PHSX 461: HW09

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Question 1

In strong coupling, the quantum states of photons 'hybridize' with the quantum states of electronic excitations ('excitons') in a material to form a superposition of light and matter quantum states (wow!). A crucial part of demonstrating this hybridization is to show "anti-crossing" behavior in the system. Let's work it out here.

the strong-coupling Hamiltonian is simplified to a two-level system composed of a photon state ($|photon\rangle$) and an exciton state ($|exciton\rangle$). the energy of the photon depends upon its wavelength: $E_p(\lambda) = (hc)/\lambda$(this is called dispersion). For the exciton, the energy is a constant, E_x. the strength of the coupling is usually referred to as Ω (assume it is real and positive). The quantum system then can be formulated as follows:

$$|\text{photon}\rangle \to \mathbf{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |\text{photon}\rangle \to \mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \hat{H} \to H = \begin{pmatrix} E_p(\lambda) & \Omega \\ \Omega & E_x \end{pmatrix}$$

a) *Determine allowed energies of the system (don't worry about the eigenvectors) in terms of $E_p(\lambda), E_x,$ and ω. Call the larger energy E_+ and the lower energy E_-. Note that both depend on λ.*

$$H\mathbf{v} = E\mathbf{v}$$

Where $\mathbf{v}$ is an eigenstate of the system.

$$0 = det \begin{vmatrix} E_p(\lambda) - E & \Omega \\ \Omega & E_x - E \end{vmatrix}$$

$$(E_p(\lambda) - E)(E_x - E) - \Omega^2 = 0$$

$$E^2 - E(E_p(\lambda) + E_x) + E_p(\lambda)E_x - \Omega^2 = 0$$

Using the quadratic formula to solve this:

$$E = \frac{(E_p(\lambda) + E_x) \pm \sqrt{(E_p(\lambda) + E_x)^2 - 4((E_p(\lambda) + E_x) - \Omega^2)}}{2}$$

$$= \frac{(E_p(\lambda) + E_x) \pm \sqrt{(E_p(\lambda))^2 + 2(E_p(\lambda)E_x) + E_x^2 - 4(E_p(\lambda) + E_x - \Omega^2)}}{2}$$

$$= \frac{(E_p(\lambda) + E_x) \pm \sqrt{(E_p(\lambda))^2 + E_x^2 - 2(E_p(\lambda) + E_x) + 4\Omega^2}}{2}$$

$$= \frac{1}{2}(E_p(\lambda) + E_x) \pm \sqrt{\left(\frac{E_p(\lambda) - E_x}{2}\right)^2 + \Omega^2}$$

From this, we get that the eigenstates of the Hamiltonian, and thus the possible energy values, is:

$$\boxed{E_+ = \frac{1}{2}(E_p(\lambda) + E_x) + \sqrt{\left(\frac{E_p(\lambda) - E_x}{2}\right)^2 + \Omega^2}}$$

$$\boxed{E_- = \frac{1}{2}(E_p(\lambda) + E_x) - \sqrt{\left(\frac{E_p(\lambda) - E_x}{2}\right)^2 + \Omega^2}}$$

b) *Assume $E_x = 2.0eV$ and $\Omega = 0.005eV$. Use your favorite plotting software to plot on $E_+(\lambda)$ and $E_-(\lambda)$ over the range $500nm \le \lambda \le 700nm$ on the same plot[1]*

c) *Make a second plot that plots the energies of the energies of the exciton (E_x) and $E_p(\lambda)$ over the same range when $\Omega = 0$ and there is no coupling between the states.*

Notice how in (c) the energies of the exciton and photon intersect when there is no coupling where as in (b) the energies $E_+(\lambda)$ and $E_-(\lambda)$ never intersect when there is coupling! This "avoided crossing" or "anti-crossing" of the levels is characteritic of coupled systems. Note that the minimum separation between the two states is Ω (this splitting is referred to as the 'Rabi splitting'). Mohammad, out TA, is working on this very subject right now!

