Foundations of Audio Signal Processing
Exercise sheet 8

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Exercise 8.1

Need to show: $\langle sinc(\cdot - k) | sinc(\cdot - \ell)\rangle = \delta_{k,\ell}$

$$
\begin{aligned}
&\langle sinc(\cdot - k) | sinc(\cdot - \ell)\rangle \\
&= \langle \widehat{rect}(\cdot - k) | \widehat{rect}(\cdot - \ell)\rangle && \text{(Def. of Box and Sinc function)} \\
&= \int_{-1/2}^{1/2} 1 e^{-2\pi i (k-\ell)t} dt \\
&= \int_{-1/2}^{1/2} rect(t) e^{-2\pi i (k-\ell)t} dt && \text{(because } rect(t) = 1 \text{ for } |t| \leq 1/2) \\
&= \widehat{rect}(k - \ell) \\
&= sinc(k - \ell) && \text{(Def. of Box and Sinc function)} \\
&= \begin{cases} sinc(k-\ell) = sinc(0) = 1 & k = \ell \\ sin(\pi(k-\ell)) = 0 \Rightarrow sinc(k-\ell) = 0 & k \neq \ell \end{cases} \\
&= \delta_{k,\ell} && \square
\end{aligned}
$$

Exercise 8.2

(a)

Following the arguments from the lecture, by performing T-sampling of f, we get the undersampled DT-signal $x(k) = f(kT)$. Applying Fourier inversion yields:

$$\begin{aligned}
f(kT) &= \int_{-\Omega'}^{\Omega'} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega \qquad \text{where } \Omega' \leq 5\Omega \\
&= \int_{-5\Omega}^{5\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega \\
&= \int_{-5\Omega}^{-\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega + \int_{-\Omega}^{\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega + \int_{\Omega}^{5\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega \\
&= \int_{-5\Omega}^{-3\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega + \int_{-3\Omega}^{-\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega + \int_{-\Omega}^{\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega \\
&\quad + \int_{\Omega}^{3\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega + \int_{3\Omega}^{5\Omega} \hat{f}(\omega) e^{-2\pi i \omega k T} d\omega
\end{aligned}$$

This results in:

$$\int_{-\Omega}^{\Omega} \underbrace{\left(\hat{f}(\omega) + \hat{f}(\omega + 2\Omega) + \hat{f}(\omega - 2\Omega) + \hat{f}(\omega + 4\Omega) + \hat{f}(\omega - 4\Omega)\right)}_{(*)} e^{-2\pi i \omega k T} d\omega$$

$$\hat{g}(\omega) = \begin{cases} (*) & \text{if } |\omega| \leq \Omega \\ 0 & \text{otherwise.} \end{cases}$$

So, all frequencies ω within $5\Omega \geq |\omega| > \Omega$ are mapped into the range $(-\Omega, \Omega)$, by adding $\pm 2\Omega$ to those frequencies with $5\Omega \geq |\omega| > 3\Omega$, and $\pm 4\Omega$ to those with $3\Omega \geq |\omega| > \Omega$.

(b)

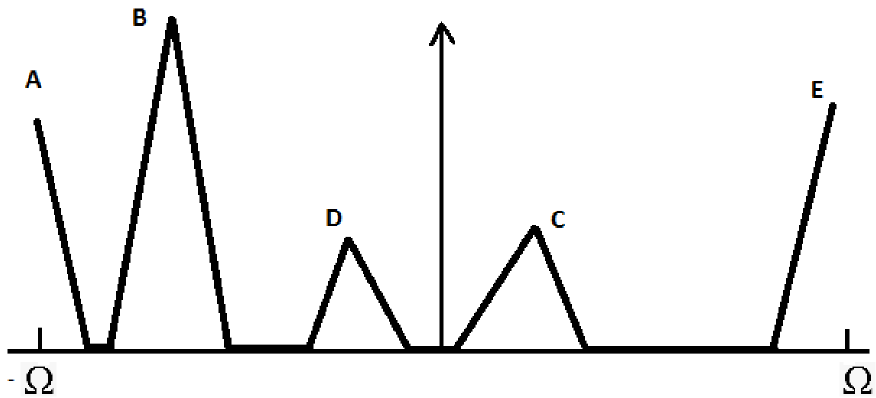

Figure 1: The folded signal $|\hat{g}|$, after T-sampling of f with $T = 2\Omega$.

