ASSIGNMENT 2

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Due date: 08:30, 19th October 2017

1. Which language does the regular expression ϵ represent?
   It represent the language $\{\epsilon\}$
2. Which language does the regular expression $\emptyset$ represent?
   It represent the the empty language $\{\} = \emptyset$
3. Which language does the regular expression a represent?
   It represent the the language {a}, respectively, where a is an element of Σ.
4. Is it always true that $R + \epsilon$, where R is a regular expression, represents the same language as R? If yes, explain. If not, give a counterexample
   Yes, becasue the resulting "new language" created by Union; $R \cup \epsilon$ still contains the same strings as in the previous "R".

5. For each of the following languages L, state whether or not L is regular. Prove your answer:
   (a) $\{a^i b^j : i, j \text{ and } i + j = 5\}$

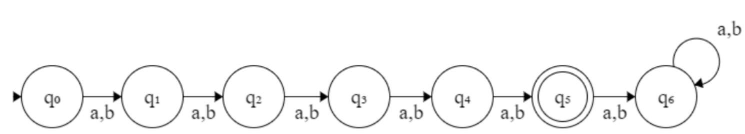

Figure 1:

   This exercise is a regular language, because if it's possible to draw a Finite Automata, the language is recognizable. Which means that the language is a regular language.

   (b) $\{a^i b^j : i, j \text{ and } i - j = 5\}$
   This exercise will be proven by Pumping Lemma.
   By $a^i b^j \to i - j = 5 \leftrightarrow$
   By assuming $k + l = i$, we can use xyz like this:
   $a^k a^l b^j$ where:
   $x = a^k$
   $y = a^l$
   $z = b^j$
   Then by pumping y 3 times: "$3 \times y = 3y$"
   Now we can tell that $k + l - j + 2l \neq k + l - j$.
   This exercise does not contain a regular language.

   (c) $\{a^i b^j : i, j \text{ and } |i - j| \cong 0 \text{ mod } 5\}$ I.e count a's (mod 5), then count b's (mod 5) and accepts iff the two counts are equal By assumtion:
   $i = 5 * k + l$
   $j = 5 * m + l$
   $i - l\%5 = 0 \wedge j - l\%5 = 0$
   $|i - j|\%5 = 0$ for every $0 \leq l \leq 5$
   $a^t a^v b^j \to t + v = i$
   $x = a^t$
   $y = a^v$
   $z = b^j$
   Then by pumping y 3 times ($3 \times y = 3y$), we end out with the function looking like:

$(t + v - j)\%5 = 0 \neq (t + 3v - j)\%5 = 0$.
By using pumping lemma, this exercise does not contain a regular language. For some expamples this will stille end as a regular language, but will fail most of the time.

(d) $\{w \in \{Y, N\}^{\star}$ w contains at least two Y's and at most two N's $\}$ Regular like exercise 5a, since its possible to make a FA for this language.