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[1] **Recall:** $\hbar c = 1240eV{\cdot}nm$

Possible energy ($\Omega = 0.005$eV)

Possible energy ($\Omega = 0$eV)
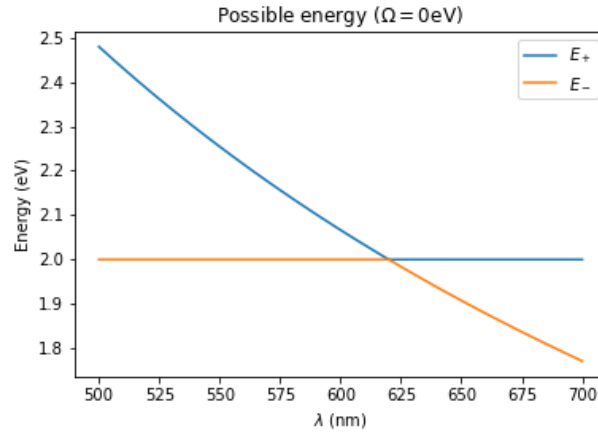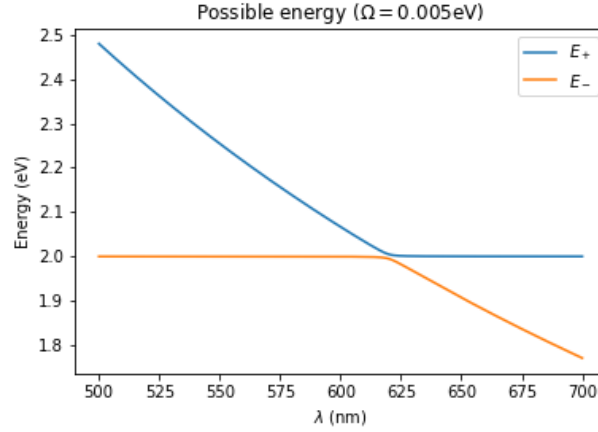
Final note, at $\lambda = 500$nm the $|-\rangle$ state is said to be "exiton-like" and the $|+\rangle$ state is said the be "photon-like" because the superpositions at this energy where $E_p \neq E_x$ are heavily waited to the $|\text{exciton}\rangle$ and $|\text{photon}\rangle$ states, respectively. In contrast, when $E_p = E_x$ the $|-\rangle$ and $|+\rangle$ states are superpositions of equal parts of the $|\text{exciton}\rangle$ and $|\text{photon}\rangle$ states. Work it out if you would like!

This is pretty easy to see from the calculations of E_+ and E_-, as when they are equal the part that differs (the square root) simplifies as: $\sqrt{\Omega^2} = \Omega$. This kinda makes sense, as there is nothing to create a bias in if the state should be the ($|\text{photon}\rangle$) state of the ($|\text{exciton}\rangle$) state. Where when they are not equal, the difference will be nonzero.

Griffiths 3.39

(*no integrals allowed - seriously! use Dirac notation*)

Oh gosh, you're really gonna be testing my patience with the physics package...

- *Find the expressions for $\hat{x}$ and $\hat{p}$ matrix elements* Before we get too far into this, let's remind outselves of a couple vital things:

$$\hat{a}_+\psi_n = \sqrt{n+1}\,\psi_{n+1} \qquad \hat{a}_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

$$\hat{x} = i\sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a}_+ + \hat{a}_-) \qquad \hat{p} = \sqrt{\frac{\hbar m\omega}{2}}\,(\hat{a}_+ - \hat{a}_-)$$

