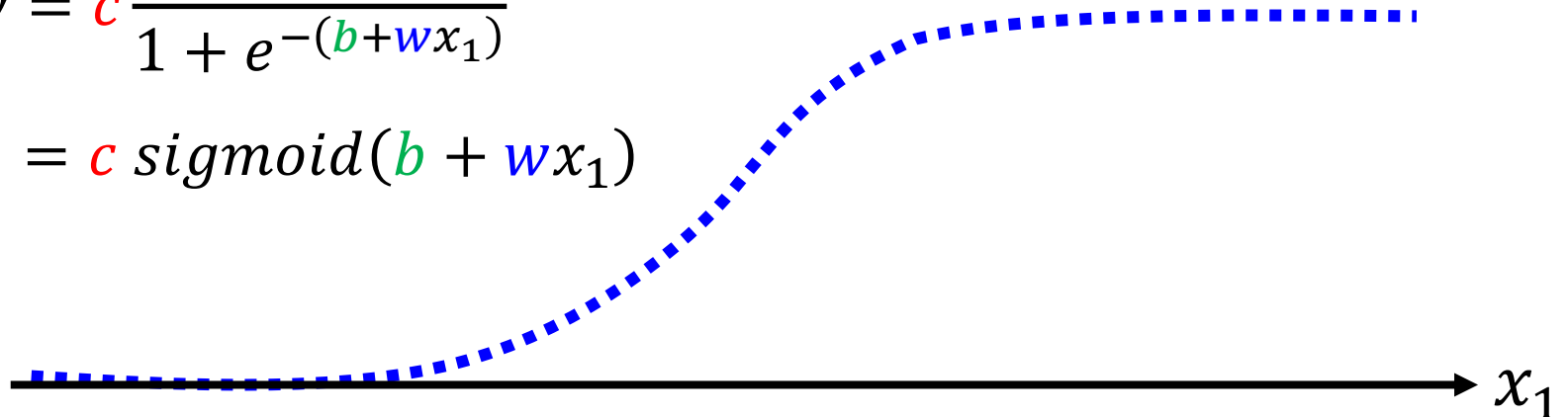
How to represent
this function?

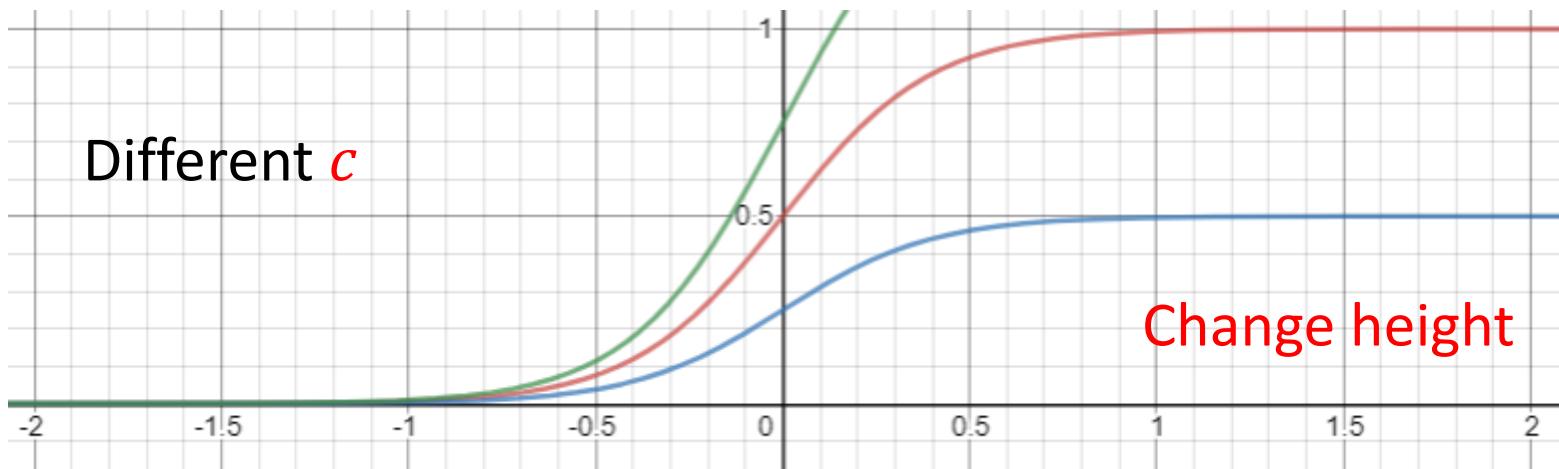
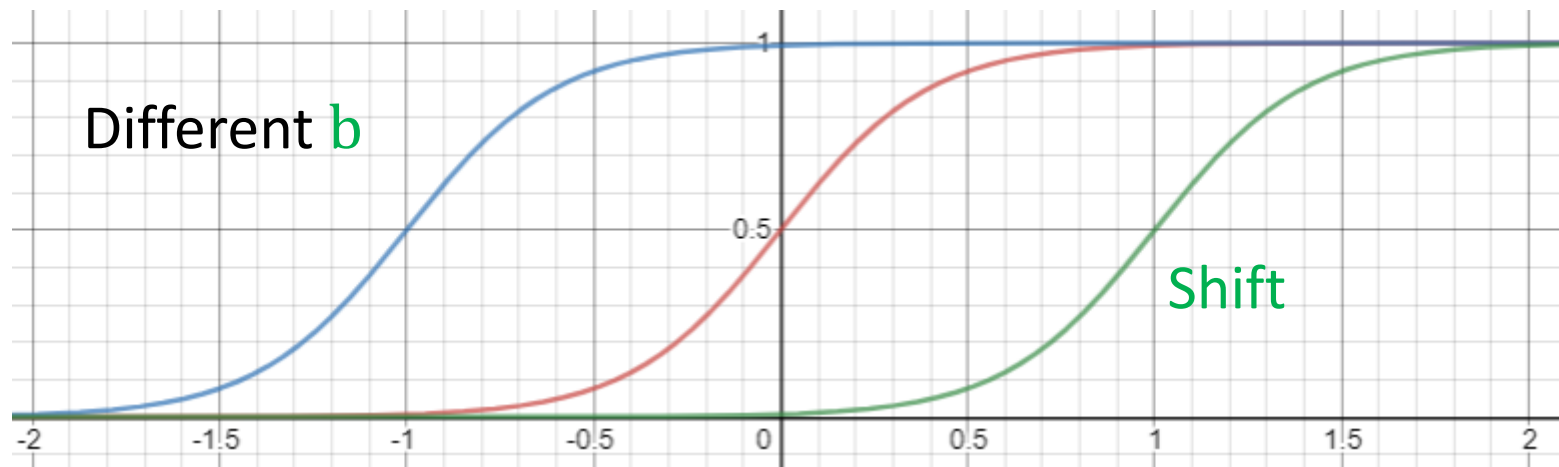
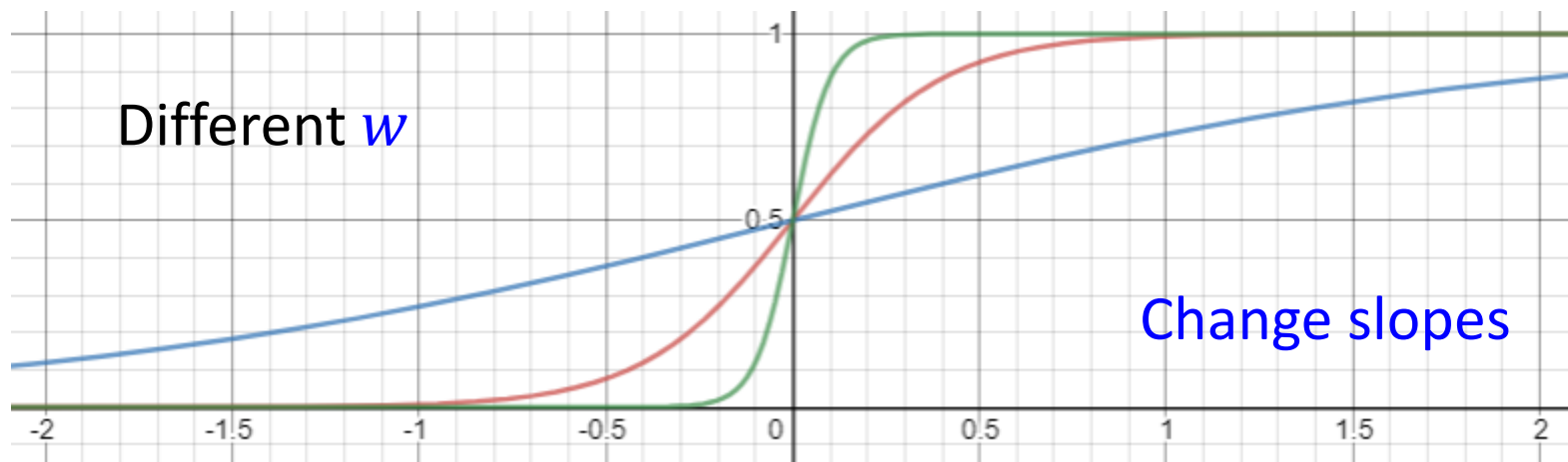


Sigmoid Function S型函数



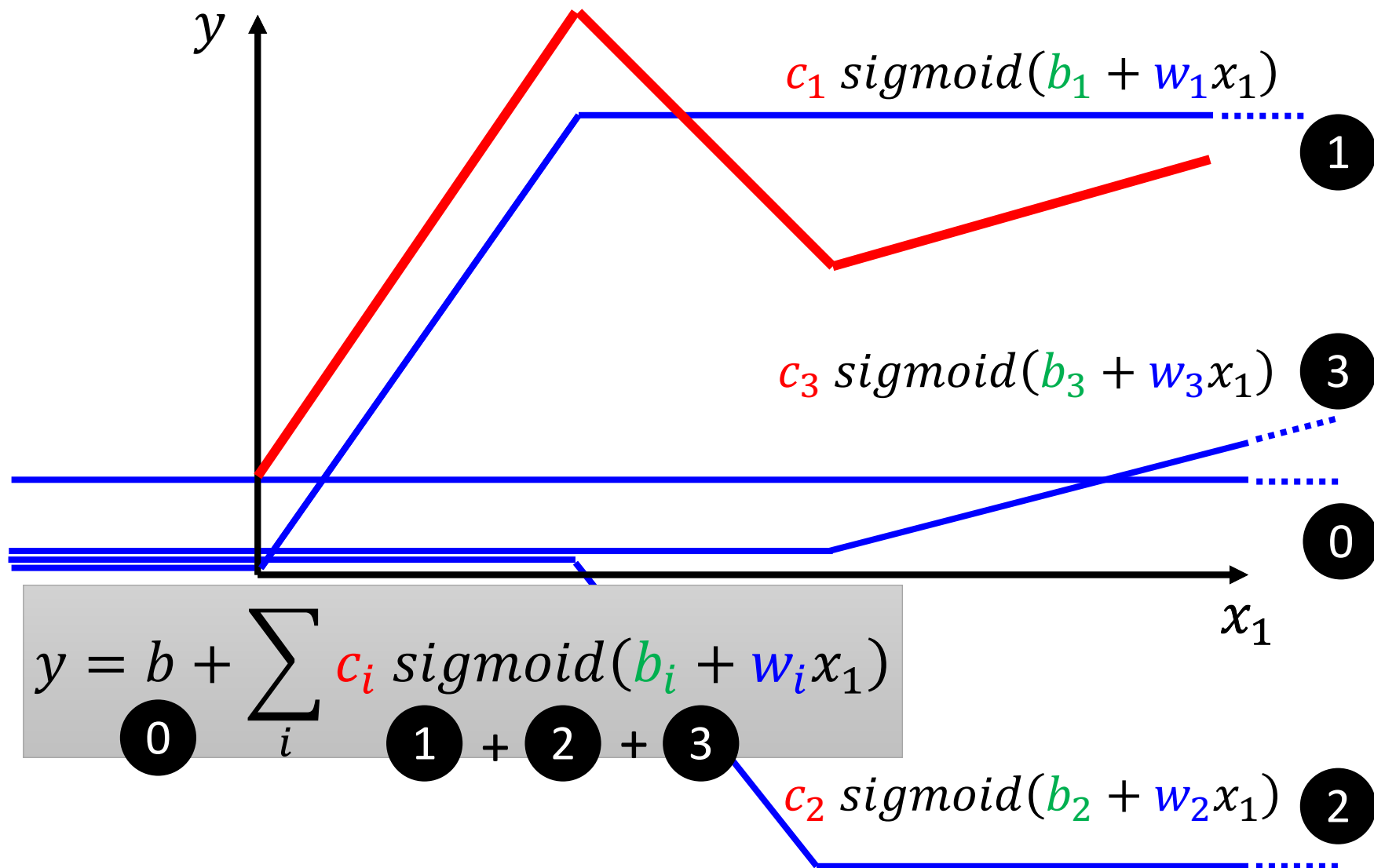
$$y = c \frac{1}{1 + e^{-(b + wx_1)}}$$
$$= c \operatorname{sigmoid}(b + wx_1)$$