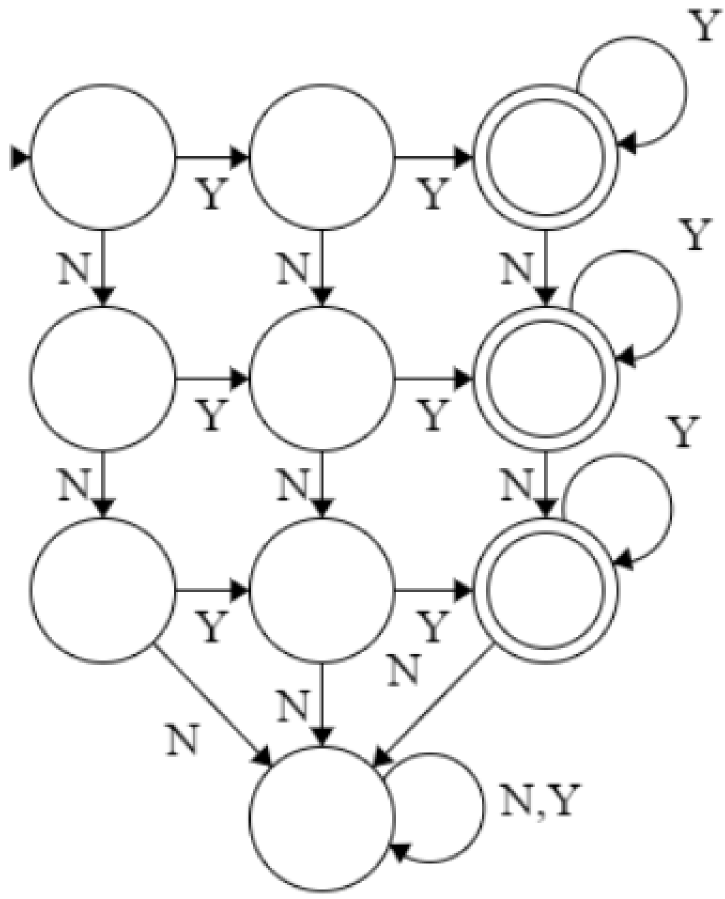

Figure 2:

(e) $\{w \in \{a, b\}^{\star}$ w contains exactly two more b's than a's$\}$
L is a infinite language and if we assume that L is regular, we'll try to apply the pumping lemma.
We'll end up with: $w = a^m b^{m+2}$. Then we can write down the xyz:
xyz $= a^k a^l b^{m+2}$ where $k + l = m$ and:
$x = a^k$
$y = a^l$
$z = b^{m+2}$
Now, by using the pumping lemma, we can tell if the language is regular or not. Because pumping lemma states that $xy^i z \in L$, even if $i = 0$. This leaves us with $xz \in L$. But for this language, that's is not the case, since $xz = a^{k-l} b^{m+2} \notin L$, $k + l = m$ and $k - l \neq m$. In this case L is not regular, since $xz \notin L$.

(f) $\{w \in \{a, b\}^{\star}$ the number of occurrences of the substring ab is equal to the number of occurrences of the substring ba.$\}$

(g) $\{w \in \{(,)\}^{\star}$ the parentheses are balances$\}$

(h) $\{ww^R \in \{a, b\}^{\star}\}$

6. Can you use the Pumping Lemma for regular languages to show that a language is regular? If yes, explain why. If no, explain why not.

   Yes. This is from the Michel Sipser book: *"Our technique for proving nonregularity stems from a theorem about regular languages, traditionally called the pumping lemma. This theorem states that all regular languages have a special property. If we can show that a language does not have this property, we are guaranteed that it is not regular."*

7. Let $\Sigma = \{a, b\}$. Consider the following grammars.

(a) $S \rightarrow aS|Sb|\epsilon$

i. a, b, ϵ, ab, aaa

ii. ba, baa, bab, baaa, baba

iii. $a^\star b^\star$

iv. L is in this exercise regular since we can write down a regular expression.

(b) $S \rightarrow aSa|bSb|a|b$

i. a, aa, aba, aabaa, bababab

ii. abaa, aaba, bbab, aaab, ab

iii. $w \in (a,b)^\star \mid w$ is symmetric.

iv. L is in this exercise not regular, this can be proven by applying Pumping Lemma.

(c) $S \rightarrow aS|bS|\epsilon$

i. a, b, ϵ, ab, aaa

ii. $\emptyset$

iii. $w \in (a,b)^\star \mid w$ is anything

DFA:

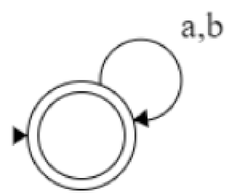

Figure 3:

iv. L is in this exercise regular since it takes every possible string and can be proven by a FA.

(d) $S \rightarrow aS|aSbS|\epsilon$

i. a, ab, aab, aaba, aaabaa

ii. b, ba bab, abba, bb

iii. $w \in (a,b)^\star \mid w$ contains either a equal amount of a's and b's or always more a's than b's.

iv. L is in this exercise not regular, this can be proven by applying Pumping Lemma.

