Quiver Viva Script

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1 Introduction

1.1 Supersymmetry

Symmetries are central object of study in theoretical physics as we can derive conservation laws from them and their irreducible representations give rise to different types of particles. The scalar potential part of the supersymmetric action is given by the derivative of the superpotential with respect to the scalar fields of the supermultiplets. (in our case vector multiplets and hypermultiplets)

2 Moduli Space

2.1 Definition and Branches

A moduli spaces is a space of the parameters of a geometric problem. In physics it is the space of vacuum expectation values of a set of scalar fields which are correlators in QFT.

The moduli space of gauge theories naturally decomposes into a Coulomb branch corresponding to VEVs of scalar fields in vector multiplets and a Higgs branch corresponding to VEVs of scalar fields in hypermultiplets of which the union gives us the full moduli space.

2.2 Hilbert Series

A Hilbert series of a coordinate ring R of the variety $\mathcal{V}$ is a polynomial whose ith coefficient is the dimension of the ith graded ring R_i which is the number of generators of polynomial with degree i.

In the context of gauge theories, moduli space is naturally a variety and the Hilbert series of it counts the number of gauge invariant operators with degree i. The complex dimension of the moduli space is the order of $t = 1$ pole of the Hilbert series.

2.3 Brane Construction

In 9+1 dimensional string theory, as shown in [the table], NS5 branes span the direction 3,4,5, D5 branes span the direction 7,8,9, D3 branes are stretched within the direction 6. Moduli space can be realised as [this]. The Coulomb branch is the space of D3 branes between NS branes and the Higgs branch is the space of the D3 branes between D5 branes.

3 Quivers

3.1 3d $\mathcal{N} = 4$

Given a theory $\mathcal{T}$, a circle node represents a gauge group i.e. a vector multiplet transforming in the adjoint representation of the gauge group, a square represents a flavour group and an edge represents a hypermultiplet transforming in the bifundamental representation of the neighbouring nodes.

If nodes of the quiver are unitary and special unitary groups, we call it a *unitary quiver* whereas if it consists of symplectic and special orthogonal groups, it is called a *orthosymplectic quiver*. From now on I will focus on unitary quivers until I do otherwise.

3.2 Branes to Quivers

rules for constructing *electric quiver*. Given a brane system, draw a projection of it in the 3,4,5 direction and the 6 direction. (An interval between NS5 branes = circle node) with (node number = number of D3 branes in the interval), if there are D5 branes then a square node is connected with a (node number = number of D5 branes).

3.3 3d Mirror Symmetry

There exists a duality between two theories. *Magnetic quiver* is obtained by taking the brane system for an electric quiver, and rotating it to 7,8,9 direction and 6 direction projection, exchanging NS5 with D5 branes. Coulomb of electric quiver = Higgs of magnetic and vice versa.

4 Computation

4.1 Coulomb Branch

4.1.1 Dimension and Global Symmetry

Dim = sum of ranks of gauge groups i.e. circle node numbers
Global symmetry = dynkin diagram of balanced gauge nodes $\times U(1)^n$ where $n =\#$ of overbalanced nodes.

4.1.2 Monopole Formula

it is for Coulomb Branch. Classical dressing factor counts degree of Casimir invariants. Compute the conformal dimension of the magnetic flux.

4.2 Higgs Branch

4.2.1 Molien-Weyl Formula

4.2.2 Dimension and Global Symmetry

Dim = # of hypers - sum of dim of adjoint rep of gauge groups where # of hypers = dimension of fundamental rep of a group * dimension of fundamental rep the other group.
Global symmetry = flavour symmetry

5 Examples

5.1 $U(1)$ with N flavours

5.2 $U(2)$ with N flavours

5.3 $Sp(1) \times Sp(2)$ with 4 flavours