

PHSX 343: Assignment 11

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+4 Problem 1

This homework seemed to be mostly plug-and-chug, so enjoy the list of equations:

$$e = \sigma T^4 = \frac{P}{A_s} = \frac{P}{4\pi r^2}$$

$$\lambda_{max}T = 2.898 \times 10^{-3} = \gamma \rightarrow \frac{\gamma}{\lambda} = T$$

$$\frac{P_{tot}}{4\pi r^2} = \sigma T^4$$

$$r^2 = \frac{P_{tot}}{4\pi\sigma\left(\frac{\gamma}{\lambda}\right)^4} = \frac{81P_{erth}}{4\pi\sigma\left(\frac{\gamma}{\lambda}\right)^4}$$

$$P_{erth} = 4\pi r_s^2 T_s^4$$

$$r^2 = \frac{81r_s^2 T_s^4}{\left(\frac{\gamma}{\lambda}\right)^4} \rightarrow r = 9r_s \left(\frac{T_s \lambda}{\gamma}\right)^2$$

Plugging in the values: $r_s = 6.96 \times 10^8 m$, $T_s = 5800 K$, $\gamma = 2.898 \times 10^{-3} m \cdot K$,
 $\lambda = 9.66 \times 10^{-9} m$

$$r = 2.341 \times 10^{10} m$$

The red giant makes perfect sense, as the light's wavelength is on the far red scale (966 nm) and the radius is so large it earns the term giant. ✓

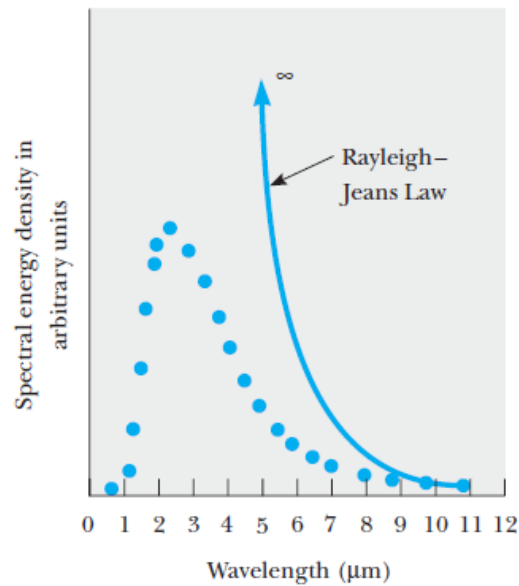
Large compared to what?

+4 Problem 2

Using the equation for $\bar{E} = \frac{hc/\lambda}{e^{hc/\lambda K_B T} - 1}$

a) $\frac{hcK_B T/10hc}{e^{10}-1} = \frac{k_B T}{10(e^{10}-1)} \approx 0.951 k_B T$ ✓

b) $\frac{k_B T}{10(e^{10}-1)} \approx 4.540 \times 10^{-4} k_B T$ ✓



As is apparent with the graph, these coefficients make complete sense. As you get smaller λ your coefficient is going to plummet, as our does for wavelength we divide by 10. When we have a larger λ our lines match up, so our coefficient will be close to one, as it is. 🍷 ✓

+4 Problem 3

$$\int_{\lambda=0}^{\infty} \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda k_B T} - 1)} d\lambda$$

We can use the substitution:

$$\frac{hc}{\lambda k_B T} = x \rightarrow \lambda = \frac{hc}{k_B T} \frac{1}{x}$$

$$d\lambda = -\frac{hc}{k_B T} \frac{1}{x^2} dx$$

$$\int_{\lambda=0}^{\infty} \frac{2\pi hc^2}{\left(\frac{hc}{k_B T} \frac{1}{x}\right)^5 (e^x - 1)} \frac{hc}{k_B T} \left(-\frac{1}{x^2}\right) dx$$

$$\lambda = 0 \rightarrow x = \infty; \lambda = \infty \rightarrow x = 0$$

$$\frac{2\pi (K_B T)^T}{h^3 c^2} \int_{x=\infty}^0 \frac{x^3}{e^x - 1} dx$$