Exercise 8.3

1. Read in the *.mat file and extract f_s and signal out of the struct
   The signal is 20 seconds long and has 160000 points
   $F_s = 8000$

2. Sample over the signal (see figure 2)
   We take one sample each second, so we have 21 samples

3. Calculate for each sample the sinc function (see figure 3)
   $t = [0 : 20]$,
   Time_sample := second where the sample was taken (shift the sinc function),
   amplitude_sample := amplitude of signal at Time_sample (weighted the sinc function)
   y = amplitude_sample * sinc(t-Time_sample)

4. Sum up all sinc functions to reconstruct the original signal (see figure 7)

5. Calculate the error of the reconstruction (see figure 8)
   error = 0.5 * abs(signal - reconstruction)^2

If we double the sample size, the reconstruction of the signal is significantly better.

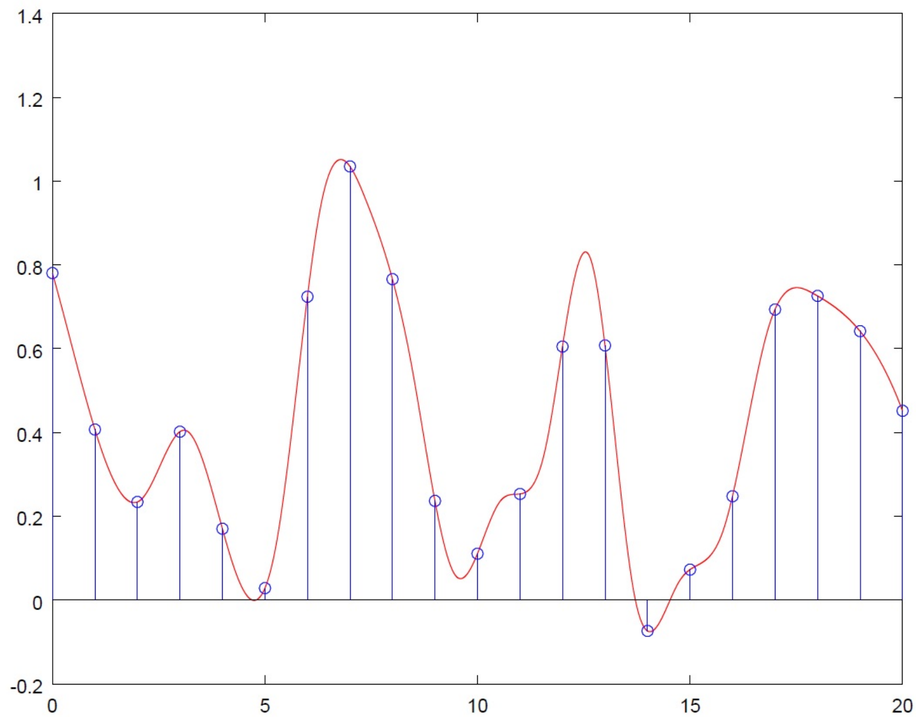

Figure 2: Signal (red) and samples (blue) for a sampling rate of 1/s

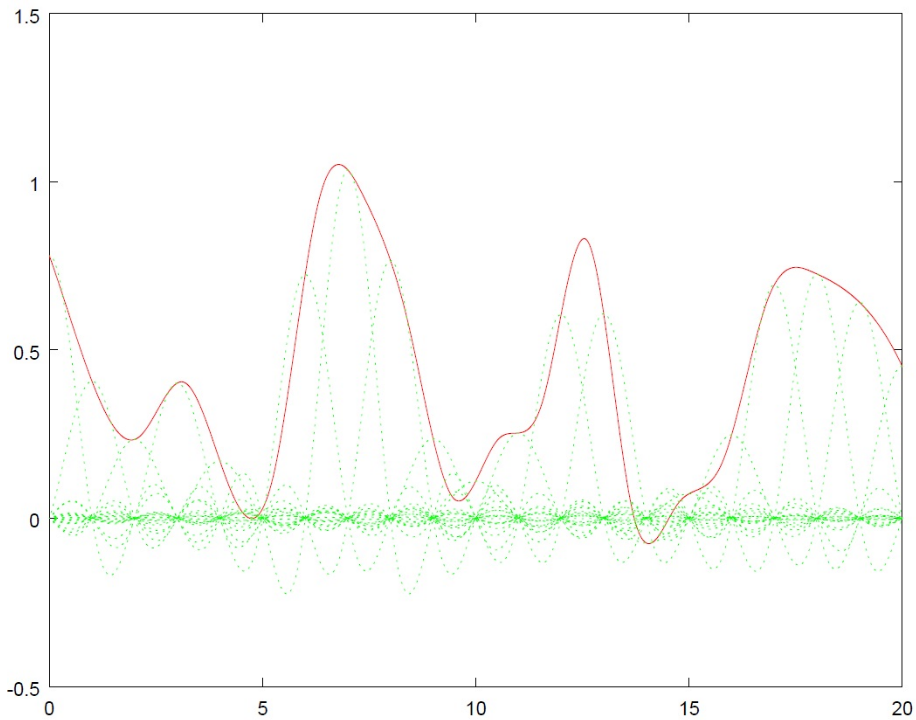

Figure 3: Signal (red) and sinc functions of samples (green)

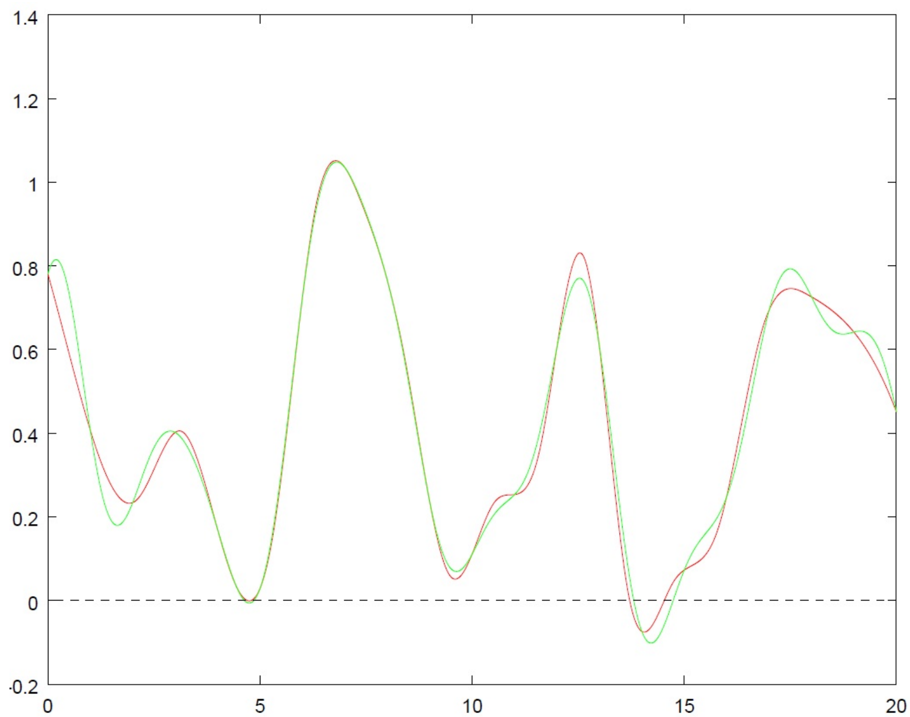

Figure 4: Signal (red) and reconstructed signal (green) for a sampling rate of 1/s

