Paper exercise:

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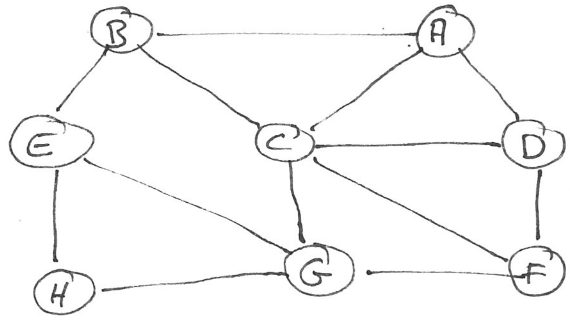

A simple graph.
All edges are bidirectional and equally weighted $(w=1)$.

Find the minimum $Cut(C_1, C_2)$:

$$\min Cut(C_1, C_2) = \min_{\substack{C_1 \dot\cup C_2 = \{A,B,C,D,E,F,G,H\} \\ C_1 \neq \emptyset \wedge C_2 \neq \emptyset}} \sum_{i \in C_1} \sum_{j \in C_2} w_{ij} = 2$$

$\Rightarrow C_1 = \{A,B,C,D,E,F,G\}$, $C_2 = \{H\}$

Find the normalized minimum $NCut(C_3, C_4)$:

$$W = \begin{pmatrix} 0&1&1&1&0&0&0&0 \\ 1&0&1&0&1&0&0&0 \\ 1&1&0&1&0&1&1&0 \\ 1&0&1&0&0&1&0&0 \\ 0&1&0&0&0&0&1&1 \\ 0&0&1&1&0&0&1&0 \\ 0&0&1&0&1&1&0&1 \\ 0&0&0&0&1&0&1&0 \end{pmatrix} ; \quad D = \begin{pmatrix} 3 & & & & & & & \\ & 3 & & & & & 0 & \\ & & 5 & & & & & \\ & & & 3 & & & & \\ & & & & 3 & & & \\ & 0 & & & & 3 & & \\ & & & & & & 4 & \\ & & & & & & & 2 \end{pmatrix}$$

degree of each node

$\Rightarrow$ generalized eigenvalue problem: $(D-W)y = \lambda D y$

$\Rightarrow D^{-1/2}(D-W)D^{-1/2} z = \lambda z$ with $z = D^{1/2} y$

Eigenvector corresponding to the second smallest eigenvalue

$\Rightarrow$ second smallest eigenvalue $\lambda = 0{,}3837$ with eigenvector

$$z = \begin{pmatrix} -0{,}3503 \\ -0{,}0434 \\ -0{,}2729 \\ -0{,}3929 \\ 0{,}4814 \\ -0{,}1647 \\ 0{,}3462 \\ 0{,}5175 \end{pmatrix} \Rightarrow D^{-1/2} z = y = \begin{pmatrix} 0 \\ 0{,}2215 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z = \begin{pmatrix} -0{,}3503 \\ -0{,}0434 \\ -0{,}2729 \\ -0{,}3929 \\ 0{,}4814 \\ -0{,}1647 \\ 0{,}3462 \\ 0{,}5175 \end{pmatrix} \Rightarrow D^{-1/2} z = y = \begin{pmatrix} -0{,}2023 \\ -0{,}0251 \\ -0{,}1221 \\ -0{,}2268 \\ 0{,}2780 \\ -0{,}0951 \\ 0{,}1731 \\ 0{,}3659 \end{pmatrix} \Rightarrow \begin{matrix} C_3 = \{A,B,C,D,F\} \\ C_4 = \{E,G,H\} \end{matrix}$$

$\Rightarrow C_1 = \{A, B, C, D, E, F, G\}, \ C_2 = \{H\}, \ C_3 = \{A, B, C, D, F\}, \ C_4 = \{E, G, H\}$

$$\left.\begin{aligned} Cut(C_1, C_2) &= 2 \\ Cut(C_3, C_4) &= 3 \end{aligned}\right\} \quad \cancel{Cut(A,B) = \sum_{i \in \cdot}} \quad Cut(X, Y) = \sum_{i \in X} \sum_{j \in Y} w_{ij}$$

$$NCut(C_1, C_2) = \frac{2}{(3+3+5+3+3+3+4)} + \frac{2}{2} = \frac{2}{24} + 1 = \frac{13}{12}$$

$$NCut(C_3, C_4) = \frac{3}{(3+3+5+3+3)} + \frac{3}{(3+4+2)} = \frac{3}{17} + \frac{3}{9} = \frac{26}{51}$$

$$NCut(X,Y) = \frac{Cut(X,Y)}{Vol(X)} + \frac{Cut\ldots}{Vol(Y)}$$

$Vol(C_1) = 24$; $Vol(C_2) = 2$; $Vol(C_3) = 17$; $Vol(C_4) = 9$