```
> ecdif1:=diff(y(x),x)=2*x*(1+(y(x))^2)
```

$$ecdif1 := \frac{d}{dx}\, y(x) = 2\, x\, (1 + y(x)^2) \tag{1}$$

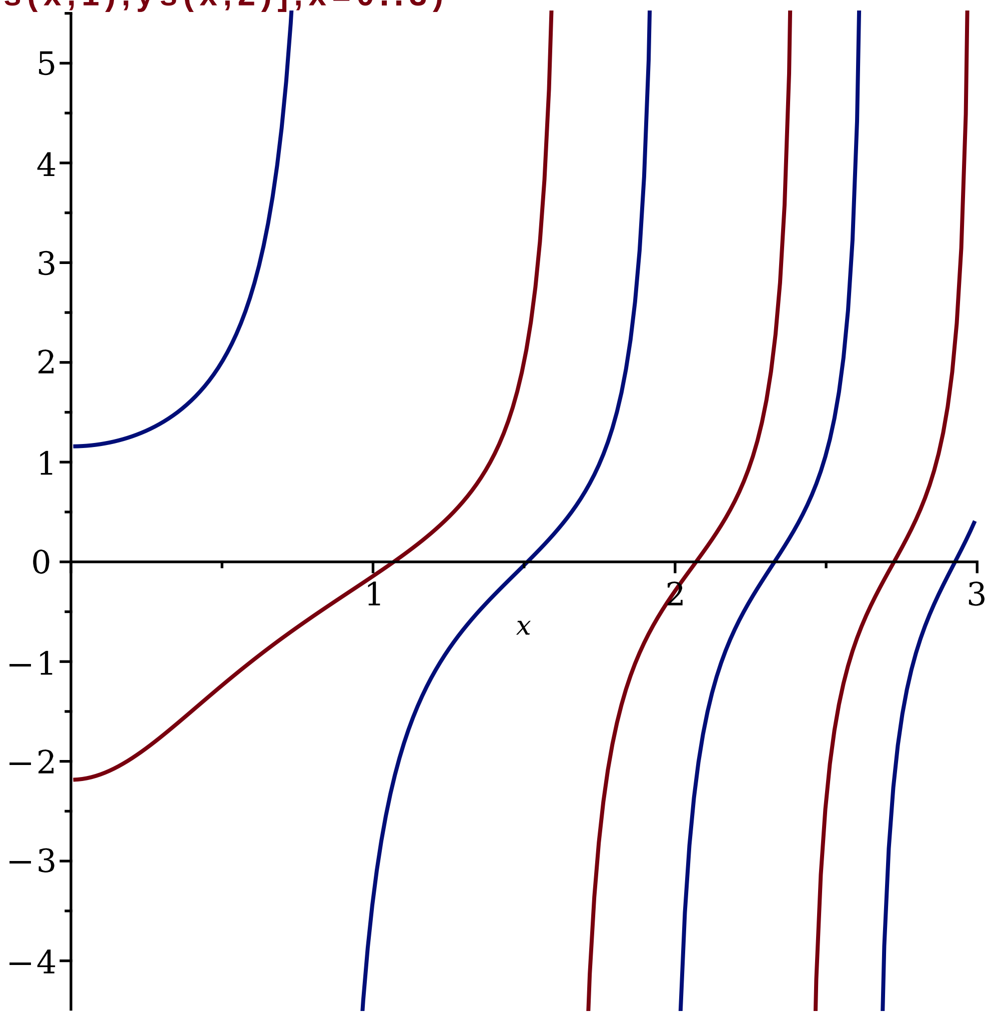
```
> sol:=dsolve(ecdif1,y(x))
```

$$sol := y(x) = \tan(x^2 + 2\, c_1) \tag{2}$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto \tan(x^2 + 2 \cdot c_1) \tag{3}$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif2:=diff(y(x),x)=(-2*x*(y(x))^2)/(x^2-1)
```

$$ecdif2 := \frac{d}{dx}\, y(x) = -\frac{2\, y(x)^2\, x}{x^2 - 1} \tag{4}$$

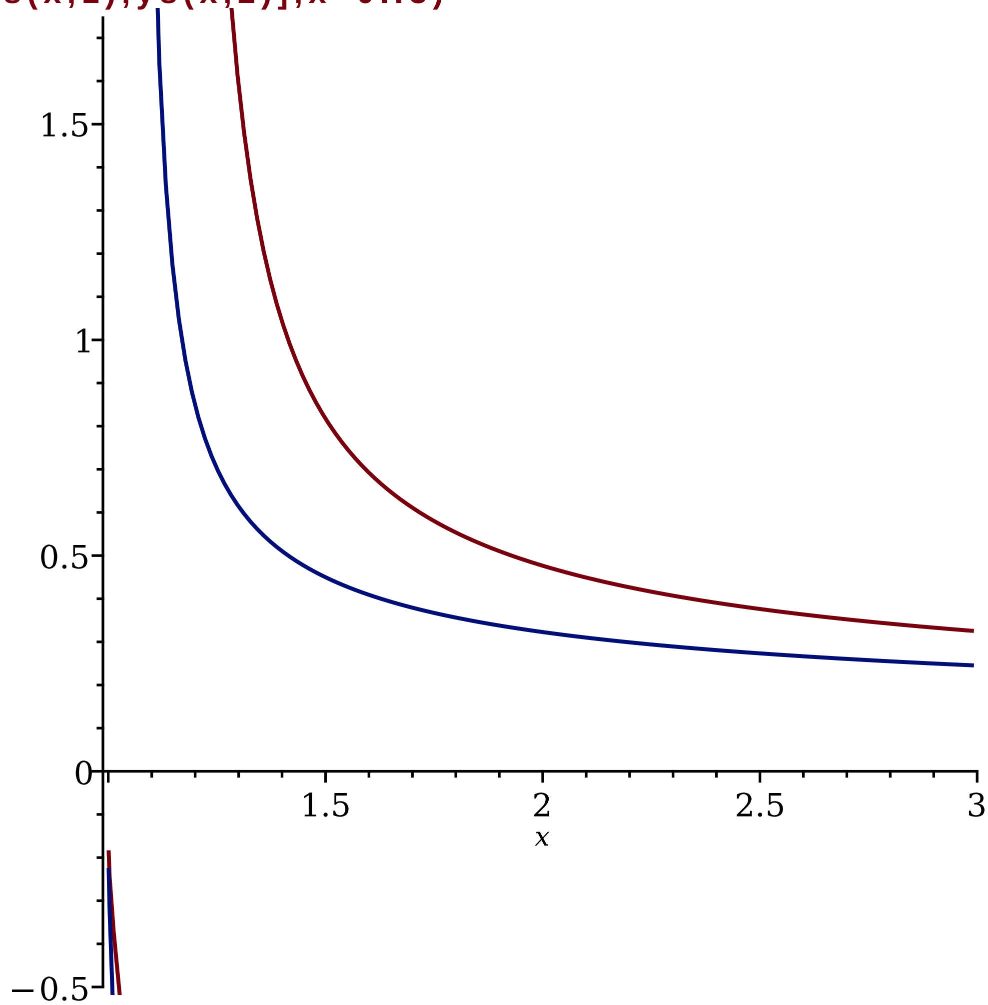
```
> sol:=dsolve(ecdif2,y(x))
```

$$sol := y(x) = \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \tag{5}$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto \frac{1}{\ln(x-1) + \ln(x+1) + c_1} \tag{6}$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif3:=diff(y(x),x)=(x^2+(y(x))^2)/(2*x^2)
```

$$ecdif3 := \frac{d}{dx}\, y(x) = \frac{x^2 + y(x)^2}{2\, x^2} \quad (7)$$

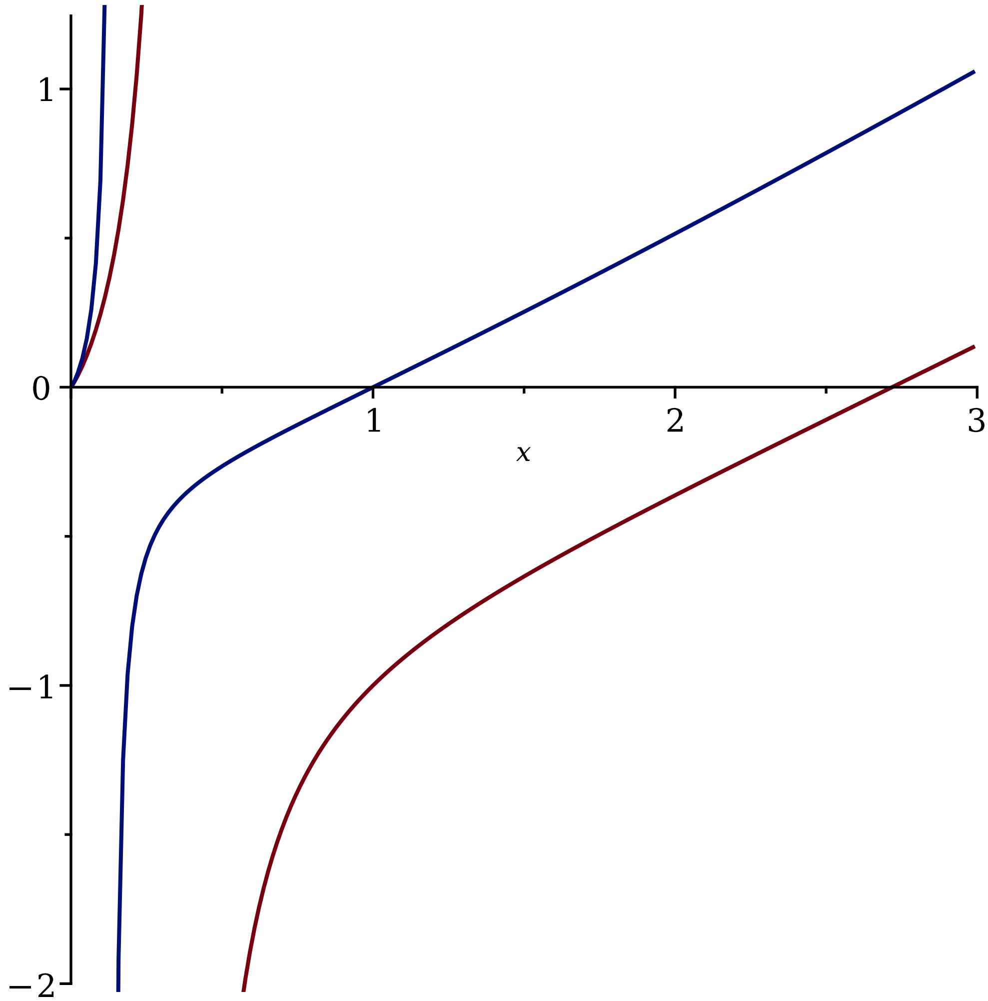
```
> sol:=dsolve(ecdif3,y(x))
```

$$sol := y(x) = \frac{x\, (\ln(x) + c_1 - 2)}{\ln(x) + c_1} \quad (8)$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto \frac{x \cdot (\ln(x) + c_1 - 2)}{\ln(x) + c_1} \quad (9)$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x)=-(x/y(x))
```

$$ecdif := \frac{d}{dx}\, y(x) = -\frac{x}{y(x)} \tag{10}$$

