```
> ec1:=diff(x(t),t)=-k*x(t);
```

$$ec1 := \frac{d}{dt}\, x(t) = -k\, x(t) \quad \textbf{(1)}$$

```
> cond_in:=x(0)=x0;
```

$$cond\_in := x(0) = x0 \quad \textbf{(2)}$$

```
> sist:=ec1;
```

$$sist := \frac{d}{dt}\, x(t) = -k\, x(t) \quad \textbf{(3)}$$

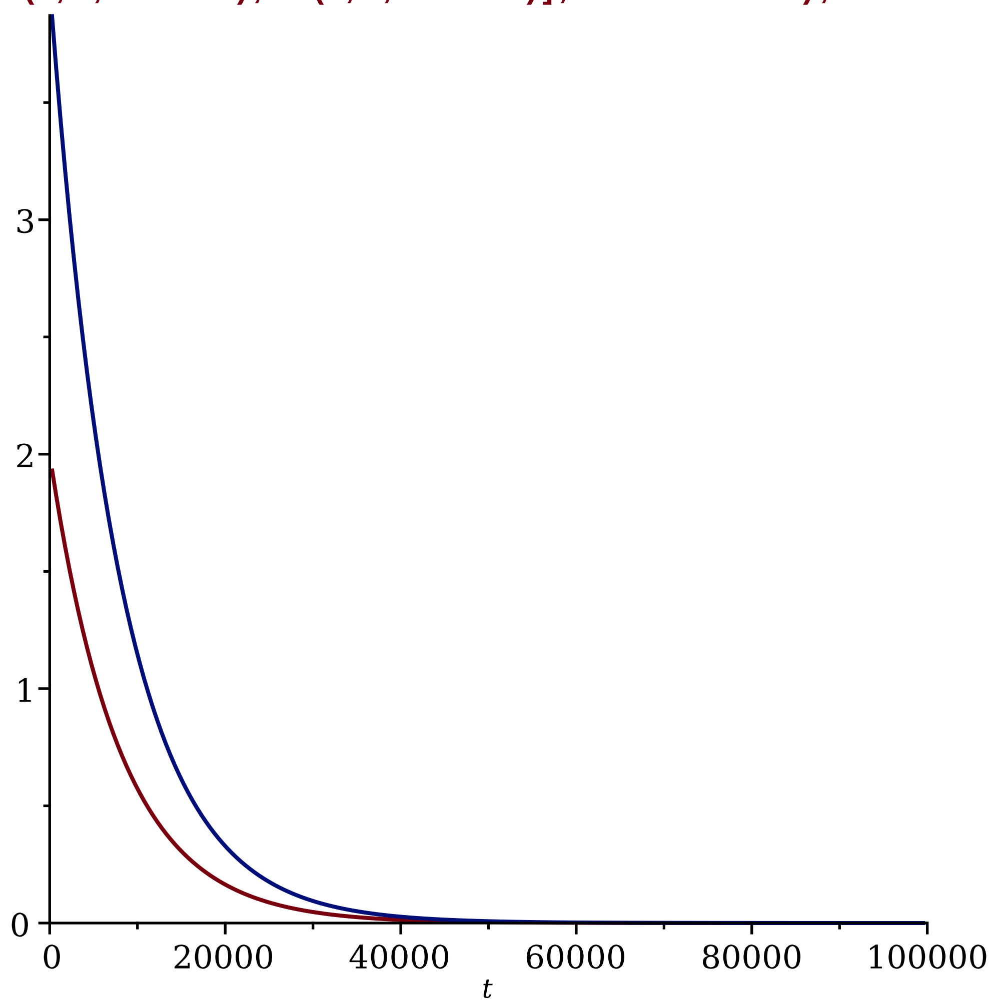
```
> with(DEtools):with(plots):
> sol:=dsolve({sist,cond_in},{x(t)});
```

