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Runaway breakdown in strong electric field as a source of terrestrial gamma flashes and gamma bursts in lightning leader steps

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Abstract

The new model of lightning step leader is proposed. It includes three main processes developing simultaneously in a strong electric field: conventional breakdown, effect of runaway electrons and runaway breakdown (RB). The theory of RB in strong electric field is developed. Comparison with the existing observational data shows that the model can serve as a background for the explanation of gamma bursts in step leader and TGF.

1. Introduction

Gamma rays preceding the lightning return stroke were first observed by Moore et al. [1]. Later in extensive experiments at the specially constructed installation Dwyer et al. have made in 2004–2005 a fundamental discovery: every step of lightning leader is accompanied by a burst of the strong gamma emission [2–4]. The time duration of gamma bursts (GB) is less than 1 µs. The emission was seen in all steps about 100 µs before the return stroke. Taking into account that Dwyer's et al. experiments were done at the Earth's surface and that during 100 µs the lightning leader pass a several hundred meters it was natural to conclude that the same gamma emission exist at every leader step—it is not seen by the installation due to gamma ray absorption in the air. And really, first observations of gamma bursts done by Fishman et al. [5] at COMPTON satellite and intensive studies fulfilled later by Smith et al. [6] at RHESSI satellite demonstrated that gamma emission from lightning is extremely strong: the full energy of terrestrial gamma flashes (TGF) reaches 20 kJ! The detailed comparison of satellite data with radio observations from the Earth [7,8] confirmed that this wonderfully strong emission is generated in lightning leader preceding the return stroke.

That made a puzzling question for the theory: how this extremely strong emission is generated? To answer this question is the main goal of the present Letter.

Note that the source of gamma emission definitely is the bremsstrahlung of high energy electrons ($\varepsilon \geqslant 0.1$–$1$ MeV). Dwyer hypotized that electrons are accelerated at the head of the leader step as runaways in neutral air in accordance with Gurevich's general theory of runaway effect in neutral gases. That claim the electric field in the head to reach values about 200 kV/cm or more (for atmosphere pressure), what is several times higher than it was established in conventional long sparks [9]. But is the Gurevich's runaway effect strong enough to explain the observed intensity of gamma flashes? The concrete calculations indicate the definite difficulties of this explanation giving the values, which are orders of magnitude less, than the observed one.

In the present work we propose a new model of electron acceleration in the lightning leader step. In this model we demonstrate that in the strong electric fields a new process become very effective—runaway breakdown (RB).

Runaway breakdown was considered previously for weak electric fields $E \sim 2$–10 kV/cm which are less, than the threshold field of the conventional air breakdown $E < E_{\rm th} \approx$

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20 kV/cm [10]. To excite RB in the weak electric field high-energy ($\varepsilon \approx 0.1$–1 MeV) electrons are needed. These electrons serve for the RB initiation. It was supposed that in the Earth atmosphere RB is initiated by the electrons generated by cosmic rays [11], most effective in cosmic ray showers (RB-EAS discharge) [12].

The strong electric field is much higher, than the threshold field of conventional breakdown $E \gg E_{\mathrm{th}}$. In this case as it will be shown runaway breakdown can be very effective also. Gurevich runaway electrons serve now for the initiation of runaway breakdown when their energy reach the RB critical energy ε_{c}.

Thus the acceleration of electrons in the leader step consists of *two stages*. First is the Gurevich runaway effect. The second stage begins when runaway electrons energy reaches the RB critical energy ε_{c}. In strong electric field $E \sim 200$ kV/cm the value of ε_{c} is only 3 keV. Besides the runaway avalanche characteristic length l_{a} becomes very small ($l_{\mathrm{a}} \approx 1$ cm). That is why at $\varepsilon > \varepsilon_{\mathrm{c}}$ due to RB the number of runaways in step leader growth up by several orders of magnitude, and electrons are accelerated up to several MeV.

In the last section of the Letter the *two stage model* will be briefly compared with existing observations and a reasonable agreement will be demonstrated.