For each language defined by the grammars, do the following:

(a) List five strings that are in L.

(b) List five strings that are not in L.

(c) Describe L concisely. You can use regular expressions, set theoretic expressions, etc.

(d) Indicate whether or not L is regular. Prove your answer.

8. Consider the following grammar $G : S \rightarrow 0S1|SS|10$.
Show a parse tree produced by G for each of the following strings:

(a) 010110

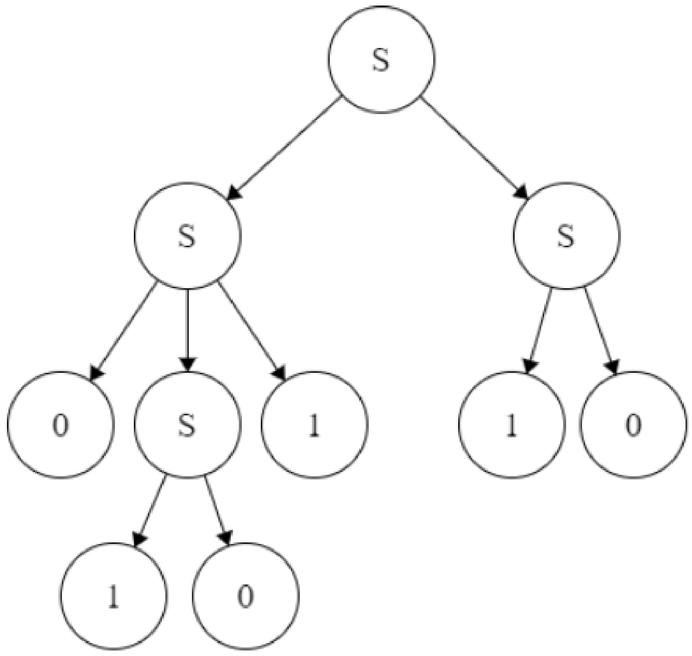

Figure 4:

(b) 00101101

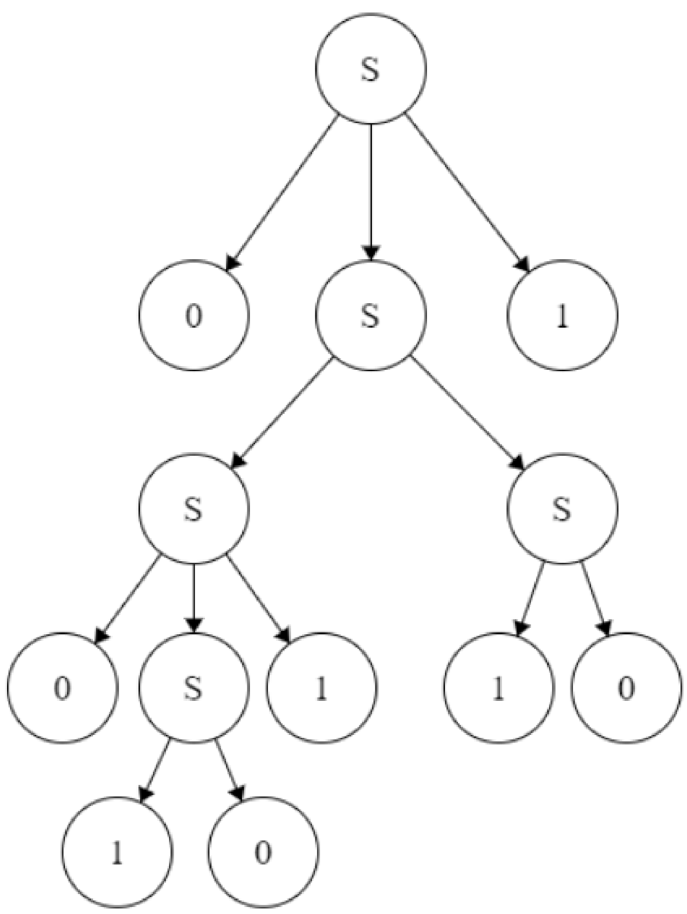

Figure 5: