

PHSX 343: Assignment 9

William Jardee

+4 Problem 1

- a) With conservation of momentum, not all the energy of the photon will go into the mass of the particle. $0.01Mc^2$ will go into the mass, but another batch will go in to bring the momentum of the particle up to the initial momentum of the photon. ✓

b)

$$E_i = E_{Mi} + E_p = Mc^2 + P_pc$$

$$E_f = E_{Mf} = \sqrt{(P_pc)^2 + (1.01Mc^2)^2}$$

For conservation of momentum:

$$E_i = E_f \rightarrow (Mc^2) + (P_pc) = \sqrt{(P_pc)^2 + (1.01Mc^2)^2}$$

$$(Mc^2)^2 + 2(Mc^2)(P_pc) + (P_pc)^2 = (P_pc)^2 + (1.01Mc^2)^2$$

$$(1.01^2 - 1)(Mc^2)^2 = 2(mc^2)(P_pc)$$

$$\frac{1.01^2 - 1}{2} Mc^2 = P_pc = E_p = 0.01005Mc^2 \quad \checkmark$$

Problem 2

- +4 a) For the conservation of 4-momentum:

For the energy of the photon, $E^2 = (Pc)^2 + (Mc^2)^2 = (Pc)^2$ and for $E = P_tc$.

$$\begin{bmatrix} E_1/c \\ P_x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} E_2/c \\ -P_x \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_p \\ P_p \\ 0 \\ 0 \end{bmatrix}$$

But it's evident that $P_x - P_x = 0 = P_p$, so the photon would have no momentum and have no energy, so there would be no photon actually created for a valid formula. ✓

- b) This time the 2nd mass will have no momentum:

$$\begin{bmatrix} E_1/c \\ P_x \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} Mc \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_p \\ P_p \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{E_1}{c} = \frac{\sqrt{(P_x c^2)^2 + (Mc^2)^2}}{c} = \sqrt{P_x^2 + (Mc)^2}$$

With conservation of momentum we can say that this P is the same as the momentum of the photon: $P_x = P_p$.

$$\frac{E_1}{c} + Mc = \sqrt{P_p^2 + (Mc)^2} + Mc = P_p$$

This is only true when $M = 0$. With that condition, we are not dealing with ~~elementary~~ particles, so this situation brings up a contradiction and is invalid. ✓

- c) If we make the situation abstract, for any starting energy and momentum (kept in 1D), then:

$$\begin{bmatrix} E_1/c \\ P_{x1} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} E_2/c \\ P_{x2} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} P_p \\ P_p \\ 0 \\ 0 \end{bmatrix}$$

Simpler than that: Just invoke the 1st postulate.

If we balance both the P_t and P_x equations:

$$\sqrt{P_{x1}^2 + (Mc)^2} + \sqrt{P_{x2}^2 + (Mc)^2} = P_p = P_{x1} + P_{x2} \quad \checkmark$$

This is only true when $(Mc)^2 = 0 \rightarrow M = 0$. This is the same issue as part ~~b~~ b

- d) Equality is a symmetric operation, so any of these derivations can be done in reverse and it remains illegal in that direction. ✓