HW14 9/12

At this point I think I'm proficient in latex and really didn't feel like typing this one out. Sorry?

Well at least LaTeX had readable contrast levels…

1) $hf = z^2 E_0 \left| \frac{1}{n^2} - \frac{1}{n'^2} \right|$ the largest wavelength will be the lowest energy of the Lynman (ends at 1) series.

+2

$$E_0 \left| \frac{1}{4} - 1 \right| = E = 9 E_0 \left| \frac{1}{n^2} - \frac{1}{n'^2} \right|$$

$$\frac{1}{9} \frac{3}{4} = \left| \frac{1}{n^2} - \frac{1}{n'^2} \right| = \frac{1}{12}$$

Now plugging these values into Desmos and playing around, I got what n and n' are.

$$0 = y = \frac{1}{\text{floor}(x)^2} - \frac{1}{\text{floor}(n)^2} - \frac{1}{12}$$

With this I found $x = 6$, $n = 4$

So the $\boxed{n=6 \text{ to } n=\cancel{4}}$ 3 transition will provide the same requency

Same time, but now with $n=3$ to $n=1$ for hydrogen

$$E_0 \left| \frac{1}{9} - 1 \right| = E = 9 E_0 \left| \frac{1}{n^2} - \frac{1}{n'^2} \right|$$

$$\frac{8}{81} = \frac{1}{n^2} - \frac{1}{n'^2}$$

we find $x = 27$ and $n = 9$ (Big jump!)

this is for an exact match

I think something went wrong with your square roots?

So the $\boxed{n=2\cancel{7} \text{ to } n=9}$ 3 transition will provide the same frequency as the 2nd hydrogen Lynman line

The Energy jumps are ~~larger~~ for hydrogen, so hydrogen will not see an energy jump equal to the $n=2$ to $n=1$ jump of Lithium

Lithium has a much deeper energy well, so the jumps for that atom are much larger.

2) Chapter 4, 37.

+3

Pulling from the book:

$E = \frac{ke^2}{2r}$ $\qquad r = n^2 a_0$

$= \frac{ke^2}{2a}\left(\frac{1}{n^2}\right) = \frac{ke^2}{2\left(\frac{n^2h^2}{(2\pi)^2 m_e k e^2}\right)}\left(\frac{1}{n^2}\right) = \frac{k^2e^4(2\pi^2)m_e}{n^2h^2}$

$hf = E_f - E_i = \frac{k^2e^4(2\pi^2)m_e}{h^2}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$

$f = \frac{k^2e^4(2\pi^2)m_e}{h^3}\left[\frac{n^2-(n-1)^2}{(n-1)^2(n)^2}\right] = \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{n^2(n-1)^2}\right]$

from lecture, we know that classically we want

$f = \frac{e}{2\pi}\sqrt{\frac{k}{m_e}}\, r^{-3/2}$ so let's get it

$\lim_{n\to\infty} \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{n^2(n-1)^2}\right] = \lim_{n\to\infty} \frac{2\pi^2 m_e k^2 e^4}{h^3}\left[\frac{2n-1}{n^4+\dots}\right] = \frac{2\pi^2 m_e k^2 e^4}{n^3}\left[\frac{1}{n^3}\right]$

$= \frac{e}{2\pi}\sqrt{\frac{k}{m_e}}\left(\frac{(2\pi)^3}{h^3} m_e^{3/2} k^{3/2} e^3\right)\left[\frac{1}{n^3}\right]$

$= \frac{e}{2\pi}\sqrt{\frac{k}{m_e}}\left(\frac{m_e e^2 k}{\hbar^2 n^2}\right)^{3/2} = \frac{e}{2\pi}\sqrt{\frac{k}{m_e}}\left(\frac{1}{r^{3/2}}\right) = \frac{e}{2\pi}\sqrt{\frac{k}{m_e}}\, r^{-3/2}$

$\rightarrow r = \frac{n^2\hbar^2}{m_e e^2 k}$ was in the book, I assume it's free to use.

The problem told you to derive the classical frequency from centripetal motion.

3) Chapter 4, 38

+4

a) Start with kinetic equations

$$\frac{1}{2} m_e V_i^2 = \frac{1}{2} M V_{mf}^2 + \frac{1}{2} m_e V_{ef}^2$$

$$m V_i = M V_{mf} - m_e V_{ef}$$

Now for some Physics 1 fun:

$$m_e V_i^2 = M V_{mf}^2 + m_e \left(\frac{M}{m_e} V_{mf} - V_i\right)^2$$

$$= M V_{mf}^2 + m_e \left(\left(\frac{M}{m_e}\right)^2 V_{mf}^2 - 2\frac{M}{m_e} V_{mf} V_i + V_i^2\right)$$

$$0 = M V_{mf}^2 + \frac{M^2}{m_e} V_{mf}^2 - 2M V_{mf} V_i$$

$$2M V_i = M V_{mf} + \frac{M^2}{m_e} V_{mf}$$

$$V_i = \frac{1}{2M}\left(M + \frac{M^2}{m_e}\right) V_{mf} = \frac{1}{2}\left(1 + \frac{M}{m_e}\right) V_{mf} = \frac{m_e + M}{2m_e} V_{mf}$$

$$V_{mf} = \frac{2m_e}{m_e + M} V_i$$ ✓

Now we can use this derivation in the problem.

$$\frac{\Delta k}{k} = \frac{\frac{1}{2} m V_{ef}^2 - \frac{1}{2} m V_i^2}{\frac{1}{2} m V_i^2} = \frac{\frac{1}{2} M V_{mf}^2}{\frac{1}{2} m_e V_i^2} = \frac{M V_{mf}^2}{m_e V_i^2}$$

$$= \frac{M\left(\frac{2m_e}{m_e + M}\right)^2 V_i^2}{m_e V_i^2} = \frac{M}{m_e}\left(\frac{4 m_e^2}{(m_e + M)^2}\right) = \frac{4 M m_e}{(m_e + M)^2} = \frac{4M}{\frac{1}{m_e^2}(m_e + M)^2}$$

$$\frac{\Delta k}{k} = \frac{4M}{\left(1 + \frac{M}{m_e}\right)^2}$$ ✓

Mass of mercury: $\frac{g}{mol} \; \frac{mol}{atom} \; \frac{MeV/c^2}{g}$

$200.59\,(1.602 \times 10^{23})(1.7827 \times 10^{-27})$

Or better yet, google gave me

$1\ amu = 931.494\ MeV/c^2$

$M = 186848\ MeV/c^2 = 186.848\ GeV/c^2$

$m_e = 0.511\ MeV/c^2$

$M >> m_e$, so let's simplify

$$\frac{\Delta k}{k} = \frac{4M}{\left(1+\frac{M}{m_e}\right)^2} = \frac{4 m_e M}{(m_e+M)^2} = \frac{4m_eM}{M^2\left(\frac{m_e}{M}+1\right)^2} = \frac{4m_e}{M\left(\frac{m_e}{M}+1\right)^2}$$

$$\frac{m_e}{M} \to 0$$

$$\frac{\Delta k}{k} \approx \frac{4m_e}{M} \Rightarrow 0.000010939 \, \cancel{\text{MeV}} \; \boxed{= 10.9 \, \cancel{\text{eV}}}$$

No units, this is a dimensionless ratio.