Error surface is rugged …

Tips for training: **Adaptive Learning Rate**

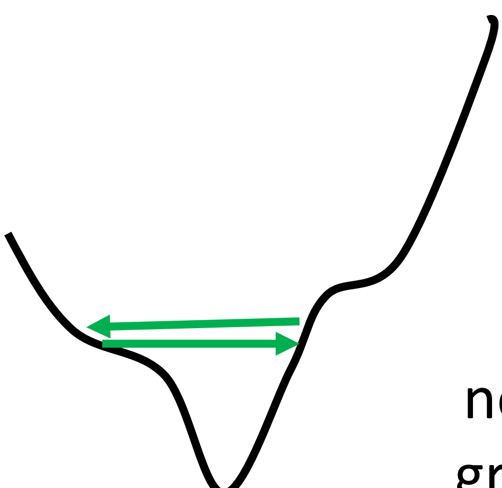
自动调整学习率

https://docs.google.com/presentation/d/1siUFXARYRpNiMeSRwgFbt7mZVjkMPhR5od09w0Z8xaU/edit#slide=id.g3532c09be1_0_382

Training stuck ≠ Small Gradient

训练卡住的话，不一定是碰到了critical points

- People believe training stuck because the parameters are around a critical point ...

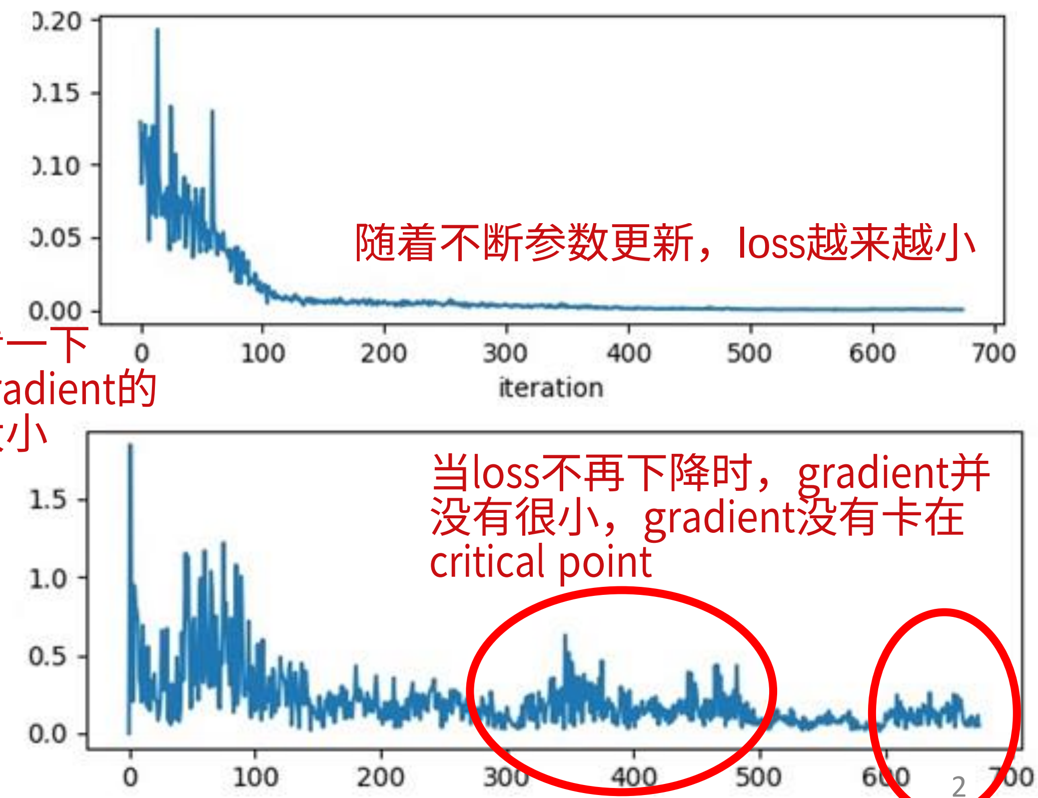


可能遇到了这种情况：
并不是到了critical point

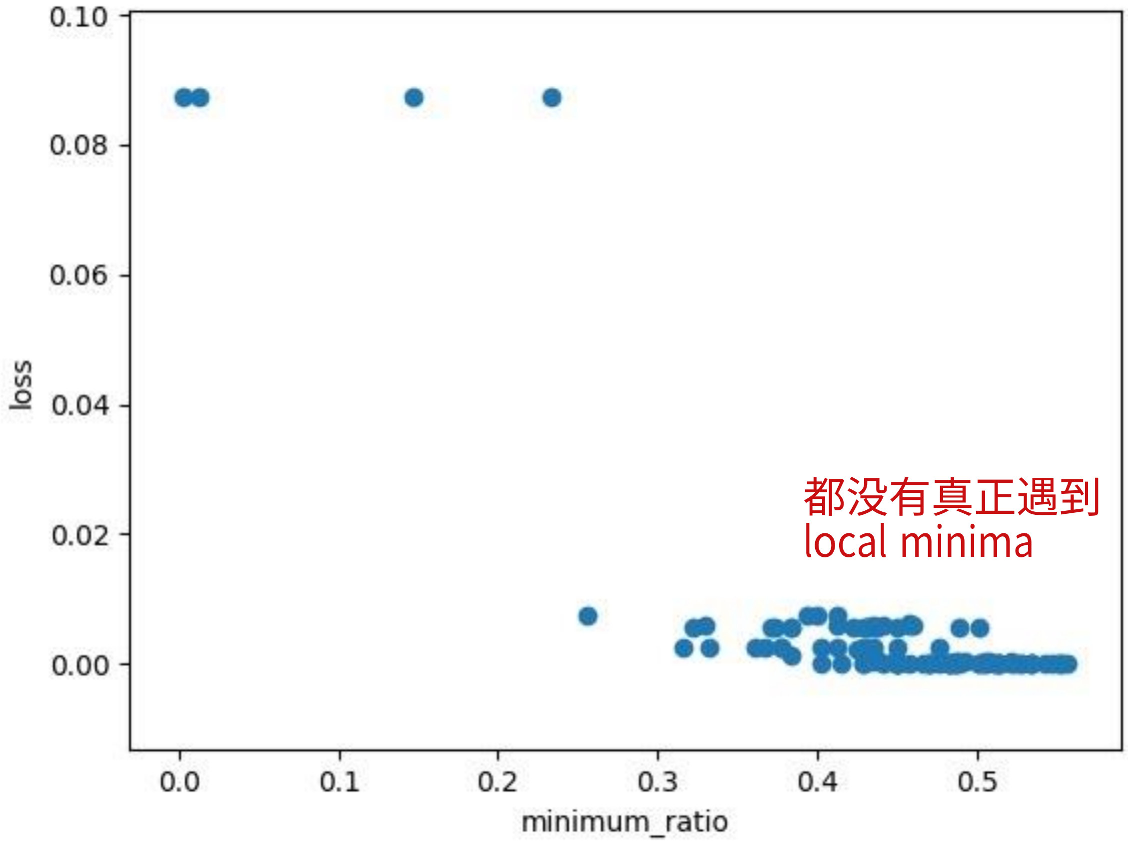
loss

看一下
gradient的
大小

norm of
gradient



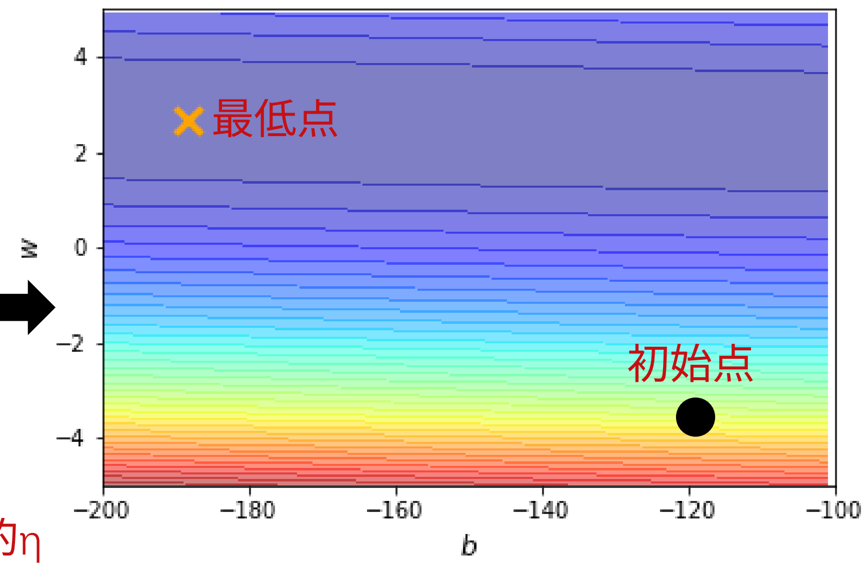
Wait a minute …

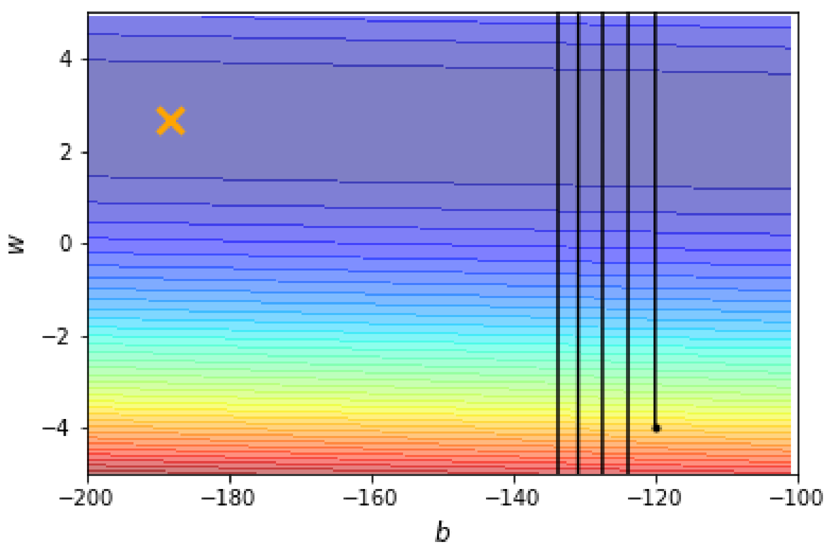


Training can be difficult even without critical points.

