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Runaway electron mechanism of air breakdown and preconditioning during a thunderstorm

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The possibility is considered of an avalanche-type increase of the number of runaway electrons leading to a new type of electric breakdown of gases. This type of breakdown could take place in the atmosphere during a thunderstorm stimulated by cosmic ray secondaries.

1. Introduction

It was first mentioned by Wilson [1] that the secondary electrons generated by cosmic rays can be influenced by the electric field of a thunderstorm. Both the theoretical and experimental aspects of this problem have been widely discussed in the literature [2–4]. A new interest is stimulated by the observation of precursors of lightning strokes coming as sharp increases of X-ray intensity [5,6]. The consistent calculations of the X-ray generation by McCarthy and Parks [7] shows that cosmic ray secondaries cannot provide the measured level of X-ray intensity, even considering the maximum impact coming from the electric field of the thunderstorm.

However, existing theories do not consider the possibility of an avalanche-type increase of the number of runaway electrons leading to the electric breakdown of the atmosphere. This paper is devoted to the investigation of this subject. Later on we will show that this phenomenon apparently determines the limiting value of the mean electric field in a thunderstorm, as well as the X-ray emission generated during the preconditioning period.

2. Atmospheric breakdown stimulated by runaway electrons

It is well known [8] that the slowing-down force of an energetic electron in the air, for $\epsilon \ll mc^2$, is described by

$$F=\frac{4\pi N_m Ze^4}{mv^2}\ln(mv^2/z\epsilon_i)\,. \quad (1)$$

Here N_m is the density of the air molecules, $Z=2z\approx 14.5$, where z is the mean nuclear charge of the nitrogen and oxygen atoms, ϵ is the electron energy, while $\epsilon_i \approx 15$ eV is the characteristic ionization energy. In the opposite case $\epsilon \gg mc^2$ this equation has the form

$$F = \frac{4\pi N_m Z e^4}{mc^2} \ln[(mc^2/\epsilon_1)\gamma] , \tag{2}$$

where the parameter $\gamma = \epsilon/mc^2 = (1-v^2/c^2)^{-1/2}$, and $\epsilon_1 \approx 270$ eV.

When $v^2 \ll c^2$ the slowing-down force decreases with an increase of the electron velocity as $F \sim v^{-2} \ln v^2$. However, in the relativistic region it begins increasing slowly as $F \sim \ln \gamma$, see fig. 1. The minimum value of F,

$$F_{\min} \approx \frac{4\pi N_m Z e^4}{mc^2} a, \quad a \approx 10 , \tag{3}$$

is reached for $\gamma_{\min} \approx 3$–4 (see ref. [7]).

Let us discuss under which conditions the electric field E is larger than $F_{\min}/e$. First, we introduce the dimensionless parameter

$$\delta_0 = \frac{Emc^2}{4\pi N_m Z e^3 a}$$
$$= 0.5 \frac{E}{1\ \text{kV/cm}} \frac{2.7 \times 10^{19}\ \text{cm}^{-3}}{N_m} \frac{10}{a} . \tag{4}$$

The following inequality holds under the conditions of interest,

$$\delta_0 > 1 . \tag{5}$$

In this case the balance equation

$$eE - F(v) = 0 \tag{6}$$

has two roots. The first one is obtained in the non-relativistic region of the electron energy,

$$v_1^2 \approx c^2/\delta_0 . \tag{7}$$

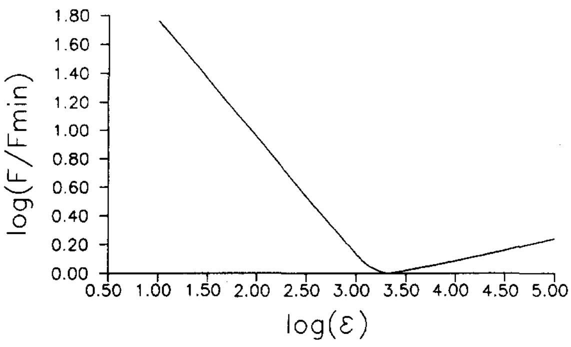

Fig. 1. Slowing-down force of fast electrons in air.