如何写出红色曲线的函数呢？

red curve = sum of a set of  + constant



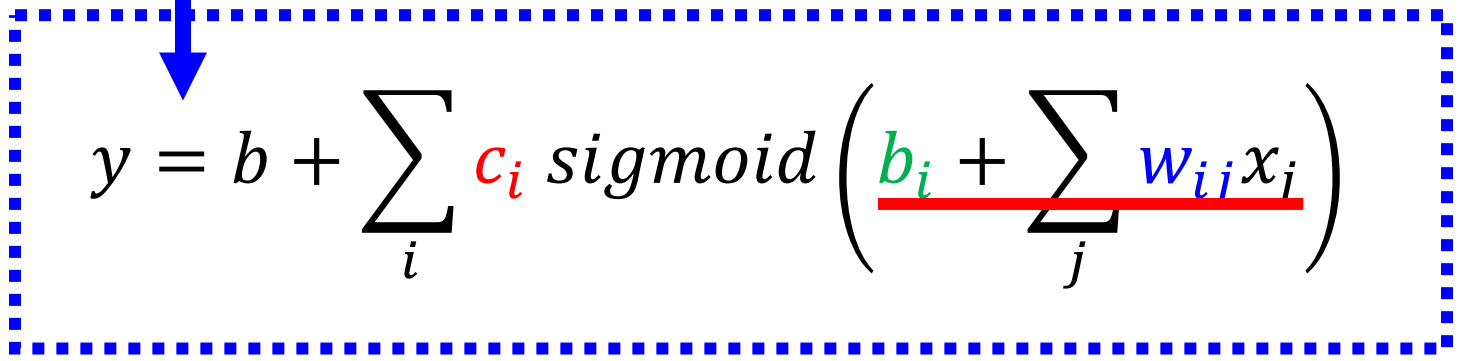

New Model: More Features

$$y = \underline{b + wx_1}$$


$$y = b + \sum_i \textcolor{red}{c_i} \textit{sigmoid}(\underline{\textcolor{green}{b_i} + \textcolor{blue}{w_i}x_1})$$

$$y = \underline{b + \sum_j w_j x_j}$$

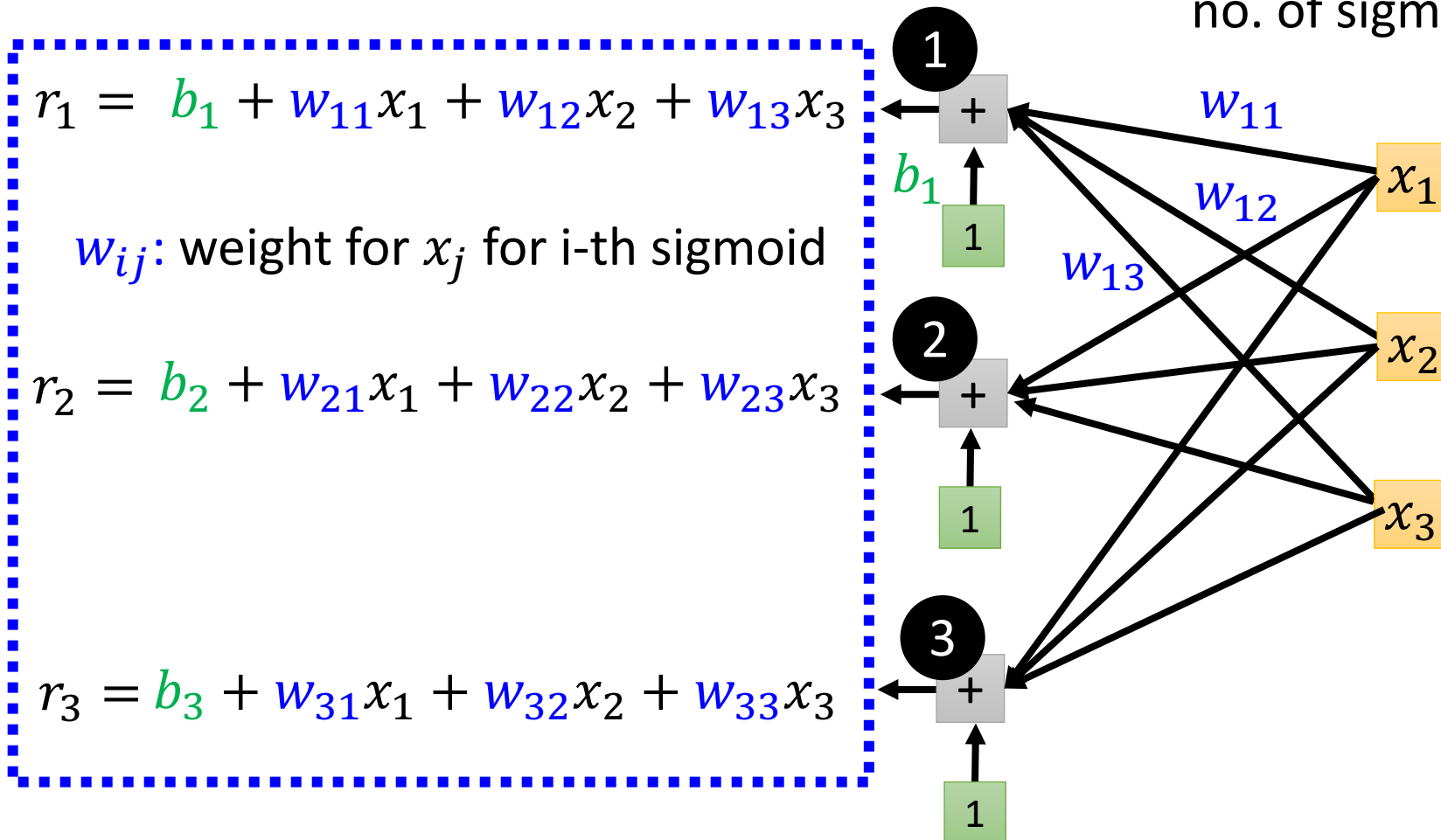
j : feature的编号


$$y = b + \sum_i \textcolor{red}{c_i} \textit{sigmoid} \left(\underline{\textcolor{green}{b_i} + \sum_j \textcolor{blue}{w_{ij}}x_j} \right)$$

把前面的函数画出来

$$y = b + \sum_i c_i \operatorname{sigmoid} \left(b_i + \sum_j w_{ij} x_j \right)$$

$j: 1, 2, 3$
no. of features
 $i: 1, 2, 3$
no. of sigmoid



$$y = b + \sum_i c_i \operatorname{sigmoid} \left(b_i + \sum_j w_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$