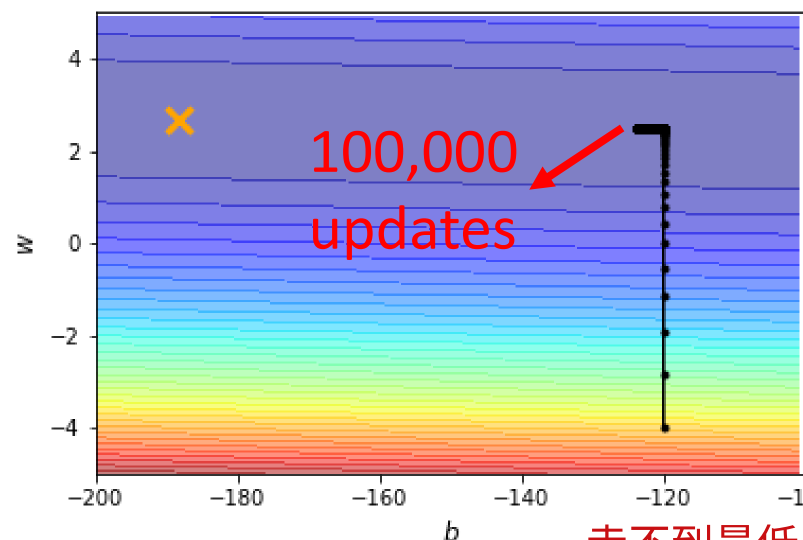
This error surface is convex.
凸

Learning rate **cannot** be **one-size-fits-all** 我们需要客制化的η





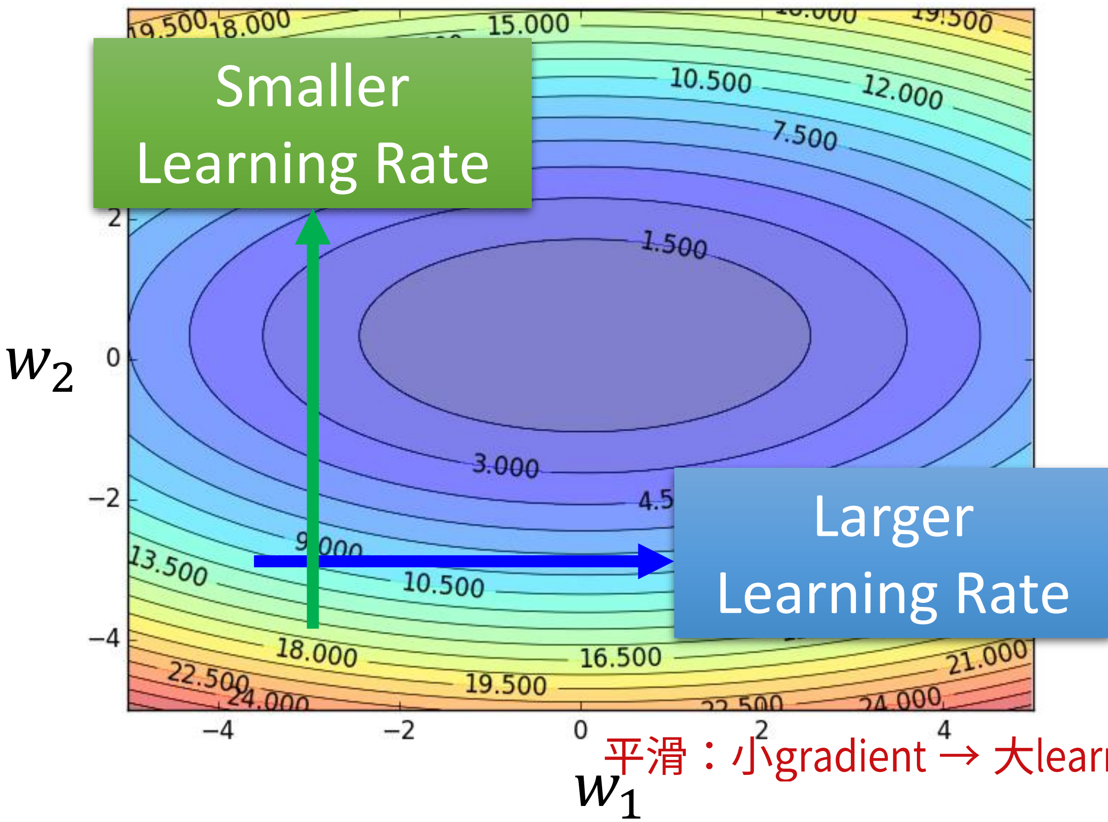
$\eta = 10^{-2}$
learning rate太大了，不断震荡



$\eta = 10^{-7}$
走不到最低点，因为learning rate太小了

Different parameters needs different learning rate

陡峭：大gradient → 小learning rate



平滑：小gradient → 大learning rate

Formulation for **one** parameter:

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \boxed{\eta}\, \boldsymbol{g}_i^t$$

普通learning rate

$$\boldsymbol{g}_i^t = \frac{\partial L}{\partial \boldsymbol{\theta}_i} |_{\boldsymbol{\theta}=\boldsymbol{\theta}^t}$$

Adaptive learning rate

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \boxed{\frac{\eta}{\sigma_i^t}}\, \boldsymbol{g}_i^t$$

Parameter dependent

i → parameter dependent
t → iteration dependent

一个常见的方法：RMS

Root Mean Square

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^t$$

$$\boldsymbol{\theta}_i^1 \leftarrow \boldsymbol{\theta}_i^0 - \frac{\eta}{\sigma_i^0} \boldsymbol{g}_i^0 \qquad \sigma_i^0 = \sqrt{\left(\boldsymbol{g}_i^0\right)^2} = \left|\boldsymbol{g}_i^0\right|$$

$$\boldsymbol{\theta}_i^2 \leftarrow \boldsymbol{\theta}_i^1 - \frac{\eta}{\sigma_i^1} \boldsymbol{g}_i^1 \qquad \sigma_i^1 = \sqrt{\frac{1}{2}\left[\left(\boldsymbol{g}_i^0\right)^2 + \left(\boldsymbol{g}_i^1\right)^2\right]}$$

$$\boldsymbol{\theta}_i^3 \leftarrow \boldsymbol{\theta}_i^2 - \frac{\eta}{\sigma_i^2} \boldsymbol{g}_i^2 \qquad \sigma_i^2 = \sqrt{\frac{1}{3}\left[\left(\boldsymbol{g}_i^0\right)^2 + \left(\boldsymbol{g}_i^1\right)^2 + \left(\boldsymbol{g}_i^2\right)^2\right]}$$

$$\vdots$$

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \frac{\eta}{\sigma_i^t} \boldsymbol{g}_i^t \qquad \sigma_i^t = \sqrt{\frac{1}{t+1}\sum_{i=0}^{t}\left(\boldsymbol{g}_i^t\right)^2}$$

