Math 242
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LaTex Write Up
Chapter 8, Proof 3.
October 28, 2021

2. For every natural number n, the integer $2n^2 - 4n + 31$ is prime.
False.
Disproof by Counterexample:
Suppose that $n = 31$. Then $2n^2 - 4n + 31 = (31)^2 - 4 * 31 + 31 = 868 = 31(28)$. Since the integer 868 is not prime, the statement does not hold for all values of n and is thus false.

8. If A, B, and C are sets, then $A - (B \cup C) = (A - B) \cup (A - C)$.
True.
Direct Proof:
Suppose A, B, and C are sets. Then $A - (B \cup C) = \{x : x \in A - (B \cup C)\}$ using set builder notation.
$= \{x : x \in A \cap x \notin (B \cup C)\}$ (Definition of Compliment)
$= \{x : x \in A \cap (x \notin B \cup x \notin C)\}$ (Definition of Union)
$= \{x : (x \in A \cap x \notin B) \cup (x \in A \cap x \notin C)\}$ (Distributive Property)
$= \{x : (x \in A - B) \cup (x \in A - C)\}$ (Definition of Compliment)
$= \{x : x \in (A - B) \cup (A - C)\}$ (Definition of Union)
Rewriting the set in the intentional definition,
$= (A - B) \cup (A - C)$
Thus we have shown that $A - (B \cup C) = (A - B) \cup (A - C)$ by means of direct proof.

QED

9. If A and B are sets, then $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$.
False.
Disproof by Contradiction:
Assume we have sets $A = \{1, 2\}$ and $B = \{2, 3\}$.
Then $\mathcal{P}(A) = \{\{\}, \{1\}, \{2\}, \{1, 2\}\}$ and
$\mathcal{P}(B) = \{\{\}, \{2\}, \{3\}, \{2, 3\}\}$.
$\mathcal{P}(A) - \mathcal{P}(B) = \{\{1\}, \{1, 2\}\}$
$A - B = \{1, 3\}$, and $\mathcal{P}(A - B) = \{\{\}, \{1\}, \{3\}, \{1, 3\}\}$
So it follows that $\mathcal{P}(A) - \mathcal{P}(B) \nsubseteq \mathcal{P}(A - B)$.
Since $\mathcal{P}(A) - \mathcal{P}(B) \nsubseteq \mathcal{P}(A - B)$, the original statement does not hold true for all possible sets, thus the hypothesis is false.

28. Suppose $a, b \in \mathbf{Z}$. If $a|b$ and $b|a$, then a=b.
False.
Disproof by Contradiction:
Since the hypothesis includes all integers, we can suppose $a = -1$ and $b = 1$. It is obvious that $-1|1$ and $1|-1$. But $a \neq b$, so the hypothesis is false.

34. If $X \subseteq A \cup B$, then $X \subseteq A$ or $X \subseteq B$.
False.
Disproof by Counterexample:
Suppose sets A, B, and X such that $A = \{1, 2\}$, $B = \{2, 3\}$, and $X = \{1, 3\}$. The union $A \cup B = \{1, 2, 3\}$. It is obvious that X is a subset of this union. However, $X \nsubseteq A$ and $X \nsubseteq B$; thus, through the definition of union the hypothesis does not hold true that if $X \subseteq A \cup B$ then $X \subseteq A$ of $X \subseteq B$.