This root is unstable, which means that an electron with velocity $v < v_1$ slows down, while an electron with $v > v_1$ accelerates, i.e. becomes "runaway" [9]. On the contrary, the second root of eq. (6), γ_2, which is obtained for relativistic electron energies, is stable,

$$\gamma_2 \approx \exp[a(\delta_0 - 1)], \quad v_2 \approx c . \tag{8}$$

An electron slows down when $\gamma > \gamma_2$, and accelerates if $\gamma < \gamma_2$. This means that an electron with a velocity larger than v_1 tends to develop into the equilibrium state γ_2. It can be seen that the electron energy is large in the stable equilibrium state. Moving in media they ionize the gas molecules. Some of the knocked-out electrons may have a velocity larger than v_1. These electrons will be accelerated by the electric field. In turn, when colliding with neutrals they can generate "runaway" electrons with $v > v_1$. An avalanche of runaway electrons, of which the number increases exponentially, will be produced by this process. This process we call air breakdown by runaway electrons. In fact, the exponential increase of the fast electrons is followed by the same exponential increase of all electron energies up to the thermal energy.

Let us consider this process quantitatively. The number of electrons with energy exceeding ϵ_1, which is produced along a unit length as a result of the ionization of molecules by a fast particle with energy $\epsilon \gg \epsilon_1$, is given by [8]

$$\frac{dN(\epsilon_1)}{ds} = \frac{\pi Z N_m e^4}{mc^2 \epsilon_1} . \tag{9}$$

It should be emphasized that according to the peculiarities of the Coulomb interaction these electrons are released almost perpendicularly to the direction of motion of the fast particles,

$$\mu \approx \epsilon_1/mc^2(\gamma - 1), \quad \mu = \cos\theta , \tag{10}$$

where θ is the angle between $\boldsymbol{v}_1$ and $\boldsymbol{v}$, i.e. between the directions of motion of the fast and of the new born particle. When calculating the number of runaway electrons one should consider also the change in the angle θ. Therefore, let us introduce the equation of motion of the electron as

$$m \frac{dv}{dt} = eE\mu - F(v) ,$$

$$\frac{d\mu}{dt} = \frac{eE}{mv}(1 - \mu^2), \quad \mu = \cos\theta = \frac{\boldsymbol{v} \cdot \boldsymbol{E}}{vE} . \tag{11}$$

It follows from eqs. (11) that the electron trajectory in the v, μ plane is described by the following equation, written in the dimensionless variables $u=v^2/c^2$ and μ,

$$\frac{\mathrm{d}u}{\mathrm{d}\mu}=\frac{2}{1-\mu^2}\left[\mu u-\frac{1}{\delta_0}\left(1+\frac{\ln u}{a}\right)\right],$$
$$a=\ln(mc^2/z\epsilon_i)\,, \tag{12}$$

where expression (1) for the slowing-down force has been used, which holds in the nonrelativistic limit. Correspondingly, the following analysis is valid for $u<1$. Eq. (12) describes two kinds of trajectories. Some cross "the runaway line", which is given by the equation

$$\mu=\frac{1}{\delta_0 u}\left(1+\frac{\ln u}{a}\right), \tag{13}$$

so for these electrons $u\to 1$ when $\mu\to 1$. Other electrons do not cross the line (13), for these $u\to O(e^{-a})$ when $\mu\to 1$ as shown in fig. 2. The separatrix $u_s(\mu)$, which separates the two types of trajectories, goes out of the end of the runaway line where

$$u_s(1)=u_0=\frac{1}{\delta_0}\left(1+\frac{\ln u_0}{a}\right),\quad \mu=1\,, \tag{14}$$

or approximately $u_0\approx\delta_0^{-1}(1-a^{-1}\ln\delta_0)$.