2. The lightning step leader and conventional breakdown in a strong electric field

2.1. Step leader

The lightning leader steps have widely varied characteristics. According to the statements in the textbooks [13,14] based on the numerous measurements one can indicate the following main features of the steps.

(1) The step length is 3–50 m. As example one can take an average value 15 m. Note, that the length is slightly growing with the atmospheric height.

(2) The step pulse duration is of the order 0.3–1 μs.

(3) The time between pulses either or between steps changes randomly from 5 to 100 μs.

(4) The pulse velocity obtained from the measurements of the luminosity movement is $\sim 5 \times 10^9$ cm/s.

(5) The width of the luminous region is up to ~ 1 m.

(6) The radio emission measured at the Earth surface demonstrate that the leader step pulse is accompanied by a significant pulse of electric current $I \sim 0.3$–5 kA.

2.2. Conventional breakdown

Let us determine the basic pulse characteristics from the model of electric discharge in a strong electric field $E \sim 10E_{\mathrm{th}} \approx 200$ kV/cm. The conventional breakdown in a given strong electric field lead to a very fast exponential growth of air ionization N_{e}

$$N_{\mathrm{e}} = N_0 \exp(\nu_{\mathrm{ion}} t). \tag{1}$$

Here N_0 is the initial density of electrons and ν_{ion} is the ionization frequency.

The breakdown in air was studied in details in numerous experimental and theoretical works (see textbooks [15,16]). The ionization frequency is growing effectively with the electric field value:

$$\nu_{\mathrm{ion}} = \nu_{\mathrm{am}} F(E/E_{\mathrm{th}}). \tag{2}$$

Here ν_{am} is the maximal attachment frequency in e–O_2 collisions:

$$\nu_{\mathrm{am}} = 7.6 \times 10^{-13} N_{\mathrm{m}}\ \mathrm{s}^{-1} \approx 2 \times 10^7 \left(\frac{N_{\mathrm{m}}}{N_{\mathrm{m0}}}\right) \mathrm{s}^{-1},$$
$$N_{\mathrm{m0}} = 2.7 \times 10^{19}\ \mathrm{cm}^{-3}. \tag{3}$$

A well-known empirical formula [15] for F is

$$F = (E/E_{\mathrm{th}})^{5.3}. \tag{4}$$

It is correct at low values of E/E_{th} up to $E/E_{\mathrm{th}} \sim 4$–5 (see [16]). At higher values of (E/E_{th}) the growth of $\nu_{\mathrm{ion}}/\nu_{\mathrm{am}}$ is diminishing reaching maximum $\sim 10^5$ approximately at $(E/E_{\mathrm{th}}) \approx 100$. In the region we are interested in

$$\frac{E}{E_{\mathrm{th}}} = 7\text{–}10, \qquad \frac{\nu_{\mathrm{ion}}}{\nu_{\mathrm{am}}} = (2\text{–}5) \times 10^3. \tag{5}$$

So, we see from (1)–(5), that due to the breakdown in a given strong electric field the degree of ionization in air grows very rapidly.

On the other hand, the growth of the free electron density leads to the fast amplification of electron conductivity σ. The last process leads, as it is well known, to the diminishing of electric field:

$$E = E_0 \exp\left(-4\pi \int \sigma\, dt\right) \tag{6}$$

which will stop the growth of the ionization.

Electrical conductivity in neutral atmosphere

$$\sigma = \frac{e^2 N_{\mathrm{e}}}{m \nu_{\mathrm{em}}}, \tag{7}$$

where ν_{em} is the electron collision frequency. Substituting (6), (7) in (1) we find that electric field keeps it's high value for the finite time $t < t^*$ when $N(t) \leqslant N(t^*)$:

$$\frac{4\pi e^2 N(t^*)}{m \nu_{\mathrm{em}} \nu_{\mathrm{am}} F(E/E_{\mathrm{th}})} \approx 1. \tag{8}$$

