Discussion of "Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 Crisis"

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Motivation and Question

Covid-19

- Big, heterogenous supply and demand shocks
- Coexistence of tight and slack labor markets
- What happens to output, unemployment, inflation?

Method

- Use disaggregated model with multiple sectors, factors, I-O linkages.
- Neoclassical framework + Nominal wage rigidities + Credit constraints (endogenous AD shocks)
- Do comparative statics, perturb initial equilibrium with shocks:
  - Supply: sectoral productivity, potential factor supply.
  - Demand: intertemporal/AD , sectoral.
- Quantitative exercise (U.S.):
  - need demand and supply shocks to match data,
  - complementarities mitigate demand shocks, amplify supply shocks.

Model

- $\mathcal{N}$ produced goods.
- $\mathcal{G}$ factors in inelastic supply.
- Homothetic final demand.
- Goods produced using other goods and factors.

Model

- **Two periods**: present and future.
  - Denote future variables with stars.
- Supply functions $L_f \in [0,1]$ for all $f \in \mathcal{G}$ in present and future.
- Present: Neoclassical setup + nominal wage rigidity.
- Future: LR equilibrium with full employment $L_f = 1$ for all factors, fixed expenditure.

Production

- A representative firm in sector $i \in \mathcal{N}$ maximizes profits subject to CRS production function F_i:

$$\pi_i = \max_{y_i, \{L_{if}\}, \{x_{ij}\}} \left\{ p_i y_i - \sum_{f \in \mathcal{G}} w_f L_{if} - \sum_{j \in \mathcal{N}} p_j x_{ij} \right.$$
$$\left. \text{subject to } y_i = A_i F_i \left(x_{i1}, \cdots, x_{i\mathcal{N}}, L_{i1}, \cdots, L_{i\mathcal{G}} \right) \right\}$$

  - x_{ij}: input j
  - L_{if}: factor f
  - A_i: total factor productivity.

Final Demand by Producer 0

- Final demand maximizes homothetic aggregator, $Y = C\left(c_1, \cdots, c_N; \omega_{\mathcal{D}}\right)$:
  - c_i: final consumption for good $i \in \mathcal{N}$
  - $\omega_{\mathcal{D}}$: preference shifter
- Budget constraint: $\sum_{i \in \mathcal{N}} p_i c_i = \sum_{f \in \mathcal{G}} w_f L_f + \sum_{i \in \mathcal{N}} \pi_i$
- Inelastic factor supplies $L_f = \sum_{i \in \mathcal{N}} L_{if}$.
- Denote p^Y as composite price index for bundle Y.

Nested CES Economies

- Write $1 + \mathcal{N} + \mathcal{G}$ for the union of the sets $\{0\}, \mathcal{N}$ and $\mathcal{G}$.
- Production and final demand function-> nested CES
  - $\theta : (1 + \mathcal{N}) \times 1$ vector of elasticities of substitution
  - ω_0: $1 \times (1 + \mathcal{N} + \mathcal{G})$ vector of sectoral demand shocks

Input-Output Definitions

- Final demand produced by producer 0 using final demand aggregator.
  - $c_i = x_{0i}$: final consumption good i.
- Notation: treat factors the same as goods:
  - $p_f = w_f$, $L_f = y_f$ as total factor supply ,
    $L_{if} = x_{if}$ as use of factor f by producer i.

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- Stack every market $1 + \mathcal{N} + \mathcal{G}$ into I-O matrix

$$\Omega_{ij} \equiv \frac{p_j x_{ij}}{p_i y_i}.$$

- Leontiff inverse matrix: $\Psi \equiv (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \cdots$.
- Nominal GDP: $GDP \equiv \sum_{i \in \mathcal{N}} p_i x_{0i} = E = p^y Y$.
- Domar weights: $\lambda_i \equiv \frac{p_i y_i}{GDP}$
  - of producer i (sales share) or of factor f (factor income share).

Household

- When some quantity of employed factor $f \in \mathcal{G}$ falls,
  - fraction L_f continue to be employed,
  - fraction $(1 - L_f)$ become unemployed,
    - ϕ_f can borrow against future income,
    - $(1 - \phi_f)$ cannot borrow -> cannot consume today.

