

PHSX 343: Assignment 7

William Jardee

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Problem 1

I guess this is the quick and dirty way of doing it. It's simple to do it with algebra.

Using Lorentz Translations, just replacing the variable with deltas:

$$\Delta x' = \gamma(\Delta x - v\Delta t) \quad \Delta t' = \gamma\left(\Delta t - \frac{v}{c}\Delta x\right)$$

- a) • If the events are simultaneous, but not collocated, $t' = 0$ and thus:

$$\Delta t' = 0 = \gamma\left(\Delta t + \frac{v}{c}\Delta x\right)$$

$$\Delta t = -\frac{v}{c}\Delta x$$

- If $x < c\Delta t$, as would be true if $v > c$, then $\Delta t'$ could be negative. This means that the event ran in reverse and would violate the first law of thermodynamics. As we know from the first postulate of relativity, that the laws of physics are consistent between any frame. ✓

- b) • If $\Delta x \leq c\Delta t$, then:

$$\Delta t' \geq \gamma\left(c\Delta t + \frac{v}{c}\Delta x\right) = \gamma\left(c + \frac{v}{c}\right)\Delta t$$

Doesn't quite work because you mangled the initial equation.

So, if $v > 0$, then $\Delta x > 0$, and consequently $\Delta t' \geq 0$

- If the object is moving in the negative x direction, when $\frac{v}{c}\Delta x < \Delta t$, then the $\Delta t' < 0$. Then it could be a logical jump that Event B caused Event A if the sign of the change was the only thing that mattered. This is an obvious contradiction to what the other frame saw, and the law of physics have to stay consistent (first postulate of relativity), so this cannot be the case.

- c) •

$$\Delta t' = \gamma\left(\Delta t + \frac{v\alpha}{c}\Delta x\right) = \gamma\left(1 + \frac{v\alpha}{c}\right)\Delta t < 0$$

The sign of the time difference isn't enough information to determine causality. You need more info about the events themselves.

More fundamentally, super-luminal information transfer causes frames to disagree on causality, which breaks the first postulate.

$$\frac{v\alpha}{c} < -1 \rightarrow v < \cancel{\frac{c}{\alpha}}$$

We can restrict our solution to this one answer because we know the velocity moves in the $+x$ direction

- We know from the Lorentz Transforms that $t' = \gamma t$. So, if $v > c$ then γ is imaginary and we get an imaginary time. We cannot have an imaginary time for a real particle in this setup, so if we take that a particle can travel faster than the speed of light then we get a time that doesn't make sense with the laws of physics. So we get a contradiction with the first law of relativity.

Problem 2

- +4** a) The Lorentz Transform will use the frame of A as un-primed, since both B and C are moving in the $-x$ direction for it. So the equation for velocity gives:

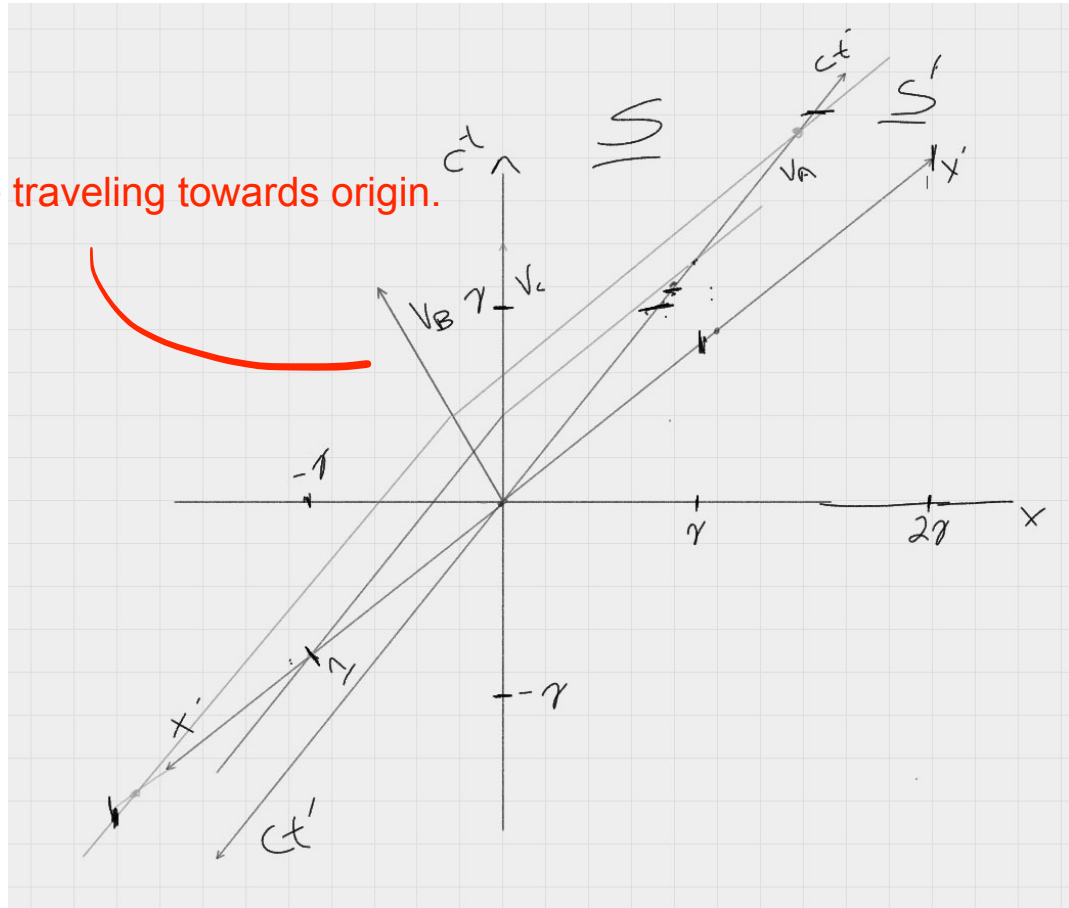
$$V_x = \frac{V'_x + \beta c}{1 + \beta \frac{V'_x}{c}}$$

$$\text{For } V_B = \frac{V'_B + \beta c}{1 + \beta \frac{V'_B}{c}} \rightarrow \frac{0.8c + \sqrt{1-0.8^2}c}{1 + \sqrt{1-0.8^2}0.8} = 0.946c \quad \checkmark$$

$$V_C = \frac{0 + \beta c}{1} = 0.8c \quad \checkmark$$

Should be traveling towards origin.

b)



Estimating the values, $\Delta x_B = -1.9\gamma$ and $\Delta t_B = 1.95\gamma$, so $v_B = 0.974c$.
 $\Delta x_C = 1\gamma$ and $\Delta t_C = 1.2\gamma$, so $v_C = 0.83c$. These numbers are moderately close to those given in part a. ✓

c) For the lab's frame, we get that:

$$\Delta p_l = (mv_{iA} + mv_{iB}) - mv_{fc} = m(0.8c - 0.8c) - m(0) = 0 \quad \checkmark$$

So the momentum is conserved.

Now for A's frame:

$$\Delta p_A = m(v_{iA}) + mv_{iB} - 2mv_{fC} = m(-0.946 - 0.8)c \neq 0 \quad \checkmark$$

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So the momentum is not conserved in A's frame.