This relation determines the maximal value of electron density reaching in breakdown:

$$N_{\mathrm{em}} = \frac{m \nu_{\mathrm{em}} \nu_{\mathrm{am}}}{4\pi e^2} F(E/E_{\mathrm{th}}) \tag{9}$$

and characteristic time of the discharge

$$t^* = \frac{C_0}{\nu_{\mathrm{am}} F(E/E_{\mathrm{th}})}, \qquad C_0 \approx \ln\left(\frac{N_{\mathrm{em}}}{N_{\mathrm{e}}}\right) \sim 10\text{–}20. \tag{10}$$

Relations (9), (10) determine the basic characteristics of a strong electric field ($E \gg E_{\mathrm{th}}$) discharge in unbounded air.

3. Runaway electrons in strong electric field

The phenomenon of runaway electrons indicated first by Wilson [17] is based on the specific features of fast particle–matter interaction. The breaking force F acting on energetic particle in matter decrease with the increasing of electron energy ε (Bethe [18]). The reason is that a fast electron interacts with the electrons and nuclei of neutral matter as if they were free particle, i.e., according to the Rutherford scattering cross section $\sigma_R \propto 1/\varepsilon^2$. That is why the breaking force

$$F \approx \varepsilon \sigma_R N_a Z \propto \frac{1}{\varepsilon} \tag{11}$$

is proportional to the atomic number density N_a, charge of a nucleus Z, and inversely proportional to the electron energy ε. Exactly the friction force decrease determines the possibility of the appearance of runaway electrons in a gas placed in an electric field. Indeed, if a strong constant electric field E is presented in the medium an electron of sufficiently high energy

$$\varepsilon > \varepsilon_0 = \frac{eE}{\sigma_R Z N_a} \tag{12}$$

will be continuously accelerated by the field (see Fig. 1). Such electrons are called runaway electrons.

If the electric field is not too strong $E \ll E_{cn}$ the main part of electrons having low energy $\varepsilon < \varepsilon_i$ (where ε_i is the ionization energy) are colliding with neutral atom as a whole. Friction force is

$$F = m\nu_a v, \tag{13}$$

where m and ν_a electron mass and collision frequency

$$\nu_a = \sigma_a N_a v_T, \tag{14}$$

$v_T = \sqrt{T_e/m}$—electron thermal velocity, σ_a—collision cross section. Eq. (13) determines usual drift of electrons under the action of electric field $E = F/e$.

Thus we see that at low electron energies $\varepsilon \leqslant \varepsilon_i$ the friction force of electrons moving in the neutral gas F is growing with their energy. But at high energies $\varepsilon > \varepsilon_m$ it begins to fall down (see Fig. 1). It means that at some middle energies $\varepsilon \sim \varepsilon_m \sim Z\varepsilon_i$ it will reach the maximal value. We emphasize that around this region the interaction of fast electron with neutrals gradually transforms from elastic and inelastic scattering at the atom as a whole to the Coulomb scattering at the electrons and the atomic nucleus. To describe this process it is necessary to use the kinetic theory.

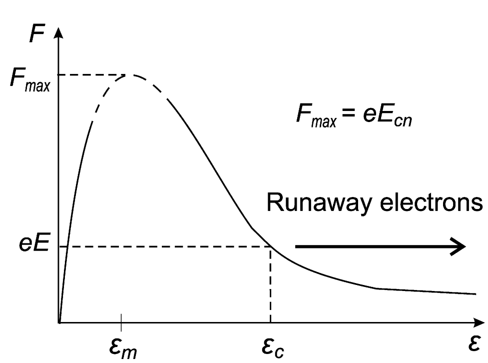

Fig. 1. The dependence of friction force of electron moving in neutral media.