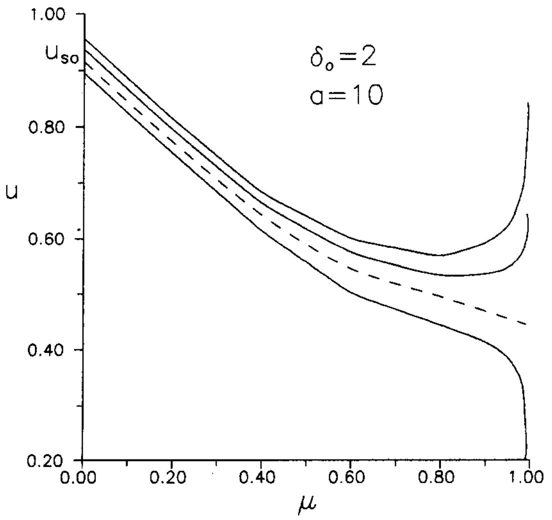

Fig. 2. Electron trajectories in the u, μ plane; the dashed line represents the separatrix $u_s(\mu)$.

Close to the saddle point $u=u_0$, $\mu=1$ the separatrix takes the form

$$u_s(\mu)=u_0+u_1(1-\mu)+u_2(1-\mu)^2+\dots,$$

$$u_1=\frac{u_0}{2-(\delta_0 a u_0)^{-1}},$$

$$u_2=\frac{u_0+u_1}{4}-\frac{u_1^2}{2\delta_0^2 a^2 u_0^2}\,. \tag{15}$$

All newborn electrons located above the separatrix will cross the runaway line, i.e. become runaway electrons. It has been mentioned above that the parameter μ is small ($\mu\to 0$) for the newborn electrons. Correspondingly, the meaning of the separatrix u_s for $\mu=0$, i.e. $u_s(0)=u_{s0}$ determines the runaway boundary. This boundary is shown in fig. 3 as function of the parameter δ_0. It determines for a given δ_0 the minimum energy of the newborn electrons to become runaways: $\epsilon_{10}=\frac{1}{2}mc^2u_{s0}$. The number of electrons with $\epsilon\geqslant\epsilon_{10}$ generated along a unit length by any fast electron, whose energy $\epsilon\gg\epsilon_{10}$, is given by eq. (9):

$$\frac{\mathrm{d}N}{\mathrm{d}s}=\frac{\pi Z N_m e^4}{mc^2\epsilon_{10}}=\frac{2\pi Z N_m e^4}{(mc^2)^2 u_{s0}}\,. \tag{16}$$

All these new electrons become runaways. Defining the characteristic length for the generation of runaways, λ,

$$\lambda=\left(\frac{\mathrm{d}N}{\mathrm{d}s}\right)_r^{-1}=\frac{(mc^2)^2u_{s0}}{2\pi Z N_m e^4}\,, \tag{17}$$

we obtain the following equation for runaway electron production,

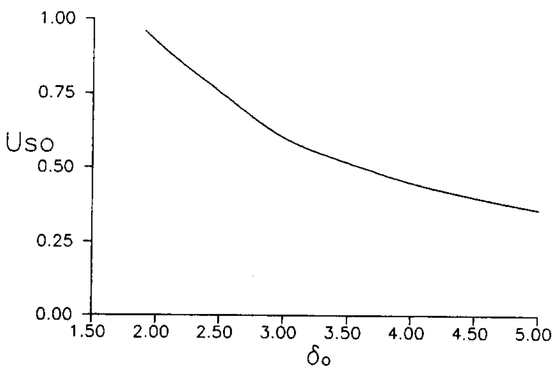

Fig. 3. Runaway boundary u_{s0}.