Now we can move onto the meat of the problem, first with $\hat{x}$:

$$\hat{x} = \left\langle \psi_n \left| \sqrt{\frac{\hbar}{2m\omega}}\,(\hat{a}_+ + \hat{a}_-) \right| \psi_{n'} \right\rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\,\langle \psi_n | \hat{a}_+ + \hat{a}_- | \psi_{n'} \rangle$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\langle \psi_n | \hat{a}_+ | \psi_{n'} \rangle + \langle \psi_n | \hat{a}_- | \psi_{n'} \rangle \right]$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\langle \hat{a}_- \psi_n | \psi_{n'} \rangle + \langle \psi_n | \hat{a}_- \psi_{n'} \rangle \right]$$

$$= \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{n}\,\langle \psi_{n-1} | \psi_{n'} \rangle + \sqrt{n'}\,\langle \psi_n | \psi_{n'-1} \rangle \right]$$

$$\boxed{\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\left[\sqrt{n}\,\delta_{n-1,n'} + \sqrt{n'}\,\delta_{n,n'-1} \right]}$$

Following a similar derivation for $\hat{p}$:

$$\hat{p} = \left\langle \psi_n \left| i\sqrt{\frac{\hbar m\omega}{2}}\,(\hat{a}_+ + \hat{a}_-) \right| \psi_{n'} \right\rangle$$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\,\langle \psi_n | \hat{a}_+ - \hat{a}_- | \psi_{n'} \rangle$$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\left[\langle\psi_n|\hat{a}_+|\psi_{n'}\rangle - \langle\psi_n|\hat{a}_-|\psi_{n'}\rangle\right]$$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\left[\langle\hat{a}_-\psi_n|\psi_{n'}\rangle - \langle\psi_n|\hat{a}_-\psi_{n'}\rangle\right]$$

$$= i\sqrt{\frac{\hbar m\omega}{2}}\left[\sqrt{n}\,\langle\psi_{n-1}|\psi_{n'}\rangle - \sqrt{n'}\,\langle\psi_n|\psi_{n'-1}\rangle\right]$$

$$\boxed{\hat{p} = i\sqrt{\frac{\hbar m\omega}{2}}\left[\sqrt{n}\,\delta_{n-1,n'} - \sqrt{n'}\,\delta_{n,n'-1}\right]}$$

- *For each operator, draw a representation of the infinite-dimensional matrix that is described by the expressions above. Describe all zero and non-zero regions of the matrix*

$$\hat{x} \implies \sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots \\ 1 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & \sqrt{3} & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

$$\hat{p} \implies i\sqrt{\frac{\hbar}{2m\omega}}\begin{bmatrix} 0 & -1 & 0 & 0 & \cdots \\ 1 & 0 & -\sqrt{2} & 0 & \cdots \\ 0 & \sqrt{2} & 0 & -\sqrt{3} & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

- *Show that $(1/2m)P^2 + (m\omega^2/2)X^2 = H$* [2]

First we must get the $\hat{x}^2$ and $\hat{p}^2$ matrices:

$$\langle\psi_n|\hat{p}^2|\psi_{n'}\rangle = -\frac{\hbar m\omega}{2}\langle\psi_n|(\hat{a}_+ - \hat{a}_-)^2|\psi_{n'}\rangle$$

$$= -\frac{\hbar m\omega}{2}\left[\langle\psi_n|\hat{a}_+\hat{a}_+\psi_{n'}\rangle - \langle\psi_n|\hat{a}_+\hat{a}_-\psi_{n'}\rangle - \langle\psi_n|\hat{a}_-\hat{a}_+\psi_{n'}\rangle + \langle\psi_n|\hat{a}_-\hat{a}_-\psi_{n'}\rangle\right]$$

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[2] I wasn't sure if we were supposed to do this part too, so I did. Good practice either way.