Kinetic theory of runaway electrons in neutral gasses was developed by Gurevich [19]. The electric field E was supposed homogeneous and the gas unbounded. According to the theory the maximal value of friction force F_{max} (Fig. 1) is characterized by the electric field E_{cn}:

$$E_{cn} = \frac{F_{max}}{e} = \frac{4\pi e^3 N_a Z}{2.72 I}, \tag{15}$$

where I is an average excitation energy. Taking into account that $I \approx 13.5Z$ eV, an astonishingly simple expression for the value of critical field in neutral gas was obtained:

$$E_{cn} \approx 7 \times 10^{-15} N_a \text{ V/cm}. \tag{16}$$

Here N_a is the number density of atoms in cm^3. For example in air

$$E_{cn} = 38\left(\frac{N_m}{2.7 \times 10^{19}\text{ cm}^{-3}}\right) \text{MV/m} \approx 20 E_{th}, \tag{17}$$

where N_m is the number density of air molecules, and E_{th}—a threshold electric field of conventional breakdown.

If the electric field is less than critical one $E < E_{cn}$ only the electrons in the tail of distribution function become runaways. The flux of runaway electrons is proportional to the exponential factor [19]:

$$S_r = \frac{dN_r}{dt} = N_e \nu_e \exp\left\{-\frac{E_{cn}}{4E} A\right\}. \tag{18}$$

Here N_e is the number density of electrons, ν_e—electron collision frequency at a characteristic runaway electron energy ε_c and A is a large constant (for air $A \approx 30$). It means that the number of runaway electrons falls down very rapidly with electric field decreasing. The basic theory was farther developed and confirmed for different gases in a number theoretical and experimental studies (see textbooks [20,21]).

4. Runaway breakdown in strong electric field

A new physical phenomenon RB was predicted in [11]. The basic physical process is the generation of a new fast electrons due to the runaway particle ionization of neutral molecules. Although the majority of newborn free electrons have low energies, some will have rather high energy $\varepsilon > \varepsilon_c$. As a result, an exponentially growing runaway avalanche can occur. Two main features of RB are:

(1) It can occur in the electric field $E > E_c$, where E_c is an order of magnitude less then the threshold field of conventional breakdown $E_c \ll E_{th}$.

(2) The presence of "seed" fast electrons having energies above the critical runaway energy ε_c is needed to initiate RB.

In the previous works the RB theory in weak electric fields $E \ll E_{th}$ was constructed and a role of RB in lightning leader initiation was studied [12,22]. But one can show that the runaway breakdown effect can take place in a strong electric field

$E \gg E_{\rm th}$ as well. The theory of strong runaway breakdown (SRB) will be considered in the present section. It will be shown that usual runaway electrons serve as a seed for SRB, when their energy reach the runaway critical energy $\varepsilon_{\rm c}$ which coincide exactly with the one determined in the weak RB theory: $\varepsilon_{\rm c} = (mc^2/2)(E_{\rm c}/E)$. Thus the usual runaway effect in neutral gases gradually transforms to SRB, at electron energies ε:

$$\varepsilon \geqslant \varepsilon_{\rm c} = \frac{mc^2}{2}\left(\frac{E_{\rm c}}{E}\right) \approx 25\ {\rm keV}\left(\frac{E_{\rm th}}{E}\right). \tag{19}$$

Here we took into account that for air $E_{\rm c}/E_{\rm th} \approx 0.1$. It is evident from (17) and (18) that SRB effect at $E \sim (1\text{–}4)E_{\rm th}$ is extremely low. It explains, that in usual long sparks $E \leqslant (3\text{–}4)E_{\rm th}$ (see [9]) the runaway electrons and gamma emission are practically absent. On the contrary, at $E/E_{\rm th} \geqslant 7\text{–}10$ the runaways could become significant what lead to the exponentially strong SRB.

4.1. *Temporal and spatial growth of runaway breakdown in strong electric field*

Let us consider now the SRB effect in a strong electric field $E/E_{\rm th} \approx 7\text{–}10$. It means that parameter $\delta = E/E_{\rm c} \approx 70\text{–}100$. In such strong fields we can consider the main part of distribution function in nonrelativistic limit. Moreover, the distribution function is directional along electric field. The kinetic equation describing runaway breakdown in this case has a form [10]

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} + \delta\frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 f\right) = \frac{1}{v^2}\frac{\partial f}{\partial v} + \frac{1}{\Lambda v^5}\int_v^\infty f(k)k\,dk. \tag{20}$$

Here z—direction along electric field, Λ—Bethe's logarithm.