$$r_1 = b_1 + w_{11}x_1 + w_{12}x_2 + w_{13}x_3$$

$$r_2 = b_2 + w_{21}x_1 + w_{22}x_2 + w_{23}x_3$$

$$r_3 = b_3 + w_{31}x_1 + w_{32}x_2 + w_{33}x_3$$

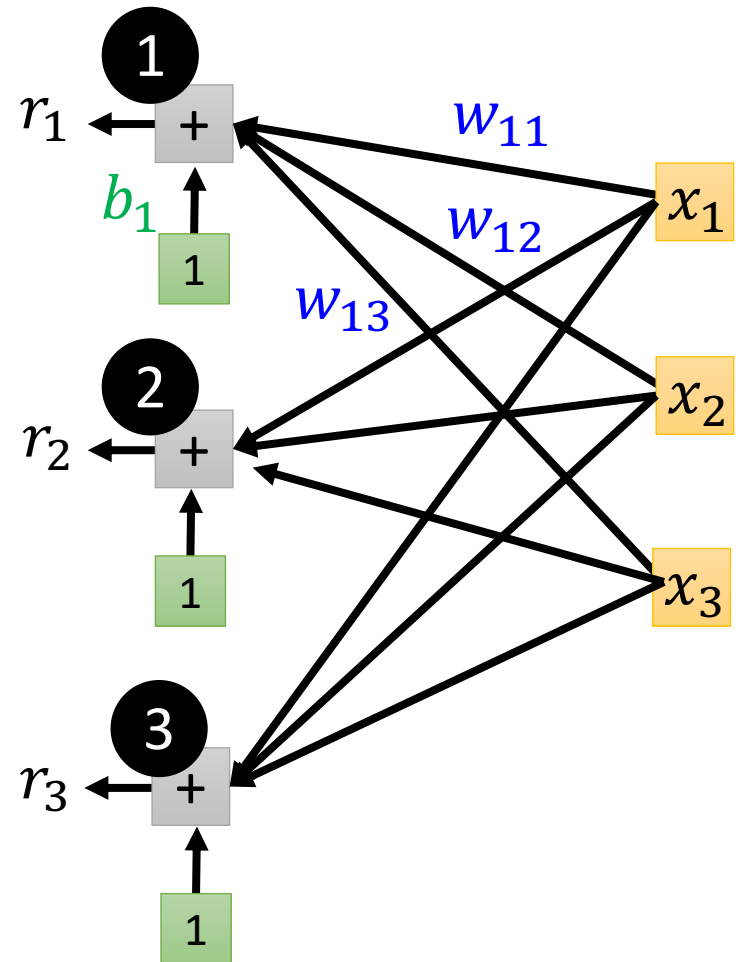
简化成向量和矩阵的相乘

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

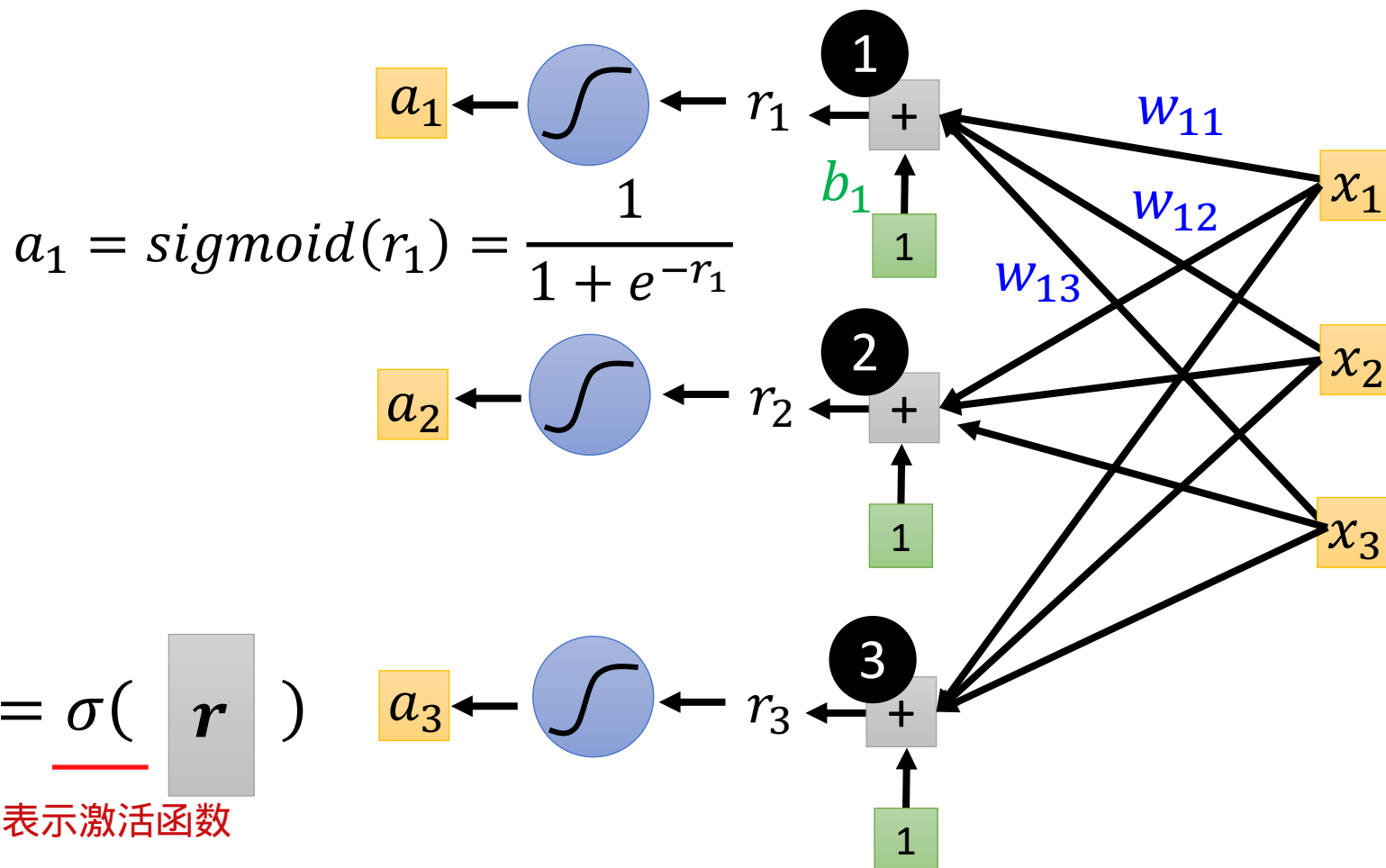
$$\mathbf{r} = \mathbf{b} + \mathbf{W} \mathbf{x}$$

$$y = b + \sum_i \textcolor{red}{c}_i \textit{sigmoid} \left(\textcolor{green}{b}_i + \sum_j \textcolor{blue}{w}_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$

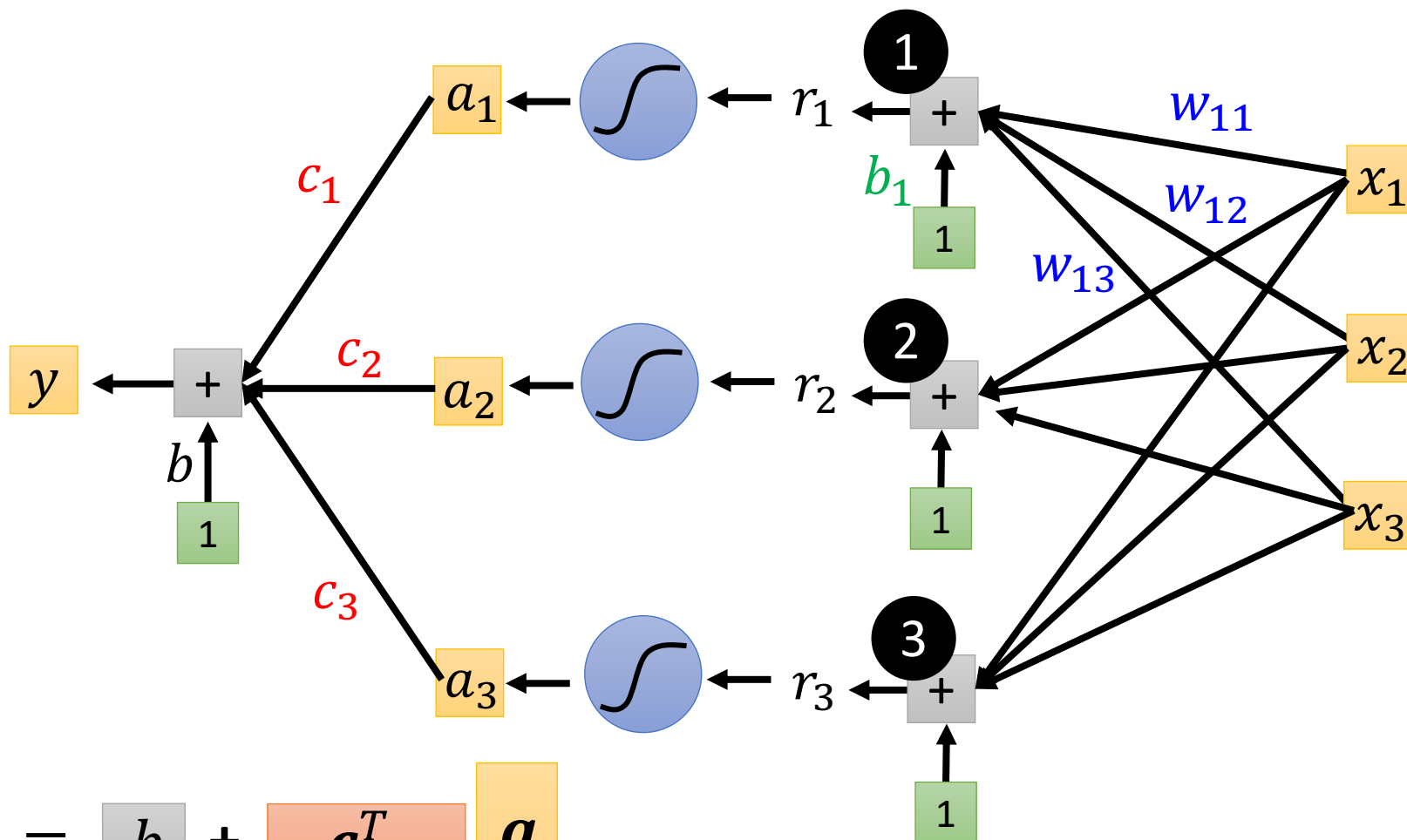
$$\mathbf{r} = \mathbf{b} + \mathbf{W} \mathbf{x}$$