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$$= -\frac{\hbar m \omega}{2} \Big[\Big\langle \psi_n \Big| \sqrt{n'+1}\sqrt{n'+2}\,\psi_{n'+2} \Big\rangle - \Big\langle \psi_n \Big| \sqrt{n'}\sqrt{n'}\,\psi_{n'} \Big\rangle$$

$$- \Big\langle \psi_n \Big| \sqrt{n'+1}\sqrt{n'+1}\,\psi_{n'} \Big\rangle + \Big\langle \psi_n \Big| \sqrt{n'}\sqrt{n'-1}\,\psi_{n'-2} \Big\rangle \Big]$$

$$= -\frac{\hbar m \omega}{2} \Big[\sqrt{n'+1}\sqrt{n'+2}\,\delta_{n,n'+2} - (2n'+1)\,\delta_{n,n'} + \sqrt{n'}\sqrt{n'-1}\,\delta_{n,n'-2} \Big]$$

$$\hat{p}^2 \implies \frac{\hbar m \omega}{2}
\begin{bmatrix}
1 & 0 & -\sqrt{1\cdot 2} & 0 & \cdots \\
0 & 3 & 0 & -\sqrt{2\cdot 3} & \cdots \\
-\sqrt{1\cdot 2} & 0 & 5 & 0 & \cdots \\
0 & -\sqrt{2\cdot 3} & 0 & 7 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

Through a very similar derivation, there is a:

$$\hat{x}^2 \implies \frac{\hbar}{2m\omega}
\begin{bmatrix}
1 & 0 & \sqrt{1\cdot 2} & 0 & \cdots \\
0 & 3 & 0 & \sqrt{2\cdot 3} & \cdots \\
\sqrt{1\cdot 2} & 0 & 5 & 0 & \cdots \\
0 & \sqrt{2\cdot 3} & 0 & 7 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

So, plugging these into the equation $(1/2m)P^2 + (m\omega^2/2)X^2 = H$:

$$\frac{1}{2m}\frac{\hbar m \omega}{2}
\begin{bmatrix}
1 & 0 & -\sqrt{1\cdot 2} & 0 & \cdots \\
0 & 3 & 0 & -\sqrt{2\cdot 3} & \cdots \\
-\sqrt{1\cdot 2} & 0 & 5 & 0 & \cdots \\
0 & -\sqrt{2\cdot 3} & 0 & 7 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

$$+\frac{m\omega^2}{2}\frac{\hbar}{2m\omega}
\begin{bmatrix}
1 & 0 & \sqrt{1\cdot 2} & 0 & \cdots \\
0 & 3 & 0 & \sqrt{2\cdot 3} & \cdots \\
\sqrt{1\cdot 2} & 0 & 5 & 0 & \cdots \\
0 & \sqrt{2\cdot 3} & 0 & 7 & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}$$

$$= \frac{\hbar\omega}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 3 & 0 & 0 & \cdots \\ 0 & 0 & 5 & 0 & \cdots \\ 0 & 0 & 0 & 7 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

Does this correspond to $\hat{H}$?

$$\langle \psi_n | \hat{H} | \psi_{n'} \rangle$$
$$= \left\langle \psi_n \middle| \hat{H} \psi_{n'} \right\rangle$$
$$= \langle \psi_n | E_{n'} \psi_{n'} \rangle$$
$$= E_{n'} \langle \psi_n | \psi_{n'} \rangle$$
$$= E_{n'} \delta_{n,n'}$$

When we remember that $E_n = \hbar\omega(n + \frac{1}{2})$ for the QHO:

$$\langle \psi_n | \hat{H} | \psi_{n'} \rangle = \frac{\hbar\omega}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 3 & 0 & 0 & \cdots \\ 0 & 0 & 5 & 0 & \cdots \\ 0 & 0 & 0 & 7 & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$

So, it would seem that $(1/2m)P^2 + (m\omega^2/2)X^2 = H$ is satisfied.