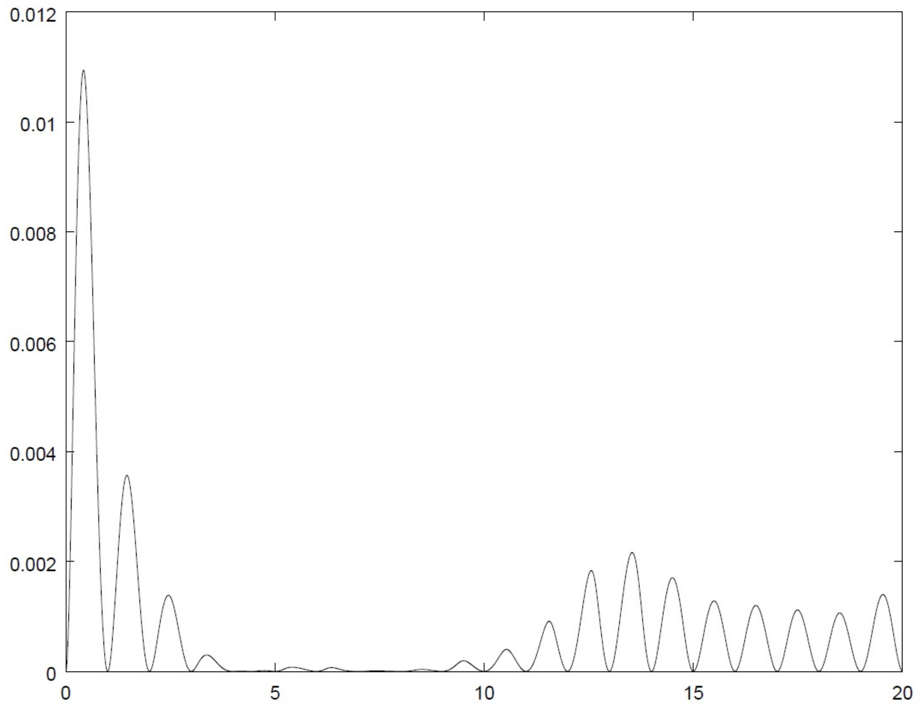

Figure 5: Reconstruction error for a sampling rate of 1/s

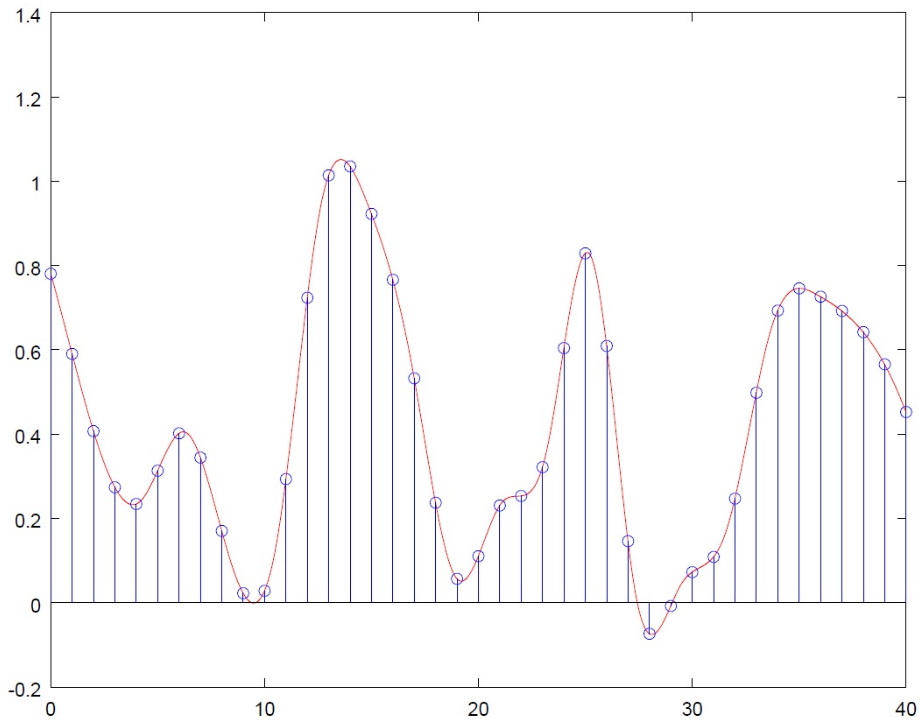

Figure 6: Signal (red) and samples (blue) for a sampling rate of 2/s

