Foundations of Audio Signal Processing
Exercise sheet 9

Group members
Christopher Schmidt
Gerhard Mund
Robert Logiewa
Maren Pielka

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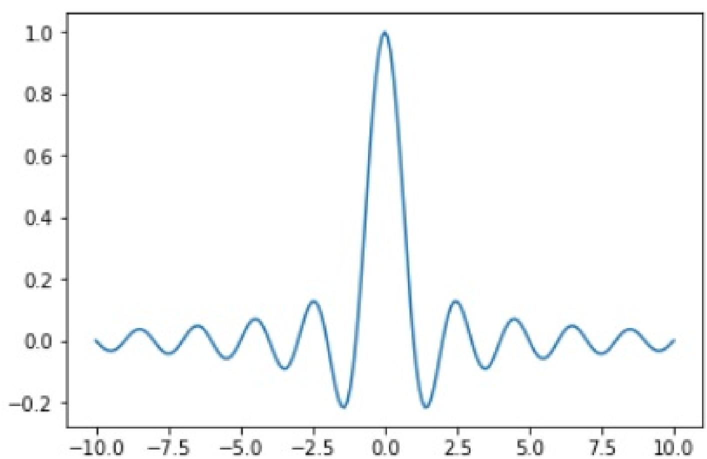
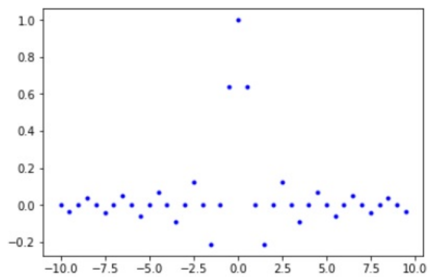
Exercise 9.1

(a)

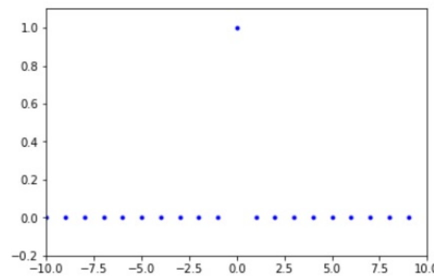
(i) The original function $f(t) = sinc(t)$:



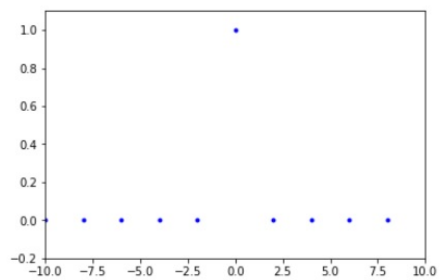
(ii) The T-sampled versions of $f(t)$ for $T = \frac{1}{2}$ (sampling rate $\frac{1}{T} = 2$), $T = 1$ (sampling rate $\frac{1}{T} = 1$) and $T = 2$ (sampling rate $\frac{1}{T} = \frac{1}{2}$):



T = 0.5



T = 1



T = 2

(b)

The Fourier Transform of the sinc function is given by

$$\hat{f}(\omega) = \begin{cases} 1 & \text{if } |\omega| < \pi \\ 0 & \text{otherwise} \end{cases}$$

Thus, the function is bandlimited with $\Omega = \pi$.

(c)

The Sampling Theorem states, that an Ω-bandlimited function f can be perfectly reconstructed using the T-sampled version of f, where $T = \frac{1}{2\Omega}$.
So we would need $T \leq \frac{1}{2\Omega} = \frac{1}{2\pi}$, to get a perfect reconstruction.
Since $\frac{1}{2\pi} < \frac{1}{2}(< 1 < 2)$, we cannot reconstruct the function without loss, using any of the three values above for T.

Exercise 9.2

(a)

For time-invariance we need to show $(\uparrow M)[x^k] = ((\uparrow M)[x])^k$

$$\begin{aligned}
(\uparrow M)[x^k](n) &= \begin{cases} x^k\left(\frac{n}{M}\right) & \text{if } M|n \\ 0 & \text{else} \end{cases} \\
&= \begin{cases} x\left(\frac{n}{M} - k\right) & \text{if } M|n \\ 0 & \text{else} \end{cases}
\end{aligned}$$

$$\begin{aligned}
((\uparrow M)[x](n))^k &= (\uparrow M)[x](n-k)) \\
&= \begin{cases} x\left(\frac{n}{M} - k\right) & \text{if } M|(n-k) \\ 0 & \text{else} \end{cases}
\end{aligned}$$

But in general $M|n \neq M|(n-k) \Rightarrow$ time variant.

(b)

For time-invariance we need to show $E_\omega[x^k] = (E_\omega[x])^k$

$$\begin{aligned}
E_\omega[x^k](n) &= e^{-2\pi i\omega n} x^k(n) \\
&= e^{-2\pi i\omega n} x(n-k) \\
&\neq e^{-2\pi i\omega(n-k)} x(n-k) \\
&= E_\omega[x](n-k) \\
&= (E_\omega[x](n))^k
\end{aligned}$$

$\Rightarrow$ time variant.