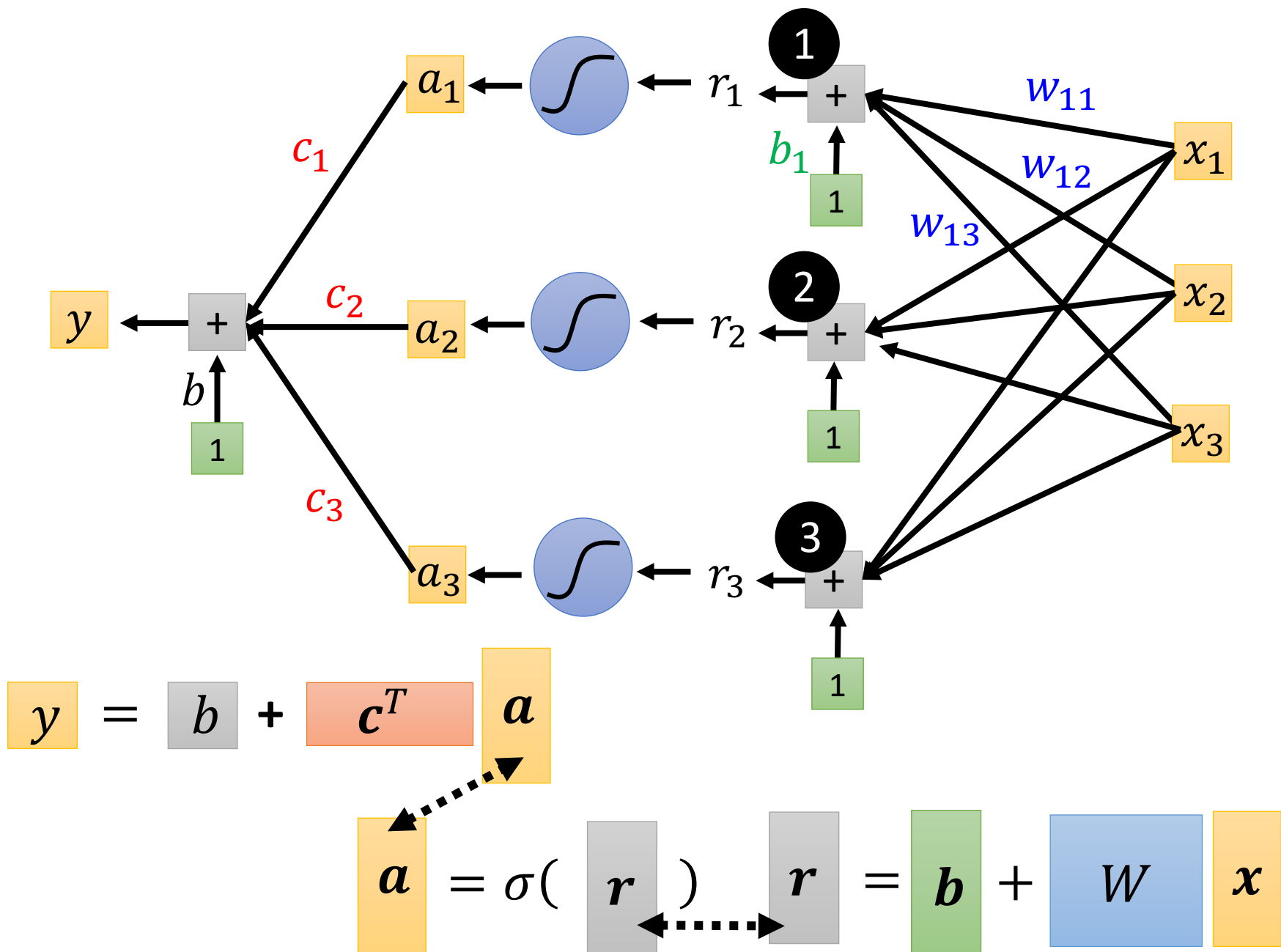


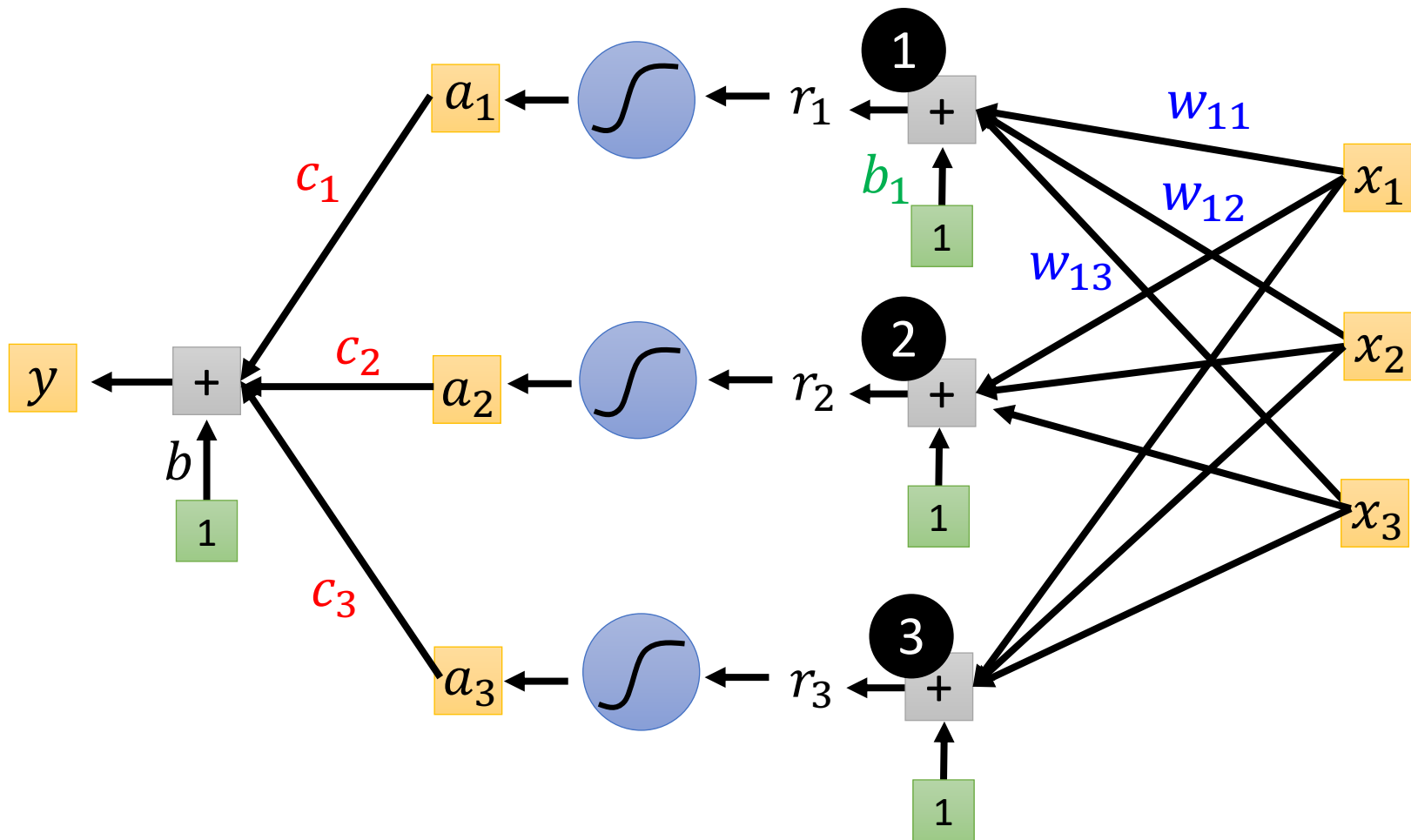
$$y = b + \sum_i c_i \text{sigmoid} \left(b_i + \sum_j w_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$



$$y = b + \sum_i \mathbf{c}_i \operatorname{sigmoid} \left(\mathbf{b}_i + \sum_j \mathbf{w}_{ij} x_j \right) \quad \begin{array}{l} i: 1,2,3 \\ j: 1,2,3 \end{array}$$