Question 3

A particle in a 1D QHO is in the state $|\alpha\rangle = \beta(|0\rangle - 3\,|1\rangle - 2i\,|3\rangle)$ where $|n\rangle$ is the nth stationary state of the !D QHO at time $t = 0$. Consider these operators:

$$\hat{Q} = \hbar\omega \sum_{n=0}^{\infty} \frac{2n+1}{2}(|n\rangle\langle n|) \quad \hat{A} = \sum_{n=0}^{\infty} \sqrt{n+1}(|n+1\rangle\langle n|) \quad \hat{B} = \sum_{n=0}^{\infty} \sqrt{n}(|n-1\rangle\langle n|)$$

a) *Calculate β*

To calculate β, let's first note that

$$\langle\alpha| = \beta^*(\langle 0| - 3\langle 1| + 2i\langle 3|)$$

So, now we can enforce the normalization constraint, that is

$$\langle\alpha|\alpha\rangle = 1$$

$$1 = \beta^*(\langle 0| - 3\langle 1| + 2i\langle 3|)\beta(|0\rangle - 3\,|1\rangle - 2i\,|3\rangle)$$
$$1 = |\beta|^2(\langle 0|0\rangle + 9\langle 1|1\rangle + 4\langle 3|3\rangle)$$
$$1 = |\beta|^2(14)$$
$$\boxed{\beta = \frac{1}{\sqrt{14}}}$$

b) *Find the $\langle Q\rangle$. What is another name for $\hat{Q}$?*

$$\langle Q\rangle = \langle\alpha|\hat{Q}|\alpha\rangle$$

$$= \langle\alpha|\,\hbar\omega \sum_{n=0}^{\infty} \frac{2n+1}{2}\,|n\rangle\langle n|\alpha\rangle$$

$$\boxed{\langle Q\rangle = \frac{\hbar\omega}{2}(1+3+7)\left(\frac{1}{14}\right) = \frac{\hbar\omega}{2}\left(\frac{11}{14}\right)}$$

$\hat{Q}$ is just the Hamiltonian.

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c) *Calculate $\hat{A}|\alpha\rangle$. What is another name for $\hat{A}$?*

$$\hat{A}|\alpha\rangle = \sum_{n=0}^{\infty} \sqrt{n+1}|n+1\rangle\langle n|\alpha\rangle$$

$$= \frac{1}{\sqrt{14}} \sum_{n=0}^{\infty} \sqrt{n+1}|n+1\rangle\langle n|(|0\rangle - 3|1\rangle - 2i|3\rangle)$$

$$= \frac{1}{\sqrt{14}}(\sqrt{1}|1\rangle - 3\sqrt{2}|2\rangle - \sqrt{4}2i|4\rangle)$$

$$\boxed{\hat{A}|\alpha\rangle = \frac{1}{\sqrt{14}}(|1\rangle - 3\sqrt{2}|2\rangle - 4i|4\rangle)}$$

$\hat{Q}$ is just the $\hat{a}_+$.

d) *Calculate $\hat{B}|\alpha\rangle$. What is another name for $\hat{B}$?*

$$\hat{B}|\alpha\rangle = \sum_{n=0}^{\infty} \sqrt{n}|n-1\rangle\langle n|\alpha\rangle$$

$$= \frac{1}{\sqrt{14}} \sum_{n=0}^{\infty} \sqrt{n}|n-1\rangle\langle n|(|0\rangle - 3|1\rangle - 2i|3\rangle)$$

$$= \frac{1}{\sqrt{14}}(\sqrt{0}|-1\rangle - 3\sqrt{1}|0\rangle - \sqrt{3}2i|2\rangle)$$

$$\boxed{\hat{B}|\alpha\rangle = \frac{1}{\sqrt{14}}(-3|0\rangle - 2\sqrt{3}i|2\rangle)}$$

$\hat{Q}$ is just the $\hat{a}_-$.

