Modern Homework 23

8/8
Excellent

1. a) $\psi_1(x) = C_1 e^{-m\omega x^2/2\hbar}(2x)$

+4

Normalization:

$$1 = \int_{-\infty}^{\infty} \left(C_1(2x) e^{-m\omega x^2/2\hbar}\right)^2 dx = \int_{-\infty}^{\infty} C_1^2 (4x^2) e^{-m\omega x^2/\hbar}\, dx$$

there is a symmetry about $x=0$

$$1 = 8C_1^2 \int_0^{\infty} x^2 e^{-m\omega x^2/\hbar}\, dx$$

using the probabilities integral page; $n=2$, $\lambda = \frac{m\omega}{\hbar}$

$$1 = 8C_1^2 I_2 = 8C_1^2\left(\tfrac{1}{4}\pi^{1/2}\lambda^{-3/2}\right) = 8C_1^2\left(\tfrac{1}{4}\pi^{1/2}\left(\tfrac{\hbar}{m\omega}\right)^{3/2}\right)$$

$$\boxed{C_1 = \left(\frac{m\omega}{\hbar}\right)^{3/4}\frac{1}{\sqrt{2}\,\pi^{1/4}}}$$

$$\psi_1(x) = \left(\frac{m\omega}{\hbar}\right)^{3/2}\frac{\sqrt{2}}{\pi^{1/4}}\, x\, e^{-m\omega x^2/2\hbar}$$

b) to keep the math simple I will keep C_1 written as C_1

$$\frac{d^2\psi}{dx^2} = \frac{d}{dx}\left[2C_1\left(e^{-m\omega x^2/2\hbar} - x\left(\frac{m\omega}{2\hbar}2x\right)e^{-m\omega x^2/2\hbar}\right)\right]$$

$$= \frac{d}{dx}\left[2C_1\left(e^{-m\omega x^2/2\hbar} - \frac{m\omega}{\hbar}x^2 e^{-m\omega x^2/2\hbar}\right)\right]$$

$$= 2C_1\left(-\frac{m\omega}{\hbar}x - \frac{2m\omega}{\hbar}x + \frac{m^2\omega^2}{\hbar^2}x^3\right)e^{-m\omega x^2/2\hbar}$$

$$= 2C_1\left(-\frac{3m\omega}{\hbar}x + \frac{m^2\omega^2}{\hbar^2}x^3\right)e^{-m\omega x^2/2\hbar}$$

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (U(x) - E)\psi$$

$$\frac{\hbar^2}{2m}\left(2C_1\left(-\frac{3m\omega}{\hbar}x + \frac{m^2\omega^2}{\hbar^2}x^3\right)e^{-m\omega x^2/2\hbar}\right) = \left(\tfrac{1}{2}m\omega^2x^2 - E\right)\left(2C_1 e^{-m\omega x^2/2\hbar}\right)$$

$$\frac{\hbar^2}{2m}\left(-\frac{3m\omega}{\hbar} + \frac{m^2\omega^2}{\hbar^2}x^2\right) = \left(\tfrac{1}{2}m\omega^2x^2 - E\right)$$

$$-\frac{3}{2}\omega\hbar + \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2x^2 - E$$

$$+\frac{3}{2}\omega\hbar = \underbrace{\left(1+\tfrac{1}{2}\right)}_{n+\frac{1}{2}\ \text{if}\ n=1}\omega\hbar = E$$

2. $\frac{\Delta E_n}{E_n} =$

+4

$$\frac{E_n - E_{n-1}}{E_n} = \frac{(n+\frac{1}{2})\hbar\omega - ((n-1)+\frac{1}{2})\hbar\omega}{(n+\frac{1}{2})\hbar\omega} = \frac{1}{n+\frac{1}{2}}$$

as $n \to \infty$, $\frac{\Delta E_n}{E_n} \to 0$

So as we get to large energies the resolution becomes nearly perfect, a condition required in Classical physics