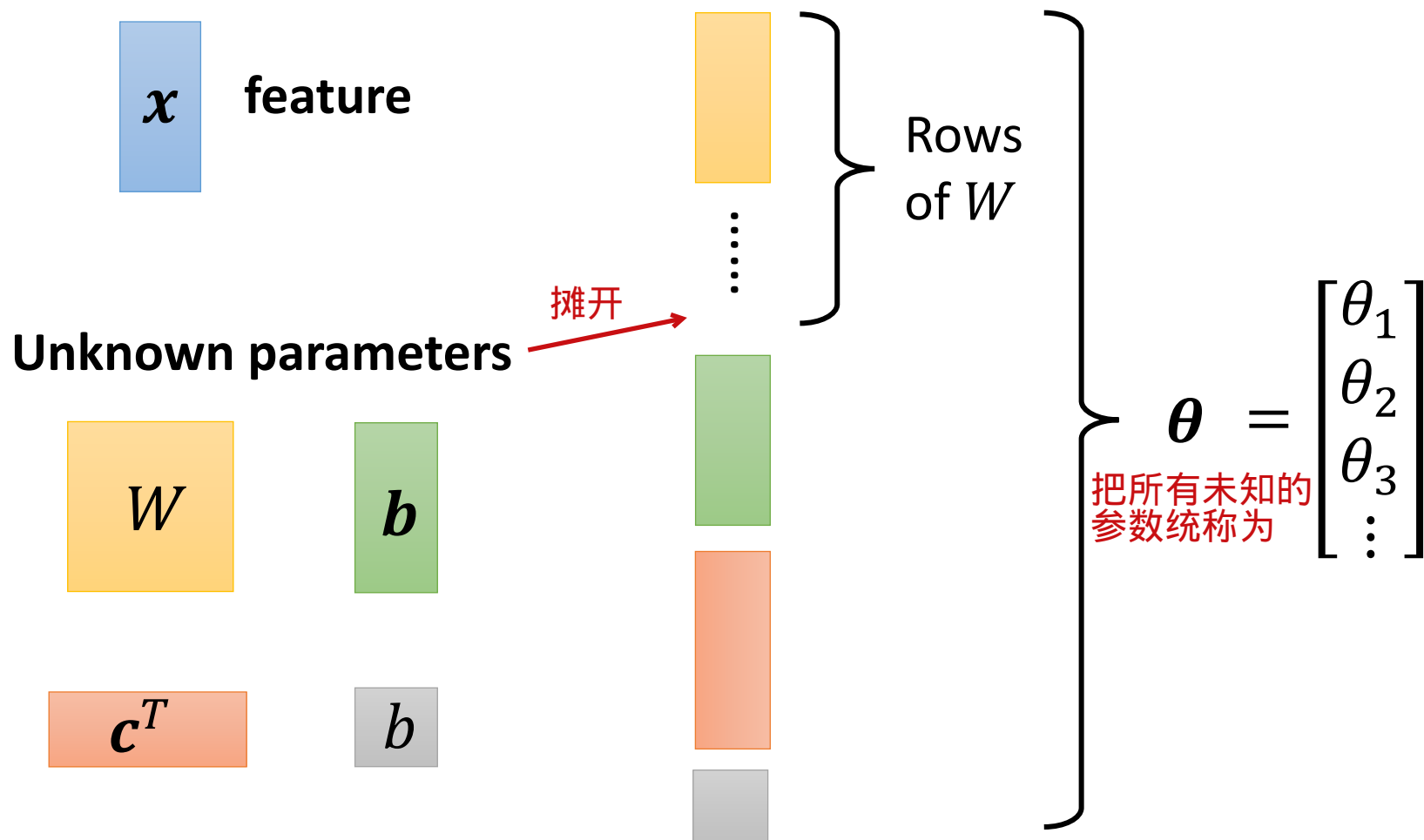


线性代数表示上面的图

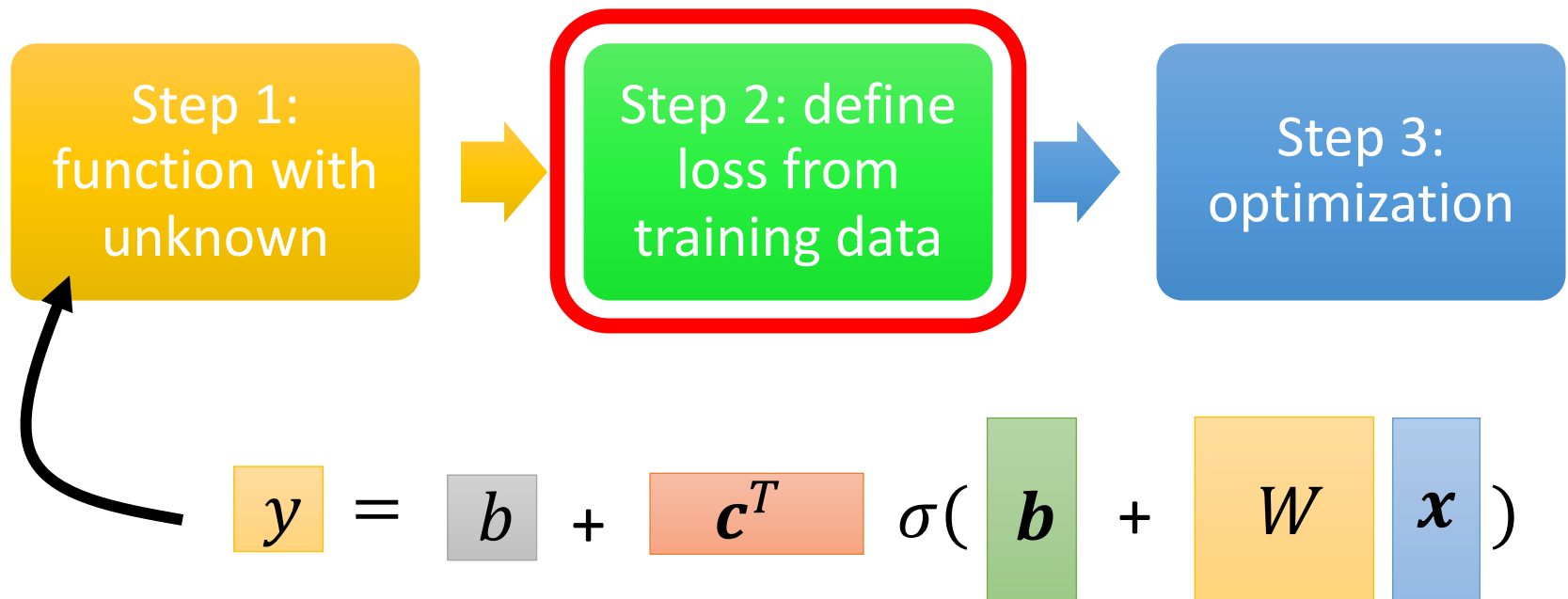
$$y = b + c^T \sigma(b + Wx)$$

Function with unknown parameters

$$y = b + c^T \sigma(b + W x)$$

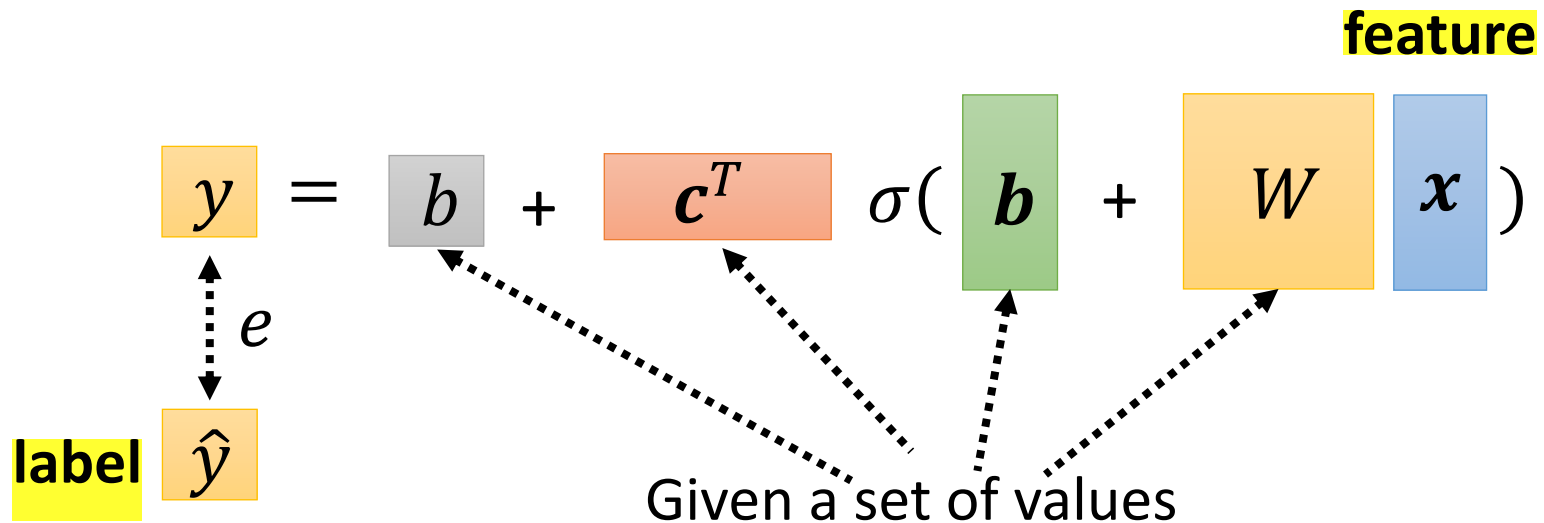


Back to ML Framework



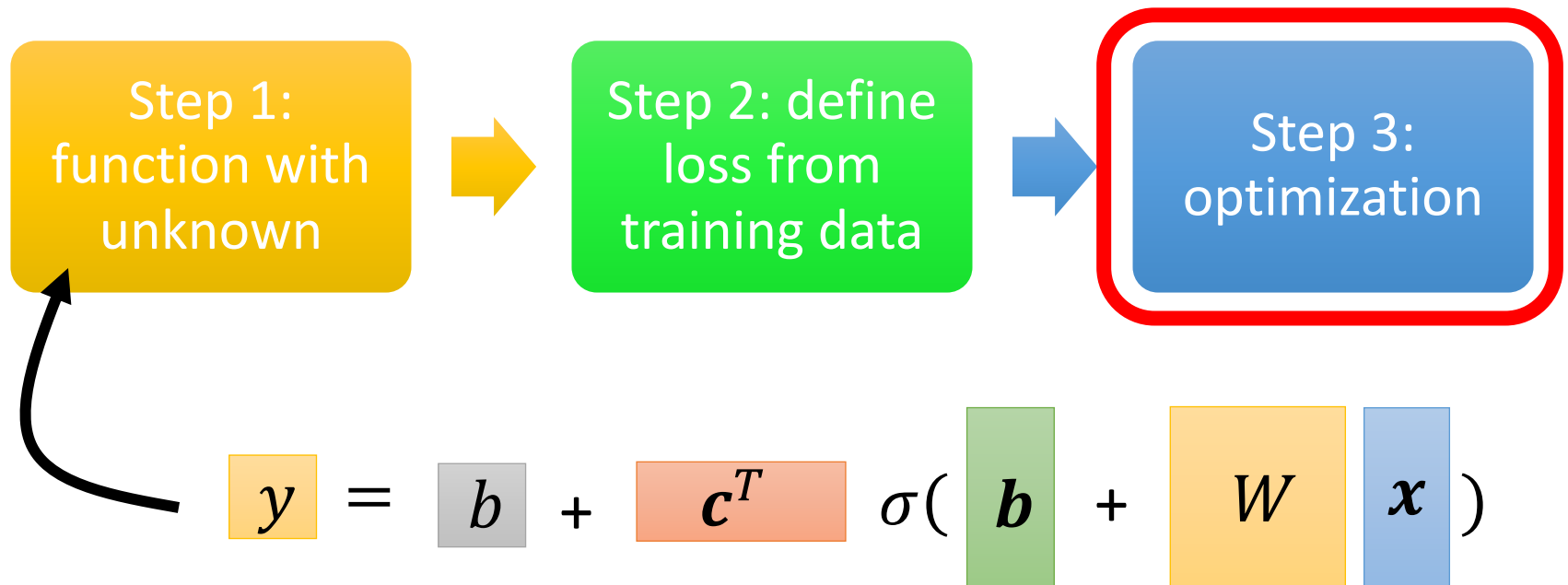
Loss

- Loss is a function of parameters $L(\theta)$
- Loss means how good a set of values is.



Loss:
$$L = \frac{1}{N} \sum_n e_n$$

Back to ML Framework



Optimization of New Model

θ^* = $\arg \min_{\theta} L$
使L最小的那组

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \vdots \end{bmatrix}$$

➤ (Randomly) Pick initial values θ^0

$$\underset{\text{gradient}}{g} = \begin{bmatrix} \frac{\partial L}{\partial \theta_1} \big|_{\theta=\theta^0} \\ \frac{\partial L}{\partial \theta_2} \big|_{\theta=\theta^0} \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \theta_1^1 \\ \theta_2^1 \\ \vdots \end{bmatrix} \leftarrow \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \\ \vdots \end{bmatrix} - \begin{bmatrix} \eta \frac{\partial L}{\partial \theta_1} \big|_{\theta=\theta^0} \\ \eta \frac{\partial L}{\partial \theta_2} \big|_{\theta=\theta^0} \\ \vdots \end{bmatrix}$$

$$g = \nabla L(\theta^0)$$

$$\theta^1 \leftarrow \theta^0 - \eta g \quad (\text{update})$$

Optimization of New Model

$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta}} L$$

➤ (Randomly) Pick initial values $\boldsymbol{\theta}^0$

➤ Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^0)$

$$\boldsymbol{\theta}^1 \leftarrow \boldsymbol{\theta}^0 - \eta \boldsymbol{g}$$

➤ Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^1)$

$$\boldsymbol{\theta}^2 \leftarrow \boldsymbol{\theta}^1 - \eta \boldsymbol{g}$$

➤ Compute gradient $\boldsymbol{g} = \nabla L(\boldsymbol{\theta}^2)$

$$\boldsymbol{\theta}^3 \leftarrow \boldsymbol{\theta}^2 - \eta \boldsymbol{g}$$

Optimization of New Model

$$\theta^* = \arg \min_{\theta} L$$

➤ (Randomly) Pick initial values θ^0

➤ Compute gradient $g = \nabla L^1(\theta^0)$

update $\theta^1 \leftarrow \theta^0 - \eta g$

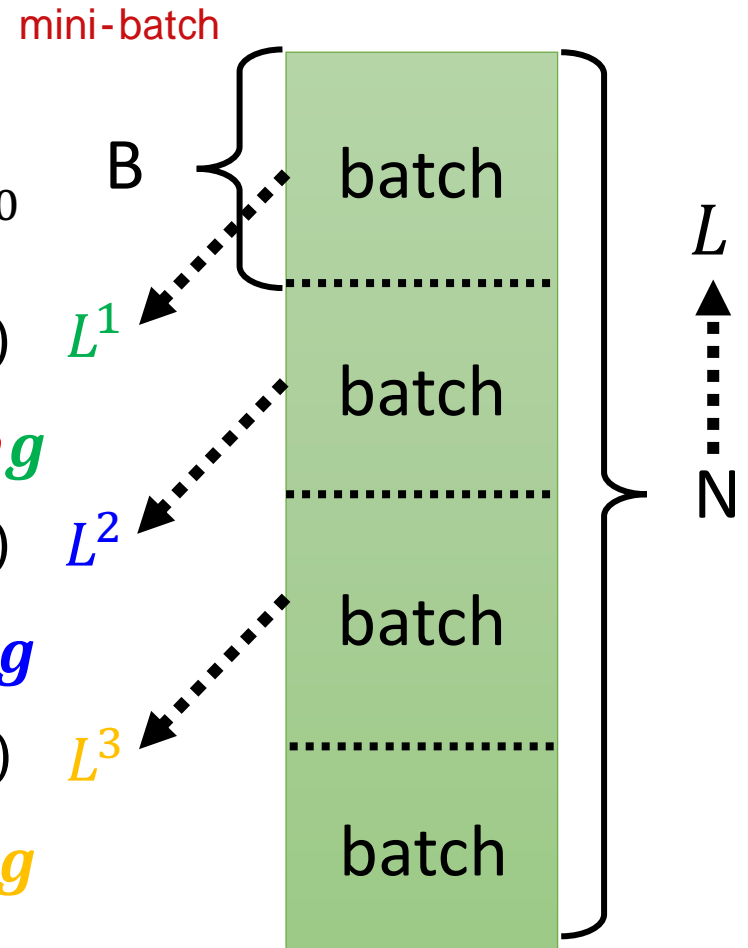
➤ Compute gradient $g = \nabla L^2(\theta^1)$

update $\theta^2 \leftarrow \theta^1 - \eta g$

➤ Compute gradient $g = \nabla L^3(\theta^2)$

update $\theta^3 \leftarrow \theta^2 - \eta g$

1 **epoch** = see all the batches once



Optimization of New Model

Example 1

- 10,000 examples ($N = 10,000$)
- Batch size is 10 ($B = 10$)

How many update in **1 epoch**?

