Modern Homework 22

C. H.

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1. a) $\lambda = \frac{2L}{n}$

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$n=1 \quad \lambda_1 = 2L$ $\quad n=3 \quad \lambda_3 = \frac{2}{3}L$

b) $\lambda = \frac{h}{p} \quad p = \frac{h}{\lambda}$

$E = K + \cancel{U}^{\to 0}$ * $U=0$ is the condition set up for an infinite well

$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2 = k$$

Where'd the n come from?

$$k: \frac{p^2}{2m} = \frac{h^2}{2\lambda^2 m^2} = \frac{h^2}{2(\frac{2}{n}L)^2 m} = \frac{\hbar^2 4\pi^2}{8L^2 m} n^2 = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

$E_n = k$ ✓

Why can we use a non-relativistic approximation here?

2. a) $$E_n = \frac{\hbar^2 \pi^2 c^2}{2(\frac{m}{c^2})L^2} n^2$$

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$$E_1 = \frac{(1240 \text{ eV nm})^2}{8(936 \times 10^6 \text{ eV})(0.001 \text{ pm})^2}$$

$= 204.9 \text{ meV}$ ~~$\approx 2.05 \times 10^{-4}$ MeV~~

Sketch of energy levels?

$E_2 = 819.6 \text{ meV}$ ~~$\approx 8.20 \times 10^{-4}$ MeV~~

$E_3 = 1844 \text{ meV}$ ~~$\approx 1.84 \times 10^{-3}$ MeV~~

* I had the wrong fm → nm conversion initially.

b) $hf = \frac{hc}{\lambda} = E_{n'} - E_n$

$$\lambda = \frac{hc}{E_{n'} - E_n}$$

i) $\lambda = \frac{1240 \text{ nm eV}}{819.6 \text{ eV} - 204.9 \text{ eV}} = 2.017 \text{ nm}$

ii) $\lambda = \frac{1240 \text{ nm eV}}{1844 \text{ eV} - 819 \text{ eV}} = 1.21 \text{ nm}$

iii) $\lambda = \frac{1240 \text{ nm eV}}{1844 \text{ eV} - 204.9 \text{ eV}} = 0.756 \text{ nm}$

c) The energy is ~~not~~ ~~even close to~~ over 1% of ~~kinetic~~ mass rest energy, so we are ~~good w/ non-relativistic~~ need to use relativistic Energy; but this is apparently very hard, so not today.

d.) Good model: Fixes Bohr model issue of destructive waves from circular orbit.

Kinda. We're looking at the nucleus, not the electrons.

Bad model: Not infinite potential (decent approx.)

3. a) the best explanation is a mathematical one

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$$|\Psi(x)|^2 = |\Psi(-x)|^2$$

$$\Psi(-x) = \pm \Psi(x)$$

b) ~~cos~~

$\cos\left(\frac{n\pi x}{L}\right) = 1$ at $n\pi$ is even values of pi

But does it actually satisfy the Schrö eq?
Is it normalized?

at $x = L$. Thus coefficient $= 0$

Same argument for $\sin\left(\frac{n\pi x}{L}\right)$

This is the logic used to remove cos in the $0 \to L$ potential well.

c) $E\Psi(x) = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2}$

$$E = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}\frac{1}{\Psi(x)} \qquad \frac{\partial^2 \Psi(x)}{\partial x^2} = \left(\frac{n\pi}{L}\right)^2 \Psi(x)$$

$$E = -\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \frac{\Psi(x)}{\Psi(x)}$$

$$= -\frac{\hbar^2\pi^2}{2mL^2}n^2$$

We have two different functional cases.

this is the same energy

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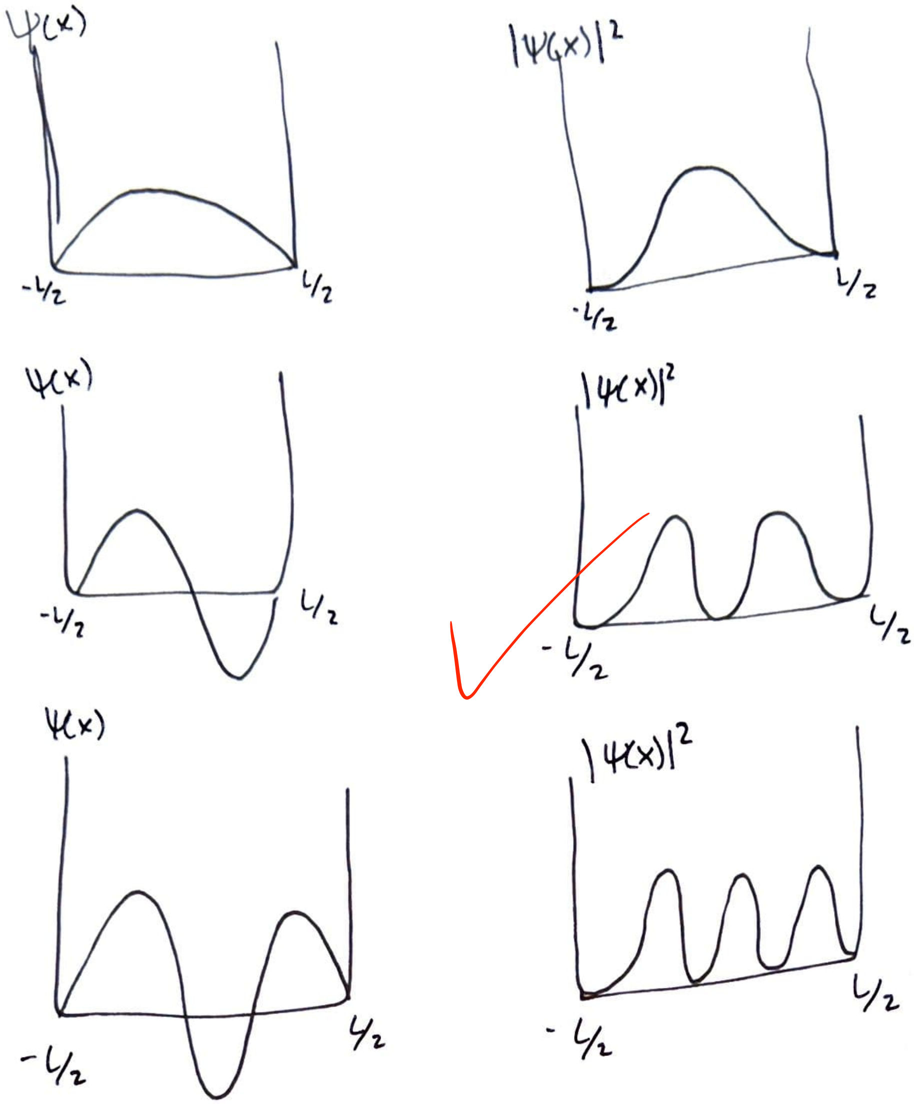
d)



$\cos\left(\frac{n\pi x}{L}\right)$ $\qquad$ $\cos^2\left(\frac{n\pi x}{L}\right)$

$\sin\left(\frac{n\pi x}{L}\right)$ $\qquad$ $\sin^2\left(\frac{n\pi x}{L}\right)$

These pretty much (exactly) match the sketches in Fig 6.9.

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