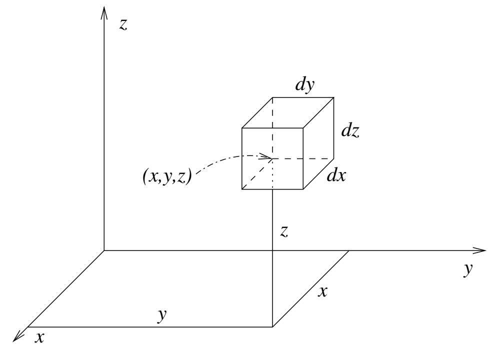
Three Useful Coordinate Systems

Cartesian Coordinates: (x, y, z)

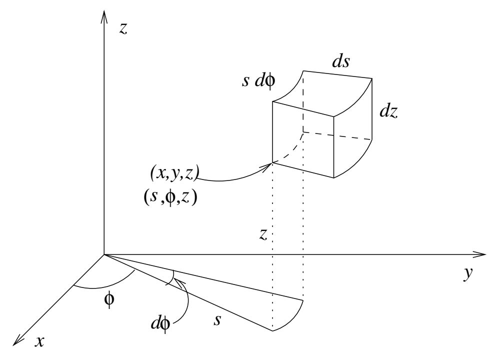


$$\begin{aligned}
d\vec{r} &= dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z} \\
d\tau &= dx\,dy\,dz
\end{aligned}$$

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial x}\,\hat{x} + \frac{\partial f}{\partial y}\,\hat{y} + \frac{\partial f}{\partial z}\,\hat{z} \\
\nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\vec{\nabla} \cdot \vec{v} &= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
\vec{\nabla} \times \vec{v} &= \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}
\end{aligned}$$

Cylindrical Coordinates: (s, ϕ, z) $\quad x = s\,\cos\phi \quad y = s\,\sin\phi$

$$\hat{s} = \cos\phi\,\hat{x} + \sin\phi\,\hat{y} \qquad \hat{\phi} = -\sin\phi\,\hat{x} + \cos\phi\,\hat{y}$$

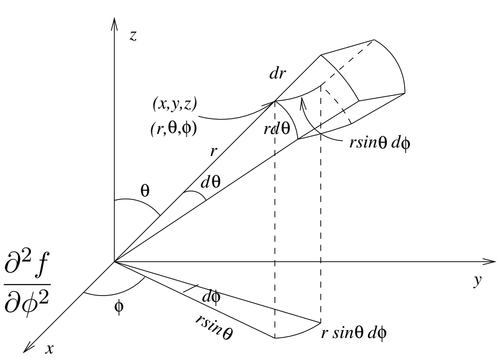


$$\begin{aligned}
d\vec{r} &= ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z} \\
d\tau &= s\,ds\,d\phi\,dz
\end{aligned}$$

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial s}\,\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\,\hat{\phi} + \frac{\partial f}{\partial z}\,\hat{z} \\
\nabla^2 f &= \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2} \\
\vec{\nabla} \cdot \vec{v} &= \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z} \\
\vec{\nabla} \times \vec{v} &= \left[\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right]\hat{s} + \left[\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right]\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{z}
\end{aligned}$$

Spherical Coordinates: (r, θ, ϕ) $\quad x = r\,\sin\theta\,\cos\phi \quad y = r\,\sin\theta\,\sin\phi \quad z = r\,\cos\theta$

$$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z} \qquad \hat{\theta} = \cos\theta\cos\phi\,\hat{x} + \cos\theta\sin\phi\,\hat{y} - \sin\theta\,\hat{z}$$



$$\begin{aligned}
d\vec{r} &= dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\,\sin\theta\,d\phi\,\hat{\phi} \\
d\tau &= r^2\,\sin\theta\,dr\,d\theta\,d\phi
\end{aligned}$$

$$\begin{aligned}
\vec{\nabla} f &= \frac{\partial f}{\partial r}\,\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\,\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\,\hat{\phi} \\
\nabla^2 f &= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2} \\
\vec{\nabla} \cdot \vec{v} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta v_\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} \\
\vec{\nabla} \times \vec{v} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial \phi}\right]\hat{r} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\theta} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial \theta}\right]\hat{\phi}
\end{aligned}$$