$$sol := x(t) = x0\, e^{-k\, t} \quad \textbf{(4)}$$

```
> xx:=unapply(rhs(sol),t,x0,k);
```

$$xx := (t, x0, k) \mapsto x0 \cdot e^{-k \cdot t} \quad \textbf{(5)}$$

```
> plot([xx(t,2,1/8000),xx(t,4,1/8000)],t=0..100000);
```




```
> ec2:=xx(5730,x0,k)=x0/2;
```

$$ec2 := x0\, e^{-5730\, k} = \frac{x0}{2} \quad \textbf{(6)}$$

```
> ksol:=solve(ec2,k);
```

$$ksol := \frac{\ln(2)}{5730} \tag{7}$$

```
> evalf(ksol)
```

$$0.0001209680943 \tag{8}$$

```
> evalf(1/8000);
```

$$0.0001250000000 \tag{9}$$

```
> ec3:=xx(t,x0,ksol)=0.2*x0;
```

$$ec3 := x0\, \mathrm{e}^{-\frac{\ln(2)\, t}{5730}} = 0.2\, x0 \tag{10}$$

```
> sol:=solve(ec3,t);
```

$$sol := 13304.64798 \tag{11}$$

```
> ece1:=xx(t,x0,ksol)=0.9157*x0;
```

$$ece1 := x0\, \mathrm{e}^{-\frac{\ln(2)\, t}{5730}} = 0.9157\, x0 \tag{12}$$

```
> ts1:=solve(ece1,t);
```

$$ts1 := 728.0141045 \tag{13}$$

```
> t1:=1988-ts1;
```

$$t1 := 1259.985896 \tag{14}$$

```
> ece2:=xx(t,x0,ksol)=0.93021*x0;
```

$$ece2 := x0\, \mathrm{e}^{-\frac{\ln(2)\, t}{5730}} = 0.93021\, x0 \tag{15}$$

```
> ts2:=solve(ece2,t);
```

$$ts2 := 598.0495293 \tag{16}$$

```
> t2:=1988-ts2;
```

$$t2 := 1389.950471 \tag{17}$$

```
> ec:=diff(T(t),t)=-k*(T(t)-Tm);
```

$$ec := \frac{\mathrm{d}}{\mathrm{d}t}\, T(t) = -k\, (T(t) - Tm) \tag{18}$$

```
> cond_in:=T(0)=T0;
```