```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \sqrt{-x^2 + c_1},\ y(x) = -\sqrt{-x^2 + c_1} \tag{11}$$

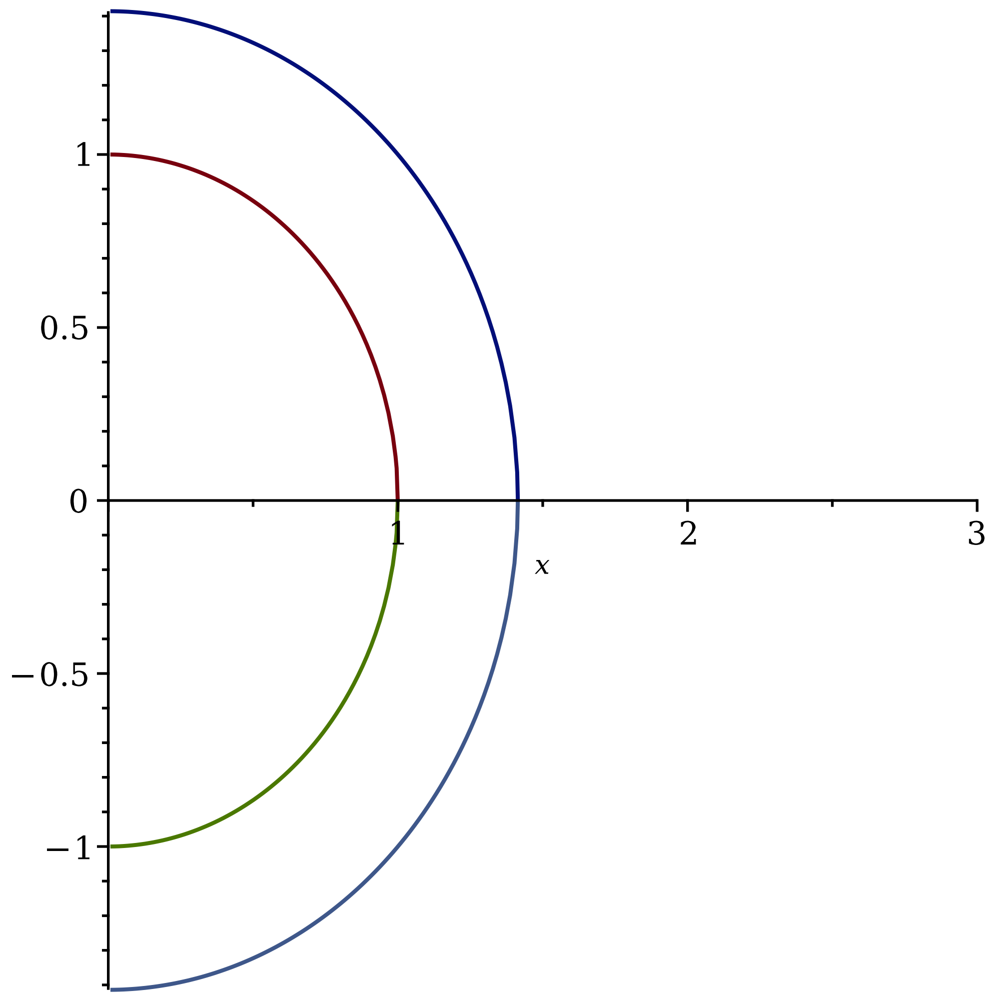
```
> ys1:=unapply(rhs(sol[1]),x,c__1)
```

$$ys1 := (x, c_1) \mapsto \sqrt{-x^2 + c_1} \tag{12}$$

```
> ys2:=unapply(rhs(sol[2]),x,c__1)
```

$$ys2 := (x, c_1) \mapsto -\sqrt{-x^2 + c_1} \tag{13}$$

```
> plot([ys1(x,1),ys1(x,2),ys2(x,1),ys2(x,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x)=-(x/y(x)^3)
```

$$ecdif := \frac{d}{dx}\, y(x) = -\frac{x}{y(x)^3} \qquad (14)$$

```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \left(-2\,x^2 + c_1\right)^{1/4},\, y(x) = -\left(-2\,x^2 + c_1\right)^{1/4},\, y(x) = -\mathrm{I}\left(-2\,x^2 + c_1\right)^{1/4},\, y(x) = \mathrm{I}\left(-2\,x^2 + c_1\right)^{1/4} \qquad (15)$$

```
> ys1:=unapply(rhs(sol[1]),x,c__1)
```

$$ys1 := (x, c_1) \mapsto \left(-2 \cdot x^2 + c_1\right)^{1/4} \qquad (16)$$

```
> ys2:=unapply(rhs(sol[2]),x,c__1)
```

$$ys2 := (x, c_1) \mapsto -\left(-2 \cdot x^2 + c_1\right)^{1/4} \qquad (17)$$

```
> ys3:=unapply(rhs(sol[3]),x,c__1)
```

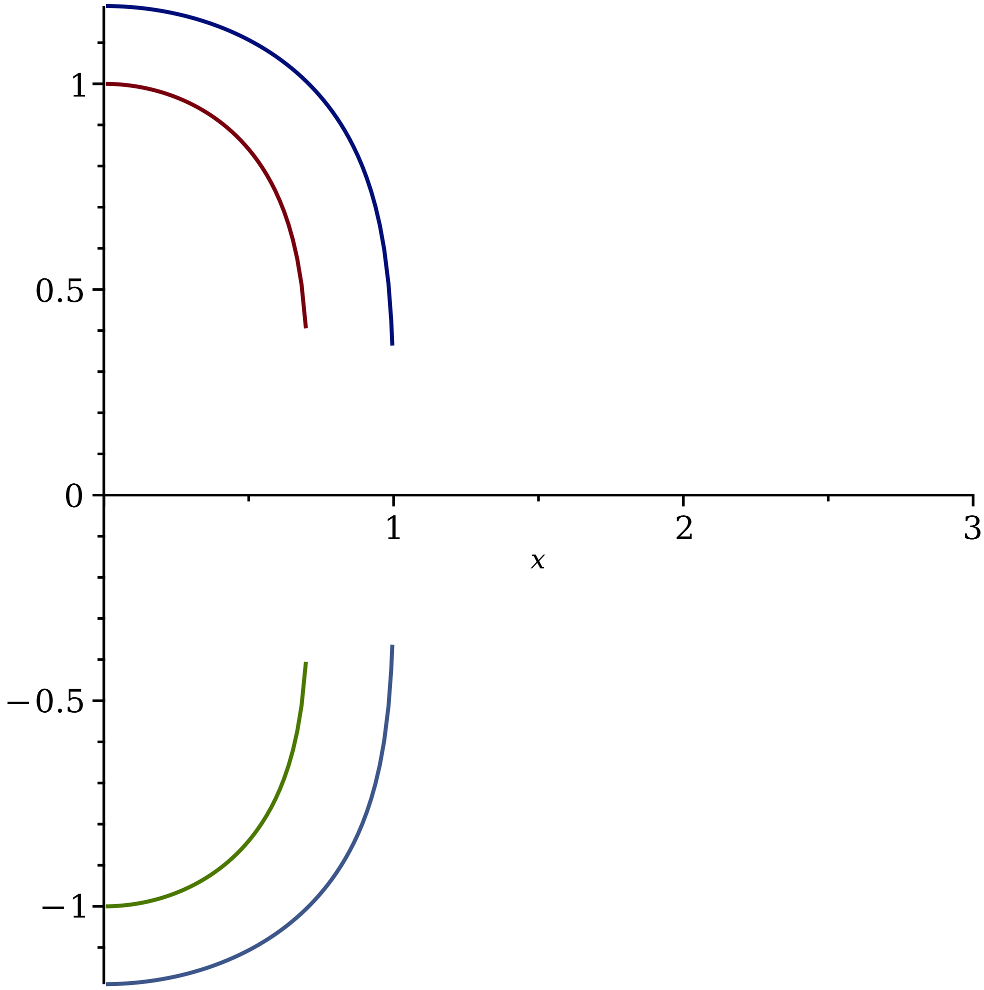
$$ys3 := (x, c_1) \mapsto -\mathrm{I} \cdot \left(-2 \cdot x^2 + c_1\right)^{1/4} \qquad (18)$$

```
> ys4:=unapply(rhs(sol[4]),x,c__1)
```

$$ys4 := (x, c_1) \mapsto \mathrm{I} \cdot \left(-2 \cdot x^2 + c_1\right)^{1/4} \qquad (19)$$

```
> plot([ys1(x,1),ys1(x,2),ys2(x,1),ys2(x,2), ys3(x,1), ys3(x,2),
  ys4(x,1), ys4(x,2)],x=0..3)
```

Warning, unable to evaluate 4 of the 8 functions to numeric values in the region; complex values were detected




