

3.C Matrices

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let $T \in \mathcal{L}(V, W)$ and $\{v_1, \dots, v_n\}$ is basis of V , $\{w_1, \dots, w_m\}$ is basis of W

$$\mathcal{M}(T) = A_{j,k} \text{ s.t. } T v_k = \sum_{j=1}^m A_{j,k} w_j$$

$$\Rightarrow T v = T \sum_{k=1}^n c_k v_k = \sum_{k=1}^n c_k \left(\sum_{j=1}^m A_{j,k} w_j \right)$$

$$T v_k = \begin{matrix} m \downarrow \\ \left(\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right) \end{matrix} \begin{matrix} n \rightarrow \\ \left(\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right) \end{matrix} \begin{matrix} \left(\begin{matrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{matrix} \right) \end{matrix} \begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \end{matrix} = \begin{matrix} \left(\begin{matrix} \vdots \\ \vdots \\ \vdots \end{matrix} \right) \end{matrix} = A_{j,k} w_j$$

← this is v_k in $\{w_j\}$ basis

← k^{th} column of A

notation: $\mathbb{F}^{m,n}$ means m by n matrix over \mathbb{F}

lemma: \mathcal{M} is linear

$$\Rightarrow \mathcal{M}(S+T) = \mathcal{M}(S) + \mathcal{M}(T) \text{ and } \mathcal{M}(\lambda T) = \lambda \mathcal{M}(T)$$

theorem: $\dim(\mathbb{F}^{m,n}) = m \cdot n$

proof: $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m) \Rightarrow \mathcal{M}(T) = \mathbb{F}^{m,n} \Rightarrow \mathbb{F}^{m,n}$ is vector space

matrix with all its entries 0 except one entry is basis,
there are $m \cdot n$ entries

definition: $(AC)_{j,k} = A_{j,i} C_{i,k}$

$$\Rightarrow (AC)_{i,k} = A C_{i,k}$$

theorem: if $T \in \mathcal{L}(U, V)$, $S \in \mathcal{L}(V, W)$

$$\Rightarrow \mathcal{M}(ST) = \mathcal{M}(S) \mathcal{M}(T)$$

$$\left(\begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \end{pmatrix} \right) \left(\begin{pmatrix} \vdots \\ \vdots \\ \vdots \end{pmatrix} \right) = \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \right)$$