$$\frac{dN}{ds}=\frac{N_f}{\lambda}. \tag{18}$$

Here N_f is the number of fast electrons; the fast electrons are accelerated runaways. Let us introduce the acceleration length l, i.e. the scale along which the particle reaches an energy ϵ of the order of 1 MeV, which is required for the effective generation of runaway electrons,

$$l=\frac{\epsilon}{eE(1-\delta_0^{-1})}$$
$$\approx 10^3 \frac{\epsilon}{1\ \mathrm{MeV}} \frac{1\ \mathrm{kV/cm}}{E} \frac{1}{1-\delta_0^{-1}}\ \mathrm{cm}. \tag{19}$$

Eq. (18) can be rewritten now in the form

$$\frac{dN}{ds}=\frac{N(s-l)}{\lambda}, \tag{20}$$

and if

$$l/\lambda \ll 1, \tag{21}$$

we obtain from (20), (21)

$$\frac{dN}{ds}=\frac{N}{\lambda_1},\quad N=N_0\exp(s/\lambda_1),$$
$$\lambda_1=\lambda(1+l/\lambda)\simeq\lambda. \tag{22}$$

Let us call the parameter λ the runaway length. It determines the breakdown potentiality stimulated by runaway electrons. It is similar to the ionization frequency in the usual breakdown theory [10]. In the air the value of λ is

$$\lambda=\frac{(mc^2)^2 u_{s0}}{2\pi Z N_m e^4}=5\times 10^3 u_{s0}\frac{2.7\times 10^{19}}{N_m}\ \mathrm{cm}. \tag{23}$$

It should be mentioned here that the same equation (22) gives the growth of electrons of any energy, not just the fast electrons.

It follows from eqs. (19), (23) and (4) that

$$\frac{l}{\lambda}=0.1\frac{\epsilon}{1\ \mathrm{MeV}}\frac{10}{a}\frac{1}{u_{s0}(\delta_0-1)}. \tag{24}$$

It follows from (24) and fig. 3 that condition (21) is usually fulfilled. Under atmospheric conditions, when the characteristic scale $L\sim 10^5$ cm, i.e. $L\gg\lambda$, the breakdown stimulated by the runaway electrons becomes quite realistic for a field $E\gtrsim 1$ kV/cm. In this case the exponential growth factor of the number of runaway electrons might exceed a value of 20–30.

Of significant interest are the peculiarities of the spatial and temporal structure of the separate formations – elementary pulses of breakdown, caused by a fast particle. If a fast particle propagates along the direction of the electric field, ahead of it moves a group of the most energetic electrons, whose velocity is close to the speed of light. These electrons form a strongly directed group. Then come particles with random velocity direction, whose energy is of the order of 0.3–0.5 MeV. They are followed by a group of particles with a smaller energy, $\epsilon=\frac{1}{2}mv^2<u_{s0}mc^2$, of which the distribution function is close to spherically symmetric. Gradually the energy of the particles decreases while their number increases. The electron avalanche becomes a high temperature plasma. This plasma cools rapidly, followed by the loss of electrons due to them getting attached to water vapour or molecules of oxygen.

In the case when the initial fast particle moves under some angle to the field direction $\boldsymbol{E}$, the generation of the runaway electrons will lead to the appearance of spurs, oriented along the field $\boldsymbol{E}$ or close to it.

3. Possible effects of runaway electrons on thunderstorm phenomena

Let us discuss briefly the possible role played by the process considered in the physical phenomena originating in the atmosphere during a thunderstorm.

(1) The threshold electric field of the atmosphere breakdown caused by the runaway electrons corresponds to a value of $\delta_0\approx 1$–2, i.e. according to eq. (4) it is equal to

$$E_{th}\approx 2\text{–}4\ \mathrm{kV/cm} \tag{25}$$

when close to the ground, which means that under any realistic conditions the mean electric field E_0 between a cloud and the ground cannot exceed E_{th}. This is in a good agreement with the results of measurements, according to which the value of E_0 does not exceed E_{th}, they even become very close sometimes [2,3]. It should be emphasized that the field E_{th} is essentially smaller than the threshold of ordinary breakdown. The last is about 23 kV/cm [11].

