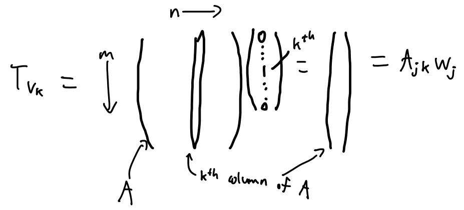
3.C Matrices

Sunday 12 May 2024 18:27

let $T \in \mathcal{L}(V,W)$ and $\{v_1,\cdots,v_n\}$ is basis of V, $\{w_1,\cdots,w_m\}$ is basis of W

$\mathcal{M}(T) = A_{jk}$ s.t. $Tv_k = \sum_{j=1}^{m} A_{jk} w_j$

$\Rightarrow Tv = T\sum_{k=1}^{n} c_k v_k = \sum_{k=1}^{n} c_k \left(\sum_{j=1}^{m} A_{jk} w_j\right)$



this is v_k in $\{w_j\}$ basis

notation: $\mathbb{F}^{m,n}$ means m (row) by n (column) matrix over $\mathbb{F}$

lemma: $\mathcal{M}$ is linear

$\Rightarrow \mathcal{M}(S+T) = \mathcal{M}(S) + \mathcal{M}(T)$ and $\mathcal{M}(\lambda T) = \lambda\mathcal{M}(T)$

theorem: $\dim(\mathbb{F}^{m,n}) = m \cdot n$

proof: $T \in \mathcal{L}(\mathbb{F}^n, \mathbb{F}^m) \Rightarrow \mathcal{M}(T) = \mathbb{F}^{m,n} \Rightarrow \mathbb{F}^{m,n}$ is vector space

matrix with all its entries 0 except one entry is basis

there are $m \cdot n$ entries ∎

definition: $(AC)_{jk} = A_{j\cdot} C_{\cdot k}$

$\Rightarrow (AC)_{\cdot k} = A C_{\cdot k}$

theorem: if $T \in \mathcal{L}(U,V)$, $S \in \mathcal{L}(V,W)$

$\Rightarrow \mathcal{M}(ST) = \mathcal{M}(S)\mathcal{M}(T)$