AdaGrad

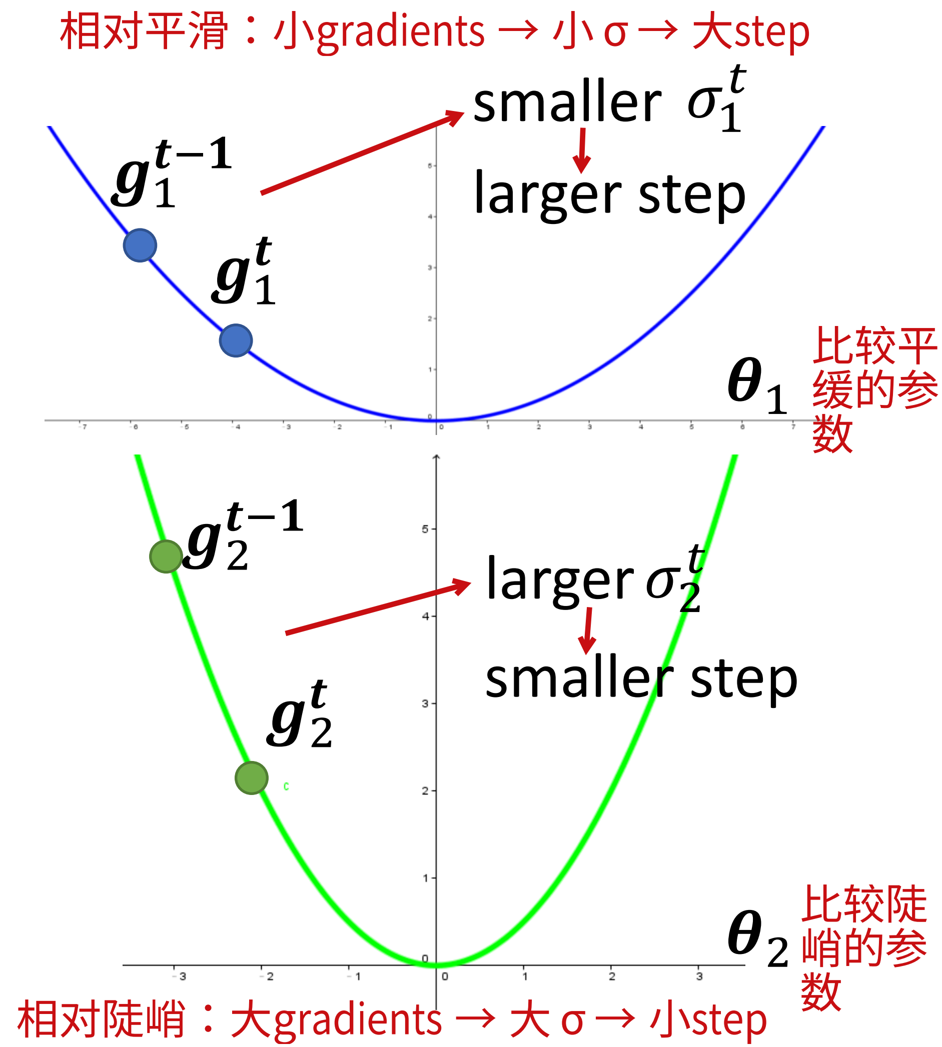
考虑前面所有的gradient

Root Mean Square

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^{t}$$

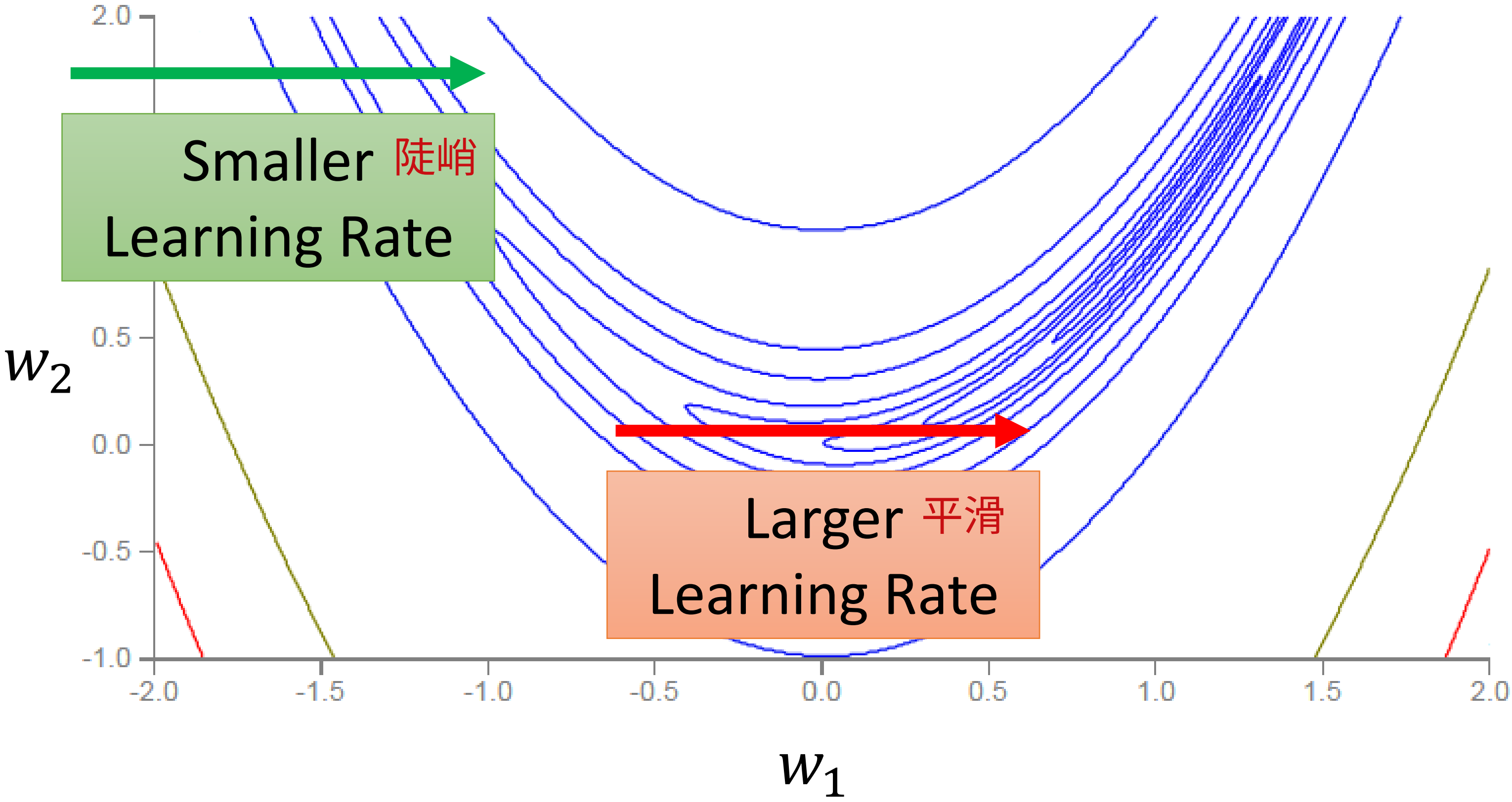
$$\sigma_i^t = \sqrt{\frac{1}{t+1}\sum_{i=0}^{t}\left(\boldsymbol{g}_i^t\right)^2}$$

Used in **Adagrad**



Learning rate adapts dynamically

Error Surface can be very complex.