```
> ecdif:=diff(y(x),x)=-((x+y(x))/y(x))
```

$$ecdif := \frac{d}{dx}\, y(x) = -\frac{x + y(x)}{y(x)} \quad (20)$$

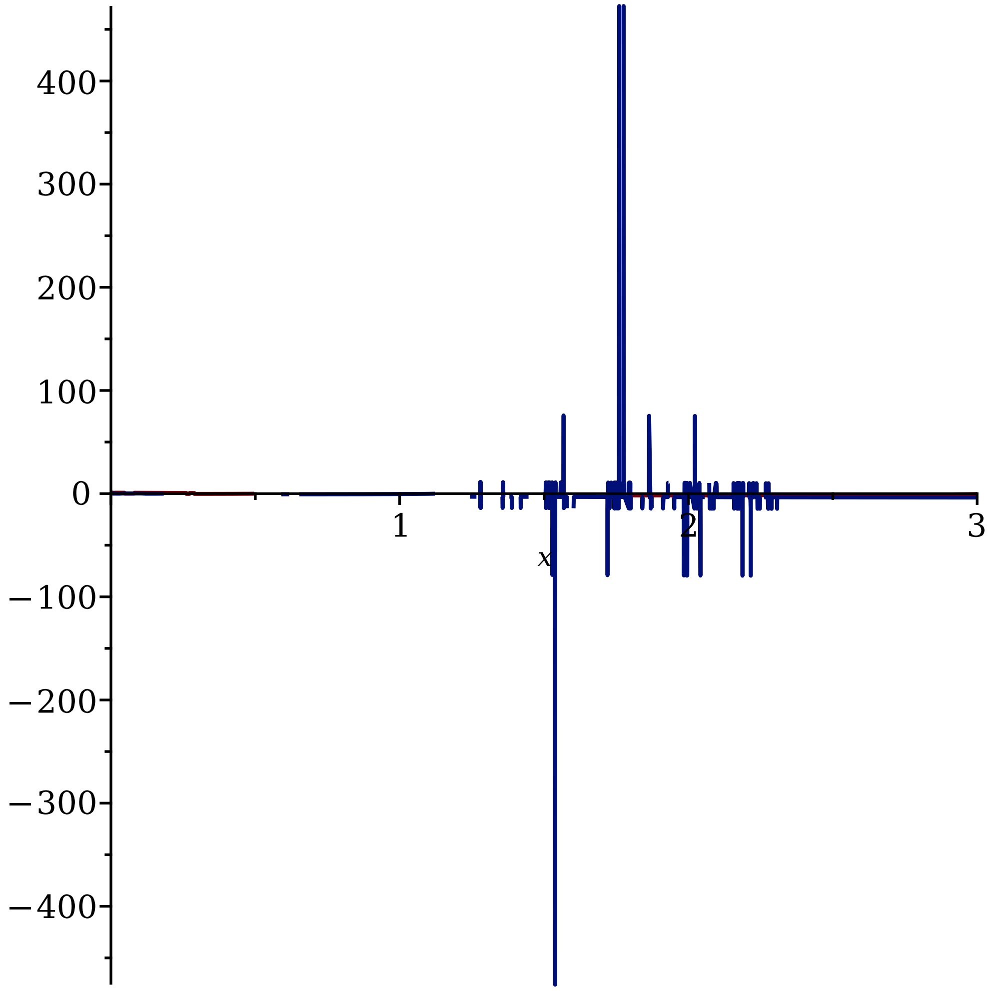
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \frac{\sqrt{3}\, x \tan\left(RootOf\left(\sqrt{3}\, \ln\left(\frac{3\, x^2}{4} + \frac{3\, x^2 \tan(\_Z)^2}{4}\right) + 2\sqrt{3}\, c_1 - 2\,\_Z\right)\right)}{2} - \frac{x}{2} \quad (21)$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto \frac{\sqrt{3}\cdot x\cdot\tan\left(RootOf\left(\sqrt{3}\cdot\ln\left(\frac{3\cdot x^2}{4} + \frac{3\cdot x^2\cdot\tan(\_Z)^2}{4}\right) + 2\cdot\sqrt{3}\cdot c_1 - 2\cdot\_Z\right)\right)}{2} - \frac{x}{2} \quad (22)$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x)=1/cos(x)-y(x)*tan(x)
```

$$ecdif := \frac{\mathrm{d}}{\mathrm{d}x}\, y(x) = \frac{1}{\cos(x)} - y(x)\tan(x) \qquad \textbf{(23)}$$

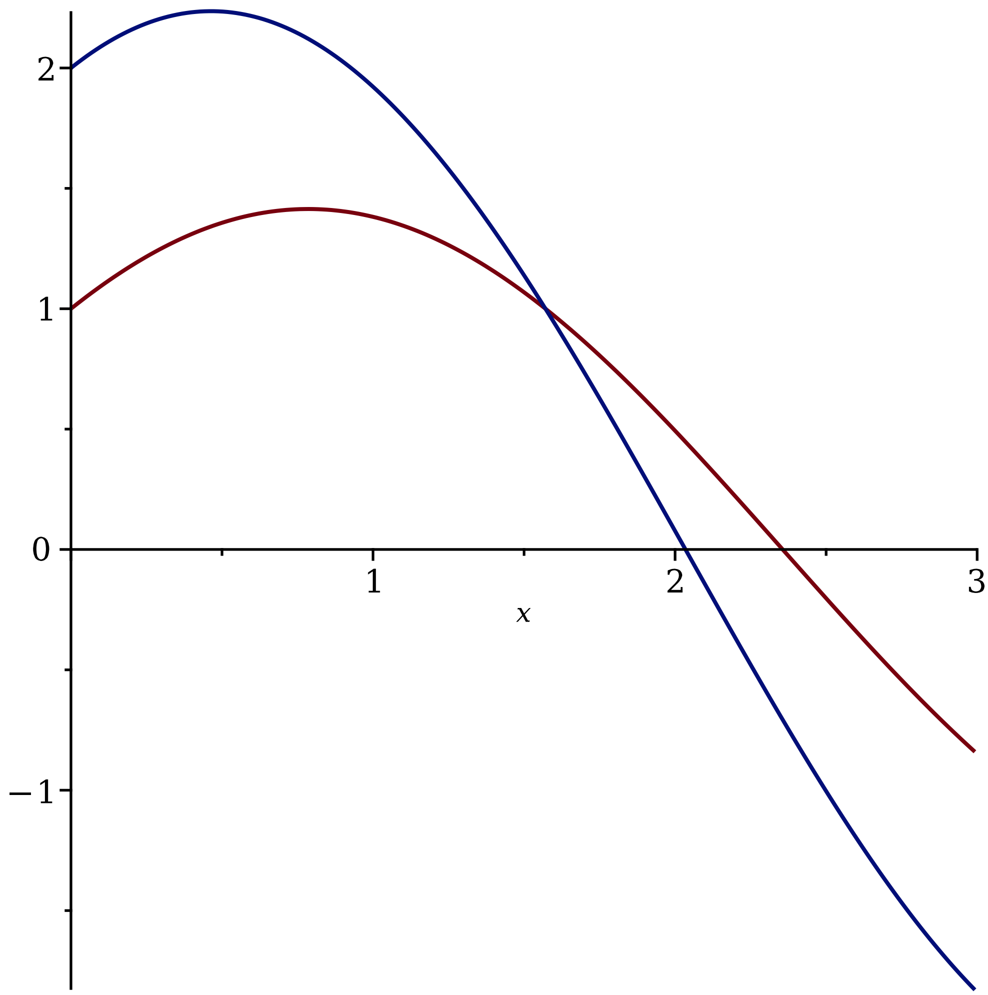
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = (\tan(x) + c_1)\cos(x) \qquad \textbf{(24)}$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto (\tan(x) + c_1)\cdot\cos(x) \qquad \textbf{(25)}$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x)=x^3-(2/x)*y(x)
```

$$ecdif := \frac{d}{dx}\, y(x) = x^3 - \frac{2\, y(x)}{x} \quad (26)$$

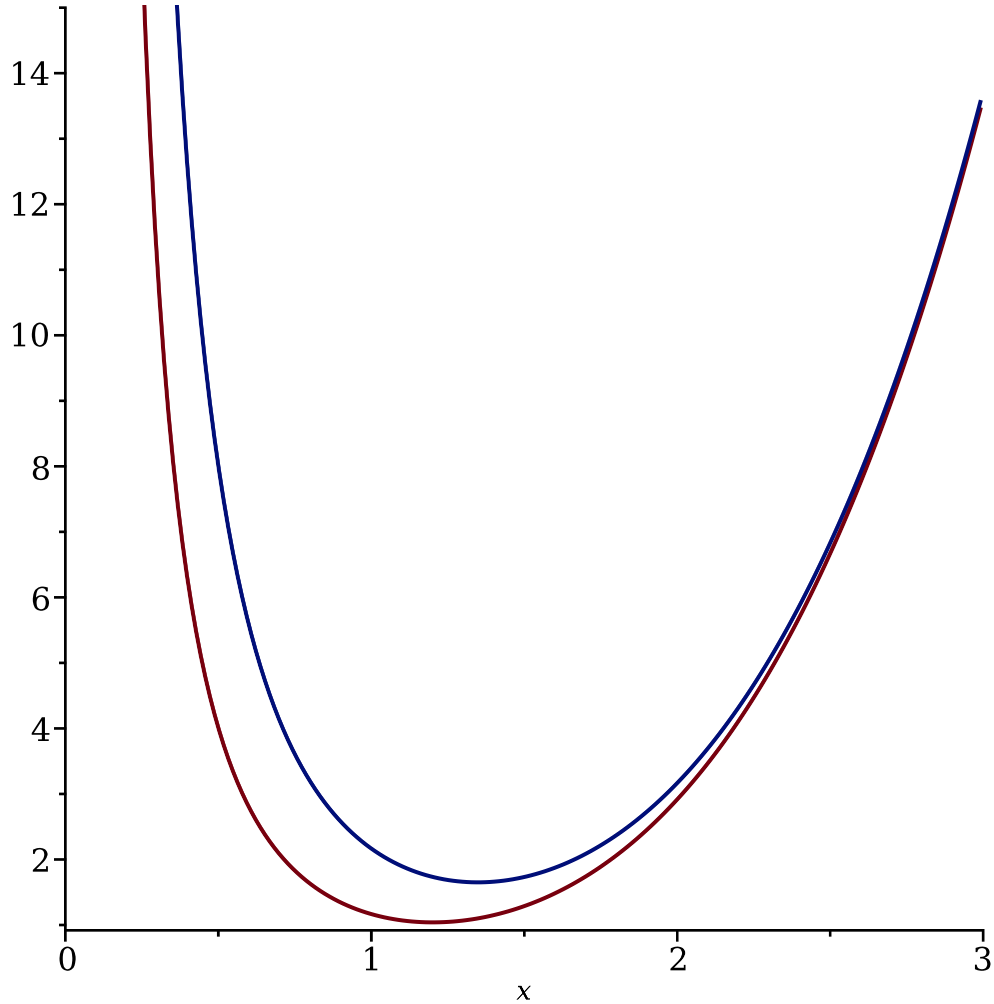
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \frac{\frac{x^6}{6} + c_1}{x^2} \quad (27)$$

```
> ys:=unapply(rhs(sol),x,c__1)
```

$$ys := (x, c_1) \mapsto \frac{\frac{x^6}{6} + c_1}{x^2} \quad (28)$$

```
> plot([ys(x,1),ys(x,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x,x)=sin(x)+cos(x)-y(x)
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) = \sin(x) + \cos(x) - y(x) \qquad (29)$$

