Modern Homework 21 William Jardee 7/8

1. +3

a) $U(x)\psi(x) = E\psi(x) + \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$

We will use the $\psi(x)$ equation instead of the $\Psi(x,t)$ equation since the probability distributions are the same and the $\psi(x)$ given is t independent.

$$U(x) = E + \frac{\hbar^2}{2m}\frac{1}{\psi(x)}\frac{d^2\psi}{dx^2}$$

$$A\frac{d}{dx}\left(-\frac{x}{L^2}e^{-x^2/2L^2}\right) = A\left(-\frac{1}{L^2}e^{-x^2/2L} + \frac{x^2}{L^4}e^{-x^2/2L^2}\right)$$

$$U(x) = E + \frac{\hbar^2}{2mA}e^{x^2/2L^2}\left(A\left(-\frac{1}{L^2}e^{-x^2/2L} + \frac{x^2}{L^4}e^{-x^2/2L^2}\right)\right)$$

$$= \frac{\hbar^2}{2mL^2} + \frac{\hbar^2}{2m}\left(-\frac{1}{L^2} + \frac{x^2}{L^4}\right) = \frac{\hbar^2 x^2}{2mL^4}$$

Sketch of the potential?

For b) it will also be helpful to have

$$U(x) = \frac{\hbar^2}{2mL^2} + \frac{\hbar^2}{2mL^4}(x^2 - L^2)$$

b) $k = E - U = \frac{\hbar^2}{2mL^4}(x^2 - L^2)$

The "turn around points" will be when $V=0$, so $k=0$

$$0 = \frac{\hbar^2}{2mL^4} \rightarrow 0 = x^2 - L^2$$

$$\boxed{x = \pm L}$$

c) if we say $E = \frac{1}{2}\hbar\omega = \frac{1}{2}\frac{\hbar^2}{mL^2}$

$$\omega = \frac{\hbar}{mL^2}$$

$$U(x) = \frac{1}{2}m\frac{\hbar^2}{m^2L^4}x^2 = \frac{1}{2}m\omega^2x^2$$

Ain't that some cute SHM

Don't you love a good oscillator?

2. a) +4

This is a tricky question. No x will be more likely, since each has a probability of zero to be found, the width is infinitesimally small at a point.

But the center of the $\psi(x)$ of a simple e^{-x^2} will be at $x=0$, so that will be the densest region as we will see.

b) $P(x)dx = |\psi(x)|^2dx = A^2e^{-2x^2/2L}dx$

i) $x=0 \rightarrow P(x)dx = A^2dx$

ii) $x=L \rightarrow P(x)dx = A^2/e\,dx$

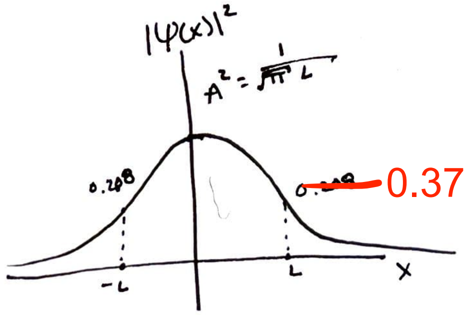
iii) $x=2L \rightarrow P(x)dx = A^2/e^4\,dx$

c) $1 = A^2\int_{-\infty}^{\infty}e^{-x^2/L^2}dx \xrightarrow{\text{even}} 1 = 2A\int_0^{\infty}e^{-x^2/L^2}dx$

$$\frac{1}{2A^2} = \int_0^{\infty}e^{-x^2/L^2}dx = I_0 = \frac{1}{2}\pi^{1/2}\lambda^{-1/2}$$

$$= \frac{1}{2}\pi^{1/2}(L)$$

$$A = \sqrt{\frac{1}{\sqrt{\pi}L}}$$



D) let us take the case $L=1$, then the area (roughly) can be estimated by a triangle from $x=1$ to $x=2$ and height 0.208. Doubling this to include neg x direction.

$$P(x) = 2(A_\Delta) = 2\left(\frac{1}{2}(0.208)(1)\right) = .208 = 0.2$$

this measures up "okay" to an integral calculator that gave $0.157 \approx 0.2$

My value much over estimated, as to be expected from the picture made by Desmos.