Modern Homework 26

~~#~~

6/8

1. a) $T(E) = \left\{ 1 + \frac{1}{4}\left(\frac{U^2}{E(U-E)}\right) \sinh^2 \alpha L \right\}^{-1}$

+3

$\alpha = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{2\pi\sqrt{2mc^2(U-E)}}{hc}$

$E = 4.40$ MeV

$U = 50$ MeV

$L = 1.00$ fm

$T(E) = 0.0686$

b) Since we are Not in realitivistic territory: $(938)0.01 > 4.4$ all the energy can be kinetic, If $U=0$

$\frac{1}{2}mv^2 \Rightarrow \quad v = \sqrt{2\frac{E}{mc^2}}\, c$

So each bounce will take $\frac{w}{v}$ time, where w = width = 5 fm.

So the rate of expulsion is $\frac{w}{v}T(E) =$

Gonna need a number for both of these.

c) if the width increases, the T(E) will decrease and consequently the rate of transmission will decrease proportionally.

D) $T(E) \approx \exp\left(-\frac{2}{\hbar}\sqrt{2m}\int\sqrt{(U(x)-E)}\,dx\right) = \exp\left(-\frac{2}{\hbar}\sqrt{2m}\sqrt{U-E}\,L\right)$

≈ 0.0516

They differ by more than 20%. The approximation isn't that good.

This general approx agrees pretty well with our direct calculation, so this supports our solution in a. Specifically the use of it as a wide tall ~~barrier~~ barrier. If anything this is a slightly thinner barrier, as the probability of transmission is 1% higher.

2. a) $\frac{\hbar^2 k_2^2}{2m} = E - U \rightarrow k_2 = \sqrt{\frac{2m(E-U)}{\hbar^2}} = \cancel{\frac{2\pi\sqrt{2mc^2 U_0}}{hc}}$

+3

Just plug E into here.

After the step down the energy is all kinetic, so:

$E + U_0 = T$

$2U_0 + U_0 = T$

$3U_0 = T$

$\boxed{\frac{3}{2}E = T}$

↳ this is total energy though.

$T_0 : 2U_0$

$T_f : \frac{3}{2}U_0$

$\frac{T_f}{T_0} = \frac{\frac{3}{2}U_0}{2U_0} = \frac{3}{2}$

$$\boxed{\frac{T_f}{T_0} = \frac{3}{2}}$$

b) pulling from lecture, as this is the same idea, just with neg U

$T = \frac{4k_1 k_2}{(k_1+k_2)^2} = \frac{4\left(\frac{2\pi\sqrt{2mc^2U_0}}{hc}\right)\left(\frac{4\pi\sqrt{2mc^2U_0}}{hc}\right)}{\left(\frac{6\pi\sqrt{2mc^2U_0}}{hc}\right)^2}$

$k_1 = \frac{2\pi\sqrt{2mc^2E}}{hc}$

$= \frac{4\pi\sqrt{mc^2U_0}}{hc}$

$= \frac{32}{36} : 8/9$

Bad math happened.

$T + R = 9/9 = 1$ ✓

$R = \frac{(k_1-k_2)^2}{(k_1+k_2)^2} = \frac{\left(2\pi\sqrt{2mc^2U_0}\right)^2}{\left(6\pi\sqrt{2mc^2U_0}\right)^2} = \frac{4}{36} = 1/9$

So: $\boxed{T = 8/9 = 0.\cancel{88}9, \quad R = 1/9 = 0.\cancel{11}}$

0.99 0.1

2

c) $T = 0.889$

$\boxed{(1e6)(T) = 8.89e5}$ particles get transferred.

Compared to the classical expectation of 1e6. It is lower than classical, as we expect, but is still a majority, which is good.