Foundations of Audio Signal Processing
Exercise sheet 7

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Exercise 7.1

For the s-scaled version of the Fourier Transform it holds:

$$\widehat{f(\frac{\cdot}{s})}(\omega) = |s| * \hat{f}(\omega s) \tag{1}$$

So:

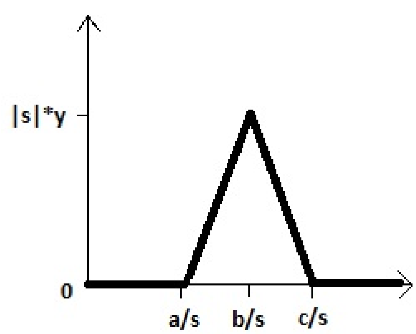

Figure 1: Visualization of the scaled Fourier Transform

Exercise 7.2

(a)

1. Need to show: $\widehat{x+y}(\omega) = \hat{x}(\omega) + \hat{y}(\omega)$.

$$
\begin{aligned}
\hat{x}(\omega) + \hat{y}(\omega) &= \sum_{t \in \mathbb{Z}} x(t) e^{-2\pi i \omega t} + \sum_{t \in \mathbb{Z}} y(t) e^{-2\pi i \omega t} \\
&= \sum_{t \in \mathbb{Z}} x(t) e^{-2\pi i \omega t} + y(t) e^{-2\pi i \omega t} \\
&= \sum_{t \in \mathbb{Z}} (x(t) + y(t)) e^{-2\pi i \omega t} \\
&= \widehat{x+y}(\omega).
\end{aligned}
$$

2. Need to show: $\lambda \hat{x}(\omega) = \widehat{\lambda x}(\omega)$.

$$
\begin{aligned}
\lambda \hat{x}(\omega) &= \lambda \sum_{t \in \mathbb{Z}} x(t) e^{-2\pi i \omega t} \\
&= \sum_{t \in \mathbb{Z}} \lambda x(t) e^{-2\pi i \omega t} \\
&= \widehat{\lambda x}(\omega).
\end{aligned}
$$

(b)

Need to show: $\widehat{x_k}(\omega) = e^{-2\pi i \omega k} \hat{x}(\omega)$, where $x_k(n) = x(n-k)$.

$$
\begin{aligned}
\widehat{x_k}(\omega) &= \sum_{n \in \mathbb{Z}} x_k(n) e^{-2\pi i \omega n} \\
&= \sum_{n \in \mathbb{Z}} x(n-k) e^{-2\pi i \omega n} \\
&= \sum_{t \in \mathbb{Z}} x(u) e^{-2\pi i \omega (u+k)} && \text{(Substitute } (n-k) \text{ by } u\text{)} \\
&= \sum_{t \in \mathbb{Z}} x(u) e^{-2\pi i \omega u} e^{-2\pi i \omega k} \\
&= e^{-2\pi i \omega k} \sum_{u \in \mathbb{Z}} x(u) e^{-2\pi i \omega u} \\
&= e^{-2\pi i \omega k} \hat{x}(\omega).
\end{aligned}
$$

(c)

Need to show: $\widehat{x^{\omega_0}}(\omega) = \hat{x}(\omega+\omega_0)$, where $x^{\omega_0}(n) := e^{-2\pi\omega_0 n}x(n)$

$$\begin{aligned}
\widehat{x^{\omega_0}}(\omega) &= \sum_{k\in\mathbb{Z}} \left(e^{-2i\pi\omega_0 k}x(k)\right) e^{-2i\pi\omega k} \\
&= \sum_{k\in\mathbb{Z}} e^{(-2i\pi\omega_0 k)+(-2i\pi\omega k)}x(k) \\
&= \sum_{k\in\mathbb{Z}} e^{-2i\pi k(\omega+\omega_0)}x(k) \\
&= \hat{x}(\omega+\omega_0) \qquad \square
\end{aligned}$$

(d)

Need to show $y = \overline{x} \Rightarrow \hat{y}(\omega) = \overline{\hat{x}(-\omega)}$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{k\in\mathbb{Z}} e^{-2i\pi\omega k}y(k) \\
&= \sum_{k\in\mathbb{Z}} e^{-2i\pi\omega k}\overline{x(k)} && \text{(because } y=\overline{x}\text{)} \\
&= \overline{\sum_{k\in\mathbb{Z}} e^{2i\pi\omega k}x(k)} \\
&= \overline{\sum_{k\in\mathbb{Z}} e^{-2i\pi(-\omega)k}x(k)} \\
&= \overline{\hat{x}(-\omega)} \qquad \square
\end{aligned}$$

(e)

Need to show: If for all $n\in\mathbb{Z}$, $y(n) = x(-n)$, then $\hat{y}(\omega) = \hat{x}(-\omega)$

$$\begin{aligned}
\hat{y}(\omega) &= \sum_{k\in\mathbb{Z}} e^{-2i\pi\omega k}y(k) \\
&= \sum_{k\in\mathbb{Z}} e^{-2i\pi\omega k}x(-k) \\
&= \sum_{u\in\mathbb{Z}} e^{-2i\pi\omega(-u)}x(u) && \text{(substituting } k \text{ with } -u\text{)} \\
&= \sum_{u\in\mathbb{Z}} e^{-2i\pi(-\omega)u}x(u) \\
&= \hat{x}(-\omega) \qquad \square
\end{aligned}$$

Exercise 7.3

(a) We know, that the wave-file contains 6 digits and the tone of each digit has the same length. So we can split the file in 6 parts of equal length, such that each part contains the tone of one dialed digit. Each tone is composed of the sum of two sine functions with different frequencies. We apply the FFT on each part of the file, to get these frequencies. Finally we look-up the frequencies in the given table, to find the dialed number.

(b) The dialed number is 251216.