(2) As was mentioned above, the electrical discharge (lightning) usually appears in the atmosphere when the mean electric field $E_0<E_{th}$. The controlling impact on the development of the stepladder phase of lightning comes from the overflow of the charge, which leads to the formations of regions with a strong local field $E_0(\boldsymbol{r})$ [2,3]. Under the condition $E_0(\boldsymbol{r})\gg E_{th}$ over a scale Δr which significantly exceeds the runaway length λ (23), local breakdown is possible. Such a type of breakdown can lead to the rapid flash of an emission over the scale Δr and stimulate a charge overflowing in the direction defined by the initial fast electron (cosmic ray secondary). Thus it is likely that the breakdown by runaway electrons stimulated by cosmic rays plays a noticeable role in the development of stepladder lightning.

(3) The occurrence of preconditioning processes can be expected, which are followed by an overflow of electric charge in the atmosphere during the preparation of the basic discharge. The multiple local breakdown by the runaway electrons followed by the generation of X-ray pulses, as well as radio pulses, can serve as a possible overflow mechanism.

Let us discuss the following simple model example. Assume that electric charge is located in a thin layer of the atmosphere at a height h_0, and the surface charge density is $-\sigma_0$. For $h<h_0$ it forms an electric field directed upward along the vertical axis,

$$E_0=\sigma_0/\varepsilon_0\,. \tag{26}$$

Here we consider the polarization charge density σ_0 formed on the conducting surface of the ground and ε_0 is the permittivity of air. Assume that $\delta_0(h_0)>2$, for instance, $\delta_0(h_0)=2.2$. According to (4) this corresponds to

$$E_0\approx 1.4\,\text{kV/cm},\quad \sigma_0\approx 1.2\times 10^{-6}\,\text{C/m}^2 \tag{27}$$

at a height of 10 km. Each cosmic ray secondary when crossing the layer, produces a local breakdown in the atmosphere at a lower height than h_0. Nevertheless, due to the increase of the air density, described by the barometric height formula: $N_m(h)=N_m(h_0)\times\exp[(h_0-h)/H]$, $H\approx 8.8$ km, δ_0 decreases with h. The breakdown stops at the height h_1 where $\delta_0\approx 2.0$. We obtain from (4) using the barometric height formula that

$$h_1=h_0-H\ln[\delta_0(h_0)/\delta_0(h_1)]\,. \tag{28}$$

The plasma produced by the micro-breakdown allows the overflow of part of the charge from layer h_0 to layer h_1. Due to the set of multiple breakdowns the charge gradually moves from h_0 to h_1.

Let us estimate the characteristic time of the charge overflow. The density of cosmic ray secondaries with energy $\epsilon>1$ MeV crossing the layer h_0 located at 10 km is $\alpha\sim 1/\text{cm}^2\,\text{s}$ [12]. Each of these particles produces a micro-breakdown with growth rate λ given by (23). Correspondingly, the number of fast electrons increases by a factor q from h_0 to h_1, where

$$q=\exp[\lambda^{-1}(h_0-h_1)]\,. \tag{29}$$

The number of secondary electrons produced by a fast electron along the scale λ is

$$n=\beta\lambda,\quad \beta\approx 30/\text{cm}\,. \tag{30}$$

Therefore, the total number of electrons produced in the layer close to h_1 by one cosmic ray secondary is

$$N=nq=\beta\lambda\exp[\lambda^{-1}(z_0-z_1)]\,. \tag{31}$$

The characteristic time of the charge overflow can be evaluated as

$$\Delta t=\sigma_0/N\alpha\,. \tag{32}$$

It is assumed here that the electric current formed in the channel of a micro-breakdown transports all newborn electrons. For the example discussed here $h_0-h_1\approx 1$ km, and correspondingly $\Delta t\sim 10$ s. Micro-breakdowns are followed by X-ray emission produced by the fast electrons. A rough agreement with observed [5,7] X-ray intensities during the preconditioning period could be easily obtained due to the exponential factor (29).

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