PHSX 425: HW10

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November 18, 2021

Question 1

In class, and also in Griffiths, the reflection and transmission coefficients at normal incidence were derived assuming that $\mu = \mu_0$. Derive the reflection and transmission coefficients without making this assumption, and then show that energy is conserved (i.e., $R + T = 1$).

Question 2

Check your results to the previous problem against the solutions derived in Griffiths for oblique incidence, with the polarization in the plan of incidence. Show that $R + T = 1$ *in this more general case.*

Question 3

We have assumed that the wave frequency, ω, is constant across an interface. Consider a wave function of the form $\psi_I = f_I(\mathbf{r})e^{-i\omega_I t}$, incident on a boundary from material 1 to material 2. The incident, relfected, and transmitted waves in these two substances must satify a generalized linear wave equation,

$$[a_j \nabla^2 + b_j \partial_t + c_j \partial^2]\psi = 0$$

Where $j = 1, 2$ denotes materials 1 and 2. Show that $\omega_I = \omega_R = \omega_T$. Were any addition assumptions necessary to show this?[1] *Would your conclusion change for a nonlinear wave equation? Why or why not?*[2]

[1] Hint: you might find it helpful to solve problem 9.16 first. Consider reflected and transmitted waves of the same form, and assume some general sort of boundary condition holds.

[2] Hint: If the wave equation were nonlinear in ψ, would solutions of the assumed form be likely to work?