传奇：Hinton的Coursera的课上提出
引用时，用slides的页数

RMSProp

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^t$$

α 是另一个hyperparameter
接近0：之前的gradient不重要
接近1：之前的gradient重要，现在算出来的gradient不重要

$$0 < \alpha < 1$$

$$\boldsymbol{\theta}_i^1 \leftarrow \boldsymbol{\theta}_i^0 - \frac{\eta}{\sigma_i^0} \boldsymbol{g}_i^0 \qquad \sigma_i^0 = \sqrt{\left(\boldsymbol{g}_i^0\right)^2}$$

$$\boldsymbol{\theta}_i^2 \leftarrow \boldsymbol{\theta}_i^1 - \frac{\eta}{\sigma_i^1} \boldsymbol{g}_i^1 \qquad \sigma_i^1 = \sqrt{\alpha\left(\sigma_i^0\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^1\right)^2}$$

$$\boldsymbol{\theta}_i^3 \leftarrow \boldsymbol{\theta}_i^2 - \frac{\eta}{\sigma_i^2} \boldsymbol{g}_i^2 \qquad \sigma_i^2 = \sqrt{\alpha\left(\sigma_i^1\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^2\right)^2}$$

$$\vdots$$

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \frac{\eta}{\sigma_i^t} \boldsymbol{g}_i^t \qquad \sigma_i^t = \sqrt{\alpha\left(\sigma_i^{t-1}\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^t\right)^2}$$

相比较Adagrad，增加了对现在gradient的敏感度

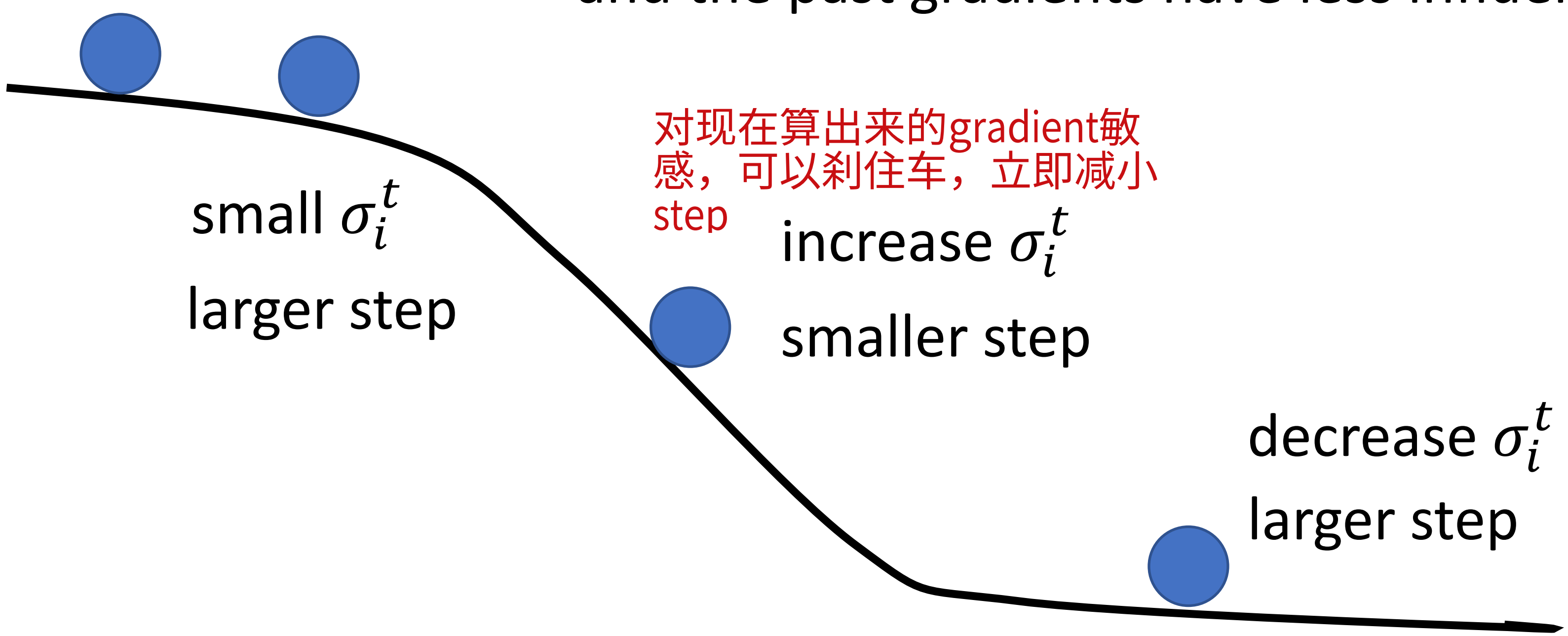
RMSProp

$$\boldsymbol{g}_i^{\mathbf{1}} \quad \boldsymbol{g}_i^{\mathbf{2}} \quad \cdots\cdots \quad \boldsymbol{g}_i^{\boldsymbol{t-1}}$$

$$0 < \alpha < 1$$

$$\boldsymbol{\theta}_i^{\boldsymbol{t+1}} \leftarrow \boldsymbol{\theta}_i^{\boldsymbol{t}} - \boxed{\frac{\eta}{\sigma_i^t}} \boldsymbol{g}_i^{\boldsymbol{t}} \qquad \sigma_i^t = \sqrt{\alpha\left(\sigma_i^{t-1}\right)^2 + (1-\alpha)\left(\boldsymbol{g}_i^{\boldsymbol{t}}\right)^2}$$

The recent gradient has larger influence, and the past gradients have less influence.