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- HH maximize intertemporal utility s.t. intertemporal budget constraint of representative Ricardian HH:

$$\max_{Y, Y_*} \left\{ (1-\beta) \frac{Y^{1-\frac{1}{\rho}} - 1}{1 - \frac{1}{\rho}} + \beta \frac{Y_*^{1-\frac{1}{\rho}} - 1}{1 - \frac{1}{\rho}} \right.$$

$$\left. \text{subject to } p^Y Y + \frac{p_*^Y Y_*}{1+i} = \sum_{f \in \mathcal{G}} w_f L_f + \sum_{f \in \mathcal{G}} \frac{w_f^* L_f^*}{1+i} \left(1 - (1 - L_f)(1 - \phi_f)\right) \right\},$$

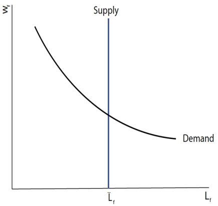
where ρ is the intertemporal elasticity of substitution, $\beta \in [0, 1]$ time preference, $(1 + i)$ nominal interest rate.

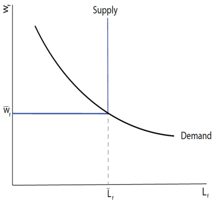
Equilibrium

- Goods: for all $i \in \mathcal{N}$, $c_i + \sum_{j \in \mathcal{N}} x_{ji} = y_i$.
- Factors: 2 cases.

1. Market is **tight**. i.e. market clears, is supply-constrained
2. Market is **slack**. i.e. market does not clear, demand constrained

Equilibrium in Factor Markets





- "Capitals" $f \in \mathcal{K}$: always flexible.
- Consider: Shocks to supply, $d \log \bar{L}_f$, and nominal demand of factor f,
  $d \log w_f L_f = d \log \lambda_f + d \log E$
- Flexible wage adjustment: $d \log w_f$
- Factor supplied: $d \log L_f = d \log \bar{L}_f$

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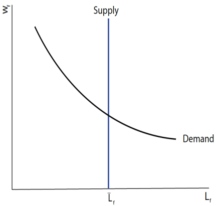

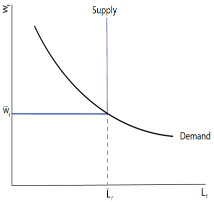

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- Flexible wage adjustment: $d \log w_f$
- Factor supplied: $d \log L_f = d \log \bar{L}_f$
- "Labors" $f \in \mathcal{L}$: flexible ($f \in \mathcal{F}$) or rigid ($f \in \mathcal{R}$) in equilibrium.
- Constrained wage adjustment: $d \log w_f \geq 0$.
- Factor supplied: $d \log L_f = \min\left\{d \log \lambda_f + d \log E, d \log \bar{L}_f\right\} \leq d \log \bar{L}_f$.
  - If demand of factor f decreases more than supply-> Keynesian unemployment.

Equilibrium in Factor Markets

- Equilibrium set of tight factor markets: $\mathcal{S} \subseteq \mathcal{G}$.
  - Share of tight markets: $\lambda_{\mathcal{S}} = \sum_{f \in \mathcal{S}} \lambda_f$
- Equilibrium set of slack factor markets: $\mathcal{R} \subseteq \mathcal{G}$.
  - $\mathcal{R} \subseteq \mathcal{L}$
  - Share of slack markets: $\lambda_{\mathcal{R}} = \sum_{f \in \mathcal{R}} \lambda_f$
  - $1 = \lambda_{\mathcal{S}} + \lambda_{\mathcal{R}}$

Local Comparative Statics

Intertemporal Euler Equations

- Consumption EE for Ricardian HH is:

$$Y = \left(\frac{1-\beta}{\beta}\right)^{\rho} \left(\frac{p^{Y}}{\bar{p}_{*}^{Y}/(1+i)}\right)^{-\rho} \bar{Y}_{*} \frac{1}{(1+i)} \left(1 - \sum_{h \in \mathcal{G}} \lambda_{h}^{*} (1 - L_{h}) (1 - \phi_{h})\right).$$

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- Log-linearizing yields AD curve:

$$d \log Y = -\rho d \log p^Y + d \log \zeta + d \log \Theta,$$

where $d \log \zeta = -\rho \left(d \log \left(1+i\right) + d \log \frac{\beta}{1-\beta} - d \log \bar{p}_*^Y \right) + d \log \bar{Y}_*$ and $d \log \Theta = \frac{E_{\lambda^*}\left[L_f (1-\phi_f) d \log L_f\right]}{1 - E_{\lambda^*}\left[(1-L_f)(1-\phi_f)\right]}$ (endog AD shock).