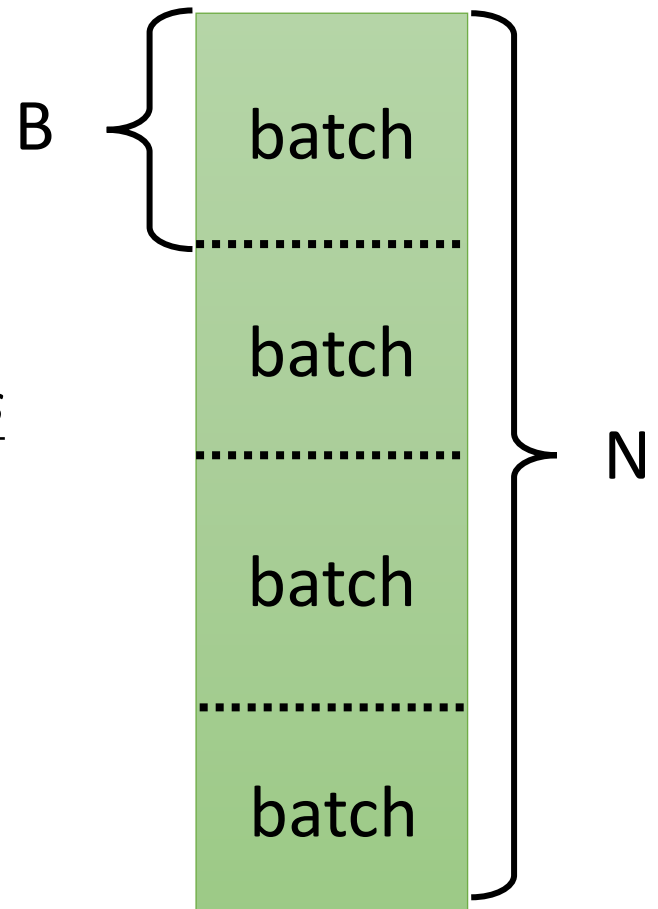
1,000 updates

Example 2

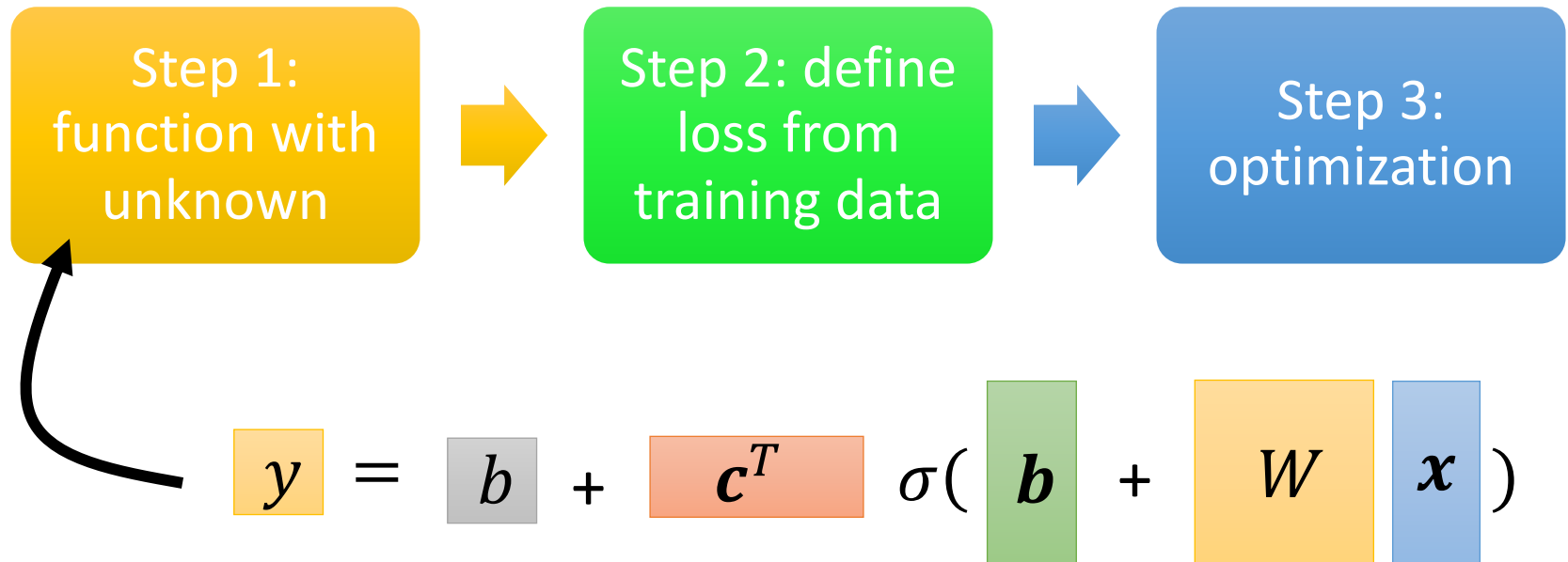
- 1,000 examples ($N = 1,000$)
- Batch size is 100 ($B = 100$)

How many update in **1 epoch**?

10 updates



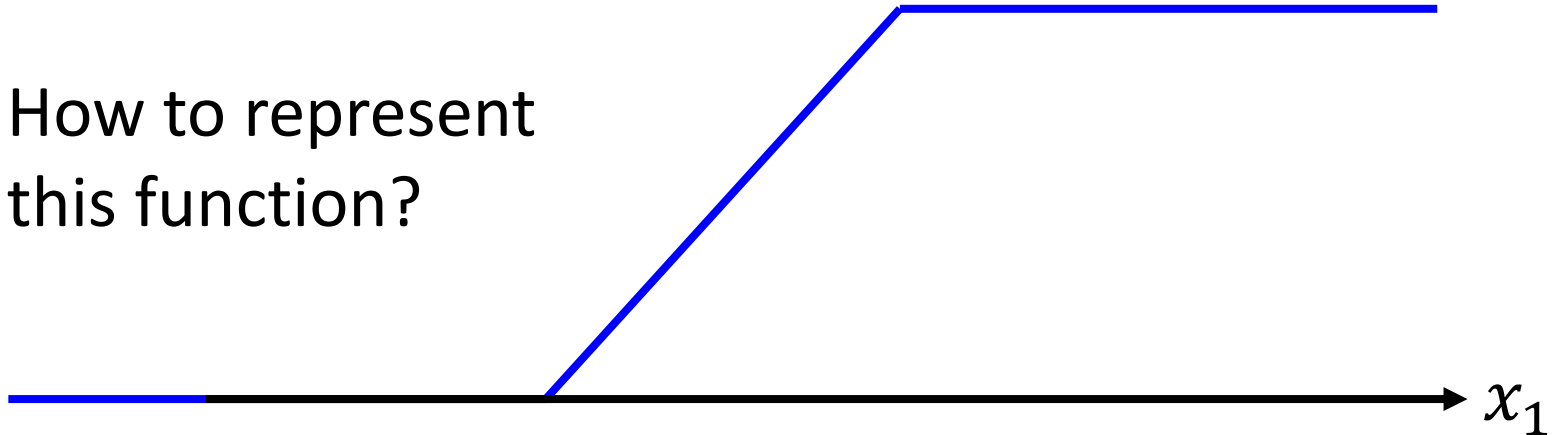
Back to ML Framework



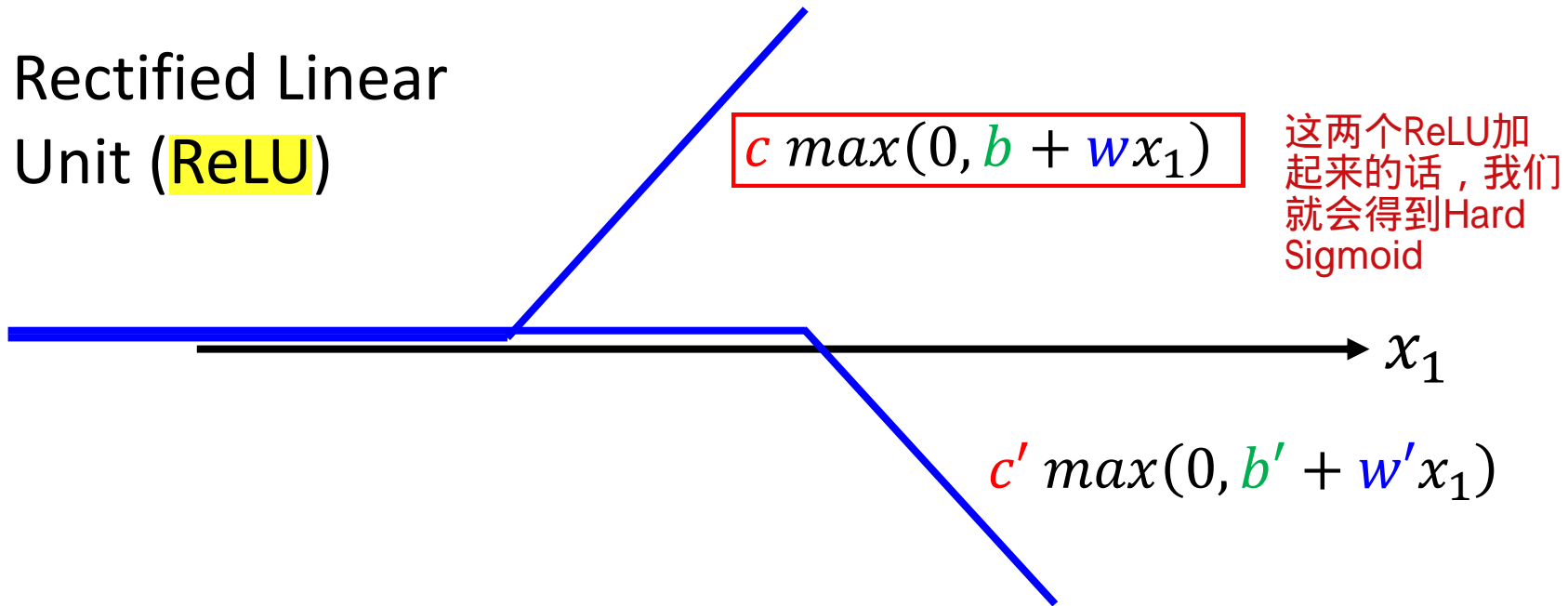
More variety of models ...

Sigmoid \rightarrow ReLU

How to represent
this function?




Rectified Linear
Unit (ReLU)




Sigmoid \rightarrow ReLU

Sigmoid

$$y = b + \sum_i c_i \text{sigmoid}\left(b_i + \sum_j w_{ij} x_j\right)$$


Activation function

ReLU

$$y = b + \sum_{2i} c_i \max\left(0, b_i + \sum_j w_{ij} x_j\right)$$


Which one is better?

Experimental Results

$$y = b + \sum_{\textcolor{red}{i}} \textcolor{red}{c}_i \max \left(0, \textcolor{green}{b}_i + \sum_j \textcolor{blue}{w}_{ij} x_j \right)$$