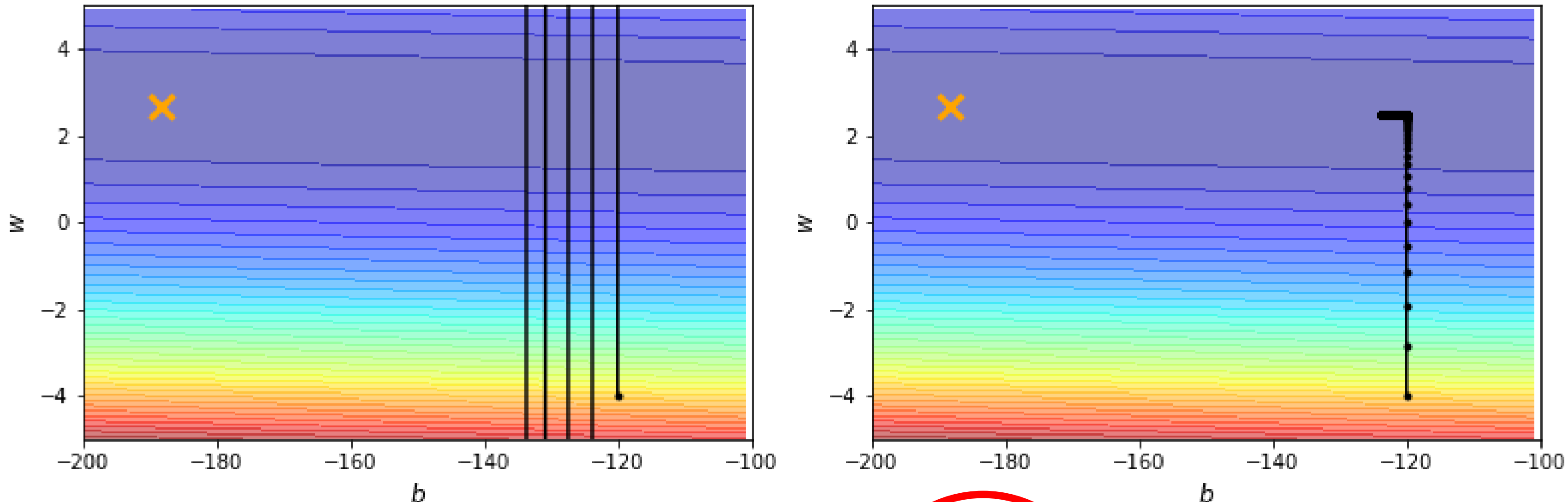
Original paper: https://arxiv.org/pdf/1412.6980.pdf

Adam: RMSProp + Momentum

Algorithm 1: *Adam*, our proposed algorithm for stochastic optimization. See section 2 for details, and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise square $g_t \odot g_t$. Good default settings for the tested machine learning problems are $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$. All operations on vectors are element-wise. With β_1^t and β_2^t we denote β_1 and β_2 to the power t.

Require: α: Stepsize
Require: $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates
Require: $f(\theta)$: Stochastic objective function with parameters θ
Require: θ_0: Initial parameter vector
  $m_0 \leftarrow 0$ (Initialize 1st moment vector) → for momentum
  $v_0 \leftarrow 0$ (Initialize 2nd moment vector) → for RMSprop
  $t \leftarrow 0$ (Initialize timestep)
  while θ_t not converged **do**
    $t \leftarrow t + 1$
    $g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$ (Get gradients w.r.t. stochastic objective at timestep t)
    $m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ (Update biased first moment estimate)
    $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ (Update biased second raw moment estimate)
    $\widehat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ (Compute bias-corrected first moment estimate)
    $\widehat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ (Compute bias-corrected second raw moment estimate)
    $\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t / (\sqrt{\widehat{v}_t} + \epsilon)$ (Update parameters)
  end while
  return θ_t (Resulting parameters)

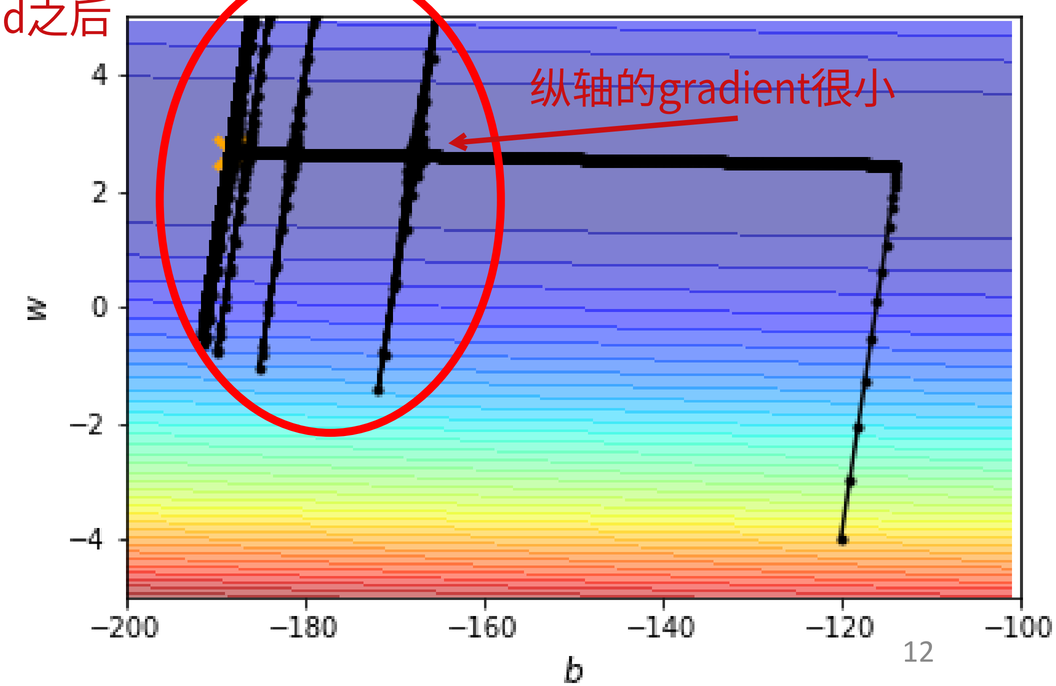
Without Adaptive Learning Rate



使用Adadgrad之后

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \boxed{\frac{\eta}{\sigma_i^t}}\boldsymbol{g}_i^t$$

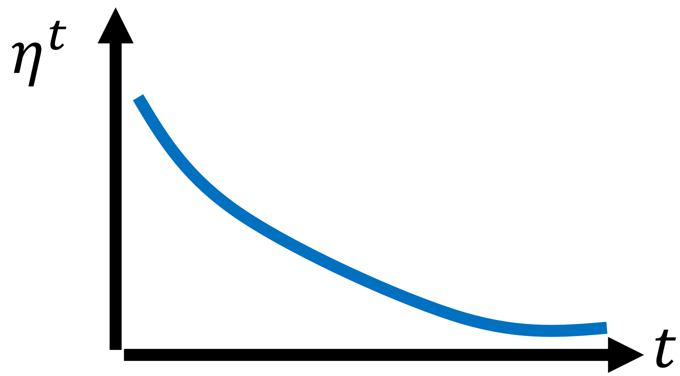
$$\sigma_i^t = \sqrt{\frac{1}{t+1}\sum_{i=0}^{t}\left(\boldsymbol{g}_i^t\right)^2}$$