Eq. (20) has a very important feature: the parameter δ could be eliminated from equation. Actually, introducing

$$F = \int_\infty^u f(k)k\,dk, \tag{21}$$

$$x = z\delta^2, \qquad \tau = t\delta^{3/2}, \qquad u = v\delta^{1/2} \tag{22}$$

we can rewrite Eq. (20) in the form

$$\frac{\partial^2 F}{\partial u \partial x} + \frac{1}{u}\frac{\partial^2 F}{\partial u \partial \tau} + \frac{1}{u^2}\frac{\partial}{\partial u}\left(u\frac{\partial F}{\partial u}\right) = \frac{1}{u^2}\frac{\partial}{\partial u}\left(\frac{1}{u}\frac{\partial F}{\partial u}\right) - \frac{F}{\Lambda u^5}. \tag{23}$$

This self-similarity of Eq. (23) on parameter δ means that we can solve it for $\delta = 1$ and automatically find from (22) the runaway breakdown growth rates for arbitrary δ.

Let us consider two limits:

(1) Spatial uniform time growth

In this case we can search for the solution of Eq. (20) in the form $f(v,t) = f(v)\exp(t/\tau_{\rm rb})$. According to (21), (23) the time of exponential growth of electrons

$$\tau_{\rm rb} = \tau_0\delta^{-3/2}. \tag{24}$$

This result is in agreement with spatially uniform runaway breakdown theory [10].

(2) Spatial dependence

In this case we can search for the solution of Eq. (20) in the form $f(v,t) = f(v)\exp(z/l_{\rm a})$. According to (21), (23) the avalanche scale $l_{\rm a}$ decrease with increasing δ

$$l_{\rm a} = l_0\delta^{-2}. \tag{25}$$

It means that the avalanche length in a strong field is diminishing dramatically. Thus, in air for $\delta = 70\text{–}100$ we have

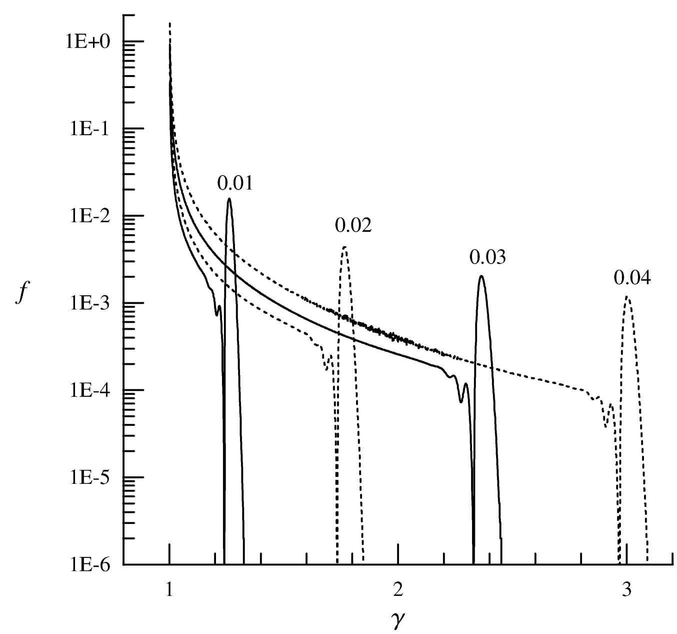

Fig. 2. The dependence of distribution function of fast electrons on energy for $\delta = 100$. Different curves correspond to time moments 0.01, 0.02, 0.03 and 0.04. Asymptotic behavior $f \propto 1/\varepsilon$ is clearly seen.