$$cond\_in := T(0) = T0 \tag{19}$$

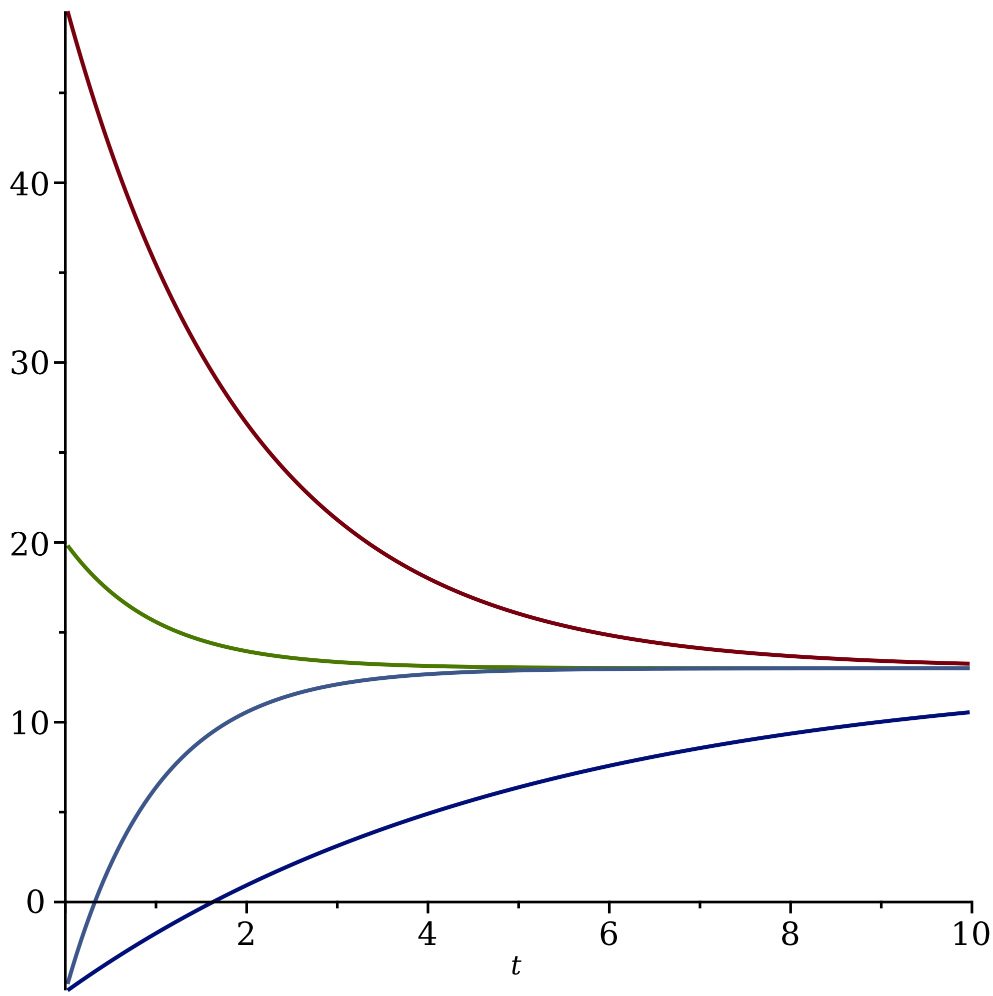
```
> sol:=dsolve({ec,cond_in},{T(t)});
```

$$sol := T(t) = Tm + \mathrm{e}^{-k\, t}\, (T0 - Tm) \tag{20}$$

```
> xt:=unapply(rhs(sol),t,T0,k,Tm);
```

$$xt := (t, T0, k, Tm) \mapsto Tm + \mathrm{e}^{-k \cdot t} \cdot (T0 - Tm) \tag{21}$$

```
> plot([xt(t,50,0.5,13),xt(t,-5,0.2,13),xt(t,20,1,13),xt(t,-5,1,13)
  ],t=0..10);
```




```
> #k-constanta de racire
> ec1:=xt(t,36,k,21)=34.22;
```

$$ec1 := 21 + 15\,e^{-k\,t} = 34.22 \tag{22}$$

```
> ec2:=xt(t+1,36,k,21)=34.11;
```

$$ec2 := 21 + 15\,e^{-k\,(t+1)} = 34.11 \tag{23}$$

```
> sist:=ec1,ec2;
```

$$sist := 21 + 15\,e^{-k\,t} = 34.22,\ 21 + 15\,e^{-k\,(t+1)} = 34.11 \tag{24}$$

```
> sol:=solve({sist},{t,k});
```

$$sol := \{k = 0.008355536648,\ t = 15.11804352\} \tag{25}$$

```
> 11.5-sol[2]+24;
```

$$-t + 35.5 = 20.38195648 \tag{26}$$

```
> ec1:=v(x)*diff(v(x),x)=-(g*R^2)/(x+R)^2;
```

$$ec1 := v(x)\left(\frac{d}{dx}\,v(x)\right) = -\frac{g\,R^2}{(x+R)^2} \tag{27}$$

```
> cond_in:=v(0)=v0;
```

$$cond\_in := v(0) = v0 \tag{28}$$

```
> sol:=dsolve({ec1,cond_in},{v(x)},implicit);
```

