530 陈斯杰 电子信息工程 第23次作业

1.假设(p,q)为y=-x+1上一点。

曲线y=y(x)。在曲线上P(x,y)的质点速度为$v = \sqrt{2gy}$

又设从原点到P的弧长为s，则

$v = \frac{ds}{dt} = \frac{\sqrt{1+y'^2dx}}{dt}$

由此可推出$T = T(y) = \frac{1}{\sqrt{2g}} \int\limits_0^p \sqrt{\frac{1+y'^2}{y}} dx$

即求一个$\hat{y}$,使得$T(\hat{y})$最小。

解得所求函数表达式为

$$\begin{cases} x = R(\theta - sin\theta) \\ y = R(cos\theta - 1) \end{cases}$$

再由

$$\begin{cases} q(\theta - sin\theta) = p(cos\theta - 1) \\ q = -p + 1 \end{cases}$$

可得$\theta = f(p)$

又可求$T = \sqrt{\frac{R}{g}}\theta = \sqrt{\frac{R}{g}}f(p)$

因此，只需寻找到p_0,使得T最小。

$\theta_0 = f(p_0)$

$R_0 = \frac{p_0}{\theta_0 - sin\theta_0}$

因此所求最速降线即为

$$\begin{cases} x = R_0(\theta_0 - sin\theta_0) \\ y = R(cos\theta_0 - 1) \end{cases}$$

2.编写求两点之间最速降线的matlab代码，命为cycloid函数

```matlab
function [T,gx,gy] = cycloid(b,H)
  n = 100;
  g = 9.8;
  h = H/n;
  minc = 0;maxc = 1/sqrt(2*g*h*n);
```

```
6    x = 0; y = 0;
7    while abs(b-x)>1e-4
8        x = 0;
9        c = (minc+maxc)/2;
10       for j= 1:n
11            y = j*h;
12            v = sqrt(2*g*y);
13            x = x+c*v*h/sqrt(1-c^2*v^2);
14            gx(j) = x;
15            gy(j) = y;
16       end
17       if(x<b)
18        minc = c;
19       else
20        maxc = c;
21       end
22   end
23   T =0;
24   for j = 1:n
25       v = sqrt(2*g*j*h);
26       if j==1
27            s = sqrt(gx(1)^2+h^2);
28       else
29            s = sqrt((gx(j)-gx(j-1))^2+h^2);
30       end
31       T = T+s/v;
32   end
33   end
```

返回值为下降时间T和最速降曲线的坐标gx,gy,在利用matlab寻找到原点到直线$x+y-1=0$上
某点下降最快的曲线，代码如下：

```
34   i = 2;T(1) = 100;j = 1;
35   for y = -3:0.1:0
36       x = 1-y;
37       T(i) = cycloid(x,-y);
```

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```matlab
38        if(T(i)<T(j))
39            j = i;
40            i=i+1;
41        end
42    end
43    y0 = -3+0.1*(j-2)    %y0 = -0.4
44    x0 = 1-y0                 %x0 = 1.4
45    [T,gx,gy] = cycloid(x0,-y0);
46    T      % T = 0.7035
47    plot(gx,-gy);
48    grid on;
```

所以自原点到直线上点(1.4,-0.4)下降时间最快，可作为自原点到直线的最速降线，曲线形状
如下：