Name: ________________________ 65 points **Show all work.**

1. (10) The star Tau Ceti is 12 light-years from our solar system. We depart our solar system on a spaceship at a constant speed of 0.80 c, headed directly toward Tau Ceti. When we have completed one-third of the journey, the light from the explosions of both our Sun and Tau Ceti arrive in our detectors.

   Construct a qualitatively accurate, spacetime-diagram depicting all of these occurrences, and use it to address two questions. (a) Which star exploded first according to observers in the frame in which both stars are at rest? (b) Can one explosion be the cause of the other?

2. (5) Describe one experiment that provides evidence in support of the de Broglie hypothesis of the wave nature of massive particles, including quantitative details.

3. (10) A photon of energy 12.40 keV is traveling in the $+x$ direction and collides with an electron with a speed 0.9500 c traveling in the $-x$ direction. After the collision, the photon has energy 371.12 keV and is traveling in the $-x$ direction. Evaluate the final kinetic energy, momentum, and velocity of the electron.

   This process is referred to as *Inverse Compton Scattering*, because the photon *gains* energy. Do you expect that a proton, of the same speed as the electron described here, would impart more or less energy to the photon?

4. A particle of mass m is a one-dimensional, quantum oscillator, described at time $t = 0$ by the normalized wave-packet

$$\Psi(x,0) = \begin{cases} 0 & \text{for } x \leq 0 \\ 2\sqrt{\frac{3}{x_\circ}}\, e^{-x/x_\circ}\left(1 - e^{-x/x_\circ}\right) & \text{for } x > 0, \end{cases}$$

where $x_\circ$ is the classical turning point for the ground state.

(a) (10) Evaluate the uncertainty in the position of the particle, and use it to determine the minimum uncertainty in the momentum of the particle. You need not carry out the calculations to completion, but everything must be fully specified.

Explain why it is critical that the wave function is normalized.

(b) (5) Is the uncertainty in the energy of $\Psi(x,0)$ zero? Justify.

5. (10) A particular excited state of the ^{4}He nucleus at rest exists for 10^{-18} s on average before it emits a photon of wavelength 51.30 fm in a gamma-decay to the ground state: $^4\text{He}^* \rightarrow {}^4\text{He} + \gamma$. The atomic mass of the ground state of ^{4}He is 4.00260325415 u. The excited state has more internal energy, so its rest energy is larger, than the ground state: $m_* c^2 = mc^2 + E_x$. Evaluate the *excitation energy* E_x, remembering, of course, to conserve momentum and account for atomic electrons.

6. A positively charged muon (spin 1/2) and a photon (spin 1) are detected emerging from the same spot in an experimental apparatus. We are asked to exclude possible explanations for this observation.

   (a) (5) The decay mode $K^+ \to \mu^+ + \gamma$ would violate conservation of angular momentum. Carefully explain why.

   (b) (5) Determine one conservation law that the decay mode $e^+ \to \mu^+ + \gamma$ would violate. Justify your answer.

   (c) (5) Assuming that the experimenters have failed to detect an outgoing neutrino, determine one conservation law that the collision $p + n \to \mu^+ + \gamma + \nu_\mu$ would violate. Justify your answer.

$c = 2.998\times10^{8}$ m/s	$e = 1.602\times10^{-19}$ C	1 eV $= 1.602\times10^{-19}$ J
$h = 6.626\times10^{-34}$ J $\cdot$ s	$hc = 1240$ eV $\cdot$ nm	$\lambda_m T = 2.898\times10^{-3}$ m $\cdot$ K
$k = 8.617\times10^{-5}$ eV/K	$\sigma = 5.670\times10^{-8}$ W/m^2 $\cdot$ K^4	$\lambda_m V = 1240$ nm $\cdot$ V
$R_\infty = 1.097373\times10^{7}$ m^{-1}	$\alpha \approx 1/137$	1 u $= 931.494$ MeV/c^2
$E_\circ = 13.6$ eV	$a_\circ = 0.0529$ nm	$\hbar = h/(2\pi)$

Particle	Lepton Number L_e	L_μ	L_τ	Mass (MeV/c^2)	Mean Life (s)
e^-	1	0	0	0.510998910	Stable
ν_e	1	0	0	$< 3\times10^{-6}$	Stable
μ^-	0	1	0	105.658367	2.197034×10^{-6}
ν_μ	0	1	0	< 0.19	Stable
τ^-	0	0	1	1776.82	290.6×10^{-15}
ν_τ	0	0	1	< 18	Stable

All leptons and quarks are spin one-half particles.

Quark	Mass (MeV/c^2)	Charge (e)	Baryon Number	U	D	S	C	B	T
u	1.7–3.3	$+\frac{2}{3}$	$\frac{1}{3}$	+1	0	0	0	0	0
d	4.1–5.8	$-\frac{1}{3}$	$\frac{1}{3}$	0	−1	0	0	0	0
s	80–130	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	−1	0	0	0
c	1180–1340	$+\frac{2}{3}$	$\frac{1}{3}$	0	0	0	+1	0	0
b	4130–4850	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	0	0	−1	0
t	172.0×10^{3}	$+\frac{2}{3}$	$\frac{1}{3}$	0	0	0	0	0	+1

Particle	Baryon Number	Spin	Strange-ness	Quark Content	Mass (MeV/c^2)	Mean Life (s)
π^+	0	0	0	$u\bar{d}$	140	2.6×10^{-8}
π°	0	0	0	$u\bar{u}, d\bar{d}$	135	8.4×10^{-17}
η	0	0	0	$u\bar{u}, d\bar{d}$	547	5.6×10^{-19}
K^+	0	0	+1	$u\bar{s}$	494	1.2×10^{-8}
K°	0	0	+1	$d\bar{s}$	498	8.9×10^{-11}
					5.2×10^{-8}	
p	1	$\frac{1}{2}$	0	uud	938.272013	Stable
n	1	$\frac{1}{2}$	0	udd	939.565346	885.7
Δ^{++}	1	$\frac{3}{2}$	0	uuu	1232	5.5×10^{-24}
Δ^+	1	$\frac{3}{2}$	0	uud	1232	5.5×10^{-24}
Δ°	1	$\frac{3}{2}$	0	udd	1232	5.5×10^{-24}
Δ^-	1	$\frac{3}{2}$	0	ddd	1232	5.5×10^{-24}
Λ°	1	$\frac{1}{2}$	-1	uds	1115.683	2.631×10^{-10}
Σ^+	1	$\frac{1}{2}$	-1	uus	1189.37	0.8018×10^{-10}
Σ°	1	$\frac{1}{2}$	-1	uds	1192.642	7.4×10^{-20}
Σ^-	1	$\frac{1}{2}$	-1	dds	1197.449	1.479×10^{-10}
Ξ°	1	$\frac{1}{2}$	-2	uss	1314.86	2.90×10^{-10}
Ξ^-	1	$\frac{1}{2}$	-2	dss	1321.71	1.639×10^{-10}
Ω^-	1	$\frac{3}{2}$	-3	sss	1672.45	0.821×10^{-10}