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PHSX 343: Assignment 5

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+4 Problem 1

a) $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ Metric Equation

$$\Delta t = \frac{\Delta x}{v}$$

$$(c\Delta s)^2 = \left(\frac{c\Delta x}{v}\right)^2 - (\Delta x)^2 \rightarrow \Delta s = \sqrt{\left(\frac{c\Delta x}{v}\right)^2 - (\Delta x)^2}$$

Where $\Delta x = 52.4m$ and $v = 0.800c$, so $c\Delta s = 39.3s$ and
$\Delta s = 1.31 \times 10^{-7} = 131ns$.

The proper time of the muon's frame is also the spacetime for the problem since the muon is inertial and $\Delta x_{muon} = 0$. So $\Delta\tau = 131ns$.

b)

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

Where $\Delta t'$ is the proper time and Δt is any other coordinate time. So, to solve for $\Delta t'$:

$$t' = \frac{\Delta x}{v}\sqrt{1 - \left(\frac{v}{c}\right)^2} = 1.31 \times 10^{-7} = 131ns$$

These two values should be the same, as they are.

But *why*?

+3 # Problem 2

a) We can use a binomial expansion on the integrad, giving us

$$\sqrt{1-\left(\frac{v}{c}\right)^2} = 1 - \frac{1}{2}\left(\frac{v}{c}\right)^2 + \frac{1}{2}\left(-\frac{1}{2}\right)\left(\frac{v}{c}\right)^4 - \dots$$

To determine how many terms to keep We have to determine how many decimal places $\left(\frac{v}{c}\right)^2$ provides. If we name $\Delta t = 1.00 \times 10^6 s$ and $a = 10\frac{m}{s^2}$, then

$$\left(\frac{v}{c}\right)^2 = \left(\frac{at}{c}\right)^2 = \left(\frac{1}{30}\right)^2 = 0.0011$$

With this analysis, it is obvious that 0.0003 to any power greater than 1 will give a value with less than 4 fig sigs, when we are adding to 1. Then out integrad becomes:

$$1 - \left(\frac{v}{c}\right)^2$$

b) To integrate the problem we can just double how long it takes to travel from A to B. Similarly we can break the integral into two calculations, one from A to the midpoint, then from the midpoint to B. We have an equation from the description of the problem for the acceleration and can use that for v(t).

$$\Delta\tau_{A\to B} = \int_0^{\Delta t} (1 - \frac{1}{2}\left(\frac{at'}{c}\right)^2 dt' + \int_0^{\Delta t} (1 - \frac{1}{2}\left(\frac{a\Delta t - at'}{c}\right)^2 dt'$$

$$\Delta\tau_{A\to B} = \left[t - \frac{1}{6}\frac{c}{a}\left(\frac{at'}{c}\right)^3\right]\Big|_0^{\Delta t} + \left[t + \frac{1}{6}\frac{c}{a}\left(\frac{a\Delta t - at'}{c}\right)^3\right]\Big|_0^{\Delta t}$$

$$\Delta\tau_{A\to B} = \left[\Delta t - \frac{1}{6}\frac{c}{a}\left(\frac{a\Delta t}{c}\right)^3 + \Delta t + \frac{1}{6}\frac{c}{a}\left(\frac{-a\Delta t}{c}\right)^3\right] = 2\Delta t - \frac{1}{3}\frac{a^2}{c^2}\Delta t^3$$

$$\Delta\tau_{A\to B} = \Delta t\left(2 - \frac{1}{3}\left(\frac{a\Delta t}{c}\right)^2\right)$$

To give the total path of $\Delta\tau_{A\to A}$, we just have to double the the value. The following proper time is for the whole path.

$$\Delta\tau = 3.9993 \times 10^6$$

Now to find the difference in time,

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$$4\Delta t - \Delta\tau = 0.0007 \times 10^6 \to 0.\cancel{001} \times 10^6 \text{ (4 sig figs)}$$

This is actually only 3 sig-figs, the first 0 is a leading 0.