- Note: No HtM HHs $\rightarrow \phi_f = 1$ for all $f \rightarrow d \log \Theta = 0$

Intertemporal Aggregation Equation

- Euler equation for aggregate nominal expenditure $E = p^Y Y$:

$$d \log E = (1 - \rho) d \log p^Y + d \log \zeta + d \log \Theta.$$

- When $\rho = 1$, and no HtM HHs ($\phi_f = 1$), Nominal expenditure is exogenous.

Intratemporal Aggregation Equation

- Changes in output are given by

$$
\begin{aligned}
d \log Y &= \sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{G}} \lambda_f d \log L_f \\
&= \sum_{i \in \mathcal{N}} \lambda_i d \log A_i + \sum_{f \in \mathcal{K}} \lambda_f d \log \bar{L}_f + \sum_{f \in \mathcal{L}} \lambda_f d \log \bar{L}_f \\
&+ \sum_{f \in \mathcal{L}} \lambda_f \min \left\{ \mathbf{d} \log \boldsymbol{\lambda}_{\mathbf{f}} + \mathbf{d} \log \mathbf{E} - d \log \bar{L}_f, 0 \right\},
\end{aligned}
$$

Intratemporal Propogation Equation

- Changes in sales and factor shares approximated by F-O:

$$d\log\lambda_k = \theta_0\, Cov_{\Omega_{(\mathbf{0})}}\left(d\log\omega_0, \frac{\Psi_{(k)}}{\lambda_k}\right) + \sum_{j\in 1+\mathcal{N}} \lambda_k\left(\theta_j - 1\right) Cov_{\Omega_{(j)}}\left(\sum_{i\in\mathcal{N}} \Psi_{(i)}\, d\log A_i - \sum_{f\in\mathcal{G}} \Psi_{(f)}\left(d\log\lambda_f - d\log L_f\right), \frac{\Psi_{(k)}}{\lambda_k}\right).$$

- Role of network (Ω, Ψ) and elasticities θ_j.

Cobb Douglas model

- Assume $\rho = \theta_j = 1$ for all $j \in \mathcal{N}+1$, uniform ϕ_f
- **Negative supply shocks**, average: $d \log \bar{L} < 0$.
  - $d \log \bar{L} = \lambda_{\mathcal{S}} d \log \bar{L}_{\mathcal{S}} + \lambda_{\mathcal{R}} d \log L_{\mathcal{R}}$
- Changes in output:

$$d \log Y = \frac{\lambda_{\mathcal{S}}}{1 - (1 - \phi)\lambda_R} d \log \bar{L}_{\mathcal{S}}.$$

- $\lambda_{\mathcal{S}} d \log \bar{L}_S$: direct effect
- $\frac{1}{1-(1-\phi)(1-\lambda_{\mathcal{S}})}$: keynesian amplification
  - stronger amplification: high HtM share parameter and low $\lambda_{\mathcal{S}}$.
- Network irrelevance (conditional on factor shares).

Other shocks

- **Complementarities in production** $\theta < 1$ + **all shocks** + no HtM, $\phi = 1$

$$d\log Y = \frac{\lambda_{\mathcal{S}}\, d\log \bar{L}_{\mathcal{S}}}{1-(1-\theta)(1-\lambda_{\mathcal{S}})} - \theta \frac{\lambda_{\mathcal{S}}\, d\log \omega_{0\mathcal{S}}}{1-(1-\theta)(1-\lambda_{\mathcal{S}})} + \left(1 - \frac{(1-\theta)\lambda_{\mathcal{S}}}{1-(1-\theta)(1-\lambda_{\mathcal{S}})}\right)(1-\lambda_{\mathcal{S}})\, d\log \zeta.$$

- First term: response of Y to supply shocks
  - multiplier captures **amplification** effect of complementarities
  - Negative supply shocks in one market can be transmitted as negative demand shocks to other factor markets
- Second: response to sectoral demand shocks
- Third: response to AD shock
  - $(1-\lambda_{\mathcal{S}})\, d\log \zeta$: effect of AD shock reduces employment in rigid sectors, $\downarrow$ Y .
  - Term in brackets: **mitigating** effect of complementarities.

Global Comparative Statics

- Multiple equilibria, can rank unique best equilibrium.
- Local comparative statics hold globally.

