# Foundations of Audio Signal Processing
# Exercise sheet 11

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## Exercise 11.1

(a)

Need to show: $(h(n))_{n\in\mathbf{Z}} = 2\omega_0 sinc(2\omega_0 n)$

$$
\begin{aligned}
h(n) &= \int_{-\omega_0}^{\omega_0} e^{2\pi i\omega n} d\omega \\
&= [\frac{1}{2\pi i n} e^{2\pi i\omega n}]_{-\omega_0}^{\omega_0} \\
&= \frac{1}{2\pi i n} e^{2\pi i\omega_0 n} - \frac{1}{2\pi i n} e^{-2\pi i\omega_0 n} \\
&= \frac{1}{2\pi i n}(e^{2\pi i\omega_0 n} - e^{-2\pi i\omega_0 n}) \\
&= \frac{1}{2\pi i n}((\cos(2\pi\omega_0 n) + i\sin(2\pi\omega_0 n)) - (\cos(-2\pi\omega_0 n) + i\sin(-2\pi\omega_0 n)) \\
&= \frac{1}{2\pi i n}(i\sin(2\pi\omega_0 n) - i\sin(-2\pi\omega_0 n)) && \text{(Cosinus even)} \\
&= \frac{1}{2\pi i n}(2i\sin(2\pi\omega_0 n)) && \text{(Sinus odd)} \\
&= \frac{\sin(2\pi\omega_0 n)}{\pi n} \\
&= 2\omega_0 sinc(2\omega_0 n) && \text{(Def. of sinc)}
\end{aligned}
$$

(b)

Need to show: $h(n) \neq 0$ for some $n > 0$.

$$\begin{aligned} h(-1) &= 2\omega_0 sinc(-2\omega_0) \\ &= \frac{1}{2} sinc(-\frac{1}{2}) && \text{Setting } \omega_0 = \frac{1}{4} \\ &\approx \frac{1}{2} * 0.9589 \neq 0 \end{aligned}$$

$\implies C_h$ is not causal.

(c)

Need to show: $h(n)$ has infinitely many coefficients different from 0.
Consider that $sinc(x) = 0 \Leftrightarrow x \in \mathbf{Z}$.
So:

$$\begin{aligned} 2\omega_0 sinc(2\omega_0 n) = 0 &\Leftrightarrow sinc(2\omega_0 n) = 0 \\ &\Leftrightarrow 2\omega_0 n \in \mathbf{Z} \\ &\Leftrightarrow \frac{1}{2} n \in \mathbf{Z} && \text{Setting } \omega_0 = \frac{1}{4} \\ &\Leftrightarrow n \text{ is even.} \end{aligned}$$

So for all odd $n \in \mathbf{Z}$, $2\omega_0 sinc(\omega_0 n) \neq 0$.
$\implies C_h$ has infinitely many nonzero filter coefficients.

# Exercise 11.2

(a)

$$\begin{aligned} H(\omega) &= \sum_{n=0}^{\infty} h(n) \cdot e^{-2\pi i n \omega} \\ &= \sum_{n=1}^{1} h(n) \cdot e^{-2\pi i n \omega} && \text{(Because } h(n) = 0 \text{ for } n \notin \{0, 1\}) \\ &= \frac{1}{2} e^{-2\pi i 0 \omega} + \frac{1}{2} e^{-2\pi i 1 \omega} \\ &= \frac{1}{2} + \frac{1}{2} e^{-2\pi i \omega} \end{aligned}$$

$$
\begin{aligned}
G(\omega) &= \sum_{n=0}^{\infty} g(n) \cdot e^{-2\pi i n\omega} \\
&= \sum_{n=1}^{1} g(n) \cdot e^{-2\pi i n\omega} && \text{(Because } g(n) = 0 \text{ for } n \notin \{0,1\}\text{)} \\
&= \frac{1}{2} e^{-2\pi i 0\omega} - \frac{1}{2} e^{-2\pi i 1\omega} \\
&= \frac{1}{2} - \frac{1}{2} e^{-2\pi i\omega}
\end{aligned}
$$

(b)

Need to show: $(\widehat{h+g}) \cdot \hat{x} = \hat{x}$

$$
\begin{aligned}
(\widehat{h+g}) &= \sum_{n=0}^{\infty} (h(n) + g(n)) \cdot e^{-2\pi i n\omega} \\
&= \sum_{n=0}^{1} (h(n) + g(n)) \cdot e^{-2\pi i n\omega} \\
&= \left(\frac{1}{2} + \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{2}\right) e^{-2\pi i 1\omega} = 1
\end{aligned}
$$

$\Rightarrow (\widehat{h+g}) \cdot \hat{x} = 1 \cdot \hat{x} = \hat{x}$ $\square$