Learning Rate Scheduling

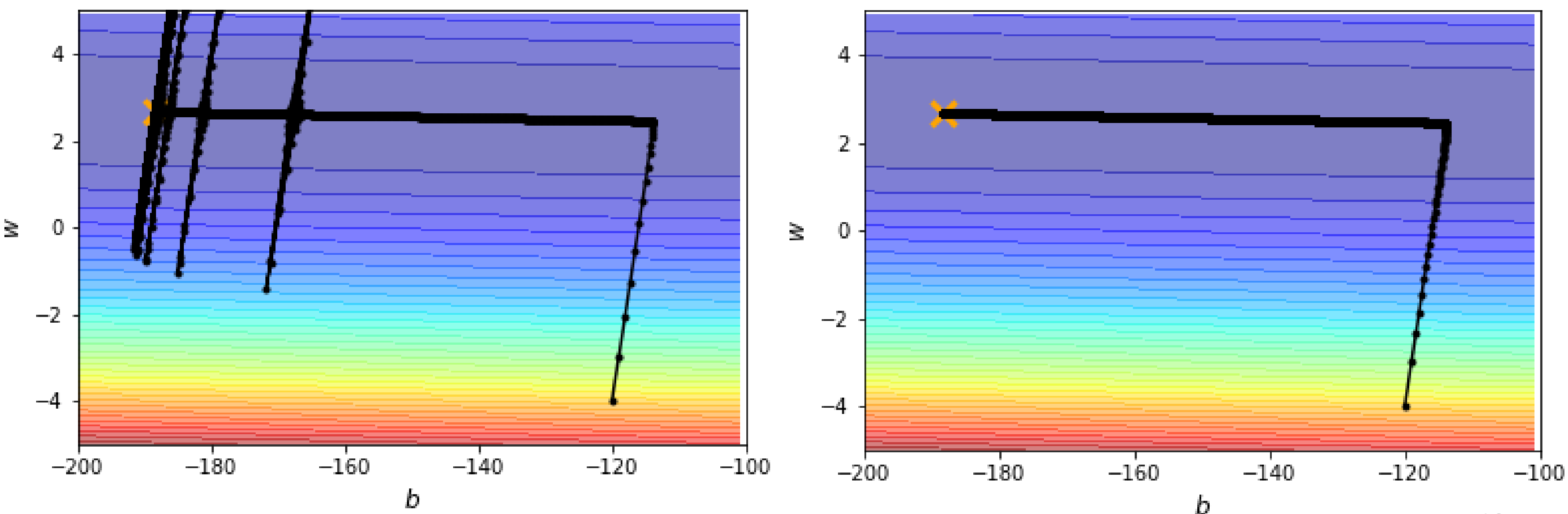
和时间有关的learning rate

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \frac{\boxed{\eta^t}}{\sigma_i^t}\boldsymbol{g}_i^t$$



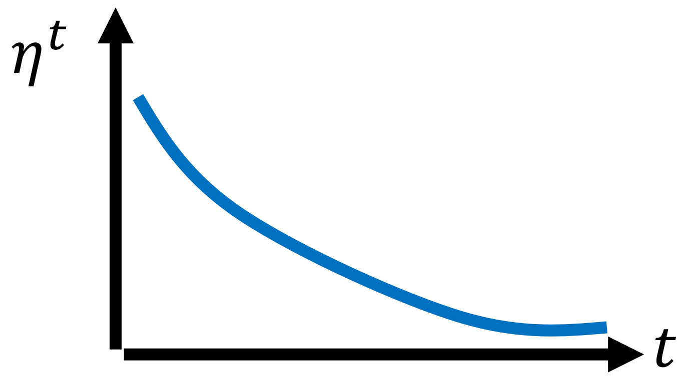
1. ***Learning Rate Decay***

As the training goes, we are closer to the destination, so we reduce the learning rate.



Learning Rate Scheduling

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^t - \frac{\boxed{\eta^t}}{\sigma_i^t} \boldsymbol{g}_i^t$$



1. ***Learning Rate Decay***

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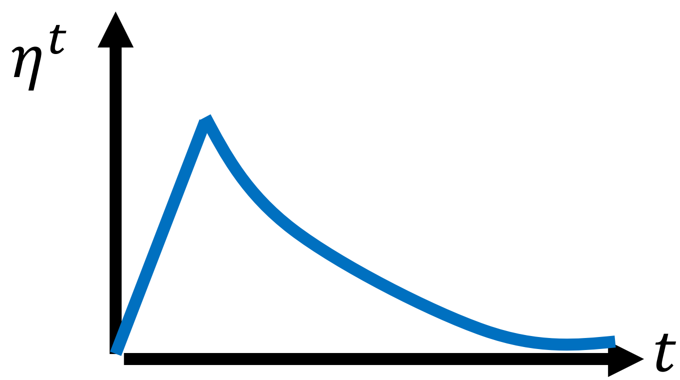


2. ***Warm Up*** （黑科技）

   Increase and then decrease?

warmup在知名论文里被提到，当作黑科技

We further explore $n = 18$ that leads to a 110-layer ResNet. In this case, we find that the initial learning rate of 0.1 is slightly too large to start converging[5]. So we use 0.01 to warm up the training until the training error is below 80% (about 400 iterations), and then go back to 0.1 and continue training. The rest of the learning schedule is as done previously. This 110-layer network converges well (Fig. 6, middle). It has *fewer* parameters than other deep and thin

[5] With an initial learning rate of 0.1, it starts converging (<90% error) after several epochs, but still reaches similar accuracy.

Residual Network

https://arxiv.org/abs/1512.03385

2015年12月

5.3 Optimizer

We used the Adam optimizer [17] with $\beta_1 = 0.9$, $\beta_2 = 0.98$ and $\epsilon = 10^{-9}$. We varied the learning rate over the course of training, according to the formula:

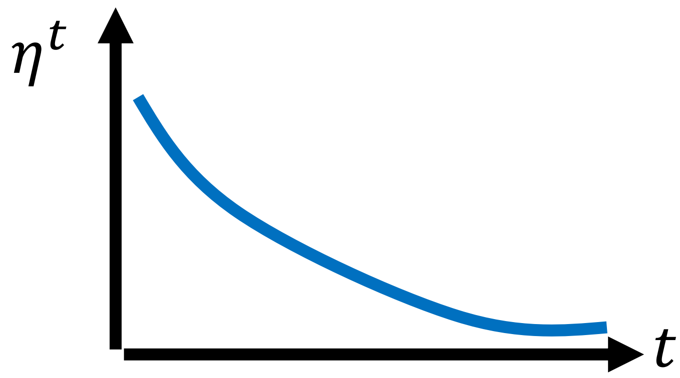
$$lrate = d_{\text{model}}^{-0.5} \cdot \min(step\_num^{-0.5}, step\_num \cdot warmup\_steps^{-1.5}) \tag{3}$$

This corresponds to increasing the learning rate linearly for the first $warmup\_steps$ training steps, and decreasing it thereafter proportionally to the inverse square root of the step number. We used $warmup\_steps = 4000$.