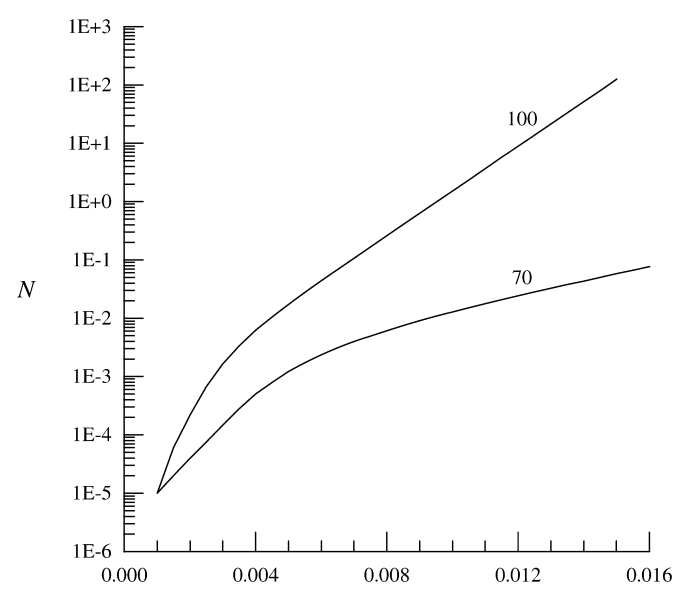

Fig. 3. The growth of total number of fast electrons on time for parameters $\delta = 100$ and $\delta = 70$.

$l_a \approx 1$ cm. The distribution function for $\delta = 70$ and the growth of runaway electron density for $\delta = 70$, 100 determined by numerical solution of Eq. (23) are presented at Figs. 2, 3. The results are in agreement with studied previously runaway breakdown for the case of a weak electric field. The distribution function is approaching rapidly to $f \propto 1/\varepsilon$.

The characteristic time-scale of SRB process in atmosphere is of the order of a few ns. Note that the density growth is stronger during the initial 0.5 ns, analogous effect was seen in the weak fields (see [10]).

5. Discussion

5.1. Step leader model

The model of the SRB step leader is shown in Fig. 4. The maximal electric field is naturally reached in the head region near the leading front. The head structure is determined by the joint effect of conventional breakdown, runaway electrons and runaway breakdown. The head length l_0 is of the order of 10 cm. Its velocity v_h is determined by the characteristic time t^* of maximal electron density creation (10)

$$v_h \sim \frac{l_0}{t^*} \sim (3\text{–}10) \times 10^9 \text{ cm/s}. \tag{26}$$

The potential drop of electric field along the head

$$\Phi \approx \int_0^{l_0} E\,dz \sim (2\text{–}4) \text{ MeV} \tag{27}$$

could be considered as the criterium of SRB streamer formation.

Below we will discuss the parameters at the head mainly. Electric field

$$E \sim (7\text{–}10) E_{th} = 150\text{–}200 \text{ kV/cm} (N_m/N_{m0}),$$
$$N_{m0} = 2.7 \times 10^{19} \text{ cm}^{-3}. \tag{28}$$

The mean energy of electrons in the air discharge at $E \sim 10E_{th}$ [16]

$$\bar{\varepsilon} = (10\text{–}20) \text{ eV}. \tag{29}$$

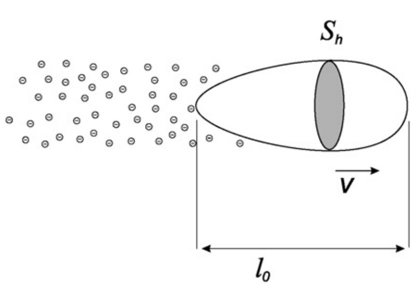

Fig. 4. The model of the SRB step leader. The maximal electric field is reached in the head region. The head structure is determined by the joint effect of conventional breakdown, runaway electrons and runaway breakdown. The head length l_0 is of the order of 10 cm. Its velocity v is determined by the characteristic time t^* of maximal electron density creation (10), S_h—head cross section (36). The head velocity v agrees well with the characteristic velocity of runaway electrons at $\varepsilon = \varepsilon_c$.