	linear
2017 – 2020	0.32k
2021	0.46k

10 ReLU

0.32k

0.45k

10 ReLU

0.28k

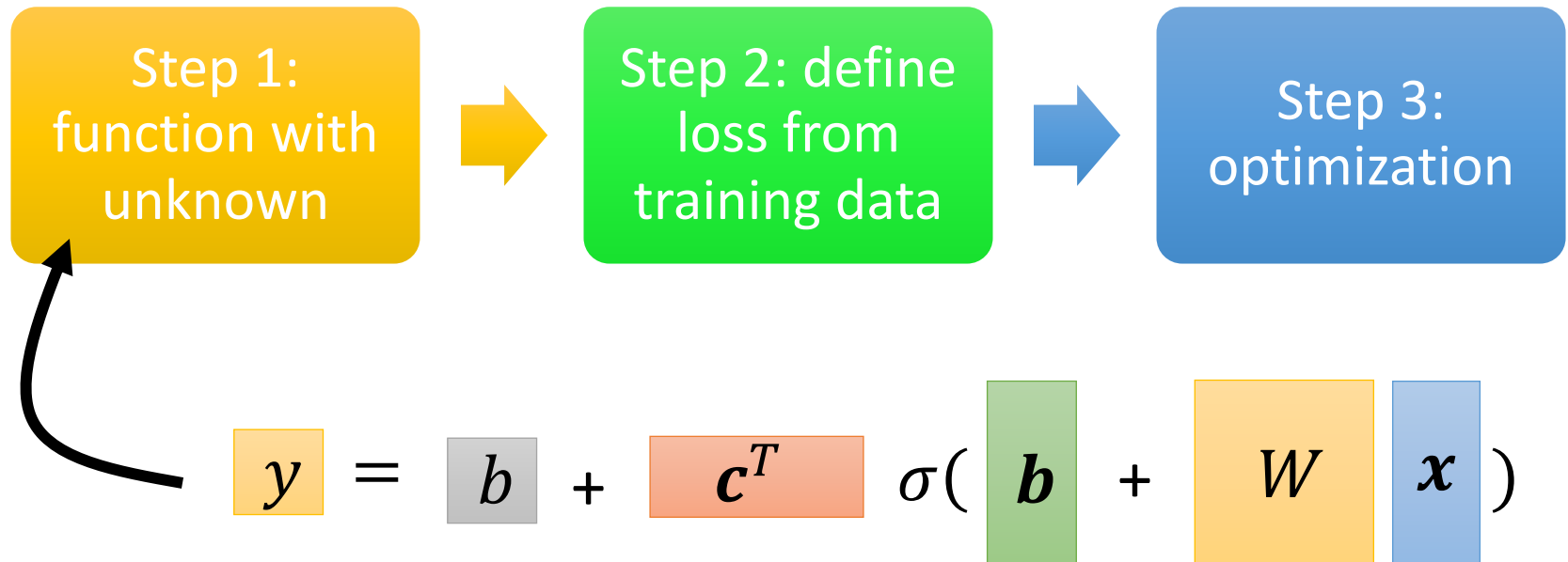
0.43k

1000 ReLU

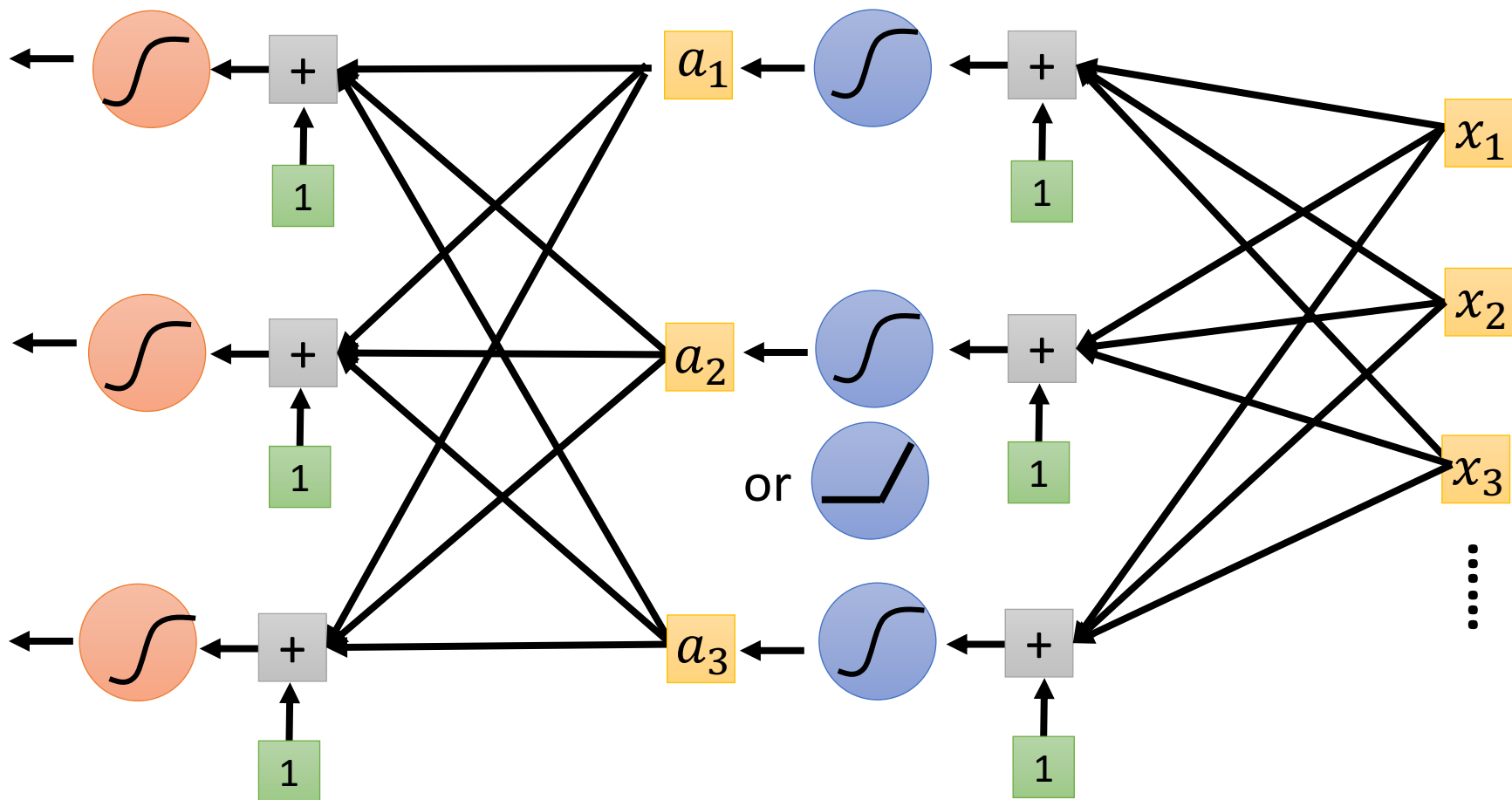
0.27k

0.43k

Back to ML Framework



Even more variety of models ...



$$\begin{aligned}
 \boxed{a'} &= \sigma \left(\boxed{b'} + \boxed{W'} \boxed{a} \right) & \boxed{a} &= \sigma \left(\boxed{b} + \boxed{W} \boxed{x} \right)
 \end{aligned}$$

A dashed arrow points from the \boxed{a} in the first equation to the \boxed{a} in the second equation, indicating the flow of information from the hidden layer output to the input of the next layer.

Experimental Results

- Loss for multiple hidden layers
 - 100 ReLU for each layer
 - input features are the no. of views in the past 56 days

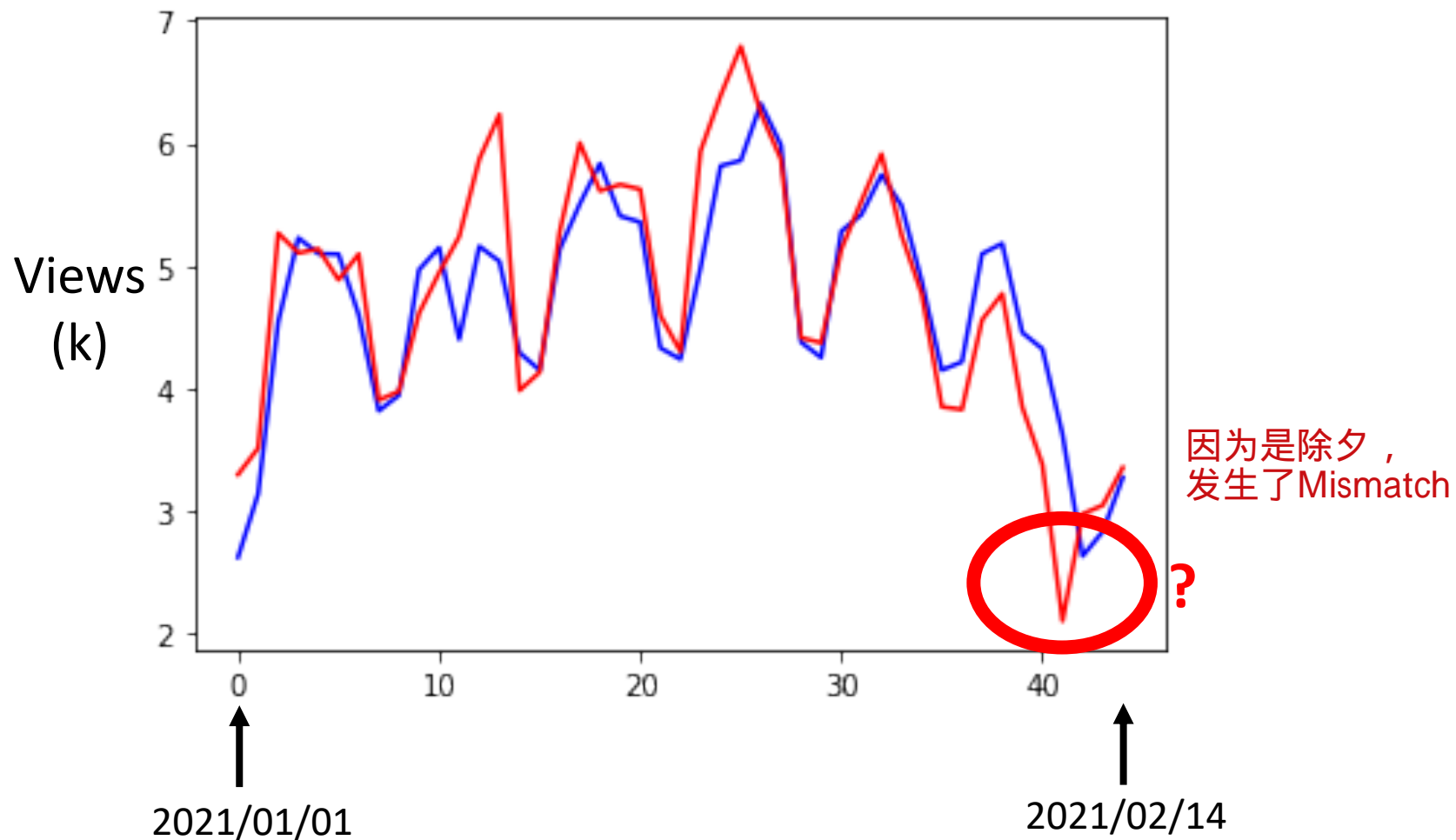
	1 layer	2 layer	3 layer
2017 – 2020	0.28k	0.18k	0.14k
2021	0.43k	0.39k	0.38k

