Modern Homework 19 William Jardee

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1. a) $f(x) = Ae^{ik_0x} e^{-\alpha|x|}$

+3

$$a(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Ae^{ik_0x} e^{-\alpha|x|} e^{-ikx}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} Ae^{i(k_0-k)x - \alpha|x|}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_0^{\infty} Ae^{i(k_0-k)x - \alpha x}\, dx + \int_{-\infty}^{0} Ae^{i(k_0-k)x + \alpha x}\, dx \right]$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{1}{i(k_0-k)-\alpha} e^{i(k_0-k)x - \alpha x}\Big|_0^{\infty} + \frac{1}{i(k_0-k)+\alpha} e^{i(k_0-k)x + \alpha x}\Big|_{-\infty}^{0} \right]$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{1}{i(k_0-k)-\alpha} e^{-\alpha x}\left(\cos(k_0-k)x + i\sin(k_0-k)x\right)\Big|_0^{\infty} + \frac{1}{i(k_0-k)+\alpha} e^{\alpha x}\left(\cos(k_0-k)x + i\sin(k_0-k)x\right)\Big|_{-\infty}^{0} \right]$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{1}{i(k_0-k)-\alpha}\left[(0-1)\right] + \frac{1}{i(k_0-k)+\alpha}(1-0) \right]$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{1}{i(k_0-k)+\alpha} - \frac{1}{i(k_0-k)-\alpha} \right] = \frac{A}{\sqrt{2\pi}} \left[\frac{\alpha - i(k_0-k)}{\alpha^2 + (k_0-k)^2} - \frac{-\alpha - i(k_0-k)}{\alpha^2 + (k_0-k)^2} \right]$$

$$= \frac{A}{\sqrt{2\pi}} \left[\frac{2\alpha}{\alpha^2 + (k_0-k)^2} \right] = \frac{2A\alpha}{\sqrt{2\pi}\,(\alpha^2 + (k_0-k)^2)}$$

[Purely real]

b)

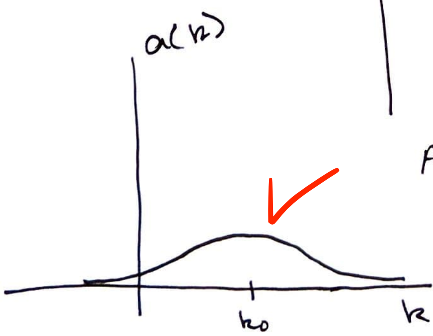
$$|f(x)|^2 = A^2 e^{-2\alpha|x|}$$

$$|a(k)|^2 = \frac{2A^2\alpha^2}{\pi^2(\alpha^2 + (k_0-k)^2)^2}$$

k_0 shifts $a(k)$ to the right

α changes the sharpness of $f(x)$ and $a(k)$, it also changes the height, by both the numerator and denominator

A just changes the amp for both.



c) $|f(x_{half})|^2 = \frac{1}{2}A^2 = A^2 e^{-2\alpha|x|}$

$\frac{1}{2} = e^{-2\alpha|x|}$

$-2\alpha|x| = \ln|1/2|$

$|x| \Rightarrow x = \frac{1}{\alpha}\ln(4)$

$\boxed{\Delta x = \frac{1}{\alpha}\ln(4)}$

$|a(k_{half})|^2 = \frac{A^2\alpha^2}{\pi^2(\alpha^2+(k_0)^2)^2} = \frac{A^2 2\alpha^2}{\pi^2(\alpha^2+(k_0-k)^2)^2}$

$(\alpha^2+(k_0-k)^2)^2 = 2(\alpha^2+k_0^2)^2$

$(k_0-k)^2 = \sqrt{2}((1-\sqrt{2})\alpha^2+k_0^2)$

$k = k_0 - \sqrt[4]{2}((1-\sqrt{2})\alpha^2+k_0^2)^{1/2}$

$\boxed{\Delta k = k_0 - \sqrt[4]{2}((1-\sqrt{2})\alpha^2+k_0^2)^{1/2}}$

$\Delta x \Delta k = \frac{1}{\alpha}\ln(4)\left[k_0 - \sqrt[4]{2}((1-\sqrt{2})\alpha^2+k_0^2)^{1/2}\right]$

