# PHSX 491: HW01 - Mass Equivalence

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Question 1

Here we will work to convince ourselves that the inertial mass and gravitational mass are indeed equivalent. Consider two objects, one near the surface of the Earth and the other out in deep space where it cannot interact with any other objects. Each object has teh same inertial mass m_i and gravitational mass m_g. For simplicity we will take the speed of light to one, $c \to 1$.
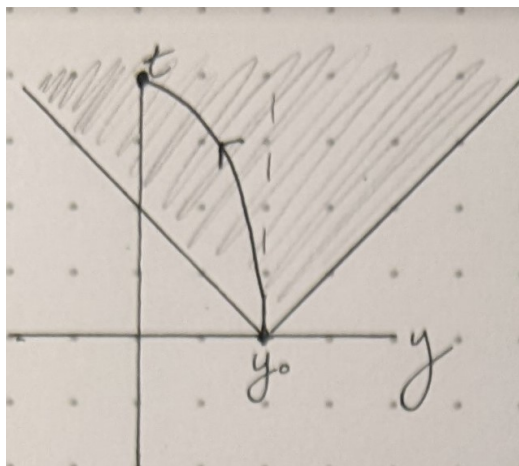
(a) The 2-dimensional spacetime diagram for part's **a)** and **b)**. The shaded region is the "light cone", those are the points that are said to be time-like (aka, they could be related by two event separated in time).
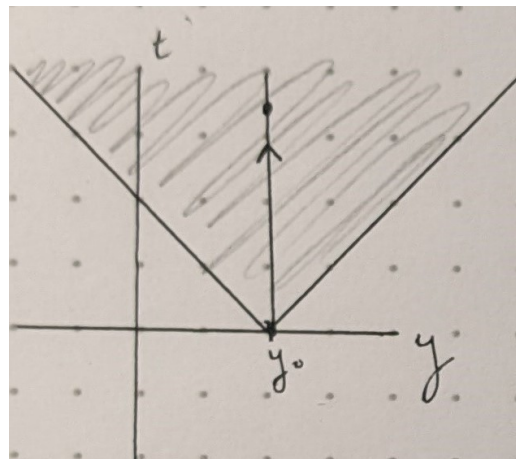
(b) Yeah, not gonna explain this for all of them. This is for **c)**

a) *Barrett's questions are quite wordy, so I'm just gonna skip copying it all down...
if I feel like it is needed to track the question I will quote it, otherwise I will
assume that you know these questions intimately enough...*

   See Fig. 1a

b) See Fig. 1a
   Isn't this just so exciting! Considering all of this is physics that we have been
   doing for numerous years, I assume that I don't need to elaborate any more on
   how to do a simple "$ct \times x$" diagram.

c) See Fig. 1b
   In this case, the path is not a geodesic. Our working definition of geodesic is
   the 'shortest path that connects two points in spacetime.' Pairing this with
   the Geodesic Hypothesis that 'A free particle will move along a geodesic', since
   this particle is no longer a free particle (we are exerting a force on it) the path
   it follows can't be the geodesic in spacetime.

d) The force of gravity is seen in Fig. 1a is going to be the particle's response to
   the gravitational field:
   $$F_g = \frac{GM}{r^2} m_G = g \cdot m_G$$

   Where we are calling
   $$g = \frac{GM}{r^2}$$

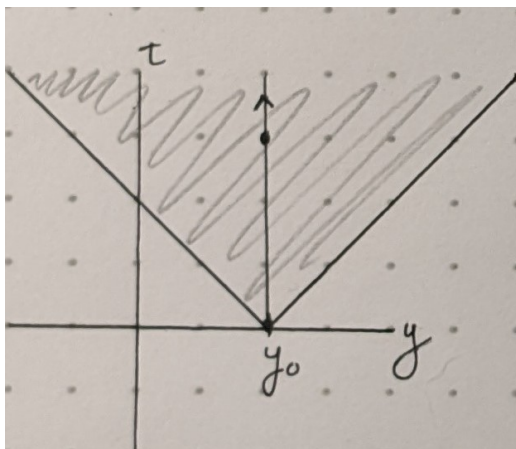   Thus, the upwards force must be

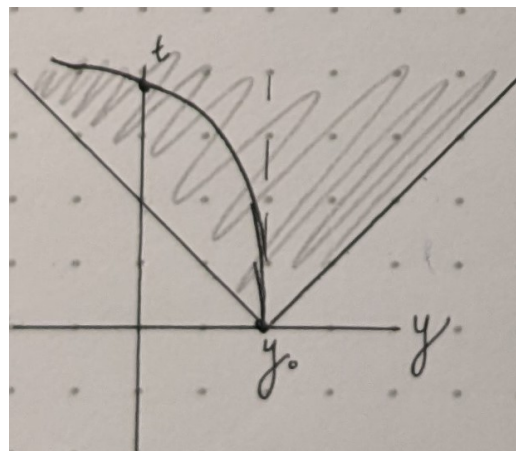 $$F'_g = -g \cdot m_G \tag{1}$$

e) See Fig. 2a

f) See Fig. 2a

g) See Fig. 2b
   Very similar to the answer in part **c)**, this path will not be the geodesic since
   the path being follow is not being done by a free particle. I will also drop this
   here to have for the next part: The force must be:

   $$F_I = g \cdot m_I \tag{2}$$

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(a) You know the drill: **e)** and **f)**

(b) Bro, have you ever tried soy sauce in mac & cheese? It is surprisingly good. You should try it sometime - just don't over do it on the salt when you're making the pasta... Oh, and: **g)**

h) Looking at Fig 1a and Fig. 2b, intuition says that their paths are the same. To justify that statement, we can simply say that the acceleration both of them see must be the same (as that was given in part **g)**). Since both have the same initial conditions and equations of motions, by uniqueness of solutions, their curves must be identical. So, assuming there is a unique way to transform a trajectory (which is a reasonable assumption, but this last week has kinda messed with some of my assumptions recently, so who knows.), then their forces must be equivalent.

i) In part **h)** we said that Eq. 1 and Eq. 2 must equal[1], so:

$$-F_G = F_I$$

$$g \cdot m_G = g \cdot m_I$$

$$\boxed{m_G = m_I}$$

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[1]That is in magnitude, their directions will be opposite because of the way I defined the forces.