(29)

$$sol := v(x)^2 - \frac{2\,g\,R^2}{x+R} + 2\,g\,R - v0^2 = 0 \quad (29)$$

```
> eq:=lhs(sol)=0;
```

$$eq := v(x)^2 - \frac{2\,g\,R^2}{x+R} + 2\,g\,R - v0^2 = 0 \quad (30)$$

```
> s:=solve(eq,v(x));
```

$$s := \frac{\sqrt{-(x+R)\,(2\,g\,R\,x - v0^2\,R - v0^2\,x)}}{x+R},\; -\frac{\sqrt{-(x+R)\,(2\,g\,R\,x - v0^2\,R - v0^2\,x)}}{x+R} \quad (31)$$

```
> vx:=unapply(s[1],x,v0,g,R);
```

$$vx := (x, v0, g, R) \mapsto \frac{\sqrt{-(x+R)\cdot(2\cdot R\cdot g\cdot x - R\cdot v0^2 - v0^2\cdot x)}}{x+R} \quad (32)$$

```
> vx(75,50,9.81,6371*10^3);
```

$$32.07050550 \quad (33)$$

```
> hmax:=solve(vx(x,50,9.81,6371*10^3)=0,x);
```

$$hmax := 127.4235475 \quad (34)$$

```
> ec5:=vx(x,v0,g,R)=0;
```

$$ec5 := \frac{\sqrt{-(x+R)\,(2\,g\,R\,x - v0^2\,R - v0^2\,x)}}{x+R} = 0 \quad (35)$$

```
> sol:=solve(ec5,v0);
```

$$sol := \frac{\sqrt{2}\,\sqrt{(x+R)\,g\,R\,x}}{x+R},\; -\frac{\sqrt{2}\,\sqrt{(x+R)\,g\,R\,x}}{x+R} \quad (36)$$

```
> vh:=unapply(sol[1],x);
```

$$vh := x \mapsto \frac{\sqrt{2}\cdot\sqrt{(x+R)\cdot g\cdot R\cdot x}}{x+R} \quad (37)$$

```
> vev:=limit(vh(x),x=infinity);
```

$$vev := \sqrt{g\,R}\,\sqrt{2} \quad (38)$$

```
> v_evadare:=(g,R)->sqrt(2*g*R)
```

$$v\_evadare := (g, R) \mapsto \sqrt{2\cdot R\cdot g} \quad (39)$$

```
> km_to_m:=x->x*10^3
```

$$km\_to\_m := x \mapsto 1000\cdot x \quad (40)$$

```
> rec_km:=6378.160
```

$$rec\_km := 6378.160 \quad (41)$$

```
> g_ec:=9.78
```

$$g\_ec := 9.78 \quad (42)$$

```
> rec:=km_to_m(rec_km)
```

$$rec := 6.378160000 \times 10^6 \quad (43)$$

```
> v_e_ecuator:=v_evadare(g_ec,rec)
```
$$v\_e\_ecuator := 11169.45879 \quad (44)$$

```
> rpol_km:=6357.778
```
$$rpol\_km := 6357.778 \quad (45)$$

```
> g_pol:=9.832
```
$$g\_pol := 9.832 \quad (46)$$

```
> rpol:=km_to_m(rpol_km)
```
$$rpol := 6.357778000 \times 10^{6} \quad (47)$$

```
> v_e_poli:=v_evadare(g_pol,rpol)
```
$$v\_e\_poli := 11181.20506 \quad (48)$$

```
> rm_km:=6371.110
```
$$rm\_km := 6371.110 \quad (49)$$

```
> g_m:=9.81
```
$$g\_m := 9.81 \quad (50)$$

```
> rm:=km_to_m(rm_km)
```
$$rm := 6.371110000 \times 10^{6} \quad (51)$$

```
> v_e_medie:=v_evadare(g_m,rm)
```
$$v\_e\_medie := 11180.39258 \quad (52)$$

```
> evalf([v_e_ecuator,v_e_poli,v_e_medie])
```
$$[11169.45879, 11181.20506, 11180.39258] \quad (53)$$