都有进步

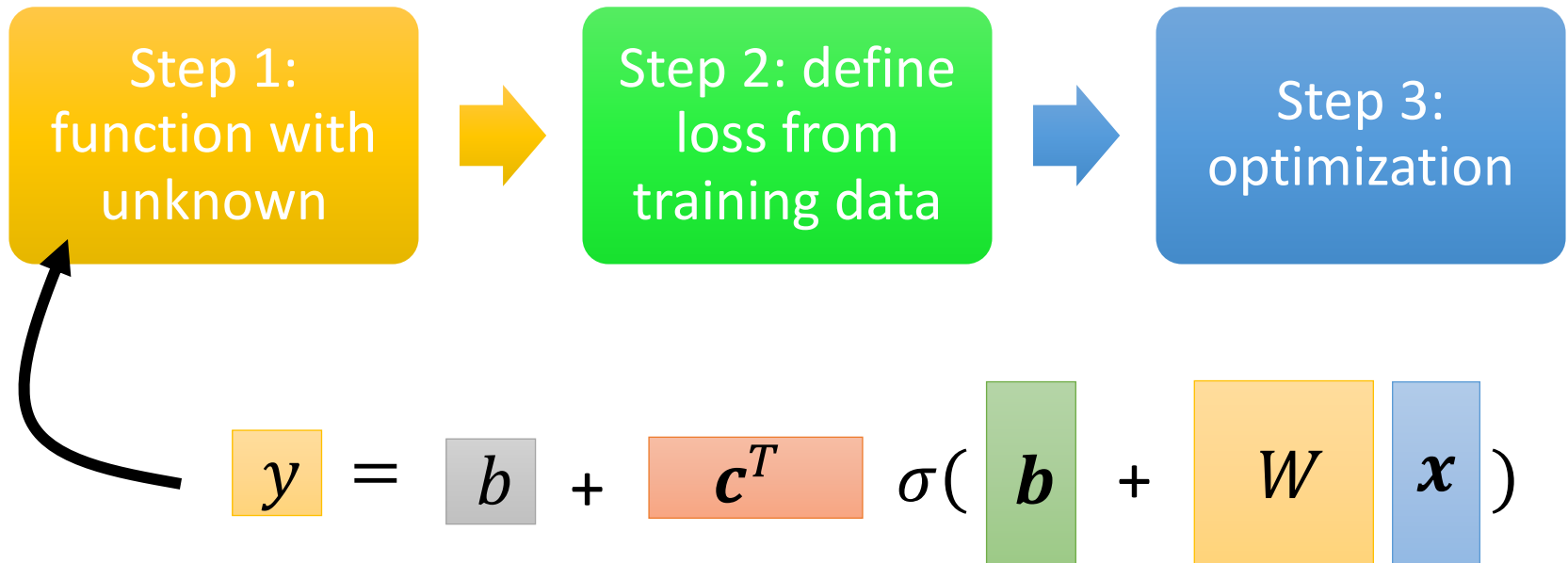
3 layers

Red: real no. of views

blue: estimated no. of views

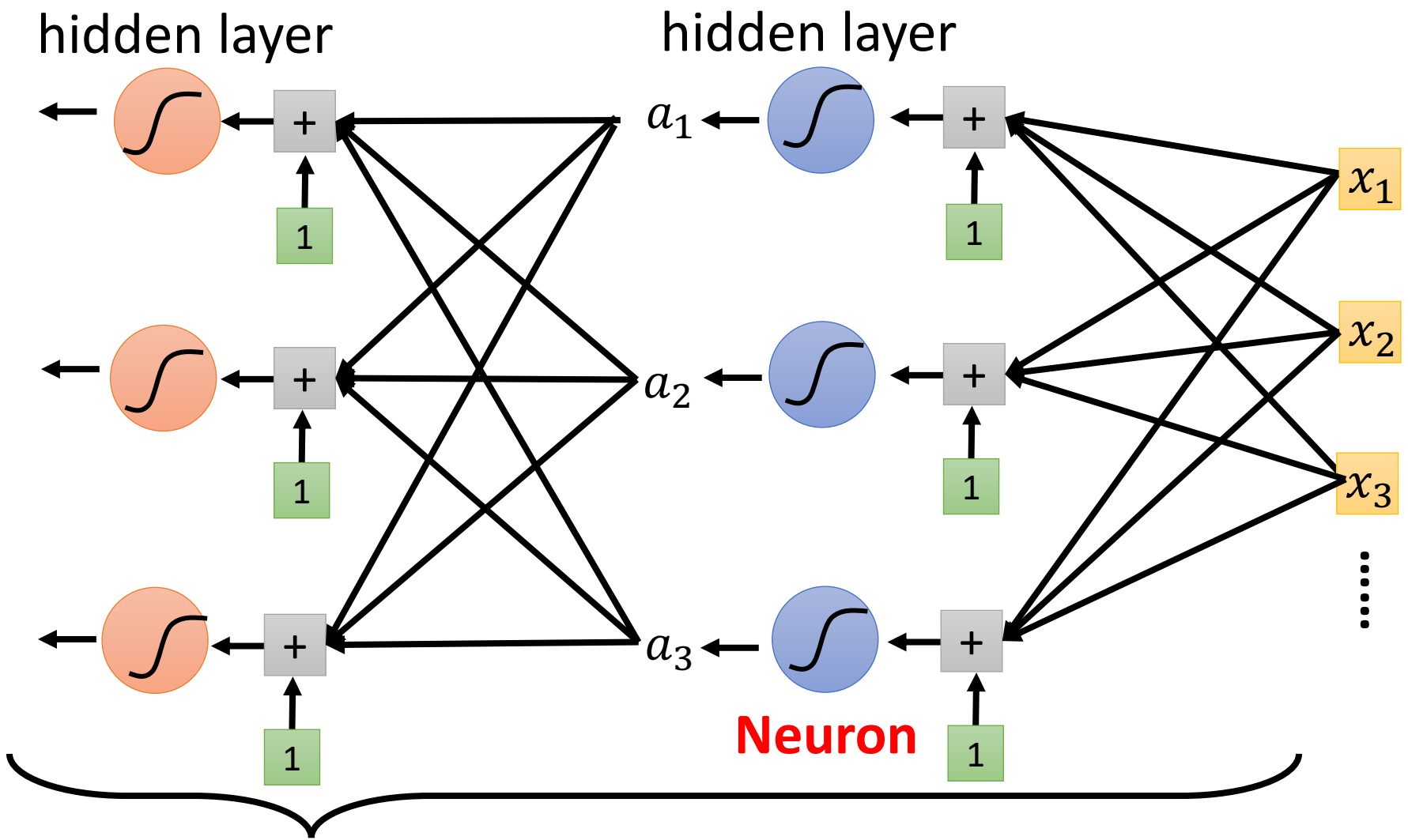


Back to ML Framework



It is not ***fancy*** enough.

Let's give it a ***fancy*** name!



Neural Network

This mimics human brains ... (???)

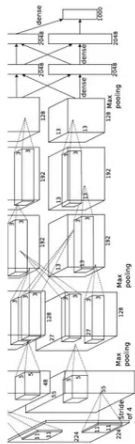
Many layers means **Deep** ➡ **Deep Learning**

Deep = Many hidden layers

http://cs231n.stanford.edu/slides/winter1516_lecture8.pdf

8 layers

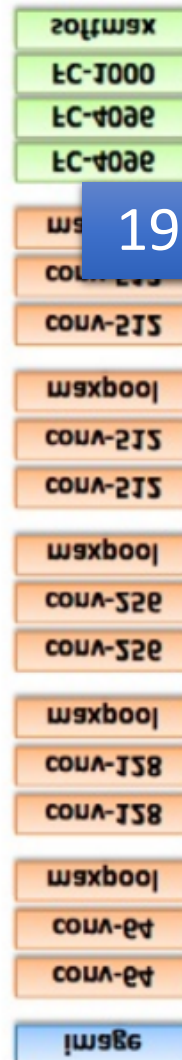
16.4%



AlexNet (2012)

19 layers

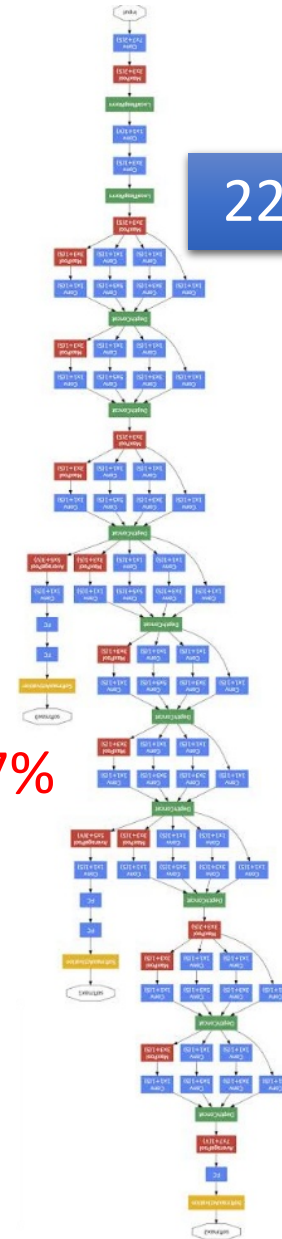
7.3%



VGG (2014)

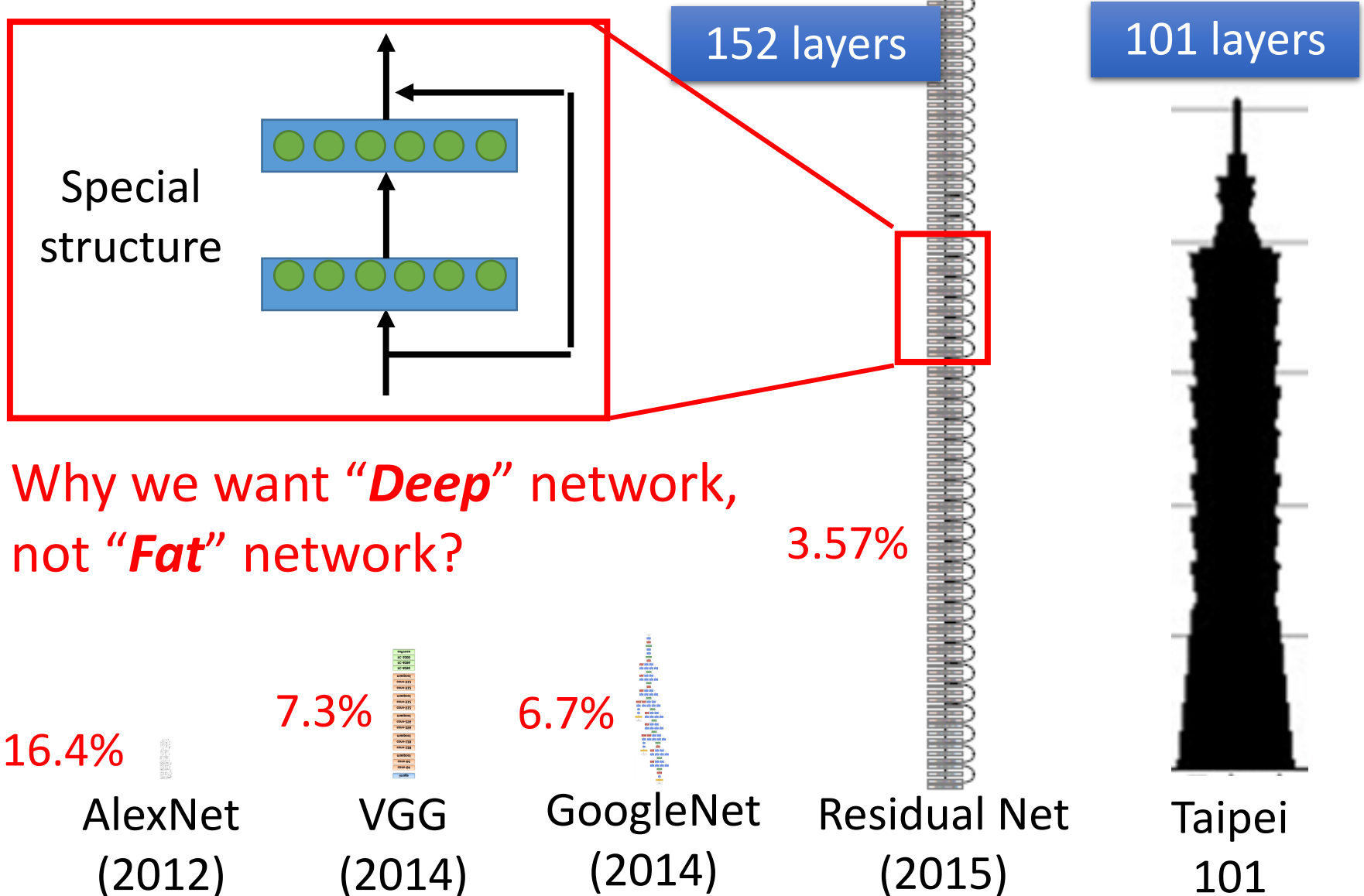
22 layers

6.7%



GoogleNet (2014)

Deep = Many hidden layers



Why don't we go deeper?

- Loss for multiple hidden layers
 - 100 ReLU for each layer
 - input features are the no. of views in the past 56 days

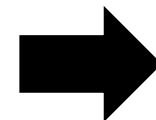
	1 layer	2 layer	3 layer
2017 – 2020	0.28k	0.18k	0.14k
2021	0.43k	0.39k	0.38k

Why don't we go deeper?

- Loss for multiple hidden layers
 - 100 ReLU for each layer
 - input features are the no. of views in the past 56 days

	1 layer	2 layer	3 layer	4 layer
2017 – 2020	0.28k	0.18k	0.14k	0.10k
2021	0.43k	0.39k	0.38k	0.44k

Better on training data, worse on unseen data



Overfitting

Let's predict no. of views today!

- If we want to select a model for predicting no. of views today, which one will you use?

	1 layer	2 layer	3 layer	4 layer
2017 – 2020	0.28k	0.18k	0.14k	0.10k
2021	0.43k	0.39k	0.38k	0.44k

We will talk about model selection next time. 😊

To learn more

Basic Introduction



<https://youtu.be/Dr-WRIEFefw>

Backpropagation

Computing gradients in
an efficient way



<https://youtu.be/ibJpTrp5mcE>