Foundations of Audio Signal Processing
Exercise sheet 10

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Exercise 10.1

(a)

$$\begin{aligned}(x * y)(0) &= x(0)y(0)\\ &= 1 \cdot 4 = 4\end{aligned}$$

$$\begin{aligned}(x * y)(1) &= x(0)y(1) + x(1)y(0)\\ &= 1 \cdot 5 + 2 \cdot 4 = 13\end{aligned}$$

$$\begin{aligned}(x * y)(2) &= x(1)y(1) + x(2)y(0)\\ &= 2 \cdot 5 + 3 \cdot 4 = 22\end{aligned}$$

$$\begin{aligned}(x * y)(3) &= x(2)y(1)\\ &= 3 \cdot 5 = 15\end{aligned}$$

So $(x*y)$ is defined by $((x*y)(0), (x*y)(1), (x*y)(2), (x*y)(3)) = (4, 13, 22, 15)$.

(c)

For the given values, the convolution yields (1,2,3,4).
We plotted the result in each convolution step (iterating over x). The last plot is the final result of the convolution.

Exercise 10.2

(a)

To show: $(x * y)(n) = (y * x)(n)$

$$\begin{aligned}(x * y)(n) &= \sum_{k=-\infty}^{\infty} x(k)y(n-k) \\ &= \sum_{k=-\infty}^{\infty} x(n-n+k)y(n-k) && \text{Substitute } k \text{ with } l = n-k \\ &= \sum_{l=-\infty}^{\infty} x(n-l)y(l) \\ &= (y * x)(n) && \square\end{aligned}$$

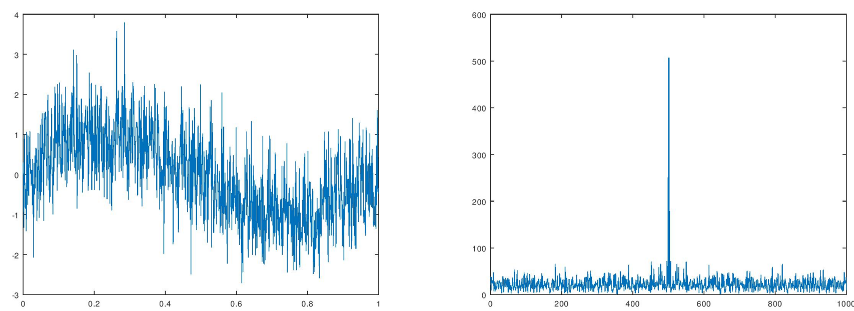
(b)

Need to show: $(\widehat{x * y})(\omega) = \hat{x}(\omega) \cdot \hat{y}(\omega)$

$$\begin{aligned}(\widehat{x * y})(\omega) &= \sum_{t\in\mathbb{Z}} (x * y)e^{-2\pi i\omega t} \\ &= \sum_{t\in\mathbb{Z}} e^{-2\pi i\omega t} \cdot \left(\sum_{k\in\mathbb{Z}} x(k) \cdot y(t-k)\right) \\ &= \sum_{t\in\mathbb{Z}} \sum_{k\in\mathbb{Z}} e^{-2\pi i\omega t} x(k)y(t-k) \\ &= \sum_{k\in\mathbb{Z}} x(k) \sum_{t\in\mathbb{Z}} e^{-2\pi i\omega t} y(t-k) \\ &= \sum_{k\in\mathbb{Z}} x(k) \sum_{t\in\mathbb{Z}} e^{-2\pi i\omega l+k} y(l) && \text{Substitute } t \text{ with } l = t-k \\ &= \left(\sum_{k\in\mathbb{Z}} x(k)e^{-2\pi i\omega k}\right)\left(\sum_{t\in\mathbb{Z}} e^{-2\pi i\omega l} y(l)\right) \\ &= \hat{x}(\omega) \cdot \hat{y}(\omega) && \square\end{aligned}$$

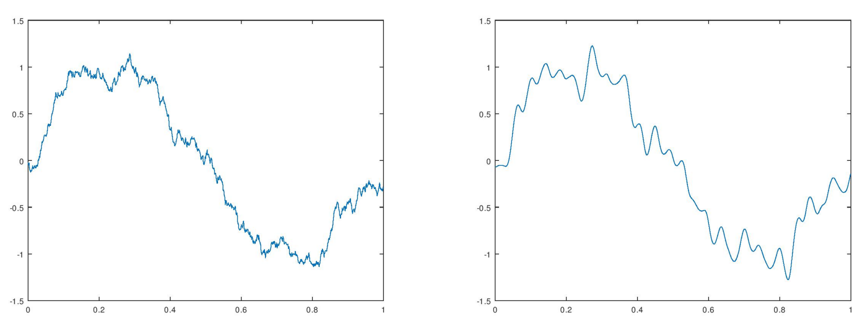
Exercise 10.3

The original, noisy sine-function is depicted in fig. 1a.
The averaging filter smoothes this noisy signal significantly (see fig. 2a) and the Hann filter (fig. 2b) smoothes it even further. In comparison the result of the averaging filter is a bit more ragged.
This can also be observed in the frequency space (fig. 3). Beside the main frequency, there are a lot of smaller peaks of different frequencies in the original signal (fig. 1b). Those are substantially smaller after applying the averaging filter (fig. 3a). The Hann filter even removes these completely, except for some neighbouring frequencies of the main peak.



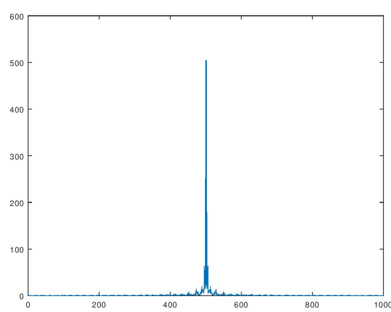
(a) Signal with noise (b) Frequency of the noisy signal

Figure 1: Original signal

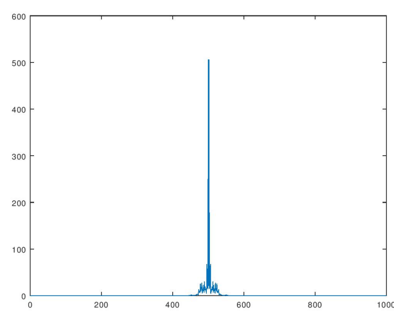


(a) Signal with averaging filter (b) Signal with Hann filter

Figure 2: Smoothed signals.



(a) Frequencies with averaging filter



(b) Frequencies with Hann filter

Figure 3: Frequencies of smoothed signals.