Quantitative Application

- U.S. economy: 66 sectors, capital, labor, and intermediates.
  - Flexible capital, rigid labor; no reallocation.
- Construct I-O matrix, match share parameters.
- Set $\rho = 1, \phi = 1$.
- SR Elasticities $(\sigma, \theta, \varepsilon, \eta) = (1.0, 0.2, 0.6, 0.5)$.
  - σ : elasticity of substitution between final goods
  - ε : elasticity of substitution between (K, L) and intermediates
  - η: elasticity of substitution between K and L
- Negative shocks to AD, sectoral demand, and potential labor supplies
  - changes in nominal GDP, sectoral composition of household spending across different sectors, changes in hours by sector

Quantitative Application, complementarities

- Negative AD shocks, composition-of-demand shocks (from PCE data), negative labor-supply shocks.
- Inflation: Baseline model performs well. CPI inflation: -0.9%

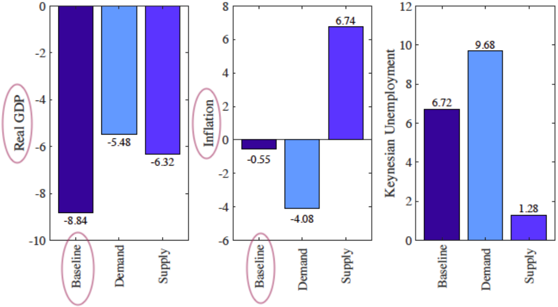


Figure 6.1: Real GDP, inflation, and Keynesian unemployment as a function of shocks for the model with complementarities. The "Baseline" line includes negative demand and supply shocks. The "Supply" bar only includes the sectoral supply shocks. The "Demand" bar only includes the demand shocks.

Quantitative Application, no complementarities

- Complete markets and no complementarities
- Supply shocks in one sector do not have Keynesian spillovers.

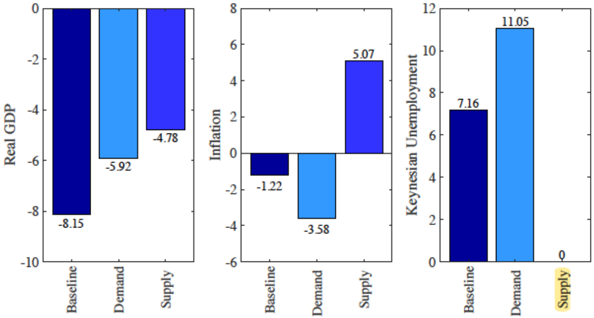
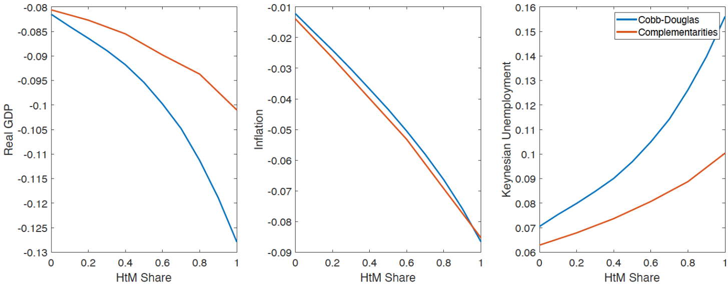


Figure 6.2: Real GDP, inflation, and Keynesian unemployment as a function of shocks without complementarities. The "Baseline" line includes negative demand and supply shocks. The "Supply" bar only includes the sectoral supply shocks. The "Demand" bar only includes the demand shocks.

Aggregate outcomes in the Cobb-Douglas model and the model with complementarities as we vary the share of potentially HtM workers

- Presence of HtM amplifies reduction/increase in aggregate outcomes $\rightarrow$ transfers important in mitigating negative demand effects associated with covid-19
- Complementarities: mitigating effect