Full collision (elastic and inelastic) cross section [16] (Fig. 2.1.)

$$\sigma_{tr} = 10^{-15} \text{ cm}^2. \tag{30}$$

From (29), (14) follows the collision frequency of electrons

$$\nu_{em} = 5 \times 10^{12} \left(\frac{N_m}{N_{m0}}\right) \text{ s}^{-1}. \tag{31}$$

Taking into account the ionization frequency ν_{ion} at strong field (2), (3) one obtain from (2)–(4) the rather high value of maximal density of electrons (9)

$$\frac{N_{emax}}{N_m} = 5 \times 10^{-6} \left(\frac{N_m}{N_{m0}}\right),$$
$$N_{emax} \sim (0.5\text{–}1) \times 10^{14} \text{ cm}^{-3}. \tag{32}$$

5.2. Pulse electric current. Pulse head cross section

The lightning initiation and step leader propagation is accompanied by a chain of bipolar radio pulses [13,23]. The pulse duration is about 0.3–1 µs, time between pulses $\sim$ 15 µs. The radio pulse is naturally connected with the pulse of electric current generated during the step of the leader. Let us estimate the maximal current in the pulse

$$I_p = e\bar{v} N_{emax} S_h, \quad \bar{v} = \frac{eE}{m\nu_{em}}. \tag{33}$$

Here $\bar{v}$ is the transport velocity of electrons, S_h is the head cross section (Fig. 4). From (28), (31) it follows

$$\bar{v} \approx 6 \times 10^7 \text{ cm/s}. \tag{34}$$

The maximum pulse current determined from observations is [13]

$$I_{ob} \sim (0.5\text{–}5) \text{ kA}. \tag{35}$$

Comparing the calculated value (33) with (35) one can find the head cross section

$$S_h \approx 1\text{–}10 \text{ cm}^2. \tag{36}$$

We used here formulas (32) and (34) for N_{emax} and $\bar{v}$.

We emphasize, that the value of the head cross section (36) is much higher than determined usually in a long spark laboratory observations $S_h \sim 10^{-2}$ cm^2 [9]. In usual laboratory sparks the energetic electrons and gamma emission are not detected as well. So, we see that the results of lightning step leader observations show significantly more energetic process. Note, that in recent special laboratory experiments using Marx generator with a giant potential gap 1.5 MV the gamma rays emission with the energy up to 150 keV was observed by Dwyer et al. [24].

5.3. Runaway electrons

As the electric field in the head is strong reaching the value about 200 kV/cm which is only one half of the critical field E_{cn} in neutral air (17) the runaway electrons should appear. Let us estimate their number. The full number of runaway electrons

n_R according to [19] is

$$n_R = \int S_r V \, dt, \tag{37}$$

where S_r is the runaway flux (18) and V is the volume of the high electric field region. Runaways in a step are generated in the head. So, N_e is N_{emax} (32), electric field $E/E_{cn} \approx 0.5$ and ν_e electron collision frequency determined in accordance with Bethe's formulae:

$$\nu_e = \frac{F}{mv} = \frac{4\pi N_m Z e^4}{m^2 v^3} \ln\left(\frac{mv^2}{I}\right), \tag{38}$$

where v is the electron velocity. Taking the electron energy $\varepsilon = mv^2/2 = 5$ keV of the order or slightly higher than ε_c we obtain from (38)

$$\nu_e = 10^{10}\left(\frac{N_m}{N_{m0}}\right). \tag{39}$$

From (18), (32) and (39) we find the runaway flux (taking constant $A = 30$)

$$S_r = N_{emax}\nu_e \exp\left(-\frac{E_{en}}{4E}A\right) = 3 \times 10^{17}\frac{el}{s\ \mathrm{cm}^3}\left(\frac{N_m}{N_{m0}}\right)^2 \tag{40}$$

and the full number of runaways during the step is

$$n_R = S_r S_h l_0 \Delta t \approx 3 \times 10^{12} el. \tag{41}$$

Here the head cross section ~ 3 cm^2, mean head length $l_0 \sim 10$ cm and step time $\Delta t \approx 0.3$ μs were used. Thus we see, that the full number of Gurevich runaways during one step is large enough. Supposing mean acceleration by the electric field of every runaway particle to 100 keV we obtain the whole energy of accelerated electrons $\varepsilon \approx 0.5$ J. Note, that this energy is small compared to the observed in TGF.