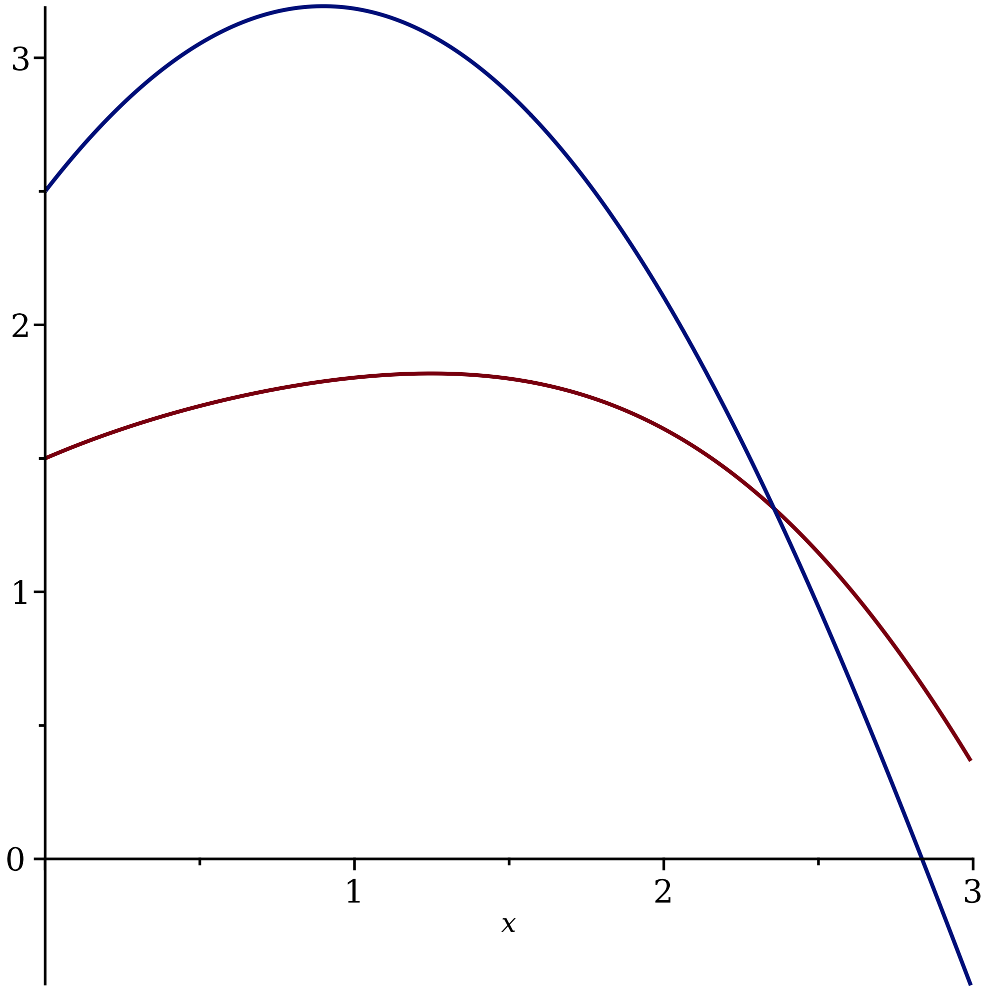
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \sin(x)\, c_2 + \cos(x)\, c_1 + \frac{(1 - x)\cos(x)}{2} + \frac{\sin(x)\, x}{2} \qquad (30)$$

```
> ys:=unapply(rhs(sol),x,c__1, c__2)
```

$$ys := (x, c_1, c_2) \mapsto \sin(x)\cdot c_2 + \cos(x)\cdot c_1 + \frac{(1 - x)\cdot\cos(x)}{2} + \frac{\sin(x)\cdot x}{2} \qquad (31)$$

```
> plot([ys(x,1,1),ys(x,2,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x,x)=exp(2*x)+y(x)
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) = e^{2\,x} + y(x) \qquad (32)$$

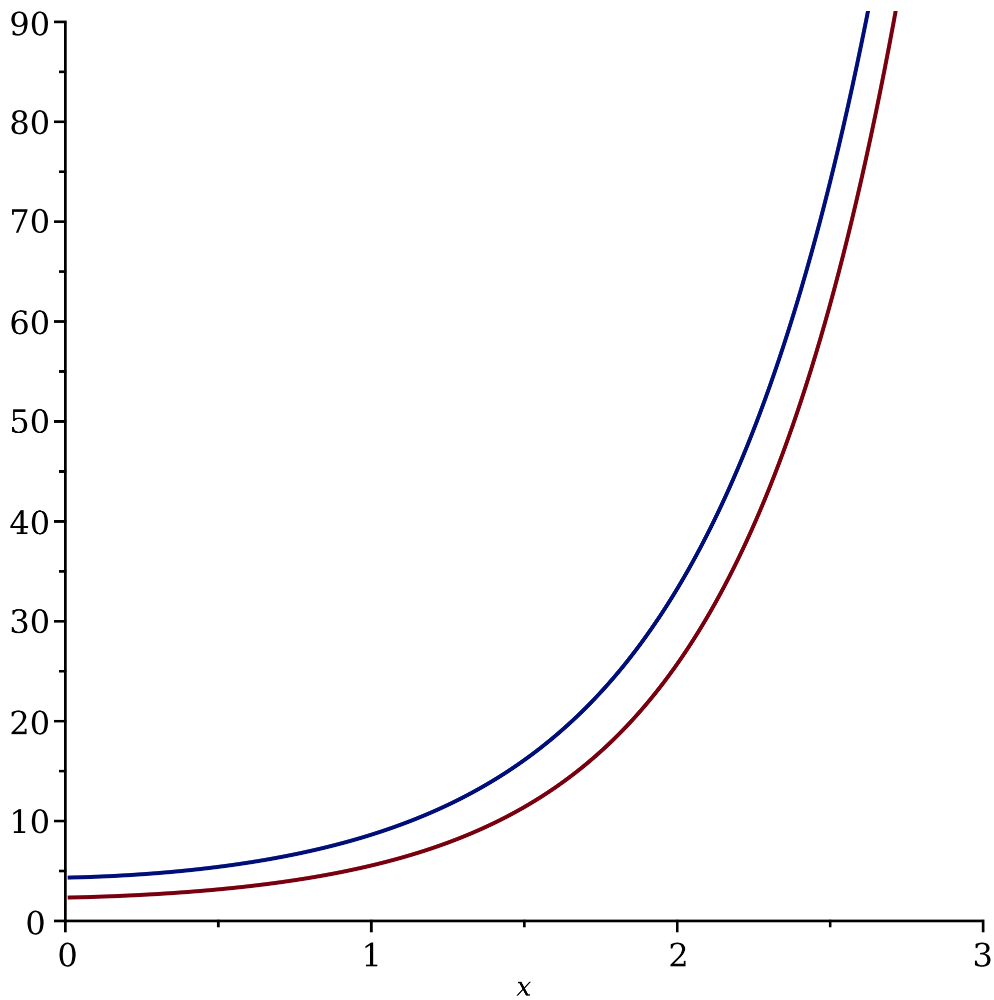
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = e^{-x}\, c_2 + e^{x}\, c_1 + \frac{e^{2\,x}}{3} \qquad (33)$$

```
> ys:=unapply(rhs(sol),x,c__1, c__2)
```

$$ys := (x, c_1, c_2) \mapsto e^{-x}\cdot c_2 + e^{x}\cdot c_1 + \frac{e^{2\cdot x}}{3} \qquad (34)$$

```
> plot([ys(x,1,1),ys(x,2,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x,x)=1/(cos(2*x))-4*y(x)
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) = \frac{1}{\cos(2\,x)} - 4\,y(x) \qquad (35)$$

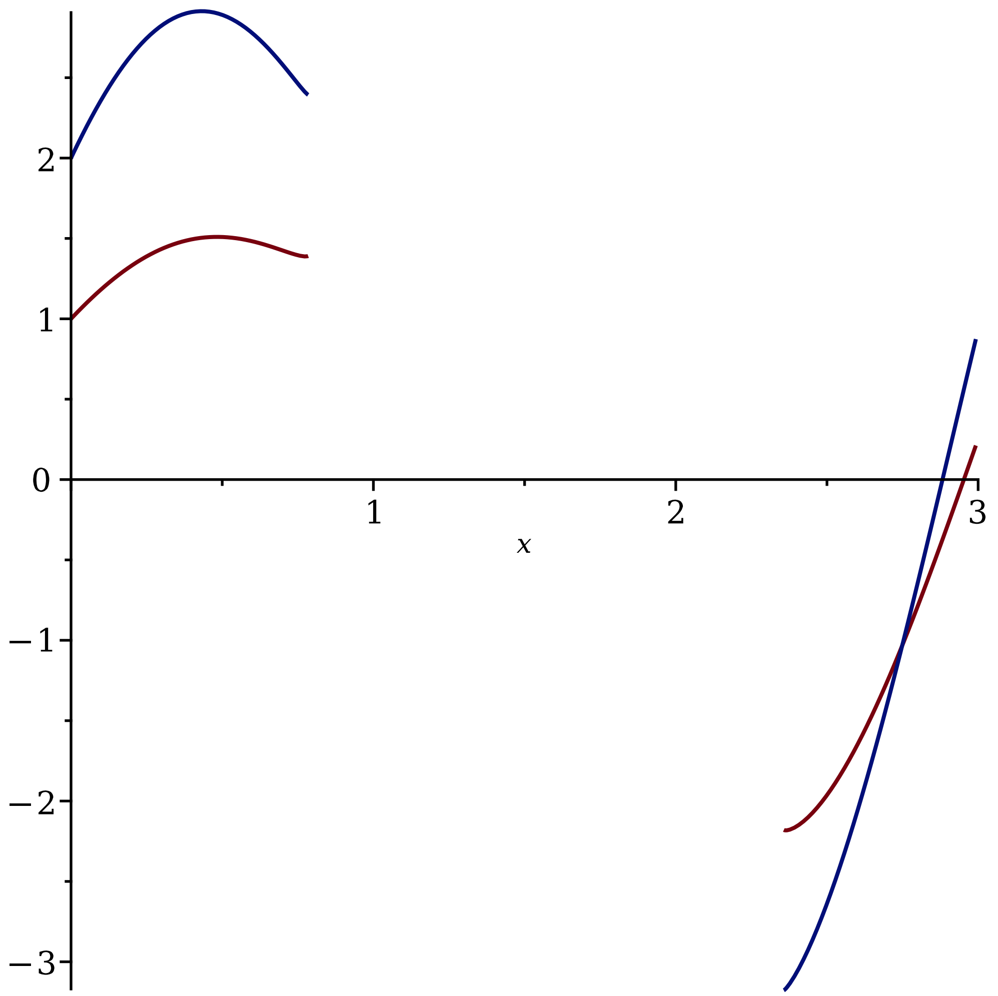
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = \sin(2\,x)\,c_2 + \cos(2\,x)\,c_1 + \frac{x\sin(2\,x)}{2} + \frac{\ln(\cos(2\,x))\cos(2\,x)}{4} \qquad (36)$$

```
> ys:=unapply(rhs(sol),x,c__1, c__2)
```

