$$H = b_1 Z_1 + b_2 Z_2 + \varepsilon X_1 X_2$$

$$H|z,0\rangle = \left(b_1(-1)^z + b_2\right)|z,0\rangle + \varepsilon|1-z,1\rangle$$

$$H|z,1\rangle = \left(b_1(-1)^z - b_2\right)|z,1\rangle + \varepsilon|1-z,0\rangle$$

$$H|z_1 z_2\rangle = \underbrace{\left(b_1(-1)^{z_1} + b_2(-1)^{z_2}\right)}_{\beta(z_1,z_2)}|z_1 z_2\rangle + \varepsilon|1-z_1, 1-z_2\rangle$$

$$\beta(z_1 z_2) = -\beta(1-z_1, 1-z_2) \equiv \beta$$

$$H|z_1 z_2\rangle = \beta(z_1 z_2) + \varepsilon|1-z_1, 1-z_2\rangle$$

$$H^2|z_1 z_2\rangle = \beta H|z_1 z_2\rangle + \varepsilon H|1-z_1\ 1-z_2\rangle$$

$$= \beta^2|z_1 z_2\rangle + \varepsilon^2|z_1 z_2\rangle$$

$$\therefore H^2 = (\beta^2+\varepsilon^2) I_2 \qquad H^{2k} = \sqrt{\beta^2+\varepsilon^2}^{\,2k}$$

$$e^{-itH} = \sum_{n=0}^{\infty} \frac{(-it)^n}{n!} H^n$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k}}{(2k)!} (H^2)^k - i \sum_{k=0}^{\infty} \frac{(-1)^k t^{2k+1}}{(2k+1)!} (H^2)^k H$$

$$= \cos\left(\sqrt{\beta^2+\varepsilon^2}\, t\right) I - i \frac{H}{\sqrt{\beta^2+\varepsilon^2}} \sin\left(\sqrt{\beta^2+\varepsilon^2}\, t\right)$$

$${}_2\langle 1| e^{-iHt} (\alpha|00\rangle + \beta|10\rangle)$$

$$= \alpha \, {}_2\langle 1| e^{-iHt} |00\rangle + \beta \, {}_2\langle 1| e^{-iHt} |10\rangle$$

$$= \alpha \left(-i \frac{\sin(\sqrt{}\, t)}{\sqrt{(b_1+b_2)^2+\varepsilon^2}} \varepsilon |1\rangle \right) + \beta \left(-i \frac{\sin(\sqrt{}\, t)}{\sqrt{(b_1-b_2)^2+\varepsilon^2}} \varepsilon |0\rangle \right)$$

is $\left|\frac{\alpha}{\beta}\right|^2 \le \left| \dfrac{\beta \frac{\sin(\sqrt{}\, t)}{\sqrt{(b_1-b_2)^2+\varepsilon^2}}}{\alpha \frac{\sin(\sqrt{}\, t)}{\sqrt{(b_1+b_2)^2+\varepsilon^2}}} \right|^2$?

$$\frac{\alpha^4}{\beta^4} \le \frac{(b_1+b_2)^2+\varepsilon^2}{(b_1-b_2)^2+\varepsilon^2} \frac{\sin^2\left(\sqrt{(b_1-b_2)^2+\varepsilon^2}\, t\right)}{\sin^2\left(\sqrt{(b_1+b_2)^2+\varepsilon^2}\, t\right)}$$

$\therefore$ if $t = \frac{n\pi}{\sqrt{(b_1+b_2)^2+\varepsilon^2}}$ & $\neq \frac{m\pi}{\sqrt{(b_1-b_2)^2+\varepsilon^2}}$

then "cooling" works.

w.p. $P = \beta^2 \varepsilon^2 \frac{\sin^2(\sqrt{}\, t)}{(b_1-b_2)^2+\varepsilon^2}$

let $b_1 = b_2 = \frac{\varepsilon}{2}$

then $t = \frac{n\pi}{\sqrt{2}\,\varepsilon} \neq \frac{m\pi}{\varepsilon}$ w.p. $P = \beta^2 \sin^2\left(\frac{n\pi}{\sqrt{2}}\right)$