Exercise Sheet 1

1. A call centre receives calls according to a Poisson process. The expected number of calls it receives in a day is 1523. On any given day, the call centre has enough staff to cope with a maximum of 1600 calls. What is the probability that on any given day the call centre receives more calls than it can cope with.

   Function you will need for this question: `ppois`.

2. Write code that simulates a homogeneous temporal Poisson process on $(0, T]$ with intensity λ. Have a go at writing it as a function `homogeneouspp(lambda,T)`, such that it can take any value of λ and T that you wish to give it.

   Simulate 100 realisations of a Poisson process on $(0, 10]$ with intensity $\lambda = 10$.

   (a) Verify that the sample mean number of events in the interval $(3, 5]$ is consistent with the expected number of events in $(3, 5]$.

   (b) Plot a histogram for the 100 realisations of the number of events $N\{(2, 8]\}$. Is this consistent with the expected distribution? HINT: consider plotting the probability mass function for the Poisson distribution on top of the histogram.

   (c) It has been stated that homogeneous Poisson processes are memoryless. That is to say, if A and B are disjoint subsets of $(0, 10]$,then $N(A)$ and $N(B)$ are independent random variables. Independent random variables are uncorrelated. Using your 100 simulations, estimate the correlation coefficient between $N\{(2, 5]\}$ and $N\{(6, 8]\}$.

   Functions you will need for this question: `rpois`, `dpois`, `runif`, `cor`.

3. Simulate a homogeneous spatial Poisson process on $[0, 10] \times [0, 10]$ with intensity $\lambda = 0.5$.

   (a) Simulate 100 realisations of this Poisson process. Verify that the sample mean number of events in the interval $[0, 10] \times [0, 10]$ is consistent with the expected number of events in $[0, 10] \times [0, 10]$.

   (b) Plot a histogram for the 100 realisations of the number of events $N\{[0, 10] \times [0, 10]\}$. Is this consistent with the expected distribution? HINT: consider plotting the probability mass function for the Poisson distribution.

   Functions you will need for this question: `rpoispp`, `dpois`.

4. An inhomogeneous Poisson process with intensity $\lambda(t)$ can be simulated using the following method:

   A. Simulate a homogeneous Poisson process with intensity $\lambda^* = \max_t \lambda(t)$. This produces a set of event times $\{\tau_1, \tau_2, ...\}$

   B. For each realised event τ_i, keep it with probability $p_i = \lambda(\tau_i)/\lambda^*$, or discard it with probability $1 - p_i$. The remaining events are a realisation of the inhomogeneous Poisson process.

   Write code that can simulate from an inhomogeneous Poisson process on $(0, 100]$ with intensity function $\lambda(t) = 20 \exp\{-t/10\}$.

5. An inhomogeneous spatial Poisson process with intensity $\lambda(\mathbf{s})$ can be simulated using the `rpoispp` function. Look at the documentation for this function, and then attempt to simulate an inhomogeneous Poisson process on $[0, 10] \times [0, 10]$ with intensity

$$\lambda(\mathbf{s}) = \lambda(x, y) = (x^2 + y^2)^{1/2}.$$

   Plot your point pattern using `plot`.

   Try experimenting with different types of intensity function.

6. The `Kest` function computes the Ripley's K-function estimator for a point pattern. Practice using the function on the datasets `data1.csv`, `data2.csv` and `data3.csv`. Experiment with using different edge correction methods (and not using any at all). Do you think these point patterns are CSR/clustered/regular?

   Consider plotting the $L(r) - r$ function as well (see `?Kest`).

   In each of these, the region of interest is $[0, 1] \times [0, 1]$.

   Functions you will need for this question: `read.table` (the separator character is comma ‘,’), `as.ppp`, `Kest` and `plot`.