$$ys := (x, c_1, c_2) \mapsto \sin(2\cdot x)\cdot c_2 + \cos(2\cdot x)\cdot c_1 + \frac{x\cdot\sin(2\cdot x)}{2} + \frac{\ln(\cos(2\cdot x))\cdot\cos(2\cdot x)}{4} \qquad (37)$$

```
> plot([ys(x,1,1),ys(x,2,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x,x)-diff(y(x),x)=1/(1+exp(x))
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) - \frac{d}{dx}\, y(x) = \frac{1}{1+e^x} \tag{38}$$

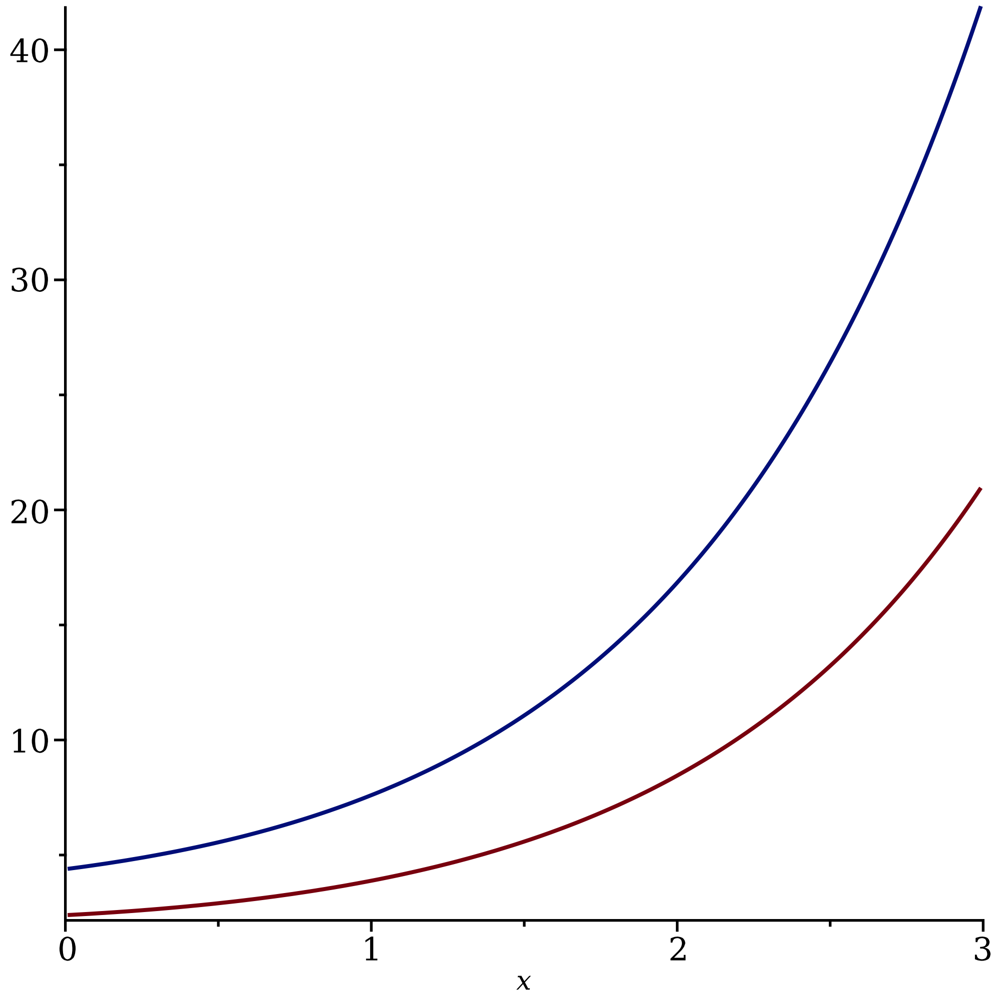
```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = -x + e^x c_1 + \ln(1+e^x)\,(1+e^x) - 1 - e^x \ln(e^x) + c_2 \tag{39}$$

```
> ys:=unapply(rhs(sol),x,c__1, c__2)
```

$$ys := (x, c_1, c_2) \mapsto -x + e^x \cdot c_1 + \ln(1+e^x)\cdot(1+e^x) - 1 - e^x \cdot \ln(e^x) + c_2 \tag{40}$$

```
> plot([ys(x,1,1),ys(x,2,2)],x=0..3)
```




```
> ecdif:=diff(y(x),x)=1+(y(x)^2)
```

$$ecdif := \frac{d}{dx}\, y(x) = 1 + y(x)^2 \quad (41)$$

```
> cond_in:=y(0)=1
```

$$cond\_in := y(0) = 1 \quad (42)$$

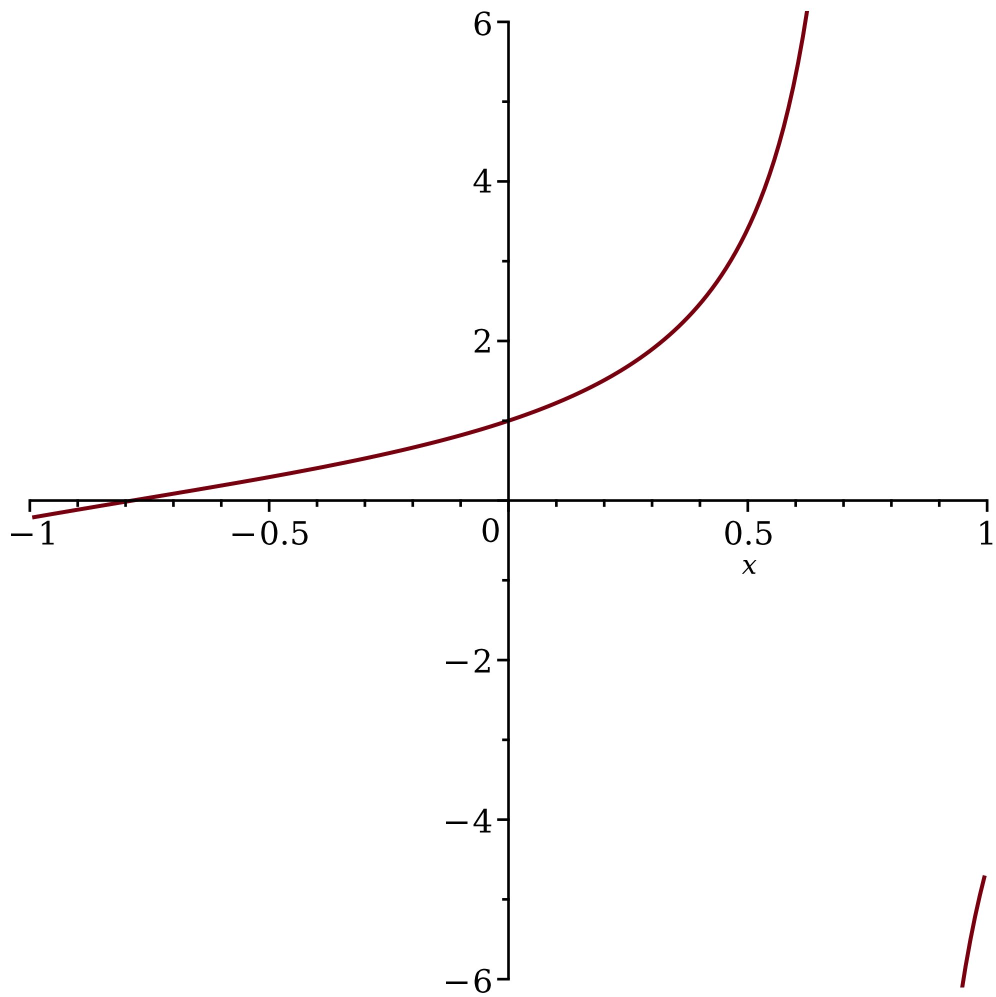
```
> sol:=dsolve({ecdif,cond_in},y(x))
```

$$sol := y(x) = \tan\left(x + \frac{\pi}{4}\right) \quad (43)$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto \tan\left(x + \frac{\pi}{4}\right) \quad (44)$$

```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x)=1/(1-x^2)*y(x)+1+x
```

$$ecdif := \frac{d}{dx}\, y(x) = \frac{y(x)}{-x^2+1} + 1 + x \quad (45)$$

```
> cond_in:=y(0)=0
```

$$cond\_in := y(0) = 0 \quad (46)$$

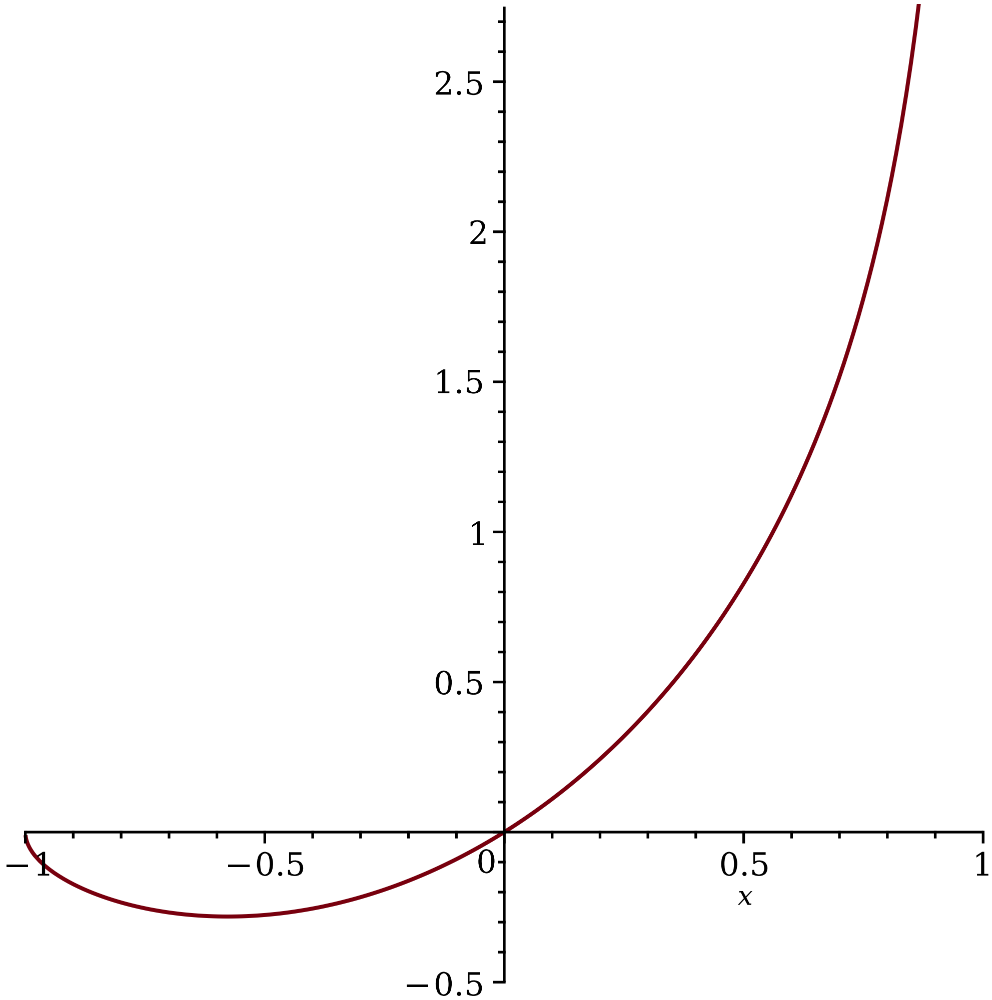
```
> sol:=dsolve({ecdif,cond_in},y(x))
```

$$sol := y(x) = \frac{\left(x\sqrt{-x^2+1} + \arcsin(x)\right)(x+1)}{2\sqrt{-x^2+1}} \quad (47)$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto \frac{\left(x\cdot\sqrt{-x^2+1} + \arcsin(x)\right)\cdot(x+1)}{2\cdot\sqrt{-x^2+1}} \quad (48)$$

```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x)=-x^2+2*y(x)
```

