Modern Homework ~~2~~3
11.

8/8 Good work!

1. $\frac{d^2\psi}{dx^2} = \frac{2mE}{\hbar^2}\psi$ $\qquad \frac{d^2\psi}{dx^2} = \frac{-2m(E-U_0)}{\hbar^2}$

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$k^2 = \frac{2mE}{\hbar^2}$ $\qquad \alpha^2 = \frac{2m(U_0 - E)}{\hbar^2}$

Starting with the odd solution:

$$\psi_{odd}(x) = \begin{cases} Be^{\alpha x} & x < -L/2 \\ A\sin(kx) & |x| < L/2 \\ Be^{-\alpha x} & x > L/2 \end{cases}$$

at. $-L/2$ $\qquad Be^{-\alpha L/2} = A\sin(-kL/2) = -A\sin(kL/2)$

$B\alpha e^{-\alpha L/2} = Ak\cos(-kL/2) = Ak\cos(kL/2)$

Where do these expressions come from?

$$\frac{B\alpha e^{-\alpha L/2}}{Be^{-\alpha L/2}} = \frac{Ak\cos(kL/2)}{-A\sin(kL/2)} \Rightarrow \alpha = -k\cot(kL/2)$$

$$\boxed{-\cot(kL/2) = \frac{\alpha}{k}}$$

2. a) tan (the even solution) intersects $\frac{\alpha}{k}$ before cot

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Why is "before" significant?

b) plotting the tan & cot equations with the Energy on the x-axis and setting the boundary condition $E_1 = 0.5eV$ the graph looks just like the one presented in the book. ~~[illegible]~~ Since there are ~~[illegible]~~ four points in this new graph, it is consistent. 0.5 eV * 4^2

c) From plugging in values with the graph, $L = \cancel{[illegible]}\ 7.28 \times 10^{-10}$ m

or $\cancel{\boxed{L = 728\text{ pm}}}$ $\qquad \boxed{L = 728\text{pm}}$ How is this different?

Estimating from $\tan(kL/2) = 3.9$ & $E = 0.5$ good

$\tan(kL/2) = 3.9$

$L = \frac{2}{k}\tan^{-1}(3.9)$ $\qquad k = \sqrt{\frac{2m(0.5)}{\hbar^2}} = \sqrt{\frac{4\pi^2 mc^2}{(hc)^2}}$

$l = 7.287 \times 10^{-10} = 729\text{pm}.$

Which is real close to my graph calc

D) using the same compter magrcs, the $n=4$ disappears at $l = 1.625 \times 10^{-10}$

and the $n=5$ doesn't show up till aftr 2.1×10^{-10}

so the $\boxed{\Delta L \approx 20\text{ pm}}$ Seems a bit tight to me.

using the point, $\tan(kL/2) = 0.32$

$$l' = \frac{2}{k}\tan^{-1}(0.32) \qquad k = \sqrt{\frac{2mE}{\hbar^2}} = \sqrt{\frac{8\pi^2 mc^2 E}{(hc)^2}}$$

$$E = 7.255$$

$$L' = 4.489 \times 10^{-11}$$

Now the uncertainty will be half this value

$\boxed{\Delta L = 22\text{pm}}$

e) as $U \to \infty$ it become an infinite square well.

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$$E_1 = \frac{(1240\text{ nm eV})^2}{8(0.511 \times 10^6 \frac{eV}{c^2}) L^2 c^2}$$

$$L = \sqrt{\frac{(1240)^2}{4(0.511 \times 10^6)}}\text{ nm} = 0.867\text{nm} \boxed{= 867\text{pm}}$$

✓

$$k = \sqrt{\frac{8\pi^2 m x}{\left(1240 \cdot 10^{-9}\right)^2}}$$

$$a = \sqrt{\frac{8\pi^2 m(8-x)}{\left(1240 \cdot 10^{-9}\right)^2}}$$

$$m = 0.511 \cdot 10^6$$

$m = 511000$

$$L = 7.28 \cdot 10^{-10}$$

$L = 7.28 \times 10^{-10}$

$$\tan\left(k \cdot \frac{L}{2}\right)$$

$$-\cot\left(k \cdot \frac{L}{2}\right)$$

$$\frac{a}{k}$$