5.4. Runaway breakdown

Let us consider now the role of runaway breakdown in strong electric field. Simultaneously in strong field the conventional breakdown is developing. But during linear stage a runaway breakdown and conventional breakdown could be excited quite independently. Condition for RB initiation is the existence of seed electrons whose energy is higher than the RB critical energy $\varepsilon > \varepsilon_c$. In our case $E \approx (7\text{–}10)E_{th}$, the critical energy $\varepsilon_c \approx 2.5\text{–}3.5$ keV. As one can see from (18) there is a significant flux of the seed electrons at $\varepsilon \geqslant 5$ keV. Thus according to calculations presented in Section 4 the SRB process can develop.

The SRB avalanche length

$$l_a \approx 1 \text{ cm}. \tag{42}$$

It is smaller than the characteristic length of streamer head

$$l_a \leqslant \sqrt{S_h} < l_0. \tag{43}$$

Here S_h—head cross section (36), and l_0—mean head length (Fig. 4). The head velocity (26) agrees well with the characteristic velocity of runaway electrons at $\varepsilon = \varepsilon_c$:

$$v_c = \sqrt{\frac{2\varepsilon_c}{m}} \approx (3\text{–}5) \times 10^9 \text{ cm/s}. \tag{44}$$

So, the strong runaway breakdown (SRB) process can develop in streamer head conditions, leading to amplification of runaways number at 2–4 orders of magnitude.

5.5. Brief comparison with observations

Let us summarize here briefly the results of our model in comparison with the new observational data. Of course, the considered process is random and only some average characteristics could be discussed.

(1) The gamma emission in the step of the leader is predicted as a pulse with time length 0.1–0.3 μs and full energy

$$\varepsilon_\gamma \sim (0.01\text{–}1) \text{ kJ}.$$

(2) The integral gamma emission of lightning in the model having 20–50 steps

$$\varepsilon_{l\gamma} \sim 1\text{–}10 \text{ kJ}.$$

In agreement with Smith et al. [6] TGF observations.

(3) The natural for runaway breakdown energy spectrum of emission is obtained in SRB theory:

$$F \propto \frac{1}{\varepsilon}.$$

It agrees with the observations of Smith et al. [6].
We emphasize that in [25] the observed TGF energy spectrum was compared with the RB theory of gamma ray emission in the weak electric field only ($E < E_{th}$). The RB in weak field cannot explain the time scale of TGF (2–3 ms) and the full intensity of the gamma pulse. The gamma emission of the leader steps [2–4] cannot be explained by the weak field theory either.

(4) Pulse electric current maximum value and its time scale is

$$J_{max} \approx 0.3\text{–}5 \text{ kA}, \qquad \Delta t \approx 0.1\text{–}0.5 \text{ μs}$$

in agreement with observations.

(5) Very significant is the strong diminishing of the avalanche length (25), predicted by the theory. It determines the SRB process in the step leader head.

(6) The number of low energy electrons generated due to SRB process in the head region is large enough. It can, besides conventional breakdown, affect the growth of electron conductivity.

(7) To match the usual runaway effect with strong RB process we need the front of the streamer head velocity (26) to be close to characteristic runaway velocity (44). This condition is fulfilled and it agrees with the observed streamer luminous pulse velocity [13,26].

So, we see that runaway breakdown effect in strong electric field (SRB) could serve as a background for the explanation of the strong gamma emission generated in every step of lightning leader and in terrestrial gamma flashes (TGF) as integral of gamma emission from all lightning steps.

Of course the farther development of nonlinear theory considering the creation of streamer head in SRB process is of great significance. Even more important are the detailed high-time resolution (nanosecond) simultaneous observations of gamma, UV, optic and radio emissions in step leader.

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