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Griffiths 3.44 (b)

Note: *remember that to construct $|S(t)\rangle$ you need to multiply each eigenstate in of $\hat{H}$ that makes up the superposition of $|S(0)\rangle$ by the correct wiggle factor. Same stuff as earlier in the semester, now just with matrix representations of $\hat{H}$ and vectors for the states!*

The Hamiltonian for a certain three-level system is represended by the matrix

$$H = \begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{bmatrix}$$

where a, b, and c are real numbers.

a) *if the system starts out in the state*

$$|S(0)\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

what is $|S(t)\rangle$

Since $|S(0)\rangle$ is an eigenstate of H with eigenvalue c, $|S(0)\rangle$ won't evolve in time except for with a "wiggle factor":

$$\boxed{|S(t)\rangle = |S(0)\rangle\, e^{ct/\hbar}}$$

b) *if the system starts out in the state*

$$|S(0)\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

what is $|S(t)\rangle$

Since $|S(0)\rangle$ is not an eigenvector of H, we must find what linear combination of eigenvectors make it up. By looking back at a), we already have one of the

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eigenvectors. And, by educated guess (from the amount of symmetry in the problem), we can also say that

$$\mathbf{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \qquad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

are eigenvectors. To show this is true, let's multiply them out.

$$\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a+b \\ 0 \\ a+b \end{pmatrix} = (a+b) \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0 \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

Conveniently, that also gave the eigenvalues. So, decomposing $|S(0)\rangle$ and applying the "wiggle factor":

$$|S(0)\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{2} \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$$

$$|S(t)\rangle = \frac{1}{2} \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{(a+b)\,t/\hbar} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} e^{0 \cdot t/\hbar} \right)$$

$$\boxed{|S(t)\rangle = \frac{1}{2} \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} e^{(a+b)\,t/\hbar} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)}$$

Griffiths 3.14 (a) and (b)

a) *Prove the commutator identities*

$$\left[\hat{A} + \hat{B}, \hat{C}\right]$$

$$= (\hat{A} + \hat{B})(\hat{C}) - (\hat{C})(\hat{A} + \hat{B})$$
$$= \hat{A}\hat{C} + \hat{B}\hat{C} - \hat{C}\hat{A} - \hat{C}\hat{B}$$
$$= (\hat{A}\hat{C} - \hat{C}\hat{A}) + (\hat{B}\hat{C} - \hat{C}\hat{B})$$

$$\boxed{\left[\hat{A} + \hat{B}, \hat{C}\right] = \left[\hat{A}, \hat{C}\right] + \left[\hat{B}, \hat{C}\right]}$$

$$\left[\hat{A}\hat{B}, \hat{C}\right] = \hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}$$

Perhaps this one is easier going the other direction

$$\hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B}$$

$$= \hat{A}(\hat{B}\hat{C} - \hat{C}\hat{B}) + (\hat{A}\hat{C} - \hat{C}\hat{A})\hat{B}$$
$$= \hat{A}\hat{B}\hat{C} - \hat{A}\hat{C}\hat{B} + \hat{A}\hat{C}\hat{B} - \hat{C}\hat{A}\hat{B}$$
$$= \hat{A}\hat{B}\hat{C} - \hat{C}\hat{A}\hat{B}$$

$$\boxed{\hat{A}\left[\hat{B}, \hat{C}\right] + \left[\hat{A}, \hat{C}\right]\hat{B} = \left[\hat{A}\hat{B}, \hat{C}\right]}$$

b) *Show that*

$$[x^n, \hat{p}] = i\hbar n x^{n-1}$$

Recall that $\hat{p} = -i\hbar\frac{\partial}{\partial x}$

$$[x^n, \hat{p}] = x^n\hat{p} - \hat{p}x^n$$
$$= x^n\hat{p} \cdot 1 - \hat{p}x^n$$
$$= x^n - i\hbar\frac{\partial}{\partial x}1 - \left(-i\hbar\frac{\partial}{\partial x}\right)x^n$$
$$= 0 + i\hbar n x^{n-1}$$

$$\boxed{[x^n, \hat{p}] = i\hbar n x^{n-1}}$$