PHSX 461: HW07

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3.7

a) *Suppose that $f(x)$ and $g(x)$ are two eigenfunctions of an operator $\hat{Q}$, with the same eigenvalue q. Show than any linear combination of f and g is itself an eigenfunction of $\hat{Q}$, with eigenvalue q.*

b) *Check that $f(x) = exp(x)$ and $g(x) = exp(-x)$ are eigenfunctions of the operator d^2/dx^2, with the same eigenvalue. Construct two linear combinations of f and g that are orthogonal eigenfunctions on the interval $(-1, 1)$.*

3.9

a) *Cite a Hamiltonian from Chapter 2 (other than the harmonic oscillator) that has only a discrete spectrum.*

b) *Cite a Hamiltonian from Chapter 2 (other than the free particle) that has only a continuous spectrum.*

c) *Cite a Hamiltonian from Chapter 2 (other than the finite square well) that has both a discrete and a continuous part to its spectrum.*

3.13

Show that

$$\langle x \rangle = \int \Phi^* \left(i\hbar \frac{\partial}{\partial p} \right) \Phi \, \mathrm{d}p$$

Notice that $x \, exp(ipx/\hbar) = -i\hbar(\partial/\partial p) exp(ipx/\hbar)$, *and use Equation 2.147. In momentum space, then, the position operator is* $i\hbar \, \partial/\partial p$. *More generally,*

3.26

Consider a three-dimensional vector space spanned by an orthonormal basis $|1\rangle$, $|2\rangle$, $|3\rangle$. *Kets* $|\alpha\rangle$ *and* $|\beta\rangle$ *are given by*

$$|\alpha\rangle = i\,|1\rangle - 2\,|2\rangle - i\,|3\rangle\,, \qquad |\beta\rangle = i\,|1\rangle + 2\,|3\rangle$$

a) *Construcct* $\langle\alpha|$ *and* $\langle\beta|$ *(in terms of the dual basis* $\langle 1|$, $\langle 2|$, $\langle 3|$*).*

b) *Find* $\langle\alpha|\beta\rangle$ *and* $\langle\beta|\alpha\rangle$, *and confirm that* $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$.

c) *Final all nine matrix elements fo the operator* $\hat{A} = |\alpha\rangle\langle\beta|$, *in this basis, and construct the matrix* A. *Is it hermitian?*

Question 5.

Prove that the momentum operator, $\hat{p}$ is Hermitian.
Hint: *you will need to assume that any functions you use are normalizable. You may also use the results from the previous homework assignment.*

3.33

An operator $\hat{A}$, representing observable A, has two (normalized) eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two (normalized) eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1 = (3\phi_1 + 4\phi_2)/5, \qquad \psi_2 = (4\phi_1 - 3\phi_2)/5$$

a) *Observable A is measured, and the value a_1 is obtained. What is the state of the system (immediately after this measurement?*

b) *If B is now measure, what are the possible results, and what are their probabilities?*

c) *Right after the measurement of B, A is measured again. What is the probability of getting a_1? (Note that the answer would be quite different if I had told you the outcome of the B measurement.*