Transformer https://arxiv.org/abs/1706.03762

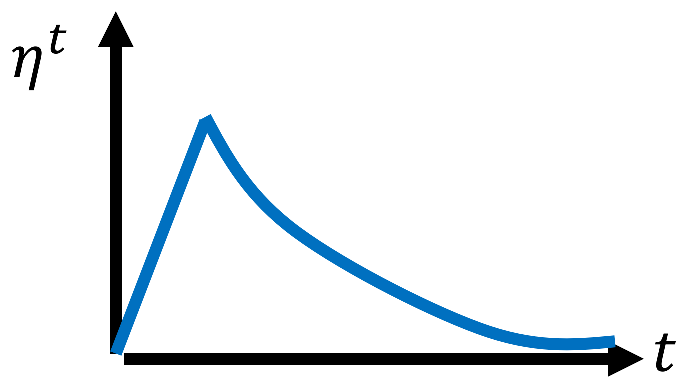
Learning Rate Scheduling

$$\theta_i^{t+1} \leftarrow \theta_i^t - \frac{\boxed{\eta^t}}{\sigma_i^t} g_i^t$$



Learning Rate Decay

After the training goes, we are close to the destination, so we reduce the learning rate.



有一解释：一开始σ不准，先让σ学一学

Warm Up

为什么warmup呢？值得研究

Increase and then decrease?

At the beginning, the estimate of σ_i^t has large variance.

Please refer to **RAdam** https://arxiv.org/abs/1908.03265

Rectified Adam

Summary of Optimization

(Vanilla) Gradient Descent

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \eta \boldsymbol{g}_i^{t}$$

Various Improvements

$$\boldsymbol{\theta}_i^{t+1} \leftarrow \boldsymbol{\theta}_i^{t} - \frac{\eta^t}{\sigma_i^t} \boldsymbol{m}_i^{t}$$

Learning rate scheduling

Momentum: weighted sum of the previous gradients

Consider direction

root mean square of the gradients

only magnitude

To Learn More ……

助教课：optimizers for deep learning

https://youtu.be/4pUmZ8hXlHM

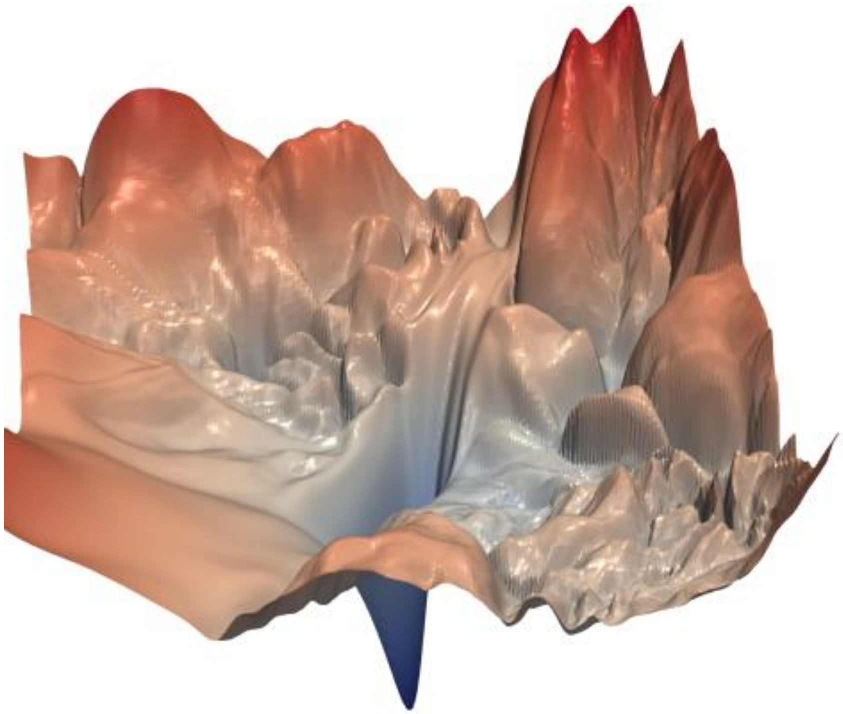
(in Mandarin)

https://youtu.be/e03YKGHXnL8

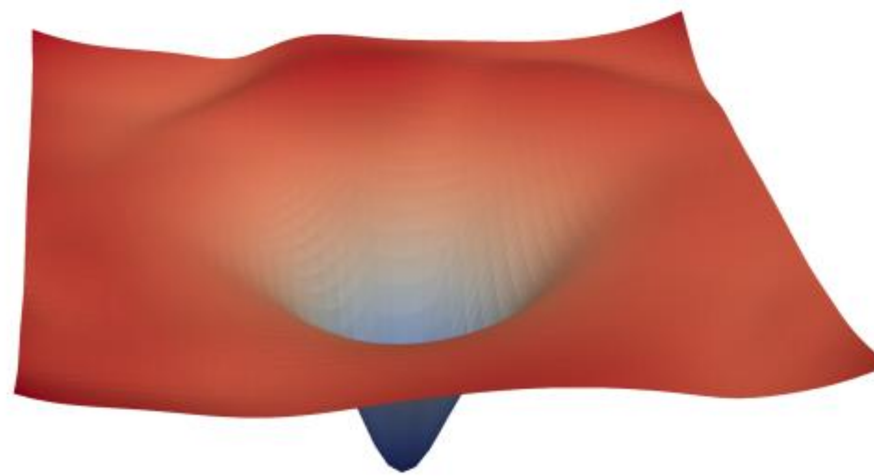
(in Mandarin)

Next Time

Source of image: https://arxiv.org/abs/1712.09913



Next time



Better optimization strategies:
If the mountain won't move, build a road around it.

Can we change the error surface?
Directly move the mountain!