Modern Homework 16

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1) $V = 4.00\,cm/s$ $\lambda = 6.00\,cm$

+4 $A = 3.00\,cm$

$k = \frac{2\pi}{\lambda}$ $\frac{\omega}{k} = v \rightarrow \omega = kv = \frac{2\pi}{\lambda} v$

$$y_R = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{\lambda}vt\right)$$

$$y_R(x = 10cm,\ t = 3.125\,s) = -2.598\,cm \boxed{\rightarrow -2.60\,cm}$$

For the $-x$ direction, $v \rightarrow -v$

$$y_R = A\cos\left(\frac{2\pi}{\lambda}x + \frac{2\pi}{\lambda}vt\right)$$

$$y_R(x = 10cm,\ t = 3.125\,s) = 2.574\times10^{-15}\,cm$$

I assume this is python trickery, so the displacement is practically

$$\boxed{y_R = 0}$$

No seriously, we actually wanted you to calculate a femtometer of displacement

;)

2) a) $y_1 = 0.00200\cos(8.00x - 400t)$ $y_2 = 0.00200\cos(7.60x - 380t)$

+3

$$y(x,t) = 2(0.00200)\left(\cos\left(\tfrac{1}{2}(0.40)x - \tfrac{1}{2}(20)t\right)\right)\cos(7.80x - 390t)$$

$$= 0.00400\cos(0.2x - 10t)\cos(7.80x - 390t)$$

b) $V_p = \frac{\bar{\omega}}{\bar{k}} = \frac{390}{7.80} \boxed{= 50\,m/s}$

c) $V_g = \frac{\Delta\omega}{\Delta k} = \frac{10}{0.20} \boxed{= 50\,m/s}$

d) the "longer" cosine is $\cos(0.2x)$, for $t=0$

$$\Delta x = \frac{2\pi}{\cancel{0.2}\ 0.4} = \cancel{10}\pi\ [m]$$

$$\Delta x\,\Delta k = 10\pi\,(0.4) \boxed{= \cancel{4\pi}}$$

2π

3.) a) using deBroglie's equations:

+4

$$\lambda = \frac{h}{p} \qquad f = \frac{E}{h}$$

the definition of wave velocity $V_p = f\lambda = \frac{E}{p}$

$$V_p = \frac{E}{p} = \frac{\sqrt{(pc)^2+(mc^2)^2}}{p} = \sqrt{c^2+\left(\frac{mc^2}{p}\right)^2} = c\underbrace{\sqrt{1+\left(\frac{mc^2}{pc}\right)^2}}_{>1}$$

so $V_p > c$

b) $$V_g = V_p\Big|_{k_0} + k_0 \frac{dV_p}{dk}\Big|_{k_0} \qquad p = \frac{h}{\lambda} = \frac{h}{2\pi/k} = \frac{hk}{2\pi} = \hbar k$$

$$= c\sqrt{1+\left(\frac{mc^2}{\hbar c k_0}\right)^2} + \frac{d}{dk}\, c\sqrt{1+\left(\frac{mc^2}{\hbar c k}\right)^2}\Bigg|_{k_0} k_0$$

$$= c\sqrt{1+\left(\frac{mc^2}{\hbar c k_0}\right)^2} + c\left[-\left(\frac{mc^2}{\hbar c}\right)^2\left(\frac{1}{k_0}\right)^3 2\left(\frac{1}{2}\right)\left(1+\left(\frac{mc^2}{\hbar c k_0}\right)^2\right)^{-1/2}\right] k_0$$

$$= c\sqrt{1+\left(\frac{mc^2}{\hbar c k_0}\right)^2} + -c\left(\frac{mc^2}{\hbar c}\right)^2\left(\frac{1}{k_0}\right)^2\left(1+\left(\frac{mc^2}{\hbar c k_0}\right)^2\right)^{-1/2}$$

$$= C \; \frac{\left(1+\left(\frac{mc^2}{\hbar c k_0}\right)^2\right) - \left(\frac{mc^2}{\hbar c k_0}\right)^2}{\sqrt{1+\left(\frac{mc^2}{\hbar c k_0}\right)^2}} = C \left[\sqrt{1+\left(\frac{mc^2}{\hbar c k_0}\right)^2}\,\right]^{-1}$$

$$= C \left[\sqrt{1+\frac{(mc^2)^2}{(pc)^2}}\,\right]^{-1} = \frac{c(pc)}{\sqrt{(pc)^2+mc^2)}} = \frac{c^2 p}{E} = \frac{c^2 m \gamma u}{\gamma m c^2}$$

$$= u \quad \checkmark$$