7/8

PHSX 343: Assignment 10

William Jardee

+3 Problem 1

Writing the problem in 4-momentum: ($m_e = m_p$)

$$\begin{bmatrix} m_e c \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} m_e c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_{p_1} \\ P_{p_1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} P_{p_2} \\ P_{p_2} \\ 0 \\ 0 \end{bmatrix}$$

Using conservation of momentum in the x-directions;

$$P_{p_1} + P_{p_2} = 0 \rightarrow P_{p_1} = -P_{p_2} = P_p$$

$$\begin{bmatrix} m_e c \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} m_e c \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_p \\ P_p \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} P_p \\ -P_p \\ 0 \\ 0 \end{bmatrix}$$

The invariant rest energy for the pair would be

$$E_{tot,inv} = m_e c^2 + m_e c^2$$

Plugging in the value for $m_e = 0.511 \frac{MeV}{c^2}$, so the rest energy is $\boxed{1.022 MeV}$.

Using conservation of momentum in the t-direction:

$$2P_p = 2m_e c \rightarrow P_p = m_e c$$

Plugging in the value for the energy, $E_{tot} = m_e c^2 + m_e c^2 \rightarrow \boxed{1.022 MeV}$.

$$P_{tot} = \frac{2m_e c^2}{c} \rightarrow \boxed{1.022 \frac{MeV}{c}}$$

The invariant mass is $M^2c^4 = E^2 - p^2c^2$. You need to explicitly show that this is a conserved quantity.

Verifying conservation of invariant mass:

$$m_e c^2 + m_e c^2 = 2m_e c^2 = 2P_p c = P_p c + P_p c$$

To ensure that I finish that thought, the invariant mass of a photon is:

$$E^2 + mc^2 = E^2 = (Pc)^2$$

Last time I needed to compare the numbers. So we also see that the invariant mass initially is the same as the invariant mass at the end:

$$2mc^2 = 1.022MeV = 2P_p c$$

So we see the invariant mass of the electron pair is the same as that of the photon pair.

+4 Problem 2

a)

$$\begin{bmatrix} Mc \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{E_1}{c} \\ P \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{E_2}{c} \\ P \\ 0 \\ 0 \end{bmatrix}$$

With invariant mass we can confirm that $E_1 = E_2$:

$$E_1^2 = (mc^2)^2 + (Pc)^2 = E_2$$

This leads to the conclusion that, by conservation of energy:

$$E_1 = E_2 = \frac{1}{2}Mc^2$$

$$\left(\frac{1}{2}Mc^2\right)^2 = (mc^2)^2 + (Pc)^2$$

$$P = \frac{1}{c}\sqrt{\frac{1}{4}(Mc^2)^2 - (mc^2)^2}$$

Using the equation for speed of a particle: $\frac{u}{c} = \frac{Pc}{E}$

$$\frac{u}{c} = \frac{2Pc}{Mc^2} = \frac{2\sqrt{\frac{1}{4}(Mc^2)^2 - (mc^2)^2}}{Mc^2} = \sqrt{1 - 4\left(\frac{m}{M}\right)^2}$$

b)

$$\begin{bmatrix} 4Mc \\ P_i \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{E_1}{c} \\ P_{f_1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \frac{E_2}{c} \\ P_{f_2} \\ 0 \\ 0 \end{bmatrix}$$

Using our equations for conservation of momentum and energy:

$$P_i = P_{f_1} + P_{f_2}$$

$$4mc^2 = E_1 + E_2 = \sqrt{(mc^2)^2 + (P_{f_1}c)^2} + \sqrt{(mc^2)^2 + (P_{f_2}c)^2}$$

Using the invariant mass equation for the initial particle:

$$(4mc^2)^2 = (Mc^2)^2 + (P_i c)^2 \to P_i = \sqrt{(4mc)^2 - (Mc)^2}$$

Substituting that into the conservation of momentum:

$$P_{f_1} = \sqrt{(4mc)^2 - (Mc)^2} - P_{f_2}$$

And, finally, plugging that into the energy equation:

$$4mc^2 = \sqrt{(mc^2)^2 + ((\sqrt{(4mc)^2 - (Mc)^2} - P_{f_2})c)^2} + \sqrt{(mc^2)^2 + (P_{f_2}c)^2}$$

$$4mc^2 = \sqrt{(mc^2)^2 + (\sqrt{(4mc^2)^2 - (Mc^2)^2} - P_{f_2}c)^2} + \sqrt{(mc^2)^2 + (P_{f_2}c)^2}$$

Thank you so much for not making us solve for this!

What, you mean you afraid of a little algebra?