$$ecdif := \frac{d}{dx}\, y(x) = -x^2 + 2\, y(x) \tag{49}$$

```
> cond_in:=y(0)=1/4
```

$$cond\_in := y(0) = \frac{1}{4} \tag{50}$$

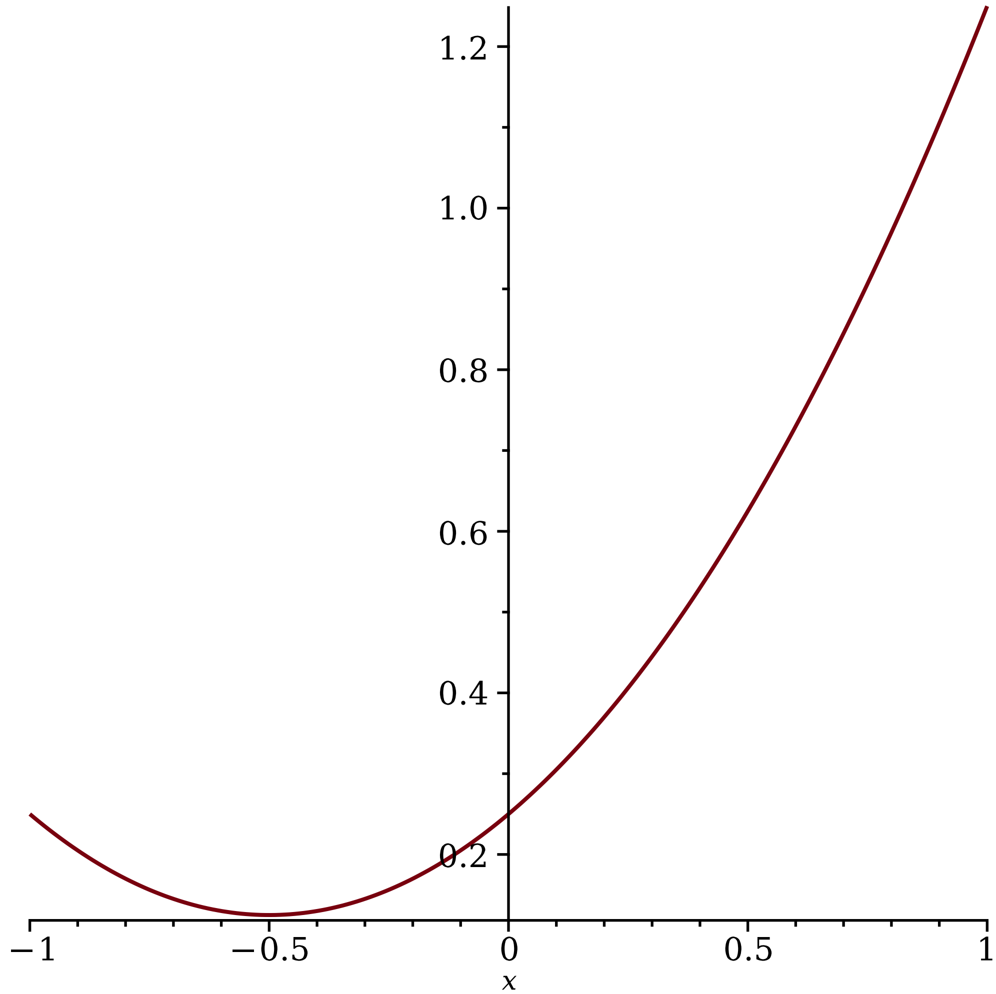
```
> sol:=dsolve({ecdif,cond_in},y(x))
```

$$sol := y(x) = \frac{1}{2}\, x^2 + \frac{1}{2}\, x + \frac{1}{4} \tag{51}$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto \frac{1}{2} \cdot x^2 + \frac{1}{2} \cdot x + \frac{1}{4} \tag{52}$$

```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x,x)-5*diff(y(x),x)+4*y(x)=0
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) - 5\,\frac{d}{dx}\, y(x) + 4\, y(x) = 0 \tag{53}$$

```
> cond_in2:=D(y)(0)=8
```

$$cond\_in2 := D(y)(0) = 8 \tag{54}$$

```
> cond_in1:=y(0)=5
```

$$cond\_in1 := y(0) = 5 \tag{55}$$

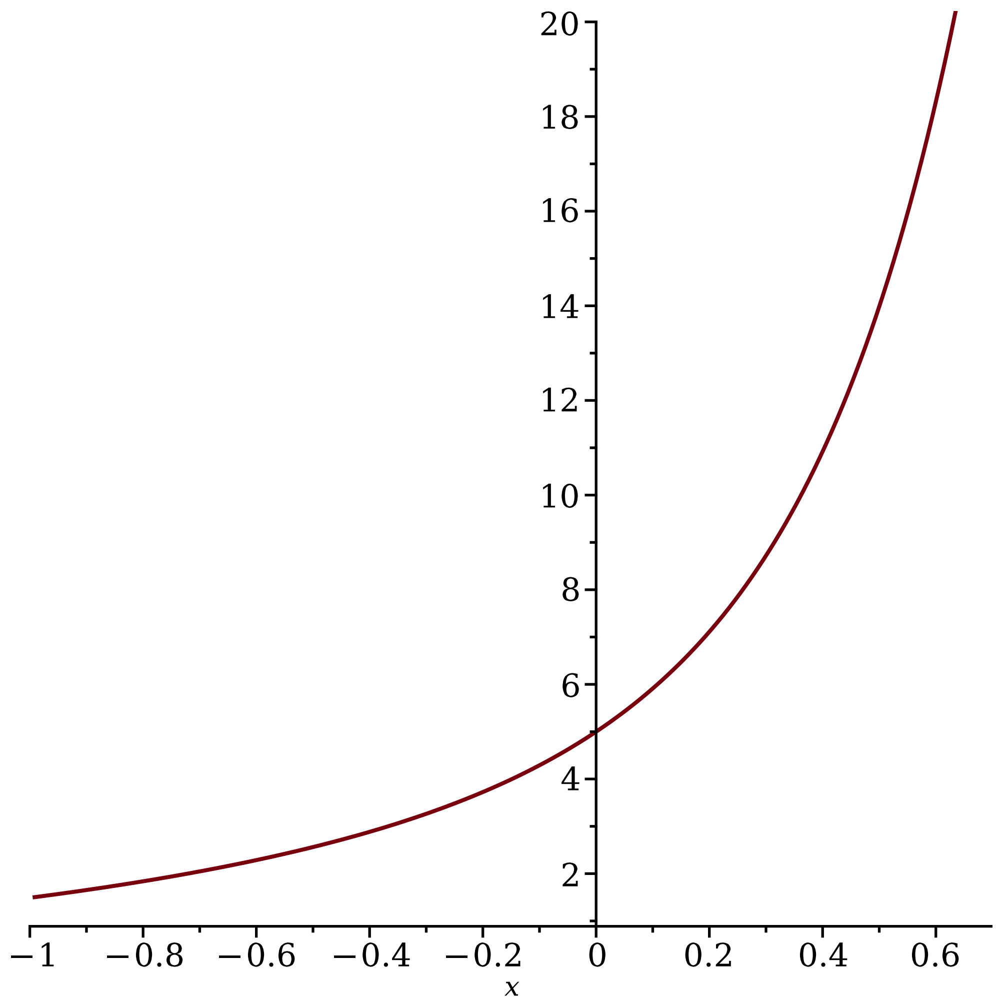
```
> sol:=dsolve({ecdif,cond_in1, cond_in2},y(x))
```

$$sol := y(x) = 4\,e^{x} + e^{4\,x} \tag{56}$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto 4 \cdot e^{x} + e^{4 \cdot x} \tag{57}$$

```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x,x)-4*diff(y(x),x)+5*y(x)=2*(x^2)*exp(x)
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) - 4\,\frac{d}{dx}\, y(x) + 5\, y(x) = 2\, x^2\, e^x \quad (58)$$

```
> cond_in1:=y(0)=2
```

$$cond\_in1 := y(0) = 2 \quad (59)$$

```
> cond_in2:=D(y)(0)=3
```

$$cond\_in2 := D(y)(0) = 3 \quad (60)$$

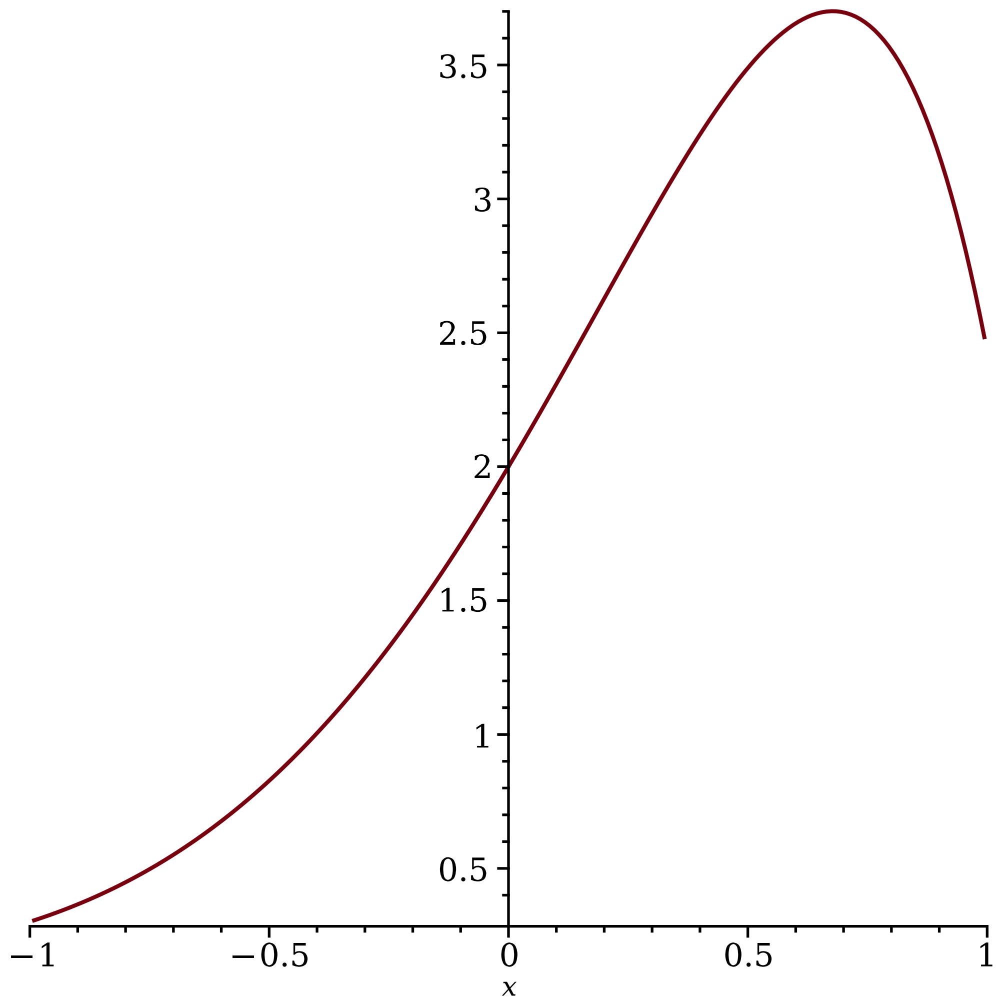
```
> sol:=dsolve({ecdif,cond_in1, cond_in2},y(x))
```