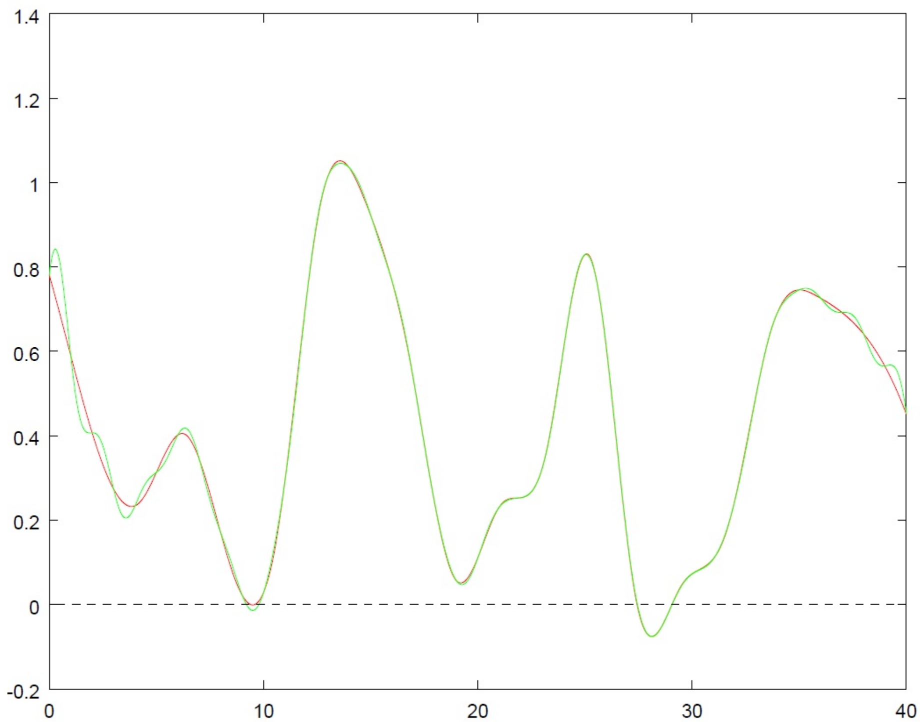

Figure 7: Signal (red) and reconstructed signal (green) for a sampling rate of 2/s

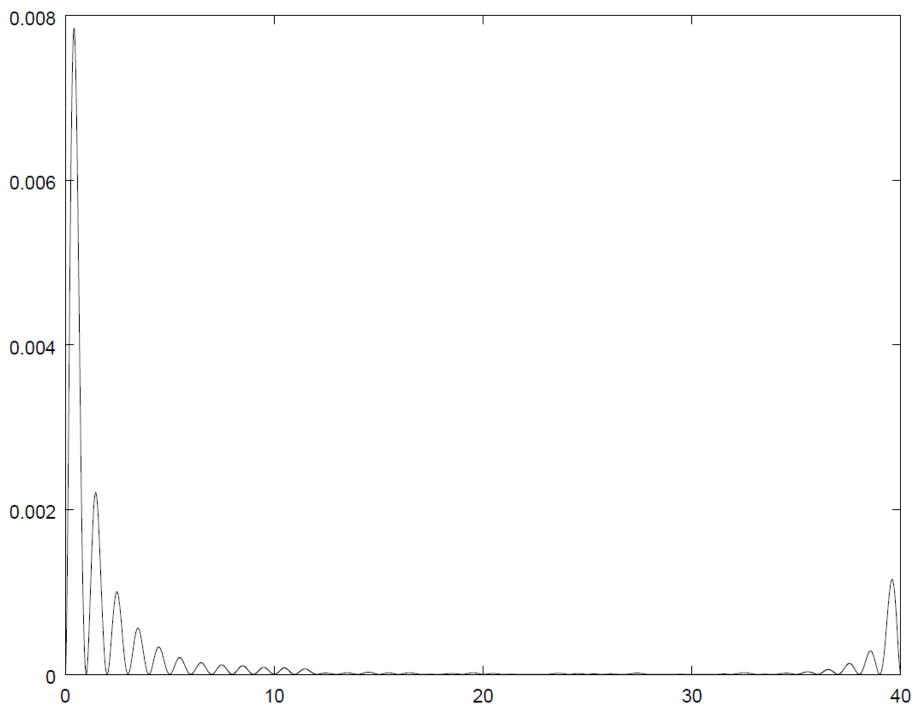

Figure 8: Reconstruction error for a sampling rate of 2/s