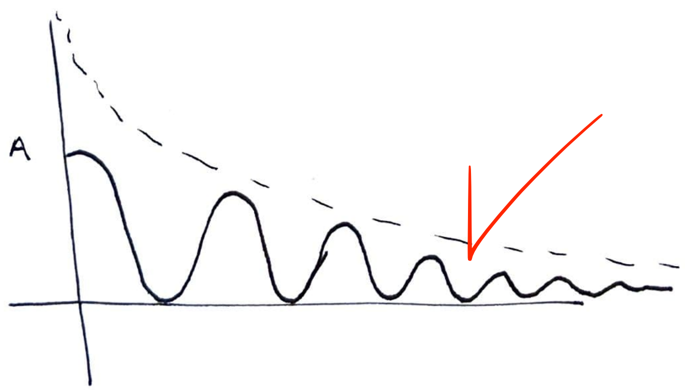
D) $g(x) = \frac{1}{2}\left[A e^{ik_0 x} e^{-\alpha|x|} + A e^{-ik_0 x} e^{-\alpha|x|}\right]$

$= \frac{A}{2} e^{-\alpha|x|}\left[\cos(k_0 x) + i\sin(k_0 x) + \cos(k_0 x) - i\sin(k_0 x)\right]$

$= \frac{A}{2} e^{-\alpha|x|}\left[2\cos(k_0 x)\right] = A e^{-\alpha|x|}\cos(k_0 x)$

the modulus squared is equal to $|g(x)|^2$

$|g(x)|^2 = A e^{-2\alpha|x|}\cos^2(k_0 x)$



Fourier transform?

2) For the set up

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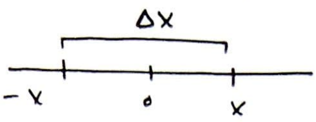


$\Delta x = 10^{-15}$ m

$$P = \frac{h}{\lambda} = \frac{h}{2x} = \frac{h}{\Delta x} = \frac{1}{c}\sqrt{(k + mc^2)^2 - (mc^2)^2}$$

$$\left(\frac{hc}{\Delta x}\right)^2 = (k + mc^2)^2 - (mc^2)^2$$

$$k = \sqrt{\left(\frac{hc}{\Delta x}\right)^2 + (mc^2)^2} - (mc^2)$$

But we have to account for uncertainty

$$\Delta x \, \Delta P \geq \frac{\hbar}{2}$$

$$\Delta P \geq \frac{\hbar}{2\Delta x}$$

We will treat m + c as known quanities

$$\frac{1}{c}\sqrt{(\Delta k + mc^2)^2 - (mc^2)^2} \geq \frac{\hbar}{2\Delta x} = \frac{h}{4\pi \Delta x}$$

$$(\Delta k + mc^2)^2 - (mc^2)^2 \geq \left(\frac{hc}{4\pi \Delta x}\right)^2$$

$$\Delta k \geq \sqrt{\left(\frac{hc}{4\pi\Delta x}\right)^2 + (mc^2)^2} - mc^2$$

So the minimum kinetic energy is

$\cancel{k-}\Delta k$ Just Δk

a) neutron: m = 938.27 MeV

$k - \Delta k = 937$ MeV → ~~900~~ MeV

20 MeV

The neutron is more massive, so it will need less kinetic energy to have the same momentum.

b) electron: m = 0.511 MeV

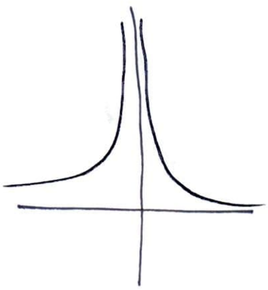
$k - \Delta k = 114$ MeV → ~~100~~ MeV

200 MeV

c) With our current nstraint, No. But if Δx was expanded ~~to~~ ~~the~~ the quation (removing a lot of ~~variables~~ constants) becomes:

$$k - \Delta k = \sqrt{\left(\frac{1}{x}\right)^2 - a} - \sqrt{\left(\frac{1}{\Delta x}\right)^2 - a}$$

if Δx is small enough then the graph goes to



The asymptotic behavior means the value can go small enough if the Δx is small enough, since the range of ΔP, from $\Delta x \Delta P \geq \frac{\hbar}{2}$ has to compensate with the knowledge of Δx