$$sol := y(x) = (-2\sin(x) + \cos(x))\, e^{2x} + (x+1)^2\, e^x \quad (61)$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto (-2\cdot\sin(x) + \cos(x))\cdot e^{2\cdot x} + (x+1)^2\cdot e^x \quad (62)$$

```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x,x)+4*y(x)=4*(sin(2*x)+cos(2*x))
```

$$ecdif := \frac{d^2}{dx^2}\, y(x) + 4\, y(x) = 4 \sin(2\, x) + 4 \cos(2\, x) \tag{63}$$

```
> cond_in1:=y(Pi)=2*Pi
```

$$cond\_in1 := y(\pi) = 2\, \pi \tag{64}$$

```
> cond_in2:=D(y)(Pi)=2*Pi
```

$$cond\_in2 := D(y)(\pi) = 2\, \pi \tag{65}$$

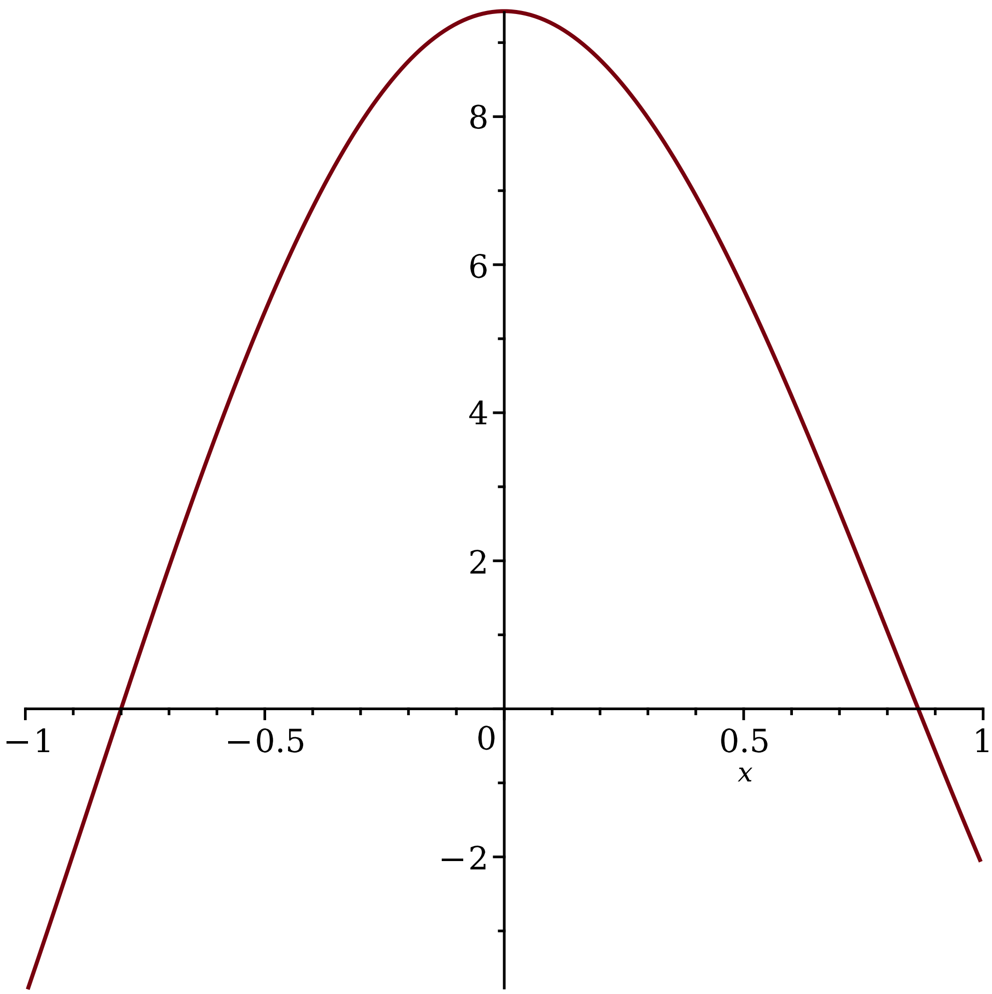
```
> sol:=dsolve({ecdif,cond_in1, cond_in2},y(x))
```

$$sol := y(x) = (-x + 3\, \pi) \cos(2\, x) + \frac{\sin(2\, x)\, (2\, x + 1)}{2} \tag{66}$$

```
> ys:=unapply(rhs(sol),x)
```

$$ys := x \mapsto (-x + 3 \cdot \pi) \cdot \cos(2 \cdot x) + \frac{\sin(2 \cdot x) \cdot (2 \cdot x + 1)}{2} \tag{67}$$

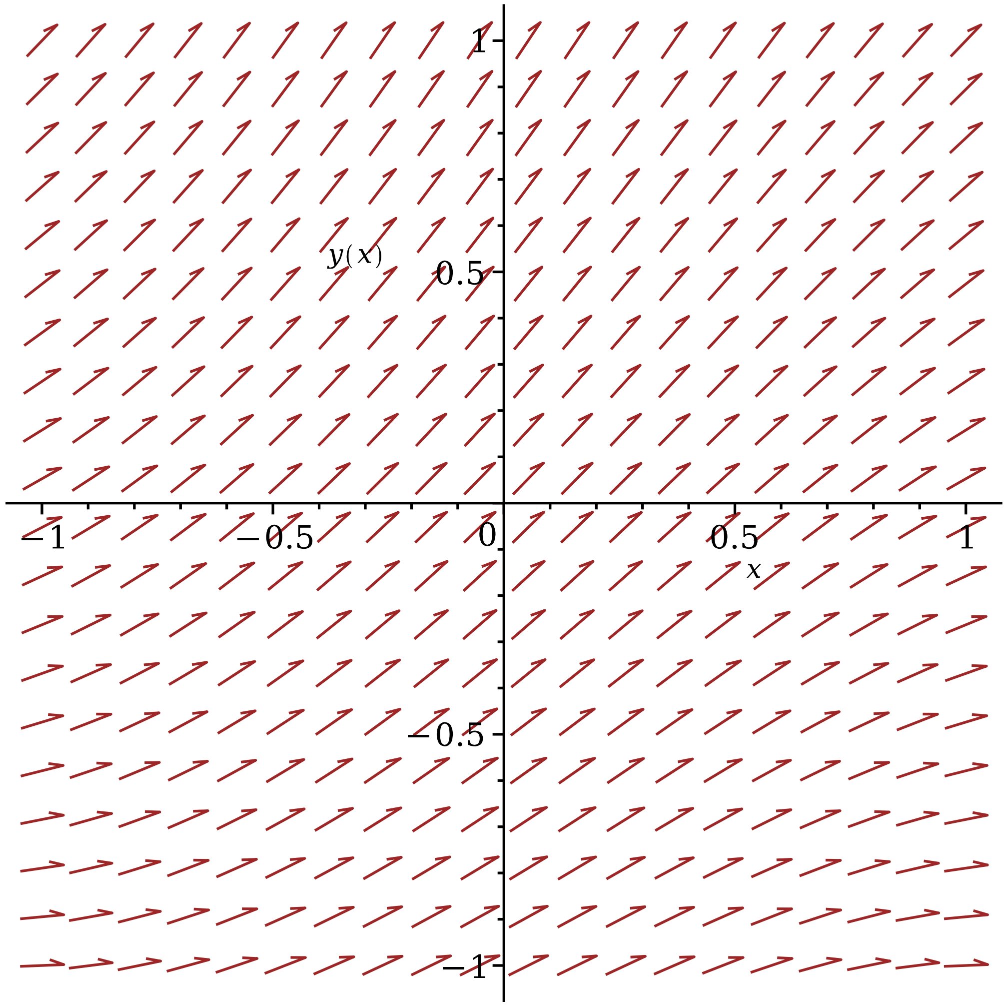
```
> plot(ys(x),x=-1..1)
```




```
> ecdif:=diff(y(x),x)-(1/2)*y(x)=cos(x)
```

$$ecdif := \frac{d}{dx}\, y(x) - \frac{y(x)}{2} = \cos(x) \qquad (68)$$

```
> with(DEtools):
> DEplot(ecdif,y(x),x=-1..1,y=-1..1);
```




```
> cond_in:=y(0)=a
```

$$cond\_in := y(0) = a \tag{69}$$

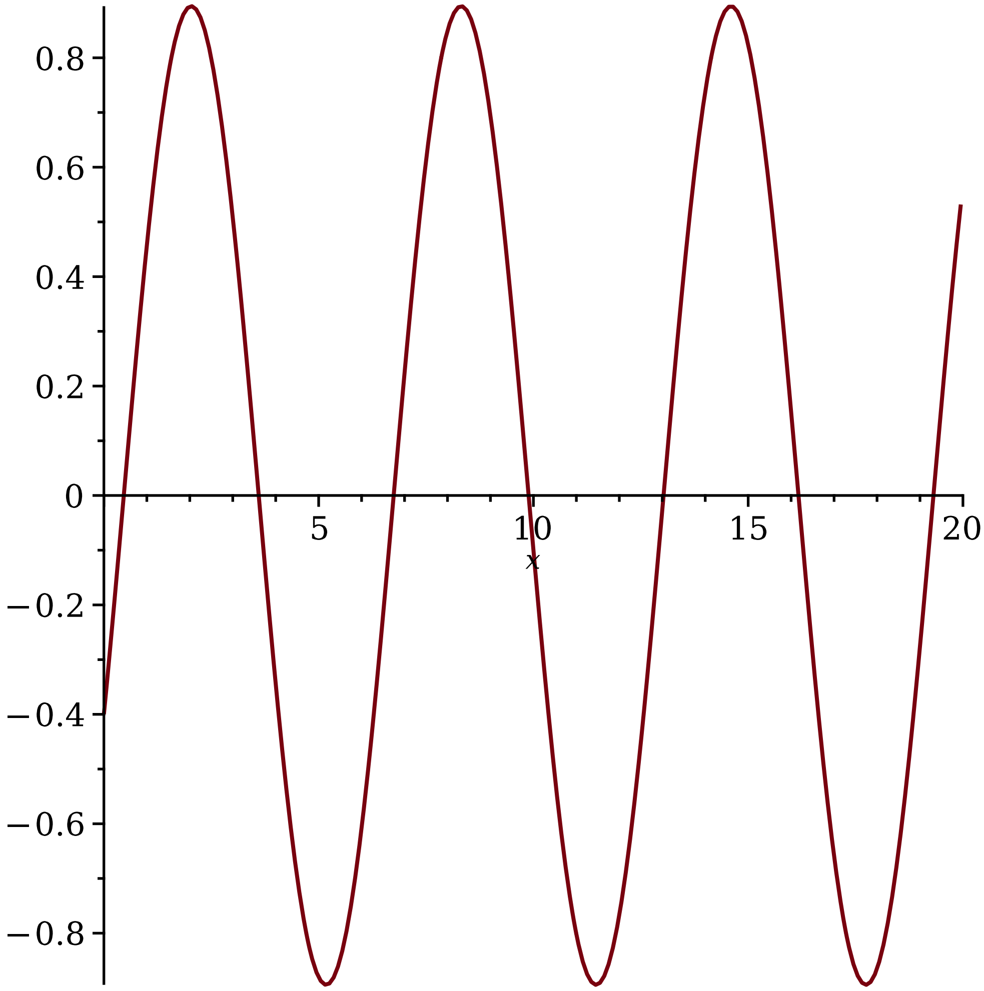
```
> sol:=dsolve({ecdif,cond_in},y(x))
```