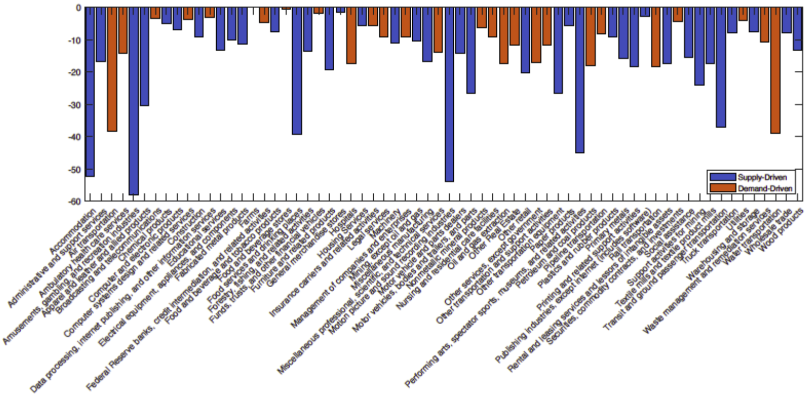


Tightness and Slackness across Sectors

Baseline case

- Almost all sectors experienced reduction in hours
- Tight/Supply constrained: 36 factor markets
  - motion pictures(-54%), food services and accomodations (-39%), food products and beverages(-8%)
  - mild inflation (1%); data (1%)
- Slack/Demand constrained: 30 factor markets
  - air transport (-40%), water transport (-43%), rail transport (-19%), petroleum and coal (-21%)
  - mild deflation (5.4%); data (2.4)

Feb-May 2020: Model implied percentage reduction in hours by sector



Policy Implications

- Distinguishing between supply and demand shortfalls important for policy
- Stimulating spending on tight labor markets is wasteful and complementarities further worsen this problem.

Questions

- Local versus Global comparative statics?
- Implementation

Nested CES Economies

- Production and final demand function-> nested CES
- Let $\bar{\omega}$ be a $(1+\mathcal{N}+\mathcal{G}) \times (1+\mathcal{N}+\mathcal{G})$ matrix of I-O parameters; θ, a $(1+\mathcal{N})$ vector of elasticities of substitution.
- Each good $i \in \mathcal{N}$ is produced according to:

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i} \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}} .$$

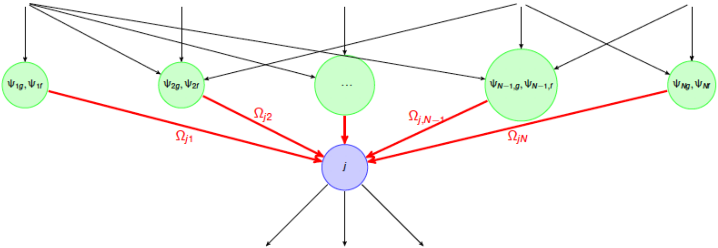
- Final demand is produced by producer 0 using:

$$\frac{Y}{\bar{Y}} \equiv \frac{y_0}{\bar{y}_0} = \left(\sum_{j \in \mathcal{N}+\mathcal{G}} \bar{\omega}_{0j} \frac{\omega_{0j}}{\bar{\omega}_{0j}} \left(\frac{x_{0j}}{\bar{x}_{0j}} \right)^{\frac{\theta_0 - 1}{\theta_0}} \right)^{\frac{\theta_0}{\theta_0 - 1}} ,$$

  where ω_{0j} are sectoral demand shocks that sum to 1.
- Let ω_0 be the $1 \times (1+\mathcal{N}+\mathcal{G})$ vector
- Note: variables with over-lines are normalizing constants.
- Factors are non-reproducible goods: $\frac{y_f}{\bar{y}_f} = 1$.

I-O Covariance Operator

- Captures substitution effects by downstream producers j, in response to a shock to g, on factor share/sales share of f
- $Cov_{\Omega^{(j)}}\left(\Psi_{(g)}, \Psi_{(f)}\right) =$
  $\sum_{l \in \mathcal{N}+\mathcal{G}} \Omega_{il} \Psi_{lg} \Psi_{lf} - \left(\sum_{l \in \mathcal{N}+\mathcal{G}} \Omega_{il} \Psi_{lg}\right)\left(\sum_{l \in \mathcal{N}+\mathcal{G}} \Omega_{il} \Psi_{lf}\right)$
  - covariance of two $1 + \mathcal{N} + \mathcal{G}$ vectors, j-th row of the I-O matrix $\Omega^{(j)}$ as a probability distribution.
  - $\Psi_{(i)}$ denotes the i-th column of matrix Ψ.
  - Ψ_{lg} : exposure of l to g



Other shocks

- **Negative aggregate demand shock**:

$$d \log Y = \frac{\lambda_{\mathcal{R}}}{1-(1-\phi)\lambda_{\mathcal{R}}} d \log \zeta.$$

  - If there are HtM HHs, AD shocks are amplified by multiplier.
  - Network irrelevance.