$$sol := y(x) = -\frac{2\cos(x)}{5} + \frac{4\sin(x)}{5} + \mathrm{e}^{\frac{x}{2}}\left(a + \frac{2}{5}\right) \tag{70}$$

```
> ys:=unapply(rhs(sol),x, a)
```

$$ys := (x, a) \mapsto -\frac{2\cdot\cos(x)}{5} + \frac{4\cdot\sin(x)}{5} + \mathrm{e}^{\frac{x}{2}}\cdot\left(a + \frac{2}{5}\right) \tag{71}$$

```
> plot([ys(x,-2/5)],x=0..20)
```




```
> restart
> with(DEtools):
> with(plots):
> ecdif:=diff(y(x),x)=a*y(x)+b
```

$$ecdif := \frac{d}{dx}\, y(x) = a\, y(x) + b \tag{72}$$

```
> sol:=dsolve(ecdif,y(x))
```

$$sol := y(x) = -\frac{b}{a} + e^{a\, x}\, c_1 \tag{73}$$

```
> m_const:=-b/a
```

$$m\_const := -\frac{b}{a} \tag{74}$$

```
> sol_cond:=dsolve({ecdif,y(0)=1},y(x))
```

$$sol\_cond := y(x) = \frac{e^{a\, x}\, (a + b) - b}{a} \tag{75}$$

```
> eq1:=eval(rhs(sol_cond),x=2)=2*exp(2)-1
```

$$eq1 := \frac{e^{2\, a}\, (a + b) - b}{a} = 2\, e^{2} - 1 \tag{76}$$

```
> eq2:=eval(rhs(sol_cond),x=3)=2*exp(3)-1
```

$$eq2 := \frac{e^{3a}(a+b)-b}{a} = 2e^3 - 1 \quad (77)$$

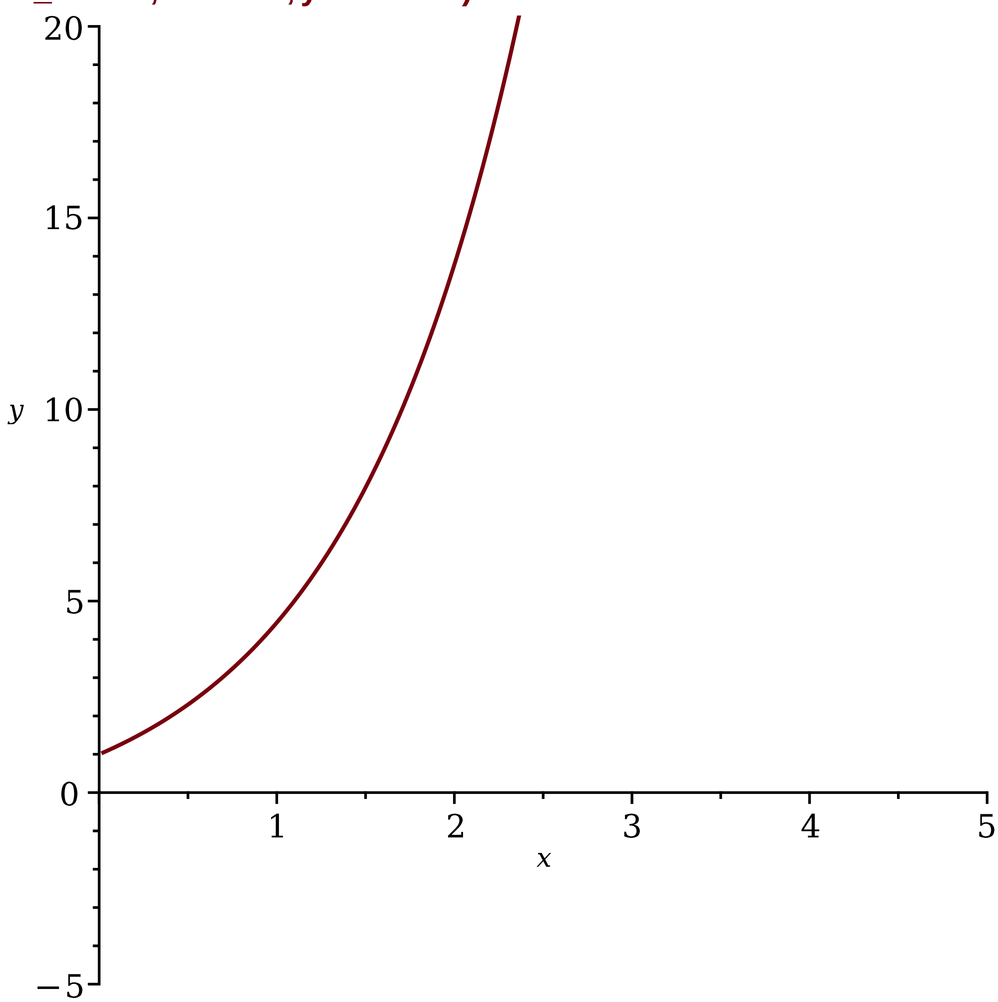
```
> sol_ab:=solve({eq1,eq2},{a,b})
```

$$sol\_ab := \{a=1, b=1\}, \left\{a = \ln\left(-\frac{1}{e+1}\right)+1, b = -\frac{\left(\ln\left(-\frac{1}{e+1}\right)+1\right)(2e^4+4e^3-2e-1)}{2e+1}\right\} \quad (78)$$

```
> sol_final:=eval(rhs(sol_cond),sol_ab[1])
```

$$sol\_final := 2e^x - 1 \quad (79)$$

```
> plot(sol_final,x=0..5,y=-5..20)
```




```
> restart
> with(DEtools):
> with(plots):
> de := diff(y(x), x, x) - diff(y(x), x) - 2*y(x) = 0:
  ics := y(0) = a, D(y)(0) = 2:
```

```
> sol := dsolve({de, ics}, y(x));
```

$$sol := y(x) = \frac{(-2+2\,a)\,e^{-x}}{3} + \frac{e^{2\,x}\,(a+2)}{3} \tag{80}$$

```
> simplify(sol);
```

$$y(x) = \frac{(-2+2\,a)\,e^{-x}}{3} + \frac{e^{2\,x}\,(a+2)}{3} \tag{81}$$

```
> eq := coeff(rhs(sol), exp(2*x)) = 0;
```

$$eq := \frac{a}{3} + \frac{2}{3} = 0 \tag{82}$$

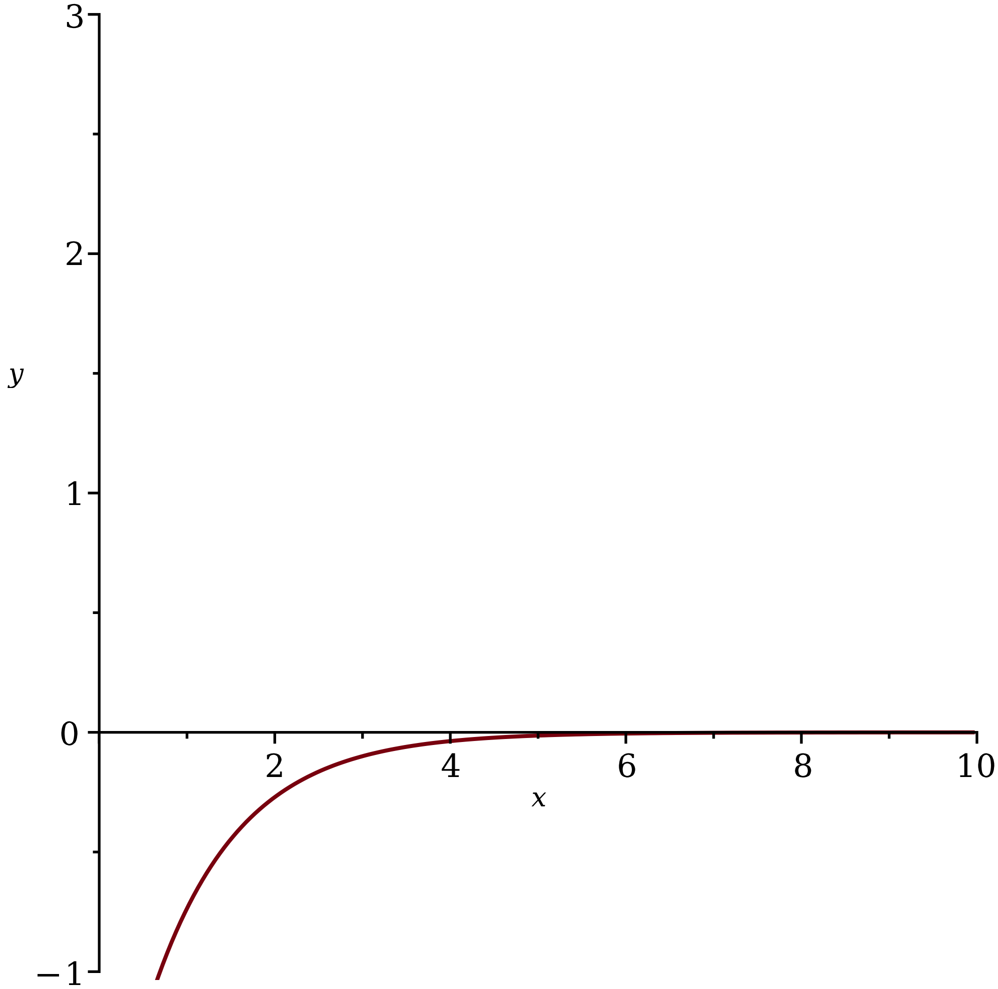
```
> a_value := solve(coeff(rhs(sol), exp(2*x)) = 0, a);

  sol_final := subs(a = a_value, sol);

  plot(rhs(sol_final), x = 0 .. 10, y = -1 .. 3)
```

$$a\_value := -2$$

$$sol\_final := y(x) = -2\,e^{-x}$$



3
2
1
0
−1
y
2
4
6
8
10
x